Optimal Bank Runs Without Self-Fulfilling Prophecies

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First draft

Abstract

This paper extends the standard Diamond-Dybvig model for a general equilibrium in which depositors make their withdrawal decisions sequentially and banks strategically choose their contracts. There is a unique Subgame Perfect Nash Equilibrium (SPNE) in the decentralized economy. Bank runs can occur when depositors perceive a low return on bank assets. When information is imperfect, bank runs can happen even when the economy is in a good state. A representative bank can earn positive profits in equilibrium due to the sequential service constraint. When there are several risky projects available, the high-risk technology may be chosen as a socially efficient solution.

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1 Introduction

Bank runs have always been of great public concern. During a bank run, depositors rush to withdraw their deposits forcing banks to fail even when they would otherwise be solvent. Because banks issue liquid liabilities but invest in illiquid assets, they have always been vulnerable to panics. In the 1980s and 1990s, 73% of the IMF's member countries suffered some form of banking crisis (see Lindgren, Garcia & Saal 1996). Recently, this phenomenon has attracted more attention because it is thought to have played an important role in the balance-of-payment (BOP) crises that hit Mexico and Argentina in 1994, Southeast Asia in 1997, Russia in 1998, and more recently Brazil and Ecuador. Kaminsky and Reinhart (1996), in fact, refer to the occurrence of both banking crises and BOP crises as the "twin crises" phenomenon. In their study, they find that 56 percent of banking crises were followed by a BOP crises within three years during the 1980s and 1990s. This evidence suggests, therefore, that a good understanding of the determinants of bank runs is not only important for bank management, but also crucial for explaining the twin crises phenomenon.

It is important to distinguish between two types of bank runs. A type-I bank run occurs when a solvent bank is forced to go into bankruptcy due to liquidity reasons. A type-II bank run, on the contrary, happens when a bank is insolvent. There are two differences between these two types of bank runs. First, in a type-I bank run the financial intermediaries are solvent but illiquid, while in a type-II bank run, we observe both illiquidity and insolvency. Thus, in a type-I bank run, there is no fundamental problem, the depositors rush to withdraw their money for non-economic reasons, such as pessimistic expectations or herd behavior. Second, a type-I bank run is a suboptimal phenomenon, while a type-II bank run is an efficient outcome in a market economy. This paper provides a framework in which both types of bank runs are possible when all agents rationally choose their behaviors.

Our starting point is the framework developed in the insightful paper of Diamond and Dybvig (1983). They present a benchmark model in which bank runs are self-fulfilling prophecies. Demand deposit contracts offered by banks provide a risk-sharing mechanism to risk-averse agents. In their simultaneous game there exist multiple-equilibria due to the illiquidity problem of banks. On the one hand, if no one expects that bank runs will happen, only "impatient" agents will withdraw their deposits. On the other hand, if all depositors anticipate a bank run, then they all have the incentive to withdraw immediately. Which one of these two equilibria occurs is determined exogenously, say, by the "sunspot."

The Diamond-Dybvig (D-D) model is very attractive in that it can explain both types of bank runs in a simple way; however, there are two major problems. First, the property that banking crises are self-fulfilling is quite controversial. Gorton (1988), Calomiris and Gorton (1991), Corsetti, Pesenti & Roubini (1998) conduct a broad range of empirical studies and conclude that the data do not support the "sunspot" view that banking crises are random events; instead, the empirical evidence suggests that bank runs are intimately related to the state of the business cycle. Second, the D-D framework is only a partial equilibrium analysis in that it does not study the impact of bank runs on the behavior of banks, either in terms of the optimal deposit contracts or their investment portfolios. It is unclear why banks are willing to offer the demand deposit contract from the beginning if bank runs could happen. Under a general equilibrium framework, it is important to consider how banks will redesign their deposit contracts and liquidity structure in response to the possibility of bank runs. Accordingly, numerous papers tried to resolve these two problems.

Most of the literature that follows the D-D study aims to unveil the "fundamentals" that trigger bank runs. Chari and Jagannathan (1988) provide a signal-extracting story. In their model, liquidity needs are uncertain and agents have asymmetric information on asset returns. Uninformed agents observe total early withdrawal and try to figure out what information the informed agents have. Chari and Jagannathan show that bank runs can happen even when no one has adverse information because uninformed agents will incorrectly infer that the future return is low when liquidity-based withdrawals turn out to be unusually high. Jacklin and Bhattacharya (1988), and Alonso (1996) study the socially optimal allocation when agents get asymmetric information on future returns in the interim period. They find that when future returns are too volatile, the social planner may choose a bank-run contract over a run-proof alternative, in which bank runs happen when the informed agents receive a bad interim signal. Allen and Gale (1998) develop a model that is consistent with the business cycle view of the origins of banking panics. In their model, they assume that bank runs occur only when they are unavoidable.[1] As a result, a bank panic occurs only when the returns of bank assets are going to be low. Bank runs are inevitable results of the standard deposit contract and can be first-best efficient in a world with aggregate uncertainty.

The other branch of research tries to combine the Diamond-Dybvig analysis with optimal contract theory. Cooper and Ross (1991, 1998) consider how banks respond to the possibility of runs in their investment decisions, particularly through the holding of excess liquidity. They prove that whether banks choose a run-preventing or bank-run allocation depends on the probability of bank runs. Chang and Velasco (1998, 1999) develop a simple model in which banks take the possibility of self-fulfilling runs into account when choosing their external debt structure and interest rates. They also show that if the probability of a run is sufficiently small, banks can deliberately choose an illiquid asset-liability position and expose themselves to a run. However, in all these papers, a bank run features to be one of the many possible equilibria. Since there is no good model available to resolve the equilibrium-selection problem, the probability of the bank-run equilibrium, which is crucial to each study, has to be assumed to be exogenously given — a sunspot equilibrium.

This paper extends the existing literature and resolves both problems. By assuming that agents sequentially choose their withdrawal decisions, this model avoids the equilibrium selection problem and yields a unique Subgame Perfect Nash Equilibrium (SPNE). Based on this result, a general equilibrium study explains how banks strategically choose their demand deposit contracts and investment decisions in a decentralized economy. In equilibrium, bank runs are only possible when agents receive a bad signal on the future returns of bank assets. When the signal is imperfect, both types of bank run are possible.

Another contribution of this paper is that it offers explanations of why high-risk investment projects are chosen in favor of low-risk alternatives.[2] The most widely cited reason is the moral hazard problem caused by a deposit insurance program (Cooper and Ross 1988,

[1]There still exists the multiple-equilibrium phenomenon in Allen and Gale's model, but they assume that if there are multiple equilibria, the equilibrium without runs will always be chosen. This assumption is the only reason why type-I bank runs are impossible in their model. In contrast, this paper eliminates the possibility of multiple-equilibria.

[2]This phenomenon is sometimes referred to as "over-investment."

Krugman 1998). Deposit insurance reduces investors' incentives to monitor banks' behavior, thus encouraging the bankers to "bet" more aggressively. Over-investment in this situation is suboptimal. In this paper, we show that the over-investment phenomenon can happen when there is no deposit insurance. When banks do not commit to their investment decisions,[3] banks always have incentive to over-invest as a result of the principal-agent problem. Even when a commitment mechanism exists, over-investment is still possible. The reason is as follows: Because banks are liquidated when the future return is low, the actual return of the asset follows a truncated distribution. Since a more volatile asset has a "fatter" tail in the high-return region, it has a higher actual return and can bring a higher payoff for depositors. Therefore, the high-risk investment can be a more efficient choice for the economy.

The paper is organized as follows. Section 2 describes the setup of the model. Section 3 analyzes the equilibrium withdrawal decisions for agents under a given demand deposit contract. Section 4 studies how banks respond to the possibility of bank runs in their choice of deposit contracts and investment structure. Section 5 discusses the properties of the equilibrium contract under a decentralized economy. Section 6 shows how banks choose from different risky projects, and section 7 provides concluding remarks.

2 Model setup

The basic framework is the Diamond and Dybvig (1983) model with aggregate uncertainty, but our model differs in that withdrawal decisions are made sequentially, and the banks play an active role by choosing their interest rate structures and investment portfolios. Under these assumptions, it can be shown that bank runs are possible in the unique equilibrium.

The model has three periods ($T = 0, 1, 2$), two types of assets, and two kinds of players: private agents and banks.

- Investment technologies

  There are two available investment technologies. One is *storage technology*, which

[3] Or if depositors are not able to monitor banks' investment behavior.

produces a certain return of 1 in period 1 or 2. The other is *risky investment*,[4] which provides a random return of $\tilde{R}$ in period 2. For simplicity, we assume $\tilde{R}$ follows a binomial distribution:

$$\tilde{R} = \begin{cases} R_H & \text{with probability } \pi \\ R_L & \text{with probability } 1-\pi \end{cases}$$

where $R_L < R_H$ and $0 < \pi < 1$.

The risky investment is more efficient in the long run[5] but less preferable in the short run because it is illiquid. In particular, the liquidation value in period 1 is $1-\tau$, where τ is the liquidation cost.[6,7]

Following the D-D model, I also assume that only financial intermediary (banks) can invest in the risky technology, but both banks and individual agents have access to the storage technology. This assumption allows a patient agent to withdraw his deposit earlier and carry it through for future consumption.

- Banks

This paper extends the standard D-D model by allowing the banks to behave strategically in choosing their deposit contracts and investment portfolios. In period 0, banks compete with each other by offering demand deposit contracts which specify a short-run interest rate, r_1, and a long-run interest rate, r_2. After receiving deposits, each bank chooses its optimal portfolio structure (allocation between liquid and illiquid assets).

[4] Throughout this paper, the *storage technology* is equivalent to *riskless* or *liquid asset*, while *risky technology* is the same as *illiquid asset*. Unless specified, *return* refers to the long-run return for risky technology.

[5] That is, a representative agent prefers $\tilde{R}$ to a certain income of 1. Obviously, a necessary condition is $E(\tilde{R}) > 1$ if agents are risk-averse.

[6] The liquidation cost was first introduced by Cooper and Ross (1991). In the D-D model, there is no liquidation cost ($\tau = 0$); therefore, the risky asset strictly dominates the riskless asset. At the other extreme, Jacklin and Bhattacharya (1988) assume $\tau = 1$.

[7] There are several papers discussing how the liquidation value is determined when a financial crisis happens. One story is provided by Krugman (1998): when a self-fulfilling crisis happens, the firms are forced to liquidate their assets early and only get a proportion of the real value. Another is developed in a more recent paper by Backus, Foresi and Wu (1999): due to a liquidity crunch and imperfect information, an idiosyncratic bank run might be contagious and banks' assets have to be liquidated at a very high cost.

