Uncertainty and the open economy: a view through two different lenses*

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Abstract

Under a Bayesian framework of model uncertainty, closed economy models of monetary policy typically suggest that policy responses should be attenuated. Conversely, under a Knightian view of uncertainty, where the policymaker cannot specify probabilities across alternative models, intensifying the policy response is recommended. In this paper, this dichotomy is found to extend to the case of a calibrated open economy new-Keynesian model. Two alternative model specifications are developed to examine the influence of the specification of the exchange rate on policy recommendations. One model specifies that the policymaker cares about the volatility of the exchange rate; a second model specifies a small exchange-rate channel relative to the standard channel of transmission. Although these alternative models produce quantitatively different results, qualitatively the models recommend standard results: policy attenuation if the policymaker maintains a Bayesian view of uncertainty and a more aggressive policy response, if the policymaker holds a Knightian view. Ultimately, resolving these policy proscriptions requires addressing the underlying beliefs about uncertainty of the policymaker, rather than the form of the underlying model.

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1 Introduction

Thinking about best practice monetary policy for policymakers means thinking about uncertainty. This is best exemplified by Alan Greenspan, Governor of the Federal Reserve Board:

> The Federal Reserve's experiences over the past two decades make it clear that uncertainty is not just a pervasive feature of the monetary policy landscape; it is the defining characteristic of that landscape.[1]

Thinking about uncertainty is not simple. Uncertainty may take many forms. Policymakers face uncertainty of types of uncertainty associated with measurement error, the magnitude of shocks, additive parameter uncertainty, multiplicative parameter uncertainty and general model uncertainty. Policy recommendations may be overturned when uncertainty is extended in a particular dimension. Policy recommendations may seem counterintuitive at first pass. This paper aims to illustrate two how alternative views of uncertainty generate alternative types of behaviour for policymakers.

The paper restricts itself to parameter and general model uncertainty, exploring these types of uncertainty from a *parametric* view and a *nonparametric* view of uncertainty. Whether policymakers should accentuate or attenuate a policy response drawn from a model with no uncertainty is addressed. For typical economic models, the literature notes that if the parameter uncertainty surrounds the transmission of the policy instrument to the state variables in the macroeconomy, policy should be less aggressive than otherwise — don't use a sledgehammer to crack a nut. However, when uncertainty surrounds the degree of persistence in particular state variables it transpires that policy should be more aggressive — a stitch (maybe two) in time saves nine.

A small open economy designed to broadly match the key macroeconomic features of data drawn from Australia, Canada and New Zealand, is used as a laboratory for examining policy under uncertainty. In particular, specific policy rules are derived from the model and from these rules, generic behavioural responses (such as whether policy should be more or less aggressive) are given.

Section 2 presents the small open economy model, but firstly parametric and nonparametric uncertainty is introduced.

1.1 Parametric uncertainty

Since Brainard (1967), it has been recognized that additive uncertainty does not affect the policy response, i.e. the magnitude of the error variance-covariance matrix in the state equation does not affect the policy rule. However, Brainard (1967) notes that multiplicative uncertainty, in particular uncertainty regarding the "bang per buck" or effectiveness of monetary policy, generates different policy outcomes. Sack (1998) compares the path of the implied optimal interest

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rate under no multiplicative uncertainty with the path of the implied optimal interest rate when there is multiplicative uncertainty associated with estimates from a VAR model of the US economy. He concludes that this uncertainty implies the Federal Reserve should mitigate its response but such a response cannot match the degree of smoothness in the historical interest rate path. Martin and Salmon (1999) obtain the same result for the United Kingdom using Sack's method but note the extent to which policy is more or less aggressive depends on the structure of the model and the magnitude of the variance-covariance matrix of the errors. Debelle and Cagliarini (2000) report a less aggressive policy rule under some parameter uncertainty for Australia.

However, Shuetrim and Thomson (1999) calculate the optimal policy response on a model estimated with Australian data and reach the conclusion that policy should be more aggressive, when there exists uncertainty with respect to all the parameters in the model. They note that their result depends crucially on the structure of their model. Policy attenuation or accentuation is thus context specific.

1.2 Non-parametric uncertainty

Knight (1924) describes an alternative form of uncertainty, considered non-parametric in the sense that the policymaker is uncertain about the form the uncertainty takes and is unable to form a probability distribution over different possible models. This uncertainty in explored in Hansen and Sargent's (2002) monograph *Robust Control and Model Uncertainty in Macroeconomics*. Their philosophy is to recognize that policymakers work with models which are regarded as approximations to some true, unknown model (Hansen and Sargent, 2002). The objective for researchers who consider this form of uncertainty (see for example, Onatski and Stock (2002), Tetlow and von zur Muehlen (2001) , Hansen *et al.* (1999) or the macroeconomic models in Hansen and Sargent (2002)) is to construct a rule that is robust across a set of models close to the policymaker's best approximation to the true model.

