Inequality, Technology Adoption and Development*

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Abstract

This paper incorporates education in a model of the Big Push where industrialization requires skilled labor. We show that initially unequal distribution of wealth can prevent the economy from taking a sustainable industrialization path. Moreover, financial imperfections, by affecting the elasticity of skill supply, imposes a constraint on the set of technologies that can be adopted. In particular, in this context, it is shown that the new organizational innovations in the North may widen the productivity gap between North and South.

Keywords: Income Distribution, Credit Constraints, Human Capital, Organizational Change, Productivity Differences

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Introduction

Why isn't the whole world developed?

The answer of Easterlin (1981) concentrates on human capital. He argues that human capital is the prime engine of development, and this is the absence of enough human capital that prevents some countries from developing. But if human capital is an important determinant of economic development, why these underdeveloped countries don't accumulate human capital? A first answer to this question is financial imperfections. Indeed, Galor and Zeira (93) show how credit market imperfections prevent the poor from investing in education, when this investment is indivisible, and lead to a steady state where the econmy is polarized, with a group of educated dynasties and an other group of unskilled dynasties. Banerjee and Newman (1993), although not concerned with human capital, show how wealth distribution, in a world with credit constraints, determines the distribution of occupations in the economy[1].

In this paper, we share the view of Easterly seriously, and show how wealth inequality, by affecting human capital accumulation, may be a barrier to technology adoption. To this purpose we assimilate economic development with industrialization (i.e. the use of increasing returns to scale technologies) and show that the success and speed of the development process is constrained by the distribution of wealth. In a sense, our paper is in the spirit of Galar and Zeira (1993) and Banerjee and Newman (1993). However, Galor and Zeira (1993) are not concerned with growth issues nor technological adoption. Banerjee and Newman (1993) are more concerned with the influence of wealth distribution on the structure of contracts in the development process and not realy on growth nor technological development.

We start from the empirical and historical evidence that industrialization of LDCs takes the form of the adoption of technologies developed in the North, and the implementation of these technologies necessitates human capital[2].

Skilled workers are needed to produce, assimilate the new technology and organize production. We assume that there are fixed costs of entry in the modern sector, which are complementary to unskilled labor[3]. We then show that this creates a positive link between the measure of educated workers

[1] Precisely, they show that wealth distribution will determine whether the economy will remain populated by small proprietors, artisans, and peasants or become a nation of entrepreneurs employing industrial workers in large factories

[2] This is the assimilationist view of development

[3] Although we don't have capital, one can think about this costs as machines and organizational capital complementary to unskilled labor.

and the wage they receive. Thus, as in Aghion and Bolton (1997), we exhibit a kind of trickle-down effect via an increase in the wage of skilled workers. A rise of the educated work force will push up the returns to education and reduce the burden of the financial imperfection on skill accumulation.

But depending on the initial distribution of wealth, industrialization can either start and be sustainable or it may fail to begin or be unsustainable.

Moreover we show that the set of technologies that can be adopted is constrained by the initial distribution of wealth. Specifically, in our model, the more equal is wealth distribution the more productive is the technology adopted. We know of no paper that links the productivity of the technology adopted by an country with its wealth distribution.

Our paper is related to the work of Murphy, Shleifer and Vishny (1989) who formalize the idea of the big push, which goes back to Rosenstein-Rodan (1953), in a simple static model with demand spillovers. But in their model, nothing permits to selec the equilibrium that the economy may reach (zero industrialization or industrialization). The vision of their Big Push is a market size one. Recently Alberto Ades and Edward Glaeser (1999) show that data support a much stronger relation between growth and initial wealth among closed economies. Their findings support the importance of the extent-of-the-market and aggregate demand in fostering growth. However, their emphasis is on initial wealth and not initial distribution of wealth.

Our model assumes a small open economy, and consequently the market size effect is absent and our economy may fail to grow even if it is rich enough to finance education for every body. So wealth distribution is really what is important, and we derive some results that are consistent with Deininger and Squire (1998). The authors use cross country data on assets distribution (land) and show that (i) there is a strong negative correlation between initial distribution of assets and long run growth and (ii) inequalities reduce the income growth of poor but not for the rich. In section 2.2, where we allow the productivity of skilled workers to depend on the level of education, we show, using the result of section 2.1, that wealth distribution affects the level of education which determines the long-run growth level.

This paper is also related to Acemoglu and Zilliboti (1999) who explain the productivity differential between the North and the South by the fact that machines produced in the North are developed for the north which has more skilled people than Southern countries. Consequently, although using the same machines, LDCs are less productive. However our focus is rather on internal organization of firms. Indeed, we consider that importing a machine is also importing an organization of work. If we consider that there is a complementarily between the quality-productivity of a machine and the quality of the organization of the firm, then on top of not being endowed with the skill level necessary to operate the machine at its best level, developing countries do not have also the right internal organizations of firms to operate the machine at its most efficient level.

In section 2.1 we look at the possible impact of these new "skill consuming" organizational innovations in the North on the development of LDCs. We show that rising organizational costs can constrain an inegalitarian economy to adopt low-productivity technologies. The main reason is that the supply of skill, under credit imperfection, can not respond enough to rising organizational costs.

Finally this paper is an argumentation in favor of redistribution of industrialization fruits in order to accelerate the process of economic development and to allow the economy to switch to more productive technologies.

It is organized as follows. Section 1 presents the model. Section 2 deals with the interaction between income distribution and technology choice. Section 3 presents an extension allowing rent seeking. All the proofs are relegated to an Appendix.

1 Framework

We build on Galor and Zeira (1993) and Murphy, Shleifer and Vishny (1989).

1.1 Firms

We consider a small open economy producing a continuum of goods. The world is endowed with a storage technology which yields r units of any good per unit stored[4].

There is a continuum of sectors each producing a good indexed by $q \in [0, 1]$. A good can be produced either with a backstage technology with the production function $y = l$, where l is physical labor, or with an industrial technology[5] which necessitates when implemented at t an entry cost $\Upsilon(\phi_t, X_t)$ in terms of unskilled labor, where ϕ_t is the measure of skilled workers available at t and X_t is the number of industrialized sectors. After the cost of entry is incurred, the production necessitates $\frac{1}{\alpha}$, with $\alpha > 1$, units of skilled labor

[4] So we don't have capital in our model. Since capital is not central to our discussion, we can abstract from it. Although capital constraint can be an other part of our story.

[5] Only one firm (a monopolist) can implement the modern technology. We take the assumption that firms live one period. So the fixed cost is incurred each period by the new monopolist, but it is decreasing in time because of the knowledge spillover.

per unit of output[6][7].

The expression of the fixed cost deserves some comments. For a mater of tractability we choose a simple form specified by $\Upsilon(\phi_t, X_t) = F(1-\phi_t)/X_t$, where F is a constant, X_t is an aggregate spillover effect which means that the greater the number of firms (or sectors) which have adopted the new technology, the lower the cost of entry of a marginal firm (or sector)[8]. So the effect of human capital is embedded in the term $1/X_t$, which is taken to be external to the firm. Thus it can be interpreted as a "learning by doing" externality *à la* Arrow (1962). As more sectors use the technology there is a public knowledge generated that increases the efficiency use of the modern technology (better understanding of the production process, more efficient organization of labor..)[9]. Here we take the number of industrialized sectors as a proxy of the knowledge generated[10]. Finally, when $X_t = 0$, the cost of entry is infinite, but this is only a simplification, and we could just have supposed that this cost of adoption is too high for a single firm when no other firm adopts the technology[11].

The term $(1-\phi_t)$ is just a way of getting vanishing fixed costs in order to permit the transition towards an economy producing with the constant

[6]So the fixed cost permits to save on the variable cost of producing one unit of output but the labor required is skilled labor. In an appendix available upon request, we show that allowing a production function in the modern technology which uses unskilled and skilled workers (for example $A(L_s)^{\beta}(L_u)^{1-\beta}$), rather than the $1/\alpha$ units of skilled labor per unit output, complicates seriously the calculus but does not change the analysis.

[7]In an appendix, available upon request, we show that using a more conventional way of modelling the knowledge externality (namely in the production function rather than in the entry cost) doesn't alter the analyses.. We found it simpler to use it in the fixed cost.

[8]The economy as a whole preforms better the new technology. But note also that the fact that F is divided by X_t means that there is a kind of "sharing" of the fixed cost between all sectors. Indeed without the term $(1-\phi_t)$, we see that the aggregate fixed cost is constant through the industrialization process and equal to F, but the effect of industrialization is to reduce the cost paid by and individual firm equal to $\frac{F}{X_t}$.

[9]Note that as we choose the fixed cost in terms of unskilled labor, it means that the cost of adoption is a cost of organization complementary to unskilled labor. As X_t will be determined by the mass of skilled workers, the knowledge externality is consequently generated by human capital which allows a better understanding of the technology and allows the reduction of organizational costs.

[10]Here we don't have R&D that reduces the costs, as in Peretto (1999a and 1999b). We only have a knowledge generated by production. The fact that this knowledge decreases the fixed cost is not restrictive. We could have chosen, as is conventional, to put this externality in front of the production function. Here we choose rather to underline that the cost of entry is an important factor in the development process, and it is rather more relevant when we suppose that the technologies are imported from the North.

[11]Or we could take an other specification for the externality, $f(X_t)$ with $f(0) > 0$ and $f' > 0$.

returns to scale technology using skilled labor $y = \alpha l$ in all sectors[12].

When a firm decides to implement the new technology it replaces the competitive fringe which produced the good with the backstage technology. As in Murphy and *alii.* (1989), because demand is inelastic the price of any good will be 1, and the wage paid to unskilled workers is 1 too (marginal productivity)[13].

