Gaps and Triangles*

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January, 1999

Abstract

We compute the optimal monetary policy in an economy with price stickiness.

1. Introduction

" ...since fluctuations about the system's equilibrium represented disequilibrium behavior, standard welfare propositions could be applied only to the average behavior of the system, and not to fluctuations about the average. This left one free to apply other criteria in evaluating stabilization policies: "gaps" instead of "triangles", as James Tobin [?] puts it. The general idea was to use policy tools to keep the actual path of the system "close" in one sense or another to its equilibrium path. Proponents of various stabilization policies were thus free of the burden under which the ordinary welfare economist labors - of "justifying" intervention by some specific "market failure" and tailoring the nature of the intervention to the nature of the failure." Robert E. Lucas, Jr (1980)

*Preliminary and incomplete. We thank Jordi Gali for very helpful discussions.

We consider a world where firms operate in a monopolistic competition framework and prices are sticky. Under commitment the government takes account of the fact that the price level can be adjusted in response to monetary policy. However, this response is a response for a single price and therefore the solution with sticky prices and commitment does not need to coincide with the solution with flexible prices. The solution with sticky prices gives an additional degree of freedom that can be used by a benevolent government.

Nonetheless, for a broad class of preferences, all those that are time-separable and simultaneously consistent with balanced growth and Gorman agregable, the flexible prices solution is the optimal solution. Thus, gaps and distortions have to be treated in the same way. The arguments are public finance arguments. You want to set the same markup over all states of the world if in the optimal taxation problem is optimal to set the same tax across goods. If the interest rates are equal across states, then the optimal allocation is the flexible prices one.

2. An economy with flexible prices and portfolio choices

Our model economy follows closely the structures in Christiano, Eichenbaum and Evans (1997), Ireland (1996) and Carlstrom and Fuerst (1998). The economy consists of a representative household, a continuum of firms indexed by $i \in [0, 1]$, financial intermediaries and a government or central bank. Each firm produces a distinct, perishable consumption good, indexed by i.

The government makes a lump-sum monetary transfer $X_t = (G_t - 1)M_{t-1}^s$ to the representative financial intermediary at each date $t = 0, 1, 2, ..$, where M_t^s represents the money supply per household at date t. The money supply evolves according to $M_t^s = G_t M_{t-1}^s$. If M_{t+1} denotes the money carried by the household into period $t + 1$, market clearing requires that $M_t^s = M_{t+1}$, for all $t = 0, 1, 2, ...$

The financial intermediaries receive loans L_t from the households and lend it out to the firms. The gross nominal interest rate on both the deposits and the loans is R_t. The financial intermediaries receive from the monetary authority the transfer of money X_t, that is also lent out to the firms.

2.1. The households

The preferences are over composite consumption C_t, and leisure $1 - N_t$, and described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(C_t, 1 - N_t \right) \right\}$$

where β is a discount factor and the composite C_t is

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

where θ is the elasticity of substitution between any two goods.

The households start period t with outstanding money balances, M_t, decide to lend out L_t in the credit market to the financial intermediaries and buy state contingent nominal securities S_{t+1}. Each security pays one unit of money at the beginning of period $t+1$ in a particular state and costs $Q_{t,t+1}$ in the beginning of period t. The expenditure in securities is $E_t Q_{t,t+1} S_{t+1}$. The households receive the labor income, $W_t N_t$ where W_t is the nominal wage rate, that can be used to purchase consumption in the same period. The purchases of consumption goods have to be made with $M_t - L_t + W_t N_t - E_t Q_{t,t+1} S_{t+1} + S_t$, so,

$$\int_0^1 P_t(i) c_t(i) di \leq M_t + W_t N_t - L_t - E_t Q_{t,t+1} S_{t+1} + S_t \tag{2.1}$$

At the end of the period, the households receive the gross returns on the loans $R_t L_t$, the dividends from the financial intermediaries, $R_t X_t$, and profits from firms, $\int_0^1 \Pi_t(i) di$.