Tetlow and von zur Muehlen (2001) study robust policies within the context of a forward-looking closed economy model, similar to the wage-contracting model of Fuhrer and Moore (1995). They conclude that under unstructured uncertainty, where model misspecification arises in the local vicinity of a single model, from a range of factors including omitted variables and misspecified dynamics, the implied policy rule is more aggressive than the case where the estimated model is assumed to be the true model. However, when structure is placed on the uncertainty, in particular, when the uncertainty is restricted to parameter uncertainty, the implied policy rule is less aggressive.

This is a similar result to Onatski and Stock (2002) who use the Rudebusch and Svensson (1999) model to compare generalized Taylor-type rules that are robust to specifications of uncertainty. Within this model, the rule that is robust to unstructured uncertainty is more aggressive than the case of no uncertainty and produces a highly volatile economy if the true model is close to the estimated model. Increasing the structure on the form of the uncertainty, generally reduces the aggressiveness of the generalized Taylor rule. Onatski and Stock (2002) state that the model with the most structure is most like the Bayesian, parametric case and that this rule is less aggressive than the case where the estimated model is treated as the true model.

Before turning to the policy implications of parametric and nonparametric uncertainty, explored in sections 4 and 5, a cohesive framework for monetary policy is detailed in the following section.

2 A Monetary Policy Framework

2.1 The linear-quadratic framework

Monetary policy is examined from the perspective of optimal control within the linear-quadratic framework. The central bank is assumed to possess a set of goals or objectives for monetary policy. These goals are achieved by setting the interest rate as a rule that responds to the variables in the model of the economy. The behaviour of the economy acts as a constraint on the ability of the central bank in achieving its goals. It is assumed that central bank preferences can be approximated by a quadratic function and further, that the economy can be approximated by a linear model. Under this set of assumptions, the optimal interest rate rule will be unique.

Söderlind (1999) presents solution techniques for linear rational expectations models where the linear model of the economy is represented by:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} + Bu_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_{2\times 1}} \end{bmatrix} \tag{1}$$

and the preferences of the central bank take the form:

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta_t (x_t' Q x_t + 2x_t' U u_t + u_t' R u_t) \tag{2}$$

Furthermore, we can consider monetary policy under two alternative scenarios: discretion and commitment to a rule. Under discretion, the central bank reoptimizes the optimal rule in each period but must take the expectation formation process of agents as given. Under commitment to a rule, the central bank does not reoptimize but is assumed to possess the ability to commit to a particular rule. Agents form their expectations conditional on the rule commitment rule the central bank uses.

Söderlind (1999) shows, in detail, solution methods for linear quadratic rational expectations methods. For completeness of exposition, these equations are reproduced. Söderlind (1999) shows that the Bellman's equation for the problem under discretion is:

$$
\begin{aligned}
x_{1t}'Vx_{1t}+\upsilon_t &= \min_{u_t^*}\left[x_{1t}'Qx_{1t}+2x_{1t}'U_tu_t+u_t'R_tu_t+\beta E_t(x_{1t+1}'V_{t+1}x_{1t+1}+\upsilon_{t+1})\right]\\
s.t.\quad E_tx_{2t+1} &= C_{t+1}E_tx_{1t+1}\\
\begin{bmatrix} x_{1t+1}\\ E_tx_{2t+1}\end{bmatrix} &= A\begin{bmatrix} x_{1t+1}\\ x_{2t+1}\end{bmatrix}+Bu_t+\begin{bmatrix}\varepsilon_{t+1}\\ 0_{n_2\times 1}\end{bmatrix}
\end{aligned} \tag{3}
$$

where the initial vector x_{1t} is specified and:

$$
\begin{aligned}
D_t &= (A_{22}-C_{t+1}A_{12})^{-1}(C_{t+1}A_{11}-A_{21})\\
G_t &= (A_{22}-C_{t+1}A_{12})^{-1}(C_{t+1}B_1-B_2)\\
A_t^* &= A_{11}+A_{12}D_t\\
B_t^* &= B_1+A_{12}G_t\\
Q_t^* &= Q_{11}+Q_{12}D_t+D_t'Q_{21}+D_t'Q_{22}D_t\\
U_t^* &= Q_{21}G_t+D_t'Q_{22}G_t+U_1+D_t'U_2\\
R_t^* &= R+G_t^{'}Q_{22}G_t+G_t'U_2+U_t'G_t
\end{aligned} \tag{4}
$$

The first order condition for the associated Bellman's equation is:

$$
u_t=-F_{1t}x_{1t} \tag{5}
$$

where F_{1t} is:

$$
F_{1t}=(R_t^*+\beta B_t^{*'}V_{t+1}B_t^*)^{-1}(U_t^{*\prime}+\beta B_t^{*'}V_{t+1}A_t^*). \tag{6}
$$