The deposit contract in this paper differs from those in the existing literature in that it specifies the long-run interest rate. Most existing models assume that the long run rate is determined by evenly distributing the remaining resources among the late consumers. This assumption is innocuous if banks make zero profit in every state; however, as we see below, this is not necessarily true.[8]

As in the D-D model, I assume that the banks pay the depositors according to a "first come, first served" rule. The difference is that I assume that the sequential service rule should be observed in all periods, while in D-D model it is only valid in period 1. This difference is the main reason that banks earn positive profits in equilibrium (see section 5).

- Agents

There are N agents in the economy, where N is large but finite.[9] Each agent is endowed with one unit of good at the beginning and must decide how much to deposit in the banks after the announcement of interest rates.

Following the standard D-D framework, I assume there are two types of agents: impatient agents and patients agents. Their utility functions are given respectively by

$$u^1(c_1, c_2) = u(c_1) \tag{1}$$

and

$$u^2(c_1, c_2) = u(c_2), \tag{2}$$

where $u(\cdot)$ satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. That is, all agents are risk-averse and impatient agents derive utility only from period 1 consumption while patient agents only care about period 2 consumption.

[8] This problem is more serious when a capital requirement is imposed. Under a capital requirement, banks have to earn profits in "good" states to compensate for their capital losses in "bad" states; therefore the choice of the long-run interest rate will be more important.

[9] The finiteness of N is necessary to derive the uniqueness of the SPNE in section 3.

The type of each agent is unknown in period 0. In period 1, the type for each agent is realized. Each agent only knows his own type; however, everyone knows that a constant fraction α of individuals are impatient.

- Information

In period 1, all agents receive a public signal, s, which correctly indicates the outcome of asset returns with a certain probability. More specifically,

$$Pr\{s = R_H | \tilde{R} = R_H\} = Pr\{s = R_L | \tilde{R} = R_L\} = p \tag{3}$$

The signal is perfect when $p = 1$ and imperfect when $0 < p < 1$.

- Timing

In the beginning period, banks compete for deposits by announcing their short-term interest rates and long-term interest rates. Individual agents decide how much endowment to put in the banks. Then banks choose their investment portfolio to maximize their expected profits.

In period 1, the type for each agent is realized and the public signal s is revealed. The decision each agent makes is simple: either to wait or to withdraw immediately. I assume agents make withdrawal decisions according to a given sequence: impatient agents make their decisions first, then patient agents make their withdrawal decisions sequentially. Each agent has complete information on the decisions made by those in front of him.[10] This procedure is shown as figure 1: after the public signal is revealed, agent 1 makes his decision whether to "wait" or to "withdraw." Agent 2, after observing the decision of agent 1, chooses his strategy. Agent 3 observes the decisions by both agent 1 and agent 2, and makes his decision accordingly. This process continues for all agents.

In period 2, banks repay the late consumers until their assets run out. The sequence of payment is randomly determined.

[10] In fact, it is sufficient to assume that each agent observes the amount of withdrawal by those agents in front of him.

3 Optimal decisions of depositors

This section analyzes how individual agents choose their optimal withdrawal decisions given a certain demand deposit contract (interest rates r_1 and r_2) and banks' investment portfolios (banks invest $1 - i$ in the liquid asset and i in the illiquid asset). For simplicity, I assume agents put all their endowment in banks in period 0.

Two important features of this model are worth noting. First, since there is no asymmetric information among agents and the proportion of impatient agents is constant, there is no signal-extracting story as in Chari & Jagannathan (1988), nor information cascade or herd behavior phenomenon. Each agent's decision solely depends on the public signal s and the withdrawal history he observes. Second, the assumptions of sequential decisions and complete information on withdrawal history rule out the usual multiple equilibria result in the D-D model. It will be shown that there is a unique SPNE outcome, and bank runs are possible in some states.

Each agent tries to maximize his expected utility based on his own type and his information set. For an impatient agent, the decision is trivial: always withdraw immediately in period 1. A patient agent's decision rule is more complex. It depends on his belief about future returns and the withdrawal history information he observes.[11] Given the sequential decision rule, each agent takes into account how his choice will affect the followers' withdrawal decisions. We analyze it in the following steps.

- Belief about future returns

  Under imperfect information, each agent adjusts his belief about the distribution of future returns based on the public signal s. Define p_H as the subjective probability of a high future return when a good signal is observed, and p_L as the subjective probability of a high future return when a bad signal is observed. Using Bayesian rule,

$$p_H \equiv Pr\{\tilde{R} = R_H | s = R_H\} = \frac{p\pi}{p\pi + (1-p)(1-\pi)} \tag{4}$$

[11] As in the standard D-D model, this paper do not consider mixed strategies.

and

$$p_L \equiv Pr\{\tilde{R} = R_H | s = R_L\} = \frac{(1-p)\pi}{(1-p)\pi + p(1-\pi)}. \tag{5}$$

- Payoff function

The payoff function is endogenously determined. If an agent chooses to withdraw early, he will get r_1 as long as the bank is still solvent.[12] If the agent chooses to wait, in period 2 he will get full payment of r_2 with a certain probability, θ. The probability is determined by the asset return $\tilde{R}$ and the aggregate early withdrawals, L:

$$\theta(\tilde{R}, L) = max[0, min(\beta(\tilde{R}, L), 1)], \tag{6}$$

where $\beta(\tilde{R}, L)$ is given by

$$\beta(\tilde{R}, L) = \begin{cases} \frac{1-i+i\tilde{R}-r_1 L}{r_2(1-L)} & \text{when } r_1 L \leq 1-i \\ \\ \frac{[1-i\tau - r_1 L]\tilde{R}}{r_2(1-L)(1-\tau)} & \text{when } r_1 L > 1-i \end{cases} \tag{7}$$

Here, $1-i$ represents the proportion of assets invested in liquid technology by the bank. The first equation describes the probability that the agent is fully paid in period 2 when the liquid assets $1-i$ are sufficient to meet early withdrawals $r_1 L$. In the second equation, short-term liquidity is insufficient to meet early withdrawals and the bank has to liquidate part of its illiquid assets, leaving only $i + \frac{1-i-r_1 L}{1-\tau} = \frac{1-i\tau-r_1 L}{1-\tau}$ of the risky asset for long-term repayment.

There are four possible cases for the $\beta(\tilde{R}, L)$ curve as shown in figure 2:

- case 1: $1 - i\tau < r_1$ and $1 - i + i\tilde{R} < r_1$.
- case 2: $1 - i\tau < r_1$ and $1 - i + i\tilde{R} > r_1$.
- case 3: $1 - i\tau > r_1$ and $1 - i + i\tilde{R} < r_1$.
- case 4: $1 - i\tau > r_1$ and $1 - i + i\tilde{R} > r_1$.

[12] If the bank is insolvent, the agent always gets nothing no matter what decision he makes. I assume the patient agent will wait when he is indifferent between waiting and withdrawing.

Note $1 - i\tau$ is the total liquidity available to the bank in period 1, and $1 - i + iR$ is the total wealth to the bank in period 2. Figure 2 shows us that: (1) when the bank needs to liquidate its assets ($r_1 L > 1 - i$), $\beta(\tilde{R}, L)$ is increasing in L if $1 - i\tau > r_1$ and is decreasing in L otherwise; and (2) when the bank does not need to liquidate its assets ($r_1 L < 1 - i$), $\beta(\tilde{R}, L)$ is increasing in L if $1 - i + iR > r_1$ and is decreasing in L otherwise.

The economic intuition behind the upward-slope of $\beta(\cdot, L)$ is as follows. Consider the case where $r_1 L > 1 - i$. $1 - i\tau$ is the total liquidity available to the bank, which is also the "share" of wealth each agent will get if the bank assets are evenly distributed among all agents. If more agents withdraw their deposits when $r_1 > 1 - i\tau$, they are taking more than their share from the bank, which leaves less wealth for each late consumer. On the other hand, if $r_1 < 1 - i\tau$, the early-withdrawal agents are in fact sacrificing part of their share of wealth to the late consumers. Therefore, when more agents withdraw in period 1, it is more likely that late consumers get a full repayment.

The subjective probability of full repayment when agents observe a signal s is written as:

$$\theta(s, L) = p_s \theta(R_H, L) + (1 - p_s)\theta(R_L, L), \quad s = H, L. \tag{8}$$

- Strategies for patient agents in period 1

Given the assumptions that patient agents make withdrawal decisions sequentially and each agent has complete information of withdrawal history, the equilibrium strategy for patient agents can be derived by using backward induction.

Let us first consider the last patient agent. When he observes an aggregate earlier withdrawal of L_N, his choice between waiting and withdrawing solely depends on which action leads to a higher expected utility. Obviously, when $u(r_1) > \theta(s, L_N)u(r_2)$, he will withdraw his deposit; otherwise, he will choose to wait.

Now consider the second-to-the-last patient agent. Being a rational agent, he knows the strategy the last agent will adopt. Accordingly, his strategy should be: withdraw if $u(r_1) > \theta(s, L_{N-1})U(r_2)$ and wait otherwise.[13]

Using backward induction, it is straightforward that the patient agent i chooses the following strategy in equilibrium:

$$A_i(s, L_i) = \begin{cases} \text{withdraw} & \text{if } u(r_1) > \theta(s, L_i)u(r_2) \\ \text{wait} & \text{otherwise} \end{cases}$$

Notice that the equilibrium strategy for a patient agent depends on the withdrawal amount upon his decision (L_i) and has nothing to do with the aggregate withdrawal in period 1 (L). This result differs from the existing literature. Since the classical D-D model (1983), most economists have assumed that agents simultaneously make their withdrawal decisions in period 1, and the rational expectations equilibrium turns out to be a natural solution. As a result, each agent's decision depends on his expectation of aggregate early withdrawal. In this model, due to the assumptions of sequential decisions and complete information, a Subgame Perfect Nash Equilibrium (SPNE) is more appropriate to define the equilibrium outcome. The equilibrium strategy no longer depends on the aggregate withdrawal (L) because each agent has taken into account how his decision will affect the decision of followers. In fact, the first patient agent has the power to choose the aggregate early withdrawal.

Proposition 1 *(uniqueness of equilibrium outcome) Under the given conditions, there is a unique SPNE in period 1.*[14] *For different parameters, there are three possible equilibrium outcomes: no panic (only impatient agents withdraw early), a complete panic (all agents withdraw early), or a partial panic (impatient agents and some patient agents withdraw early).*

Proof: given the above equilibrium strategy for each agent, it is straightforward to find the unique SPNE. For a patient agent i, there exists a critical θ^* such that he chooses to

[13] When $U(r_1) > \theta(s, L_{N-1})U(r_2)$, we use the assumption that N is a large number, therefore $U(r_1) \geq \theta(s, L_N)U(r_2)$, where $L_N = L_{N-1} + \frac{1}{N}$. Considering the strategy for the last agent, "withdraw" is at least as good as "wait."

