

Tests for the Error Component Model in the Presence of Local Misspecification

Anil K. Bera
Department of Economics
University of Illinois at Urbana-Champaign

Walter Sosa Escudero[✉]
Department of Economics
National University of La Plata, Argentina

Mann Yoon
Department of Economics
California State University at Los Angeles

This Version: January 2000

Abstract

It is well known that most of the standard specification tests are not valid when the alternative hypothesis is misspecified. This is particularly true in the error component model, when one tests for either random effects or serial correlation without taking account of the presence of the other effect. In this paper we study the size and power of the standard Rao's score tests analytically and by simulation when the data is contaminated by local misspecification. These tests are adversely affected under misspecification. We suggest simple procedures to test for random effects (or serial correlation) in the presence of local serial correlation (or random effects), and these tests require ordinary least squares residuals only. Our Monte Carlo results demonstrate that the suggested tests have good finite sample properties and are capable of detecting the right direction of the departure from the null hypothesis. We also provide some empirical illustrations to highlight the usefulness of our tests.

[✉]Corresponding author: Walter Sosa Escudero, Departamento de Economía, Facultad de Ciencias Económicas, UNLP, Calle 6 entre 47 y 48, oficina 516, 1900 La Plata, Argentina. Phone-fax: (54-221)-4229383. E-mail: wsosa@feedback.net.ar

1 Introduction

The random error component model introduced by Balestra and Nerlove (1966) was extended by Lillard and Willis (1978) to include serial correlation in the remainder disturbance term. Such an extension, however, raises questions about the validity of the existing specification tests such as the Rao's (1948) score (RS) test for random effects assuming no serial correlation as derived in Breusch and Pagan (1980). In a similar way doubts could be raised about tests for serial correlation derived assuming no random effects. Baltagi and Li (1991) proposed a RS test that jointly tests for serial correlation and random effects. One problem with the joint test is that, if the null hypothesis is rejected, it is not possible to infer whether the misspecification is due to serial correlation or to random effects. Also, as we will discuss later, because of higher degrees of freedom the joint test will not be optimal if the departure from the null occurs only in one direction. More recently, Baltagi and Li (1995) derived RS statistics for testing serial correlation assuming fixed/individual effects. These tests require maximum likelihood estimation of individual effects parameters.

For a long time econometricians have been aware of the problems that arise when the alternative hypothesis used to construct a test deviates from the data generating process (DGP). As emphasized by Haavelmo (1944, pp. 65-66), in testing any economic relations, specification of a given fixed set of possible alternatives, called the priori admissible hypothesis, H_0 ; is of fundamental importance. Misspecification of the priori admissible hypotheses was termed as type-III error by Bera and Yoon (1993). Welsh (1996, p. 119) also points out a similar concept in the statistics literature. Typically, the alternative hypothesis may be misspecified in three different ways. In the first one, which we shall call "complete misspecification," the set of assumed alternatives, H_0 , and the DGP, H_1 , say, are mutually exclusive. This happens, for instance, if one tests for serial independence when the DGP has random individual effects but no serial dependence. The second case occurs when the alternative is underspecified in that it is a subset of a more general model representing the DGP, i.e., $H_0 \subset H_1$. This happens, for example, when both serial correlation and individual effects are present, but are tested separately (one at a time). The last case is "overtesting" which results from overspecification, that is, when $H_0 \supset H_1$. This can happen when, say, Baltagi and Li (1991) joint test for serial correlation and random individual effects is used when only one effect is present. [For a detailed discussion of the concepts of undertesting and

overtesting, see Bera and Jarque (1982)]. In this paper, we study analytically the asymptotic effects of misspecifications on the one-directional and joint tests for serial dependence and random individual effects. These results compliment the simulation results of Baltagi and Li (1995). Then, applying the modified RS test developed by Bera and Yoon (1993), we derive a test for random effects (serial correlation) in the presence of serial correlation (random effects). Our tests can be easily implemented using ordinary least squares (OLS) residuals from the standard linear model for panel data.

The plan of the paper is as follows. In the next section we review a general theory of the distribution and adjustment of the standard RS statistic in the presence of local misspecification. In Section 3, the general results are specialized to the error component model. In Section 4, we present two empirical illustrations. Section 5 reports the results of an extensive Monte Carlo study. These results, along with the empirical examples, clearly demonstrate the inappropriateness of one-directional tests in identifying the specific source of misspecification(s), and highlight the usefulness of our adjusted tests. Section 6 provides some concluding remarks.

2 Effects of misspecification and a general approach to testing in the presence of a nuisance parameter

Consider a general statistical model represented by the log-likelihood $L(\theta; \tilde{A}; \hat{A})$. Here, the parameters \tilde{A} and \hat{A} are taken as scalars to conform with our error component model, but in general they could be vectors. Suppose an investigator sets $\hat{A} = \hat{A}_0$ and tests $H_0 : \tilde{A} = \tilde{A}_0$ using the log-likelihood function $L_1(\theta; \tilde{A}) = L(\theta; \tilde{A}; \hat{A}_0)$, where \hat{A}_0 and \tilde{A}_0 are known values. The RS statistic for testing H_0 in $L_1(\theta; \tilde{A})$ will be denoted by $RS_{\tilde{A}}$. Let us also denote $\mu = (\theta_0; \tilde{A}_0; \hat{A}_0)'$ and $\hat{\mu} = (\hat{\theta}_0; \tilde{A}_0^0; \hat{A}_0^0)'$, where $\hat{\theta}$ is the maximum likelihood estimator (MLE) of θ when $\tilde{A} = \tilde{A}_0$ and $\hat{A} = \hat{A}_0$. The score vector and the information matrix are defined, respectively, as

$$d_a(\mu) = \frac{\partial L(\mu)}{\partial a} \quad \text{for } a = \theta; \tilde{A}; \hat{A}$$

and

$$J(\mu) = -E \frac{1}{n} \frac{\partial^2 L(\mu)}{\partial \mu \partial \mu'} = \begin{matrix} & \begin{matrix} \text{2} & & \text{3} \end{matrix} \\ \begin{matrix} \text{1} \\ \text{4} \\ \text{5} \end{matrix} & \begin{matrix} J_{\theta\theta} & J_{\theta\tilde{A}} & J_{\theta\hat{A}} \\ J_{\tilde{A}\theta} & J_{\tilde{A}\tilde{A}} & J_{\tilde{A}\hat{A}} \\ J_{\hat{A}\theta} & J_{\hat{A}\tilde{A}} & J_{\hat{A}\hat{A}} \end{matrix} \end{matrix};$$

where n denotes the sample size. If $L_1(\cdot; \tilde{A})$ were the true model, then it is well known that under $H_0 : \tilde{A} = \tilde{A}_0$,

$$RS_{\tilde{A}} = \frac{1}{n} d_{\tilde{A}}(\hat{\beta})^0 J_{\tilde{A}^c}^{-1}(\hat{\beta}) d_{\tilde{A}}(\hat{\beta}) \overset{D}{\rightarrow} \hat{A}_1^2(0);$$

where $\overset{D}{\rightarrow}$ denotes convergence in distribution and $J_{\tilde{A}^c}(\mu) = J_{\tilde{A}^c} = J_{\tilde{A}} \circ J_{\tilde{A}}^{-1} J_{\tilde{A}}$. And under $H_1 : \tilde{A} = \tilde{A}_0 + \frac{P}{n}$,

$$RS_{\tilde{A}} \overset{D}{\rightarrow} \hat{A}_1^2(\lambda_1); \tag{1}$$

where the noncentrality parameter λ_1 is given by $\lambda_1 = \lambda_1(\mu) = \mu^0 J_{\tilde{A}^c}$. Given this set-up, asymptotically the test will have correct size and will be locally optimal. Now suppose that the true log-likelihood function is $L_2(\cdot; \tilde{A})$ so that the alternative $L_1(\cdot; \tilde{A})$ becomes completely misspecified. Using a sequence of local values $\tilde{A} = \tilde{A}_0 + \frac{P}{n}$, Davidson and MacKinnon (1987) and Saikkonen (1989) obtained the asymptotic distribution of $RS_{\tilde{A}}$ under $L_2(\cdot; \tilde{A})$ as

