Authority and Communication in Organizations*

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Abstract

This paper studies delegation as an alternative to communication. We show how a principal may give up control rights in order to foster the efficient use of information present in an organization. We point out how the benefits of delegation vary with the congruence between agent and principal, the risk-aversion of the principal and the uncertainty of the environment, and we identify an important role for intermediaries to limit the loss of control under delegation and yet preserve sufficient communication.

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1 Introduction

This paper is concerned with the old saying that ‘knowledge is power’. In organizations, much of the information used in decision making is dispersed in the hierarchy. Lower-level managers, for example, are often much better informed about consumer needs, competitive pressures, specialized technologies or market opportunities than their superiors. The financial press is full of stories about how companies have pushed decision rights lower in the hierarchy in order to profit from this local knowledge.\footnote{Among firms decentralizing decision rights in the 1990’s are AT&T, General Electric, Eastman Kodak, Fiat, Ford, Motorola, United Technologies, Xerox,...} For the same reason, newly acquired subsidiaries are often left with substantial autonomy. The goal of this paper is to better understand why an uninformed principal (the company owners, senior management,...) may grant formal decision rights to an agent (senior or middle management,...) who is better informed but has different objectives. We argue that a principal often delegates authority in order to avoid the noisy communication, and hence the loss of information, which stems from these differences in objectives.

At first sight, it may seem a puzzle why keeping authority and letting the agent report would not always weakly dominate delegating authority. At worst, the principal can always rubber-stamp the proposals of the agent, and by keeping control the principal may refrain him from implementing projects which are obviously not in the interest of the organization. By delegating authority, on the other hand, the principal commits to never reverse the agent’s decisions. A very convincing rationale for delegation is provided by Aghion and Tirole (1997), hereafter referred to as AT. They show that a principal may delegate formal authority to an agent in order to give the latter better incentives to acquire information. While the focus of AT is on the impact of decision rights on the information structure, we, in contrast, take the information structure as given - the agent is assumed to be better informed than the principal - and look at how the allocation of authority affects the use and communication of this information. Key to our analysis is that differences in objectives between principal and agent are not just 'random', as in AT, but systematic and predictable. It is for example well documented that managers may be short-term biased, status-quo biased, risk-averse, empire builders,... But whenever the principal and the agent systematically disagree on a certain action dimension, the principal will not rubber-stamp a naive recommendation by the agent of his preferred action, but correct for the 'bias' in objectives. As the agent is not naive but anticipates this, communication
is inherently strategic and - in equilibrium - noisy. Hence, the central trade-off in our paper is one between a loss of information, which occurs when the principal keeps authority and which is due to the noisy communication game, and a loss of control which is the consequence of delegating decisions rights to an agent with different objectives.

Model & Results. — In order to analyze this trade-off, we develop a stylized model in which the principal (she) must screen among a range of projects which differ from each other on one dimension. The agent (he) has superior information on which project is best for the principal, but his objectives differ in a systematic way. He could, for example, always prefer a larger project than the principal (size-bias). For simplicity, this bias is constant and positive. Section 2 provides a discussion of the kind of biases we have in mind. Given these differences in objectives, one may expect the agent to find it difficult to convince the principal of which project optimally should be undertaken. Most likely, what is done best depends on a range of factors which the principal may find hard to verify or to understand. We assume therefore that the agent can only make loose and unverifiable recommendations, that is information is soft. Further, following Grossman and Hart (1986) and Hart and Moore (1990), we posit that projects (actions) cannot be contracted upon and that to engage in a project, the use of some resources, which are controlled by the principal, is necessary. This implies that the agent normally needs the fiat of the principal to implement a project. The principal, however, can also delegate decision rights to the agent by granting him the authority over the use of the critical resources.

In our organization, the principal thus faces the choice between fully delegating a task to a better informed agent or to order the latter what to do after having consulted him. If she keeps decision rights and consults the agent, a game of strategic communication takes place in which each equilibrium is characterized by a partition of all possible states of nature and where the agent introduces noise into his signal by only specifying to which partition element the realized state of nature belongs. Such a strategic information transmission has been first analyzed by Crawford and Sobel (1982), hereafter referred to as CS. While communication always involves a loss of information as long a preferences are not perfectly congruent, a central result of their paper is that the closer the preferences of agent and principal, the better is communication. As shown by Spector (1999), the loss of information even goes asymptotically to zero when differences in objectives disappear. However, also the delegation of authority becomes less costly when the agent’s preferences are closer to those of the principal. At first
sight, it is thus not clear whether the principal should delegate authority or not when the agent’s bias is small, and how, if at all, the optimal allocation of decision rights varies with this bias.

To resolve this trade-off, we first focus on the case where the prior of the principal is uniformly distributed. We find that when the equilibrium partition becomes finer, which, as we show, is when the number of partition elements increases, then this partition becomes also more coarse relative to the bias. Reason is that in order for the agent to reveal to which partition element the state of nature belongs, partition elements must be increasing in size (must be increasingly noisy), where the increase between two subsequent elements is proportional to the bias of the agent. As a consequence, if communication is good, which implies a large number of partition elements, the size of the highest partition elements is very large relative to the bias. Hence, the smaller the bias and thus the more informative is communication, the better it is to delegate authority to the agent and avoid communication. Even more, it turns out that whenever the agent is able to send an informative signal, the principal optimally delegates authority: no communication ever occurs in equilibrium.

A first prediction of our model is thus that delegation will occur when preferences of principal and agent are sufficiently congruent. Since the principal keeps control if and only if an uninformed decision yields a higher expected utility than a biased decision, our analysis predicts further that delegation will be observed for larger biases when the principal faces more uncertainty. Finally, since delegation yields a constant bias whereas the deviation from the optimal action has a large variance if the principal keeps control, delegation is also more likely when the principal is more risk-averse.

Although it is in general very hard to characterize the equilibrium partition for more general distributions than the uniform one, we could obtain some results which support these predictions: for any continuous distribution, the principal optimally delegates authority whenever the differences in objectives are sufficiently small and/or the utility function is sufficiently concave.

While this provides a strong rationale for full delegation of decision rights as opposed to no delegation at all, the principal can, for a large range of preferences, even do better by delegating decision rights to a middleman with intermediate objectives between hers and her agents’. To see this, suppose first that the principal could restrict the discretion of the agent to a subset of actions. Obviously, the principal may reduce the average deviation from the optimal project by putting an upper bound on the projects which the agent is
allowed to undertake. Further limiting the discretion of the agent, however, only harms the principal if her prior is uniformly distributed. While she may reduce the bias in some states of nature, this will always lead to an increase in the bias for other states of nature, generating a mean-preserving spread of the deviation which is better avoided given her concave preferences.

In practice, however, it may be difficult to commit to a policy of only reversing the largest projects, since the principal will also be tempted to intervene when the agent wants to implement small or medium-sized projects.\(^2\) Hence, by giving control to an intermediary with objectives between hers and her agent’s, the principal may prevent these very large projects from being initiated and at the same time foster communication concerning small and medium projects. Thus, when the bias of the agent is 'intermediate’, we show that with a uniform distribution and a quadratic loss function, delegating authority to a middleman dominates full or no delegation. For small biases, full delegation is still optimal.

Though it is sometimes optimal to have communication between an intermediary and the agent, direct communication between principal and agent never occurs with a uniform distribution. The uniform distribution, however, is somehow special in the sense that it is the limit case of how agnostic the principal can be about which action she optimally should take. If, on the other hand, the principal has quite a precise idea of what should be done, that is when her prior has a sufficiently steep single-peaked distribution, she might sometimes be better off by relying on communication than on delegation. This will be true when the bias is 'intermediate’: while delegation then always leads to substantial deviation from the optimal action, the principal can take an action which is mostly less off the mark than the ones taken by the agent and count on the agent to warn her in the rare case a small project is optimal. Hence, the principal optimally keeps control and some communication, though limited, occurs in equilibrium.

We conclude by pointing out that also changing the nature of information may generate (far) more communication than is suggested by our results. While the agent always has the option to produce an unverifiable report, in some cases he may be able to 'prove’ that his information is true. In the extreme case where the agent is always able to do this, there is a unique fully revealing equilibrium so that the principal never delegates authority.

\(^2\) However, if in order to carry out a specific actions, a certain amount of money, man-power, time,... is needed, the principal may be able to put an upperbound on the actions of the agent by granting him full decision rights but only a limited budget, a limited man-power, a limited time-period,....
Literature. — Although we are, to the best of our knowledge, the first to analyze delegation as an alternative to communication, the aggregation and revelation of information has been a very productive research area in economics.\textsuperscript{3} Hence, our work is related to a number of literatures:

By assuming an incomplete contracting environment, we clearly depart from the standard principal-agent model in which the principal elicits private information by designing a mechanism. While this approach may explain many institutions, in particular when the private information concerns a characteristic of the agent, we feel that the underlying premise that the principal can perfectly and without costs commit herself to any mechanism is not very realistic in many organizational contexts, especially when projects are urgent or complex or when superiors face an important work-load.\textsuperscript{4}

In sharp contrast with the mechanism design literature, the literature on strategic communication, initiated by \textit{CS}, assumes that no commitment at all is possible. When we think about communication in organizations, this is also unrealistic. Indeed, it is an insight of the property rights literature that ownership or more generally the control over critical resources confers authority to its holder.\textsuperscript{5} Hence, the principal can commit to never overrule the decisions of the agent by choosing an appropriate organizational structure, as we argue in section 2. Note that by analyzing an adverse selection problem without moral hazard, we distinguish ourselves from most of the incomplete contracting literature which typically analyzes the impact of authority on relationship-specific investments under symmetric information.\textsuperscript{6}

\textsuperscript{3}After this version of the paper was finished, however, I learned about a related project by de Groot-Thorson and Ottaviani which is closely related and focuses also on delegation versus communication. Their preliminary draft includes results that are similar to my propositions 2 and 5.

\textsuperscript{4}For a discussion on the foundations of incomplete contracts, see Hart (1995) and Tirole (1999) and the references therein. Jeon (1999) and Wells (1998) are recent papers which look at information aggregation and the internal organization of the firm from a complete contracting perspective.

\textsuperscript{5}As this paper, several recent articles (see for instance \textit{AT} and Rajan and Zingales (1998)) have applied the framework of the property rights literature to the internal organization of the firm and have argued that authority not only stems from ownership of assets. For an overview of the property rights literature, see Bolton and Scharfstein (1998) and Hart (1995).

