Non-Fungibility and Mental Accounting: A Model of Bounded Rationality with Sunspot Equilibria

Aditya Goenka †
(University of Essex and National University of Singapore)

2 October 2003

†Department of Economics, National University of Singapore, Block AS2 #06-10, 1 Arts Link, Singapore 117570. *e-mail* ecsadity@nus.edu.sg. Fax: (65) 6775 2646. Tel (65) 6874 3961.
This paper grew out of a project on bounded rationality and sunspots with Karl Shell. I would like to thank him for generating my interest in the subject, and for many stimulating discussions and insights. All errors are my own.

Abstract: In this paper we consider a model where some consumers act in a boundedly rational way by treating money as non-fungible (Kahneman and Tversky (1979) and (1984), Thaler (1987) and (1990). The budget is broken up into different expenditure groups (cookie-jars). Given the amount of resources allocated to a given expenditure group, boundedly rational consumers then decide how to spend the resources on commodities in that expenditure group. We study the general equilibrium effects of these 'mental accounting systems'. An important implication of such behaviour is that consumers can act *as if* they are credit constrained even when they are not. It is shown that such environments are prone to self-fulfilling fluctuations. In three polar cases: (i) Where nearly every consumer is rational; (ii) Where the consumers are either rational or nearly rational; or (iii) If every consumer is boundedly rational and has an expenditure weight for each commodity, there are no self-fulfilling fluctuations. We also characterize properties of the demand functions so the demand of boundedly rational consumers can be distinguished from that of consumers whose first best behaviour is to have fixed expenditure weights.

Keywords: Bounded rationality, Sunspot Equilibrium, Fungibility, Mental Accounting, Behavioural Economics, Economic Fluctuations.

JEL Classification Numbers: E32, D11, D51, D84, D81.

1 Introduction

For the most part modern economic theory treats agents as rational – fully maximizing objectives subject to economic and technological constraints. However, it has increasingly become apparent that this is not an entirely satisfactory behavioural postulate. Not only is there experimental and emperical evidence that consumers routinely violate the axioms of rational behaviour (see Kahneman and Tversky [14] and [15], Thaler [24] and [25], and Benartzi and Thaler [4]) but also on theoretical grounds it seems too strong to assume that every consumer behaves in the prescribed fashion (see Radner [17], [18], Simon [22]). In this paper we model one kind of boundedly rational decision making and examine its consequences on general equilibrium outcomes.

The situation we have in mind is one where consumers use mental accounting systems to construct different expenditure groups ('cookie-jars'), e.g., one for housing, heating and transportation; for food; for entertainment; for savings and insurance; etc., and optimize within each expenditure group but not across the expenditure groups. Thus money is treated as non-fungible and has a label (see Kahneman and Tversky [14], [15], and Thaler [24] and [25]). The consumers after assigning the level, perhaps using a rule of thumb, for each 'cookie-jar', then optimize given the level in the 'cookie-jar.' Different consumers may have different levels in the 'cookie-jars' and different commodities in each 'cookie-jar.' We move beyond studying only the decision problem and ask what are the general equilibrium consequences of such behaviour.

The problem of mental accounting is an important one. Kahneman and Tversky [15] give the results of the following experiment. Subjects were asked to imagine that they have decided to see a play, admission to which is \$ 10 per ticket. As they enter the theatre thet discover they have lost a \$ 10 note. They were asked whether they would still pay \$ 10 for the ticket to the play? 88% of the people said yes, while 12% said no. The same individuals were confronted with a different situation. They were asked to imagine that they had decided to see a play and paid admission price of \$ 10 for ticket. As they enter the theatre they discover that they have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would they pay $ 10 for another ticket? The wealth effect of both the situations are the same if money is non-fungible. However, now only 46% said they would purchase a new ticket while, 54 % said they would not purchase a new ticket. This suggests that the individuals are mentally coding expenditures. Thaler [25] presents other evidence of this behaviour. He points out consumers seem to treat different sources of income as non-fungible. Consumers have different marginal propensities to consume (MPC) from different sources of income: current income close to 1; future income close to 0; asset income somewhere in between. In Japan, empirical evidence shows that MPC from regular bonus income is lower than MPC from regular salary income (Ishikawa and Ueda [13]) even though from an economic point of view both sources are the same. Mental coding of expenditures seem to characterize behaviour even of college professors in the USA (see Benartzi and Thaler [4]). TIAA-CREF data shows that on average a 50-50 split is made in contributions to CREF (which are equity based pension funds) and TIAA (which are bond based pension funds), with the value rising to 66% in CREF funds due to higher growth in stocks. If the individuals prefer a 50-50 split in the values then they should rebalance contributions so that this split in value of their pension funds is maintained. If they prefer a 66-33% split then the contributions should be tailored so that this is maintained from the start. The behaviour is characterized by inertia in choosing the levels of contributions to bonds and stocks, but then the portfolios within these class of securities are managed much more closely. This has also been pointed out by Samuelson and Zeckhauser [19] who report that the typical TIAA-CREF participant makes one asset allocation decision and never changes it. We focus on mental coding of expenditures.

Different mechanisms can give rise to this behaviour. The rules of thumbs in assigning expenditure levels could arise from loss aversion, cultural mores, costs of computation, or even long term contracts. One of the key aspects of bounded rationality is that agents follow *simple decision rule* (Gigerenzer and Selten [9]). Mental accounting is an example of such decision rule. We can also think of such behaviour as an example of 'fast and frugal heuristics' (Todd [26]). The crucial behavioural postulate is that the allocation of expenditures is predetermined and is not responsive to changes in the prices and income. The model is different from other models of bounded rationality and economic fluctuations, e.g., Akerlof and Yellen [1], in that money is not fungible. This mental accounting has the effect that the consumers do not fully use financial markets to insure themselves against risks. As pointed out by Thaler [25], mental accounting has the effect that consumers appear to be credit-constrained, even if they are not.

The decision making procedure in our paper also has a connection with the two-stage budgeting procedure. In the two stage-budgeting procedure, if certain restrictions on preferences are satisfied (see Blackorby and Russell [5]), then one can view consumers as assigning expenditure levels to different commodity classes based on income level and price indices for the commodity classes (*price aggregation*), and then optimizing within each class given the expediture levels and price levels for the goods the class (*decentralisability*). The condition we impose on preferences for some of our results, separability across the different classes, is sufficient only for decentralisability (Gorman [12]). Thus, in our model, the boundedly rational consumers are not 'solving' the first stage in an optimal way. Even if this was solved optimally at some stage, the solution is held fixed, and hence, consumers do not change the expenditure weights once prices (and hence, the price indices) change.