$$RS_{\tilde{A}} \overset{D}{\rightarrow} \hat{A}_1^2(\lambda_2); \tag{2}$$

where the non-centrality parameter λ_2 is given by $\lambda_2 = \lambda_2(\pm) = \pm^0 J_{\tilde{A}\tilde{A}^c} J_{\tilde{A}^c}^{-1} J_{\tilde{A}\tilde{A}^c} \pm$ with $J_{\tilde{A}\tilde{A}^c} = J_{\tilde{A}\tilde{A}} \circ J_{\tilde{A}^c}^{-1} J_{\tilde{A}^c}$. Due to this non-centrality parameter, $RS_{\tilde{A}}$ will have power in the model $L(\cdot; \tilde{A}; \tilde{A})$ even when $\tilde{A} = \tilde{A}_0$; and, therefore, the test will have incorrect size. Notice that the crucial quantity is $J_{\tilde{A}\tilde{A}^c}$ which can be interpreted as the partial covariance between $d_{\tilde{A}}$ and $d_{\tilde{A}}$ after eliminating the effect of $d_{\tilde{A}}$ on $d_{\tilde{A}}$ and $d_{\tilde{A}}$. If $J_{\tilde{A}\tilde{A}^c} = 0$, then the local presence of the parameter \tilde{A} has no effect on $RS_{\tilde{A}}$.

Turning now to the case of underspecification, let the true model be represented by the log-likelihood $L(\cdot; \tilde{A}; \tilde{A})$: The alternative $L_1(\cdot; \tilde{A})$ is now underspecified with respect to the nuisance parameter \tilde{A} ; leading to the problem of undertesting. In order to derive the asymptotic distribution of $RS_{\tilde{A}}$ under the true model $L(\cdot; \tilde{A}; \tilde{A})$, we again consider the local departures $\tilde{A} = \tilde{A}_0 + \frac{P}{n}$ together with $\tilde{A} = \tilde{A}_0 + \frac{P}{n}$. It can be shown that [see Bera and Yoon (1991)]

$$RS_{\tilde{A}} \overset{D}{\rightarrow} \hat{A}_1^2(\lambda_3); \tag{3}$$

where

$$\begin{aligned} \lambda_{3,3}(\lambda; \pm) &= (\pm^0 J_{\tilde{A}\tilde{A}c} + \lambda^0 J_{\tilde{A}c}) J_{\tilde{A}c}^{-1} (J_{\tilde{A}\tilde{A}c} \pm + J_{\tilde{A}c} \lambda) \\ &= \lambda_{1,1}(\lambda) + \lambda_{2,2}(\pm) + 2\lambda^0 J_{\tilde{A}\tilde{A}c} \pm \end{aligned}$$

Using this result, we can compare the asymptotic local power of the underspecified test with that of the optimal test. It turns out that the contaminated non-centrality parameter $\lambda_{3,3}(\lambda; \pm)$ may actually increase or decrease the power depending on the configuration of the term $\lambda^0 J_{\tilde{A}\tilde{A}c} \pm$:

The problem of overtesting occurs when multi-directional joint tests are applied based on an overstated alternative model. Suppose we apply a joint test for testing hypothesis of the form $H_0 : \tilde{A} = \tilde{A}_0$ and $\tilde{A} = \tilde{A}_0$ using the alternative model $L(\cdot; \tilde{A}; \tilde{A})$. Let $RS_{\tilde{A}\tilde{A}}$ be the joint RS test statistic for H_0 : To find the asymptotic distribution of $RS_{\tilde{A}\tilde{A}}$ under overspecification, i.e., when the DGP is represented by the likelihood either $L_1(\cdot; \tilde{A})$ or $L_2(\cdot; \tilde{A})$, let us consider the following result, which could be obtained from (1) by replacing \tilde{A} with $[\tilde{A}^0; \tilde{A}^0]$. Assuming correct specification, i.e., under the true model represented by $L(\cdot; \tilde{A}; \tilde{A})$ with $\tilde{A} = \tilde{A}_0 + \lambda^0 \mathbf{P}_{\tilde{A}}$ and $\tilde{A} = \tilde{A}_0 + \pm \mathbf{P}_{\tilde{A}}$;

$$RS_{\tilde{A}\tilde{A}} \stackrel{D}{\rightarrow} \hat{A}_2^2(\lambda, 4); \quad (4)$$

where

$$\lambda_{4,4}(\lambda; \pm) = \begin{bmatrix} \lambda^0 & \pm^0 \end{bmatrix} \begin{bmatrix} J_{\tilde{A}c} & J_{\tilde{A}\tilde{A}c} \\ J_{\tilde{A}\tilde{A}c} & J_{\tilde{A}c} \end{bmatrix} \begin{bmatrix} \lambda \\ \pm \end{bmatrix};$$

Using this fact, we can easily find the asymptotic distribution of the overspecified test. Consider testing $H_0 : \tilde{A} = \tilde{A}_0$ and $\tilde{A} = \tilde{A}_0$ in $L(\cdot; \tilde{A}; \tilde{A})$ where $L_1(\cdot; \tilde{A})$ represents the true model. Under $L_1(\cdot; \tilde{A})$ with $\tilde{A} = \tilde{A}_0 + \lambda^0 \mathbf{P}_{\tilde{A}}$, we obtain by setting $\pm = 0$ in (4)

$$RS_{\tilde{A}\tilde{A}} \stackrel{D}{\rightarrow} \hat{A}_2^2(\lambda, 5); \quad (5)$$

where $\lambda_{5,5}(\lambda) = \lambda^0 J_{\tilde{A}c}$:

Note that the non-centrality parameter $\lambda_{5,5}(\lambda)$ of the overspecified test $RS_{\tilde{A}\tilde{A}}$ is identical to $\lambda_{1,1}(\lambda)$ of the optimal test $RS_{\tilde{A}}$ in (1). Although $\lambda_{5,5} = \lambda_{1,1}$; some loss of power is to be expected, as shown in Das Gupta and Perlman (1974), due to the higher degrees of freedom of the joint test $RS_{\tilde{A}\tilde{A}}$.

Using the result (2), Bera and Yoon (1993) suggested a modification to $RS_{\tilde{A}}$ so that the resulting test is valid in the local presence of \hat{A} . The modified statistic is given by

$$RS_{\tilde{A}}^{\square} = \frac{1}{n} [d_{\tilde{A}}(\hat{\beta}) + J_{\tilde{A}\hat{A}^c}(\hat{\beta}) J_{\hat{A}^c}^{-1}(\hat{\beta}) d_{\hat{A}}(\hat{\beta})]^0 [J_{\tilde{A}^c}(\hat{\beta}) + J_{\tilde{A}\hat{A}^c}(\hat{\beta}) J_{\hat{A}^c}^{-1}(\hat{\beta}) J_{\hat{A}\tilde{A}^c}(\hat{\beta})]^{-1} [d_{\tilde{A}}(\hat{\beta}) + J_{\tilde{A}\hat{A}^c}(\hat{\beta}) J_{\hat{A}^c}^{-1}(\hat{\beta}) d_{\hat{A}}(\hat{\beta})] \quad (6)$$