[14] More strictly, there is a *unique SPNE outcome*. It is possible to have multiple SPNEs but the equilibrium outcomes (aggregate early withdrawal, expected utility for each agent) are exactly the same. The uniqueness is a simple result of Zermelo's Theorem. See Mas-Coleu, Whinston and Green (1995), page 272.

withdraw if $\theta(s, L_i) < \theta^*$ and to wait otherwise.[15] The unique SPNE has three possible outcomes (see figure 3):

- case 1: $\theta(s, \alpha) \geq \theta^*$.

  All patient agents will choose to wait under this circumstance. The first patient agent will not withdraw his deposit because he knows that if he chooses to wait, the other patient agents will also wait. In this way he will have the probability of $\theta(s, \alpha)$ to get full payment in period 2, which is better than getting the short-term interest rate. Similarly, the other patient agents will adopt the same strategy. In this case, no bank run happens.

- case 2: $\theta(s, \alpha) < \theta^*$ and there is no $L \geq \alpha$ such that $\theta(s, L) = \theta^*$.

  All patient agents will choose to withdraw and force the bank into bankruptcy because withdrawing strictly dominates waiting in this case. A bank run is inevitable.

- case 3: $\theta(s, \alpha) < \theta^*$ and there exist $L^* \in (\alpha, 1)$ such that $\theta(s, L^*) = \theta^*$.

  In this case, some patient agents choose to withdraw and the others choose to wait. The equilibrium aggregate withdrawal in period 1 is L^*.[16] A partial bank run happens.

Proposition 2 $1 - i\tau > r_1$ *is a sufficient condition for a run-proof equilibrium when* $r_1 \leq r_2$.

Proof: A complete bank run happens only when there is no $L \geq \alpha$ such that $\theta(s, L) \geq \theta^*$. When $1 - i\tau > r_1$, $\beta(s, L)$ is as shown in cases (3) and (4) in figure 2. It is obvious that $\theta(s, L)$ is 1 when L is close to 1. The only possible results are no bank run or a partial bank run.

3.1 Numerical example

Here I use an example to illustrate how individual agents make their withdrawal decisions in the SPNE and discuss the feasibility of the socially optimal contract in a decentralized

[15] θ^* is defined by $\theta^* = \frac{u(r_1)}{u(r_2)}$. A more general definition is $u[x + (1-x) * r_1] = \theta^* u[x + (1-x) r_2] + (1-\theta^*) u(x)$, where $1 - x$ is the amount of deposits.

[16] When there are several Ls that satisfy the condition, L^* refers to the smallest one.

economy.

- Parameters

  - Return distribution for risky investment: $R_H = 1.3$, $R_L = 0.9$, $\pi = Pr\{\tilde{R} = R_H\} = 0.5$.

  - Liquidation cost: $\tau = 0.5$.

  - Utility function: $u(c) = \ln(c+1)$, which has the property of u(0)=0, $u'(\cdot) > 0$, $u''(\cdot) < 0$.

  - Proportion of impatient agents: $\alpha = 0.4$.

  - Signal quality: for simplicity, I assume everyone receives perfect interim information ($p = 1$).

- Optimal allocation without intermediaries

  For comparison, I first study the case in which there are no financial intermediaries in the economy and individual agents allocate their endowments between the two types of investment technologies. Let i be the amount of endowment placed in the illiquid investment. Agents choose i to solve:

$$
\begin{aligned}
\max_i \quad & \alpha u(1 - i\tau) + (1-\alpha)\pi u(1 - i + iR_H) \\
& \qquad\qquad\qquad +(1-\alpha)(1-\pi)u(1 - i + iR_L), \\
\text{s.t.} \quad & 0 \le i \le 1.
\end{aligned} \tag{9}
$$

  The choice of i has two opposite welfare effects. More liquidity holdings (smaller i) reduces the liquidation cost when the agent turns out to be an impatient consumer, but hurts the investor when the agent is a patient consumer because more endowment is placed in the less productive investment. This tradeoff cannot be resolved due to the absence of an *ex ante* risk-sharing instrument.

In this example, the optimal decision can be easily solved: i^*=0. That is, individual agents will invest all endowments in the liquid asset and get a maximum expected utility of $U_1 = 0.6931$. The high liquidation cost prevents the agents from utilizing the more productive technology in the no-intermediary economy.

- Socially optimal allocation with intermediaries

  Financial intermediaries provide an *ex ante* insurance arrangement for the economy. By solving the social planner's problem, we can find out the most efficient allocation of endowments. In the planning period, the social planner chooses how to allocate the endowments across the two technologies; in period 1, the planner determines the consumption levels for each type of agent subject to the resource constraint. Let i represent the investment in the illiquid asset, the planner solves the following problem:

$$\max_{i} \quad \alpha u(r_1) + (1-\alpha)u(r_2), \tag{10}$$

$$\text{s.t.}$$

$$r_1 = \tfrac{1-i}{\alpha}$$

$$(1-\alpha)r_2 = i \cdot E(\tilde{R})$$

$$0 \leq i \leq 1.$$

  The first order condition is

$$u'(r_1) = E(\tilde{R})u'(r_2). \tag{11}$$

  In this example, the optimal solution is: $i^o = 0.62$. Accordingly, the short-run interest rate is $r_1^o = 0.95$, the long-run interest rate is $r_2^o = 1.1367$, and expected utility is $U^o = 0.7227$.[17]

- Feasibility of the socially optimal solution in a decentralized economy

[17] Diamond and Dybvig (1983) proved that a sufficient condition for $r_1^o > 1$ when CRRA utility function is used is that the relative risk aversion coefficient is greater than 1.

In a decentralized economy, the socially optimal allocation cannot be sustained in the unique SPNE. Under the optimal contract, only impatient agents get repaid in the first period, that is, $L^o(R = R_H) = L^o(R = R_L) = 0.4$. However, using equation (7),

$$\theta^* = \frac{u(r_1^o)}{u(r_2^o)} = 0.8796$$

$$\theta(R_H, 0.4) = 1, \quad \theta(R_L, 0.4) = 0.8182$$

$$\theta(R_L, L) = \frac{1.8(0.69-0.95L)}{1.1367(1-L)} < \theta^* \text{ when } L > 0.4.$$

That means, when the economy is in a good state, all patient agents are willing to wait and the socially optimal outcome is realized (case 1 in figure 3); but when the economy is in a bad state, all agents choose to withdraw their deposits and a bank run happens (case 2 in figure 3), which violates the socially optimal contract requirement. Therefore, the socially optimal outcome cannot be supported by the withdrawal decisions in the decentralized economy when the future return is low.

The underlying reason for the welfare loss is the negative externality (more liquidation costs) caused by early withdrawal. When the social planner makes his decision, he takes into account this externality and will ask the patient agents to wait to minimize the liquidation costs. While in a decentralized economy, each agent maximizes only his own welfare and neglects the negative externality he will bring to the whole economy. As a result, those depositors who make decisions first have the incentive to withdraw early to beat the followers, forcing the banks to suffer huge liquidation costs and go into bankruptcy.

In the next section, I will discuss how the interest rates and portfolio structure are determined in the decentralized economy when banks respond to the possibility of bank runs.

4 Equilibrium in decentralized economy

Up to now, I have taken the banks' interest rates and investment portfolios as given. In a general equilibrium framework, I need to extend the partial equilibrium results and analyze how banks design their contracts accordingly. Using the same notation as in section 2 and 3, and define $1-x$ as the amount of bank deposit for each individual agent. The equilibrium should satisfy the following conditions:

- Banks' portfolio choice

  Banks choose the optimal portfolio structure to maximize their expected profits. In period 1, each patient agent makes his withdrawal decision based on his individual information set. Proposition 1 states that the aggregate early withdrawal depends on the public signal and other contract variables (interest rates and investment portfolio). Assume in the SPNE, the aggregate early withdrawal is $L_H(r_1, r_2, x, i)$ when the signal is "good" and $L_L(r_1, r_2, x, i)$ when the signal is "bad." Since the signal is imperfect, there are four possible cases:

  - Real return is high, and signal is good;
  - Real return is high, but signal is bad;
  - Real return is low, but signal is good;
  - Real return is low, and signal is bad.

  The expected profit for the bank is:

$$\begin{aligned} E(\Pi) &= (1-x)\{p\pi\Pi(R_H, L_H) + (1-p)(1-\pi)\Pi(R_L, L_H) \\ &\quad +\pi(1-p)\Pi(R_H, L_L) + (1-\pi)p\Pi(R_L, L_L)\}, \end{aligned} \tag{12}$$

  where the profit function Π is defined as

$$\Pi(R, L) = \begin{cases} \max(0, 1-i-r_1L+iR-r_2(1-L)), & \text{if } 1-i \geq r_1L. \\ \max(0, \frac{(1-i\tau-r_1L)R}{1-\tau} - r_2(1-L)), & \text{if } 1-i < r_1L. \end{cases} \tag{13}$$

  The optimal portfolio choice $i^* = i(r_1, r_2, x)$ for banks is the solution to this maximization problem.

- How agents allocate their endowments

This paper differs from the existing bank-run models in that investors are allowed to deposit only part of their endowment in banks.[18]

The problem to be solved is

$$\max_{x\in[0,1]} \quad E[U(r_1, r_2, x, i^*(r_1, r_2, x))] \tag{14}$$

$$= \sum_{i=H,L} \sum_{j=H,L} \mathrm{pr}(\tilde{R} = R_i, L = L_j) U(R_i, L_j),$$

where

$$U(\tilde{R}, L) = \begin{cases} Lu(c_1) + (1-L)[\theta(\tilde{R}, L)u(c_2) + (1-\theta(\tilde{R}, L))u(x)], & \text{if } 1 - i\tau \geq r_1 L \\ \\ L\rho\, u(c_1) + (1 - L\rho)u(x), & \text{if } 1 - i\tau < r_1 L \end{cases}$$

where $c_1 = x + (1-x)r_1$ and $c_2 = x + (1-x)r_2$ are agents' consumption in period 1 and period 2, respectively, when they get full interest payment, and $\rho = \frac{1-i\tau}{r_1 L}$ represents the proportion of early consumers who can get full payment. The two equations refer to three possible cases: (1) banks have no liquidity problem in either period (if $\theta(R, L) = 1$ in the first equation); (2) banks do not have enough liquidity in period 2 ($\theta(R, L) < 1$ in the first equation); (3) banks are out of liquidity in the interim period (second equation).

The optimal deposit amount is determined by the deposit contract: $x^* = x^*(r_1, r_2)$. Accordingly, $i^* = i(r_1, r_2, x^*(r_1, r_2)) = i^*(r_1, r_2)$.