Iterating until convergence, using numerical methods, effectively solves the model backwards in time. Assuming that F_{1t} and C, the solution for the expectation process, converge, the dynamics of the model can be expressed as:

$$
x_{1t+1}=Mx_{1t}+\varepsilon_{t+1} \tag{7}
$$

where:

$$
M=(A_{11}+A_{12}C-B_1F_1) \tag{8}
$$

and:

$$
x_{2t}=Cx_{1t}. \tag{9}
$$

In contrast commitment to a simple rule is described by:

$$u_t = -F_s x_t \tag{10}$$

where F_s denotes the simple rule with some elements in F restricted. This generates system dynamics:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = (A - BF_s) \begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 \times 1} \end{bmatrix}. \tag{11}$$

To optimize the simple rule, the central bank chooses the restricted elements of F_s such that the goals of objectives for the central bank are minimized.

2.2 A structural new-Keynesian model for policy analysis

The modelling philosophy is underpinned by three goals: (i) to use a model where the key parameters can be considered structural, and as such, policy invariant; (ii) to use a small model yet a model capable of replicating the key features of output gap, inflation, exchange rate and interest rate data; and (iii) to use a model with a transmission mechanism consistent with a practitioner's view of the lags of the transmission mechanism.

The first key equation is the output gap. McCallum and Nelson (1999) show how an IS equation, derived from a consumption Euler equation, implies that the output gap is a function of agents' expectations of the output gap in addition to the real interest rate. However, this view of the output gap process is generally inconsistent with the finding that the output gap displays substantial persistence in the data. Amato and Laubach (2001) show that the addition of habit formation to the utility function for consumers implies that the lag of output enters the optimizing IS equation. If we appeal to inertia on the part of decision making on the part of consumers and lag the real interest rate, we obtain a closed economy output gap equation largely constructed from structural parameters yet sufficiently flexible to replicate the persistence in output gap data.

In a closed economy, drawing inference about the behaviour of the output gap from the behaviour of consumption implicitly assumes that investment and government expenditure are not important for explaining the short to medium movements in output.

A key feature of an open economy is that the short term dynamics of output through the exchange rate's effect on net exports — expenditure switching on the part of domestic residents and changes in export returns. McCallum and Nelson (1999) derive an open economy version of their optimizing closed economy IS equation that models the output gap as a function of the real exchange, foreign output, the expectation of the real exchange rate and the expectation of foreign output gap.

If we assume that interest rates affect the exchange rate contemporaneously, including the contemporaneous exchange rate in the output gap equation allows policy to affect the output gap contemporaneously. Modelling the output gap as a function of the lag of the real exchange rate lets the model capture net exports yet preserves the lags of the transmission mechanism. Thus the output gap equation takes the following form:

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} + (1-\beta) E_t \tilde{y}_{t+1} - \beta_2 r_{t-1} - \beta_3 q_{t-1} + \varepsilon_{yt} \tag{12}$$

where $\tilde{y}_t$ represents the output gap, r_t is a long term real interest rate and q_t represent the real exchange rate — an increase in q_t represents an exchange rate appreciation. All the coefficients are positive according to theory. The long term *ex ante* real interest rate is defined using a risk neutral arbitrage condition so that the long rate is the sum of the sequence of expected short term interest rates, that is:

$$r_t = \frac{1}{d} \sum_{s=0}^{\infty} \left(\frac{d}{1+d} \right)^s E_t (i_{t+s} - \pi_{t+1+s}) \tag{13}$$

with $d = 40$ such that the long term interest rate is a ten year rate under where t is assumed to represent one quarter.

A Phillips equation is used to model domestic inflation. Structural models of the Phillips curve can be derived from wage-contracting behaviour on the part of firms and workers (for example, Fuhrer (1997) or the model of Batini and Haldane (1999)). These models suggest that workers form wage demand as an average of the expected real wage and observed past real wages with a mark-up in good times and a lower real wage in bad times, based on the realization of the output gap. Alternatively, pricing behaviour on the part of firms (see Calvo (1983) and Galí and Gertler (1999)) can be used to derive structural equations for inflation that contain forward and backward-looking components. These two behavioural generate hybrid domestic inflation equations,similar in form to:

$$\pi_t^d = \alpha_1 E_t \pi_{t+1}^d + (1-\alpha_1) \pi_{t-1}^d + \alpha_2 \tilde{y}_{t-1} + \varepsilon_{\pi^d t} \tag{14}$$

where π_t^d represents domestic inflation. Many inflation equations developed from microfoundations predict a contemporaneous relationship between the output gap and inflation, yet it is difficult to reconcile these equations with policy practitioners views of the transmission mechanism. Svensson (2000) develops an optimizing model that motivates inertia to generate appropriate lags. Here, inertia is assumed so that the lag of the output gap enters the Phillips equation. This assumption implies that policy takes two periods to impact on domestic inflation through the standard aggregate demand channel.