This new test essentially adjusts the mean and variance of the standard $RS_{\tilde{A}}$. Bera and Yoon (1993) proved that under $\tilde{A} = \tilde{A}_0$ and $\hat{A} = \hat{A}_0 + \pm \frac{P}{n} RS_{\tilde{A}}^{\square}$ has a central \hat{A}_1^2 distribution. Thus, $RS_{\tilde{A}}^{\square}$ has the same asymptotic null distribution as that of $RS_{\tilde{A}}$ based on the correct specification, thereby producing an asymptotically correct size test under locally misspecified model. Bera and Yoon (1993) further showed that for local misspecification the adjusted test is asymptotically equivalent to Neyman's $C^{(R)}$ test and, therefore, shares the optimality properties of the $C^{(R)}$ test. There is, however, a price to be paid for all these benefits. Under the local alternatives $\tilde{A} = \tilde{A}_0 + \pm \frac{P}{n}$

$$RS_{\tilde{A}}^{\square} \stackrel{D}{\rightarrow} \hat{A}_1^2(\cdot, \cdot); \quad (7)$$

where $\cdot, \cdot = \pm \frac{P}{n} (J_{\tilde{A}^c} + J_{\tilde{A}\hat{A}^c} J_{\hat{A}^c}^{-1} J_{\hat{A}\tilde{A}^c})$. Note that $\cdot, \cdot \geq 0$, where \cdot, \cdot is given in (1). Result (7) is valid both in the presence or absence of the local misspecification $\hat{A} = \hat{A}_0 + \pm \frac{P}{n}$, since the asymptotic distribution of $RS_{\tilde{A}}^{\square}$ is unaffected by the local departure of \hat{A} from \hat{A}_0 . Therefore, $RS_{\tilde{A}}^{\square}$ will be less powerful than $RS_{\tilde{A}}$ when there is no misspecification. The quantity

$$\cdot, \cdot = \cdot, \cdot + \pm \frac{P}{n} J_{\tilde{A}\hat{A}^c} J_{\hat{A}^c}^{-1} J_{\hat{A}\tilde{A}^c} \quad (8)$$

can be regarded as the premium we pay for the validity of $RS_{\tilde{A}}^{\square}$ under local misspecification. Two other observations regarding $RS_{\tilde{A}}^{\square}$ are also worth noting. First, $RS_{\tilde{A}}^{\square}$ requires estimation only under the joint null, namely $\tilde{A} = \tilde{A}_0$ and $\hat{A} = \hat{A}_0$. Given the full specification of the model $L(\cdot; \tilde{A}; \hat{A})$ it is, of course, possible to derive a RS test for $\tilde{A} = \tilde{A}_0$ in the presence of \hat{A} . However, that requires MLE of \hat{A} which could be difficult to obtain in some cases. Second, when $J_{\tilde{A}\hat{A}^c} = 0$, $RS_{\tilde{A}}^{\square} = RS_{\tilde{A}}$. In practice this is a very simple condition to check. As mentioned earlier, if this condition is true, $RS_{\tilde{A}}$ is an asymptotically valid test in the local presence of \hat{A} .

3 Tests for error component model

We consider the following one-way error component model introduced by Lillard and Willis (1978), which combines random individual effects and first order autocorrelation in the disturbance term:

$$\begin{aligned} y_{it} &= x_{it}'\beta + u_{it}; \quad i = 1; 2; \dots; N; t = 1; 2; \dots; T; \\ u_{it} &= \alpha_i + \varepsilon_{it}; \\ \varepsilon_{it} &= \rho_{i;t-1} \varepsilon_{i;t-1} + \eta_{it}; \quad |\rho| < 1; \end{aligned} \quad (9)$$

where β is a $(k \times 1)$ vector of parameters including the intercept, $\alpha_i \gg \text{IIDN}(0; \sigma_\alpha^2)$ is a random individual component, and $\varepsilon_{it} \gg \text{IIDN}(0; \sigma_\varepsilon^2)$. The α_i and ε_{it} are assumed to be independent of each other with $\varepsilon_{i;0} \gg N(0; \sigma_\varepsilon^2/(1 - \rho^2))$. N and T denote the number of individual units and the number of time periods, respectively. For the validity of the tests discussed here, we need to assume that the regularity conditions of Anderson and Hsiao (1982) are satisfied. Also, testing for ρ involves the issue of the parameter being at the boundary. Although for the nonregular problem of testing at the boundary value, both the likelihood ratio and Wald test statistics do not have their usual asymptotic chi-squared distribution, the RS test statistic does [see, e.g., Bera, Ra and Sarkar (1998)].

Let us set $\mu = (\beta; \tilde{\alpha}; \tilde{\alpha})' = (\sigma_\alpha^2; \sigma_\varepsilon^2; \rho)'$. Consider the problem of testing for the existence of the random effects ($H_0: \tilde{\alpha} = 0$) in the presence of serial correlation ($\tilde{\alpha} \in 0$). To derive our $RS_{\tilde{\alpha}}^\mu$, which will now be denoted as $RS_{\tilde{\alpha}}^\mu$, we note that it is sufficient to consider the scores and the information matrix evaluated at $\mu_0 = (\beta_0; \tilde{\alpha}_0; \tilde{\alpha}_0)' = (\sigma_\alpha^2; 0; 0)'$ because of the block-diagonality of the information matrix involving the β and μ parameters. These quantities have been derived in Baltagi and Li (1991):

$$\begin{aligned} \frac{\partial L}{\partial \sigma_\alpha^2} &= d_\beta = \frac{1}{2} \frac{NT}{\sigma_\alpha^2} + \frac{u^0 u}{2\sigma_\alpha^4}; \\ \frac{\partial L}{\partial \sigma_\varepsilon^2} &= d_\beta - d_{\tilde{\alpha}} = \frac{1}{2} \frac{NT}{\sigma_\varepsilon^2} - \frac{1}{2} \frac{u^0 (I_N - e_T e_T') u}{\sigma_\varepsilon^4}; \\ \frac{\partial L}{\partial \rho} &= d_{1/2} - d_{\tilde{\alpha}} = NT \frac{u^0 u_{j-1}}{\sigma_\varepsilon^4}; \end{aligned} \quad (10)$$

where I_N is an identity matrix of dimension N , e_T is a vector of ones of dimension T , $u^0 = (u_{11}; \dots; u_{1T}; \dots; u_{N1}; \dots; u_{NT})$ and u_{j-1} is an $(NT \times 1)$ vector containing $u_{i;t-1}$. To

have simplify notation, here the score for the parameter β_1 is denoted as d_1 . We will continue to follow this convention for the elements of the information matrix and for expressing our test statistics. Denoting $J = (NT)^{-1} E(\dot{\mu} \dot{\mu}')$ evaluated at μ_0 , we have

$$J = \frac{1}{2^{3/4}} \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & T & \frac{2(T-1)^{3/4}}{T} & \frac{2(T-1)^{3/4}}{T} \\ 1 & T & \frac{2(T-1)^{3/4}}{T} & \frac{2(T-1)^{3/4}}{T} \\ 0 & \frac{2(T-1)^{3/4}}{T} & \frac{2(T-1)^{3/4}}{T} & \frac{2(T-1)^{3/4}}{T} \end{bmatrix}.$$

This implies that

$$\begin{aligned} J_{1,1} &= J_{11} = \frac{T-1}{T^{3/4}}; \\ J_{1,2} &= J_{12} = \frac{T-1}{2^{3/4}}; \\ J_{1,3} &= J_{13} = \frac{T-1}{T}; \end{aligned} \quad (11)$$

where \cdot stands for the parameter β_1 . Since $J_{1,1} > 0$, indicating the asymptotic positive correlation between the scores d_1 and d_2 , the one-directional test for the random effects reported in Breusch and Pagan (1980) is not valid asymptotically in the presence of serial correlation. For this case our RS_1^* can be easily constructed, from equation (6), as

$$RS_1^* = \frac{NT(A+2B)^2}{2(T-1)(1+\frac{2}{T})}; \quad (12)$$

where A and B denote, as in Baltagi and Li (1991),

$$A = \frac{1}{N} \frac{\mathbf{u}'(I_N - e_T e_T') \mathbf{u}}{\mathbf{u}' \mathbf{u}};$$

and

$$B = \frac{\mathbf{u}' \mathbf{u}_1}{\mathbf{u}' \mathbf{u}};$$

Note that \mathbf{u} are the OLS residuals from the standard linear model $y_{it} = x_{it}' \beta + u_{it}$ without the random effects and serial correlation. Also notice that A and B are closely related to the estimates of the scores d_1 and d_2 , respectively. It is easy to see that the RS_1^* adjusts the conventional RS statistic given in Breusch and Pagan (1980), i.e.,

$$RS_1 = \frac{NTA^2}{2(T-1)}; \quad (13)$$

by correcting the mean and variance of the score d_1 for its asymptotic correlation with $d_{1/2}$.