\textsuperscript{6}Some exceptions, though, analyze a moral hazard problem in combination with asymmetric information. Farrell and Gibbons (1995) consider a problem in which a buyer has private information about the efficient scale or nature of a relation-specific investment by a producer. They show that giving some bargaining power to the buyer can improve welfare (by improving communication) even though it exacerbates the hold-up problem. Rabin (1993) analyzes when an informed agent will gain control over productive assets as a response to difficulties in selling information with respect to this production. Gertner
A few studies, such as Holmström (1984) and Melumad and Shibano (1991), take an intermediate position by positing that the principal can commit to a decision rule but not to monetary transfers. As noted by Holmström, such a decision rule only limits the discretion of the agent to a subset of actions and thus boils down to a partial form of delegation. In contrast to our paper, which stresses the loss of information which occurs when a commitment to (full) delegation fails, Holmström derives some general properties of the optimal partial delegation while Melumad and Shibano emphasize that the sender (the agent) does not always benefits from communication and hence may try to avoid it. The main difference with our paper, however, is that both presume that the principal can reverse some actions (which she determines ex ante) and at the same time is able to commit never to reverse others.

Finally, by explicitly taking into account the incentives to distort information, our paper departs from the traditional team theoretic framework of the literature on communication and information processing costs.\(^7\)

While our paper considers communication in a broad organizational context, most models which apply the CS communication framework, do this to specific political settings. Gilligan and Krehbiel (1986, 1989, 1990), for instance, study the role of congressional committees as providers of expertise to the House and draw implications for the organization of Congress and the composition of committees.\(^8\) A notable exception is Banerjee and Somanathan (1999) which studies voice, that is the voluntary expression of people’s views, as a means of information aggregation in organizations. As in our model, one of their conclusions is that there is often very little communication in equilibrium.\(^9\)

Outline.— The paper is organized as follows: Section 2 describes the model. Section 3 solves the equilibrium for given decision rights and iden-

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\(^7\) See, for example, Bolton and Dewatripont (1994), Radner (1992, 1993), and more recently, Van Zandt (1999).

\(^8\) Particularly related to our work is (1986), which, like AT, argues that agenda-setting power may be given to a committee in order to enhance its incentives to acquire information. Another line of research (see for example Austen-Smith (1990) and Banks (1990)) asks whether communication prior to voting has any effect on the outcome of voting.

\(^9\) Other recent papers which extend the Crawford-Sobel framework are Krishna and Morgan (1998) and Ottaviani and Sorensen (1998).
tifies the central trade-off of the paper. In section 4, we first illustrate the benefits of delegation with a simple example in which the principal faces the choice between three potential projects. We then generalize the setup to a continuum of projects. In section 5, we go on to look at how intermediate forms of delegation may even do better, while section 6 points out some limits of our results. We conclude in section 7.

2 The Model

A (profit or non-profit) organization has the opportunity to engage in a valuable project. There are an infinity of potential projects, but only one project can be undertaken. While an agent (he) is hired to implement this project, a principal (she) initially controls the critical resources of the organization which are needed to initiate any of these projects. The principal can be the CEO or the owner of a firm, but in principle, any hierarchical relationship in an organization could fit our model.

Preferences.— Projects differ from each other on one dimension and can be represented by a real number $y \in \mathcal{R}$. (Alternatively, projects may have different dimensions, but agent and principal agree on all but one dimension). With each project $y$ is associated a monetary gain and/or private benefit $U_P = U_P(y, m)$ for the principal and a private benefit $U_A \equiv U_A(y, m, b)$ for the agent, where $m$ is a random variable and $b$ a parameter of discongruence between agent and principal. The utility of the principal reaches a unique maximum for $y = m$ and can be rewritten as

$$U_P(y, m) = U_P(m, m) - \ell(|y - m|)$$

where $\ell''(.) > 0$ and $\ell'(0) = 0$. The principal always prefers the project which is, in expectations, most valuable to her to the status quo (no project at all). Similarly, the utility of the agent is maximized for $y = m + b$ and can be rewritten as

$$U_A(y, m) = U_A(m + b, m) - \ell_A(|y - (m + b)|)$$

where $\ell''_A(.) > 0$ and $\ell_A'(0) = 0$. The agent prefers $y = m + b$ to the status quo. Wlog, we assume $b > 0$.

We will often refer to $b$ as the bias of the agent. Systematic biases in agency relationships are well documented, both theoretically and empirically. It is well accepted, for instance, that managers have a propensity to cause their department, division or firm to grow beyond the optimal size, i.e.
they are empire builders and undertake too much investments.\textsuperscript{10} They further seldom take externalities on future managers into account and, hence, are excessively oriented towards short-term profitability and results. Employees, concerned with their career perspectives, will favor projects with a high visibility or a close contact with senior management; they may, for the same reason, prefer projects which allow the acquisition or improvement of important skills or avoid risky projects. Managers also often internalize too much the interests of their subordinates. Bertrand and Mullainathan (1999), e.g., provide evidence that managers have a preference for paying high wages. In the same vein, managers with close ties to their personnel may fire too few employees during a restructuring. Anecdotal evidence of other biases abound: employees are claimed to be effort-averse, status-quo biased...

It is worth noting that biases often arise endogenously as the product of inherently imperfect incentive schemes. Division managers’ salaries depend in general on the performance of their division, which distorts incentives if projects involve externalities on other divisions. Similarly, shareholders may partially control managerial short-termism by the way remuneration depends on reported earnings and changes in stock-market value.\textsuperscript{11} In order to simplify the analysis, however, we will treat the bias as exogenous.

\textit{Information Structure}.— Only the agent observes \( m \); the other parameters of the utility functions are common knowledge. We assume that \( m \) has a differentiable probability distribution function \( F(m) \) which has support on \([-L, L]\) and whose density \( f(m) \) is non-increasing at the upper end of the support: \( \exists L' < L : m \geq L', f'(m) \leq 0.\textsuperscript{12} \)

Though not made explicit in the model, the superior information from the agent can be seen as an externality from implementing actions in previous periods or from his ‘proximity’ to the business environment (clients, suppliers, competitors).

Given the incongruence in objectives, one may expect the agent to find

\textsuperscript{10}This size-bias drives the free-cash flow theory of Jensen (1986), and the models of Stulz (1990), Hart and Moore (1995) and Zwiebel (1996), among others. In his study of 12 large Fortune 500 firms, Gordon Donaldson (1984), cited by Jensen, concludes that managers of these firms were not driven by maximization of the value of the firm, but rather by the maximization of “corporate wealth”, defined as “the aggregate purchasing power available to management for strategic purposes during any given planning period” (p.3).

\textsuperscript{11}This presumes that the stock-market price reflects the long-term profitability of the firm. As shown by Stein (1988), though, managerial short-termism may also result from the desire to boost the stock-market price in the short-term.

\textsuperscript{12}If the bias were negative, we would need the assumption that \( f(m) \) is non-decreasing at the lower-end of the support, i.e. \( \exists L' < L : m < -L', f'(m) \geq 0.\)
it difficult to convince the principal of which project should be optimally undertaken. Most likely, the best thing to do depends on a wide range of factors which the principal may find hard to verify or to understand.\textsuperscript{13} During most of the paper, we assume therefore that the agent can only make loose and unverifiable recommendations, that is information is soft. At the end of the paper, however, we explore the opposite assumption, i.e., that the agent, in any state of nature, can make his information hard at no cost.

\textit{Authority & Contracts}.— We adopt an incomplete contracting approach by positing that projects (actions) cannot be contracted upon and that to engage in a project, the use of some resources, which are initially controlled by the principal, is needed. Resources which we have in mind, are (a) the assets of the organization, (b) the name of the organization and more generally the right to contract on behalf of the organization with third parties, and, (c) to some extent, the human resources of the organization. Hence, following several recent articles,\textsuperscript{14} we argue that authority not only stems from ownership of assets. What is needed is the effective control over the critical resources at the initiating stage of the project:

By default, the owner of the organization owns the assets of the organization, the name of the organization and the right to contract on behalf of the organization. Since the owner can contract with employees, that is hire, promote, demote and fire them, he controls also to a large extent the human resources of the organization.\textsuperscript{15} While projects cannot be contracted upon, we assert that the principal may grant subordinates authority over the use of the resources needed to initiate a project. This can be done by contracts, job-descriptions, corporate charters, customs or, in the extreme case, by selling some of the assets of the organization to the agent.\textsuperscript{16} A production manager,

\textsuperscript{13}Jensen and Meckling (1992) provide an interesting discussion on the cost of transferring all the "knowledge" which is used in decision making and invoke these communication costs as a reason for decentralizing formal decision rights. While we agree very much with the idea that it may be a difficult and time-consuming process for a subordinate to provide and explain all the underlying reasons for a decision in a credible way, the communication costs of a 'soft' recommendation are very low. In this sense, Jensen and Meckling do not directly explain why decision rights should be decentralized since, a priori, the principal can always rubber-stamp the 'soft' recommendations of the agent.

\textsuperscript{14}See, for example, AT and Rajan and Zingales (1997). AT argue that "Authority may more generally result from an explicit or implicit contract allocating the right to decide on specified matters to a member or group of members of the organization" (p.2). Rajan and Zingales stress access, defined as the the ability to use, or work with, a critical resource, as an alternative mechanism to allocate power.

\textsuperscript{15}One can easily construct a model where these contracting possibilities enable a principal to control substantially the behaviour of his personnel.

\textsuperscript{16}Besides the control over some resources and hence, some decisions of the organization,
for example, may be given the right to change the production process (batch-size, inventories, production method) and to decide which machines will be bought by the organization without prior approval of top-management. A sales manager may be granted the right to sign binding contracts with clients on behalf of the organization, a purchasing manager may be able to do the same with suppliers... Whether or not an agent has control over some human resources, that is whether employees follow up his instructions, depends to a large extent on the job descriptions of these employees and the hierarchical structure of the organization. A manager of a division with its own personnel or purchasing office, for example, has a large control over the purchasing or personnel policy in his division. In contrast, this control is substantially reduced if the head of the personnel or purchasing office directly reports to top-management.

If an agent is granted authority over the use of the critical resources, he can initiate a project without assistance of the principal; he has formal decision rights. If the principal keeps authority, on the other hand, the agent needs her fiat. This fiat can take the form of some 'signatures', may require some concrete actions by the principal, or may imply that the principal (or her staff) takes fully care of the initiation stage. In any case, the principal then fully controls the project choice. Once a project is initiated, it still needs to be implemented by the agent, but cannot be reversed anymore.

The timing is as follows. (i) The principal decides whether or not to delegate the agent authority over the use of the critical resources. (ii) The agent learns $m$ and initiates her preferred project if she has authority. If the principal has not delegated authority, she may ask the agent to make a recommendation, and then initiates a project. (iii) The agent implements the project. We assume that control rights over resources can only be allocated to the agent at the initial date and, hence, are always unconditional.\textsuperscript{17}

\section{Equilibrium for Given Decision Rights}

In this section we characterize the equilibrium for a given allocation of decision rights and identify the trade-off between keeping authority, which results in a loss of information, and delegation, which results in a loss of control.

\footnote{\textsuperscript{17}More generally, one could consider control rights being allocated ex post on the basis of the parties’ announcements of willingnesses to pay for these rights (see, e.g., Hart (1995) and Maskin-Tirole (1999)). This, however, is outside the scope of the paper.}
3.1 Delegation

If the agent has formal decision rights in the second period, he will implement 
\( y = m + b \) which yields

\[
U_P(m + b, m) = U_P(m, m) - \ell(b)
\]

to the principal. The utility of the principal thus equals her utility under 
perfect information, minus a loss \( \ell(b) \) which depends on the difference in 
objectives between the principal and the agent but is independent \( m \).