In a different context Vayanos [27] has examined the optimal portolio allocation problem in an organization when there are costs of information processing. He shows that there is an optimal hierarchy of the organization of the portfolio where at the lower levels some subset of securities are chosen by each decision maker, and these sub-portfolios are aggregated in *fixed levels* by decision makers at higher levels. This behaviour, in fact, characterizes decision making in investment companies (Sharpe [20]). The insights of our model can be extended to cover effects of actual portfolio management practices on asset prices.

If we interpret the states as time periods, then one can re-interpret the model to address the issue of mortgage lending and the volatility of the housing market. In many countries, for example in the U.K., lenders use a rule of thumb in lending to potential borrowers - 3 to 3.5 times the annual income. For a significant proportion of borrowers, this constraint is binding. As borrowers often do not have the option to underpay on their mortgage committment, this has the effect of making the income non-fungible, and the borrowers isomorphic to our boundedly rational consumers.

The focus of this paper is not the solution of the decision problem, but to assess the effect of such behaviour on equilibrium outcomes: particularly what effect does it have on the possibility of extrinsic uncertainty or sunspot equilibria? (See Cass and Shell [6], and Shell [21]) Given the structure of the economy existence of equilibrium is not an issue. The more interesting question is whether sunspots affect equilibrium outcomes in a non-trivial way. We show that if some of the consumers are boundedly rational and some are rational, then extrinsic uncertainty can have non-trivial effects. Thus, unlike many models of extrinsic uncertainty, if one were to look at the economy not taking into account the effects of extrinsic uncertainty, the impact of bounded rationality would not be discernable, as we could rationalize the economy as consisting of consumers who are Walrasian and have some 'optimally chosen expenditure weights. However, once we take extrinsic uncertainty into account the nature of the equilibrium outcomes changes as not only are there excess fluctuations in equilibrium outcomes but the bounded rationality constraints become binding as these consumers do not optimally insure against the endogenous price risk.

If one can show that boundedly rationality can give rise to non-trivial sunspot equlibria, a relevant question is to what extent is model with only rational consumers (the Walrasian model) robust to introductions of boundedly rationality. In other words how large has to be the bounded rationality before qualitatively new equilibria with endogenous fluctuations can emerge. We perform various perturbations of the model to see if small deviations from the Walrasian model either by introducing a small proportion of boundedly rational consumers or by varying the degree of bounded rationality for a fixed number of consumers can lead to non-trivial sunspot equilibria. We show that in contrast to Akerlof and Yellen [1] 'small deviations from rationality' will not lead to excess volatility. The equilibria in the Walrasian economy are generically robust to introduction of a small number of boundedly rational consumers. To see the effect of varying the extent of bounded rationality, but holding the number of these consumers fixed, we can proceed in two different ways. The first is by letting the constraint on expenditures go to 'zero' keeping the endowments fixed and the second is keeping the expenditures fixed but perturbing the endowments. For the former the stability to non-trivial sunspot equilibria is generic and for the latter the answer is still an open question. We also show that if each of the consumer is boundedly rational and has a different expenditure group for each commodity then sunspots cannot matter. The behaviour of each consumer is now consistent with each consumer having Cobb-Douglas preferences and who acts rationally, a situation where sunspots cannot matter.

In the last section we examine the consequence of such behaviour on the demand functions so that one can distinguish boundedly rational behaviour with situations where the structure of preferences (Cobb-Douglas utility functions) is generating the fixed expenditure weights.

The plan of the paper is as follows. After outlining the economy, in the next two sections we examine the case where some consumers are rational and some are not, and the behaviour of the economy when 'nearly all' the consumers are rational. We then look at the polar case where each consumer is boundedly rational either with or without an expenditure group for each commodity. In the last section the effect on the demand functions of boundedly rational behaviour is studied.

2 The Economy

We consider a pure exchange economy with two equiprobable states of nature, $s = \alpha, \beta$. (A finite number of states with arbitrary non-zero probabilities can be considered without any substantive change in the analysis.) The states are intrinsically the same as made precise below. There are two groups of consumers: the boundedly rational consumers, $j = 1, \ldots, J$, and the fully rational consumers, $i = 1, \ldots, I$. In each of the states there are L commodities, $x^{\ell}(s)$, $\ell = 1, \ldots, L$. In order to avoid problems associated with boundaries we take the entire Euclidean space in the relevant dimension to be the consumption set. For both types of consumers, the utility functions are separable and symmetric (with respect to the states of nature). $U_h : \Re^{2L} \rightarrow \Re$ is given by $\sum_s u_h(x_h(s))$ for $h = i, j$. While the specification allows for non-expected utility maximization behaviour, this is not the focus, and a special case for which the entire analysis is valid is that of the von Neumann - Morgenstern utility functions. The utility functions, $u_h(\bullet)$, are smooth, strictly increasing, and strictly concave. The indifference surfaces are bounded from below. The absence of intrinsic uncertainty apart from the symmetry of preferences requires that the endowments be state symmetric as well, $\omega_h(\alpha) = \omega_h(\beta) = \omega_h^*,\ h = i, j$. Thus, we have $\omega_h = (\omega_h^*, \omega_h^*)$. The prices vector is p.

The fully rational consumers act in the usual way. They treat the income (wealth) to be fungible, and thus, have a single budget constraint for each state. As they may choose to tranfer income (insure) across the states of nature by using the insurance markets, they face a single budget constraint across all the states of nature. Their maximization problem is given as follows:

$$\begin{array}{cc} Max & \sum_s (u_j(x_j^1(s), \ldots, x_j^L(s)) \\ \text{s.t.} & \sum_{s=\alpha,\beta} \sum_{l=1}^{L} (p^l(s) x_j^l(s)) = \sum_{s=\alpha,\beta} \sum_{l=1}^{L} (p^l(s) \omega_j^{l*}) \end{array}$$

The boundedly rational consumer treat their income (wealth) as non-fungible. Thus, they assign it to "cookie-jars" or assign "mental accounts" whereby money assigned to a specific account is used only for specified purposes. This, theoretically, has the effect of partitioning the commodities into different groups each with its own budget constraint. As money is non-fungible, it will also imply that they do not transfer resources across the states of nature making the group budget constraints state specific. Thus, these consumers act *as if* they are credit constrained due to their bounded rationality, even though there is no inherent reason that they are. Thus, the motivation of the restriction from participation in the credit markets is different from that in Cass and Shell [6]. For modelling purposes, for each j consumer partition the L commodities into two different groups, θ_j and θ_j^*.[1] Without any loss of generality the first $L_j < L$ commodities are in the first group and the remaining commodities in the second group. The expenditure groups thus are not common across the boundedly rational consumers. These two groups are the expenditure groups within which the consumer optimizes but does not necessarily optimize across the groups.