To see the behavior of RS_1 let us first consider the case of complete misspecification, i.e., $\beta_1^2 = 0$ but $\beta_2 \neq 0$. Using (2) and (11), the noncentrality parameter of RS_1 for this case is:

$$\lambda_2(\beta_2) = \pm J_{1/2}^0 \beta_2 J_{1/2}^1 J_{1/2}^0 \pm = \beta_2^2 \frac{2(T-1)}{T^2}; \quad (14)$$

where for simplicity we use β_2 in place of \pm . In this case, the use of RS_1 will lead to rejection of the null hypothesis $\beta_1^2 = 0$ too often. For local departures RS_1^* will not have this drawback when $\beta_2 \neq 0$ since under $\beta_1^2 = 0$, RS_1^* will have a central $\hat{\Lambda}^2$ distribution. Let us now consider the underspecification situation i.e., when we have both $\beta_1^2 > 0$ and $\beta_2 \neq 0$, and we use RS_1 to test $H_0 : \beta_1^2 = 0$. From (1), (3) and (11), we see that the change in the noncentrality parameter of RS_1 due to nonzero β_2 is given by

$$\begin{aligned} \lambda_3(\beta_1^2; \pm) - \lambda_1(\beta_1^2) &= \lambda_2(\beta_2) + 2\beta_1^0 J_{1/2}^0 \beta_2 \\ &= \beta_2^2 \frac{2(T-1)}{T^2} + 2\beta_1^2 \beta_2 \frac{T-1}{T^{3/2}} \\ &= \frac{2(T-1)}{T} \left[\frac{\beta_2^2}{T} + \frac{\beta_1^2 \beta_2}{\beta_2^2} \right]; \end{aligned} \quad (15)$$

where we use β_1^2 in place of β_1 . From (15), it is easy to see that when $\beta_2 > 0$, the presence of autocorrelation will add power to RS_1 ; but when $\beta_2 < 0$ it can lose power if the individual effect is very high and β_2^2 is low. In this situation, the noncentrality parameter of RS_1^* is not affected. From (7) and (11), the noncentrality parameter of RS_1^* under $\beta_1^2 > 0$ and $\beta_2 \neq 0$, can be written as

$$\begin{aligned} \lambda_6 - \lambda_6(\beta_1^2) &= \beta_1^4 (J_{1/2}^0 - J_{1/2}^0 J_{1/2}^1 J_{1/2}^0) \\ &= \beta_1^4 \frac{T-1}{2^{3/2}} - \frac{(T-1)^2}{T^{2^{3/2}}} \frac{T-1}{T-1} \\ &= \frac{\beta_1^4}{2^{3/2}} (T-1) \left(\frac{1}{2} - \frac{1}{T} \right); \end{aligned} \quad (16)$$

which does not depend on β_2 . There is, however, a cost in applying RS_1^* when β_2 is indeed zero. From (8) the cost is

$$\lambda_7 - \lambda_7(\beta_1^2) = \beta_1^4 J_{1/2}^0 J_{1/2}^1 J_{1/2}^0 = \frac{\beta_1^4}{2^{3/2}} \frac{T-1}{T}; \quad (17)$$

Note that this cost is present only under $\frac{3}{4}\epsilon^2 > 0$. That is, there is a cost only in terms of the power of $RS_{1/2}^{\square}$; the size is unaffected. Later we will provide an interesting interpretation of this cost of $RS_{1/2}^{\square}$ in terms of the behavior of the unadjusted test $RS_{1/2}$ under $\frac{3}{4}\epsilon^2 > 0$.

As mentioned before, Baltagi and Li (1995) derived a RS test for serial correlation in the presence of random individual effects. Naturally, the test requires MLE of $\frac{3}{4}\epsilon^2$: Our procedure gives a simple test for serial correlation in the random effects model. In this situation $RS_{1/2}^{\square}$ is obtained simply by switching $\frac{3}{4}\epsilon^2$ and $\frac{1}{2}$ to yield

$$RS_{1/2}^{\square} = \frac{NT^2(B + \frac{A}{T})^2}{(T - 1)(1 - \frac{2}{T})}; \quad (18)$$

If we assume that the random effects are absent throughout, then $RS_{1/2}^{\square}$ in (18) reduces to

$$RS_{1/2} = \frac{NT^2B^2}{T - 1}; \quad (19)$$

This conventional RS statistic (19) is also given in Baltagi and Li (1991).

As we have done for RS_1 , we can also study the performance of $RS_{1/2}$ under various misspecifications. When there is complete misspecification, i.e., when $\frac{1}{2} = 0$ but $\frac{3}{4}\epsilon^2 > 0$, the noncentrality parameter of $RS_{1/2}$ is

$$s_2(\frac{3}{4}\epsilon^2) = \frac{0 J_{1/2} J_{1/2}^2 J_{1/2}^2 \epsilon^2}{\frac{3}{4}\epsilon^2} = \frac{\frac{3}{4}\epsilon^2 T - 1}{\frac{3}{4}\epsilon^2 T}; \quad (20)$$

where we have used $\frac{3}{4}\epsilon^2$ in place of ϵ^2 . Therefore, $RS_{1/2}$ will reject $H_0 : \frac{1}{2} = 0$ too often when $\frac{3}{4}\epsilon^2 > 0$. Similarly, when there is underspecification, i.e., $\frac{1}{2} \neq 0$ with $\frac{3}{4}\epsilon^2 > 0$, the change in the noncentrality parameter due to the presence of the random effect, is

$$\begin{aligned} s_3(\epsilon^2; \pm) - s_1(\pm) &= s_2(\frac{3}{4}\epsilon^2) + \frac{2\pm^0 J_{1/2} \epsilon^2}{\frac{3}{4}\epsilon^2} \\ &= \frac{T - 1}{T} \frac{\frac{3}{4}\epsilon^2}{\frac{3}{4}\epsilon^2} + 2\frac{1}{2}; \end{aligned} \quad (21)$$

Therefore, we have an increase in (or a possible loss of) power when $\frac{1}{2} > 0$ (or $\frac{1}{2} < 0$). The noncentrality parameter of $RS_{1/2}^{\square}$ will not be affected at all under $\frac{3}{4}\epsilon^2 > 0$. On the other hand, we do, however, pay a penalty when $\frac{3}{4}\epsilon^2 = 0$ and we use the adjusted test $RS_{1/2}^{\square}$. The penalty is

$$s_7(\frac{1}{2}) = \frac{1}{2} J_{1/2} J_{1/2}^2 J_{1/2}^2 \epsilon^2 = 2\frac{1}{2} \frac{T - 1}{T^2}; \quad (22)$$

Due to this factor the power of $RS_{1/2}^{\alpha}$ will be somewhat less than that of $RS_{1/2}$ when α^2 is indeed zero; the size of $RS_{1/2}^{\alpha}$, however, remains unaffected. It is very interesting to note that

$$J_7(\alpha^2) = J_2(\alpha^2) \quad (23)$$

given in (14). Similarly, from (17) and (20)

$$J_7(\alpha^2) = J_2(\alpha^2): \quad (24)$$

An implication of (23) is that the cost of using $RS_{1/2}^{\alpha}$ when $\alpha^2 = 0$ is the same as the cost of using RS_1 when $\alpha^2 \neq 0$. Similarly, (24) implies that the loss in the noncentrality parameter of RS_1 when $\alpha^2 = 0$ is equal to the unwanted gain in the noncentrality parameter of $RS_{1/2}$ when $\alpha^2 > 0$. We will explain these seemingly unintuitive phenomena after we find a relationship among the four statistics, RS_1^{α} , RS_1 , $RS_{1/2}^{\alpha}$, and $RS_{1/2}$. It should be noted that the equalities of equations (23) and (24) are not specific for the error component model, and they hold in general. This can be seen by comparing $J_2(\pm)$ below (2) with J_7 in equation (8), where \tilde{A} swaps position with \hat{A} and \gg is replaced by \pm .