3.2 No Delegation (Crawford - Sobel)

If the principal keeps formal authority, on the other hand, she will under-
take the project \( y \) which maximizes her expected utility conditional on her 
beliefs upon \( m \). These beliefs may be affected by a communication of infor-
mation, a ‘recommendation’, of the agent. In our model, as in many (most?) 
real organizations, communication is not structured by contracts but occurs 
simply through ‘informal’ talk. The resulting communication game between 
agent and principal is formally equivalent to a sender-receiver game in which 
a better-informed sender (the agent) may reveal some of his information by 
sending a possibly noisy signal to a receiver (the principal), who then takes 
an action (initiating a project) which determines the welfare of both. The 
only constraint on the information transmission is that both the agent’s mes-
sage and the principal’s subsequent decision are an optimal response to the 
ponent’s strategy, which takes into account its implications in the light 
of his/her beliefs. In other words, the recommendation of the agent and the 
response given to it by the principal, must be a \textit{Perfect Bayesian equilibrium}. 

Formally, an equilibrium is characterized by (i) a family of \textit{signalling rules} 
\( q(n|m) \) for the agent, where for every \( m \in [-L, L] \), \( q(n|m) \) is the conditional 
probability of sending message \( n \) given state \( m \), and (ii) a \textit{decision rule} \( y(n) \) 
for the principal, where \( y(.) \) is a mapping from the set of feasible signals \( N \) 
to the set of actions \( \mathcal{R} \), such that
- for each \( m \in [-L, L] \), if \( n^* \) is in the support of \( q(.|m) \), then \( n^* \) maximizes 
  the expected utility of the agent given the principal’s decision rule \( y(.) \).
- for each \( n \), \( y(n) \) maximizes the expected utility of the principal, taking into 
  account the agent’s signalling strategy and the signal she receives in order to 
  update her prior of the distribution of \( m \).

As shown by \textit{CS}, all equilibria in this communication game are charac-
terized by a \textit{partition} of \([-L, L] \), where the sender (the agent) introduces 
noise into his signal by only specifying to which partition element the real-
ized state of nature belongs. As their model encompasses ours, the following
proposition, which is a variant of Theorem 1 in CS, can be shown to hold. Let \( a \equiv (a_0, \ldots, a_N) \) denote a partition of \([-L, L]\) with \( N \) steps and dividing points between steps \( a_0, \ldots, a_N \), where \( -L = a_0 < a_1 < \ldots < a_N = L \). Define, for all \( a, \tilde{a} \in [-L, L] \), \( a < \tilde{a} \),

\[
\bar{y}(a, \tilde{a}) \equiv \arg \max \int_a^{\tilde{a}} U_P(y, m)f(m)dm
\]

**Proposition 1 (Crawford and Sobel: Communication Equilibrium)**

If \( b > 0 \), then there exists a positive integer \( N(b) \) such that, for every \( N \) with \( 1 \leq N \leq N(b) \), there exists at least one equilibrium \((y(n), q(n|m))\), where

\[
q(\bar{y}(a_{i-1}, a_i)|m) = 1 \quad \text{if} \quad m \in (a_{i-1}, a_i), \quad (1)
\]

\[
U_A(\bar{y}(a_i, a_{i+1}), a_i, b) - U_A(\bar{y}(a_{i-1}, a_i), a_i, b) = 0, \quad (A)
\]

\[
(i = 1, \ldots, N - 1),
\]

\[
y(n) = \bar{y}(a_{i-1}, a_i) \quad \text{if} \quad n \in (a_{i-1}, a_i) \quad (2)
\]

\[
a_0 = -L, \quad \text{and} \quad a_N = L. \quad (3)
\]

Further, all other equilibria have relationships between \( m \) and the principal’s induced choice of \( y \) that are the same as those in this class for some value of \( N \) with \( 1 \leq N \leq N(b) \); they are therefore economically equivalent.\(^{18}\)

**Proof.** The proof follows directly from CS, Theorem 1. ■

In equilibrium, only a finite number of actions \( 1 \leq N \leq N(b) \) are thus implemented with positive probability, and the states of nature for which each of these actions is best for the agent form an interval, and these intervals form a partition of \([-L, L]\). The partition \( a \) is determined by \((A)\), a well-defined second-order nonlinear difference equation in the \( a_i \)'s, and \((3)\) and \((4)\), its initial and terminal conditions. Equation \((A)\) is an ‘arbitrage’ condition which says that for states of natures who fall on the boundaries between steps, the agent is indifferent between the associated values of \( y \). Given our assumptions about \( U_A \), this condition is necessary and sufficient for the agent’s signalling rule to be a best response to \( y(n) \). Similarly, \((2)\) gives a best response of the principal to the signalling rule \((1)\).

\(^{18}\) CS Theorem 1, for instance, proposes that \( q(n \mid m) \) is uniform, supported on \( [a_{i-1}, a_i] \) if \( m \in (a_{i-1}, a_i) \).
Equilibrium Selection: Although there are, in general, several essentially different equilibria, CS provide some sufficient conditions under which the expected utility of both the principal and the agent are ex ante (that is, before the agent knows the state of nature) maximized by the same equilibrium.\footnote{\textit{While these conditions are quite stringent, they are always satisfied when }F(m)\textit{ is uniform.}} Under these conditions, this is the equilibrium with the largest number, \(N(b)\), of partition elements. As CS, we assert that it is reasonable for the players to coordinate on this (ex ante) Pareto-superior equilibrium. Doing this, \textit{we allow communication to be as powerful as possible.} Whenever this assumption may affect our results, we discuss this in the text.

Comparative Statics: Under the same conditions as above, CS establish a sense in which communication gets better when the receiver (the principal) and the sender (the agent) have more similar preferences They show that (i) for a given number of partition elements, the principal then always strictly prefers equilibrium partitions with more similar preferences (i.e., a smaller value of b), (ii) for a given b, the principal always strictly prefers equilibrium partitions with more elements and (iii) the largest possible number of partition elements \(N(b)\) weakly decreases with b. In a much more general setting (that is without the restrictive conditions which CS needs to prove their monotonicity result), Spector (1999) shows that as the preferences of sender and receiver tend to coincide, the noise in the sender’s message tends to zero in the most informative equilibrium. In the limit, as b goes to zero, communication thus becomes perfect.

\subsection{3.3 Loss of Control versus Loss of Information}

Delegation of formal authority clearly leads to a \textit{loss of control} in our model: the decision taken by the agent has a constant bias relative to what is optimal for the principal and if the latter, unexpectedly, were given control rights ex post, she definitely would want to reverse the action of the agent.

In contrast with this, if the principal does not delegate authority, the action taken in equilibrium is on average unbiased, that is \(E(y(n)|m) = m\), which is a direct consequence of the rational-expectations character of the Bayesian Nash equilibrium. If, for example, in any state of nature the principal prefers to spend \(x\) dollar less than the agent, then in equilibrium she will \textit{on average} spend \(x\) dollars less than the agent would have spent. The Bayesian character of the equilibrium thus destroys completely the real authority of the agent. However, even after the transmission of information, the principal has only incomplete information: the agent only specifies a
region to which the state belongs. Keeping authority thus results in a loss of information.

CS show that, at least under some conditions, this loss of information becomes smaller when preferences are more similar. They conclude that "direct communication is more likely to play an important role, the more closely related are agent’s goals". As proved by Spector (1999), communication even becomes perfect in the limit. On the other hand, the cost of delegation also decreases as b gets smaller. At first glance, when preferences are close, it is not clear whether it is better for the principal to keep authority and thus whether communication actually will occur when preferences are close, as predicted by CS.

4 The Rationale for Delegation

We are now ready to endogenize the allocation of formal decision rights. To provide some preliminary intuitions, we start with a simple example in which there are only three potential projects. We then generalize the framework by considering a continuum of projects. As there exists in general no tractable solution to the second-order linear equation (A) of proposition 1, unless the distribution of information is uniform,\(^\text{20}\) we first focus on that simple case. We subsequently discuss to what extent our results carry over to more general distributions.

4.1 A Simple Example with Three Projects

A principal faces the choice between a small, a medium and a large project. The profitability of the projects depends on demand conditions, characterized by a scalar \(m\). The larger the demand, the better it is to engage in a large project. An agent of the principal is in charge of implementing the project. Through his proximity to the market and his experience in implementing previous projects, this agent has a precise estimate about market demand \(m\). While ceteris paribus, this agent prefers a more profitable project, he also derives a private benefit of heading large projects since this provides him with more experience and increases his human capital.

\(^{20}\)To the best of our knowledge, there exists no application of a sender-receiver game in which the action space is continuous and where the distribution of information is assumed to be both continuous and non-uniform. Even to prove the above mentioned comparative static results, CS need quite strong technical conditions. While CS show that these conditions are always satisfied whenever \(F(m)\) is uniform, they find no other well known distribution for which the same is true.
Formally, $y$ can take only three values, $y_1$, $y_2$ and $y_3$, and preferences over these projects satisfy the assumptions made in section 2. We suppose that each project is as likely to be optimal for a perfectly informed principal. If we assume that $m$ is uniformly distributed on $[-1, 1]$,$^{21}$ this implies that

$$y_1 = -\frac{2}{3}, \quad y_2 = 0, \quad y_3 = \frac{2}{3}$$

Hence, the following table summarizes the demand conditions for which each project is preferred by the principal and by the agent:

<table>
<thead>
<tr>
<th></th>
<th>$y = -\frac{2}{3}$</th>
<th>$y = 0$</th>
<th>$y = \frac{2}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>principal</td>
<td>$m \in [-1, -\frac{1}{3}]$</td>
<td>$m \in [-\frac{1}{3}, \frac{1}{3}]$</td>
<td>$m \in [\frac{1}{3}, 1]$</td>
</tr>
<tr>
<td>agent</td>
<td>$m \in [-1, -\frac{1}{3} - b]$</td>
<td>$m \in [-\frac{1}{3} - b, \frac{1}{3} - b]$</td>
<td>$m \in [\frac{1}{3} - b, 1]$</td>
</tr>
</tbody>
</table>

Depending on the bias $b$, the communication equilibrium and optimal allocations of authority is as follows.$^{22}$

1) If $b < \frac{1}{3}$, the principal optimally rubber-stamps a recommendation of the agent if she believes that the agent recommends his preferred project. Since the agent has no incentive to misrepresent his preferences given these beliefs, this is indeed an equilibrium - and even the most informative equilibrium the principal can hope for. As the agent has effective control over the project choice (he has real authority), the allocation of decision rights is irrelevant.$^{23}$

2) Second, if $\frac{1}{3} < b < \frac{2}{3}$, only $y_1$ and $y_2$ are implemented with strictly positive probability in equilibrium: the agent recommends a 'large' project whenever he prefers $y_2$ or $y_3$ and a 'small' project in the rare case that he prefers $y_1$. No other informative equilibria exist. Indeed, if the principal were to believe that the agent always reveals his preferred project, she would rubber-stamp $y_1$ and $y_3$ but not $y_2$. Since the expected value of $m$ equals $-b$ if the agent prefers $y_2 = 0$, she then initiates $y_1 = -\frac{2}{3}$. This is neither an equilibrium, however. If the agent anticipates a proposal of $y_1$ and $y_3$ will be followed, but $y_2$ never will be implemented, he recommends $y_3$ whenever

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$^{21}$ Assuming that $m$ is uniformly distributed on $[-L, L]$ yields the same qualitative results, but makes notation heavier.