The foreign good component of inflation is assumed to a direct mark-up over the exchange rate with incomplete pass-through such that foreign good inflation is:

$$\pi_t^f = \kappa\pi_{t-1}^f + (1-\kappa)\Delta q_t \tag{15}$$

where π_t^f is foreign inflation and the parameter κ calibrates the degree of exchange rate pass-through. Finally, consumer price inflation is a combination of domestic price inflation and foreign good inflation, weighted according to ϕ,the proportion of foreign goods in the consumer price index:

$$\pi_t = \phi\pi_t^f + (1-\phi)\pi_t^d. \tag{16}$$

The no arbitrage condition that is the basis of uncovered interest rate parity forms an appealing structural relationship for modelling the real exchange rate. However, this condition does not appear to capture the predilection of the exchange rate to move through large cycles. To account for this feature of the data, we allows for autocorrelated exchange rate errors. Thus the real exchange rate equation is modelled as:

$$q_t = E_t q_{t+1} + (i_t - E_t\pi_{t+1}) - (i_t^f - E_t\pi_{t+1}^f) + \varepsilon_{qt}. \tag{17}$$

while the exchange rate errors are modelled as AR(1) process:

$$\varepsilon_{qt} = \rho\varepsilon_{qt-1} + \xi_t \tag{18}$$

where ξ_t is a standard normal error process.

2.3 Calibrating the model

The aim of the calibration is to formulate a model with generic dynamics that can serve as a baseline model to explore robust control for open economy inflation targeters.

Firstly, within the aggregate demand equation $\beta_1 = 0.8$ is calibrated with a high degree of persistence that corresponds to a large role for habit formation of consumers. This calibration is lower than the calibration of 0.5 in Söderström et al. (2002) and the estimate of 0.3 in Fuhrer (2000), yet several authors (Söderlind (1999) , Rudebusch (2002), Ball (1999) and Batini and Haldane (1999)) specify a zero weight on the contribution of output gap expectations to the output gap. The sensitivity of the output gap to the real interest rate is parameterized to 0.2. This is identical to the calibrated value in Ball (1999), lower than the calibration of 0.5 in Batini and Haldane (1999) but slightly higher than the parameter estimated in Söderlind (1999) and Rudebusch and Svensson (1999), and thus seems an appropriate choice in the middle of a range of estimates. The coefficient on the lag of the real exchange rate is set to 0.1, identical to the parameterisation in Ball (1999).

As Dennis and Söderström (2002) note, the literature has not settled on an appropriate calibration for any forward-looking component in the Phillips equation. Completely forward-looking inflation equations have difficulty explaining

Table 1: Model versus Data Standard Deviations

	σ_π	$\sigma_{\tilde{y}}$	σ_q	σ_i
Model	1.73	1.63	2.79	2.66
Australia	1.21	0.97	1.18	2.90
Canada	1.03	1.24	2.24	2.69
New Zealand	1.06	1.65	2.11	2.47

the persistence in inflation data. Ball (1999) and Rudebusch and Svensson (1999) assume that the Phillips equation has no forward-looking component. Other researchers, for example Fuhrer (1997), Galí and Gertler (1999), Roberts (2001), Lindé (2002) and suggest estimates on the forward-looking component to be in the range of 0.1 to 0.7. A weighting of 0.3 on the forward-looking component is pursued here given this range of estimates.

In addition, the effect of the output gap on inflation is calibrate to 0.2. Combined with the calibration of the affect of the real interest rate on the output gap, this provides a relatively standard transmission channel from interest rate to domestic inflation.

Incomplete pass-through from the exchange rate to the price of foreign goods is modelled by setting $\kappa = 0.5$. The baseline economy we work with has an equal weight on domestic and foreign goods within the consumer price index so $\mu = 0.5$.

Finally, it is assumed that setting $\rho_q = 0.75$ allows for sufficient serial correlation in the exchange rate errors to explain some of the persistent deviations of the real exchange rate from uncovered interest rate parity. The variance of the shocks to the inflation, output gap and exchange rate error equations are specified to be standard normal.

This model forms the basis for the experiments that follow. Thus the standard deviation and autocorrelation function of the variables implied by the baseline model are compared to the data for three small open economies, Australia, Canada and New Zealand, over the period 1990:1 to 2003:4.

Table 1 below depicts the standard deviations for inflation, HP-filtered output gap, real exchange rate and the nominal interest rate. The first row of the table presents the standard deviations in 9,000 observations of data generated by the model. The next three rows show the empirical standard deviations of these variables observed over the period 1990:1 to 2003:4.[2]

Looking at the first column of the table, it appears that the baseline model overstates the volatility of inflation. The standard deviation of inflation implied by the model is about 50% larger than the volatility observed in the Australian datasample and about 80% larger than the volatility of inflation for Canada and New Zealand. Again the volatility in the output gap is much larger in the model relative to the Australian data but matches the observed volatility in New

[2]Australia, Canada and New Zealand went through a disinflation process at the beginning of the 1990s. A data sample beginning in the first quarter of 1993 would show slightly lower volatilities in the data across all three countries.