We do not model the choice of distribution of expenditure across the two expenditure groups but treat it as given. One could view this as being chosen according to some criteria at some date, but is now being held fixed. If these shares were being chosen optimally in each state then there would be no boundedly rational behaviour. The expenditure share of the first group of commodities in each state is $\lambda_j \in (0, 1)$ and the share of the second group is $(1 - \lambda_j)$.

For the boundedly rational consumers the choice problem is given by:

$$\begin{array}{ll} \text{Max} & \sum_s (u_j(x_j^1(s), \ldots, x_j^{L_j}(s), x_j^{L_j+1}(s), \ldots, x_j^L(s)) \\ \text{s.t.} & \sum_{l \in \theta_j} (p^l(s) x_j^l(s)) = \lambda_j (p(s) \omega_j^*) \\ \text{and} & \sum_{l \in \theta_j^*} (p^l(s) x_j^l(s)) = (1 - \lambda_j)(p(s) \omega_j^*) \\ \text{for} & s = \alpha,\ \beta \text{ and } 0 < \lambda_j < 1 \end{array}$$

The necessary and sufficient condition for optimization within each group is that the preferences are weakly separable, see Deaton and Muellbauer [7]. For some parts of the analysis we assume the stronger condition that preferences are block additive across the groups, $U_j : \Re^{2L} \rightarrow \Re$ is given by $\sum_s [u_j(x_j^1(s), \ldots, x_j^{L_j}(s)) + v_j(x_j^{L_j+1}(s), \ldots, x_j^L(s))]$. This is not sufficient for the optimality of a two-stage budgeting procedure (see Blackorby and Russell [5]). If preferences are additively separable across expenditure groups, then from the specification of the choice problem it is clear that we can "break up" this into four sub-problems of maximizing an objective function subject to a single constraint. Thus, we break up each boundedly rational

[1] Multiple expenditure groups can be analyzed at the cost of only notational inconvenience.

cosumer j into four "quasi-rational" consumers who maximize a utility function subject to a single budget constraint. Each of these quasi-rational consumers consumes a strict subset of the commodities and has 'endowments' which are proportional to the expenditure weights (see Balasko [2], Balasko, Cass, and Shell, [3], Goenka [10], [11]). Thus, with each consumer j associate consumers $j1\alpha, j2\alpha, j1\beta, j2\beta$. Consumer $j1s$ consumes only goods in group θ_j with preferences represented by $(u_j(x_j^1(s), \ldots, x_j^{L_j}(s))$ and has endowments $\lambda\omega_j^*$ in state s. Consumer $j2s$ consumes only goods in group θ_j^* in state s, has preferences represented by $v_j(x_j^{L_j+1}(s), \ldots, x_j^L(s))$ and has endowments $(1-\lambda)\omega_j^*$. The economy with $I+4J$ consumers is called the "expanded model." This is an Arrow-Debreu economy with a special endowment structure and with some consumers consuming only subsets of the commodity. We will also refer to a "certainty economy" or a "reduced model." This is the economy with only one state (no extrinsic uncertainty) and $I+2J$ consumers derived naturally from the original model.

3 Sunspot Equilibrium

To study the possibility of economic fluctuations under bounded rationality we consider only extrinsic uncertainty. For this economy we can define a sunspot equilibrium in the usual way – where markets clear in both states given the demand of the consumers. There are two questions on the existence of sunspot equilibrium. First, do they always exist, and second, are the effect of sunspots non-trivial. It is easy to show the equilibria will always exist using a standard argument and this is, thus, not addressed in the paper.

Definition 1

(p, ω) is a Sunspot Equilibrium relative to the distribution of the expenditure weights $\lambda = (\lambda_1, \ldots, \lambda_J)$ if

$$\sum_i f_i + \sum_j f_j = \sum_i \omega_i + \sum_j \omega_j, \tag{1}$$

where $(f_h),\ h = i, j$ are solutions to the respective maximization problems.
Let the set of Sunspot Equilibria relative to the distribution of expenditure weights, λ, (for variable ω) be denoted as

$$E(\omega, \lambda) = \{(p, \omega) : \sum_i f_i + \sum_j f_j = \sum_i \omega_i + \sum_j \omega_j\}. \tag{2}$$

Definition 2

Extrinsic uncertainty (sunspots) does not matter or sunspots have a trivial effect if:

$$x_h(\alpha) = x_h(\beta)\ \ \forall h = i, j.$$

The issue is to establish whether sunspots have non-trivial effects or not. One can distinguish the general case where both types of consumers are present from the polar cases where consumers of only one type are present. If there are no boundedly rational consumers then the economy is an Arrow- Debreu economy and sunspots do not matter (see Cass and Shell [6]). If there are only boundedly rational consumers then either sunspot equilibria are just randomizations over the underlying certainty equilibria (see Theorem 4) or sunspots do not matter (see Theorem 5 below for this latter result). These two results depend on the number of expenditure groups relative to the number of commodities in each state.

To look at the general proof of the general case instead of trying to analyze the allocations directly, we examine properties of the Price and Income Equilibria (see Balasko [2], Goenka [10], [11]). The strategy is to work with the expanded model with $I + 4J$ consumers. It is sufficient to examine whether prices and incomes of the consumers (rational and quasi-rational) are state symmetric. The allocations will be state symmetric if, and only if, the prices and incomes are state symmetric. This property follows from the properties of demand functions. The crucial feature of the derived quasi-rational consumers (which distinguishes the economy from a simple restricted market participation economy as in Cass and Shell [6]) is that their incomes are related. This follows from the fact that the 'endowments' and hence 'incomes' given to the two quasi-rational consumers in a state depend on the expenditure weight. This factor of proportionality will be the same for the other two corresponding quasi-rational consumers in the other state. This relationship is given below:

$$w_{j1s} = \frac{\lambda_j}{(1-\lambda_j)} w_{j2s} \quad s = \alpha, \beta, \tag{3}$$

where $w_{j1s} = \sum_{l \in \theta_j} p^l(s) x_j^l(s)$ and $w_{j2s} = \sum_{l \in \theta_j^*} p^l(s) x_j^l(s)$, the expenditure on the two groups in the two states. There is no loss of generality in working with the expanded model (the economy with $I + 4J$ consumers) rather than with the original one.