$^{22}$ Only for $b < \frac{1}{3}$, our assumption that the most informative communication equilibrium prevails will influence the optimal allocation of authority. If communication is less informative, the principal then strictly prefers to delegate authority.

$^{23}$ As we will see shortly, this irrelevance is driven by the discrete number of projects and will disappear if the projects form a continuum.
$m > -b$, but then the principal optimally implements $y_2 = 0$. Hence there exists no equilibrium in which $y_3$ may be implemented.

If the principal keeps authority, she is able to avoid that $y_3$ is taken too often. This, however, comes at an important information loss: though both agent and principal prefer $y_3$ to $y_2$ whenever $m > 1/3$, the agent cannot credibly communicate this information to the principal. In contrast, if the agent has authority, he will use it to implement $y_3$ when this is mutual beneficial, but also abuse it to implement $y_3$ when $m \in [1/3 - b, 1/3]$ and $y_2$ is optimal. As long as $b < 2/3$, one can verify that this bias towards $y_3$ is less harmful than the loss of information under centralization: the principal optimally delegates authority.\footnote{For $m < 1/3 - b$, the same actions are implemented with or without delegation. In contrast, for $m > 1/3 - b$, $y_2 = 0$ is implemented under centralization and $y_3 = 2/3$ under delegation. Since $E(m|m > 1/3 - b) = 2/3 - b/2$, the principal obtains a higher utility under delegation if and only if $b < 2/3$.}

3) Finally, if $b > 2/3$, no informative communication is possible and the principal always initiates project 2. Since the loss of information is now smaller than the loss of control the principal optimally keeps authority.

The previous example shows how the principal weakly prefers to delegate authority as long as informative communication is possible. By delegating authority, the principal basically solves a commitment problem: for $1/3 < b < 2/3$, he cannot refrain himself from implementing $y_1$ if the agent recommends $y_2$. The agent responds to this by distorting his information, so that finally the principal is not able to tell whether the agent prefers $y_2$ or $y_3$. The resulting loss of information turns out to be more damaging than the loss of control under delegation, which allows the agent to implement too frequently projects of a medium or a large size. In the next section, we show that with a continuum of projects and a uniform distribution, the principal strictly prefers to delegate authority as long as communication is possible - even for very congruent preferences.

4.2 The Uniform Distribution

With a continuum of projects, all communication equilibria are fully characterized by a partition of $[-L, L]$, where the agent tells the principal to which partition element the state of nature $m$ belongs. The amount of information which is communicated depends thus on how fine this partition is. It will be useful to take as a measure for the \textit{minimal loss of information}, the minimal
average size of the partition elements, denoted by $\bar{A}(b)$:

$$\bar{A}(b) \equiv 2L/N(b)$$

where $N(b)$ is the maximum number of partition elements in equilibrium given $b$. Dividing this measure by $b$, we obtain a proxy, denoted by $D(b)$, for how attractive delegation is relative to keeping authority:

$$D(b) \equiv \bar{A}(b)/b$$

Note that $D(b) > 4$ is a sufficient condition for delegation to be optimal, since the expected absolute deviation from the optimal action, $E(|y^* - m|)$, is then larger or equal than $b$.

In the remainder of this section, we show that although communication improves and $\bar{A}(b)$ decreases weakly if preferences become more aligned, $D(b)$ tends to infinity a $b$ goes to zero. Moreover if $N(b) > 2$, then $D(b) > 4$ and hence delegation is optimal. We conclude by showing that the principal prefers to delegate authority to the agent as long as informative communication is possible.

Common sense tells us that if a person with a preference towards large projects recommends a 'large' project, then his message is not very informative. In terms of our model, the size of the partition element associated with this message will be very large. If, on the other hand, the same person proposes a 'small' project, we will be much more prone to believe him and the associated partition element will have a small size. This is exactly what we observe in our three project example. When the agent recommends a 'large' project, the principal deduces that perhaps a large project is effectively optimal, but maybe ($b < 1/3$) or probably ($b > 1/3$) a medium action is better. Indeed, for $b > 1/3$, the principal does not know whether a 'high' signal implies that the agent prefers a large or a medium size project. On the other hand, if this same agent proposes the small project, the principal trusts him and knows that demand is low. We show now that with a continuum of projects, we obtain the same intuitive feature of the communication equilibrium that partition elements are increasing in size.

Suppose that $y_1$ and $y_2$ are two subsequent actions which are taken with a positive probability in equilibrium and denote by $a_0$, $a_1$ and $a_2$ the dividing

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\textsuperscript{25}Note that this measure underestimates the real loss of information if partition elements are unequal in size.

\textsuperscript{26}Note that this proxy underestimates the profitability of delegation since it only takes the average value of $|y^* - m|$ into account. Given the concavity of preferences, the variance of $|y^* - m|$ under communication implies that delegation is more attractive than reflected by $D(b)$.  

---
points of the equilibrium partition respectively preceding $y_1$, following $y_1$ and following $y_2$.

$$\begin{array}{cccc}
a_0 & Y_1 & a_1 & (Y_1 + Y_2)/2 \\ & \vdots & & \vdots \\ & \vdots & & \vdots \
\end{array}$$

Since the principal is not restricted in her project choice, she always initiates the project which is equal to the average state of nature of a partition element, that is $y_1 = (a_0 + a_1)/2$ and $y_2 = (a_1 + a_2)/2$. At the dividing point $m = a_1$, the agent must be indifferent between $y_1$ and $y_2$, otherwise he would not always truthfully reveal to which partition element $m$ belongs. As a result, we also have that $a_1 = (y_1 + y_2)/2 - b$. As can easily be seen on the figure above, the latter two conditions imply that

$$a_2 - a_1 = a_1 - a_0 + 4b$$

**Lemma 1** The size of a partition element is always $4b$ larger than the size of the preceding one:

$$a_{i+1} - a_i = a_1 - a_0 + 4ib, \quad i = 1, ..., N(b) - 1$$

Note that the increase in size of subsequent partition elements is proportional to $b$. Intuitively, if we fix the size of the first partition element and increase $b$, then at the dividing point $m = a_1$, the agent will strictly prefer $y_2$ and thus misrepresent his information for some values of $m < a_1$, unless the distance between $a_1$ and $y_2$, and hence the size of the whole second partition element, also increases.

The simple result of lemma 1 implies that, when preferences become closer, (i) communication improves in absolute terms, but (ii) communication performs worse and worse when compared to delegation:

(i) The comparative static results of $CS$, which we discussed in section 2, follow almost directly from lemma 1. One sees immediately that for given preferences, the most informative communication equilibrium has $N = N(b)$ partition elements, and $N(b)$ increases weakly as $b$ decreases. As $b$ tends to 0, $N(b)$ goes to infinity and hence $\bar{A}(b)$ tends to 0: in the limit communication becomes perfect.

(ii) On the other hand, the fact that each additional partition step is $4b$ larger than the preceding one is in particular worrying for communication as
it implies that the noise in the agent’s signal decreases very slowly with \( N(b) \) when high actions are optimal. Since the best way to extract information from the agent is to have an as high as possible number of partition elements, the average size of a partition element, \( A(b) \), will be very large relative to the bias \( b \) if communication is very good. From lemma 1, we obtain after some calculations that

\[
D(b) = \frac{A(b)}{b} \geq 2(N(b) - 1) \tag{5}
\]

As a result, the larger is \( N(b) \) and thus the better is communication, the worse communication performs relative to delegation. In the limit \( D(b) \) goes to infinity when \( b \) tends to 0.

From (5), if \( N(b) > 2 \), that is if an equilibrium exists with at least three partition elements, then \( D(b) > 4 \) and the principal optimally delegates authority to the agent. Indeed, if \( A(b) \geq 4b \), then \( E(|y^* - m|) \geq b \), (where \( y^* \) denotes the project which is finally implemented in equilibrium). Given the concavity of preferences, the principal strictly prefers to delegate authority in which case \( |y^* - m| = b \) in any state of nature:

**Lemma 2** The principal optimally delegates authority if \( N(b) > 2 \).

The next proposition tells us that as long as informative communication between agent and principal is possible, the principal is always better off by delegating full decision rights to the agent than by keeping authority and relying on this communication.

**Proposition 2** Suppose \( F(m) \) is uniformly distributed over \([-L, L] \), then there exists a threshold \( \nu' \) such that the principal delegates control rights to the agent if and only if \( b < \nu' \), where \( \nu' \) is such that no (informative) communication is possible.

**Proof.** From lemma 2, the principal optimally delegates authority whenever \( N(b) > 2 \). Suppose now that \( N = 2 \) and denote \( A_1 = a_1 - a_0 \) and \( A_2 = a_2 - a_1 \). From lemma 1, \( A_2 = A_1 + 4b \) so that if the principal keeps authority, she implements \( y = a_0 + A_1/2 \) if \( m \in (a_0, a_1) \) and \( y = a_1 + A_1/2 + 2b \) if \( m \in (a_1, a_2) \). Given the uniform distribution, this implies that \( E(|y^* - m|) \geq b \), where the equality holds if and only if \( A_1 = 0 \). As long as informative communication is possible \( (N(b) > 1) \), the principal thus optimally delegates authority.

Finally, in a babbling equilibrium (\( N = 1 \)), the principal orders the agent to implement \( y = 0 \) which yields an expected loss \( E[\ell(y - m)] = \int_{-L}^{L} \ell(m) dm \), which is independent of \( b \). Delegating authority, on the other hand, yields \( \ell(b) \), which is strictly increasing in \( b \). There exists thus a cut-off value \( \nu' > \frac{L}{2} \) for which \( \ell(\nu') = \int_{-\nu'}^{\nu'} \ell(m) dm \). The principal optimally keeps authority if and only if \( b > \nu' \). ■
Corollary 1 If \( F(m) \) is uniform, there occurs no (informative) communication in equilibrium.

Proposition 2 provides a first determinant of the allocation of authority, i.e., delegation occurs when preferences between agent and principal are not too far apart. A second prediction, which follows indirectly from proposition 2, is that delegation is more likely when the principal faces a lot of uncertainty. A measure for this uncertainty is given by the variance of \( m \), \( \sigma_m^2 = L^2/3 \). Since the cut-off value \( \mathcal{B}' \) below which delegation is optimal equals

\[
\ell(\mathcal{B}') = \frac{1}{L} \int_0^L \ell(m)dm
\]

\( \mathcal{B}' \) increases with \( \sigma_m^2 \) and goes to infinity as \( \sigma_m^2 \) goes to infinity. The proof of the following corollary is now direct:

Corollary 2 Given \( b \), there is a \( \sigma_m^2 \) such that \( P \) delegates authority if and only if \( \sigma_m^2 > \sigma_m^2 \).

