Forecasting Australian GDP Growth Using Coefficients Constrained by A Term Structure Model

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Abstract

Yield spread between long and short bonds has been used to forecast economic activity for a long time and has yielded some positive results, particularly for the U.S. data. Recently it has been shown that the forecast can be improved by incorporating the economic activity variable into a term structure model with observable factors. The idea is to constrain the parameters of the system by the term structure model and see whether the constrained model produces better forecasts for the economic activity. This has been done for the U.S. We test this model on Australian GDP growth. We find the forecast results using constrained parameters are quite poor compared to those for the unconstrained model. The reason is that in the traditional affine yield model, researchers normally assume a mean reverting process for factors such as short rate. When this is not supported by the data, the forecast results could be quite poor. To overcome this problem, one might want to twist the affine factor model so that it can accommodate non-mean reverting processes for factors such as the short rate.

1 Introduction

Yield spread is the difference between rates for long-term bonds and short-term bonds. It has been found that the yield spread has very good predictive power for the growth rate of GDP for many countries. Some early literature that illustrates this result include Fama (1984), Hardouvelis (1988) and Mishkin (1988). Stock and Watson (1988) included the yield spread in their list of leading indicators for economic activity.

Estrella and Hardouvelis (1991, hereafter EH) developed a linear model for the growth rate. In their model, the right hand side variables include yield spread which is the difference between five year bond yield and one quarter yield. Using post-war U.S. data, the authors found the yield spread has significant predictive power for GDP growth. In particular, the yield spread has a positive relationship with GDP growth and has three to four year predictive power for GDP growth. Moreover, it outperforms many other possible candidates such as, lagged output growth, lagged inflation and the index of leading indicators. Estrella and Mishkin (1997) extended this work to four European countries and found similar results.

Why does the yield spread have a positive relationship with GDP growth? A common explanation is given using the Expectation Theory. The Expectation Theory states that the long term interest rate is the average of the expected future short term rates. A monetary contraction raises the current short rate and at the same time, makes the long rate rise less than the short rate. Therefore, the yield spread curve is flattened. The increase in the short rate will eventually have an impact on interest sensitive sectors in the economy and cause a contraction of the economy. Thus, a positive relationship between GDP growth and the yield spread exists.

The above analysis shows why the yield spread may affect GDP growth. On the other hand, since the term structure of interest rates is determined by economic factors, GDP growth should also affect the yield spread. This observation tempts people to study the interrelationship between the two. Ang, Piazzesi and Wei (2003, APW hereafter) is one of these attempts.

The details of their model will be discussed in the next section. The key feature of their model is, that unlike Estrella and Hardouvelis' model, which just regresses the GDP growth on the yield spread and other predetermined variables, such as lagged GDP growth and inflation, in APW's model, the coefficients in the growth regression are determined by the parameters in the term structure model of interest rates. In other words, the coefficients in the GDP growth regression are constrained. These constraints ensure the GDP growth and yields are jointly determined and all the yield terms used in the regression are consistent with the no-arbitrage requirement.

APW used quarterly US data from 1964:Q1 - 2001:Q4 to estimate their model and compared the out-of-sample forecasts from their yield implied model with other GDP forecasting models. Their forecast period is 1990:Q1 - 2001:Q4. The main findings of their study are (1), the yield implied model seems to outperform EH's unconstrained model in terms of RMSE. (2), however, the differences between the two models are not large.[1] (3), APW found that predictive power for GDP growth mainly comes from the short rate rather than the yield spread. After further analysis, they found that the forecasting power is actually from the inflation rate. The short rate acts like a proxy for the inflation rate over the sample used in the model.

The purpose of this paper is to apply APW's model to the Australian economy and determine whether the yield implied forecast model performs better than the unconstrained yield models. (EH type). Previous studies on Australian GDP growth using yield spread found similar results. Karunaratne (1999) found that the yield spread is a robust predictor of the future economic activity.