- Equilibrium interest rate structure

Banks make their interest rate offer based on two considerations. First, banks maximize their expected profits. Second, because the market is competitive, each bank offers the

[18] It will simplify the problem if I assume that agents only have two choices: either put all their endowment in the banks or make no deposit. But using our method might lead to more fruitful results in the future research when we study the amount of capital flow and how it changes with respect to different policies.

interest rates that maximizes the expected utility for a representative agent to attract more deposits.

The banks' problem is to solve:

$$\max_{r_1, r_2} E[U(r_1, r_2, x^*(r_1, r_2), i^*(r_1, r_2))], \tag{15}$$

subject to

$$E[\Pi(r_1, r_2, x^*(r_1, r_2), i^*(r_1, r_2))] \geq 0. \tag{16}$$

Definition 1 *In a decentralized economy, the equilibrium contract* (r_1^*, r_2^*, x^*, i^*) *should solve the above maximization problems (12)-(16).*

Unfortunately, this is a very complicated non-linear optimization problem and an analytical solution is beyond our ability. In the next section, I will discuss some properties of the equilibrium contract (r_1^*, r_2^*, x^*, i^*). Also, some numerical examples are provided to help in understanding these properties.

5 Properties of the equilibrium

In a decentralized economy, the equilibrium contract (r_1^*, r_2^*, x^*, i^*) should have the following properties.

Lemma 1 *in equilibrium,* $r_1^* \leq r_2^*$.

Proof: This is the familiar incentive compatibility constraint (see Jacklin and Bhattacharya 1988 and Alonso 1996). Suppose $r_1^* > r_2^*$ in equilibrium, then in period 1 all individual agents will have the incentive to withdraw early because it strictly dominates the waiting strategy. Therefore, bank runs always happen no matter whether the future return is high or low. Banks have to invest all deposits in the liquid asset to minimize the liquidation cost. Obviously, the best interest rates banks can offer are $r_1^* = 1$, $r_2 < 1$, which is no better than the no-intermediary case. Under such a situation, there is no need for the intermediaries to exist.

Lemma 2 *in equilibrium, $L_H \leq L_L$. More generally, if $Pr[\tilde{R} = R_H|s = s_1] > Pr[\tilde{R} = R_H|s = s_2]$ for two signals s_1, s_2, then $L(s_1) \leq L(s_2)$;*

Proof: First, given a certain contract (r_1, r_2, x, i), te function $\theta(R, L)$ is nondecreasing in R. From equation (7), I get

$$\frac{\partial \beta(\tilde{R}, L)}{\partial \tilde{R}} = \begin{cases} \frac{i}{r_2(1-L)} > 0 & \text{when } L \leq \frac{1-i}{r_1} \\ \\ \frac{1-i\tau-r_1L}{r_2(1-L)(1-\tau)} & \text{when } L > \frac{1-i}{r_1}. \end{cases} \tag{17}$$

β is increasing in $\tilde{R}$ unless $1 - i\tau - r_1L \leq 0$. But if $1 - i\tau - r_1L \leq 0$, the banks have already gone into bankruptcy and $\theta = 0$ for all Rs. Therefore, $\theta(\tilde{R}, L)$ is always nondecreasing in $\tilde{R}$. From equation (8), $\theta(s_1, L) \geq \theta(s_2, L)$ for all L because signal s_1 corresponds to a higher probability of a high return.

Second, from the proof of proposition 1, we know the aggregate early withdrawal L is determined by $L_s^* = \min\{L \in [\alpha, 1] : \theta(s, L) \geq \theta^*\}$. Since the critical value θ^* remains the same and $\theta(s_1, L) \geq \theta(s_2, L)$, it is obvious that $L_(s_1) \leq L(s_2)$.

Lemma 3 *The optimal liquidity $1 - i^* \in [r_1L_H^*, r_1L_L^*]$.*

Proof: this conclusion is quite intuitive. First, the liquidity cannot be less than $r_1L_H^*$ because otherwise in period 1 the banks always have to liquidate part of their illiquid assets. By increasing the liquid asset holdings, the banks can reduce the liquidation costs. Second, when $L_H^* < L_L^*$, the banks may have incentive to hold extra liquidity (an amount above $r_1L_H^*$). Holding extra liquidity has two effects. (1) When the signal is bad, it can reduce the liquidation costs; and (2) when the signal is good, the banks suffer a loss because the liquid asset is less productive than th illiquid asset in the long run. How much extra liquidity the banks are willing to hold depends on which effect plays a dominant role. Third, the maximum liquidity holding is the maximum interim repayment $r_1L_L^*$. More liquid assets are undesirable because they have to be carried over to the last period and they are less productive than illiquid assets in the long run.

5.1 Equilibrium properties under perfect information ($p = 1$)

As a benchmark, I first discuss the equilibrium properties when the public signal is perfect ($p = 1$).

Lemma 4 *Under perfect information, banks can earn positive profits only when there is no bank run.*

Proof: The property is obvious from the proof in proposition 1 (figure 3). In the three cases, the profit and utility are:

- *Case 1*: $L = \alpha$.

$$\begin{aligned} \Pi &= \max(0, 1 - i - r_1\alpha + iR - r_2(1-\alpha)) \\ U &= \alpha u(c_1) + (1-\alpha)[\theta(R,\alpha)u(c_2) + (1-\theta(R,\alpha))u(x)] \end{aligned}$$

- *Case 2*: $L = 1$.

$$\begin{aligned} \Pi &= 0 \\ U &= \frac{1 - i\tau}{r_1}u(c_1) + (1 - \frac{1-i\tau}{r_1})u(x) \end{aligned}$$

- *Case 3*: $\alpha < L < 1$.

  At L, patient agents are indifferent between early withdrawal and late withdrawal.

$$\begin{aligned} \Pi &= 0 \\ U &= u(c_1) \end{aligned}$$

Combining all these results, banks can earn positive profits only when no bank runs happen.

Proposition 3 *Under perfect information ($p = 1$), we have the equilibrium property $L_H^* = \alpha$ in the decentralized economy. That is, bank runs can happen only when the economy is in a bad state.*

Proof: See Appendix A.

Proposition 3 implies that a type-I bank run is impossible under a perfect information world. When the economy is healthy, no patient agent has the incentive to misreport his type and withdraw his deposit early because he can gain from earning the long term interest rate. The only reason for a bank run is because there are fundamental problems in the economy which makes bank deposits an unattractive investment tool.

Proposition 4 *In equilibrium it is possible that banks can earn positive profits when the economy is in the good state. Under perfect information, the necessary condition for $E(\Pi) > 0$ is that there is no bank run at all in equilibrium ($L_H^* = L_L^* = \alpha$).*

Proof: Appendix B shows that it is impossible for banks to earn positive profits when there exist partial or complete bank runs in equilibrium, but in a no bank-run equilibrium, banks can earn positive profits when the economy is in the good state. This seems quite counter-intuitive since the banking sector is competitive. The underlying reason is that withdrawal decisions are endogenously determined in this model and a small change in the contract might lead to dramatic changes in depositors' withdrawal decisions. Therefore there are two important properties for the expected utility and expected profit functions: (1) they are not continuous in contract variables (r_1, r_2, x, i): a small change in the contract might lead to huge jumps in banks' expected profits and a representative agent's expected utility; and (2) The two functions may not be inversely related. More specifically, higher interest rates reduce banks' profits but do not necessarily increase individual agents' expected utility. It is possible that the best contract which maximizes a representative agent's expected utility can bring the banks some positive profits. We provide two possible cases below.

- Consider case 1 in figure 4, where $L_H = L_L = \alpha$, $\theta(\tilde{R} = R_L, L = \alpha) = \beta(\tilde{R} = R_L, L = \alpha) = 1$ and $\beta(\tilde{R} = R_H, L = \alpha) > 1 = \theta(\tilde{R} = R_H, L = \alpha)$. Obviously, there is no bank run in equilibrium, banks' profits are positive when the return is high and zero when the return is low.

  Now consider an increase in the long run interest rate r_2. There will be two effects. First, if the return is high, then every late consumer is better off. However, if the return is low, the remaining resources will be less evenly distributed among late consumers and it hurts them. Whether the new interest rate can lead to a higher expected utility for agents depends on which effect dominates. If the second effect dominates, the higher interest rate reduces both banks' profits and the representative agent's expected utility. Under these conditions the initial contract with positive profits is chosen.

- Another possibility of positive profits in equilibrium is shown in case 2 in figure 4, where $\theta(\tilde{R} = R_L, L = \alpha) = \theta^*$ and $\beta(\tilde{R} = R_H, L = \alpha) > 1 = \theta(\tilde{R} = R_H, L = \alpha)$. There are also two effects when the interest rate r_2 is increased. First, if the return is high, the depositors are better off because some of banks' profits are transferred to them. Second, if the return is low, individual agents will change their withdrawal decisions to $L'_L = 1$ because $\theta'(\tilde{R}, L = \alpha) < \theta^{*\prime}$ (as shown in Appendix A, $\frac{\theta(\tilde{R}, L=\alpha)}{\theta^*}$ is decreasing in r_2). The risky assets are liquidated at a high cost and the representative agent suffers a huge welfare loss. If the second effect dominates, banks will stick to the initial contract which brings positive profits.[19]

5.2 Properties under imperfect information $(0 < p < 1)$

Under imperfect information, there are similar propositions.

Proposition 5 *When information is imperfect ($0 < p < 1$), we still have the equilibrium property $L^*_H = \alpha$ in a decentralized economy, but the economic implications are different: both type-I and type-II bank runs are possible under imperfect information.*

Proof: This proposition can be proved following the same steps as under perfect information.

Although the conclusions are the same, they have different economic meanings. As I point out earlier, under perfect information, it implies that only type-II bank runs are possible in

[19]The result, in fact, comes from our assumption that in period 2 all late consumers should get their repayment according to a "first come, first served" rule. If I assume that in period 2 the remaining assets are evenly distributed among late consumers (as in the existing literature), the second effect no longer exists in both cases and the initial contract cannot be optimal. In fact, in another paper, I show that the expected profit is zero under the new assumption.

the equilibrium. Yet under imperfect information, both type-I bank runs and type-II bank runs are possible.

Suppose in equilibrium a complete bank run happens when the market receives a bad signal. Because the information is imperfect, it includes two possible cases. (1) The signal correctly reflects the bad economic state. In this case the bank run is the second type. (2) The economy is in fact in a good state but everyone receives a bad signal and a bank run happens based on the pessimistic expectation. Under this situation it is a type-I bank run because banks are *de facto* solvent.