Given that no communication occurs in equilibrium, this result is very intuitive: an increase in uncertainty decreases the pay-off of an uninformed decision while it has no impact on the pay-off under delegation. Without the prior knowledge of proposition 2, however, this corollary is less straightforward. One can easily verify that informative communication is possible for a larger range of values of \( b \) when \( \sigma_m^2 \) increases; more generally, from lemma 1, the number of partition elements in equilibrium increases weakly with \( L \) and thus \( \sigma_m^2 \). More uncertainty makes delegation nevertheless more attractive, because this increased communication is not sufficient to compensate for the loss of information.

Finally, (6) also implies that there will be more delegation if the principal is more risk-averse in the sense of Arrow-Pratt. If the principal keeps authority, she takes on average an unbiased action, but the deviation from the optimal action has a large variance. Hence, the more concave her utility function, the more attractive is the constant bias which prevails under delegation.

Corollary 3 If preferences of the principal shift from \( U_P(y, m) \) to \( \hat{U}_P(y, m) \), where \( \hat{U}_P(y, m) \) is more concave in \( |y - m| \) in the sense of Arrow-Pratt, then the principal will delegate authority for a larger range of values of \( b \).

Proof. Since the principal’s utility depends negatively on \( |y - m| \), a necessary and sufficient condition for a utility function \( \hat{U}_P(y, m) \) to be more concave than \( U_P(y, m) \), is that it has a larger certainty equivalent for any lottery \( F(.) \) over the distance \( |y - m| \). Hence, \( \hat{U}_P(y, m) \) has a larger cutoff value \( \mathcal{B}' \) than \( U_P(y, m) \).
4.3 General Distributions

It is clear from proposition 1 that it is very hard to characterize equilibrium partitions for other distributions than the uniform one. While the results we could obtain are therefore weaker, they support the main intuitions from the previous section.

First of all, proposition 2 predicts that delegation is more likely to be observed when preferences are more congruent and there are more partition elements in the most informative communication equilibrium. Intuitively, also for more general distributions, the larger the number of partitions elements, the larger the size of the 'higher' partition elements relative to the bias and the more profitable it becomes for the principal to delegate authority. Since for $b$ small, communication is 'very good' and 'good' communication requires a fine partition with a large number of partition elements, logically, the principal then delegates authority:

**Proposition 3** Given any distribution $F(m)$, it is optimal for the principal to delegate control rights to the agent if $b$ is small enough.

**Proof.** See Appendix

The question is of course how large $b$ may be. To have an idea on this, we performed numerical simulations for a class of truncated normal distributions, and a quadratic loss function ($\ell(x) = x^2$). For a standard deviation equal to $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 2$, we found that there is a threshold $b'$ such that the principal delegates authority as long as $b < b'/\sigma$, where $b'$ is independent $\sigma$. While for $b = b'/\sigma$, communication between agent and principal is still possible, this communication is not very informative: only two actions are implemented with positive probability in equilibrium ($N(b) = 2$), where in 98.6% of the states of nature, the agent recommends the highest of these two action. Regardless of the variance of the normal distribution, the obtained results are thus quite close to the ”no communication” result. At least for the normal distribution, we may conclude that $b$ must be relative large for the principal to keep authority. Note that the condition $b < b'/\sigma$ supports our second prediction that delegation is more likely when the principal faces a lot of uncertainty.

Finally, the next proposition suggests that also the last prediction concerning the impact of the principal’s risk-aversion is robust:

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27 Concretely, $m \sim N(0, \sigma)/c^x$, supported on $[-4\sigma, 4\sigma]$ where $c^x = \int_{-4\sigma}^{4\sigma} dN(0, \sigma) \approx 0.99994$.

28 In section 6.1 we provide the intuition why effective communication, though not very informative here, may occur in equilibrium when $m$ has a single-peaked distribution.

29 Concretely, $b$ must be larger than 95.45% of the standard deviation $\sigma$. 

---
Proposition 4 Given $b < L/2$, if $f'(m) \leq 0$ for $m \in (L - 4b, L)$, then there exists a function $\hat{U}_P(y, m)$ such that the principal optimally delegates authority if $U_P(y, m)$ is more concave in $|y - m|$ than $\hat{U}_P(y, m)$ in the sense of Arrow-Pratt.

Proof. See appendix □

As in the uniform case, the intuition behind this result is that delegation avoids the variance in the distance from the optimal action.

5 Intermediate Forms of Delegation

Our analysis so far has concentrated on the benefits of full delegation of the right to decide upon the course of action. In practice, however, one commonly observes intermediate patterns of delegation, with decision rights being restricted to a subset of actions or being delegated to an intermediary with objectives between the principal’s and the agent’s.

5.1 Restricted Delegation

In reality, the discretion of managers is often restricted to a subset of actions. One way for the principal to do this is to simply keep authority. From proposition 1, communication between agent and principal is a noisy signalling game in which a finite number of actions $(y_1, \ldots, y_N)$ are implemented with positive probability. Since in this signalling equilibrium, the optimal response of the principal is always a pure strategy, the agent knows exactly which action will be implemented for which signal. The agent thus possesses a limited form of real authority: he is able to implement any action $y \in (y_1, \ldots, y_N)$.

Of course, if the principal could commit to carry out certain pre-specified actions whenever they are proposed by the agent - where this set of actions does not have to satisfy the conditions of proposition 1 - she would do at least as well. Sometimes such a commitment can be achieved by granting the agent authority over a ‘limited’ amount of the resources which are needed to implement projects (a limited budget, limited manpower, a limited capacity...). In a dynamic setting, the principal may also try to build a reputation of endorsing all recommendations of actions which belong to this subset (but no others) or, alternatively, a reputation of punishing the agent severely if it were discovered that the latter had implemented an ‘illegal’ project.

If we rule out random mechanisms, optimally limiting the number of actions on which the agent has discretion is equivalent to designing an optimal
mechanism under the constraint of no monetary incentives.30 Given that no transfers are to be specified, this follows directly from the revelation principle. The following proposition shows that if \( F(m) \) is uniformly distributed, then the optimal mechanism consists of putting an upper bound on the actions the agent can implement.

**Proposition 5** If \( F(m) \) is uniformly distributed on \([-L, L]\) and monetary transfers are not feasible or agents do not respond to monetary incentives, the optimal mechanism to extract information consists of putting an upper bound \( y' \) on the actions which the agent may implement, where \( y' = L - b \) if \( b < L \) and \( y' = 0 \) if \( b > L \).

**Proof.** See appendix

The intuition of the proof is simple. Obviously, with a finite support, it pays off to limit the largest action which may be taken by an agent with a positive bias. Further limiting the discretion of the agent, however, only harms the principal. While for some states of nature, this limited discretion reduces the bias in the undertaken action, for other states it increases the bias, generating thus a mean-preserving spread which is better avoided given the concave preferences of the principal.

If in order to carry out a specific action, a certain amount of money, manpower, capacity,... is needed and the agent is biased to use too much of these resources, the principal can thus easily implement the optimal mechanism by giving the agent only a limited budget, limited manpower, a capacity constraint... While this result is appealing, it will not generally hold if \( F(m) \) is not uniform. If \( f(x) \) is not too convex, the principal may then be better off by restricting the discretion of the agent to a set of non-connected actions for values of \( m \) where \( f(m + b) \) is decreasing. This way, the principal reduces the bias for states of nature with a higher probability weight while she increases it for states with a lower probability.

A similar result has been found by Melumad and Shibano (1991) which extends the Crawford-Sobel framework by allowing for commitment and for differences in the sensitivity to information.31 While the focus of their paper is on whether the sender always benefits from communication (he does not), they find that for some parameter values, the optimal mechanism without monetary transfers consists of limiting the discretion of the agent to a continuum of actions. Melumad and Shibano (1994) uses this result to

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30 See also Holmström (1984) which noted this already.
31 They assume that preferences of principal and agent are respectively \(-(y - (am + b))^2\) and \(-(y - m))^2\), where \( a \) is a parameter of sensitivity to the private information \( m \), which is uniformly distributed.
study the optimality of the veto-based delegation arrangement between the Securities and Exchange commission (SEC) and the Financial Accounting Standards Board (FASB).

5.2 Delegation to an Intermediary

The previous section shows how a principal optimally allows the agent to implement any project which is not "too large". In practice, however, it may be difficult to commit too such a policy, since the principal will also be tempted to intervene when the agent wants to implement small or medium-sized projects. Here, an intermediary may play an important role. By delegating decision rights to supervisors or managers with intermediate objectives between hers and her agent’s, the principal may prevent these very large projects from being initiated and at the same time foster communication concerning small and medium-sized projects.\(^{32}\) This provides a potential rationale for why decisions affecting agents (including difficult ones such as restructuring a company, laying off workers,...), are sometimes being delegated by company owners to managers.\(^{33}\)

We illustrate this idea first with a variant of our example with three projects. Suppose that there is also fourth potential project, \(y_4 = 4/3\). Obviously, since \(m \in [-1, 1]\) and \(y_3 = 2/3\), the principal always prefers \(y_3\) to \(y_4\). Since the principal never considers \(y_4\), communication equilibria are not affected by this additional project. If the principal fully delegates authority, however, \(y_4\) is implemented with positive probability. Hence, full delegation becomes far less attractive. It is easy to verify that for \(1/3 < b < 2/3\), the principal then optimally delegates authority to a middleman with an intermediate bias \(b_I\), where \(b_I\) satisfies

\[
b - 1/3 < b_I < b/2
\]

The upper bound on \(b_I\) ensures that the intermediate on average prefers \(y_3\) to \(y_4\) when the agent prefers \(y_4\) to \(y_3\). The lower bound on \(b_I\) guarantees good

\(^{32}\)The assumption that the 'best' communication equilibrium always prevails will be crucial to our results, however. If there were a babbling equilibrium between the middleman and the agent, for instance, delegating authority to an (uninformed) middleman is always dominated by keeping authority.