Our approach is to compare RMSEs from the APW model and the EH model with RMSEs from a simple AR(1) model and decide which model gives us the best result over our sample period. We

[1] APW proposed a test by Diebold and Mariano (1995) and found that the differences between the two models are not statistically significant, though the power of the test is low.

calculate the k-period GDP growth rate as

$$g_{t\to t+k} = \frac{100}{k} \times (\ln(GDP_{t+k}) - \ln(GDP_t)), \tag{1}$$

and for the one period growth rate[2] $g_t \equiv 100 \times ((\ln(GDP_t) - \ln(GDP_{t-1}))$.

One important finding of this paper is that, if one has to use a smaller sample size of observation on the short rate, the short rate could behave like an non-mean reverting process. In this case, using a traditional affine yield model, like APW's model, does not help in forecasting GDP growth, since it puts an incorrect mean reverting restriction on the short rate. Our result shows that the forecast bias could be very large using an incorrect mean reverting process for the short rate.

The plan of the paper is as follows: Section 2 outlines the APW model. Section 3 details data and estimation results from APW model. Section 4 lists the out-of-sample forecast results. Section 5 discusses the results and Section 6 concludes the paper.

2 The Model

The yield model in APW is an affine factor model. The structure of the affine factor model is well developed by Duffie and Kan (1996) and Dai and Singleton (2000). The general steps of setting up an affine model include specifying the dynamics of state variables, short rate and the market price of risk. One can then use the bond pricing model provided by Duffie and Kan (1996) to solve for bond price and yield.

One of the difficulties of an affine model is how to obtain state variables, which are used as factors in the model. These variables determine the production opportunities in an economy (Cox, Ingersoll and Ross, 1985) and are not directly observable. A general approach is to assume a certain number of yields have measurement errors and assume a number of yields without measurement errors. The number of yields without measurement errors are the same as the number of the state variables in

[2] We did not annulize the growth rate by multiplying it by 4.

the model. Thus, one can solve for the state variables in terms of those yields without measurement errors. In this way, the unobserved variables can be substituted out and the estimation can then be carried out. The disadvantage of using this approach is obvious, one needs to arbitrarily assume which yields have measurement errors. APW took another approach to overcome this problem. They found the first two principal components of their yield samples account for 99.7% of the variation of yields in the sample. In addition, the correlation between the first principal component and short rate is -95.6% and the correlation between the second principal component and yield spread (difference between 5-year and one quarter rate) is -86.5%. Therefore, most variation in yields can be proxied by short rate and yield spread. This observation prompts APW to use short rate and the yield spread as proxies for the unobserved state variables. It should be noted that unlike Knez, Litterman and Scheinkman who use three factors to capture level, slope and hump for the yield structure, the APW model is a two factor model, which uses short rate to capture the level factor and yield spread to capture the slope factor.

The discrete time APW model is as follows:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{2}$$

where X_t is a 3×1 vector containing current short rate (y_t^1), yield spread ($y_t^n - y_t^1$) and the one period growth rate g_t, μ, Φ and Σ are parameter vector and matries, $\varepsilon_t \sim iid\ N(0, I)$. (2) defines the dynamics of the state variables, the short rate and the yield spread in the model. g_t is included for forecasting purpose. Note this specification implies that the short rate is a mean reverting process. As we shall see later that this assumption becomes crucial in yielding desirable results.

To complete the yield model, we need to specify the process of market price of risk λ_t. APW assume,

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{3}$$

where λ_0 is a vector of constants, that is, the market price of risk is also affine in state variables. The advantage of this specification is it is relatively simple. However, recent studies by Duffee (2000) and Dai and Singleton (2002) found that this specification is too rigid to match practical yield patterns.