At period t each firm takes as given aggregate demand d_t, the wage paid to skilled workers w_t^s, the measure ϕ_t of skilled workers available at t, and the fraction of industrialized sectors X_t. Then the profit of a firm producing an industrialized good q is

$$\pi_t(q, \phi_t, X_t) = a_t d_t - \frac{F(1-\phi_t)}{X_t} \ , \tag{1}$$

where $a_t = 1 - \frac{w_t^s}{\alpha}$. Thus aggregate profits are[14]

$$\Pi_t(\phi_t, X_t) = \int_0^{X_t} \pi_t(q, \phi_t, X_t)\, dq = aX_t d_t - F(1-\phi_t) \ , \tag{2}$$

The equilibrium condition on the labor market for skilled people is $X_t d_t/\alpha = \phi_t$. Using the expression of the profit of a firm producing an industrialized good q (equation (1)) and aggregate profits (equation (2)), we get

$$\Pi_t(\phi_t, X_t) = a_t \alpha \phi_t - F(1-\phi_t) = \left(1 - \frac{w_t^s}{\alpha}\right) \alpha \phi_t - F(1-\phi_t) \ , \tag{3}$$

which we will write as $\Pi(w_t^s, \phi_t)$.

1.2 Households

The economy is composed of overlapping generations, and each generation contains a continuum of size 1 of individuals. An agent lives two periods and

[12] All the results derived in the paper are still holding if we take an entry cost of the form F/X_t, that is to say without the term $(1-\phi_t)$. The difference will simply be that there will still remain a cost F of entry at the new stationary equilibrium (we will come back to this case later on), and F will determine the fraction of unskilled people in the long run (the details are available upon request). Just for a matter of analytical convenience, we keep the form $F(1-\phi_t)/X_t$.

[13] Alternatively as we deal with a small open economy, it could be more convenient to assume that prices are given by the rest of the world.

[14] As the identity of the good does not matter, we assume that those sectors which first industrialize are those with lower indexes.

is endowed each period with one unit of labor which he supplies inelastically. The utility of an agent born at $t-1$ is

$$(1-\beta)\int_0^1 \ln x_t(q)dq + \beta \ln e_t \ , \tag{4}$$

where $x_t(q)$ is the consumption of good q and e_t is the bequest he gives to his offspring. Then when income[15] is y_t, utility maximization simply yields[16] $x_t(q) = (1-\beta)y_t$ and $e_t = \beta y_t$. The indirect utility is therefore $U(y_t) = v + 2\ln y_t$, where $v = (1-\beta)\ln(1-\beta) + \beta \ln \beta$.

During the first period of his life he decides wether to acquire education (to become skilled) or not. If he decides not to become skilled he works as an unskilled worker (n) during both periods of life. Becoming skilled requires a fixed indivisible investment h. When the young decides to become skilled, he has to spend h, devote all his "youth-time" to learn and he works as a skilled worker (s) when adult[17][18]. During the second period of his life the agent has an offspring and allocates his income between consumption and a bequest to his child. For simplicity we assume that agents only consume in the second period. When the bequest is sufficient to pay h, the young will go to school (this will be the case in the model given that income is sufficiently higher for skilled people), when it is not the case the young will have to borrow. But financial imperfections will prevent poor people to borrow, and they will stay poor as their parents.

Noting $y_t^n(e_{t-1})$ and $y_t^s(e_{t-1})$, the expected incomes in the second period of life respectively when unskilled and skilled and when the bequest received is e_{t-1}, then a young will decide to become skilled if $U[y_t^s(e_{t-1})] > U[y_t^n(e_{t-1})]$, which is equivalent to $y_t^s(e_{t-1}) > y_t^n(e_{t-1})$.

The world interest rate is r (independent of time), and we assume that because of financial imperfections the borrowing interest rate is $i > r$ (see Zeira and Galor (1993) to see how to get simply an $i > r$).

1.3 Education decision

A young individual will face the choice in the first period of his life between going to school or working as a laborer. Given that the indirect utility

[15] Recall that agents consume only in second period of life, thus what we call income is first plus second period earnings

[16] As in Murphy et al (1989), an individual spends an equal fraction of income on each good. Thus demand is the same for all sectors.

[17] The amount h is redistributed in the economy by, say, wages to teachers.

[18] h is in terms of all the goods, that is to say $\int_0^1 h = h$

function he gets depends only on his lifetime income, he will choose the occupation that gives him the higher expected lifetime income.

A young agent born at date $t-1$ who received a bequest $e_{t-1} \geq h$, an who expects future wage of skilled workers w_t^s, will compare the incomes he will get as a skilled and unskilled

$$y_t^s(e) = w_t^s + (1+r)(e-h) , \quad (5)$$

$$y_t^n(e) = 1 + (1+r)(e+1) , \quad (6)$$

When he decides to acquire education, he pays the cost h, saves the remain amount of his bequest $e-h$ at the rate r and receives the expected wage w_t^s. When he chooses to become a laborer, he saves his bequest and earns the wage equal to 1 each period of his life. Thus he will choose education iff $y_t^s(e) \geq y_t^n(e)$, which is equivalent to

$$w_t^s - \{(2+r) + (1+r)h\} \geq 0 , \quad (7)$$

We will note $\widehat{w} = (2+r) + (1+r)h$.

We examine now the decision of someone who received $e < h$: the incomes he will get if he is respectively unskilled and skilled are

$$y_t^n(e) = 1 + (1+r)(e+1) , \quad (8)$$

$$y_t^s(e) = w_t^s + (1+i)(e-h) , \quad (9)$$

When he chooses to become a laborer, he saves his bequest and earns the wage equal to 1 each period of his life. When he decides to acquire education, he has to borrow the difference between his inherited wealth e and the cost of education h, but at a higher interest rate than r, and receives the expected wage w_t^s. Thus he will choose education iff

$$w_t^s - \widehat{w} + (i-r)(e-h) \geq 0 , \quad (10)$$

Which is equivalent to

$$e \geq \tilde{e}(h) = h - \frac{w_t^s - \widehat{w}}{i-r} , \quad (11)$$

Equation (11) traduces the fact that the investment of h on education must give him a reward greater than the extra cost of the funds he gets to finance education.

Lemma 0 Profits are null and the wage of skilled workers at t is given by

$$w_t^s = w(\phi_t) = \alpha - \frac{F(1-\phi_t)}{\phi_t} , \quad (12)$$

Lemma 0 says that entry in the modern sectors will drive profits to zero. One has to note that profits are null because of the competition between firms who want to attract skilled workers in order to use the modern technology. Profits will be absorbed by wages of skilled workers and the fixed costs. First, because an extra entry of firms into the modern sectors requires skilled workers. But as individuals are unequally endowed with wealth, attracting an extra fraction of young into the education sector necessitates higher wages. So wages will increase until profits are exhausted. Second, as the fixed cost is decreasing, skilled workers' wages will indeed absorb a higher fraction of profits until all individuals have acquired education, and thus the fixed cost has disappeared (at the new stationary equilibrium if industrialization is achievable)[19].

Assumption 1 $\widehat{w} < \alpha$

This assumption is necessary to eliminate the trivial case where $\phi = 0$, i.e. the case where industrialization is "*technically*" impossible. We now state *Proposition 1* which sumarizes the discussion above, using equations (7), and (11).

Proposition 1 *Given G_{t-1} the distribution of wealth at date $t-1$ and the arbitrage conditions* (7) *and* (11), *the measure of young agents who will choose education at t exists and is the fixed point of the mapping $x \longrightarrow 1 - G_{t-1}\left[h - \frac{w(x)-\widehat{w}}{i-r}\right]$.*

Indeed, in Appendix we show that the solution of *Proposition 1* may be $\phi_t = 0$ if the initial distribution of wealth is too unequal. We also show that there are conditions under which multiple equilibria are possible. In particular we show that an equilibrium defined as follows exists.

Definition of equilibrium *A perfect foresight equilibrium is a sequence $(G_{t-1}, \phi_t, X_t, w(\phi_t))_{t=1}^{\infty}$, where G_{t-1} is the distribution of wealth at $t-1$, such that*

i) Given G_{t-1}, and future expected wage $w(\phi_t)$ there are ϕ_t young agents at t-1 who decide to become skilled

ii) Given ϕ_t , X_t sectors industrialize

iv) ϕ_t is the fixed point of the mapping $x \longrightarrow 1 - G_{t-1}\left[h - \frac{w(x)-\widehat{w}}{i-r}\right]$, and $w(\phi_t)$ by (12)

(v) Markets clear

[19]Remark : in the case we take rather $\frac{F}{X_t}$, the rise of the wage of skilled workers will be limited by the remaining fixed cost F.

One can at this stage look at the impact of wealth distribution on the equilibrium value of ϕ. More precisely, let us take an example. Assume $G(x) = (\frac{x}{\bar{h}})^{\gamma}$ where $\bar{h}$ represents the maximum wealth level in the economy. Define $f(x) = 1 - G\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$ and $g(x) = h - \frac{w(x)-\widehat{w}}{i-r}$. Then we easily get the following relation $-x\frac{f''}{f'} = 2\left[1 + (1-\gamma)\frac{x}{2}\frac{g'}{g}\right]$.

So take $G(x) = (\frac{x}{\bar{h}})^{\gamma}$ and $H(x) = (\frac{x}{\bar{h}})^{\gamma'}$ with $1 > \gamma > \gamma'$, consequently G is less concave than H. Define $f_{\gamma}(x) = 1 - G\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$ and $f_{\gamma'}(x) = 1 - H\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$, then we get $-x\frac{f''_{\gamma}}{f'_{\gamma}} > -x\frac{f''_{\gamma'}}{f'_{\gamma'}}$. Assuming[20] a range of parameters such that $f_{\gamma'}$ is concave (so it will be for f_{γ}), then $f_{\gamma'}$ is less concave[21] than f_{γ}. As f_{γ} is more concave than $f_{\gamma'}$, one would expect to get a lower value of ϕ under f_{γ} than under f_{γ}. But, as will be more extensively discussed, f_{γ} is above $f_{\gamma'}$ and consequently the equilibrium value of ϕ under the G distribution is above its value under the H distribution. However, the result that f_{γ} is above $f_{\gamma'}$ comes from the first order stochastic dominance of G, *i.e.* $\forall x\ G(x) \leq H(x)$. In section 1.5, we will show that the first order stochastic dominance can be weakened.