Combining the results, it is obvious that:

Proposition 6 *A type-I bank run happens only when the market information is imperfect.*

Proposition 7 *Under imperfect information, banks can earn positive profits in equilibrium.*

Proof: The economic intuition why positive profits are possible under imperfect information is the same as the explanation under perfect information. In a competitive market, each bank tries to offer the contract that maximizes depositors' expected utility subject to a non-negative-profit constraint. When the banks can make profits in an initial contract, they might have incentive to increase the interest rates, sacrificing part or all of their profits to offer a better contract to investors, but increasing interest rates has two opposite effects. On the one hand, it gives the investors a higher payment in some situations; on the other hand, a higher interest rate might lead to less even distribution of assets among depositors, or it might lead to a bank run which initially does not happen. When the second effect dominates, increasing the interest rate is not a wise decision for the banks because it reduces their profits yet does not benefit the investors.

Notice that under imperfect information, lemma 4 is no longer valid. Banks might be able to make profits when a partial bank run happens. Suppose a partial bank run happens when the market observes a public signal s ($L_s \in (\alpha, 1)$). From proposition 1, we know that $E(\theta(R, L_s|s)) = \theta^*$. It is possible that $\beta(R_H, L_s) > 1 = \theta(R_H, L_s) > \theta^* > \theta(R_L, L_s)$. Under this situation, the banks can earn positive profits because

$$E(\Pi) = \mathrm{pr}(\tilde{R} = R_H|s) \cdot r_2[\beta(R_H, L_s) - 1](1 - \alpha) > 0.$$

As a result, the necessary condition for positive profits in Proposition 4 is no longer valid under imperfect information.

5.3 Numerical example

Although an analytical solution is beyond our ability, I can use some numerical methods to get the solutions and illustrate the equilibrum properties. Because none of the functions (withdrawal decision, profit function, and expected utility function) are continuous in contract variables, I use the grid-searching method to find out the best contract in the decentralized economy. The method is as follows:

(1) divide the range of r_1 into a large number of small segments with a step of 0.01;

(2) divide the range of r_2: $(r_1, (1 - r_1\alpha)R_H)$ into M segments ($M = 50$);

(3) divide the range of x into 100 segments with a step of 0.01;

(4) for given r_1, r_2, and x, find the profit-maximization investment structure $i^*(r_1, r_2, x)$;

(5) find optimal x^* for given r_1, r_2;

(6) choose the interest rates (r_1, r_2) which maximize a representative agent's expected utility.[20]

Table 1 shows the equilibrium contracts under four different situations. In all four examples, I assume $\alpha = 0.4$, $\tau = 0.5$, and $u(c) = \ln(1 + c)$.

5.3.1 Example 1: benchmark

I first study the example in section 3.1, in which $R_H = 1.3$, $R_L = 0.9$, $\pi = 0.5$, and signal quality $p = 1$. It is easy to calculate that $E(\tilde{R}) = 1.1$, $\text{Var}(\tilde{R}) = \pi(1 - \pi)(R_H - R_L) = 0.2^2$.

Using the grid-searching method, I can find the equilibrium contract in a decentralized economy is as follows:

- interest rate structure: $r_1 = 0.89$, $r_2 = 1.3953$;
- deposit amount: each agent puts 62% of his endowment into the banks;

[20] I do not impose the non-negative-profit constraint here. Banks always make non-negative profits because there is no capital requirement in this model.

- banks' portfolio structure: banks invest 35.6% of the deposits in the liquid asset, which is just enough for their interim liability payment;
- agents' withdrawal decision: no bank run happens;
- payoff: E(U)=0.7037; $E(\Pi) = 0$.

As pointed out in section 3.1, the socially optimal contract cannot be supported in a decentralized economy. Therefore, the banks have to find a new contract to maximize agents' welfare. In this example, banks choose a run-proof contract and earn zero profit in equilibrium. Not surprisingly, the equilibrium outcome in the decentralized economy is better than the no-intermediary case but less efficient than the socially optimal contract ($0.6931 < 0.7037 < 0.7227$).

5.3.2 Example 2

The second example has the same signal quality ($p = 1$) and expected return ($E(\tilde{R}) = 1.1$), but is more risky ($\mathrm{Var}(\tilde{R}) = 0.6^2 > 0.2^2$).

The equilibrium outcome has different properties from the benchmark example. Because the future return is more volatile, the cost to maintain a no-run equilibrium is very high. It turns out that banks finally deliberately choose a bank-run contract, in which bank runs happen when the economy is in a bad state. Despite the existence of bank runs, all agents will gain from the intermediaries' risk-sharing role when the economy is in the good state. The gain is high enough to compensate the losses and induce the banks to choose the bank-run contract in equilibrium.

As proposition 3 states, only type-II bank runs are possible under perfect information.

5.3.3 example 3

Example 3 illustrates that banks can earn positive profits in equilibrium.[21] The return profile has the same mean as examples 1 and 2, but has a different distribution. While examples

[21]Although earning positive profits is possible, it turns out that in most cases banks earn zero profit in equilibrium.

1 and 2 each have a symmetric return distribution, this example has a highly asymmetric distribution. The return is most likely to be at near its mean level ($\tilde{R} = R_L \approx E(\tilde{R})$ with probability 90%), and is very high in some cases ($\tilde{R} = R_H \gg E(\tilde{R})$ with probability 10%).

In the equilibrium contract, banks can earn positive profits ($E(\Pi) = 0.0123$) and no bank runs happen. The reason banks cannot offer a better contract by sacrificing their profits is stated in section 5.1.[22] If banks offer a higher interest rate, the investors will benefit when the return is high, but suffer a welfare loss when the return is low. In the specific example, the return is highly asymmetrically distributed and $R = R_H$ is only an occasional case. Therefore the second effect plays a dominant role in banks' contract choice.

Comparing examples 1 and 3, in which banks both choose a no-run contract, it is not surprising that the less risky (case 3) project yields a higher expected return to investors. The reason is simple. When no bank run happens, banks only pay back impatient agents in the interim period and invest the rest of deposits in the risky asset. Since both projects have the same expected return, the low-risk technology must be superior.

5.3.4 example 4

Example 4 is the same as example 2 except the signal is imperfect. The public signal only correctly reveals the future return with a probability of 99%. For similar reasons, banks choose a bank-run contract in equilibrium, but the economic implications are different. Because the information is imperfect, the bank run can be either a type-I run or a type-II run. Specifically, the probability that a type-II bank run happens is $0.5 \times 0.99 = 0.495$, and the probability of type-I bank run is $0.5 \times 0.01 = 0.005$. One percent of bank runs are a suboptimal outcome due to imperfect information.

6 Response to the return volatility

Another interesting question is when there are several risky investment technologies, how will the banks choose among them? Krugman (1998) suggests that one possible reason for

[22] Draw the $\theta(\tilde{R}, L)$ function for equilibrium contract, it is the same type as case 1 in figure 4.

the 1997 Asian crisis is that banks over-invest or invest in the high-risk project due to the moral hazard problem caused by implicit deposit insurance.

This paper provides two explanations for the "over-investment" phenomenon when no deposit insurance exists. I still use numerical examples to illustrate the underlying reasoning. First, using the grid searching method, I study the change in equilibrium contracts when the return becomes more volatile. Using the parameters in the benchmark example: $\alpha = 0.4$, $\tau = 0.5$, $p = 1$. There are a lot of risky assets which have the same expected return but have different variance. More specifically, $R_H = 1.1 + d$, $R_L = 1.1 - d$, and $\pi = 0.5$, where $d = 0$, 0.02, 0.04, $\cdots$, 1.08, 1.1. It is obvious that $E(\tilde{R}) = 1.1$ and $\text{Var}(\tilde{R}) = d^2$.

Figure 5 illustrates how the equilibrium contracts (r_1^*, r_2^*, x^*, i^*) change when the illiquid technology becomes more risky. First, banks prefer a no-run equilibrium for low-risk projects; but when the investment becomes too risky ($d \geq 0.52$), banks will switch to a bank-run contract to take advantage of the high return in good state. Second, the variance of future returns affects the equilibrium interest rate structure. When the return is more volatile, banks will offer a lower short-term interest rate and a higher long-term interest rate. Third, a representative agent's expected utility is inversely related with volatility of returns when banks choose a no-bank-run contract. In contrast, if banks choose a bank run contract, the expected utility increases when banks invest in a more risky project. This is the "U-shaped" utility response function we observed, and the expected utility when banks invest in the most risky asset (d=1.1) is even higher than when banks invest in a riskless asset ($d = 0$). Accordingly, the bank deposit is characterized by a "U-shaped" curve because the more an agent can gain from putting money in the bank, the more endowment he is willing to deposit.

Based on these results, I can discuss how the banks will choose between a high-risk project and a low-risk project. Suppose banks announce their choice of investment technology at the same time they announce the interest rates. There are two possible cases: (1) the announcement of investment technology is not a binding commitment; therefore, banks are free to choose the investment projects and investment portfolios after receiving deposits; (2) the banks make a commitment that they will not use the deposits for other projects. In both cases, banks may have incentives to choose the high-risk investment over the low-risk one.

- If there is no binding commitment for banks' investment choice, banks will always choose the high-risk project after they receive the deposits. This is a result of so-called "time-inconsistency" problem. Suppose banks can invest in either of two illiquid investments which have standard deviation of return of $d_1 = 0.08$ and $d_2 = 0.2$, respectively. The equilibrium contract for the two projects are: (1) $d_1 = 0.08$: $r_1 = 0.97$, $r_2 = 1.2036$, $x = 0$, $i = 0.612$, $L_H = L_L = 0.4$, $E(U) = 0.7131$, $E(\Pi) = 0$; (2) for $d_2 = 0.2$: $E(U) = 0.7037$. The low-risk project leads to a higher expected utility in equilibrium and should be chosen to improve investors' welfare, but are the banks willing to do this?

  Suppose banks announced that they will invest in the low-risk project and accordingly offer the interest rates $r_1 = 0.97$ and $r_2 = 1.2036$ at the beginning. After they receive the money, will they still keep the promise? The answer is no. Given the interest rates, the banks will switch to the high-risk project because they lose nothing when the return is low but gain some positive profits when the return is good. Taking this into account, agents will not put their money in the bank. The only equilibrium that can avoid the "time-inconsistency" problem is the equilibrium contract under the high-risk project; therefore, the high-risk project is always chosen under the no-commitment case.

- It is welfare improving if banks make binding commitments to which project they will choose before they receive deposits, but even under this situation, banks might still want to choose the high-risk project due to the "U-shaped" expected utility function. For example, consider two projects that have return variance of $d_1^2 = 0.4^2$ and $d_2^2 = 0.72^2$, respectively. Figure 7 indicates that the high-risk project leads to a higher expected utility in equilibrium and therefore should be chosen.

Although in both cases banks choose the high-risk project, their underlying reasons are distinct. In the first case, the underlying reason lies in the conflict of interests between banks an depositors: Banks want to maximize their own profits after they receive the deposits, which contradict with the interests of depositors. This phenomenon is also widely cited as the "principal-agent" problem or "moral hazard" problem. In the second case, the high-risk asset is chosen because it benefits both the banks and the depositors and, therefore, is an efficient result.