We start with the space of all potential prices and incomes and then impose restrictions so as to be able to focus on a restricted set which will be consistent with the special structure of the model. The set of all potential prices and incomes will be a set of dimension $2L+I+4J$: $2L$ prices and $I+4J$ incomes. As there is only extrinsic uncertainty, the aggregate resources have to be state symmetric, thus imposing L restrictions. As the incomes of the 2 derived quasi-rational consumers have to be proportional *within* each state, there are $2J$ additional restrictions. Thus, the set of equilibrium prices and incomes should have dimension $L+I+2J$. The set of equilibria where sunspots do not matter will be where the income of the quasi-rational consumers are symmetric *across* states. This set has J additional restrictions and thus is a lower dimensional subset of the set of all equilibria (should be of dimension $L + I + J$). All one now has to do is check that firstly, asymmetric price and income equilibria are consistent with symmetric endowments, and that these actually exist. This is stated in Theorem 1. The proof of this is given in the appendix.

Theorem 1

In the economy with some rational and some boundedly rational consumers, sunspots can matter.

4 Near-Rationality

We see that if there is a non-trivial proportion of boundedly rational consumers and rational consumers, then self-fulfilling fluctuations can affect the economy. A natural question is to see the robustness of the Walrasian model to small perturbations to include bounded rationality. We want to take a process where in the limit the economy is Walrasian with all consumers fully rational. Akerlof and Yellen [1] show in a different context that a competitive model may not be robust to small deviations from rationality. The perturbation of the bounded rationality "going to zero" can be done in two different ways. First, consider a situation where consumers choose the optimal consumption plan without any mental accounting constraints. The optimal levels for the cookie-jars can be determined given their endowments. Now perturb the endowments so that the pre-assigned cookie-jar levels need not be optimal anymore. Second, hold the cookie-jar levels of the different boundedly rational consumers fixed but change the *proportion* of these consumers. In particular, let the proportion go to zero, so that nearly everyone in the economy is rational. Generically, the the economy is robust to the second type of perturbation.

To carry out this perturbation, let there be M types of consumers, with $\gamma_h, h = 1, \ldots, M$ proportion of consumers of each type being bounded rational with some fixed expenditure weights γ_h. If $\gamma = 0$, this corresponds to a Walrasian economy. Then one can show that there will be a neighbourhood of $\mathbf{0}$, i.e., the Walrasian economy, where sunspots do not matter. The key intuition is that the equilibria are locally constant if the economy is regular. Generically, the economies are indeed regular. Thus, if we were to perturb the Walrasian economy by introducing small fractions of boundedly rational consumers, the equilibria will not be affected by sunspots. Denote the equilibrium set with γ_h proportion of boundedly rational consumers with each of the boundedly rational consumers of type h allocating λ_h of their income on first group as $E(\omega, \lambda, \gamma)$. We will hold ω, λ constant and perturb γ in the neigbourhood of $\mathbf{0}$.

Theorem 2:

Let $\overline{\omega}$ be a regular economy (of the certainty model). Then for this economy there exists an open neighbourhood V of $\mathbf{0} \in \Re^M$, such that for $\gamma \in V$, sunspots do not matter.

Proof:

The proof follows in three steps. First, show that $E(\omega, \lambda, 0) = E(\omega^*) \times E(\omega^*) \cap \Delta$, where Δ is the diagonal in the of the Cartesian product of the space of endowments of the certainty economy. Second, $E(\omega, \lambda, 0) \subset E(\omega, \lambda, \gamma)$. Then apply an adapted version of Theorem 2.4 of Balasko, Cass, and Shell [3] which establishes that there are no other branches in the equilibrium set $E(\omega, \lambda, \gamma)$ other than the constant branches emanating from $E(\omega, \lambda, 0)$ for a regular economy in the certainty economy. The trick to use this result is that in the limit economy, it does not matter what is the level of λ. *Q.E.D.*

To examine the effect of varying the expenditure shares, we will modify the model slightly: the boundedly rational consumers now do not spend more than a constant fraction of their income in any state on the second group. Thus, they are not fully constrained in their allocation of expenditure. We also restrict the consumption set of the consumers to the strictly positive orthant (of the relevant dimension). The maximization problem for a consumer $j = 1, \dots, J$ is:

$$\begin{array}{ll} \text{Max} & \sum_s (u_j(x_j^1(s), \dots, x_j^{L_j}(s)) + v_j(x_j^{L_j+1}(s), \dots, x_j^L(s)) \\ \text{s.t.} & \sum_s \sum_l (p^l(s) x_j^l(s)) = \sum_s \sum_l (p(s)\omega_j^*) \\ \text{and} & \sum_{l \in \theta_j^*} (p^l(s) x_j^l(s)) \leq (1 - \lambda_j)(p(s)\omega_j^*) \\ \text{for} & s = \alpha,\ \beta \text{ and } 0 < \lambda_j < 1 \end{array}$$

Thus, consumers can potentially transfer income across states, but they limit themselves to not spending more than a fixed proportion of their income in each state on the second group of commodities. The reason for the change in the model is twofold.

In the orginal specification λ_j^* becomes a knife-edge case. More importantly, in the original specification there is no continuous way to consider a transformation of the economy with boundedly rational consumers into a Walrasian economy by varying some parameter. The boundedly rational consumers will be unable to transfer income in the appropriate limit economy. Here $\lambda = 0$ corresponds to a Walrasian economy as the second constraint becomes redundant. Note, that in this economy sunspots will have non-trivial effects as the economy is isomorphic to the case of value rationing considered in Goenka [11]. There are three perturbations that could be considered. First, start from $\lambda_j = 0$, and consider small perturbations of these. We show sunspots will generically (in endowment space) not matter. Note that here the numbers of the boundedly rational consumers and all endowments are being held fixed but we are varying the expenditure weights λ. Second, fix λ but perturb the endowment. The idea is to see for a fixed set of expenditure weights, and utility functions, how large is the potential set of endowments consistent with non-trivial sunspot equilibria. We could consider a third perturbation of both λ and the endowments simultaneously. For this, start of with an economy where every one is rational and hence there is only the single budget constraint for each of the consumers, i, j. In this economy we can solve for the optimal allocation of expenditure weights λ_j^* for the 'boundedly rational' consumers. Now hold these λ_j^* fixed, i.e., the consumers treat this as a constraint, then and perturb the endowments. Thus, we can consider a small perturbation from the first best situation holding the number of the boundedly rational consumers fixed. It is an open question whether we will get emergence of non-trivial sunspot equilibria in the last two cases.