The households face the budget constraints

$$M_{t+1} \leq \left[M_t + W_t N_t - L_t - E_t Q_{t,t+1} S_{t+1} + S_t - \int_0^1 P_t(i) c_t(i) di \right] + R_t \left[L_t + X_t \right] + \int_0^1 \Pi_t(i) di \tag{2.2}$$

The households' problem is defined as

$$V(M_t, S_t) = \text{Max } \left\{ u \left(C_t, 1 - N_t \right) + \beta E_t V(M_{t+1}, S_{t+1}) \right\}$$

subject to

$$\int_0^1 P_t(i) c_t(i) di \leq M_t + W_t N_t - L_t - E_t Q_{t,t+1} S_{t+1} + S_t \tag{2.3}$$

$$M_{t+1} \leq \left[M_t + W_t N_t - L_t - E_t Q_{t,t+1} S_{t+1} + S_t - \int_0^1 P_t(i) c_t(i) di\right] + R_t \left[L_t + X_t\right] + \int_0^1 \Pi_t(i) di \tag{2.4}$$

Let λ_t and μ_t, be the multipliers of the budget constraint, (2.4), and the c.i.a. constraint, (2.3). For $R_t > 1$, $t = 0, 1, 2, ...$, the following are first order conditions of this problem

$$u_{Ct} \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{1}{\theta-1}} c_t(i)^{\frac{-1}{\theta}} - \left(\lambda_t + \mu_t\right) P_t(i) = 0 \tag{2.5}$$

$$-u_{1-Nt} + \left(\lambda_t + \mu_t\right) W_t = 0 \tag{2.6}$$

$$R_t \lambda_t = \lambda_t + \mu_t \tag{2.7a}$$

$$-\lambda_t + \beta E_t V_{Mt+1} = 0 \tag{2.8a}$$

$$\beta \Pr(s^{t+1}/s^t) V_{St+1} - \left(\lambda_t + \mu_t\right) \Pr(s^{t+1}/s^t) Q_{t,t+1} = 0 \tag{2.9a}$$

Using the envelope theorem

$$V_{Mt} = \lambda_t + \mu_t \tag{2.10}$$

and

$$V_{St} = \lambda_t + \mu_t \tag{2.11}$$

Therefore, using (2.8a) and (2.10)

$$\lambda_t = \beta E_t \left[\lambda_{t+1} + \mu_{t+1}\right] \tag{2.12a}$$

and using (2.9a) and (2.11),

$$\beta(\lambda_{t+1} + \mu_{t+1}) - (\lambda_t + \mu_t) Q_{t,t+1} = 0 \tag{2.13a}$$

Equation (2.5) can be written as

$$u_{Ct} C_t^{\frac{1}{\theta}} c_t(i)^{\frac{-1}{\theta}} = \left(\lambda_t + \mu_t\right) P_t(i) \tag{2.14}$$

Let $P_t = \left[\int P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$. Then, using conditions (2.14) for all i,

$$\left[\int P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} = \frac{u_{Ct} C_t^{\frac{1}{\theta}}}{\lambda_t + \mu_t} \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{-\frac{1}{\theta-1}}$$

and so

$$u_{Ct} = (\lambda_t + \mu_t) P_t \tag{2.15}$$

From (2.5) and (2.14) we obtain

$$\frac{c_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{2.16}$$

To summarize we have the following marginal conditions

$$u_{Ct} = (\lambda_t + \mu_t) P_t$$

$$u_{1-Nt} = (\lambda_t + \mu_t) W_t$$

$$\mu_t = (R_t - 1)\lambda_t$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} + \mu_{t+1}\right]$$

$$\beta(\lambda_{t+1} + \mu_{t+1}) - (\lambda_t + \mu_t) Q_{t,t+1} = 0$$

and therefore

$$u_{Ct} = R_t \lambda_t P_t \tag{2.21}$$

$$u_{1-Nt} = R_t \lambda_t W_t \tag{2.22a}$$

$$\lambda_t = \beta E_t \left[R_{t+1} \lambda_{t+1}\right] \tag{2.23a}$$

$$\beta R_{t+1} \lambda_{t+1} - R_t \lambda_t Q_{t,t+1} = 0 \tag{2.24a}$$

Using (2.15) and (2.6) we have

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{W_t}{P_t} \tag{2.25}$$

From (2.21) and (2.23a), we have that

$$\frac{u_{Ct}}{P_t} = R_t E_t \left[\frac{\beta u_{Ct+1}}{P_{t+1}}\right] \tag{2.26}$$

and from (2.21) and (2.24a), we have

$$Q_{t,t+1}\frac{P_{t+1}}{P_t} = \frac{\beta u_{Ct+1}}{u_{Ct}} \tag{2.27}$$

From (2.26) and (2.27) we obtain

$$E_t\left[Q_{t,t+1}\right] = \frac{1}{R_t}$$

Condition (2.16) defines the demand for each of the intermediate goods i and condition (2.25) sets the intratemporal marginal rate of substitution between consumption and leisure equal to the real wage. Notice that the household can use the labor income to consume the good in the same period. Condition (2.26) is a requirement for the optimal savings decision. One additional unit of L_t implies the marginal cost of $\frac{U_{ct}}{P_t}$, and the benefit of $R_t E_t\left[\frac{\beta U_{ct+1}}{P_{t+1}}\right]$, since the returns can only be used for consumption the following period.