7 Conclusion and extensions

In this paper I analyze the general equilibrium when agents sequentially make their withdrawal decisions and banks strategically choose their interest rate structure and investment portfolios. In equilibrium, banks may choose a bank-run contract. I show that imperfect information is the only reason that type-I bank runs occur in this model. Banks can earn positive profits in equilibrium due to the "sequential service" constraint.

Another contribution of this paper is that it offers a possible explanation for why banks sometimes choose a high-risk project over a low-risk alternative. Besides the moral hazard problem that is widely cited in existing literature, I put forward another explanation that it might be an efficient choice for the economy. When a bank-run equilibrium is chosen, a more volatile return can lead to higher expected utility for two reasons. First, when the return is low, banks are liquidated and the low return is never realized. The low return is, in fact, truncated to its liquidation value. However, the high-risk asset has a fatter tail in high return region and therefore is a more efficient investment tool. As a result, a high-risk investment can be a socially efficient choice in this model.

We see the basis for further work on this problem in several directions. In this paper I make a simplifying assumption that agents make their decisions according to an exogenously determined sequence. This assumption simplifies the problem but is not necessary. In Appendix C, I show that proposition 1 is still valid when the withdrawal sequence is random, or when the withdrawal sequence is endogenously determined.

A second extension, which is more interesting, can be made on the information set. In this paper, everyone observes the same public signal. This is a very strong assumption and is only reasonable under certain situations. A more reasonable assumption is that individual agents have different information sets regarding the future return. Each agent can choose whether and when to withdraw his deposit. Under the new setting, "information cascade" or "herd behavior" phenomena arise naturally. Banerjee (1992) provides a modern framework for this kind of problem. More recent works by Gul and Lundholm (1995), Zhang (1997), Chari and Kehoe (1999), and Calvo (1999) shed more light on the future research along this direction.

Finally, this model provides a framework for the future study of policy implications. After the 1997 East Asian crisis, the deposit insurance policy was severely attacked for causing a moral hazard problem. Is this the only demon, or are there any other problems in the bank sector? The other controversial topic is the role of the IMF. Some economists (Sachs 1998, Radelet and Sachs 1998) argue that a lender-of-last-resort is an efficient way to prevent self-fulfilling financial panics; therefore the IMF should be expanded and a larger amount of funds should be provided more quickly when financial crises occur. At the other extreme, Schwartz (1998) and Calomiris (1998) criticize the IMF acting as lender-of-last-resort as causing a serious moral hazard problem and increasing the fragility of the world financial system in the long run. They suggest that the IMF should be banished. In my second paper, I will extend this model and discuss how deposit insurance and capital requirement policies may affect the bank sector's behavior and what is the efficient policy to prevent a bank run.

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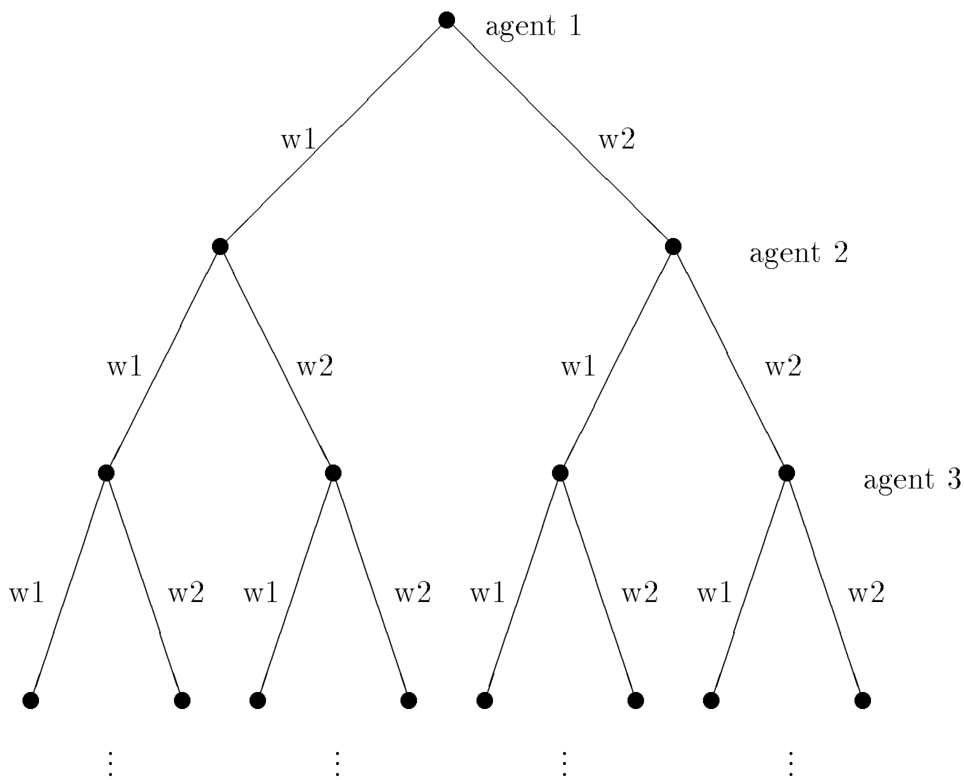
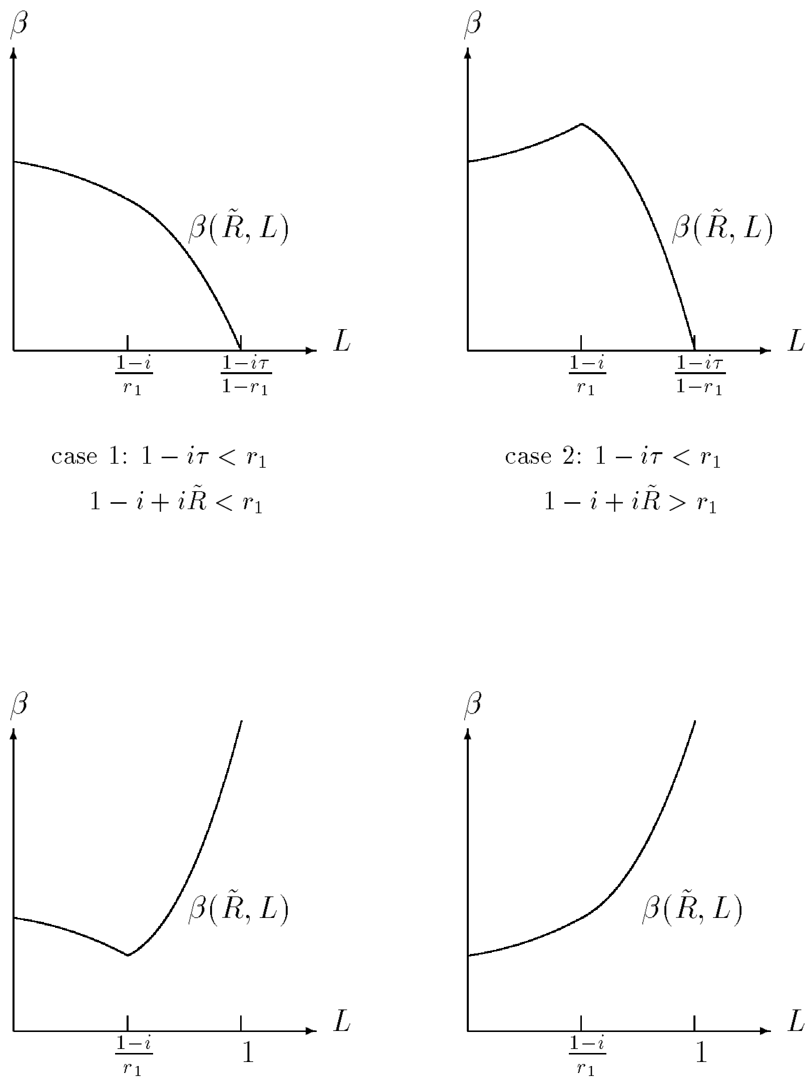


Figure 1: subgame for patient agents in period 1 (n=3)
w1 – wait; w2 – withdraw



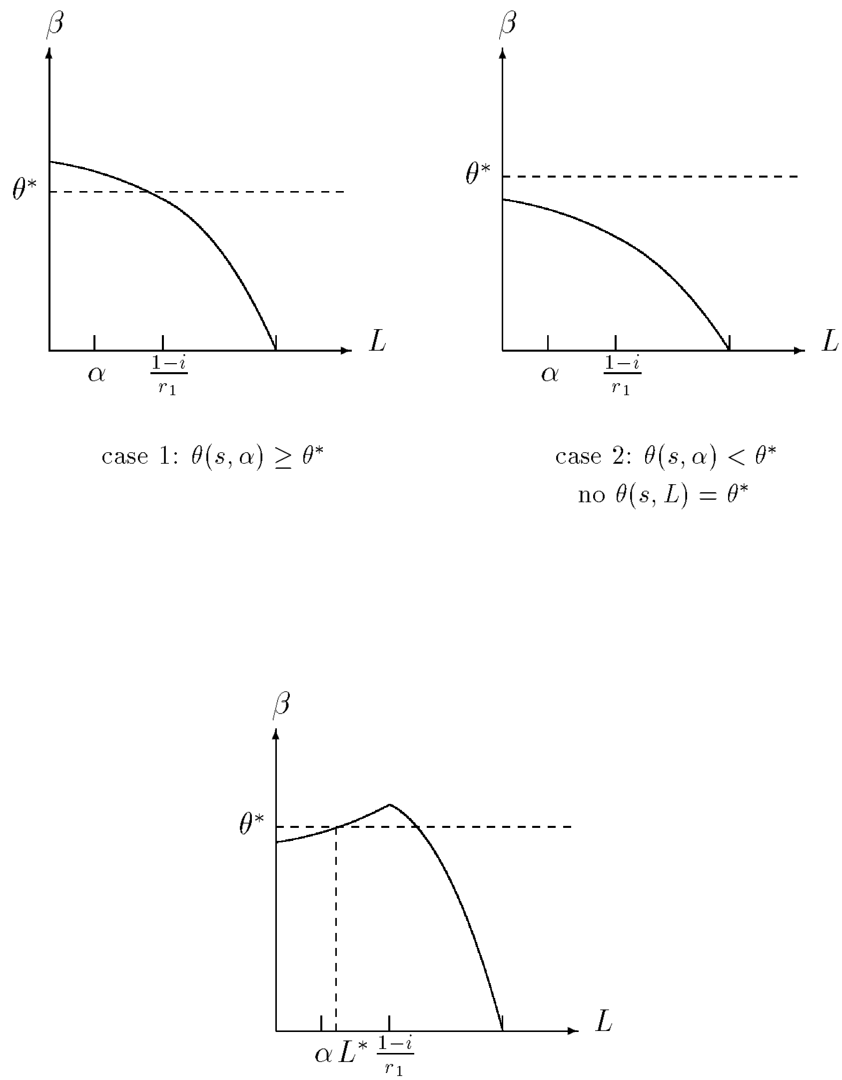
case 1: $1 - i\tau < r_1$
$1 - i + i\tilde{R} < r_1$