Theorem 3:

Let ω^* be a regular economy of the certainty economy when there are no constraints on expenditures for the J consumers, i.e., $\lambda = \mathbf{0} \in \Re^J$. Then there exists an open neighbourbood U of $\mathbf{0} \in \Re^J$, such that for all $\lambda \in U$, $E(\omega, \lambda) = E(\omega, 0)$.

Proof:

First, consider the economy where there are no constraints on allocating expenditure.

For this economy, $E(\omega, 0) = E(\omega^*) \times E(\omega^*) \cap \Delta$. In equilibrium the budget constraint is satisfied, $\sum_s \sum_l (p^l(s) x_j^l(s)) = \sum_s \sum_l (p(s)\omega_j^*)$. Thus, from the symmetry of equilibrium allocations, $\sum_l (p^l(s) x_j^l(s)) = \sum_l (p(s)\omega_j^*)$, $s = \alpha,\ \beta$. Now from the strict positivity of consumption, $\sum_{l \in \theta_j^*} (p^l(s) x_j^l(s)) < (1 - \lambda_j)(p(s)\omega_j^*)$for $s = \alpha,\ \beta$. Thus, there exists a $\lambda_j^* > 0$ such that $\sum_{l \in \theta_j^*} (p^l(s) x_j^l(s)) \leq (1 - \lambda_j)(p(s)\omega_j^*)$. Define $\lambda^* = \min$ $(\lambda_1^*, \ldots, \lambda_J^*)$. This will be strictly positive. As the markets clear in the economy where for each consumer $\lambda_j^* > \lambda^*$ (the constraints are non-binding), they will also clear in the Walrasian economy. Define the J dimensional vector $\lambda^* = (\lambda_j^*, \ldots, \lambda_j^*)$. Thus, for $\lambda < \lambda^*$, we have $E(\omega, 0) \subset E(\omega, \lambda)$. Then a suitably modified application of Theorem 2.4 of Balasko, Cass, and Shell [3] will establish the desired result. *Q.E.D.*

5 Full Bounded Rationality

In this section we examine the polar case where all the consumers are boundedly rational ($I = 0$). First, we consider the case where the consumers have the number of expenditure classes less than the number of goods in each state. Then we consider the case where the consumers have a different expenditure weight for each of the commodities, so that there are L groups for each consumer. In the previous sections the number of groups did not play any special role as long are there were both rational and boundedly rational consumers. The results in this section do not depend on the strong regularity or separability assumptions made on the preferences. However, it is important to ensure that the consumers have non- zero income in each state (this did not play any special role earlier as any possible endowment was in the interior of the consumption set).

The consumption set is $\Re_+^{2L}$, the utility function defined over it is separable across states and symmetric: $u_j(x_j(\alpha)) = u_j(x_j(\beta))$. The utility functions are continuous, strictly increasing, and strictly concave. The endowments are state symmetric and lie in the interior of the consumption set.

In the case of the number of groups less than the number of commodities, we can look at the entire economy as the Cartesian product of two identical certainty economies. As there are transfers of income across the states are not possible, the equilibrium set in one state is identical to that in the other. Thus, corresponding to the results in [6], the equilibria in the entire economy are just randomizations over the certainty equilibria.

Theorem 4

If all the consumers are boundedly rational and the number of commodities is strictly greater than the number of expenditure classes, then the sunspot equilibria are only randomizations over the underlying certainty equilibria.

The more interesting case is where the number of expenditure classes equal to the number of commodities. Here, the maximization problem for each consumer is:

$$\begin{array}{ll} \text{Max} & \sum_s (u_j(x_j(s)) \\ \text{s.t.} & \sum_l \ p^l(s)x_j^l(s) = \sum_l \ p(s)\omega^* \\ \text{and} & \dfrac{p^l(s)x_j^l(s)}{\sum_l \ p(s)\omega^*} \equiv \lambda_{jl} \\ \text{for} & s = \alpha,\ \beta \text{ and } l = 1, \ldots, L. \end{array}$$

Define $A_{lk} = \sum_j \ \lambda_{jl}\omega_j^k$ and let A be the $(L \times L)$ matrix A_{lk}. From the market clearing condition using the definition of λ_{jl} we obtain:

$$\begin{array}{llll} \dfrac{\sum_j \lambda_{kl} p(s)\omega_j}{p^l(s)} & = \sum_j \omega_j^l & \equiv & \omega^l \\ \text{or} & \dfrac{\sum_j(\sum_k \lambda_{kl} p(s)^k \omega_j^k)}{p^l(s)} & = & \omega^l \\ \text{or} & \dfrac{\sum_k (p^k A_{lk})}{p^l(s)} & = & \omega^l \\ \text{and} & \sum_k p^k(s) A_{lk} & = & p^l \omega^l. \end{array}$$

Let W be the diagonal matrix (ω^l). Then, the last equation above implies, $Ap(s) = Wp(s)$ or

$$(A - W)p(s) = 0.$$

Consider the matrix $(A - W)$:

$$\begin{pmatrix} A_{11}-\omega^1 & A_{12} & \cdot & \cdot & \cdot & A_{1L} \\ A_{21} & A_{22}-\omega^2 & \cdot & \cdot & \cdot & A_{2L} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{L1} & \cdot & \cdot & \cdot & \cdot & A_{LL}-\omega^L \end{pmatrix}$$

Since $A_{ll} = \sum_j \lambda_{jl}\omega_j^l$ this matrix has diagonal terms which are negative and all off-diagonal terms that are positive. Further, summing along each column we obtain,

$$\sum_l A_{lk} - \omega^l = 0.$$

Therefore, the $(L-1) \times (L-1)$ upper-left submatrix of $(A-W)$ possesses a dominant diagonal (see Takayama [23, p. 359]. Hence, this submatrix is invertible and Rank $(A-W) = L-1$.