2.2. Firms

The firms need to borrow the wage bill from the financial intermediaries. Define $B_t(i)$ as the demand for loans by firm i, y_t as the production of firm i and $n_t(i)$ as the labor demanded by firm i. Flexible price monopolistic competitive firms maximize the stock market value. This is equivalent to choosing prices every period so that profits are maximized. Notice that the cash flow of the firm obeys

$$B_t(i) = B_{t-1}(i)R_{t-1} - P_{t-1}(i)y_{t-1}(i) + W_t n_t(i) + \Pi_{t-1}(i)$$

At the beginning of period t the firm decides on the loans $B_t(i)$ in order to pay the loans made last period and pay the wage bill in period t. The revenue from selling the output of period $t-1$ is received at the end of the period which is equivalent to being received at the beginning of period t. The same applies for profits.

The problem of the firm is to choose the price in order to maximize profits that can be used for consumption in period $t+1$ taking the demand function as given.The value of profits in units of utility is $E_t\frac{\beta u_{Ct+1}}{P_{t+1}}\Pi_t(i)$. Alternatively could have $E_t Q_{t,t+1}\Pi_t(i)$ in units of money at the beginning of the period t, or yet $E_t\frac{Q_{t,t+1}}{P_t}\Pi_t(i)$ in units of consumption in period t. Maximizing any of these functions is equivalent to maximizing

$$\Pi_t(i) \equiv P_t(i)y_t(i) - W_t n_t(i) - (R_t - 1)B_t(i)$$

satisfying the demand function

$$\frac{y_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{2.28}$$

obtained from the households problem(2.16), the technology

$$y_t(i) \leq s_t n_t(i)$$

where s_t is the level of technology, and the cash-in-advance restriction

$$W_t n_t(i) \leq B_t(i).$$

If the technology and cash-in-advance restrictions are both satisfied with equality, the profits can be written as

$$\Pi_t(i) = P_t(i)y_t(i) - R_t W_t \frac{y_t(i)}{s_t}$$

where $P_t(i)$ satisfies the demand function (2.28). The first order condition of this problem is

$$P_t(i)\left[1 + \frac{d\ln P_t(i)}{d\ln y_t(i)}\right] - \frac{R_t W_t}{s_t} = 0$$

where $\frac{d\ln P_t(i)}{d\ln y_t(i)} = -\frac{1}{\theta}$, since θ is the demand elasticity. Therefore, it must be that

$$P_t = P_t(i) = \frac{\theta}{\theta - 1}\frac{W_t R_t}{s_t}$$

The firms set a common price, which is a constant mark-up over marginal cost. As the elasticity of demand θ gets larger, the mark-up converges down to 1.

Then we have

$$\frac{W_t}{P_t}R_t = \frac{(\theta - 1)s_t}{\theta} \tag{2.29}$$

So when R_t increases, costs increase so that the firms will set a higher price relative to the wage. Anf if the labor supply is a positive function of the wage rate a reduction of the real wage induces a reduction in labor.

2.3. Market clearing:

$$L_t + X_t = B_t$$

where $B_t = \int_0^1 B_t(i)di$

$$c_t(i) = y_t(i)$$

$$N_t = \int_0^1 n_t(i)\ di$$

$$S_t = 0$$

2.4. Equilibrium allocations:

The equilibrium allocations in this environment with flexible prices and portfolios decisions can be summarized by the following equations that determine in every period $C^*(s_t, R_t)$, $N^*(s_t, R_t)$ and $\left(\frac{W_t}{P_t}\right)^* (s_t, R_t)$:

$$\frac{U_{1-N}(C_t, 1-N_t)}{U_c(C_t, 1-N_t)} = \frac{W_t}{P_t} \tag{2.30}$$

$$C_t = s_t N_t \tag{2.31}$$

and (2.29). This system can be solved for N_t

$$\frac{U_{1-N}(s_t N_t, 1-N_t)}{U_c(s_t N_t, 1-N_t)} = \frac{(\theta-1)s_t}{\theta R_t} \tag{2.32}$$

3. When prices are set in advance

We consider now an environment where firms set the prices one period in advance and sell the output on demand in period t at the previously chosen price. These prices are $P_t(i)$.