case 2: $1 - i\tau < r_1$
$1 - i + i\tilde{R} > r_1$

case 3: $1 - i\tau > r_1$
$1 - i + i\tilde{R} < r_1$

case 4: $1 - i\tau > r_1$
$1 - i + i\tilde{R} > r_1$

Figure 2: possible cases for $\beta(L)$



case 1: $\theta(s,\alpha) \geq \theta^*$

case 2: $\theta(s,\alpha) < \theta^*$
no $\theta(s,L) = \theta^*$

case 3: $\theta(s,\alpha) < \theta^*$, $\theta(s,L^*) = \theta^*$

Figure 3: SPNE possible outcomes

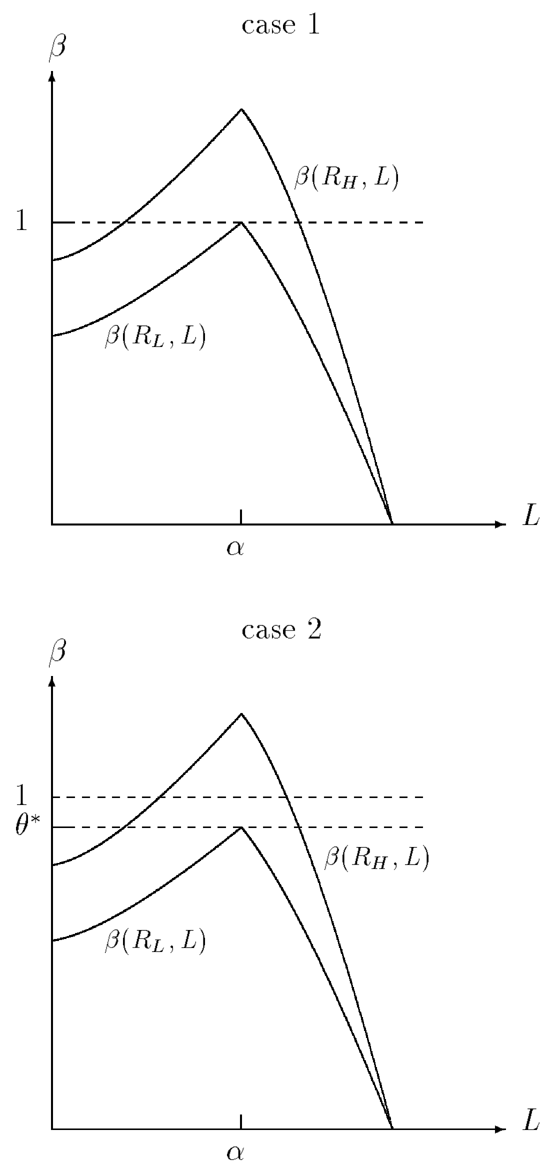
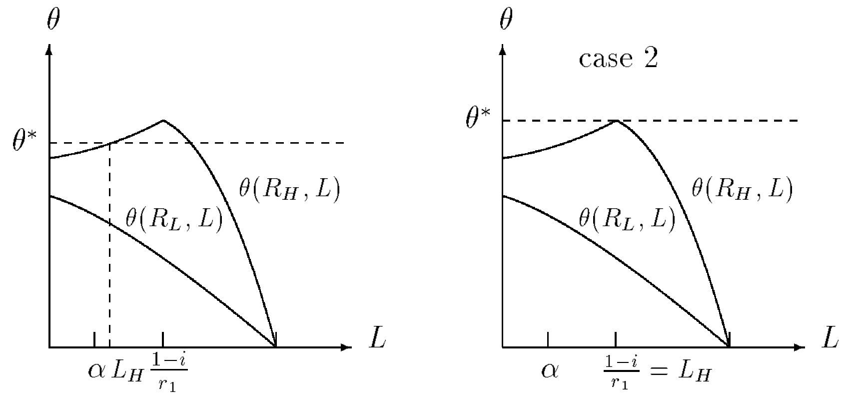


figure 4: possible cases in which $E(\Pi) > 0$



case 1: $\theta(R_H, \frac{1-i}{r_1}) > \theta^*$ case 2: $\theta(R_H, \frac{1-i}{r_1}) = \theta^*$

Figure 6: two possible cases for $L_H \in (\alpha, 1)$, $L_L = 1$

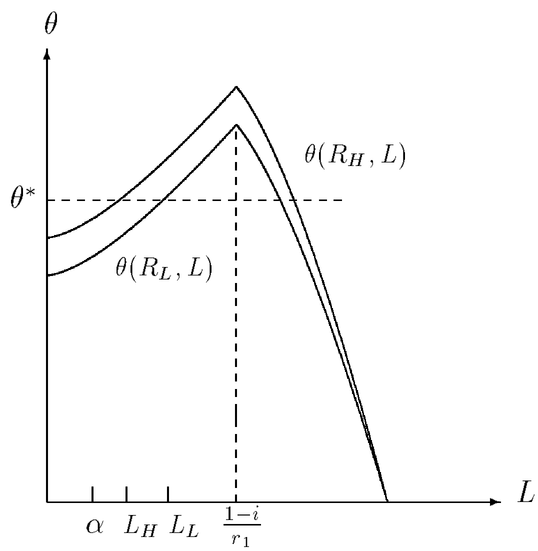


Figure 7: $\alpha < L_H < LL < 1$

Table 1: Four Numerical Examples:
Equilibrium Contracts Under Different Situations

Input Variables:

$\alpha = 0.4; \qquad \tau = 0.5; \qquad u(c) = log(c+1)$

	Case 1	case 2	Case 3	case 4
R_H	1.3	1.7	1.28	1.7
R_L	0.9	0.5	1.08	0.5
π	0.5	0.5	0.1	0.5
$E(\tilde{R})$	1.1	1.1	1.1	1.1
$Var(\tilde{R})$	0.2^2	0.6^2	0.06^2	0.6^2
p	1	1	1	0.9

Equilibrium Contract:

	Case 1	Case 2	case 3	case 4
r_1^*	0.89	0.74	0.96	0.84
r_2^*	1.3953	1.9947	1.1088	1.8607
x^*	0.38	0.62	0	0.66
i^*	0.644	0.704	0.616	0.664
$L(s = R_H)$	0.4	0.4	0.4	0.4
$L(s = R_L)$	0.4	1	0.4	1
E(U)	0.7037	0.6997	0.7168	0.6979
$E(\Pi)$	0	0	0.0123	0.0021
Prob(Type I bank run)	0	0	0	0.5%
Prob(Type II bank run)	0	50%	0	49.5%

Notations:

π: prob($R = R_H$);
p: signal quality;
r_1: short-term interest rate;
r_2: long-term interest rate;
$1 - x$: deposit amount;
i: banks' holding of illiquid assets;
L: aggregate early withdrawal;
U: agents' utility;
Π: banks' profits.

Appendix

A Proof for Proposition 3

Proof: define $L_i = L(s = R_i)$, $i = H, L$. If $L_H \neq \alpha$, then there are two other possibilities: either $L_H = 1$ or $0 < L_H < 1$. In the following steps I show neither of them is possible in equilibrium.

1. $L_H = 1$ is impossible.

   If $L_H = 1$, then $L_L = 1$ from Lemma 2. Bank runs happen in both states. The optimal investment strategy for banks is to invest everything in the safe asset to minimize liquidation costs. It is straightforward that $r_1 = 1$ and $E(U) = u(1)$ for individual agents. This result is attainable under the no-intermediary case. Therefore there is no need for the bank sector to exist. There is a contradiction.

2. $L_H \in (\alpha, 1)$, $L_L = 1$ is impossible.

   If $L_H \in (\alpha, 1)$ and $L_L = 1$, then $E(U) < u(c_1)$. Because $E(U) \geq 1$ must be satisfied in equilibrium, c_1 must be no less than 1. Therefore, $r_1 \geq 1$ and $1 - i\tau < r_1$.

   There are two possible cases as shown in figure 6: (1) $\theta(s = R_H, L = \frac{1-i}{r_1}) > \theta^*$; (2) $\theta(s = R_H, L = \frac{1-i}{r_1}) = \theta^*$.

   Step 1: case 1 is not an optimal result.

   Consider a new portfolio structure $i' = i - \epsilon$. When ϵ is small enough, it is easy to verify that the new contract will have the same property: a partial bank run happens when the return is good and a complete bank run happens when the return is bad. Banks still get zero profit. Under the new scheme, however, $U'(\tilde{R} = R_H) = u(c_1) = U(\tilde{R} = R_H)$, $U'(\tilde{R} = R_L) = \frac{1-i'}{r_1}u(c_1) + (1 - \frac{1-i'}{r_1})u(x) > \frac{1-i}{r_1}u(c_1) + (1 - \frac{1-i}{r_1})u(x) = U(\tilde{R} = R_L)$. Therefore a representative agent can get a higher expected utility under the new contract.

   Step 2: case 2 is not an optimal result.

   First, I prove that for $r_2' = r_2 - \epsilon$, the bank will still be subject to a partial bank run when the signal is good and a complete bank run when the signal is bad. Besides, $\theta'(s = R_H, L = \frac{1-i}{r_1}) > \theta^{*\prime}$.[23]

   When ϵ is small enough, it is trivial that $L_L = 1$ and $\theta(s = R_H, L = \alpha) < \theta^*$ can still be satisfied. I only need to prove that $\theta'(s = R_H, L = \frac{1-i}{r_1}) > \theta^{*\prime}$ when $r_2\prime = r_2 - \epsilon$.

   For simplicity, I write $\theta(r_2)$ to represent $\theta(s = R_H, L = \frac{1-i}{r_1}, r_1, r_2, x, i) = \frac{iR_H}{r_2(1-L)}$. From the assumption, $\frac{\theta(r_2)}{\theta^*} = 1$.

[23]The change in r_2 affects agents' withdrawal decision in two ways. First, when r_2 decreases, a late consumer is more likely to get full repayment in period 2. Second, the threshold value (θ^*) at which an individual agent is willing to wait is higher. We need to determine which effect dominates in this case.

$$
\begin{aligned}
\frac{\partial \frac{\theta(r_2)}{\theta^*}}{\partial r_2} &= \frac{\partial \frac{iR_H[u(c_2)-u(x)]}{r_2(1-L)[u(c_1)-u(x)]}}{\partial r_2} \\
&= A\frac{(1-x)u'(c_2)r_2-u(c_2)+u(x)}{r_2^2}
\end{aligned}
$$

Where $A = \frac{iR_H}{(1-L)[u(c_1)-u(x)]}$ is positive. Define $F(r_2,x) \equiv (1-x)u'(c_2)r_2 - u(c_2) + u(x)$, it is easy to show that $F(r_2,x) < 0$ for $0 \leq x < 1$ by using the following properties:

i) $F(r_2,0) = u'(r_2)r_2 - u(r_2) < 0$;

ii) $F(r_2,1) = u(1) - u(1) = 0$;

iii) $\frac{\partial F(r_2,x)}{\partial x} = u''(c_2)(1-r_2)r_2(1-x) - u'(c_2) + u'(x) > 0$ because $u''(\cdot) < 0$, $u'(x) > u'(c_2)$ and $r_2 \geq r_1 \geq 1$.