Baltagi and Li (1991, 1995) derived a joint RS test for serial correlation and random individual effects which is given by

$$RS_{1/2} = \frac{NT^2}{2(T-1)(T-2)} [A^2 + 4AB + 2TB^2]: \quad (25)$$

Under the joint null $\alpha^2 = \beta^2 = 0$; $RS_{1/2}$ is asymptotically distributed as \hat{A}_2^2 : Use of this will result in a loss of power compared with the proper one-directional tests when only one of the two forms of misspecification is present, as we noted while discussing (5). For example, when $\beta^2 = 0$ and $\alpha^2 > 0$, the noncentrality parameter of both RS_1 and $RS_{1/2}$ is [see (1) and (5)]

$$J_1(\alpha^2) = \alpha^4 J_{1c} = \frac{\alpha^4 T-1}{\alpha^2 2}: \quad (26)$$

Since for RS_1 and $RS_{1/2}$ we will use respectively \hat{A}_1^2 and \hat{A}_2^2 critical values, $RS_{1/2}$ will be less powerful. An interesting result follows from (12), (13), (18), (19) and (25), namely,

$$RS_{1/2} = RS_1^{\alpha} + RS_{1/2} = RS_1 + RS_{1/2}^{\alpha}, \quad (27)$$

i.e., the two directional RS test for $\frac{3}{4t}^2$ and $\frac{1}{2}$ can be decomposed into the sum of the adjusted one-directional test of one type of alternative and the unadjusted form for the other one. Using (27) we can easily explain some of our earlier observations. First, consider the identities in (23) and (24). From (27), we have

$$RS_{\frac{1}{2}} \text{ ; } RS_{\frac{1}{2}}^{\text{a}} = RS_{\frac{1}{2}} \text{ ; } RS_{\frac{1}{2}}^{\text{a}}: \quad (28)$$

Let us consider the case of $\frac{3}{4t}^2 = 0$ and $\frac{1}{2} \notin 0$. Then the left-hand side of (28) represents the "penalty" of using $RS_{\frac{1}{2}}^{\text{a}}$ (instead of $RS_{\frac{1}{2}}$) while the right-hand side amounts to the "cost" of using $RS_{\frac{1}{2}}$. (28) implies that these penalty and cost should be the same, as noted in (23). A reverse argument explains (24). Secondly, the local presence of $\frac{1}{2}$ (or $\frac{3}{4t}^2$) has no effect on $RS_{\frac{1}{2}}^{\text{a}}$ (or $RS_{\frac{1}{2}}^{\text{a}}$); therefore, from (5) and (27), we can clearly see why the noncentrality parameter of $RS_{\frac{1}{2}}$ will be equal to that of $RS_{\frac{1}{2}}$ (or $RS_{\frac{1}{2}}$) when $\frac{3}{4t}^2 = 0$ (or $\frac{1}{2} = 0$).

4 Empirical illustrations

In this section we present two empirical examples that illustrate the usefulness of the proposed tests. The first is based on a data set used by Greene (1983, 1997). The equation to be estimated is a simple, log-linear cost function:

$$\ln C_{it} = \beta_0 + \beta_1 \ln R_{it} + u_{it};$$

where R_{it} is measured as output of firm i in year t in millions of kilowatt-hours, and C_{it} is the total generation cost in millions of dollars, $i = 1; 2; \dots; 6$, and $t = 1; 2; 3; 4$. The second example is based on the well-known Grunfeld (1958) investment data set for 16 US manufacturing firms measured over 20 years which is frequently used to illustrate panel issues. It has been used in the illustration of misspecification tests in the error-component model in Baltagi, Chang and Li (1992), and in recent books such as those by Baltagi (1995, p.20) and Greene (2000, p.592). The equation to be estimated is a panel model of firm investment using the real value of the firm and the real value of capital stock as explanatory variables:

$$I_{it} = \beta_0 + \beta_1 F_{it} + \beta_2 C_{it} + u_{it};$$

where I_{it} denotes real gross investment for firm i in period t , F_{it} is the real value of the firm and C_{it} is the real value of the capital stock, $i = 1; 2; \dots; 5$, and $t = 1; 2; \dots; 20$.

We estimated the parameters of both models by OLS and implemented the following tests based on OLS residuals: the Breusch-Pagan test for random effects (RS_1), the proposed modified version (RS_1^{\square}), the LM serial correlation test ($RS_{1/2}$), the corresponding modified version ($RS_{1/2}^{\square}$), and the joint test for serial correlation and random effects ($RS_{1+1/2}$). The test statistics for both examples are presented in Table 1; the p-values are given in parentheses.

All of the test statistics were computed individually, and the equalities in (28) are satisfied for both data sets. In the example based on Greene's data the unmodified tests for serial correlation ($RS_{1/2}$) and, to some extent, for random effects (RS_1) reject the respective null hypothesis of no serial correlation and no random effects, and the omnibus test rejects the joint null. But our modified tests suggest that in this example the problem seems to be serial correlation rather than the presence of both effects. For Grunfeld's data, applications of our modified tests point to the presence of the other effect. The unmodified tests soundly reject their corresponding null hypotheses. The modified version of the random effect test (RS_1^{\square}) also rejects the null but the modified serial correlation test ($RS_{1/2}^{\square}$) barely rejects the null at the 5% significance level. It is interesting to note the substantial reduction of the autocorrelation test statistic, from 73.351 to 3.712. So in this example the misspecification can be thought to come from the presence of random effects rather than serial correlation. As expected, the joint test statistic is highly significant.

In spite of the small sample size of the data sets, these examples seem to illustrate clearly the main points of the paper: the proposed modified versions of the test are more informative than a test for serial correlation or random effect that ignores the presence of the other effect. In the first case, serial correlation spuriously induces rejection of the no-random effects hypothesis, and in the second case the opposite happens: the presence of a random effect induces rejection of the no-serial correlation hypothesis. The joint test $RS_{1+1/2}$ rejects the joint null but is not informative about the direction of the misspecification.

$RS_{1+1/2}$ provides a correct measure of the joint effects of individual component and serial correlation. The main problem is how to decompose this measure to get an idea about the true departure(s). From a practical standpoint if $RS_{1+1/2} = RS_1 + RS_{1/2}$ does not hold, that should be an indication of the presence of an interaction between random effects and serial correlation; and the unadjusted statistics RS_1 and $RS_{1/2}$ will be contaminated by the presence of other departures. For example, for the Grunfeld data

$$RS_1 + RS_{1/2} - RS_{1/2} = RS_1 - RS_{1/2} = RS_{1/2} - RS_{1/2} = 69:638:$$

This provides a measure of the interaction between $\frac{3}{4}$ and $\frac{1}{2}$ and is also equal to the correction needed for each unadjusted test.

It is important to emphasize that the implementation of the modified tests is based solely on simple OLS residuals. It could be argued that a more efficient testing procedure could be based on the estimation of a general model that allows for both serial correlation and random effects, and could then test the hypothesis of no-serial correlation and no-random effects as restrictions on this general model (either jointly or individually). But this would require the maximization of a likelihood function whose computational tractability is substantially more involved than computing simple OLS residuals. Hsiao (1986, p.55) commented that the "computation of the MLE is very complicated." For more on the estimation issues of the error component model with serial correlation see Baltagi (1995, pp. 18-19), Majumder and King (1999) and Phillips (1999).

5 Monte Carlo results

In this section we present the results of a Monte Carlo study to investigate the finite sample behavior of the tests. To facilitate comparison with existing results we follow a structure similar to the one adopted by Baltagi, et al. (1992) and Baltagi and Li (1995).