When $\gamma \geq 1$, f_{γ} is always concave, and in this case the results are more obvious, and the higher γ the higher the level of ϕ.

We turn now to the dynamics of wealth and industrialization.

1.4 Wealth and Industrialization dynamics

To study the dynamics of industrialization, that is to say the dynamics of ϕ_t, using *Proposition 1*, we see that we have to derive the dynamics of wealth distribution function G_t.

At period $t-1$ the distribution of wealth G_{t-1} is predetermined. In order to determine the period t distribution G_t, we must characterize the dynamics of $(e_t)_{t\in N}$. Let us define $v_t = e_t - h$, $R = 1+r$, $I = 1+i$, $\theta(\phi_t) = \beta w(\phi_t) - h$, $\widehat{\theta} = \beta\widehat{w} - h$ and note $v_t = v_t^i$ if wealth belongs to category (i) defined by : (1) is the child of a wealthy parent who worked as a skilled, (2) is the child of a parent who borrowed to become skilled (an indebted parent), (3) is the

[20] See Appendix B.2.1 for a more detailed discussion.

[21] Note that this reversion of concavity is intuitive. Indeed, since $g(x)$ is a decreasing function and $\lim\limits_{0} g(x) = +\infty$, one gets that the density function of G decreases less rapidly, as the level of wealth increases, than the density of H. Consequently as around $\phi = 0$, $g(\phi)$ takes its values on the upper levels of wealth, the reversed concavity is obtained. In Appendix we show that around $\phi = 0$ we have $f_{\gamma'}(\phi) << f_{\gamma}(\phi)$.

child of an unskilled. The term $\theta(\phi_t)$ is the difference between the bequest an educated old individual gives to his child and the cost of education h, and $\widehat{\theta}$ is the difference between the amount an educated old laborer bequeath to his child and h.

Then using the fact that $e_t = \beta y_t$ and using expressions (5), (8) and (9) we get

$$v_t^1 = \beta R v_{t-1}^1 + \theta(\phi_t) \qquad \Longleftrightarrow \qquad v_{t-1} > 0 \ , \tag{13}$$

$$v_t^2 = \beta I v_{t-1}^2 + \theta(\phi_t) \qquad \Longleftrightarrow \qquad v_{t-1} \in \left[-\frac{w(\phi_t) - \widehat{w}}{i - r}, 0 \right] \ , \tag{14}$$

$$v_t^3 = \beta R v_{t-1}^3 + \widehat{\theta} \qquad \Longleftrightarrow \qquad v_{t-1} \in \left] -\infty, -\frac{w(\phi_t) - \widehat{w}}{i - r} \right] \ , \tag{15}$$

We then make the following assumption.

Assumption 2 $\beta\alpha - h > 0$ *and* $\beta\widehat{w} - h < 0$

The first inequality ensures that the equilibrium where every body has acquired the level h of education is a stationary equilibrium (it ensures that the transmission of wealth between generations is enough to finance h). The second inequality guaranties that a dynasty of unskilled is trapped in a "low wealth" point (otherwise the model is irrelevant!).

Assumption 3 $h - \frac{\alpha - \widehat{w}}{I - R} > 0$

This inequality is necessary in order not to get the trivial case where $\phi = 1$ whatever wealth distribution.

Further, with some abuse of language, we will talk about "moving attraction point" for a sequence of real numbers with reference to another sequence of points towards which the first sequence is attracted. For example in equation (17), if ϕ_t was constant then the sequence $(v_t^2)_t$ would converge towards $\theta(\phi_t)/1 - \beta R$.

The three sequences defined by equations (17), (18) and (19) have 3 moving attraction points which are respectively

$$\frac{\widehat{\theta}}{1 - \beta R} < \frac{\theta(\phi_t)}{1 - \beta R} < \frac{\theta(\phi_t)}{1 - \beta I} \ , \tag{16}$$

The first one is the moving attraction point of $(v_t^3)_{t \in IN}$, the second is the moving attraction point of $(v_t^1)_{t \in IN}$ and the third one is the moving attraction point of $(v_t^2)_{t \in IN}$.

Figure 2 in appendix draws the evolution[22] of $(v_t)_{t\in IN}$.

Please insert FIGURE 2

Looking directly at the figure one sees that when v_{t-1} is above $\theta(\phi_t)/(1-\beta R)$, then the bequest is decreasing but remains above h. When v_{t-1} is under $\theta(\phi_t)/(1-\beta R)$, then the bequest is increasing. When $v_{t-1} \geq \theta(\phi_t)/(1-\beta I)$ then $v_t \leq v_{t-1}$, and if $v_{t-1} \leq \theta(\phi_t)/(1-\beta I)$ then $v_t \geq v_{t-1}$. Finally if $v_{t-1} \geq \widehat{\theta}/(1-\beta R)$ then $v_t \leq v_{t-1}$, and if $v_{t-1} \leq \widehat{\theta}/(1-\beta R)$ then $v_t \geq v_{t-1}$.

This will be the starting point of the study of wealth dynamics. We will prove two lemma and a proposition that will enable us to describe entirely the pace of industrialization without explicitly determining point by point the sequence of distribution functions. Before we can intuitively describe what happens. At the beginning of the first period, the economy begins with a continuum of old unskilled workers among whom wealth is distributed according to the concave distribution function G_0^o, the upper indexed designing old. Then these old individuals bequeath to their child a fraction β of their wealth, and the distribution function of the bequest is G_0 (concave).

Using *Proposition 1*, G_0 will determine ϕ_1, and using Figure 5 one also gets that the old of period 1 (those who were young at the beginning of the world) who inherited more than $\bar{e_1}= h - \frac{w(\phi_1)-\widehat{w}}{i-r}$ will bequest more than they received, and those who received less than $\bar{e_1}$ will bequest more if their wealth was under the poverty attraction point and less if above the poverty attraction point. Thus, intuitively, we need to know how is $\bar{e_1}$ relatively to the poverty attraction point $h + \frac{\widehat{\theta}}{1-\beta R}$ (a dynasty of poor will converge towards this wealth level). Fortunately, given the assumptions we made, $\bar{e_1}$ will be under $h+\frac{\widehat{\theta}}{1-\beta R}$ when industrialization is sustainable, and consequently wealth will be globally increasing (to be defined soon).

We will then use a convenient and intuitive result that states that if one distribution dominates in the first stochastic order sense an other distribution, then it will be associated with a higher level of industrialization. This will enable us to show that the sequence $(\phi_t)_t$ is increasing, and then using recursively the lemmas, one proves that it converges towards one. But all this discussion holds only, as stated by *Proposition 3*, when industrialization can begin and is sustainable.

[22] That is to say : the difference between the bequest an old laborer received when he was young and the bequest he gives to his child. The difference between the bequeath an old in debt skilled received when he was young and the bequest he gives to his child. And finally the difference between the bequest an old wealthy skilled received when he was young and the bequest he gives to his child

However, we will show that we can compare two industrialization paths under two different initial distributions of wealth that cannot be compared in the first stochastic order sense.

We will say further that wealth is "globally increasing" if for the relevant range of wealth levels, *i.e.* around h, each date more people cross the line defined by h. Lemma 2 below shows that wealth is globally increasing if $\theta(\phi_t) \geq 0$ and $-[w(\phi_t) - \widehat{w}]/(i-r) \leq \widehat{\theta}/(1-\beta R)$. This last inequality expresses that the threshold level for education i.e. $h - [w(\phi_t) - \widehat{w}]/(i-r)$ is under the fixed point of the wealth of "unskilled dynasties" i.e. $h + \widehat{\theta}/(1-\beta R)$. In appendix it is shown that this last inequality is always holding for taking-off economies. The first inequality $\theta(\phi_t) \geq 0$ is required for industrialization to be sustainable, because if $\theta(\phi_t) < 0$ then wealth is decreasing and *lemma 2* below shows that it means "de-industrialization".

Lemma 1 *When* $\theta(\phi_t) \geq 0$ *wealth is globally increasing and* $-[w(\phi_t) - \widehat{w}]/(i-r) \leq \widehat{\theta}/(1-\beta R)$

The first part of the lemma says that each individual receives from his parent more than his parent received from his grand parents. The second part says that the threshold level of education is under the poverty trap.

Lemma 2 *Let* $H(x)$ *be a real valued function decreasing and convex, and* G *and* T *two distribution functions. If* $T(x) \leq G(x)$ $\forall x$, *then the solutions* ϕ_G *and* ϕ_T *of the following equations* $1 - \phi_G = G(H(\phi_G))$ *and* $1 - \phi_T = T(H(\phi_T))$ *verify* $\phi_T > \phi_G$

Lemma 2 demonstrates an intuitive result, which is that when one distribution function T dominates an other distribution function G in the sense of first order stochastic dominance, then industrialization is higher when wealth is distributed according to H.

Then, as announced, Lemma 1 and lemma 2 enable us to study the dynamics of industrialization without deriving explicitly G_t.

The fact that $-[w(\phi_t) - \widehat{w}]/(i-r) \leq \widehat{\theta}/(1-\beta R)$ is sufficient to show that ϕ_t converges towards 1 because this inequality and the fact that $(\phi_t)_t$ is increasing guaranty that there will not subsist any "dynasty of unskilled". This is simply because the poor are attracted by the point $\widehat{\theta}/(1-\beta R)$ and the threshold level for education is under this point, then as $(\phi_t)_t$ is increasing the threshold level will decrease and as wealth levels are attracted by the moving points $h + \frac{\theta(\phi_t)}{1-\beta R}$ and $h + \frac{\theta(\phi_t)}{1-\beta I}$, every dynasty of poor would cross one day a threshold level and become a dynasty of wealthy and educated individuals[23].

But this will hold only if certain conditions are satisfied, and this is the object of the following proposition.