Combining these results, when $r_2' = r_2 - \epsilon$, the new contract leads to the same type of equilibrium outcome as in case 1.

Second, when $r_2' = r_2 - \epsilon$, it is obvious that the new contract will lead to the same expected utility as the original contract because under both cases, $U(\tilde{R} = R_H) = u(c_1)$ and $U(\tilde{R} = R_L) = \frac{1-i\tau}{r_1}u(c_1) + (1 - \frac{1-i\tau}{r_1})u(x)$.

But I have already shown in step 1 that there exists another contract that yields a higher expected utility than the new contract; therefore, the initial contract is not an equilibrium contract.

3. $\alpha < L_H \leq L_L < 1$ is impossible. (See figure 7.)

   Step 1: Using the same argument as before, $r_1 \geq 1$ must be satisfied.

   Step 2: $i' = 1 - r_1\alpha$ is a better contract.

   Because $\theta(\tilde{R}, L = \frac{1-i}{r_1}) = \frac{i\tilde{R}}{r_2(1-\frac{1-i}{r_1})} = \frac{i\tilde{R}r_1}{r_2(r_1-1+i)}$ is increasing in i when $r_1 \geq 1$. For $i' = 1 - r_1\alpha > i$, $\theta'(\tilde{R}, L = \alpha) = \theta(\tilde{R}, L = \frac{1-i'}{r_1}) > \theta(\tilde{R}, L = \frac{1-i}{r_1}) \geq \theta^*$. Therefore, no bank runs happen under the new contract, $U'(R_H,\alpha) > \theta^* u(c_2) = u(c_1)$ and $U'(R_L,\alpha) > u(c_1)$. The new contract yields a higher expected utility.

   Combining all of the above arguments, under perfect information bank runs can only happen when the economy is in a bad state. This conclusion is important because it implies that when the information is perfect and banks are allowed to choose their interest rates and portfolio structure strategically, only type-II banks runs are possible.

B Proof for Proposition 4

Proof: In this part I only show that banks cannot make positive profits when bank runs happen ($L_L > \alpha$). For the possibility of positive profits in equilibrium, both theoretical analysis and numerical examples are provided in the text.

Step 1: if $E(\Pi) > 0$, then $i = 1 - r_1\alpha$.

This conclusion comes from lemma 4 and the fact that i should be chosen to maximize the banks' expected profits. If $E(\Pi > 0)$ and $i < 1 - r_1\alpha$, then banks can get more profits by increasing their holding of risky assets.

- If $\Pi(\tilde{R} = R_H) > 0$ and $\Pi(\tilde{R} = R_L) = 0$, by increasing i, banks can earn more profits when $\tilde{R} = R_H$ and are at least as good as before when $R = R_L$.
- If $\Pi(\tilde{R} = R_H) > 0$ and $\Pi(\tilde{R} = R_L) > 0$, then $L_H = L_L = \alpha$ (from lemma 4), $E(\Pi) = 1 - i + iE(\tilde{R}) - r_1\alpha - r_2(1-\alpha)$, which is increasing in i because $E(\tilde{R}) > 1$. Banks should hold as much of the risky asset as possible.

Step 2: $(\alpha, 1)$, $(+, 0)$[24] is impossible in equilibrium.

Consider $r_2' = \frac{(1-r_1\alpha)R_H}{1-\alpha}$. It is easy to prove $i = 1 - r_1\alpha$ still maximizes the banks' profits (in fact, any i will lead to zero profit). Under the new contract, the withdrawal and profit profile should be $(\alpha, 1)$ and $(0, 0)$, and the expected utility $E(U') = \pi[\alpha u(c_1) + (1-\alpha)u(c_2')] + (1-\pi)[\frac{1-i\tau}{r_1}u(c_1) + (1 - \frac{1-i\tau}{r_1})u(x)] > E(U)$. Banks will have the incentive to use the new contract to attract more deposits.

Step 3: (α, α_1), $(+, 0)$, where $\alpha < \alpha_1 < 1$, is impossible.

Because $E(\Pi) > 0$, we know from step 1 that $i = 1 - r_1\alpha$. For $\alpha < L_L < 1$ to be satisfied, it must be a run-proof contract ($1 - i\tau > r_1$). Now consider a new contract $r_2' = \frac{(1-r_1\alpha)R_H}{1-\alpha}$. It is easy to show that the new contract leads to a higher expected utility for investors and reduces the banks' profits to zero.

C Loosening Assumption on Decision Sequence

In this paper, I assume that agents make their withdrawal decisions according to an exogenously given sequence: impatient agents first, then patient agents make decisions one after another. This assumption is not necessary. In this part I show that Proposition 1 is still valid when this assumption is relaxed.

C.1 Random decision sequence

If the withdrawal sequence is randomly determined, the Subgame Perfect Nash Equilibrium is still unique, and the equilibrium outcome is the same as under the pre-determined sequence. I still use figure 3 for illustration.

- Case 1 contract: Using backward induction, it is trivial that each impatient agent will choose to withdraw, and each patient agent will choose to stay because they know all other patient agents will also choose the same strategy. There is no bank run.

[24] The first pair of numbers represent the aggregate early withdrawal in equilibrium, and the second pair of signs represent the amount of bank profits when the return is high and low, respectively.

- Case 2 contract: in this situation, each agent knows withdraw is a better strategy than wait, therefore he will take his opportunity whenever it is his turn to make his decision. A complete bank run happens. The only difference is that some patient agents might get repayment before some of the impatient agents.

- Case 3 contract: the equilibrium strategy for patient agents is a little different in this case. They will choose to wait if they observe that less than $1 - L^*$ agents did not withdraw their deposits, and choose to withdraw otherwise. However, the same partial bank run happens under the new assumption.

C.2 Endogenized Decision Sequence

Even if all agents are allowed to choose their decision timings, the property still does not change. Following the same methodology, it is not difficult to show that in all three cases, the equilibrium outcomes are the same as in the benchmark model.

The above proof still use the assumption that each agent has complete information on withdrawal history. If this assumption is relaxed and withdrawal sequence is random, however, the problem will become much more complicated and the conclusion might no longer be valid. Intuitively, each agent's decision should depend on the return signal and his expectation on the types of the following agents. A Bayesian Nash Equilibrium might be an insightful tool for the study in this direction. See Banerjee (1992) for related discussion.

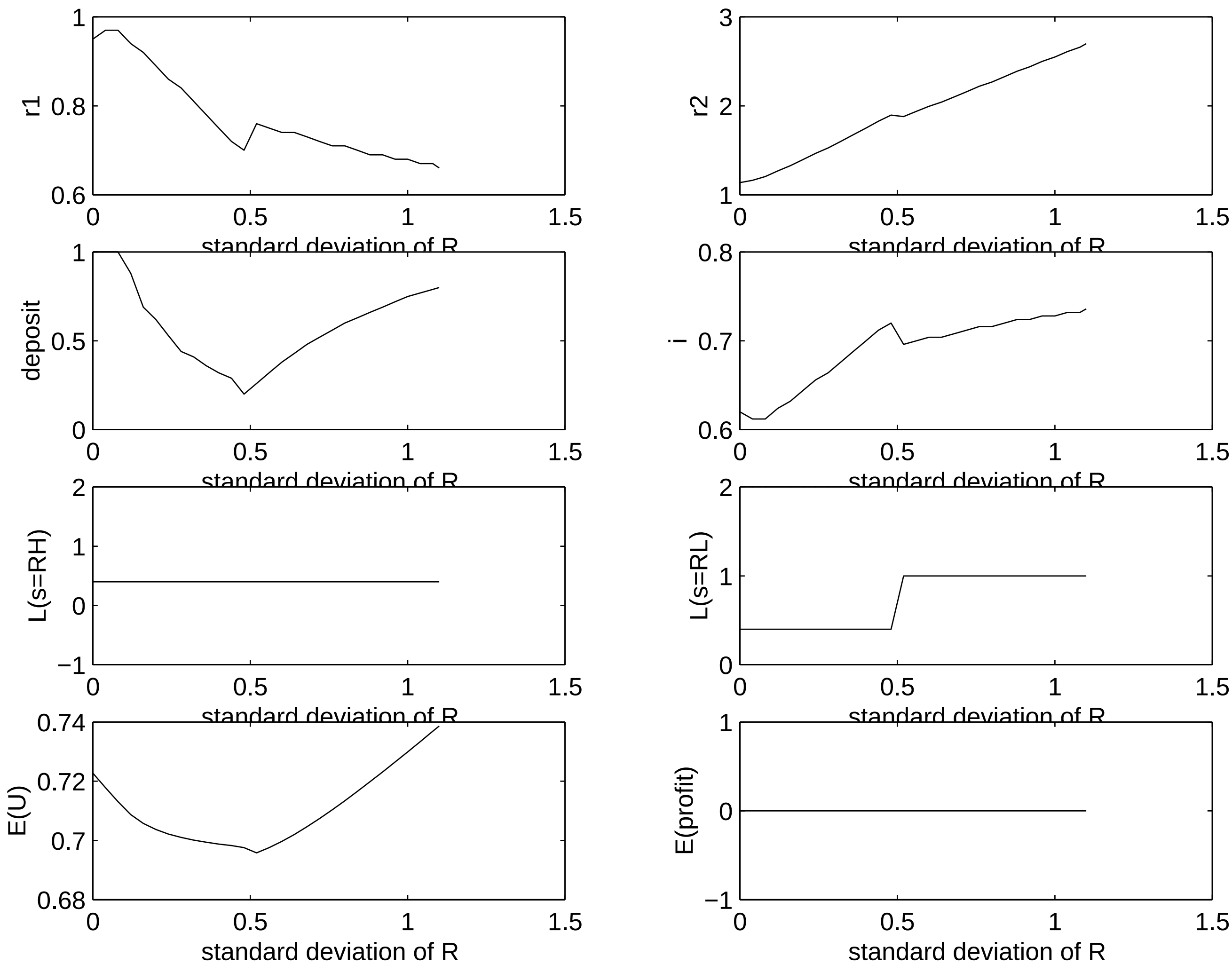


Figure 5: optimal contract vs. return volatility