The model was set as a special case of (9):

$$\begin{aligned} y_{it} &= \alpha + \beta x_{it} + u_{it}; \quad i = 1; 2; \dots; N; t = 1; 2; \dots; T \\ u_{it} &= \gamma_i + v_{it}; \\ v_{it} &= \rho v_{i;t-1} + \epsilon_{it}; \quad |\rho| < 1; \end{aligned}$$

where $\alpha = 5$ and $\beta = 0.5$: The independent variable x_{it} was generated following Nerlove (1971):

$$x_{it} = 0.1t + 0.5x_{i;t-1} + \eta_{it};$$

where η_{it} has the uniform distribution on $[0; 0.5; 0.5]$. Initial values were chosen as in Baltagi, et al. (1992). Let $\frac{3}{4}$; $\frac{3}{4}$; $\frac{3}{4}$ and $\frac{3}{4}$ represent the variances of u_{it} ; γ_i ; v_{it} and η_{it} ,

respectively, and let $\zeta = \frac{3}{4}\sigma^2 = \frac{3}{4}\sigma^2$; which represents the "strength" of the random effects. Here, $\frac{3}{4}\sigma^2 = \frac{3}{4}\sigma^2 + \frac{3}{4}\sigma^2$, and we set $\frac{3}{4}\sigma^2 = 20$: ζ and $\frac{1}{2}$ were allowed to take seven different values (0; 0.05; 0.1; 0.2; 0.4; 0.6; 0.8); and three different sample sizes (N; T) were considered: (25; 10); (25; 20) and (50; 10): Since for each i ; v_{it} follows an AR(1) process, $\frac{3}{4}\sigma^2 = \frac{3}{4}\sigma^2 = (1 - \frac{1}{2})$: Then, according to this structure, the random effect term and the innovation were generated as:

$$v_i \sim \text{IIDN}(0; 20(1 - \zeta))$$

$$v_{it} \sim \text{IIDN}(0; 20(1 - \zeta)(1 - \frac{1}{2}^t)):$$

For each sample size the model described above was generated 1,000 times under different parameter settings. Therefore, the maximum standard errors of the estimates of the size and powers would be $\frac{P}{0.5(1 - 0.5)^{1000}} = 0.015$. In each replication the parameters of the model were estimated using OLS, and \bar{v} -ve test statistics, namely, RS_1 ; RS_1^{\square} ; $RS_{1/2}$; $RS_{1/2}^{\square}$ and $RS_{1/2}$, were computed. The tables and graphs are based on the nominal size of 0.05. Our simulation study was quite extensive; we carried out experiments for all possible parameter combinations for the three sample sizes. We present here only a portion of our extensive tables and graphs; the rest is available from the authors upon request.

Calculated statistics under $\zeta = \frac{1}{2} = 0$ were used to estimate the empirical sizes of the tests and to study the closeness of their distributions to \hat{A}^2 through Q-Q plots and the Kolmogorov-Smirnov test. From Table 2 we note that both RS_1 and RS_1^{\square} have similar empirical sizes, but these are below the nominal size 0.05 for $N = 25; T = 10$ and $N = 25; T = 20$: However, when N increases to 50 with $T = 10$, the sizes are higher than 0.05 but are still within acceptable limits. The results for the other three tests $RS_{1/2}$; $RS_{1/2}^{\square}$; $RS_{1/2}$ are not good. All of them reject the null too frequently, and the empirical sizes do not improve as we increase N and T . The performances of $RS_{1/2}$ and $RS_{1/2}^{\square}$ are quite similar. This will enable us to make a valid power comparison between them.

The results of Table 2 are consistent with the Q-Q plots in Figure 1 for $N = 25, T = 10$. To save space figures for the other two combinations of (N; T) are not included. From the plots note that the empirical distributions of the test statistics diverge from that of the \hat{A}_1^2 at the right tail parts. For RS_1 and RS_1^{\square} the points are below the 45° line, particularly for the high values, and that leads to sizes being below 0.05 as we just noted from Table

2. However, the number of points (out of 1,000) that are far away from the 45° line at the tail parts are not many. For $RS_{1/2}$ and $RS_{1/2}^{\square}$ we observe a higher degree of departure from the 45° line in the opposite direction, and this leads to much higher sizes of the tests. Results from the Kolmogorov-Smirnov test, not reported here, accept the null hypothesis of the overall distribution being the same as \hat{A}^2 for all τ -ve statistics. For the true sizes of the tests, however, it is only the tail part, not the overall distribution, that matters.

Let us now turn into the performance of tests in terms of power. For $N = 25$ and $T = 10$, the estimated rejection probabilities of the tests are reported in Table 3, and are also illustrated in Figures (2a)-(2d). Let us first concentrate on $RS_{1/2}$ and $RS_{1/2}^{\square}$, which are designed to test the null hypothesis $H_0 : \lambda_{1/2}^2 = 0$. When $\lambda_{1/2} = 0$, $RS_{1/2}$ is the optimal test. This is clearly evident looking at all the rows in Table 3 with $\lambda_{1/2} = 0$; $RS_{1/2}$ has the highest powers among all the tests. The power of $RS_{1/2}^{\square}$ is less than that of $RS_{1/2}$ when $\lambda_{1/2} = 0$. The losses in power are, however, not very large, as can also be seen from Figure 2(a). When $\lambda_{1/2}$ exceeds 0.2 (or $\lambda_{1/2}^2$ exceeds 4, since we set $\lambda_{1/2}^2 = 20\lambda_{1/2}$) both tests have power equal to 1. The amount of loss in using $RS_{1/2}^{\square}$ when $\lambda_{1/2} = 0$ was characterized by (17) in terms of the decrease in the noncentrality parameter. That loss increases with $\lambda_{1/2}(\lambda_{1/2}^2)$. However, the overall power of $RS_{1/2}^{\square}$ is guided by the noncentrality parameter in (16):

$$p_{1/2}(\lambda_{1/2}^2) = \frac{\lambda_{1/2}^4}{2^{3/4}}(T + 1) + \frac{\lambda_{1/2}^4}{3/4} \frac{T + 1}{T};$$

where the second term is the amount of penalty in using $RS_{1/2}^{\square}$ when $\lambda_{1/2} = 0$, and it is given in (17). Since the first term dominates, the relative value of the loss is negligible. While $RS_{1/2}^{\square}$ does not sustain much loss in power when $\lambda_{1/2} = 0$, we notice some problems in $RS_{1/2}$ when $\lambda_{1/2}^2 = 0$ but $\lambda_{1/2} \notin 0$. $RS_{1/2}$ rejects $H_0 : \lambda_{1/2}^2 = 0$ too frequently. For example, when $\lambda_{1/2} = 0$ (i.e., $\lambda_{1/2}^2 = 0$) and $\lambda_{1/2} = 0.4$, $RS_{1/2}$ has a rejection probability 0.860. For other values of $\lambda_{1/2}$ the proportion of rejections of $\lambda_{1/2}^2 = 0$ (when it is true) can be seen in Figure 2(b). As we discussed in Section 3, this unwanted power is due to the noncentrality parameter $\lambda_{1/2}(\lambda_{1/2}^2)$ in (14), which is "purely" a function of the degree of departure of $\lambda_{1/2}$ from zero. $RS_{1/2}^{\square}$ also has some unwanted power but the problem is less severe. For the above case of $\lambda_{1/2} = 0$ and $\lambda_{1/2} = 0.4$, $RS_{1/2}^{\square}$ has power 0.356. Figure 2(b) gives the power of $RS_{1/2}^{\square}$ when $\lambda_{1/2} = 0$ for different values of $\lambda_{1/2}$. As we mentioned earlier, $RS_{1/2}^{\square}$ is designed to be robust only under local misspecification, i.e, for low values of $\lambda_{1/2}$. From that point of view, it does a very good job | its performance deteriorates only when $\lambda_{1/2}$ takes high values.

From Table 3 and Figure 2(c), we note that when $\zeta > 0$, an increase in $\frac{1}{2}(\zeta > 0)$ enhances the power of $RS_{\frac{1}{2}}$. For example, when $\zeta = 0.05$ the powers of $RS_{\frac{1}{2}}$ for $\frac{1}{2} = 0.0$ and 0.2 are, respectively, 0.307 and 0.702. This can be explained using the expression (15), which gives the changes in the noncentrality parameter of $RS_{\frac{1}{2}}$ due to $\frac{1}{2}$. From (16) we see that the noncentrality parameter of $RS_{\frac{1}{2}}^{\square}$ does not depend on $\frac{1}{2}$. This result is, of course, valid only asymptotically and for local departures of $\frac{1}{2}$ from zero. Figure 2(d) shows that there is some gain in power of $RS_{\frac{1}{2}}^{\square}$, but it is prominent only when $\frac{1}{2} = 0.4$.