When firm i sets prices one period in advance it solves the problem of choosing the price $P_t(i)$ that maximizes

$$E_{t-1}\left[\beta^2 \frac{u_{Ct+1}}{P_{t+1}} y_t(i) \left(P_t(i) - \frac{R_t W_t}{s_t}\right)\right] \tag{3.1}$$

where u_{Ct+1} is the marginal valuation of consumption at period $t+1$ when the profits can be used for consumption. Alternatively could have profits as

$$E_{t-1}\left[Q_{t-1,t}Q_{t,t+1}y_t(i)\left(P_t(i)-\frac{R_tW_t}{s_t}\right)\right] \tag{3.2}$$

in units of money at $t-1$. Since

$$Q_{t,t+1}=\frac{\beta u_{Ct+1}}{u_{Ct}}\frac{P_t}{P_{t+1}} \tag{3.3}$$

we obtain the expression for profits above.

The expression results from

$$\text{Max } E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}\left(P_t(i)y_t(i)-R_tW_tn_t(i)\right)\right]$$

$$\begin{aligned} \text{s.t.} \quad &: \\ \frac{y_t(i)}{C_t} &= \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \\ y_t(i) &\leq sn_t(i) \end{aligned}$$

Expression (3.1) can be written as

$$E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta}P_t\left(\frac{P_t(i)}{P_t}-\frac{R_tW_t}{s_tP_t}\right)\right]$$

The first order conditions are

$$E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t(-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}\left(\frac{P_t(i)}{P_t}-\frac{R_tW_t}{s_tP_t}\right)+\frac{u_{Ct+1}}{P_{t+1}}C_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\right]=0$$

or, dividing through by $P_t(i)^{-\theta}$

$$E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t(-\theta)\frac{P_t(i)^{-1}}{P_t^{-\theta-1}}\left(\frac{P_t(i)}{P_t}-\frac{R_tW_t}{s_tP_t}\right)+\frac{u_{Ct+1}}{P_{t+1}}\frac{C_t}{P_t^{-\theta}}\right]=0$$

or, yet,

$$E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}\frac{C_t(1-\theta)}{P_t^{-\theta}}+\frac{u_{Ct+1}}{P_{t+1}}C_t\theta\frac{P_t(i)^{-1}}{P_t^{-\theta}}\frac{R_tW_t}{s_t}\right]=0$$

The solution is given by

$$P_t(i) = \frac{\theta E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}} C_t P_t^{\theta} \frac{R_t W_t}{s_t}\right]}{(\theta - 1) E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}} C_t P_t^{\theta}\right]}$$

This can be written as

$$P_t(i) = P_t = \frac{\theta}{(\theta - 1)} E_{t-1}\left[\upsilon_t \frac{R_t W_t}{s_t}\right]$$

where $\upsilon_t = \frac{\frac{u_{Ct+1}}{P_{t+1}} C_t}{E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}} C_t\right]}$.

4. The Ramsey problem

4.1. Flexible prices

The following are first order conditions of the private problem:

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{W_t}{P_t} \tag{4.1}$$

$$\frac{u_{Ct}}{P_t} = R_t E_t \left[\frac{\beta u_{Ct+1}}{P_{t+1}}\right] \tag{4.2}$$

$$\frac{W_t}{P_t} R_t = \frac{(\theta - 1) s_t}{\theta} \tag{4.3}$$

$$C_t = s_t N_t \tag{4.4}$$

Conditions (4.1), (4.3) and (4.4) determine the allocations for C_t and N_t as functions of s_t and R_t. Can define the problem

$$\text{Max } E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t\right) \right\}$$

s.t. (4.4) and

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{(\theta - 1) s_t}{\theta R_t} \tag{4.5}$$

obtained from (4.1) and (4.3). Let

$$\mu \equiv \frac{(\theta-1)}{\theta R_t} \tag{4.6}$$

Then we have the problem

$$\text{Max } E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1-N_t\right) \right\}$$

s.t.

$$C_t = s_t N_t \tag{4.7}$$

and

$$\frac{u_{1-Nt}}{u_{Ct}} = \mu s_t \tag{4.8}$$

and we can determine the marginal distortions from the multipliers of the restrictions (4.8). These multipliers are not necessarilly the same across states. This will leave room to improve upon the allocations in the sticky prices solution.