With these specifications, one can use the existing formulas developed by Duffie and Kan (1996) to solve for the price function of yield. APW adopt Campbell, Lo and MacKinlay's (1997) method to solve for the price of an n-period bond P_t^n

$$\begin{aligned} P_t^n &= \exp\{A_n + B_n' X_t\}, \\ A_{n+1} &= A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2} B_n' \Sigma\Sigma' B_n, \\ B_{n+1} &= (\Phi - \Sigma\lambda_1)' B_n - e_1, \end{aligned} \tag{4}$$

where $e_1 = \ [1\ 0\ 0]'$, and $A_1 = 0$ and $B_1 = -e_1$. The yield function can then be obtained as

$$y_t^n = -\frac{1}{n} \ln P_t^n = a_n + b_n' X_t \tag{5}$$

where $a_n = -A_n/n$ and $b_n = -B_n/n$.

Our purpose is to forecast the k-period GDP growth rate and the model we use is

$$g_{t \to t+k} = \alpha_k^n + \beta_{k,1}^n y_t^1 + \beta_{k,2}^n (y_t^n - y_t^1) + \beta_{k,2}^n g_t + \epsilon_{t+k,k}^n. \tag{6}$$

This is just an EH model. However, all yields must satisfy (5), therefore one can solve for α and the βs in (6) in terms of parameters in the yield model (μ, Φ, Σ, λ_0, λ_1). In this way, all yields in (6) must satisfy equation (5) and thus the non-arbitrage requirement in yields is met. APW give the formula for α and β in (6) as follows,

$$\begin{aligned} \alpha_k^n &= \alpha_k - (e_3' \Phi \widetilde{\Phi_k})(\widetilde{B}_n')^{-1} \widetilde{A}_n \\ \beta_k^n &= (\widetilde{B}_n')^{-1} [\widetilde{\Phi_i}' \Phi' e_3] \\ \widetilde{\Phi_i} &= \frac{1}{k} \sum_{j=0}^{i-1} \Phi^j \\ \alpha_k &= e_3' (I - \Phi)^{-1} (I - \Phi \widetilde{\Phi_k}) \mu \\ \widetilde{A}_n &= [0 \ \ a_n - a_1 \ \ 0]' \end{aligned}$$

$$\widetilde{B}_n = [e_1 \;\; b_n - b_1 \;\; e_3]', \tag{7}$$

where $\beta_k^n = [\beta_{k,1}^n \; \beta_{k,2}^n \; \beta_{k,3}^n]'$, a_n and b_n are defined in equation (5), $e_1 = [1 \; 0 \; 0]'$ and I is a 3×3 identity matrix.

Our focus is on equation (6). We compare the forecasts from unconstrained estimates of (6) with the forecasts using constrained α and β estimates obtained from (7). In the next section, we will illustrate data used in our study first, then discuss the estimation method and results.

3 Data and Estimation Result

Our data is from 1980:Q1 - 2002:Q2. The GDP data is seasonally adjusted and under constant price. The 13-week (1-quarter) and 26-week (2-quarter) rates are Commonwealth Treasury note yields. The 2-year, 5-year and 10-year rates are Treasury bond rates. Rates are divided by 4 to make them quarterly. We use the yield spread from 10-year bond to 1-quarter bond to be one of the factors in the yield model. Using the principal component analysis, we found the first and the second principal components account for 96.5% and 3.3% of the total variation in the yields, respectively. Therefore, similar to APW, we think 2 factors are enough in our term structure models. In addition, the correlation between the first principal component (the second principal component) and the short rate (the yield spread) is 97.7% (87.1%). Hence, we use the observable short rate and yield spread as proxies for the unobservable 2 factors in our term structure model. Thus, our $X_t = [y_t^1 \; y_t^{40} - y_t^1 \; g_t]'$.

In general, estimating a yield model is no easy task, even if one assumes the joint distribution between yields and factors. The main difficulty comes from unobserved market price of risk. Normally, researchers use iteration or simulation methods to estimate parameters in the model. However, because of the specification of our model, we follow APW's two step estimation method.