Now as $p(s)$ is a solution to $(A-W)p(s) = 0$, any other solution p' must satisfy $p' = \mu p(s)$, $\mu \in \Re_{++}$. Therefore, for each s there is a $\mu(s) > 0$ such that the prices are collinear. This, in turn will imply:

$$\begin{aligned} x_j^l(s') &= \lambda_{jl}\frac{p(s')\omega_j^*}{p^l(s')} \\ &= \lambda_{jl}\frac{\mu(s')p(s)\omega_j^*}{\mu(s')p^l(s)} \\ &= \lambda_{jl}\frac{p(s)\omega_j^*}{p^l(s)} \\ &= x_j^l(s). \end{aligned}$$

This establishes that all the equilibria are symmetric so that sunspots can have only trivial effects. The intuition of the result is that once separate expenditure weights are used for each economy the allocations essentially are price independent, and hence the prices have to be collinear. Note, the above result is for the more general case where the consumers are choosing their optimal expenditure weights rather than using fixed exogenous weights. In the latter case, the result is even easier to establish.

Theorem 5

If all the consumers are boundedly rational and there is a weight for each commodity, then there can be no sunspots that matter.

6 Implications for Demand Functions

In this section some restrictions on the demand function that arise due to the boundedly rational behaviour are discussed. The restrictions are stronger than those placed by the block additivity assumption on the preferences. In particular, bounded rationality has definite restrictions on the sign and magnitude of some partial derivatives. To make the discussion self-contained, some definitions and results are re-collected here [2] (see Pollak [16] for details).

Definition 7

A utility function $u(x^1, \ldots, x^L)$ is said to be block additive if there exists a partition of commodities into m subsets, m functions $u_r(x^r)$, and a function F with $F' > 0$ such that

$$F[u(x)] = \sum_{r=1}^{m} u_r(x^r), \quad m \geq 2.$$

For block additive utility function, the demand functions can be written as a function of prices of commodities in that block and the expenditure on that block. For simplicity suppose the partition consists of two sets θ and θ^*. In this case

$$f^{ri}(p, Y) = g^{ri.\theta^*}(p_\theta, \kappa^\theta(p, Y)), \;\; r \neq s$$

where f^{ri} is the demand for the ith commodity in the rth block, $g^{ri.\theta^*}$ is the conditional demand for the same commodity given the consumption levels in the other blocks $s \neq r$, Y is the total income, p_θ is the vector of prices in the rth block, and $\kappa^\theta(p, Y)$ is given by $\kappa^\theta(p, Y) = Y - \sum_{k\in\theta^*} p^k f^k(p, Y))$. The following equations can be derived in a straightforward way:

$$\begin{aligned}
\frac{\partial f^{ri}}{\partial p^{sj}} &= \frac{\partial g^{ri.\theta^*}}{\partial A^\theta} \cdot \frac{\partial \kappa^\theta}{\partial p^j}, \; r \neq s \\
\frac{\partial f^{ri}}{\partial Y} &= \frac{\partial g^{ri.\theta^*}}{\partial A^\theta} \cdot \frac{\partial \kappa^\theta}{\partial Y}
\end{aligned}$$

where $A^\theta = \sum_{i\in\theta} p^i x^i$. If $\dfrac{\partial \kappa^\theta}{\partial Y} \neq 0$, then by eliminating $\dfrac{\partial g^{ri.\theta^*}}{\partial A^\theta}$ between the two equations we obtain:

[2] In the subsequent discussion the subscript pertaining to the consumer is suppressed.

$$\begin{aligned}\frac{\partial f^{ri}}{\partial p^{sj}} &= \frac{\partial \kappa^{\theta}/\partial p^{sj}}{\partial \kappa^{\theta}/\partial Y} \cdot \frac{\partial f^{ri}}{\partial Y} \quad r \neq s \\ &= \mu^{sj} \frac{\partial f^{ri}}{\partial Y}\end{aligned}$$

where $\mu^{sj} = \dfrac{\partial \kappa^{\theta}/\partial p^{sj}}{\partial \kappa^{\theta}/\partial Y}$. From this it follows that

$$\frac{\partial f^{ri}/\partial p^{sj}}{\partial f^{tk}/\partial p^{sj}} = \frac{\partial f^{ri}/\partial Y}{\partial f^{tk}/\partial Y} \quad r \neq s,\ t \neq s.$$

These are the restrictions for block additive preferences. Once the constraint of bounded rationality is added we have in addition, $\kappa^{\theta} = \lambda \sum_{l=1}^{L} p^{l} \omega^{l}$. Thus, we have $\dfrac{\partial \kappa^{\theta}}{\partial p^{sj}} = \lambda \omega^{j}$, and $\dfrac{\partial \kappa^{\theta}}{\partial Y} = \lambda$. It then follows:

$$\frac{\partial f^{ri}}{\partial p^{sj}} = \omega^{j} \frac{\partial f^{ri}}{\partial Y}, \quad r \neq s.$$

The cross price effects are still proportional to the income effect, but the factor of proportionality is the endowment of the commodity whose price has changed.

This restriction is, however, the same that will be generated if the consumer had a Cobb-Douglas utility function. In this case, as $f^{j}(p, Y) = \dfrac{\kappa_k}{\sum_{i=1}^{L} \kappa^{i}} \dfrac{Y}{p^{j}}$, the above restriction falls out. Thus, it seems that once we have block additivity and bounded rationality, the restrictions on cross-partial derivatives are the same as in the Cobb-Douglas case. The Cobb-Douglas case, however, places a stronger restriction on the income effect – it is linear in income.

The similarity can be pushed further to understand the model. If the maximization problem had allowed consumers to choose their expenditure share, instead of having these constant, and each of the consumers had Cobb-Douglas preferences, we would be in the first best situation, and sunspots could only have a trivial effect. In general, for the preferences to be consistent with two stage budgeting, we need a very particular structure for the preferences. Either the sub-utility functions are homothetic, or the sub-utility functions have a particular structure which includes the fact that the generated expenditure function for the groups are additive and each is homogeneous of degree one in the prices of the commodities in that group (Blackorby and Russell [5]. While the price aggregation (demand for a group depending only on an index of prices for each of the other groups) is not important for our purposes, this stage is important as it generates the optimal expenditure weight for each portfolio.

7 Appendix

Definition 3

$(p, w) \in \Re^{2L}_{++} \times \Re^{I+4J}$ is a price and income equilibria for fixed resources, r, if

$$\sum_{h=i,\ j1\alpha,\ j2\alpha\ j1\beta,\ j2\beta} f_h(p, w_h) = r. \tag{4}$$

Let the set of price and income equilibrium be

$$B = \{b = (p, w_h) : \sum_{h=i,\ j1\alpha,\ j2\alpha\ j1\beta,\ j2\beta} f_h(p, w_h) = r\} \tag{5}$$

The absence of aggregate uncertainty requires that we restrict our attention to the set of price and income equilibria with symmetric resources, B_s.