Proposition 2 *For Industrialization to begin and to be sustainable it is necessary and sufficient that (i)* $w(\phi_1) \geq \widehat{w} \Leftrightarrow \phi_1 \geq \phi^* = \frac{F}{\alpha - \widehat{w} + F}$ *and (ii)* $\theta(\phi_1) \geq 0 \Leftrightarrow \phi_1 \geq \phi^{**} = \frac{\beta F}{\beta\alpha - h + \beta F}$

So it is equivalent to $\phi_1 \geq \Omega = \max(\phi^{**}, \phi^*) = \phi^{**}$

So when wealth is very unequally distributed, the economy cannot industrialize. Note also that, if the economy as a whole is not constrained, in our model all wealth above h could be efficiently redistributed to the poor and make industrialization possible. One can also argue that, given the equation that gives the equilibrium value of ϕ, we do not necessarily need to get an important class of "rich", what we rather need is a sufficient concentration of people at any "non too low level of wealth". Indeed, look at the most favorable case where at the beginning of the world every body has a bequest greater or equal to $h - (\alpha - \widehat{w})/(I - R)$, then industrialization will be complete as soon as the first period.

Concretely, for any $\tilde{\phi} \in [0, 1]$, we can find a distribution $G_{\tilde{\phi}}$ such that $\tilde{\phi}$ is the unique solution of the equation[24] $x = 1 - G_{\tilde{\phi}}(h - (w(x) - \widehat{w})/(I - R))$. One can argue that the higher the level ϕ targeted, the less unequal has to be the initial distribution of wealth.

When the economy takes off, it converges towards one with the more productive technology $y = \alpha l$, but with l representing an educated work force. So what we modeled is the transition from an economy producing with a constant returns to scale technology $y = l$ using unskilled labor to an economy using a more productive constant returns to scale technology $y = \alpha l$ using a better educated work force. This transition necessitates the payment of fixed costs which are progressively reduced as the fraction of skilled workers

[23] The speed of industrialization will depend on the initial degree of inequality in wealth distribution. When wealth is initially too unequally distributed, industrialization will not at all begin or be sustainable (see proposition 3). When wealth is not too unequally distributed, then $\bar{e_1}$ will be near the poverty trap, and if there remain a lot of people under this level, industrialization will be slow.

[24] The proof is intuitive : suppose this is not the case, then $\forall G$ a distribution function on $[0, \bar{h}]$, either $\tilde{\phi} > 1 - G(g(\tilde{\phi}))$ or $\tilde{\phi} < 1 - G(g(\tilde{\phi}))$. Then take two distributions G_1 and G_2 such that $\tilde{\phi} < 1 - G_1(g(\tilde{\phi}))$ and $\tilde{\phi} > 1 - G_2(g(\tilde{\phi}))$. Then $\exists \lambda \in]0, 1[$ such that $\tilde{\phi} = \lambda(1 - G_1(g(\tilde{\phi}))) + (1 - \lambda)(1 - G_2(g(\tilde{\phi}))) =$
$1 - \lambda G_1(g(\tilde{\phi})) + (1 - \lambda) G_2(g(\tilde{\phi}))$. Then Take $G_{\tilde{\phi}} = \lambda G_1 + (1 - \lambda) G_2$, it is a distribution function (concave).

grows (because of the learning by doing effect). During this transition the wage of skilled workers progressively attain the marginal product of skilled workers i.e. α. But as it was discussed in subsection 1.1, the fact that the fixed cost is null at the full industrialization equilibrium is not essential for the results derived in this paper. The fixed cost could still be present with an other specification, for example $\frac{F}{X_t}$ and the economy would converge to an other stationary equilibrium described by $(X_\infty = 1, \phi_\infty = 1 - F)$[25].

1.5 Comparing two economies

We want here to show that we can compare two economies starting with two different wealth distributions, without the need to assume a strong condition of first order stochastic dominance.

In fact until now the first stochastic dominance criteria was used mainly to show that under a successful take off, we have $G_t(x) \leq G_{t-1}(x)\ \forall x$ and $\forall t \geq 1$.

It is a trivial result that starting with two different distribution functions G and H, if $G(x) \leq H(x)\ \forall x$, then industrialization will be higher and faster under G than under H.

However, the following lemma states that if G is less concave than H in the upper levels of wealth, then industrialization may be higher under the G distribution.

Lemma 3 *Let $g(x)$ be a real valued function decreasing and convex, and G and H any two distribution functions. If $-x\frac{G''}{G'}$ is sufficiently lower than $-x\frac{H''}{H'}$ for $x \in [\tilde{x}, \infty[$, where $\tilde{x}$ verifies $H'(g(\tilde{x})) << G'(g(\tilde{x}))$, then it is possible that the solutions ϕ_G and ϕ_H of the following equations $1 - \phi_G = G(g(\phi_G))$ and $1 - \phi_H = H(g(\phi_T))$ verify $\phi_G > \phi_H$*

The intuitive explanation of Lemma 3 is illustrated by the following figures.

[25] Details are available

Figure a

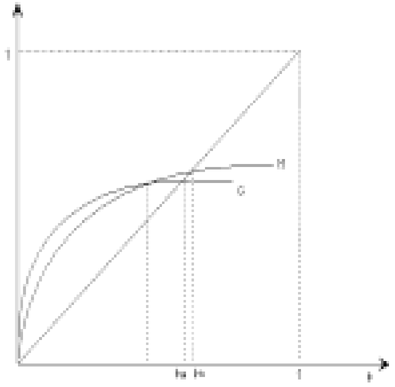

Figure b

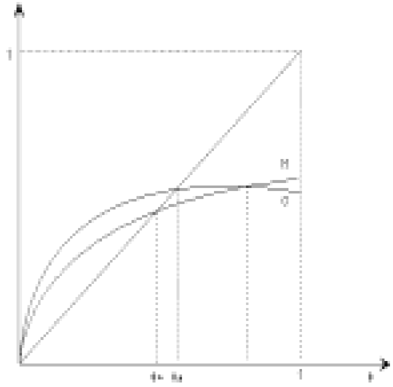

Figure a depicts the case where the concavity[26] of G is not low enough relatively to the concavity of H for $1 - G(g(\phi))$ to cross the $45°$ line after $1 - H(g(\phi))$. Figure b represents the favorable case where the concavity of G is sufficiently lower than the concavity of H for $1 - G(g(\phi))$ to cross the $45°$ line after $1 - H(g(\phi))$. The condition $H^{'}(g(\tilde{x})) << G^{'}(g(\tilde{x}))$ guaranties that the derivative of $\phi \longrightarrow 1 - G(g(x))$ at $\tilde{x}$ is greater than the derivative at $\tilde{x}$ of $\phi \longrightarrow 1 - H(g(x))$. This condition is necessary since otherwise $\phi \longrightarrow 1 - G(g(x))$ would be always under $\phi \longrightarrow 1 - H(g(x))$. Adding the condition that $-xG^{''}/G^{'}$ is sufficiently lower than $-xH^{''}/H^{'}$ for $x \in [\tilde{x}, \infty[$, guaranties that $H^{'}(g(x)) < G^{'}(g(x))$ for a sufficiently large interval $[\hat{x}, \tilde{x}]$ in order that despite its greater concavity, the function $\phi \longrightarrow 1 - G(g(x))$ remains above[27] $\phi \longrightarrow 1 - H(g(x))$.

2 Wealth inequality and technology choice

In this section we use the previous analysis to show the perverse effect of inequality on technology adoption.

2.1 Rising organizational costs

In this section, we consider a situation where the cost of adoption, F, increases with the level of productivity α. What we have in mind, is to consider the impact of new organizational innovations in the North on technology adoption by the South. To be clear, we consider that importing a machine is

[26] We talk about $1 - \gamma$

[27] That is to say despite the fact that the density decreases more rapidly, the distribution remains above.

also importing an organization of work. If we consider that there is a complementarily between the quality-productivity of a machine and the quality of the organization of the firm, then on top of not being endowed with the skill level necessary to operate the machine at its best level, developing countries do not have also the right internal organizations of firms to operate the machine at its most efficient level. So what we argue is that the concern is not only a problem of a lack of the right people who have the necessary skills to operate a machine. We rather think that the organization "around the machine" is as much important than the workers who directly operate the machine. Think of machine operation as one task among many others that are complementary. Although you have someone who knows how works the machine, if the workers who are performing the other tasks do not have the appropriate skills, then the productivity will be low. Note also that production is no longer, in developed countries, the main activity in the firm. If work evolves towards more interaction between workers, more communication, more involvement (see Gant, Ichinovski and Shaw (1999)), then LDCs will have some trouble to be as productive as the North. There is some evidence that new organizational innovations are more "skill consuming". These facts are documented in Bernan, Bound and Griliches (1994), Caroli and Van Reenen (1999), Greenan and Mairesse (1999).

In our modelling strategy, as the costs of organization increase, more skilled workers are needed to reduce them, and the knowledge generated by the utilization of the technology contributes to the reduction of the costs of adoption. To this purpose, the knowledge externality, modeled here by X_t, has to seen as a social-organizational knowledge.

The threshold level of ϕ defined in *Proposition 3* by $\Omega(\alpha, F) = \max(\phi_1^{**}, \phi_1^{*})$ is decreasing in α and increasing in F. As ϕ_1 depends on the initial distribution of income, we can argue that inegalitarian economies will be forced to adopt technologies with a low F.

Suppose that there is a menu of technologies described by the set

$$\left\{(\alpha, F(\alpha)) \diagup \alpha \in R^{+}\right\} \text{ with } F^{'}(\alpha) > 0$$

and suppose that F is convex, then we can show that Ω is an increasing function of α for $\alpha \geq \hat{\alpha}$, where $\hat{\alpha}$ is a constant which depends on[28] F. Thus using *Proposition 3* we get that inegalitarian economies may de constrained to adopt technologies with low return to education, namely technologies with low α.