Zealand particularly closely. According to the standard deviation metric, the volatility in the real exchange rate data is twice that of the Australian data sample but not much more volatile than the Canadian and New Zealand datasets. For these countries the model appears to mimic the observed volatility in the real exchange rate without recourse to a specific real exchange rate argument in the model's loss function. Finally, the volatility in the nominal interest rate implies by the model gives a close match to the data and is in fact nested by the higher volatility in the Australian dataset and the slightly lower volatilities for Canada and New Zealand.

Next, the persistence implied by the model is compared to the observed persistence in the data. This is shown in Table 2. For inflation, the model appears to overstate the degree of persistence in quarter on quarter inflation relative to weighted median measure of inflation for Australia, Canada and New Zealand. According to the model, inflation is highly persistent and inflation is still correlated with its initial value after four lags (the autocorrelation coefficient is 0.46). The data shows a much lower level of correlation in the data, with inflation for Australia and New Zealand largely uncorrelated with its initial value after four lags. The persistence in the output gap matches the data better, slightly understating the persistence in the output gap. This is also true of the real exchange rate which is highly persistent in the data but only moderately so in the model. The high degree of persistence in the nominal interest rate is closely matched by the model.

In summary, the baseline model can be criticized for overcooking the volatility and persistence of inflation, slightly understating the persistence of the output gap and the real exchange rate. This suggests that some of the quantitative results may be misleading for particular economies. However, the broad fit of the model to the data is sufficient to suggest that the qualitative results can be applied across the different economies with a degree of certainty and credibility. Impulse response functions for the baseline model are compared to impulse response functions for policy rules that address uncertainty, later in the paper.

3 Simple rules as robust rules

One approach to model uncertainty that has some history in the literature is the following. Take a set of models that the policymaker believes are a reasonable representation of the economy, calculate an optimal rule for each model and evaluate the performance of each optimal rule across the full range of models rather than for the specific model for which the rule is optimal. The literature has noted the frequent poor performance of optimal rules when applied to other models and that simple policy rules, that responds to a subset of the state variables in the model perform across models.[3]

Forecasting serves as a useful analogy here. Forecasts that overfit the model may work well in sample but poorly out of sample. Including additional vari-

[3]See the conference volume Monetary Policy Rules (1999) for other examples of this type of policy experiment.

Table 2: Model versus Data Autocorrelation Functions

Lag length	1	2	3	4	5	6	7	8
Panel 1 AC function for baseline model								
π_t	0.84	0.68	0.56	0.46	0.39	0.32	0.26	0.22
$\tilde{y}_t$	0.64	0.35	0.16	0.05	0.01	0.00	0.01	0.02
q_t	0.69	0.36	0.12	-0.02	-0.08	-0.09	-0.07	-0.05
i_t	0.86	0.65	0.47	0.34	0.25	0.19	0.16	0.13
Panel 2 AC function for Australian data								
π_t	0.46	0.36	0.27	0.09	0.13	0.06	-0.02	-0.05
$\tilde{y}_t$	0.74	0.46	0.25	0.07	-004	-0.15	-0.23	-0.28
q_t	0.84	0.74	0.59	0.49	0.35	0.24	0.15	0.04
i_t	0.84	0.71	0.56	0.43	0.31	0.20	0.09	0.00
Panel 3 AC function for Canadian data								
π_t	0.75	0.47	0.29	0.18	0.22	0.29	0.22	0.06
$\tilde{y}_t$	0.85	0.56	0.30	0.07	-0.07	-0.17	-0.24	-0.31
q_t	0.94	0.87	0.80	0.74	0.66	0.58	0.49	0.41
i_t	0.89	0.74	0.58	0.42	0.30	0.21	0.11	0.07
Panel 4 AC function for New Zealand data								
π_t	0.55	0.30	0.21	0.11	-0.04	-0.08	-0.13	-0.16
$\tilde{y}_t$	0.84	0.66	0.43	0.20	0.04	-0.12	-0.26	-0.38
q_t	0.94	0.85	0.73	0.59	0.43	0.26	0.11	-0.03
i_t	0.87	0.69	0.50	0.33	0.21	0.14	0.07	0.00

ables helps with in sample match to the data but may prove detrimental to the out of sample performance of the forecast. Similarly, including additional state variables within the optimal rule may improve performance within the context of the specific model under consideration but may prove detrimental to rule performance across alternative models. Rules that do perform well across different models are referred to as robust, in the sense that the rule provides good performance across alternative models. Simple policy rules, that respond to a limited set of state variables, have often been found to yield robust performance relative to optimal rules that appear to overfit to the model.