Definition 4

The set of price and income equilibria with symmetric resources is:

$$B_s = \{b = (p, w_h) \in B : r \text{ is symmetric}\}. \tag{6}$$

As mentioned above, there are additional restrictions on the equilibria, the incomes of the quasi-rational consumers have to be related in order for the equilibria to be consistent with the expenditure weights.

Definition 5

The set of price and income equilibria with symmetric aggregate demand consistent with the distribution of expenditure weights, $\lambda = (\lambda_1, \ldots, \lambda_J)$, is:

$$B_s(\lambda) = \{b = (p, w_h) \in B_s :\ w_{j1s} = \frac{\lambda_j}{(1-\lambda_j)} w_{j2s}\ s = \alpha, \beta,\ j = 1, \ldots, J\}. \tag{7}$$

The set of equilibria where extrinsic uncertainty does not matter will be the subset where the prices and incomes are state symmetric.

Definition 6

The set of price and income equilibria consistent with the distribution of expenditure weights, λ, where extrinsic uncertainty does not matter is:

$$\overline{B_s(\lambda)} = \{b \in B_s(\lambda) : (p, w_h) \text{ is symmetric}\} \tag{8}$$

The analysis of the effect of extrinsic uncertainty involves studying properties of these sets, and relating them to the underlying endowments. Thus, it is a part of the general problem of symmetry breaking Balasko [2]. Define the following map, $\xi : E(\omega, \lambda) \to B$:

$$\xi(p, (\omega_i)_i, (\omega_{j1s})_{j1s}, (\omega_{j2s})_{j2s}) = (p,\ (p \cdot \omega_i)_i, (p \cdot \omega_{j1s})_{j1s}, (p \cdot \omega_{j2s})_{j2s}) \tag{9}$$

The set $E(\omega, \lambda)$ is the set of sunspot equilibrium consistent with the expenditure weights λ. Now, $\xi(E(\omega, \lambda)) \subset B_s(\lambda)$. Consider, $\xi : \overline{E(\omega, \lambda)} \to B$, where the set $\overline{E(\omega, \lambda)} = \{(p, \omega_h) \in E(\lambda) : p(\alpha) = p(\beta)\}$. Note that we have $\xi(\overline{E(\omega, \lambda)}) \subset \overline{B_s(\lambda)}$. The strategy is to show that there exist points in $B_s(\lambda)$ that belong to image of $E(\omega, \lambda)$ but *not* to the image of $\overline{E(\omega, \lambda)}$ through the map ξ. The map ξ it should be noted is neither onto nor one-to-one. As in the definition of the price and income equilibria nothing has been said about the endowments, it needs to be checked that the equilibria are consistent with symmetric endowments. The sufficient condition is given below.

Proposition 1

If $p(\alpha) \neq \nu p(\beta),\ \nu \in \Re_{++}$, then the asymmetric price and income equilibria are consistent with symmetric endowments.

Proof

First, "endowments" for $j1\alpha, j1\beta, j2\alpha$ and $j2\beta$, $j = 1, \ldots, J$ are constructed and then used to find the symmetric endowments for the consumers j. The endowments of $jk\alpha$

and $jk\beta,\ k=1,2$ should be equal. These will be a solution to the linear equations

$$\begin{array}{rcl} p(\alpha)\omega^*_{jk} &=& w_{jk\alpha} \\ p(\beta)\omega^*_{jk} &=& w_{jk\beta} \end{array}$$

A solution exists if and only if $P(p) = [(p(\alpha),(p(\beta)]^T$ and $R(p) = [P(p) \quad w_j]$ have the same rank (Kronecker-Capelli Theorem). A sufficient condition is that $L \geq 2$, and rank $P(p) = 2$.

Now let $\omega_j = (\omega^*_{j1} + \omega^*_{j2},\ \omega^*_{j1} + \omega^*_{j2})$. The essential thing to notice is that the constraints on the expenditures have been subsumed in the definition of "income".

For the $i = 2, \ldots, I$ rational consumers, the endowments are derived as follows. First solve

$$(p(\alpha) + p(\beta))\omega^*_i = w_i \tag{10}$$

and then set $\omega_i = (\omega^*_i, \omega^*_i)$. Define, $\omega_1 = (\omega^*_1, \omega^*_1) = r \ - \ \sum_{i\neq 1}\omega_i \ - \ \sum_j \omega_j$. For the distribution of the endowments to be consistent with the price and income equilibria, it must be the case that

$$\begin{aligned} &(p(\alpha), p(\beta))\omega_1 = \\ &\quad (p(\alpha), p(\beta))(r \ - \ \sum_{i\neq 1}\omega_i \ - \sum_j \omega_j) \\ &\quad = w_1 \end{aligned}$$

which is true by Walras' Law. ♠

To study the existence of non-trivial sunspot equilibria, first one shows that $\overline{B_s(\lambda)}$ is a lower dimension subset of$B_s(\lambda)$. Then one shows that the complement of $\overline{B_s(\lambda)}$ in $B_s(\lambda)$ is non-empty. The intuition behind the dimensionality of the equilibria is as follows. One starts with $2L$ prices and $I + 4J$ incomes. There are L restrictions on the symmetry of aggregate demand, and $2J$ restrictions on proportionality of the incomes of the quasi-Walrasian consumers. Once these are imposed, one obtains $B_s(\lambda)$ which has dimension $L+I+2J$. In looking at $\overline{B_s(\lambda)}$ there are J additional restrictions on the symmetry of the incomes of the quasi-Walrasian consumers, thus the dimension of $L + I + J$.

Proposition 2

For a given distribution of expenditure weights, λ, $B_s(\lambda)$ is a smooth manifold of dimension $(L + I + 2J)$, and $\overline{B_s(\lambda)}$ is a smooth submanifold of dimension $(L + I + J)$ embedded in $B_s(\lambda)$.

Proof

A sketch of the proof is given, it is not difficult to fill in the details. First show that the aggregate demand map in the expanded economy, $F : \Re^{2L}_{++} \times \Re^{I+4J} \to \Re^{2L}$, is a submersion. It is sufficient to show that the Jacobian has maximal rank (see Balasko [2], Goenka [10], [11]). This will imply that F is transverse to Δ, $\Delta = \{(r(\alpha), r(\beta)) : r(\alpha) = r(\beta)\}$. This gives us the property that B_s is a smooth submanifold of $\Re^{2L}_{++} \times \Re^{I+4J}$ of dimension $(L+K+I+4J)$ as $codim\ F^{-1}(\Delta) = codim\ (\Delta)$, or $2L+I+4J - dim\ F^{-1}(\Delta) = L$. Now, $F^{-1}(\Delta) = B_s$.