Proposition 3 *Suppose a menu of technologies* $\{(\alpha, F(\alpha)) \diagup \alpha \in R^{+}\}$ *with* $F^{'}(\alpha) > 0$ *and* $F^{''}(\alpha) > 0$*, is available to a country. Then when initial wealth*

[28]In the examples given below, when $F(\alpha) = \alpha^2/2$, $\hat{\alpha} = 2\frac{h}{\beta}$

distribution G_0 is too unequal, the economy is constrained to adopt a low α technology

In Appendix $B.6.2$, we show that what maters is the difference between the speed at which $G^{'}$ (that is to say the density) decreases and the size of $F^{'}(\alpha)$, that is to say the marginal cost of adoption. For high values of α, the cost of adoption rises to much, and consequently more skilled workers are needed to absorb these more productive technologies. But when wealth distribution is too concave, the supply of skill can not respond sufficiently, and as a consequence these more productive technologies will not be adopted. Consequently an upward shift of the curve F (i.e. a rise of the costs of adoption because of organizational change in the North) would be detrimental for productivity in LDCs if the supply of skill is too inelastic.

Example :

Let us take[29] $F(\alpha) = A.\alpha^2/2$, $A < 1$, and $G(x) = \left(\frac{x}{h}\right)^{\gamma}$. We know that $\phi(\alpha)$ is implicitly defined by

$$\phi(\alpha) = 1 - G\left[h - \frac{\alpha - \frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I - R}\right] , \tag{17}$$

and from Appendix $B.5.3$, we know that $\phi(\alpha)$ admits a maximum[30]. Then differentiating[31] the equation above and taking $\phi^{'}(\alpha) = 0$ we get that

$$\phi^{'}(\alpha) = 0 \Longleftrightarrow \phi(\alpha) = \frac{F^{'}(\alpha)}{1 + F^{'}(\alpha)} ,$$

and putting this in equation (17), one gets that the optimal α noted α^* is defined by

$$\frac{A.\alpha^*}{1 + A.\alpha^*} = 1 - G\left(h - \frac{\frac{1}{2}\alpha^* - \widehat{w}}{I - R}\right) . \tag{18}$$

The function $\alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ is concave and one verifies that the function

[29] If we rather take $F(\alpha) = A(\alpha - \alpha_o)^2/2$, such that to make the cost of adoption lower, then the reluts of the example willbe modified such that ϕ will be higher. The constant α_o, could be taken equal to the lowest possible value of α i.e. $\frac{h}{\beta}$.

[30] We show that $\phi(\alpha)$ is $S-$shaped

[31] See appendix B.5.1 for more details

$\zeta : \alpha \longrightarrow 1 - G\left(h - \frac{\frac{1}{2}\alpha - \widehat{w}}{I-R}\right)$ is convex. If we restrict ourself to situations where α^* is defined[32][33], it is immediate that the higher γ the higher will be α^*, because of first order stochastic dominance. But one has to note that first order dominance is not necessary, as it was suggested in section 1.5. In this example, what is important is that the elasticity of the density function be not too low at wealth levels above h (by analogy with $figure\ b$ in section 1.5).

This is not only an example, although we do not really specify the "choice" of technology. One could argue that as wealth increases (this is the case in both models) sectors could choose an other technology $(\alpha', F(\alpha'))$ with a higher α'. But if there are high social costs to changing the technology, for example the "learning by doing" effect can be technology-specific, then switching to an other technology may be very costly for one sector. This effect can be even higher with an input-output structure, that is to say if each technology is associated with different sets of intermediate inputs, then the degree of irreversibility of technology choice is increased since it entails even a greater need for coordination between input suppliers and final goods producers (an also between input suppliers) etc. So what we advance is that with a kind of irreversibility in the technology choice (or at least high cost to changing the technology), wealth inequality may force the economy to adopt a low-productivity technology. Note also that when the economy is constrained to choose low-α technologies, it implies that it has less wealth to redistribute to foster industrialization, since the redistributed wealth comes from the pool of educated people extra wealth i.e. $\beta\alpha - h$ (which is increasing with α). Another consequence is also a reduced supply of skilled labor.

2.2 Decreasing returns to human capital

Let us suppose that the productivity parameter α is a function of h, and that there are decreasing returns to human capital in the production function. So let us take $\alpha(h) = Bh^{\upsilon}$ with $\upsilon < 1$. First we look at the program solved by a social planner. So we first assume that only one level of human capital is chosen.

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[32] To the extent that the intersection is in the range authorized for the productivity parameter α. Indeed, productivity is bounded above and below. First we have from assumption 2, $\alpha \geq \frac{h}{\beta}$, and second from assumption 3, we have $\alpha < (I-R)h + \hat{w}$.

Note that Assumption 3 garanties $\alpha < 2\hat{w}$ (otherwise it is immediate, looking at (17) that $\phi = 1$ is a trivial solution).

[33] See appendix B.7

2.2.1 Centralized equilibrium with invariable entry costs

For a given G, the level h that maximizes[34] ϕ will be given by taking ϕ as a function of h and differentiating the equation that determines ϕ. We obtain $\phi(h) = 1 - G\left[h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right]$ and consequently

$$\phi^{'}(h) = \left[1 - \frac{1}{I-R}\left(\phi^{'}(h)\frac{\partial w}{\partial \phi} + \alpha^{'}(h)\frac{\partial w}{\partial \alpha}\right)\right] G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right),$$

and taking $\phi^{'}(h) = 0$, one obtains $1 - \frac{1}{I-R}(\alpha^{'}(h) - R) = 0$. The level of human capital chosen is thus given by

$$h^{*} = \left(\frac{\upsilon B}{1+i}\right)^{1/(1-\upsilon)} = \left(\frac{\upsilon B}{1+r+(i-r)}\right)^{1/(1-\upsilon)}, \tag{19}$$

Note that it is the level which minimizes[35] $\tilde{e}(h)$ the threshold level of wealth that permits education. This is not surprising to obtain a negative relation between h^{*} and the "level" of financial imperfections measured here by $(i-r)$. The higher the financial imperfection the lower the level h of education and the slower the path of industrialization, and also the steady state total factor productivity $\alpha(h^{*})$. One can note that in this particular context, wealth distribution does not at all influence the level of education h^{*}, and only the extent of credit imperfection determines h^{*}. But now we can use *Proposition 2* to argue that in the case where the credit imperfection is severe (so h^{*} is low) and the entry cost F is too high, then wealth distribution can prevent the economy from taking a sustainable industrialization path.

2.2.2 Centralized equilibrium with variable entry costs

For a given G, the level h that maximizes ϕ will be given by taking ϕ as a function of α which is a function of h and differentiating the equation

[34] One would argue that this not obvious that the objective of the social planner is to maximize ϕ. Taking $\exp((1-\beta)\int_0^1 \ln x_i di + \beta \ln e)$ rather than $(1-\beta)\int_0^1 \ln x_i di + \beta \ln e$, all the calculus would remain unchanged, and the utility of the agents would be linear in lifetime income. So the social planner can choose to maximise the discounted aggregate welfare from $t = 0$ to infinity. This compatible with maximizing ϕ, first because the productivity of labor is higher in moden sectors, second because of the trickle -down growth effect of an increase in ϕ, the mass of individuals who will have access to education will increase and third proposition 1 has to be satisfied.

[35] This is indeed a minimum, since $\frac{d^2\tilde{e}(h)}{dh^2} = -\frac{1}{i-r}\alpha^{''}(h) > 0$

that determines ϕ. We obtain $\phi(h) = 1 - G\left[h - \frac{\alpha(h) - \frac{F(\alpha(h))(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right]$ and consequently

$$\begin{aligned}\phi^{'}(h) &= \left[1 - \frac{1}{I-R}\left(\phi^{'}(h)\frac{\partial w}{\partial \phi} + \alpha^{'}(h)\frac{\partial w}{\partial \alpha} - R\right) + \frac{1}{I-R}\alpha^{'}(h)F^{'}(\alpha(h))\frac{(1-\phi(h))}{\phi(h)}\right] \\ &\quad \times G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right),\end{aligned}$$

and taking $\phi^{'}(h) = 0$, one obtains $1 - \frac{1}{I-R}(\alpha^{'}(h) - R) + \frac{1}{I-R}\alpha^{'}(h)F^{'}(\alpha(h))\frac{(1-\phi(h))}{\phi(h)} = 0$, and rearranging one finally gets that the optimal level of education h_R^* verifies

$$1 + i = \alpha^{'}(h_R^*)\left[1 - F^{'}(\alpha(h_R^*))\frac{(1-\phi(h_R^*))}{\phi(h_R^*)}\right] . \tag{20}$$

In this context of rising organizational costs, we can show that wealth distribution will influence the optimal level of skill. Indeed, the term in brackets in equation (20) is positive only when $\phi(h_R^*) \geq \frac{F^{'}(\alpha(h_R^*))}{1+F^{'}(\alpha(h_R^*))}$. Equation (20) can be used to derive the expression of $\phi(h_R^*)$

$$\phi(h_R^*) = \frac{F^{'}(\alpha(h_R^*))}{1 - \frac{I}{\alpha^{'}(h_R^*)} + F^{'}(\alpha(h_R^*))} , \tag{21}$$

We assume that $F(\alpha(h^*)) = F$ the cost taken in section 2.2.1. Whereas in section 2.2.1 industrialization is not possible if the optimal level of skill is such that the distribution function does not permit a sufficient supply of skill, here equation (20) may admit a solution[36]. Looking at (20) one sees that $h_R^* < h^*$, and as we have assumed that $F(\alpha(h^*)) = F$, we understand why the inequality $h_R^* < h^*$. This is simply because by decreasing the level of skill, although it decreases also α, it also reduces the cost of adoption.

Note that taking $F(\alpha) = \alpha^2/2$, one gets that the function of h_R^* in the right hand side of equation (21) is an increasing function of h_R^*. Using all the previous results, one can argue that the more unequal is wealth distribution[37] the lower will be h_R^* and consequently the steady state factor productivity $\alpha(h_R^*)$.