\(^{33}\)A similar idea is mentioned though not formalized in AT, where the principal may foster the agent’s initiative by delegating to a middleman. A somehow different rationale for avoiding direct communication between top-management and agents at lower levels of the hierarchy is provided by Friebe and Raith (1999). They show how the chain of command may be an effective way of securing the incentives for superiors to recruit the best possible subordinates.
communication between agent and principal on projects 1, 2 and 3. Indeed, for $b - b_T < 1/3$, the intermediary rubber stamps proposals to implement $y_1, y_2$ or $y_3$.\footnote{If there is no intermediary, one can show that for any convex loss-function $\ell(x)$, the principal optimally delegates authority for $1/3 < b \leq 5/12$. On the other hand, given $\ell(x)$, there exists some $\varepsilon > 0$, such that for $b > 2/3 - \varepsilon$, the principal optimally keeps authority.}

Let us now consider the leading example of CS, where the principal has quadratic preferences, $U_P(y, m) = U_P(m, m) - (y - m)^2$, $F(m)$ is uniformly distributed on $[0, 1]$ and there is a continuum of projects. The agent and the intermediary prefer respectively the projects $y = m + b$ and $y = m + (1 - k)b$, where $0 \leq k \leq 1$. Their utility function exhibits the general properties we have been assuming throughout the paper. Denoting by $\sigma^2_m(kb)$ the residual variance of $m$ the intermediary expects to have after hearing the equilibrium signal of the agent, it is easy to verify that delegation to the intermediary yields

$$EU_P = U_P(m, m) - \sigma^2_m(kb) - (1 - k)^2b^2$$

This expression reflects the fact that quadratic loss equals variance plus the square of the bias and that the rational expectations character of the Bayesian Nash equilibrium eliminates all unconditional bias from the middle man’s interpretation of the agent’s signal. Solving for the communication equilibrium between agent and intermediary, we find\footnote{See CS, section 4, for computations.}

$$\sigma^2_m(kb) = \frac{1}{12N^2} + \frac{(kb)^2(N^2 - 1)}{3}$$

where $N = N(kb)$ is the largest number of partition elements given their discongruence $b - b_T = kb$. Keeping authority is for the principal equivalent to delegating to an intermediary with bias $b_T = 0$ ($k = 1$) and thus yields

$$U_P(m, m) - \sigma^2_m(b) = U_P(m, m) - 1/12N^2 - b^2(N^2 - 1)/3$$

while full delegation, equivalent to $b_T = b$ (or $k = 0$), results in an expected utility $U_P(m, m) - b^2$. It follows that the principal prefers delegation to the agent to no delegation if and only if $b < 1/2\sqrt{3}$. Since for $b \geq 1/4$, $N(b) = 1$, this confirms proposition 2: no communication occurs in equilibrium. Delegation to an intermediary with 'intermediate' preferences, however, often does better than both full delegation and no delegation. One can verify that whenever $1/2\sqrt{6} < b < \sqrt{2}/4$, the principal obtains a higher expected utility by delegating decision rights to an intermediary with preferences $b_T = b/2$.\footnote{If there is no intermediary, one can show that for any convex loss-function $\ell(x)$, the principal optimally delegates authority for $1/3 < b \leq 5/12$. On the other hand, given $\ell(x)$, there exists some $\varepsilon > 0$, such that for $b > 2/3 - \varepsilon$, the principal optimally keeps authority.}
Indeed, from (7) and (8), she then gets\(^{36}\)

\[ EU_P = U_P(m, m) - 1/48 - \frac{b^2}{2} \]

The following proposition shows how the principal optimally delegates authority to an intermediary:

**Proposition 6** If \( F(m) \) is uniformly distributed on \([0, 1]\) and the principal has quadratic preferences, that is \( \ell(y - m) = (y - m)^2 \), then

- For \( b > \sqrt{2}/4 \), there is neither delegation nor communication.

- For \( b \in \left[ 1/6, \sqrt{2}/4 \right] \), the principal delegates authority to an intermediary with bias \( b_I = (1 - k^*)b \), where \( 1 - k^* \) is nonincreasing in \( b \): \( 1 - k^* \) equals \( \frac{3}{2} \) for the largest part of the interval, that is for \( b > \frac{1}{6} \sqrt{\frac{11}{6}} \), but tends to 1 when \( b \) comes closer to 1/6, that is \( \lim_{b \to 1/6} (1 - k^*) = 1 \).

More precisely, we have \( k^* = \frac{3}{q^2 + 2} \) where \( q \) is the highest integer smaller than

\[
\frac{1}{2} + \frac{1}{2} \sqrt{\frac{36b^2 + 7 + 4\sqrt{6\sqrt{36b^2 - 1/3}}}{36b^2 - 1}} \quad (9)
\]

There is always communication between the intermediary and the agent, with the number of partition elements being equal to \( q > 1 \).

- For \( b \leq 1/6 \), the principal delegates authority to the agent.

**Proof.** See appendix □

The next figure illustrates how \( k^* \) varies as a function of \( b \), with \( 1 - k^* = 1 \) being full delegation and \( 1 - k^* = 0 \), no delegation:

**FIGURE 2**

The following table compares the optimal allocation of authority with and without intermediary. Interestingly, the presence of an intermediary may reduce as well as increase the real authority of the agent: for \( 1/6 < b < \sqrt{3}/6 \), the agent loses control over the implemented action, while his impact on decisions increases whenever \( \sqrt{3}/6 < b < \sqrt{2}/4 \):

\(^{36}\)For \( \frac{1}{4\sqrt{6}} < b < \frac{\sqrt{2}}{8} \), \( N(b) \) equals 2.
TABLE 1

That an intermediary is only useful for intermediate preferences is intuitive. When preferences are close, the potential benefits of limiting the loss of control are small. Indeed, for $b$ small, the upper bound $L - b$, which is optimally set on the discretion of the agent, is large and the range of states for which this upper bound is binding, $m \in [L - 2b, L]$, is very limited. In contrast, the costs of an intermediary in terms of loss of information on projects smaller than $L - b$ are very large; from section 4.2, we know that in particular for small discongruences, the partition is very coarse relative to the bias. When the agent has an intermediate bias, on the other hand, communication performs not so badly anymore relative to delegation while the potential benefits of limiting the projects to be smaller than $L - b$ become substantial.

In sharp contrast with corollary 1, these results show how communication may play an important role in organizations: in the leading example of $CS$, the region in which authority is delegated to an intermediary and thus information is communicated, is larger than the one in which there is full delegation. At first sight, this suggests that middlemen may be a very useful organizational tool to foster the aggregation of information and nevertheless limit the real authority of agents. Some caution is warranted, though: while in our four projects example, the intermediary loses no information concerning the three smallest projects, and thus strictly prefers to keep authority, it follows from proposition 2 that the intermediary has an incentive to delegate authority to the agent in the leading example of $CS$. The principal may try to avoid this by defining clearly the decision rights and by delegating authority but not the right to delegate this authority.\footnote{She can, for instance, specify that only the intermediary is allowed to contract with third parties on behalf of the organization, she can make a signature of the intermediary obligatory for intra-firm transactions to be valid,...} In a dynamic context, however, the intermediary can still build a reputation towards his subordinates of rubber-stamping their recommendations. After a period of time, the principal may therefore end up with an intermediary who internalizes too much the interests of his agents.

6 Limits to Delegation

While communication may occur between an intermediary and the agent, we have argued up to now that direct communication between agent an principal is rather inefficient. In this section, we point out the limits of this result by
looking at specific information structures and by changing the nature of the information. We show how the principal is sometimes better off relying on the information communicated by the agent than delegating decision rights to the latter.

6.1 Single-peaked Distributions

For a given support \([L, -L]\), the uniform distribution is the limit case of how agnostic the principal can be about which action she optimally should take. It is therefore intuitive that particularly in this case, delegation of authority does very well. If, on the other hand, the principal has quite a precise idea of what is the optimal thing to do in most states, though in some unlikely cases a totally different course of action is required, keeping authority is at first sight not such a bad idea. While delegating authority to the agent always leads to a substantial bias, by keeping authority, the principal can take an action which is mostly less off the mark than the ones taken by the agent, while count on the agent to ring the alarm-bell in the unlikely case a very 'low' action is optimal.

The idea is readily demonstrated with an extension of our three projects example. Suppose that \(F(m)\) is symmetric and single-peaked with support on \([-1, 1]\), implying that \(y_2 = 0\) is more likely to be preferred by an informed principal than \(y_1 = -2/3\) or \(y_3 = 2/3\). For \(b\) close to \(2/3\), communication is then still possible, though limited: in the most informative equilibrium, the agent recommends a 'high' project if he prefers \(y_2\) or \(y_3\), that is if \(m > -1/3 - b\), and a 'low' project if \(m < -1/3 - b\).\(^{38}\) Nevertheless, for \(b\) close enough to \(2/3\), the principal prefers this poor communication to delegation. If she keeps control, she initiates \(y_2\) for \(m > 1/3 - b\), whereas the agent would have initiated \(y_3\). Given that \(b\) is sufficiently close to \(2/3\) and \(\Pr(m \in (-1/3, 1/3)) > \Pr(m \in (1/3, 1))\), the probability that the optimal project is implemented is then higher than under delegation. For \(m < 1/3 - b\), delegation and communication yield the same outcome.

We now generalize this intuition for a continuum of projects. Denote by \(\delta\) the total probability weight in the second and third quartiles of the distribution of \(F(m)\):

\[
\delta = F\left(\frac{L}{2}\right) - F\left(-\frac{L}{2}\right),
\]

The following proposition tells us that if the distribution \(F(m)\) is single-peaked and symmetric, and \(\delta\) is sufficiently high, then there exists a range of

\(^{38}\)Hence under communication, the principal implements \(y_1 = -2/3\) whenever \(m < -1/3 - b\) and \(y_3 = 0\) if \(m > -1/3 - b\). Under delegation, on the other hand, \(y_3 = 2/3\) is implemented instead of \(y_2 = 0\) whenever \(m > 1/3 - b\).
parameter values of $b$ for which informative communication occurs in equilibrium:

**Proposition 7** Suppose $F(m)$ is a single-peaked symmetric distribution, then for $\delta > \delta'$, with $1/2 < \delta' < 1$ the unique positive solution of

$$\left[1 - \frac{\delta'}{2}\right]\ell\left(\frac{L}{2}\right) - \frac{\delta'}{2}\ell\left(\frac{L}{4}\right) = (1 - \delta')\frac{2}{L}\int_{L/2}^{L} \ell(m) dm$$

(10)

there exists a $\bar{b} < L/2$ such that for $b \in \left[\bar{b}, \frac{L}{2}\right]$, it is optimal for the principal to keep control and there is informative communication.

**Proof.** See Appendix ■

**Example 1:** Assume a quadratic loss function, $\ell(y - m) \equiv (y - m)^2$, then $\delta' = \frac{32}{47}$. Proposition 7 thus tells that if more than 78% of the probability weight is on $\left[-\frac{L}{2}, \frac{L}{2}\right]$, there exists some $\bar{b}$ such that for $b \in \left[\bar{b}, \frac{L}{2}\right]$, the principal keeps authority and informative communication is possible.