4.2. Sticky prices

The following are first order conditions of the private problem:

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{W_t}{P_t} \tag{4.9}$$

$$\frac{u_{Ct}}{P_t} = R_t E_t \left[\frac{\beta u_{Ct+1}}{P_{t+1}} \right] \tag{4.10}$$

$$E_{t-1} \left[\upsilon_t \frac{\theta}{(1-\theta)} \frac{R_t W_t}{s_t P_t} \right] = 1 \tag{4.11}$$

where

$$\upsilon_t = \frac{\frac{u_{Ct+1}}{P_{t+1}} C_t}{E_{t-1} \left[\frac{u_{Ct+1}}{P_{t+1}} C_t \right]}$$

.

$$C_t = s_t N_t \tag{4.12}$$

Can define the problem

$$\text{Max } E_0\left\{\sum_{t=0}^{\infty}\beta^t u\left(C_t, 1-N_t\right)\right\}$$

s.t.

$$C_t = s_t N_t \tag{4.13}$$

and

$$E_{t-1}\left[\upsilon_t \frac{1}{\mu s_t}\frac{u_{1-Nt}}{u_{Ct}}\right] = 1 \tag{4.14}$$

obtained from (4.9) and (4.11), where $\upsilon_t = \frac{\frac{u_{Ct+1}}{P_{t+1}}C_t}{E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t\right]}$.

If we set

$$R_t = R$$

where R is a constant then we can write (4.14) as

$$E_{t-1}\left[\upsilon_t \frac{1}{s_t}\frac{u_{1-Nt}}{u_{Ct}}\right] = \mu \tag{4.15}$$

We can determine the marginal distortion from the multiplier of the restrictions (4.14).

Can replace the P_{t+1} using the intertemporal condition. The constraints will be functions of the quantities and the interest rates.

$$E_{t-1}\left[\frac{\frac{u_{Ct+1}}{P_{t+1}}C_t}{E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t\right]}\frac{1}{\mu s_t}\frac{u_{1-Nt}}{u_{Ct}}\right] = 1 \tag{4.16}$$

Using the law of iterated expectations, we have

$$E_{t-1}\left[E_t\left(\frac{\frac{u_{Ct+1}}{P_{t+1}}C_t}{E_{t-1}\left[\frac{u_{Ct+1}}{P_{t+1}}C_t\right]}\frac{1}{\mu s_t}\frac{u_{1-Nt}}{u_{Ct}}\right)\right] = 1 \tag{4.17}$$

and therefore

$$E_{t-1}\left[\frac{E_t\left(\frac{u_{Ct+1}}{P_{t+1}}\right)C_t}{E_{t-1}\left[E_t\left(\frac{u_{Ct+1}}{P_{t+1}}\right)C_t\right]}\frac{1}{\mu s_t}\frac{u_{1-Nt}}{u_{Ct}}\right] = 1 \tag{4.18}$$

From

$$\frac{u_{Ct}}{P_t} = R_t E_t\left[\frac{\beta u_{Ct+1}}{P_{t+1}}\right] \tag{4.19}$$

can replace $E_t\left[\frac{\beta u_{Ct+1}}{P_{t+1}}\right]=\frac{u_{Ct}}{R_tP_t}$ and so

$$E_{t-1}\left[\frac{\frac{u_{Ct}}{R_tP_t}C_t}{E_{t-1}\left[\frac{u_{Ct}}{R_tP_t}C_t\right]}\frac{1}{\frac{(\theta-1)}{\theta R_t}s_t}\frac{u_{1-Nt}}{u_{Ct}}\right]=1 \tag{4.20}$$

Therefore

$$E_{t-1}\left[\frac{\frac{u_{Ct}}{R_t}C_t}{E_{t-1}\left[\frac{u_{Ct}}{R_t}C_t\right]}\frac{1}{\frac{(\theta-1)}{\theta R_t}s_t}\frac{u_{1-Nt}}{u_{Ct}}\right]=1 \tag{4.21}$$

This can be rewritten as

$$E_{t-1}\left[\frac{u_{Ct}}{R_t}C_t\frac{1}{\frac{(\theta-1)}{\theta R_t}s_t}\frac{u_{1-Nt}}{u_{Ct}}-\frac{u_{Ct}}{R_t}C_t\right]=0 \tag{4.22}$$

or

$$E_{t-1}\left[\frac{1}{s_t}u_{1-Nt}C_t-\frac{u_{Ct}}{\frac{\theta R_t}{(\theta-1)}}C_t\right]=0 \tag{4.23}$$

We can now compare the two problems, with flexible prices and with sticky prices. The problem with flexible prices is

$$\text{Max } E_0\left\{\sum_{t=0}^{\infty}\beta^t u\left(C_t,1-N_t\right)\right\}$$

s.t.