A number of simple interest rate rules have been suggested in the literature. Some of them have been found to perform well (see for example, Henderson and McKibbin (1993), Taylor (1993), Levin, Wieland and Williams (1999) and Williams (1999)). The following section reviews some of these rules. Subsequently, simulations of the model are conducted to compare the performance of the different simple interest rate rules with the optimal rule, which serves as the baseline for comparisons between the performance of different rules.

3.1 Some simple rules

3.1.1 The Taylor Rule

The Taylor rule is the most well-known simple interest rate rule. Under the Taylor rule, the nominal interest rate is set as a linear function of the contemporaneous realization of the output gap and the rate of annual inflation. Taylor (1993)suggested a parameterisation such that the weight on the output gap is 0.5 and the weight on inflation is 1.5. This represented a synthesis of research towards a rule that performed well for the US economy but also matched the historical path of the federal funds rate relatively closely.

While the Taylor rule appears to perform well as a policy rule for the United States, it is not clear *a priori* that such a parameterisation would perform well for an economy with a different structure. Typically, the parameterisation of the Taylor rule is estimated or calibrated to the structure of the economy.[4]

Taylor-type rules are commonly evaluated in the literature on monetary policy rules.[5] Rudebusch and Svensson (1999) conclude that a Taylor-type rule performs well with the implied variances of inflation and the output gap close to those of an optimal rule. Svensson (2000) suggests that within a calibrated open-economy model, the Taylor-type rule is outperformed by simple rules that respond to a wider range of variables. However, he states that the parameterisation of the Taylor rule — for both domestic and CPI inflation — does not create high volatility in any variable except the interest rate. It is interesting to evaluate the extent to which Taylor-type rules are suboptimal relative to unrestricted, optimal rules in an open-economy model. The Taylor-type rule is applied to annualized quarterly inflation where the rule takes the form:

$$i_t = f_{\tilde{y}}\tilde{y}_t + f_\pi \pi_t \tag{19}$$

where $\tilde{y}_t$ is the output gap and π_t is annualized quarterly inflation.

3.1.2 The Lagged Taylor Rule

McCallum and Nelson (1999) suggest that it is unreasonable to assume that policymakers have contemporaneous realizations of the output gap and inflation available. They suggest that it may be more appropriate to restrict the policymaker to viewing the output gap and inflation with a lag.

Other researchers have countered that while the policymaker may not be able to view the contemporaneous data exactly, they have available to them a plethora of other data,that allows the central bank to make an accurate prediction about the contemporaneous values of the output gap and inflation. The

[4]Technically, the class of rules where the variables that enter the simple rule are restricted to the output gap and the lag of inflation only, are typically referred to as Taylor-type rules while the particular parameterisation, due to Taylor (1993), of 0.5 on the output gap and 1.5 on the annual rate of inflation, is reserved for the Taylor rule.

[5]Note that these rules, commonly referred to as "Taylor-type" rules such as in de Brouwer and Ellis (1998), are restricted versions of Bryant-Hooper-Mann (1993) rules which are simple rules that respond to inflation and the output gap only. See McKibbin (1997) for commentary on this body of research.

implication is that a rule that includes contemporaneous values of inflation and the output gap is more realistic than restricting the central bank to act with a lag.

McCallum and Nelson (1999) find that while restricting the central bank to respond only to contemporaneous realizations of the output gap and inflation, , yields results inferior to the contemporaneous specification of the Taylor-type rule the deterioration in performance in not dramatic. Thus this paper evaluates the performance of a lagged Taylor-type rule of the form:

$$i_t = f_{\tilde{y}}\tilde{y}_{t-1} + f_\pi \pi_{t-1}.$$

3.1.3 A simple rule that includes the exchange rate

Several authors have noted that the Taylor rule will be suboptimal in an open economy setting (see for example Ball (1999) and Taylor (2001)). However, Clarida, Gali and Gertler (2001) suggest that with direct exchange rate pass through, the optimal rule will be analogous to the closed economy case with the optimal parameterisation now being a function of the degree of openness in the economy. This result does not hold for the model calibrated in section 3 because the exchange rate affects inflation with a different lag structure compared to the Clarida, Gali and Gertler (2001) model.

A simple rule that allows the interest rate to respond to the change in the real exchange rate, in addition to output and inflation, is proposed. Taylor (2001) notes that an interest rate that responds to the change in the exchange rate is consistent with the rule of thumb, suggested in Obstfeld and Rogoff (1995), that an appreciation of the exchange rate should be met with an easing of monetary policy. Since it is the change in the real exchange rate that affects inflation, the restriction implied by responding to the change in the real exchange rate (that the coefficient on the contemporaneous rate should be of equal magnitude but opposite in sign to the lag of the exchange rate) should not matter too much. Furthermore allowing the rule to respond to both the contemporaneous and lagged values of the real exchange rate may result in a rule that is overparameterized and not simple.