**Example 2:** Suppose the loss function is quadratic and $m$ has a truncated normal distribution, $m \sim \mathcal{N}(0, \sigma)/c^\sigma$, supported on $[-4\sigma, 4\sigma]$.$^{39}$ Since $\delta = \int_{-4\sigma}^{4\sigma} dF(m) \cong 0.95 > \frac{32}{47}$, it follows from the previous example that informative communication occurs in equilibrium for some parameter values of $b$. Indeed, numerical simulations for $\sigma = 0.5, \sigma = 1$ and $\sigma = 2$, show that if $0.9545 * \sigma < b < 2\sigma$, the principal keeps authority and a communication equilibrium with two partition elements exist. While this parameter range is fairly large, communication is not very informative: in more than 98.6% of the states of nature, the agent recommends the highest project. Hence, this highest project equals approximately the expected value of $m$, $y_2 \cong E(m) = 0$. Whenever a 'better' communication is possible, the principal prefers to delegate authority.

### 6.2 Hard Information

So far, our analysis has focused on the communication of soft information. As a motivation for this assumption we invoked that the profitability of projects often depends on a wide range of factors, which the principal may find very hard to foresee, of whom she may not understand their relevance and significance, let alone verify their magnitude or nature. Hence, the agent can justify almost any decision by picking the right arguments.

In some settings, however, it is realistic to assume that the agent can somehow 'prove' or 'certify' his local knowledge and that the principal has

$^{39}$Hence $c^\sigma = \int_{-4\sigma}^{4\sigma} d\mathcal{N}(0, \sigma) \approx 0.99994$. 

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the time and the background to assert this information. In very stable environments, for example, it may pay off for the principal to let the agent produce numerous reports and to study these reports extensively. Similarly, in core activities, the principal sometimes has sufficient background knowledge to quickly know whether information matters and to verify roughly its content.

In order to illustrate the limits of our results, this section takes the polar assumption to soft information and assumes that in any state of nature, the agent can certify his information instantaneously and at no cost for him or the principal. This brings us close to the literature on disclosure, which typically assumes that a person, although restricted to truthful announcements of the state of nature, can nevertheless withhold his information completely or partially.\footnote{Particularly close Crawford-Sobel’s and hence ours model is Milgrom and Roberts (1986).} In our framework it seems unnatural to restrict the messages of the agent to contain only verifiable information. We assume therefore that the principal has the choice between producing a verifiable ‘hard’ report or unverifiable ‘soft’ report, where the latter can be distinguished from a ‘hard’ report by the principal but not by a third party.\footnote{Hence, the agent cannot be penalized contractually for producing a soft report.} We find the standard result of the disclosure literature that all information is revealed in equilibrium:\footnote{An exception is Okuno-Fujimura, Postlewaite and Suzumura (1990), which shows that the full disclosure result crucially depends on the assumption that the agent can certify his information in any state of nature.}

**Proposition 8** Suppose that the principal knows that the agent can make his information hard, then for any distribution $F(m)$, there exists a unique equilibrium in which the agent reveals all his information.

**Proof.** See Appendix \[\square\]

**Corollary 4** If the agent can make his information hard, the principal never delegates authority.

Intuitively, if information is soft and the principal keeps authority, the Bayesian character of the communication equilibrium completely destroys the real authority of the agent: the actions finally taken by the principal are on average unbiased relative to what is optimal to her. Since his preferences are concave over the action space (larger deviations from the optimal action are increasingly more costly), the agent would thus strictly prefer to reveal correctly his information to the principal. Proposition 8 shows that when the agent is given the option to do so, this is the unique equilibrium.
For a *given allocation of authority*, being able to make his information hard is thus a blessing for the agent, since anyway he cannot systematically bias the decision of the principal and communication of soft information involves a lot of noisy signalling. When the *allocation of authority is endogenous*, however, the possibility of 'certification' tends to be a curse for the agent: if the information were soft, the principal often would have delegated authority to the agent, in which case he could have gotten exactly his way. This provides a last twist: if the agent cannot certify his information instantaneously but only gradually, if there is a 'deadline' for the decision and if the principal can delegate decision rights quickly, then there may exist a 'bad' equilibrium in which the agent always waits to certify his information until it is too late and hence the principal still delegates authority.\textsuperscript{43}

7 Concluding Remarks

This paper has stressed the limits of the use of soft information in organizations. To the extent that a senior manager cannot verify the claims of a better informed subordinate, she is in general better off delegating decision rights to this subordinate than relying on the information she can induce from his claims. Intuitively, while the subordinate may not tell her what she should do, he will use all his information when he himself takes the decision.

This simple result has potentially important implications for the design of organizations. It suggests that centralization of authority is only optimal if top-management has the information which is important to the main decisions, or is able to check and verify the information provided by lower levels of the hierarchy. At first sight, this is in line with the tendency of firms to focus on core activities, i.e. activities on which they have a profound knowledge, and to outsource other activities. Similarly, the trend of the last two decades towards more decentralization and empowerment, highlighted by the business press, may find its origin in a rapidly changing business environment which causes the knowledge of top-management to become quickly obsolete. In response to an increased foreign competition in the 1980’s and 1990’s, for instance, many large American cooperations (ITT, IBM, General Motors, Eastman Kodak, and Xerox,...) have changed the organizational design of their organization and frequently pushed decision rights lower in the organization.\textsuperscript{44} An incentive based theory of communication costs in hierarchies,

\textsuperscript{43}There is also a 'good' equilibrium in which the principal always believes that $m = -L$ if the agent waits to certify his information and the agent always reveals his information.

\textsuperscript{44}A discussion of some case studies on this can be found, e.g., in Brickley, Smith and Zimmerman (1997).
where these costs arise endogenously from the necessity to check and understand reports provided by agents prone to distort their information, will be needed to throw further light on these issues.

8 Appendix

8.1 The rationale for delegation

The proof of proposition 3 and 4 is provided in an attachment to this paper and can be downloaded on http://homepages.ulb.ac.be/~wdessein.

8.2 Restricted Delegation

Proof of proposition 5

The proof has three steps:
- Step 1: We first show that the actions upon which the agent has discretion must form a continuum. Suppose the agent has discretion to implement two actions \( y_i \) and \( y_{i+1} \), with \( y_{i+1} - y_i > \varepsilon > 0 \) and \( \varepsilon \) strictly positive, but is not allowed to implement any action \( y \in [y_i, y_{i+1}] \) (though of course he could be allowed to implement other actions \( y \notin [y_i, y_{i+1}] \)). Giving the agent additional discretion on \([y_i, y_{i+1}]\) affects the implemented action if and only if \( m \in ]y_i - b, y_{i+1} - b[\). Suppose first that \( y_{i+1} - y_i \leq 2b \). Without the additional discretion, the agent then recommends \( y_i \) if \( m \in \left[ y_{i+1} - b, \frac{y_i + y_{i+1} - b}{2}, y_{i+1} - b \right], \) resulting in a bias \( |y_i - m| < b \). On the other hand, if \( m \in \left[ \frac{y_i + y_{i+1} - b}{2}, y_{i+1} - b \right], \)

\( y_{i+1} \) is recommended, resulting in a bias \( |y_{i+1} - m| > b \). Adding up both effects results in an average bias \( b \). In contrast, if the agent could choose any action \( y \in [y_i, y_{i+1}], \) then for \( m \in [y_i - b, y_{i+1} - b[\), the bias in the action taken would be always \( b \). Given the concave preferences of the principal, the lack of discretion thus decreases the expected utility of the principal. If \( y_{i+1} - y_i > 2b \), one can verify that for \( m \in ]y_i - b, y_{i+1} - b[\), the absolute deviation from \( m \) is on average even larger than \( b \).

- Step 2: Secondly, we show the principal never strictly prefers to put a lower bound on the actions the agent can implement. Suppose the agent is not allowed to implement any action \( y < y_i \). Giving additional discretion to the agent on \( y \in [\infty, y_i] \) would affect the implemented action if and only if \( m \in [-L, y_i - b]. \) For this subset of \( M \), the agent then chooses \( y = m + b \) instead of \( y_i \). As for \( m \in [-L, y_i - b], \) \( \ell(b) \leq \ell(y_i - m), \) the principal is always weakly better off by allowing this discretion.

- Step 3: The latter two observations imply that the optimal mechanism consists of putting a (possibly infinite) upper bound on the actions the agent may implement. We conclude by showing that his upper bound is given by \( x = L - b. \) Obviously, \( x \leq L \) if the agent is allowed to implement any action
\[ y < x, \text{ where } x - b > -L \text{ and } x \leq L, \text{ the principal’s expected utility equals} \]
\[
U_P = U_P(m, m) - \frac{1}{2L} \left[ \int_{-L}^{x-b} \ell(b) d(m) + \int_{x-b}^{L} \ell(x-m) dm \right] \\
= U_P(m, m) - \frac{1}{2L} \left[ (L + x - b) \ell(b) + \int_{x-b}^{L} \ell(x-m) dm \right]
\]
Taking the first order conditions with respect to \( x \) yields
\[
\ell(b) - \left[ \ell(x-m) \right]_{x-b}^{L} - \ell(b) = 0 \Leftrightarrow \ell(x - L) = \ell(b) \Leftrightarrow x = L - b
\]
As indicated, the latter solution supposes that \( x - b > -L \), which will only be true if \( b < L \). Whenever \( b \geq L \), it is easy to verify that the principal optimally limits the implementable actions to \( y' = 0 \). Since the agent then always chooses \( y' = 0 \), this is economically equivalent to a situation in which the principal keeps full decision rights and no transmission of information occurs. \( \blacksquare \)

### 8.3 Delegation to an intermediary

**Proof.** A principal delegates authority to an intermediary with bias \( b_I = (1 - k)b \). Substituting (8) in (7), we can rewrite the expected utility of the principal as
\[
EU_P(k, b) = U_P(m, m) - \frac{1}{12N^2} - \frac{k^2b^2(N^2 - 1)}{3} - (1 - k)^2b^2 \tag{11}
\]
where \( N \) is the number of partition elements in the most informative communication equilibrium between agent and intermediary. From lemma 1, \( N \) is given by
\[
N = N(kb) \equiv \left\lfloor -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2}{kb}} \right\rfloor, \tag{12}
\]
where \( \lfloor z \rfloor \) denotes the smallest integer greater than or equal to \( z \). Let us denote by \( b^*_I \equiv [1 - k^*(b)] b \) the bias of the intermediary which maximizes (11), and by \( N^* \equiv N(k^*b) \) the number of partition elements in the corresponding communication equilibrium. Define further \( k(q) \) as
\[
k(q) \equiv \frac{3}{q^2 + 2}
\]
A) We first show that for \( b \in \left[ \frac{1}{6}, \frac{\sqrt{7}}{4} \right] \), \( k^* = k(q) \), where \( q \) is the largest integer smaller than (9) and \( q \) is also the number of partitions in the corresponding communication equilibrium:
Suppose that \( N^* = q \) and that given \( k = k(q) \), an equilibrium with \( q \) partition elements exists, i.e. \( N(k(q)b) \geq q \). From (11), then \( k^* = k(q) \), which yields an expected utility of

\[
eu_P(q, b) = U_P(m, m) - \frac{1}{12q^2} - \frac{q^2 - 1}{q^2 + 2}b^2
\]

Moreover, even if \( N(k(q)b) < q \), we always have that

\[
\forall b : EU_P(k^*(b), b) \leq eu_P(N^*(k^*(b)b), b)
\]

Define now the correspondence \( \hat{b}(q) : \)

\[
b \in \hat{b}(q) \iff eu_P(q, b) = eu_P(q - 1, b)
\]

Thus \( b \in \hat{b}(q) \) is a bias for which the principal is indifferent between delegating authority to an intermediary with bias \([1 - k(q)]b\) and bias \([1 - k(q - 1)]b\), hereby assuming that a communication equilibrium with \( q \), respectively \( q - 1 \) partition elements then effectively exists. Substituting (13) in (15) and imposing \( b \geq 0 \), we find that \( \hat{b}(q) \) is a singleton; abusing notation, we have

\[
\hat{b}(q) = \frac{1}{6} \sqrt{(q^2 + 2)(q^2 - 2q + 3)} \frac{(q - 1)q}{(q - 1)q}
\]

Moreover, from (13), if \( b = \hat{b}(q) \), then for \( b > \hat{b}(q) \), \( eu_P(q - 1, b) > eu_P(q, b) \), while for \( b < \hat{b}(q) \), \( eu_P(q - 1, b) < eu_P(q, b) \). Since \( \hat{b}(q) \) is strictly decreasing in \( q \), the following lemma thus holds

**Lemma 3** If \( b \in \left[ \hat{b}(q + 1), \hat{b}(q) \right] \), then \( \forall n \in \mathcal{N}, n \neq q : eu_P(q, b) > eu_P(n, b) \).