$$C_t=s_tN_t \tag{4.24}$$

and

$$\frac{u_{Ct}}{\frac{\theta R_t}{(\theta-1)}}C_t-\frac{1}{s_t}u_{1-Nt}C_t=0 \tag{4.25}$$

or

$$\frac{u_{Ct}}{\frac{\theta R_t}{(\theta-1)}}C_t-u_{1-Nt}N_t=0 \tag{4.26}$$

This looks like a standard implementability condition. One must apply the standard optimal taxation rules. In the problem with fixed prices the implementability condition is replaced for its expected value

$$E_{t-1}\left[\frac{u_{Ct}}{\frac{\theta R_t}{(\theta-1)}}C_t-\frac{1}{s_t}u_{1-Nt}C_t\right]=0 \tag{4.27}$$

Let $\lambda_t \Pr$ and $\varphi_t \Pr$ be the multipliers of the resources constraint (4.24) and the implementability condition (4.25). Pr are the probabilities of the the real shocks. The first order conditions of the Ramsey problem are

$$u_{Ct} - \lambda_t + \varphi_t \frac{(\theta-1)}{\theta R_t}\left(u_{Ct} + u_{CtCt}C_t\right) - \varphi_t \frac{1}{s_t} u_{1-Nt} - \varphi_t \frac{1}{s_t} u_{1-Nt,Ct} C_t = 0 \quad (4.28)$$

$$-u_{1-Nt} + \lambda_t s_t - \varphi_t \frac{(\theta-1)}{\theta R_t} u_{Ct,1-Nt} C_t + \varphi_t \frac{1}{s_t} u_{1-Nt,1-Nt} C_t = 0 \quad (4.29)$$

or

$$\lambda_t = \frac{1}{s_t} u_{1-Nt} - \varphi_t \frac{1}{s_t} u_{1-Nt,1-Nt} N_t + \varphi_t \frac{(\theta-1)}{\theta R_t s_t} u_{Ct,1-Nt} C_t \quad (4.30)$$

$$\begin{aligned} 0 &= u_{Ct} - \frac{1}{s_t} u_{1-Nt} + \varphi_t \frac{1}{s_t} u_{1-Nt,1-Nt} N_t - \varphi_t \frac{(\theta-1)}{\theta R_t s_t} u_{Ct,1-Nt} C_t + \\ &\quad \varphi_t \frac{(\theta-1)}{\theta R_t} u_{Ct}\left(1 + \frac{u_{CtCt}C_t}{u_{Ct}}\right) - \varphi_t \frac{1}{s_t} u_{1-Nt} - \varphi_t \frac{1}{s_t} u_{1-Nt,Ct} C_t \end{aligned} \quad (4.31)$$

where $\sigma = -\frac{u_{CtCt}C_t}{u_{Ct}}$. Dividing through by u_{1-Nt}

$$\begin{aligned} 0 &= \frac{u_{Ct}}{u_{1-Nt}} - \frac{1}{s_t} + \varphi_t \frac{1}{s_t} \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}} N_t - \varphi_t \frac{(\theta-1)}{\theta R_t s_t} \frac{u_{Ct,1-Nt}}{u_{1-Nt}} C_t + \\ &\quad \varphi_t \frac{(\theta-1)}{\theta R_t} \frac{u_{Ct}}{u_{1-Nt}} \left(1 + \frac{u_{CtCt}C_t}{u_{Ct}}\right) one - \varphi_t \frac{1}{s_t} - \varphi_t \frac{1}{s_t} \frac{u_{1-Nt,Ct}}{u_{1-Nt}} C_t \end{aligned} \quad (4.32)$$

Using $\frac{u_{1-Nt}}{u_{Ct}} = \frac{(\theta-1)s_t}{\theta R_t}$ and multiplying through by s_t

$$\frac{\theta R_t}{(\theta-1)} - 1 + \varphi_t \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}} N_t - \varphi_t \frac{(\theta-1)}{\theta R_t} \frac{u_{Ct,1-Nt}}{u_{1-Nt}} C_t + \varphi_t \left(1 + \frac{u_{CtCt}C_t}{u_{Ct}}\right) - \varphi_t - \varphi_t \frac{u_{1-Nt,Ct}}{u_{1-Nt}} C_t = 0 \quad (4.33)$$

and so

$$\varphi_t = \frac{\frac{\theta R_t}{(\theta-1)} - 1}{-\frac{u_{CtCt}C_t}{u_{Ct}} - \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}} N_t + \left(\frac{(\theta-1)}{\theta R_t} + 1\right) \frac{u_{Ct,1-Nt}}{u_{1-Nt}} s_t N_t} \quad (4.34)$$