As we indicated earlier there could be some loss of power of $RS_{\frac{1}{2}}$ when $\frac{1}{2} < 0$. We performed a small-scale experiment for this case, results of which are reported in Table 4. First note that when $\zeta = 0$, an increase in the absolute value of $\frac{1}{2}$ leads to an increase in the size of $RS_{\frac{1}{2}}$. For example, when $N = 25$, $T = 10$ and $\zeta = 0$, the rejection frequencies for $\frac{1}{2} = 0$ and $\frac{1}{2} = \pm 0.4$ are, respectively, 0.040 and 0.573. This is due to the noncentrality parameter (14) which is a function of $\frac{1}{2}^2$. When $\zeta > 0$ ($\frac{3}{4}\zeta^2 > 0$), the changes in the noncentrality parameter could be negative, and there could be a substantial loss in power of $RS_{\frac{1}{2}}$. For instance, for the above (25,10) sample size combinations, and $\zeta = 0.05$, the powers of $RS_{\frac{1}{2}}$, for $\frac{1}{2} = 0.0$ and -0.4 are, respectively, 0.307 and 0.039. $RS_{\frac{1}{2}}^{\square}$ does not suffer from these detrimental effects as we see from Table 4. Its size remains small for all $\frac{1}{2} < 0$, and power even increases as the absolute value of $\frac{1}{2}$ becomes larger.

In a similar way, we can explain the behavior of $RS_{\frac{1}{2}}$ and $RS_{\frac{1}{2}}^{\square}$ using Table 3 and Figures 3(a)-3(d). From Table 3 we note that, as expected, when $\frac{3}{4}\zeta^2 = 0$, $RS_{\frac{1}{2}}$ has the highest powers among all the tests. The powers of $RS_{\frac{1}{2}}^{\square}$ are very close to those of $RS_{\frac{1}{2}}$. Therefore, the premium we pay for the wider validity of $RS_{\frac{1}{2}}^{\square}$ is minimal.

The real benefit of $RS_{\frac{1}{2}}^{\square}$ is noticed when $\frac{1}{2} = 0$ but $\zeta > 0$; the performance of $RS_{\frac{1}{2}}^{\square}$ is quite remarkable, as can be seen from Figure 3(b). $RS_{\frac{1}{2}}$ rejects $H_0 : \frac{1}{2} = 0$ too often, whereas, quite correctly, $RS_{\frac{1}{2}}^{\square}$ does not reject H_0 so often. For example, when $\zeta = 0.2$ and $\frac{1}{2} = 0$, the rejection proportions for $RS_{\frac{1}{2}}$ and $RS_{\frac{1}{2}}^{\square}$ are 0.766 and 0.046, respectively. Even when we increase ζ to 0.8, the rejection proportion for $RS_{\frac{1}{2}}^{\square}$ goes up to 0.084 only whereas $RS_{\frac{1}{2}}$ rejects 100% of the time. In a way, $RS_{\frac{1}{2}}^{\square}$ is doing more than it is designed to do, that is, not rejecting $\frac{1}{2} = 0$ when $\frac{1}{2}$ is indeed zero even for large values of ζ .

From Figure 3(c), we observe that the power of $RS_{\frac{1}{2}}$ is strongly affected by the presence of random effects, while there is virtually no effect on the power of $RS_{\frac{1}{2}}^{\square}$ as seen from Figure 3(d) even for large values of ζ . This performance of $RS_{\frac{1}{2}}^{\square}$ is exceptionally good. For negative

values of $\frac{1}{2}$ in Table 4, we see that the presence of ζ has a less detrimental effect on $RS_{\frac{1}{2}}^{\square}$. For example, when $\frac{1}{2} = 0.10$, powers of $RS_{\frac{1}{2}}$ are 0.396 and 0.184 for $\zeta = 0.0$ and 0.05, respectively; for the same situations, the powers of $RS_{\frac{1}{2}}^{\square}$ are, respectively, 0.346 and 0.314.

Comparing the performance of $RS_{\frac{1}{2}}^{\square}$ and $RS_{\frac{1}{2}}^{\square}$, we see that the former is even more "robust" in the presence of ζ , both in terms of size and power, than is the latter in the presence of serial correlation. To see this from a theoretical point of view, let us consider (17) and (22), which are, respectively, the penalties of using $RS_{\frac{1}{2}}^{\square}$ and $RS_{\frac{1}{2}}^{\square}$. From (17), $\frac{3/4^4}{3/4^4} \frac{T+1}{T}$, the penalty in using $RS_{\frac{1}{2}}^{\square}$, also depends on $\frac{1}{2}$ through $3/4^2 = 20(1 + \zeta)(1 + 1/2^2)$, while (22), $2^{1/2}(T + 1) = T^2$, is a function of $\frac{1}{2}$ only and is of smaller magnitude in terms of T .

Finally, we discuss briefly the performance of the joint statistic $RS_{1, \frac{1}{2}}$ in the light of our results (4) and (5). This test is optimal when $3/4^2 > 0$ and $\frac{1}{2} \in 0$. As we can see from Table 3, in this situation $RS_{1, \frac{1}{2}}$ has the highest power most of the time. However, when the departure from $3/4^2 = 0; \frac{1}{2} = 0$ is one-directional (say, $3/4^2 > 0; \frac{1}{2} = 0$); $RS_{\frac{1}{2}}$ and $RS_{1, \frac{1}{2}}$ have the same non-centrality parameter [see (26)]. Since $RS_{1, \frac{1}{2}}$ and $RS_{\frac{1}{2}}$ use the \hat{A}_2^2 and \hat{A}_1^2 tests, respectively, there will be a loss of power in using $RS_{1, \frac{1}{2}}$. For example, when $\zeta = 0.05$ and $\frac{1}{2} = 0$, the powers for $RS_{\frac{1}{2}}$ and $RS_{1, \frac{1}{2}}$ are 0.307 and 0.248, respectively. Similarly, when $\zeta = 0; \frac{1}{2} = 0.2$; the power of $RS_{\frac{1}{2}}$ and $RS_{1, \frac{1}{2}}$ are respectively, 0.863 and 0.813. These results are consistent with those of Baltagi and Li (1995). Although $RS_{1, \frac{1}{2}}$ has overall good power, it cannot help to identify the exact source of misspecification when there is only a one-directional departure.

The qualitative performance of all the tests do not change when we increase the sample sizes to $N = 25; T = 20$; and $N = 50; T = 10$ and they further illustrate the usefulness of our modified tests. These results are not presented but are available from the authors upon request.

6 Conclusions

In this paper we have proposed some simple tests, based on OLS residuals for random effects in the presence of serial correlation, and for serial correlation allowing for the presence of random effects. These tests are obtained by adjusting the existing test procedures. We have investigated the finite sample size and power performance of these and some of the available tests through a Monte Carlo study. We have also provided some empirical examples. The Monte Carlo study, along with the examples, clearly show the usefulness of our procedures

to identify the exact source(s) of misspecification. One drawback of our methodology is that we allow for only local misspecification. For non-local departures, efficient tests could be obtained after estimating full model(s) by maximum likelihood; that, however, will lose the simplicity of our tests using only OLS residuals.

Acknowledgements

We would like to thank an associate editor and two anonymous referees for many pertinent comments that helped us to improve the paper. Thanks are also due to Miki Naoko for her help in preparing the manuscript. An earlier version of this paper was presented at Texas A&M University, the Midwest Econometric Group Meetings; the University of Wisconsin at Madison, November 1996; the Economics seminar at University of San Andres, Argentina, November 1997; and the Annual Meeting of the Argentine Association of Political Economy, Bahia Blanca, Argentina. We wish to thank the participants and Badi Baltagi for helpful comments and discussion. However, we retain responsibility for any remaining errors.