If for \( b \in \left[ \hat{b}(q + 1), \hat{b}(q) \right] \), a communication equilibrium with \( q \) partition elements exists, that is if

\[
\forall b \in \left[ \hat{b}(q + 1), \hat{b}(q) \right] : q \leq N\left( \frac{3}{q^2 + 2}b \right)
\]

then from lemma 3 and inequality (14), \( k^* = k(q) = \frac{3}{q^2 + 2} \).

From (12) and given that \( q \) is an integer, (16) holds if and only if

\[
q < \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2(q^2 + 2)}{3b(q)}}
\]

Substituting the value of \( \hat{b}(q) \), which tends to \( 1/6 \) if \( q \) becomes very large, one can verify that inequality (17) is satisfied for any \( q \in \mathcal{N} \).
Lemma 4 \( \forall q \in \mathcal{N}, q > 1 : b \in \left[ \hat{b}(q + 1), \hat{b}(q) \right] \implies k^* = k(q) \)

The function \( \hat{b}(q) \) is decreasing in \( q \), where \( \hat{b}(2) = \frac{\sqrt{3}}{4} \) and \( \lim_{q \to \infty} \hat{b}(q) = \frac{1}{6} \). Inverting \( \hat{b}(q) \) on \( \left[ \frac{1}{6}, \frac{\sqrt{3}}{4} \right] \) and taking into account that \( q \) must be an integer, it follows from lemma 4 that for \( b \in \left[ \frac{1}{6}, \frac{\sqrt{3}}{4} \right] \), \( k^* = k(q) \) where \( q \) is given by

\[
\frac{-1}{2} + \frac{1}{2} \sqrt{\frac{36b^2 + 7 + 4\sqrt{6}\sqrt{36b^2 - 1/3}}{36b^2 - 1}}
\]

Note finally that we necessarily have that the number of partition elements in the resulting communication game cannot be larger than this \( q \). Indeed, suppose \( N^*(k^*b) > q \). Since for a given \( b \) and \( b_l \), the principal is always better off by more informative communication between agent and intermediary, \( EU_F(k^*, b) > e_{up}(q, b) \). However, from (14) also \( e_{up}(N^*(k^*b), b) \geq EU_F(k^*, b) \). Since, by construction, \( b \in \left[ \hat{b}(q + 1), \hat{b}(q) \right] \), this in contradiction with lemma 3. Hence, \( N(k^*b) = q \).

B) To conclude the proof, we show that (i) for \( b \leq 1/6 \), \( k^* = 0 \) and (ii) for \( b > \frac{\sqrt{3}}{4} \), \( k^* = 1 \).

(i) One can easily verify that for \( b \leq 1/6 \), \( \frac{1}{12q^2} + \frac{q^2 - 1}{q^2 + 2}b > b^2 \) as long as \( q \) is a finite number. Hence, from (13) and (14), the principal optimally delegates to full authority to the agent (or \( k^* = 0 \)) for \( b < 1/6 \) in which case she obtains a utility \(-b^2\).

(ii) One can easily verify that for \( b > \sqrt{3}/4 \), \( \frac{1}{12q^2} + \frac{q^2 - 1}{q^2 + 2}b^2 > 1/12 \) for any \( q \in \mathcal{N} \). Hence, from (13) and (14), the principal optimally keeps authority (or \( k^* = 1 \)) for \( b > \sqrt{3}/4 \), in which case she obtains an expected utility of \(-1/12\).

8.4 Single-peaked distributions

Proof of proposition 7:
A sufficient condition for informative communication to occur in equilibrium is that (i) \( N(b) > 1 \) and (ii) the babbling equilibrium yields a higher utility than delegation of authority to the agent.

(i) Given that \( F(m) \) is symmetric and single-peaked, \( N(b) > 1 \) if and only if \( b \leq \frac{L}{2} \): For \( b \leq L/2 \), it is easy to see that one can always find a communication equilibrium with at least two partitions, and from CS, corollary 1, the only equilibrium is the babbling equilibrium if for any \( a \in [-L, L] \)

\[
U_A(\bar{y}(a, L), a, b) > U_A(\bar{y}(-L, a), a, b)
\]

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For $b > L/2$, this condition is satisfied.

(ii) A lower bound on the utility of the principal in any communication equilibrium is given by the utility she gets in a babbling equilibrium. As $F(m)$ is a single-peaked and symmetric, the principal then initiates $y = 0$. Hence if $\ell(L/2) > E[\ell(m)]$, the principal strictly prefers the babbling equilibrium to delegation for $b = L/2$. We now derive sufficient conditions for $E[\ell(L/2) - \ell(m)]$ to be strictly positive:

(a) Conditional on $m \in [-b, b]$, we have

$$E[\ell(b) - \ell(m) \mid m \in [-b, b]] = 2 \int_0^b [\ell(b) - \ell(m)] f(m) dm$$

$$> 2 \int_0^b [\ell(b) - \ell(m)] f(m) dm > 2 \int_0^{b \over 2} \left[ \ell(b) - \ell(\frac{b}{2}) \right] f(m) dm$$

$$= \left[ \ell(b) - \ell(\frac{b}{2}) \right] \left[ 1 - 2F(-b/2) \right]$$

Since $F(m)$ is symmetric and single peaked, $1 - 2F(-L/4) \geq \delta/2$ where $\delta \equiv F(\frac{b}{2}) - F(-\frac{b}{2}) = 1 - 2F(-\frac{b}{2})$. Hence

$$E[\ell(L/2) - \ell(m) \mid m \in [-L/2, L/2]] > [\ell(L/2) - \ell(L/4)] \frac{\delta}{2}$$

(b) Suppose $b = L/2$, conditional on $m \in [-L, L] \setminus [-b, +b]$, and given that $F(m)$ is single peaked then

$$E[\ell(b) - \ell(m) \mid m \in [-L, -b] \cap [b, L]] = 2E[\ell(b) - \ell(m) \mid m \in [b, L]]$$

$$\geq 2 \int_b^L (\ell(b) - \ell(m)) \frac{1 - \delta}{2(L - b)} dm = (1 - \delta) \left[ \ell(b) - \frac{1}{\tau - \delta} \int_b^L \ell(m) dm \right]$$

From (a) and (b),

$$E \left[ \ell(L/2) - \ell(m) \right] > \delta \over 2 \left[ \ell(L/2) - \ell(L/4) \right] + (1 - \delta) \left[ \ell(L/2) - \frac{\delta}{\tau} \int_{L/2}^L \ell(m) dm \right]$$

The principal strictly prefers to keep authority for $b = L/2$ if the RHS of (18) is non-negative. Define by $\delta'$ the solution of

$$\left[ 1 - \frac{\delta'}{2} \right] \ell(L/2) - \frac{\delta'}{2} \ell(L/4) = (1 - \delta') \frac{\delta}{\tau} \int_{L/2}^L \ell(m) dm$$

Since the RHS of (18) is strictly negative for $\delta = 0$, strictly positive for $\delta = 1$ and strictly increasing in $\delta$, $\delta'$ is uniquely defined and $0 < \delta' < 1$. Hence, as (18) is a strict inequality, for $b = L/2$ and $\delta \geq \delta'$ the principal strictly prefers the babbling equilibrium to delegation. By continuity, there is then a $\delta' < \delta$, such that for $b \in \left( \delta', \frac{L}{2} \right)$, the principal keeps authority, and, from (i), communication occurs in equilibrium. ■
8.5 Hard Information

Proof of proposition 8:
Suppose \( \{y_1, ..., y_n\} \) are the actions which are implemented with a positive probability in the communication equilibrium which prevails when an agent does not produce a hard report. Define \( a_o = -L, a_n = L \) and \( a_i \in \{a_1, a_2, ..., a_{n-1}\} \) to be the states of nature for which the agent is indifferent between \( y_i \) and \( y_{i+1} \). By definition of a communication equilibrium, for every \( y \in \{y_1, ..., y_n\} \), the agent can send a message to the principal such that \( y \) is implemented.\(^{45}\) On the other hand, the agent can also produce a hard report in which case \( y = m \) is implemented. If follows that whenever \( m \in \{a_{i-1}, a_i\} \), \( y_i \) will be implemented in equilibrium if

\[
|m + b - y_i| < b
\]

whereas the agent will certify his information and \( y = m \) will be implemented if \( |m + b - y_i| > b \). Hence, in a Bayesian equilibrium, \( y_i \) maximizes the principal’s expected utility conditional on \( m \in \{a_{i-1}, a_i\} \cap [y_i - 2b, y_i] \), which is only possible if \( y_i = a_{i-1} \). Let \( y_{i-1} \) and \( y_i \) be two subsequent actions which are implementable whenever the agent does not produce hard report, then \( y_k = a_{i-1} = (y_{i-1} + y_i)/2 \), from which \( y_{i-1} = y_i \). There is thus only one action \( y_i \) implementable in the strategic communication subgame, where \( y_i = a_0 = -L \). Since the agent strictly prefers \( y = m \) to \( y_i = -L \) whenever \( m > -L \), and is indifferent if \( m = -L \), it is a dominant strategy for the agent to make his information hard, where the dominance is strictly whenever \( m > -L \). In equilibrium, all information is then revealed to the principal. 

References


\(^{45}\)A mixed strategy cannot be optimal for the principal, since for any conditional distribution of \( m \), there is a unique action \( y \) which maximizes the expected utility of the agent.


[34] Rabin, Matthew, ”Information and the Control of Productive Assets”, *Journal of Law, Economics and Organizations*, 9, April 1993, 51-76.


FIGURE 2:

TABLE 1:

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