[36] Indeed, α will decrease up to the point where G permits a sustainable industrialization, i.e. proposition 2 is verified. Consequently h will decrease also as α is an increasing function of h.

[37] See Appendix B.6

2.2.3 Decentralized equilibrium

Non constrained[38] individuals (i.e.e those who have a $e \geq h^*_{nc}$) would choose h^*_{nc} such that $\alpha^{'}(h^*_{nc}) = 1 + r$ and constrained ones ((i.e. those who have a $e < h^*_{nc}$) would choose h^*_c such that $\alpha^{'}(h^*_c) = 1 + i$. But as $1 + i > 1 + r$ it results that $h^*_c = h^* < h^*_{nc}$. Note that when the social planer decides on a unique education level, he chooses h^*_c, since it is the level that minimizes the number of constrained individuals. This is simply because the central planer takes into account the effect of h on the supply of skill ϕ.

This may help us assert that there has to be a strategic choice of education policy and technology adoption policy. Adopting a technology that requires a high degree of education is not a good policy in a inegalitarian country since the supply of skill will not respond. Choosing a medium technology may be better, although it is not the most efficient, as it would permit education of more people and a rapid increase of productivity[39].

We think that this is a relevant aspect of the link between inequality and development. Moreover, the positive interaction between technology adoption and income redistribution can be used to explain part of the rapid growth of Asian countries like Korea. Keller (1996) argues, for example, that trade liberalization has to be accompanied by more human capital accumulation in order to absorb the higher arrival rate of new technologies. One can then argue that the argument is not only a problem of level of human capital. Although more complex technologies necessitate higher skilled workers, we think that it also necessitates more skilled workers[40]. We then claim that income redistribution is necessary to increase the proportion of skilled workers in the economy and thus to be able to absorb more productive technologies.

3 Extension : industrialization and rent seeking

In this section we look at the impact of the existence of rent seekers in the modern sector. We add to the previous model a mass m of individuals at the bottom of the wealth distribution, who have the political power and extract

[38] We do not detail the model in this case, but the free entry will give the wage as a function of h similar to equation (12), if we assume that the skill level is observable.

[39] Remember that the speed of development depends on the interaction of wealth distribution and the cost of education.

[40] This may be a more important aspect if we consider the impact of new organizational change in the north, which is skill-intensive.

rents from the modern sector. We choose[41] a very simple way of introducing a rent seeking activity in our economy. The point is just made to show that on top of diverting productive resources, rent seeking distorts the reward of other more socially productive activities and consequently slows the pace of economic development by constraining technology adoption.

We just assume that a fraction τ of the output in modern sectors is extracted by the rent seekers and consumed. Rent seekers don't work, they spend time controlling and extracting rents from the modern firms.

It is then immediate to establish the equivalent of *lemma 0.*

Lemme 4 *Profits are null and the wage of skilled workers is given by*

$$w_t^s = w(\phi_t) = (1-\tau)\alpha - \frac{F(1-\phi_t)}{\phi_t} \tag{22}$$

The income of a rent seeker at t is then $\tau\alpha\phi_t/m$. It remains to see wether rent seeking is more profitable than working as a skilled worker in the modern sector. The condition is that

$$\tau\alpha\phi_t/m \geq (1-\tau)\alpha - \frac{F(1-\phi_t)}{\phi_t} \tag{23}$$

For small values of ϕ_t this inequality is likely to be satisfied. As ϕ_t is inversely related to τ, one can argue that rent seeking and low industrialization are positively related.

Looking at *(21)* for $\phi = 1$, we see that it is satisfied if $\tau \geq m/(1+m)$. This means that when the rent extracting group is able to "tax" a fraction of output higher than its relative size in the population, then rent seeking will not disappear with full industrialization. But if $\tau < m/(1+m)$, then rent seeking will vanish at some degree of industrialization.

The results of section 2 can be applied here. Equation (20) shows that rent seeking reduces the reward of education, and consequently the supply of skill. With endogenous choice of technology, then rent seeking will increase the burden of wealth distribution on technology adoption.

[41] A rather more rigorous treatment of the effect of the political regime on technology adoption and human capital accumulation is under examination in an other paper.

Conclusion

This paper has tried to model the consequences of wealth inequality on industrialization. Industrialization was assimilated to the use of increasing returns to scale technologies. We assumed that the industrial technologies required a fixed cost in terms of unskilled labor and then each unit of output required $1/\alpha$ units of skilled labor. We showed that when industrial profit are exhausted by entry, there is a positive correlation between the size of the skilled work force and returns to education. This positive correlation permits, as the size of the last generation educated work force increases, to the poor to progressively get access to education. When initial distribution of wealth is not too unequally distributed, the economy can take-off an *converge* to an economy with a constant returns to scale technology using skilled labor and more productive than the initial backstage technology. When the distribution is inegalitarian, then without redistribution, the economy cannot industrialize. More over as wealth above h is *socially unproductive,* redistribution is valuable and has a kind of "trickle-down" effect.

We finally emphasized something we think to be a relevant question for developing countries, which is the perverse effect of wealth inequality on technology adoption. In the paper the threshold level of ϕ which permits the take-off is positively correlated to the size of the fixed cost F. Consequently, in a world where more productive technologies necessitates a higher fixed cost, a too inegalitarian distribution of wealth may force the economy to adopt low productivity technologies.

We think that the combination of a non well functioning labor market with financial imperfections are of great importance for studying economic development and is an interesting direction for further research. For education to be stimulated, the returns to skill have to reflect the true social value of education. Indeed, growing education in the absence of a well functioning modern sector has no effect. We thus think that a further inquiry into the link between corporate governance and the labor market conditions could be an interesting direction for further research. For example, the internal organization of firms is of great importance to study human capital accumulation. Another important aspect is the determinant of surplus sharing between owners and workers in the context of financial imperfections. When financial imperfections are strong, human capital is more tied to the physical assets because workers cannot quit their employer to build their own firm, and this may create a higher distortion of the returns to education.

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B Appendix

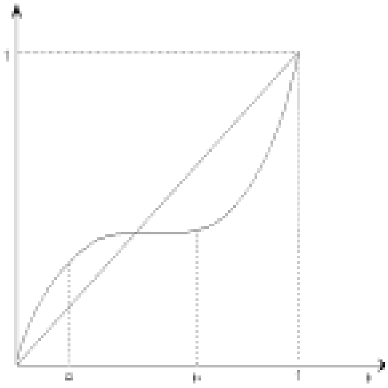
B.1 Proof of proposition 1

B.1.1 Existence and multiple equilibria

Define the function g of ϕ as $g(\phi) = h - \frac{w(\phi)-\widehat{w}}{i-r}$. Then ϕ is the intersection of the line $y = x$ and the function $y = 1 - G(g(x))$. First to see when ϕ_1 exists, we have to show that the function $u(x) = x - 1 + G(g(x))$ crosses the horizontal axe in other points than $x = 0$ or $x = 1$. Note first that first $u(0) = 0$ is always holding.

(i) We have $u(1) = G\left(h - \frac{\alpha-\widehat{w}}{i-r}\right)$. Assume that $G\left(h - \frac{\alpha-\widehat{w}}{i-r}\right) = 0$, then complete industrialization may be possible. Then there are two possible equilibria $\phi_1 = 0$, $\phi_1 = 1$. Furthermore, it is possible to have more than two equilibria if the concavity of $G(g(x))$ is not constant, $i.e$ if the concavity of this function changes by crossing the first 45° line. To see that this case is possible note that the second derivative of $G(g(x))$ is $g^{''}G^{'}(g) + g^{'2}G^{''}(g)$, and we easily get that $g'^2 = \delta g^{'}$, with δ a constant, then we get that $\frac{d^2G(g(x))}{dx^2} = g^{''}(x)\left(G^{'}(g(x)) + \delta G^{''}(g(x))\right)$, so $sign\left(\frac{d^2G(g(x))}{dx^2}\right) = sign\left(G^{'}(g(x)) + \delta G^{''}(g(x))\right)$, thus with a G_0 concave and as g is a decreasing function it is possible that $sign\left(G^{'}(g(x)) + \delta G^{''}(g(x))\right)$ changes at a unique point[42], let us call it ϕ_c.

Figure 1:



(ii) The general case we consider is when $G\left(h - \frac{\alpha-\widehat{w}}{i-r}\right) > 0$. Define $f(x) = 1 - G(g(x))$. For analytical convenience, let us take G of the following form $G(x) = (\frac{x}{h})^{\gamma}$. Then $f^{'} = -g^{'}G(g)$ and $f^{''} = -g^{''}G^{'}(g) - g^{'2}G^{''}(g)$, and we easily get the following relation $-x\frac{f^{''}}{f'} = 2\left[1 + (1-\gamma)\frac{x}{2}\frac{g^{'}}{g}\right]$. Note that in the general where we don't specify G, γ would have to be replaced by $1 - x\frac{G^{''}}{G'}$. One shows that around $x = 0$ we have $x\frac{g^{'}}{g} \approx -1$, so $-x\frac{f^{''}}{f'} \approx 1 - (1-\gamma)\frac{1}{2}$ which is strictly positive, and consequently around f is strictly concave around 0. More over $f^{'} = -g^{'}G(g) = \frac{F}{I-R}\frac{\gamma}{h^{\gamma}}x^{\gamma-3}\left(xg(x)\right)^{1-\gamma}$, and using the expression of g one sees that $xg(x) \approx F$ around 0, and consequently $f^{'}$ is infinite "around 0". In sum, as $1 - G\left(h - \frac{\alpha-\widehat{w}}{i-r}\right) < 1$, and as f is concave and has an infinite

[42] See (ii) to understand when it can happen more precisely

derivative in 0, f is first above the curve $y=x$ and it is under at $x=1$, and as f is continuous it must intercept the curve $y=x$ at some point between 0 an 1.