References

- Andereson, T.W and C. Hsiao, 1982, Formulation and estimation of dynamic models using panel data, *Journal of Econometrics*, 18, 47-82.
- Balestra, P. and M. Nerlove, 1966, Pooling cross-section and time-series data in the estimation of a dynamic model: the demand for natural gas, *Econometrica*, 34, 585-612.
- Baltagi, B., 1995, *Econometric Analysis of Panel Data*, New York: Wiley & Sons.
- Baltagi, B., Y. Chang and Q. Li, 1992, Monte Carlo results on several new and existing tests for the error component model, *Journal of Econometrics*, 54, 95-120.

Baltagi, B. and Q. Li, 1991, A joint test for serial correlation and random individual effects, *Statistics & Probability Letters*, 11, 277-280.

Baltagi, B. and Q. Li, 1995, Testing AR(1) against MA(1) disturbances in an error component model, *Journal of Econometrics*, 68, 133-151.

Bera, A.K. and C. M. Jarque, 1982, Model specification tests: A simultaneous approach, *Journal of Econometrics*, 20, 59-82.

Bera, A.K. and M.J. Yoon, 1991, Specification testing with misspecified alternatives, Bureau of Economic and Business Research Faculty Working Paper 91-0123, University of Illinois, and presented at the Econometric Society Winter Meeting, Washington DC, December 1990.

Bera, A.K. and M.J. Yoon, 1993, Specification testing with locally misspecified alternatives, *Econometric Theory*, 9, 649-658.

Bera, A.K., S.S. Ra and N. Sarkar, 1998, Hypothesis testing for some nonregular cases in econometrics, In S. Chakravorty, D. Coondoo and R. Mukherjee, Ed., *Econometrics: Theory and Practice*, New Delhi: Allied Publishers, pp. 319-351.

Breusch, T.S. and A.R. Pagan, 1980, The Lagrange multiplier test and its applications to model specification in Econometrics, *Review of Economic Studies*, 47, 239-253.

Das Gupta, S. and M.P. Perlman (1974), Power of the noncentral F-test: Effect of additional variate on Hotelling's T^2 -Test, *Journal of the American Statistical Association*, 69, 174-180.

Davidson, R. and J.G. MacKinnon, 1987, Implicit alternatives and local power of test statistics, *Econometrica*, 55, 1305-1329.

Greene, W., 1983, Simultaneous estimation of factor substitution, economies of scale and non-neutral technical change, In A. Dogramaci, ed., *Econometric analyses of productivity*, Boston: Kluwer-Nijhoff, pp. 121-144.

Greene, W., 2000, *Econometric Analysis*, 4th Edition, New Jersey: Prentice Hall.

Grunfeld, Y., 1958, The determinants of corporate investment, Unpublished Ph.D thesis, Department of Economics, University of Chicago.

Grunfeld, Y. and Z. Griliches, 1960, Is aggregation necessarily bad?, *Review of Economics and Statistics*, 42, 1-13.

Haavelmo, T., 1944, The probability approach in econometrics, *Supplement to Econometrica*, 12.

Hsiao, C., 1986, *Analysis of Panel Data*, Cambridge: Cambridge University Press.

Lillard, L.A. and R.J. Willis, 1978, Dynamic aspects of earning mobility, *Econometrica*, 46, 985-1012.

Majumder, A.K., and M.L. King, 1999, Estimation and testing of a regression with a serially correlated error component, Working Paper, Department of Econometrics and Business Statistics, Monash University, Australia.

Nerlove, M., 1971, Further evidence of the estimation of dynamic economic relations from a time-series of cross-sections, *Econometrica*, 39, 359-382.

Phillips, R.F., 1999, Estimation of mixed-effects and error components models with an AR(1) component: Some new likelihood maximization procedures and Monte Carlo evidence, Working Paper, Department of Economics, George Washington University.

Rao, C.R., 1948, Large sample tests of statistical hypothesis concerning several parameters with application to problems of estimation, *Proceedings of the Cambridge Philosophical Society*, 44, 50-57.

Saikkonen, P., 1989, Asymptotic relative efficiency of the classical test statistics under misspecification, *Journal of Econometrics*, 42, 351-369.

Welsh, A. H., 1996, *Aspects of Statistical Inference*, New York: John Wiley & Sons.

TABLE 1
Empirical illustration
Tests for random effects and serial correlation

| Data | RS_1 | RS_1^{α} | $RS_{1/2}$ | $RS_{1/2}^{\alpha}$ | $RS_{1/2}$ | RSO_1 | RSO_1^{α} |
|----------|--------------------|--------------------|-------------------|---------------------|--------------------|------------------|-------------------|
| Greene | 5.872 (0.015) | 0.269 (0.604) | 15.569 (0.000) | 9.966 (0.002) | 15.838 (0.000) | 2.423 (0.007) | 0.518 (0.3020) |
| Grunfeld | 453.822 (0.000) | 384.183 (0.000) | 73.351 (0.000) | 3.712 (0.054) | 457.535 (0.000) | 21.303 (0.00) | 19.605 (0.000) |

Note: p-values are given in parenthesis.

TABLE 2
Empirical size of tests
(nominal size=0.05)

| Sample size | Tests | | | | | | |
|-------------|--------|-----------------|------------|---------------------|------------|---------|------------------|
| | RS_1 | RS_1^{α} | $RS_{1/2}$ | $RS_{1/2}^{\alpha}$ | $RS_{1/2}$ | RSO_1 | RSO_1^{α} |
| (25,10) | 0.047 | 0.048 | 0.087 | 0.072 | 0.062 | 0.045 | 0.051 |
| (25,20) | 0.050 | 0.051 | 0.060 | 0.056 | 0.057 | 0.052 | 0.058 |
| (50,10) | 0.043 | 0.040 | 0.065 | 0.062 | 0.059 | 0.046 | 0.053 |

TABLE 4
Estimated Powers of Different Tests

| \hat{c} | $\frac{1}{2}$ | RS_1 | RS_1^a | $RS_{\frac{1}{2}}$ | $RS_{\frac{1}{2}}^a$ | $RS_{\frac{1}{2},1}$ |
|-----------------------------|---------------|--------|----------|--------------------|----------------------|----------------------|
| Sample size: N = 25; T = 10 | | | | | | |
| 0.00 | -0.05 | 0.039 | 0.031 | 0.173 | 0.170 | 0.118 |
| 0.00 | -0.10 | 0.044 | 0.019 | 0.396 | 0.346 | 0.285 |
| 0.00 | -0.20 | 0.162 | 0.016 | 0.902 | 0.857 | 0.833 |
| 0.00 | -0.40 | 0.573 | 0.048 | 1.000 | 1.000 | 1.000 |
| 0.05 | -0.05 | 0.254 | 0.289 | 0.097 | 0.130 | 0.269 |
| 0.05 | -0.10 | 0.202 | 0.340 | 0.184 | 0.314 | 0.365 |
| 0.05 | -0.20 | 0.097 | 0.369 | 0.680 | 0.830 | 0.770 |
| 0.05 | -0.40 | 0.039 | 0.679 | 0.997 | 1.000 | 1.000 |
| Sample size: N = 25; T = 20 | | | | | | |
| 0.00 | -0.05 | 0.041 | 0.025 | 0.247 | 0.217 | 0.168 |
| 0.00 | -0.10 | 0.049 | 0.025 | 0.640 | 0.600 | 0.520 |
| 0.00 | -0.20 | 0.136 | 0.010 | 0.999 | 0.999 | 0.992 |
| 0.00 | -0.40 | 0.610 | 0.018 | 1.000 | 1.000 | 1.000 |
| 0.05 | -0.05 | 0.652 | 0.707 | 0.090 | 0.200 | 0.665 |
| 0.05 | -0.10 | 0.613 | 0.758 | 0.244 | 0.557 | 0.806 |
| 0.05 | -0.20 | 0.507 | 0.829 | 0.882 | 0.987 | 0.992 |
| 0.05 | -0.40 | 0.303 | 0.963 | 1.000 | 1.000 | 1.000 |