The rule takes the form:

$$i_t = f_{\tilde{y}}\tilde{y}_t + f_\pi \pi_t + f_{\Delta q}\Delta q_t. \tag{20}$$

A specification of this type, while restrictive in the sense that the interest rate responds to the real exchange and its lag with equal but opposite coefficients, seems sensible given the behaviour of the exchange rate in Australia, Canada and New Zealand over the past decade. For all three economies the change in the real exchange rate is a useful forecast of inflation, at least in part because the exchange rate acts as an asset to capture information about future price changes

Taylor (2001) suggests that labelling the rule described in equation (20) an open-economy rule is misleading because for some models, it may be optimal to set the response coefficient on the real exchange rate equal to zero. In this paper, a rule that includes the change in the real exchange rate is referred to as an open rule in the sense that the rule contains a variable that will be present in an open economy model, namely the exchange rate. In addition, the benefits of allowing the central bank to respond to the real exchange rate and its lag, rather than the change in the real exchange rate, are examined.

3.1.4 Flexible Simple Rules

A frequently observed behaviour of central banks is that the interest rate is shifted in a series of small movements in the same direction. This behaviour has been attributed to: (i) a fear by central banks of being seen to have made a mistake if shifts in the interest rate are frequently in opposite directions; (ii) a desire to help the expectation process on behalf of the public; and (iii) uncertainty regarding the true model of the economy leading to conservatism in policy formation.

The simple interest rate rules detailed previously restrict the response on the lag of the nominal interest rate to 0. Here, that restriction is lifted for a class of interest rate rules referred to as flexible rules, where the central bank is able to smooth the path of the instrument by responding to the lag of the instrument, the nominal interest rate.[6] Thus a flexible Taylor-type rule refers to a rule of the following form:

$$i_t = f_{\tilde{y}} \tilde{y}_t + f_\pi \pi_t + f_i i_{t-1}. \tag{21}$$

Flexible interest rate rules are considered for the lagged Taylor-type rule and the open economy rule. Rules that do not contain a lag of the nominal interest rate are referred to as standard rules rather than smoothed rules throughout the paper.

3.2 Simple Rules performance

This section evaluates the performance of simple rules relative to an optimal rule. In addition to the baseline model two additional models are used. The first variant simplifies the baseline model by reducing the model towards a closed economy variant. Specifically, the weight on tradables inflation is reduced from one half to one third of consumer price inflation. In addition, the impact of the real exchange on the output is halved, reducing the parameter from -0.2 to -0.1.

The second variant alters the preferences of the central bank to include a weight on real exchange rate stabilisation. An argument that penalises the sum of squared deviations of the real exchange rate from equilibrium, is added to the loss function with the same weight attributed to the sum of squared deviations of output from trend.

[6] The observed volatility of the interest rate actually depends on all the coefficients in the interest rate rule. However, *ceteris paribus*, a rule that contains a positive response to the lag of the interest rate will exhibit a smoother path than a rule that does not contain such a coefficient.

Table 3: Evaluating Simple Rules: Baseline Model

Baseline model	*Variances* σ_i	$\sigma_{\tilde{y}}$	σ_q	$\sigma_{\bar{\pi}}$	*Loss fn.*	*Rank*
Optimal discretion	2.66	1.63	2.79	1.73	11.93	1
Closed economy rules						
$i_t=0.80\tilde{y}_t+1.55\pi_t$	3.46	1.78	3.29	2.19	17.00	7
$i_t=0.97\tilde{y}_{t-1}+1.33\pi_{t-1}$	4.02	2.14	4.31	2.72	23.16	13
$i_t=0.72\tilde{y}_t+0.89\pi_t+0.48i_{t-1}$	3.26	1.74	3.57	2.11	15.00	5
$i_t=0.96\tilde{y}_{t-1}+1.29\pi_{t-1}+0.03i_{t-1}$	4.03	2.15	4.34	2.73	23.10	12
Open economy rules						
$i_t=0.87\tilde{y}_t + 1.55\pi_t-0.10\Delta q_t$	3.47	1.76	3.34	2.21	16.92	6
$i_t=0.92\tilde{y}_{t-1}+1.38\pi_{t-1}+0.19\Delta q_t$	3.73	2.02	3.88	2.51	21.56	11
$i_t=0.67\tilde{y}_t+0.86\pi_t+0.51i_{t-1}+0.07\Delta q_t$	3.25	1.74	3.55	2.09	14.95	4
$i_t=0.65\tilde{y}_{t-1}+0.87\pi_{t-1}+0.41i_{t-1}+0.36\Delta q_t$	3.54	1.98	3.92	2.36	19.44	10
Some more open economy rules						
$i_t=2.94\tilde{y}_t+3.04\pi_t-2.03q_t+0.42q_{t-1}$	2.86	1.65	2.61	1.84	13.77	3
$i_t=2.26\tilde{y}_{t-1}+2.08\pi_{t-1}-1.16q_t-0.18q_{t-1}$	3.14	1.96	2.82	2.23	17.90	9
$i_t=2.36\tilde{y}_t+1.57\pi_t+0.99i_{t-1}-1.20q_t-0.03q_{t-1}$	2.68	1.60	2.74	1.79	12.39	2
$i_t=1.61\tilde{y}_{t-1}+1.14\pi_{t-1}+0.43i_{t-1}+0.02q_t-0.35q_{t-1}$	3.17	1.88	3.55	2.11	17.25	8