To be specific, since factor dynamics in (2) do not involve market price of risk λ_t, we can estimate parameters in this equation first. Since (2) is a VAR(1) model, obtaining estimates of μ, Φ and Σ is relatively easy. Once these parameters are estimated, we treat them as constant and use them to estimate λs. To do that, we minimize sum of squared errors from yields and forecasted yields. That is,

$$\min_{\{\lambda_0,\lambda_1\}} \sum^{i} \sum^{t} (\widehat{y_t^i} - y_t^i)^2,$$

where $\widehat{y_t^i}$ is forecasted yields calculated from (5). The two, eight and twenty quarter yields are used in estimation. This method may not yield efficient estimates. However, it is relatively simple and fast. The results are listed below.

{Table one is in here}

To examine the estimates, we calculated the unconditional mean and standard error for 2, 8 and 20-quarter forecasted yields and compared them with their counterparts of the actual data. Table 2 summarizes the results.

{Table two is in here}

The results from forecasted yields and actual data are very similar. It indicates that our estimates are reasonable. We are now ready to use those estimates to forecast GDP growth.

4 Forecasting Results

The main purpose is to compare forecasting results from constrained (6) and unconstrained (6). We name them Yield Implied Model and OLS Model, respectively. We perform out-of-sample forecasts. That is, we use our sample up to 1997:Q2 to do estimation and use the rest of the sample period to compare the forecasts. We conduct four different forecast horizons for the growth rate of GDP, namely 1-quarter, 4-quarter, 8-quarter and 12-quarter. Because of this, the number of out-of-sample forecasts decreases as the forecast horizon increases. For example, when forecasting the 1-quarter growth rate, we have 20 out-of-sample forecasts, when forecasting the 4-quarter growth rate, we have 16 out-of-sample forecasts. The number decreases to 12, when forecasting the 8-quarter growth rate and 8 when forecasting the 12-quarter growth rate.

Figure one plots Australian GDP growth and lagged yield spread from 1980:Q1 - 1997:Q2. It seems the lagged yield spread moves along with the GDP growth quite well, except for a short period in 1987 and 1994. Therefore, we shall expect to find a good predictive power from the yield spread.

{Figure one is here}

For each forecasting horizon, we make different forecasts using different yield spreads. From our data, we can calculate 4 different yield spreads. They are $y_t^2 - y_t^1$, $y_t^8 - y_t^1$, $y_t^{20} - y_t^1$ and $y_t^{40} - y_t^1$. We name them the 2-qtr, 8-qtr, 20-qtr and 40-qtr yield spreads respectively. We then calculate RMSE for each of those forecasts.

Instead of making a direct comparison between the Yield Implied Model and the OLS model, we compared them relative to the forecast from the simple AR(1) model. That is, we forecast each GDP growth rate from the AR(1) model and calculate the RMSE. We then take a ratio of the RMSE from the Yield Implied Model to that of the AR(1) model and do the same for the OLS model. This allows us to compare these ratio results and determine which model performs better. In addition, if the ratio is smaller than 1, it implies that the model performs better than the AR(1) model and vice versa.

Finally, we also calculate the forecast from an unconstrained VAR(1) model using 40-qtr yield spread. We compare its results with those of the AR(1) model in the same manner of the other two models. Note that because the 40-qtr yield spread is used as one of our factors in (2), the forecasts using the 40-qtr yield spread from the Yield Implied Model should be exactly the same as those from VAR(1) model.

The results is summarized in table 3.

{Table three is in here}

The first feature from the table is that all the ratios are greater than one, indicating that all the models perform worse than the AR(1) model. This result is different from previous studies such as Harding and Pagan (2001), who found a random walk with drift model performs well in capturing the shape of the business cycle in Australia. Secondly, unlike APW who found the Yield Implied Model outperforms other models in US, the Yield Implied Model gives the worst result among the three models for Australian data. Moreover, APW found the advantage of using the Yield Implied Model increases as the forecasting horizon increases. In our study, as the forecasting horizon increases, the Yield Implied Model's performance worsens comparing to the AR(1) model. Thirdly, the OLS model performs relatively well compared to the AR(1) model.