If $G\left(h-\frac{\alpha-\widehat{w}}{i-r}\right)=0$, one shows, that around $x=1$ we have $-x\frac{f''}{f'}\approx 2-\frac{(1-\gamma)F}{(I-R)h+\widehat{w}-\alpha}$ and if this expression is negative (and nothing prevents this from happening) then f is convex around $x=1$, and consequently as announced in (i) three equilibria may exist.

Now take $G(x)=(\frac{x}{\overline{h}})^{\gamma}$ and $H(x)=(\frac{x}{\overline{h}})^{\gamma'}$ with $1>\gamma>\gamma'$. Then $\forall x\ G(x)\leq H(x)$ and G is less concave than H. Define $f_{\gamma}(x)=1-G\left(h-\frac{w(x)-\widehat{w}}{i-r}\right)$ and $f_{\gamma'}(x)=1-H\left(h-\frac{w(x)-\widehat{w}}{i-r}\right)$, then we get $-x\frac{f''_{\gamma}}{f'_{\gamma}}<-x\frac{f''_{\gamma'}}{f'_{\gamma'}}$. When f_{γ} is concave (so it will be for $f_{\gamma'}$), then $f_{\gamma'}$ is more concave than f_{γ} and consequently the equilibrium value of ϕ under the G distribution is higher than under the H distribution. We have seen that f_{γ} is concave around $x=0$ whatever the value of γ. When $\gamma\geq 1$, f_{γ} is always concave, and in this case the results are more obvious, and the higher γ the higher the level of ϕ. When $\gamma<1$, if γ is sufficiently close to one then f_{γ} is concave, but otherwise depending on the sign of $2-\frac{(1-\gamma)F}{(I-R)h+\widehat{w}-\alpha}$, the concavity may change, but if $G\left(h-\frac{\alpha-\widehat{w}}{i-r}\right)>0$ there are only two possible equilibria (o or $\phi>0$).

For the existence of ϕ_t for $t\geq 2$, note simply that wealth at $t+1$ is the transformation by the continuous function piecewise linear f_t defined by

$$f_t(x)=\begin{cases}\beta R(x-h)+\Theta(\phi_t)+h & \text{if } x\geq h\\ \beta I(x-h)+\Theta(\phi_t)+h & \text{if } x\in\left[-\frac{w(\phi_t)-\widehat{w}}{i-r}+h,h\right]\\ \beta R(x-h)+\overset{\wedge}{\Theta}+h & \text{if } x\in\left]\underline{h},-\frac{w(\phi_t)-\widehat{w}}{i-r}+h\right]\end{cases}\tag{24}$$

The continuity is clear except for the point $-\frac{w(\phi_t)-\widehat{w}}{i-r}+h$, but one easily verifies the following equality $\beta I\left(-\frac{w(\phi_t)-\widehat{w}}{i-r}\right)+\Theta(\phi_t)=\beta R(-\frac{w(\phi_t)-\widehat{w}}{i-r})+\overset{\wedge}{\Theta}$.

B.1.2 Markets clearing

The condition of market clearing for skilled labor is

$$\frac{X_t d_t}{\alpha}=\phi_t\tag{(*)}$$

The condition of market clearing for unskilled labor is

$$1-\phi_t = (1-X_t)d_t + X_t\left(F\frac{(1-\phi_t)}{X_t}\right) \qquad ((**))$$

Then eliminating d_t from $(*)$ and putting it in $(**)$ we get the equilibrium value of X_t

$$X_t = \frac{\alpha\phi_t}{(1-\phi_t)(1-F)+\alpha\phi_t}$$

And one verifies that when $\phi_t = 0$ then $X_t = 0$, and when $\phi_t = 1$ then $X_t = 1$.

B.2 Proof of proposition 2

The first inequality (i) is necessary because otherwise, using *proposition 1*, we see that nobody would want to acquire education. The second (ii) follows from lemma 1 and lemma 2 and the discussion above. First when (ii) is verified the second period wealth distribution verifies the conditions of lemma 2. Consequently when lemma 1 is verified, $(\phi_t)_t$ is a strictly increasing sequence. Finally as the threshold level for education (which is a decreasing function of ϕ) is under the poverty attraction, one can state that $\lim \phi_t = 1$.

B.3 Proof of Lemma 1

B.3.1 First part : wealth is "globally increasing"

Recall the function f_t defined above, we then just have to show that it is increasing, and this point is clear, and that $\forall x \geq \frac{\Theta(\phi_t)}{1-\beta R}$, $f_t(x) \geq x$. This last point comes from lemma 1 (the proof of which is obvious).

B.3.2 Second part : $-\left[w(\phi_t) - \widehat{w}\right]/\left(i-r\right) \leq \widehat{\theta}/\left(1-\beta R\right)$ for taking off economies

Recall that $\widehat{\theta} = \beta\widehat{w} - h$. Then $\frac{-[w(\phi_t)-\widehat{w}]}{I-R} - \frac{\widehat{\theta}}{1-\beta R} = -\frac{(w(\phi_t)-\widehat{w})(1-\beta R)+(I-R)(\beta\widehat{w}-h)}{(I-R)(1-GbR)}$. The numerator of this fraction is equal to $-\left[(1-\beta R)w(\phi_t) - (1-\beta I)\widehat{w} - (I-R)\right]$, and it remains to determine the sign of the expression in brackets. For this recall that $-\frac{w(\phi_t)-\widehat{w}}{i-r} \leq 0$ and by *proposition 3* $-\frac{\beta w(\phi_t)-h}{1-\beta I} \leq 0$, so $-\left[\frac{\beta w(\phi_t)-h}{1-\beta I} + \frac{w(\phi_t)-\widehat{w}}{i-r}\right] \leq 0$ so $\frac{\beta w(\phi_t)-h}{1-\beta I} + \frac{w(\phi_t)-\widehat{w}}{i-r} \geq 0$, which is equivalent to $(1-\beta R)w(\phi_t) - (1-\beta I)\widehat{w} - (I-R) \geq 0$. This completes the proof.

B.4 Proof of Lemma 2

As $T(x) \leq G(x)\ \forall x$, then $1 - T(H(x)) \geq 1 - G(H(x))$, consequently as the functions $x \longrightarrow 1 - G(H(x))$ and $x \longrightarrow 1 - T(H(x))$ are concave and above the 45° line around $x = 0$, one can state the result..

B.5 Proof of proposition 3

When Ω is an increasing function of α, then using *Lemma 1* and the proof of *proposition 1* we easily get that there is a negative relation between the degree of inequality and the productivity of the adopted technology.

B.5.1 Conditions for "Ω is an increasing function of α"

Let us start with a general expression for $F(\alpha)$. Let us take $F(\alpha) = q\alpha^v$. Then $\Omega(\alpha) = \frac{q\alpha^v}{\alpha - \frac{h}{\beta} + q\alpha^v}$ and $\Omega^{'}(\alpha) = -\frac{(1-v)q\alpha^{v-1}\left(\alpha + \frac{v}{1-v}\frac{h}{\beta}\right)}{\left(\alpha - \frac{h}{\beta} + q\alpha^v\right)^2}$.

- Suppose $v < 1$ (so F is a concave function), then $sign(\Omega^{'}(\alpha)) = -sign(\alpha + \frac{v}{1-v}\frac{h}{\beta}) < 0$. Consequently, wealth distribution is not a constraint.

- Suppose that $v > 1$, then $sign(\Omega^{'}(\alpha)) = sign(\alpha + \frac{v}{1-v}\frac{h}{\beta})$, so it depends on the relative positions of $\beta\alpha$ and $\frac{v}{v-1}h$. We have supposed only that $\beta\alpha > h$. We can then say that when $\alpha \leq \frac{v}{v-1}\frac{h}{\beta}$ then $\Omega^{'}(\alpha) \leq 0$, and consequently over the range of productivities $[\frac{h}{\beta}, \frac{v}{v-1}\frac{h}{\beta}]$ the distribution of wealth doesn't not constrain "productivity" gains. For $\alpha > \frac{v}{v-1}\frac{h}{\beta}$, as $\Omega^{'}(\alpha) > 0$, then wealth distribution is a constraint for productivity gains. For $v = 2$, wealth distribution becomes a constraint for $\alpha > 2\frac{h}{\beta}$. We have to verify that this parameters configuration is plausible in our model. First we have supposed that $h > \beta\hat{w} = \beta(2 + Rh) \Longleftrightarrow h > \frac{2\beta}{1-\beta R}$ (1^*). Second we must verify that $h - \frac{2\frac{h}{\beta} - \hat{w}}{I-R} > 0$ (otherwise full industrialization is immediate), which is equivalent to $(1 - \frac{\beta I}{2})h < \beta$ (1^{**}). For (1^*) and (1^{**}) to be compatible, we must have $(1 - \frac{\beta I}{2})\frac{2\beta}{1-\beta R} < 1 \Longleftrightarrow \beta^2 I - \beta(2 + R) + 1 > 0$, which is not a constraint for not too high values of β. Then, take the specification of $\alpha(h) = Bh^v$, then $\alpha(h^*) > 2\frac{h^*}{\beta} \Longleftrightarrow \beta\left(\frac{I-R}{Bv}\right)^v > 2$, and we can choose B, v and β such as to verify this inequality.

B.5.2 Proof

As ϕ is likely to be an increasing function of α, then it is possible that as α increases, although the threshold level of ϕ for a successful industrialization rises, ϕ may increase.

Let us then pose that ϕ is a function of α, $\phi(\alpha)$, and study the sign of its derivative.