The problem with sticky prices is

$$\text{Max } E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t\right) \right\}$$

s.t.

$$C_t = s_t N_t \tag{4.35}$$

and

$$E_{t-1}\left[\frac{u_{Ct}}{\frac{\theta R_t}{(\theta-1)}}C_t - \frac{1}{s_t}u_{1-Nt}C_t\right] = 0 \tag{4.36}$$

Let $\lambda_t \Pr$ and $\overline{\varphi_t}$ be the multipliers of the resources constraint and the implementability condition. The first order conditions of the Ramsey problem

$$u_{Ct} - \lambda_t + \overline{\varphi_t}\frac{(\theta-1)}{\theta R_t}\left(u_{Ct} + u_{CtCt}C_t\right) - \overline{\varphi_t}\frac{1}{s_t}u_{1-Nt} - \overline{\varphi_t}\frac{1}{s_t}u_{1-Nt,Ct}C_t = 0 \tag{4.37}$$

$$-u_{1-Nt} + \lambda_t s_t - \overline{\varphi_t}\frac{(\theta-1)}{\theta R_t}u_{Ct,1-Nt}C_t + \overline{\varphi_t}\frac{1}{s_t}u_{1-Nt,1-Nt}C_t = 0 \tag{4.38}$$

When the multipliers are identical across states the two problems, with flexible and fixed prices coincide.

5. Characterizing the rule

We have seen that the mark ups should be constant across states, that is, the flexible price allocation should be replicated through policy when the expresion of the marginal distortion in the flexible price allocation, given by

$$\varphi_t = \frac{\frac{\theta R_t}{(\theta-1)} - 1}{-\frac{u_{CtCt}C_t}{u_{Ct}} - \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}}N_t + \left(\frac{(\theta-1)}{\theta R_t} + 1\right)\frac{u_{Ct,1-Nt}}{u_{1-Nt}}s_t N_t} \tag{5.1}$$

is constant across states. The characterization of this policy rule requires the discussion of the cases in which this is true. As R_t is by definition constant for each period, this implies the study of the conditions under which the denominator does not depend on the state, that is it does not depend on the allocation, or the allocation is constant across states. Let us rewrite the denominator as

$$D = -\frac{u_{CtCt}C_t}{u_{Ct}} + \frac{u_{Ct,1-Nt}}{u_{Ct}}N_t - \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}}N_t + \frac{u_{Ct,1-Nt}}{u_{1-Nt}}C_t$$

Let us concentrate on the class of "reasonable" preferences as the ones that qualify for being able of representing the aggregation of heterogeneous agents, that is being Gorman aggregable, and the ones consistent with balanced growth.

For time separable preferences we are left with the following classes, defined by the momentary utility function:

$$\begin{aligned} 1)\ u &= C^{1-\sigma}/1-\sigma-\alpha N \\ 2)\ u &= \frac{(C^{\alpha}(1-N)^{\beta})^{1-\sigma}}{1-\sigma}-1,\ \sigma>0 \\ 3)\ u_t &= \frac{(C-\gamma_s\zeta N^{\beta})^{1-\sigma}}{1-\sigma}-1,\ \sigma>0,\beta>1 \end{aligned}$$

We can prove that for every one of these functions the denominator is constant across states:

1) $D=\sigma$.

2) $D=\frac{u_{Ct,1-Nt}}{u_{Ct}}-\frac{u_{1-Nt,1-Nt}}{u_{1-Nt}}=\frac{1}{1-N}$. Because N is constant across states for these preferences, the denominator is constant.

3) Because D is invariant for monotonic transformation of preferences, we prove that D is constant for the function $C-\gamma_s\zeta N^{\beta}$. In this case it is immediate to see that $D=1-\beta$.

There are some functions that satisfy the constancy of D but that we dismiss for not being "reasonable" such as

4) $u=\log C-\zeta N^{\beta}$, that is non aggregable.

5) $u=F(C,N)$, where F is homogeneous in C and N, that is compatible with equilibria with $N=0$ or $N=\infty$.

6. Monetary Policy

In this section we show that any allocation that satisfies the solution of the social planner problem as defined in the previous section can be decentralized through a suitable monetary policy. We also charactherize the appropriate monetary policy.