Next consider $B_s \cap B(\lambda)$. This set is $B_s(\lambda)$. The intersection if transverse is a smooth submanifold of dimension B_s+ dimension $B(\lambda)$ - dimension B. The intersection is transverse if the tangent spaces to the two manifolds together span the tangent space to B (see Dubrovin, Fomenko and Novikov [8]). As this is the case the dimension of $B_s(\lambda)$ can be calculated to be $(L + I + 2J)$.

To study $\overline{B_s(\lambda)}$ define the following maps, $\chi\ : \overline{B_s(\lambda)} \to B^*(\lambda)$, where $B^*(\lambda)$ is the set of price and income equilibria consistent with the distribution of expenditure weights in the reduced economy. $\chi(p, w_i, w_{j1}) = (p^*, (w_i^*), (w_{j1}^*))$, where $p^* = p(\alpha) = p(\beta)$, $w_i^* = w_i$, and $w_{j1}^* = w_{j1\alpha} = w_{j1\beta}$. The map χ is bijective. The inverse map ψ is defined as follows: $\psi\ : B^*(\lambda) \to \overline{B_s(\lambda)}$, with $\psi(p^*, w_i^*, w_{j1}^*) = (p, w_i, w_{j1})$, where $p = (p^*, p^*)$, $w_i = w_i^*$, $w_{j1\alpha} = w_{j1\beta} = w_{j1}^*$. This map is proper, injective and an immersion, and takes its values in $\overline{B_s(\lambda)}$. It is an embedding, thus showing the required property. The dimensions can now be easily calculated. ♠

Corollary 1

The set of asymmetric price and income equilibria $B_s(\lambda)\backslash\overline{B_s(\lambda)}$ is an open subset of $B_s(\lambda)$.

One would like to know if this set is non-empty, i.e., there exist asymmetric price and income equilibria. A *sufficient* condition is that there are multiple equilibria in the certainty economy (the argument parallels the one in Balasko, Cass, and Shell [3]). As we know the conditions for uniqueness of certainty equilibrium are very strong (gross substitutability in the expanded economy), in a robust class of environments the above theorem would be true. One can alternatively follow the construction of a non-trivial equilibrium in Goenka [10] which involves showing that we can find preferences such that the price vector in the states are not collinear. Given Proposition 1, we would have proved the desired result.

References

[1] G.A. Akerlof and J.L. Yellen, A near-rational model of the business cycle, with wage and price inertia, *Quarterly J. Econ.* **100**, Supplement (1985), 823-838.

[2] Y. Balasko, Equivariant general equilibrium theory, *J. Econ. Theory* **52** (1990), 380–396.

[3] Y. Balasko, D. Cass and K. Shell, Market participation and sunspot equilibria, *Rev. Econ. Stud.* **62** (1995), 491-512.

[4] S. Benartzi and R.H. Thaler, Myopic loss aversion and the equity premium puzzle, *Q.J.E.* (1995), 73-92.

[5] C. Blackorby and R.R. Russell, Two-stage budgeting: An extension of Gorman's theorem, *Economic Theory* **9**, 185-193.

[6] D. Cass and K. Shell, Do sunspots matter? *J. Pol. Econ.* **91** (1983), 193-227.

[7] A. Deaton and J. Muellbauer, *Economics and Consumer Behaviour*, Cambridge: Cambridge University Press, 1980.

[8] B.A. Dubrovin, A.T. Fomenko, and S.P. Novikov, *Modern Geometry - Methods and Applications, Part II*, New York: Springer-Verlag, 1985.

[9] G. Gigerenzer and R. Selten, Rethinking rationality, in G. Gigerenzer and R. Setlen (Eds), *Bounded Rationality: The adaptive toolbox*, Cambridge: MIT Press, 2001.

[10] A. Goenka, Fiscal rules and extrinsic uncertainty, *Econ. Theory* **4** (1994), 401-416.

[11] A. Goenka, Rationing and sunspot equilibria, *J. Econ. Theory* **64** (1994), 424-442.

[12] W.M. Gorman, Two Stage Budgeting, in C. Blackorby and A.F. Shorrocks (Eds.) *Separability and Aggregation: The Collected Works of W.M. Gorman, Volume I*, Oxford: Oxford University Press, 1996.

[13] T. Ishikawa and K. Ueda, The bonus payment system and Japanese personal savings, In M. Aoki (Ed.) *The Economic Analysis of the Japanese Firm*, Amsterdam: North Holland 1984.

[14] D. Kahneman and A. Tversky, Prospect theory: An analysis of decision under risk, *Econometrica* **47** (1979), 263-291.

[15] D. Kahneman and A. Tversky, Choices, values and Frames, *American Psychologist* **39** (1984), 341-350.

[16] R.A. Pollak, Conditional demand functions and the implications of separable utility, *Southern Econ. J.* **37** (1971), 423-433

[17] R. Radner, Satisficing, *J. Math. Econ.* **2** (1975), 253-262.

[18] R. Radner, The organization of decentralized information processing, *Econometrica* **61** (1993), 1109- 1146.

[19] W. Samuelson and R.J. Zeckhauser, Status quo bias in decision making, *Journal of Risk and Uncertainty* **1** (1988), 7-59.

[20] W. Sharpe, *Investments*, 3rd Ed., Prentice-Hall, 1985.

[21] K. Shell, Monnaie et allocation intertemporelle, Seminaire d'Econometrie de M. Edmond Malinvaud, C.N.R.S., Paris, 1977.

[22] H.A. Simon, Theories of bounded rationality, in C.B. McGuire and R. Radner (Eds.) *Decision and Organization*, Amsterdam: North-Holland, 1972.

[23] A. Takayama, *Mathematical Economics*, 2nd Ed., Cambridge: Cambridge University Press, 1985.

[24] R. Thaler, The psychology of choice and the assumptions of economics, in A.E. Roth (Ed.) *Laboratory Experimentation in Economics*, Cambridge: Cambridge University Press, 1987.

[25] R. Thaler, Savings, fungibility, and mental accounts, *J. Econ. Perspectives* **4** (1990), 193-205.

[26] P.M. Todd, Fast and frugal heuristics for environmentally bounded minds, in G. Gigerenzer and R. Setlen (Eds) , *Bounded Rationality: The adaptive toolbox*, Cambridge: MIT Press, 2001.

[27] D. Vayanos, The decentralization of information processing in the presence of synergies, mimeo., 1999.