We know that $\phi(\alpha)$ is implicitly defined by the equation

$$\phi(\alpha) = 1 - G\left[h - \frac{\alpha - \frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I-R}\right] , \tag{25}$$

So taking the derivative on both sides one gets

$$\phi^{'}(\alpha) =$$

$$\frac{1}{I-R}\left[1 - \frac{F^{'}(\alpha)\left(1-\phi(\alpha)\right)}{\phi(\alpha)} + \phi^{'}(\alpha)\frac{F}{\phi^2(\alpha)}\right] G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w}}{I-R}\right)$$

Rearranging, one gets

$$\phi^{'}(\alpha) = \frac{1}{I-R}\frac{\left(1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}\right) G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w}}{I-R}\right)}{1 - \frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w}}{I-R}\right)}. \tag{26}$$

Consequently the sign of $\phi^{'}(\alpha)$ is equal to

$$\frac{sign\left(1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}\right)}{sign\left(1 - \frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w}}{I-R}\right)\right)}.$$

First $1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} \geq 0 \Longleftrightarrow \phi(\alpha) \geq \frac{F^{'}(\alpha)}{1+F^{'}(\alpha)}$. As F is convex, one gets that $\frac{F^{'}(\alpha)}{1+F^{'}(\alpha)}$ is increasing with α. For analytical convenience, let us take the form $F(\alpha) = \alpha^2/2$. Then one gets that $\frac{F^{'}(\alpha)}{1+F^{'}(\alpha)} \geq \frac{F(\alpha)}{\alpha - \frac{h}{\beta} + F(\alpha)} \Longleftrightarrow$ $\alpha \geq \sqrt{2(1+\frac{h}{\beta})}$ (note that it is not a high constraint on the parameters, in light of appendix $B.6.1$ above, since $2\frac{h}{\beta} \geq \sqrt{2(1+\frac{h}{\beta})}$ as soon as $\frac{h}{\beta} \geq 1$).

Consequently, when $\alpha \geq \sqrt{2(1+\frac{h}{\beta})}$, and the solution $\phi(\alpha)$ of equation (22) lies between $\frac{F(\alpha)}{\alpha-\frac{h}{\beta}+F(\alpha)}$ and $\frac{F'(\alpha)}{1+F'(\alpha)}$, then $1-\frac{F'(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} < 0$.

It remains to evaluate $\frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G'\left(h-\frac{\alpha-\frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}-\widehat{w}}{I-R}\right)$. To this purpose let us assume the following specification for G, $G(x)=\left(\frac{x}{\bar{h}}\right)^{\gamma}$. Then one gets

$$\frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G'\left(h-\frac{\alpha-\frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}-\widehat{w}}{I-R}\right)$$
$$= \gamma\frac{1}{\bar{h}^{\gamma}}\left(\frac{F(\alpha)}{I-R}\frac{\phi(\alpha)^{-1-\gamma}}{\left(\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R}\right)^{1-\gamma}}\right)$$

The term $\left(\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R}\right)$ is positive and is bounded below for any $\alpha \geq 1$, $\exists Z_1 \in R$, and $Z_1 > 0$, such that $\forall\alpha$, $\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R} > Z_1$.

So let us assume that $\phi(\alpha) \geq \frac{F(\alpha)}{\alpha-\frac{h}{\beta}+F(\alpha)}$ (i.e. the economy is on a successful industrialization path) and $\alpha \geq \sqrt{2(1+\frac{h}{\beta})}$ (since we always assume that $\alpha \geq \frac{h}{\beta}$, then for $\frac{h}{\beta} > 2.73$, which can be easily satisfied here, we have $\frac{h}{\beta} > \sqrt{2(1+\frac{h}{\beta})}$).

Then we have $\phi(\alpha)^{-1-\gamma} \leq \left(\frac{1}{\frac{F(\alpha)}{\alpha-\frac{h}{\beta}+F(\alpha)}}\right)^{1+\gamma}$, and from $B.6.1$ we know that $\exists Z_2 \in R$, and $Z_2 > 0$, such that $\forall\alpha \geq \sqrt{2(1+\frac{h}{\beta})}$, $\frac{F(\alpha)}{\alpha-\frac{h}{\beta}+F(\alpha)} \geq Z_2$. Consequently $\gamma\frac{1}{\bar{h}^{\gamma}}\left(\frac{F(\alpha)}{I-R}\frac{\phi(\alpha)^{-1-\gamma}}{\left(\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R}\right)^{1-\gamma}}\right) < \gamma\frac{1}{\bar{h}^{\gamma}}\frac{F(\alpha)}{I-R}\frac{1}{Z_1^{1-\gamma}}\frac{1}{Z_2^{1+\gamma}}$.

Let us note $P(\gamma,\alpha)=\gamma\frac{1}{\bar{h}^{\gamma}}\frac{F(\alpha)}{I-R}\frac{1}{Z_1^{1-\gamma}}\frac{1}{Z_2^{1+\gamma}}$. We have $\lim_{\gamma\longrightarrow 0} P(\gamma,\alpha)=0$, we have also that for high values of $\bar{h}$, $P(\gamma,\alpha)$ is small. Consequently for any α, $\exists(\gamma,\bar{h})\ /\ P(\gamma,\alpha)<1$. In this configuration, then $sign(\phi'(\alpha))$ is negative. So wealth distribution is a constraint for the adoption of a higher productivity technology. As $F(\alpha)$ is increasing, we can argue that the higher α the lower has to be γ for $P(\gamma,\alpha)$ to be small. And at γ fixed, the higher α the higher has to be $\bar{h}$.

In fact the assumptions needed to get this result can be weakened. First, we do not need to assume the special form for G, indeed we can just take the approximation of G at the equilibrium value of $h-\frac{\alpha-\frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}-\widehat{w}}{I-R}$ for any α.

In this approximation, γ will be the elasticity of G' at this point. Then one gets that if this elasticity is small, then $\phi'(\alpha)$ will be negative. And as for values of α greater than $\frac{2h}{\beta}$ the threshold level $\frac{F(\alpha)}{\alpha-\frac{h}{\beta}+F(\alpha)}$ is increasing, then switching to more productive technologies is impossible.

B.5.3 Concavity of $\phi(\alpha)$

Recall the function $f(x) = 1-G(g(x))$, we know that it is concave. We know that $\phi(\alpha)$ verifies

$$\phi(\alpha) = f(\phi(\alpha)) \; .$$

Let us take the derivative of this relation at the second order. One obtains $\phi''(\alpha) = \phi''(\alpha)f'(\phi(\alpha)) + \phi'^2(\alpha)f''(\phi(\alpha))$ and then we get $\phi''(\alpha) = \frac{\phi'^2(\alpha)f''(\phi(\alpha))}{1-f'(\phi(\alpha))}$. For small values of ϕ we know that $f' >> 1$, then for small values[43] of α we have $\phi''(\alpha) > 0$ and $\phi'(\alpha) > 0$. For higher values of α (and consequently for ϕ) we have proved that $f' < 1$. Then for higher values of α we have $\phi''(\alpha) < 0$. So $\phi(\alpha)$ is invert-S shaped.

B.6 Appendix of Section 2.2.2

Putting (21) into the equation which defines implicitly $\phi(h)$ one gets

$$\frac{F'(\alpha(h_R^*))}{1-\frac{I}{\alpha'(h_R^*)}+F'(\alpha(h_R^*))} = 1-G\left(h_R^* - \frac{\alpha(h_R^*) - \frac{F(\alpha(h_R^*))}{F'(\alpha(h_R^*))}\left(1-\frac{I}{\alpha'(h_R^*)}\right) - \widehat{w(h_R^*)}}{I-R}\right) ,$$

and taking $F(\alpha) = \alpha^2/2$ one gets

$$\frac{\alpha(h_R^*)}{1-\frac{I}{\alpha'(h_R^*)}+\alpha(h_R^*)} = 1-G\left(h_R^* - \frac{\frac{1}{2}\alpha(h_R^*) - \widehat{w(h_R^*)}}{I-R} - \frac{1}{2}\frac{\alpha(h_R^*)}{\alpha'(h_R^*)}\right) . \quad (27)$$

Let us use again the following specification for $\alpha(h) = Bh^v$. Then equation (22) becomes

$$\frac{B(h_R^*)^v}{1-\frac{I}{vB(h_R^*)^{v-1}}+B(h_R^*)^v} = 1-G\left(\left(1-\frac{1}{2v}\right)h_R^* - \frac{\frac{1}{2}B(h_R^*)^v - \widehat{w(h_R^*)}}{I-R}\right) . \quad (28)$$

[43]Note that α small means α near the lower bound $\frac{h}{\beta}$. We then restrict ourself to situations where the distribution admits small ϕ as solutions around the lower bound.

B.7 Appendix of the example in section 2.1

We have to show that the equation

$$\frac{A.\alpha^*}{1+A.\alpha^*} = 1 - G\left(h - \frac{\frac{1}{2}\alpha^* - \widehat{w}}{I-R}\right) . \tag{29}$$

admits a solution for well chosen parameters. First remember that the function $\zeta_1 : \alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ is concave and the function $\zeta_2 : \alpha \longrightarrow 1 - G\left(h - \frac{\frac{1}{2}\alpha - \widehat{w}}{I-R}\right)$ is convex.

Although not in the range of values allowed for α, the derivative of $\alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ at

$\alpha = 0$ is equal to A. The derivative of ζ_2 at $\alpha = 0$ is equal to $\frac{\gamma}{2}\frac{1}{((I-R)h+\widehat{w})}\left(\frac{h+\frac{\widehat{w}}{I-R}}{\bar{h}}\right)^{\gamma}$ which can be made as small as desired. We can choose A and $\bar{h}$ and γ such that $\frac{\gamma}{2}\frac{1}{((I-R)h+\widehat{w})}\left(\frac{h+\frac{\widehat{w}}{I-R}}{\bar{h}}\right)^{\gamma} < A$. Then if $1 - G\left(h + \frac{\widehat{w}}{I-R}\right) = 0$ then we know that ζ_1 and ζ_2 will cross each other at a unique other point $\alpha^* > 0$. If $1 - G\left(h + \frac{\widehat{w}}{I-R}\right) > 0$, then if ζ_2 is too convex then there may be no intersection. When we evaluate the convexity of ζ_2, we get that the higher γ the less convex is ζ_2.