We start by decentralizing the optimal solution to the social planner's problem using the monetary policy. Take an allocation $(c_t^*(s^t),N_t^*(s^t))$ that satisfies the primal problem of the social planner. Money balances $M_t^*(s^t)$ that satisfy

$$p_t^*(s^t)c_t^*(s^t)=M_t^*(s^t)$$

and imply prices such that $\frac{u_c\left(c_t^*(s^t),N_t^*(s^t)\right)}{p_t^*(s^t)}=k$, for $k\geq 0$, and for all $s^t|s^{t-1}$ can be chosen. In that case the intertemporal foc of households can be written as

$$\frac{u_c\left(c_t^*(s^t),N_t^*(s^t)\right)}{p_t^*(s^t)}=\beta R_t\frac{u_c(c_{t+1}^*(s^{t+1}),N_{t+1}^*(s^{t+1}))}{p_{t+1}^*(s^{t+1})}.$$

Now we want to verify if the allocation $(c_t^*(s^t), N_t^*(s^t))$ can satisfy the intertemporal foc of households with a different price vector such that,

$$\frac{p_t^*(s^t)\gamma(s^t)}{p_t(s^t|s^{t-1})} = 1,$$

for all $s^t|s^{t-1}$ and with $p_t(s^t|s^{t-1})$ a constant for all $s^t|s^{t-1}$. The equation shows that the values for $\gamma(s^t)$ depend only on $p_t(s^t|s^{t-1})$. Observe that this price, $p_t(s^t|s^{t-1})$, is the price firms will choose at date $t-1$ since the allocation is consistent with the firms' foc.

Next, we compute the value for $p_t(s^t|s^{t-1})$. We know that when state $(s^t|s^{t-1})$ happens prices, $p_t^*(s^t)$, get multiplied by $\gamma(s^t)$. This can be achieved by changing the money level according to the cash-in-advance. The intertemporal foc of households is

$$\frac{u_c\left(c_t^*(s^t), N_t^*(s^t)\right)}{\gamma(s^t)p_t^*(s^t)} = \beta R_t \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)\frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{\gamma(s^{t+1})p_{t+1}^*(s^{t+1})},$$

or

$$\frac{u_c\left(c_t^*(s^t), N_t^*(s^t)\right)}{\gamma(s^t)p_t^*(s^t)} = \beta R_t \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{p_{t+1}^*(s^{t+1})} \sum_{s^{t+1}|s^t} \frac{\pi(s^{t+1}|s^t)}{\gamma(s^{t+1})}.$$

This implies that,

$$\gamma(s^t) = \left[\sum_{s^{t+1}|s^t} \frac{\pi(s^{t+1}|s^t)}{\gamma(s^{t+1})}\right]^{-1},$$

and that the price vector

$$p_t(s^t|s^{t-1}) = p_t^*(s^t)\left[\sum_{s^{t+1}|s^t} \frac{\pi(s^{t+1}|s^t)}{\gamma(s^{t+1})}\right]^{-1},$$

or

$$p_t(s^t|s^{t-1}) = p_t^*(s^t)p_{t+1}(s^{t+1}|s^t)\left[\sum_{s^{t+1}|s^t} p_{t+1}^*(s^{t+1})\pi(s^{t+1}|s^t)\right]^{-1}.$$

There is freedom in the choice of the first price level in the economy, and thus we may choose $p_0 = p_0^*$, for instance. Given the initial price level, the remaining price levels are obtained according to the equation above.

Next we characterize the monetary policy that implements the allocation $(c_t^*(s^t), N_t^*(s^t))$ and the price system $p_t(s^t|s^{t-1})$. From the equation above we get

$$\frac{p_{t+1}(s^{t+1}|s^t)}{p_t(s^t|s^{t-1})} = \frac{\left[\sum_{s^{t+1}|s^t} p_{t+1}^*(s^{t+1})\pi(s^{t+1}|s^t)\right]}{p_t^*(s^t)},$$

using the cash-in-advance conditions we can rewrite it as

$$\frac{M_{t+1}(s^{t+1})}{M_t(s^t)} = \frac{c_{t+1}^*(s^{t+1})}{c_t^*(s^t)} \frac{\left[\sum_{s^{t+1}|s^t} p_{t+1}^*(s^{t+1})\pi(s^{t+1}|s^t)\right]}{p_t^*(s^t)},$$

or as

$$\frac{M_{t+1}(s^{t+1})}{M_t(s^t)} = \frac{c_{t+1}^*(s^{t+1})}{c_t^*(s^t)} \left[\beta R_t \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{u_c\left(c_t^*(s^t), N_t^*(s^t)\right)}\right].$$

The value of $M_0 = p_0^* c_0^*$, and M_t for $t > 0$ given by the equation above.

7. Concluding Remarks

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