These findings are very interesting. We need to find out why the Yield Implied Model performs so badly. Does yield spread cause this poor forecasting performance? Figure one certainly does not suggest that. Then it must be the short rate which causes the poor forecasts. In the next section we discuss this in detail.

5 Discussion

The short rate in the APW model plays a crucial role, as it is found that it has more predictive power than the yield spread. However, we found that this is not true in our model. To examine this, we recalculate the forecasts from a VAR(1) model, excluding the short rate. In other words, we only include yield spread $(y_t^{40} - y_t^1)$ and GDP growth in the VAR. (We call it VAR2(1) model compared to the three variable VAR model in the previous section, denoted VAR3(1).) We then calculate RMSEs from the forecasts and compare them with the RMSEs from the AR(1) and the VAR3(1) models calculated from the previous section. Table four lists the details.

{Table four is in here}

Table four clearly shows that RMSEs from VAR2(1) and AR(1) are very similar. In fact, VAR2(1) is slightly better than AR(1). However, forecasts from VAR3(1) are much worse than those of other two models in terms of RMSE. This experiment shows that yield spread still has predictive power in GDP growth. The problem comes from the inclusion of the short rate. In our model, it certainly makes the forecasts worse. This is the opposite to what APW found. They found that the reason for their yield implied model to outperform an unconstrained OLS model is that the yield implied model places more weights on the short rate than the yield spread. In fact, the coefficient of the yield spread in their yield implied model becomes smaller and smaller as the forecast horizon increases whereas the coefficient of the short rate in their model is less varied. Consequently, they found that as the forecast horizon increases, it becomes more clear that their model beats the unconstrained OLS model.

Why does short rate help forecasts from the APW model but make forecasts worse in our model? The answer lies in the next figures In figure 2, we plot the US short rate used by APW[3] and the Australian short rate used in our model.

{Figure two is here}

It can be seen that although Australian data follows a similar pattern to the US rate, the US short rate over the sample period seems to be mean reverting. On the other hand, for the data range we use, Australian short rate is not. As a result, the level factor- the short rate used in our model cannot be specified as a VAR(1) model in (2). The estimates we obtained from (2) force the short rate to be mean reverting, which cannot be matched by the data. Therefore, the forecasts obtained from the estimates are biased. On the other hand, the unconstrained OLS model does not have this problem, hence the forecasts from OLS are not as bad as those from the Yield Implied Model.

[3] APW use data from 1952:Q2 - 2001:Q4. Our plot is from 1953:Q2 - 2001:Q4.

6 Conclusion

In this study, we applied the APW model in Australia. Unlike APW, who found the Yield Implied Model outperforms the unconstrained OLS model (EH type) in forecasting GDP growth in US, we found that the Yield Implied Model performs badly in forecasting Australian GDP growth. The reason is that unlike the US short rate used by APW, the short rate in our sample period does not revert to a long term mean. Consequently, the dynamics of the short rate in the affine yield model cannot be VAR(1). Our study shows again that it is important to specify the short rate as a non-mean reverting process.[4] It happens to be that the short rate sample used by APW looks like a mean reverting process. However, as argued by Pagan, Hall and Martin (1996), it is still better to treat the short rate as a non-mean reverting process, since it moves very slowly towards to its long term mean.

As for further study, we plan to re-estimate the model using a more appropriate process for the short rate, in which the mean reverting assumption does not hold. Models such as random walk with a drift might be useful in this case. However, the existing literature on affine yield models seem to focus almost excluxively on mean reverting processes. Therefore, how to accommodate a random walk process into an affine yield model is a new task and the results remain to be seen.

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[4] Many authors found the similar results, see for example, Hall, Anderson and Granger (1992) and Pagan, Hall and Martin (1996).

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