3.2.1 Baseline model

Before turning to these alternative environments the performance of simple rules are compared to the optimal rule under the baseline model. The optimal rule is calculated under the assumption that optimisation takes place under discretion whereby the central bank acts second and cannot influence expectations initially. The alternative is commitment to the simple rules outlined in the preceding section. The results for the baseline rule are presented in table 3 with the relative ranking of each rule's performance listed in the column in the far right of the table.

In the table above, we see that optimal rule is the best performing rule. This is, of course, completely unsurprising since the rule is unrestricted and can respond to all the state variables. However, the performance of the second best rule, the rule that responds to the output gap, inflation and the real exchange rate contemporaneously and in addition, to the lag of the real exchange rate and the nominal interest rate, is very close to the performance of the optimal rule. Under this rule, the standard deviations of all the state variables — that generate the concomitant loss for the central bank — are relatively similar.

Removing interest rate smoothing from this best performing simple rule increases the loss by approximately 10%. Alternatively, restricting the monetary authority to respond to the change in the real exchange rather than the level and lag of the real exchange separately, reduces the effectiveness of the rule by about 20% according the loss function metric.

Restricting the monetary authority to respond to lagged rather than contemporaneous variables has a sizeable impact on the loss. Throughout the table, the lagged counterpart to the specific contemporaneous rule suffers a loss of about 30%, on average. The worst performing rule for the model turns out to the lagged Taylor rule, which yields a loss of 23.16 — twice the loss ensuing from using the optimal rule.

Finally, the contemporaneous Taylor rule performs about 50% worse than the optimal rule. Interestingly, the open economy model suggests coefficients of 0.8 and 1.55 on the output gap and inflation respectively — relatively close to the coefficients suggested by Taylor for the US closed economy.

3.2.2 Alternative Model

Of course, these results are a function of the model and the preference set that underlie the analysis. The reality of the policy landscape is that central banks face uncertainty with regard to the true model of the economy. This necessitates evaluating the efficacy of simple rules relative to optimal rules in alternative models. In addition, the efficacy of simple rules may be different under alternative representations of central bank preferences. The table below details the relative performance of simple rules in an alternative model framework where the role of the exchange rate is mitigated.

Table 4: Evaluating Simple Rules: Alternative Preferences

	Variances				*Loss fn.*	*Rank*
	σ_i	$\sigma_{\tilde{y}}$	σ_q	$\sigma_{\bar{\pi}}$		
Optimal discretion	2.72	1.71	2.18	2.03	22.15	1
Closed economy rules						
i_t=0.80$\tilde{y}_t$+1.37π_t	3.59	1.80	3.10	2.56	32.93	9
i_t=0.95$\tilde{y}_{t-1}$+1.21π_{t-1}	4.24	2.14	4.07	3.20	50.15	13
i_t=0.73$\tilde{y}_t$+1.08π_t+0.21i_{t-1}	3.50	1.78	3.17	2.52	32.50	7
i_t=1.06$\tilde{y}_{t-1}$+1.47π_{t-1}+0.21i_{t-1}	4.21	2.10	3.91	3.13	48.92	12
Open economy rules						
i_t=0.75$\tilde{y}_t$+1.37π_t+0.09Δq_t	3.59	1.81	3.07	2.54	32.80	8
i_t=0.86$\tilde{y}_{t-1}$+1.26π_{t-1}+0.31Δq_t	3.85	2.00	3.53	2.83	42.96	11
i_t=0.64$\tilde{y}_t$+1.02π_t+0.27i_{t-1}+0.15Δq_t	3.49	1.79	3.14	2.48	32.16	6
i_t=0.62$\tilde{y}_{t-1}$+0.91π_{t-1}+0.30i_{t-1}+0.45Δq_t	3.71	2.03	3.55	2.67	41.49	10
Some more open economy rules						
i_t=2.93$\tilde{y}_t$+2.76π_t-2.47q_t+0.14q_{t-1}	2.91	1.74	2.15	2.13	23.55	3
i_t=3.37$\tilde{y}_{t-1}$+2.48π_{t-1}-2.67q_t-0.36q_{t-1}	3.27	2.05	2.24	2.57	28.58	5
i_t=3.29$\tilde{y}_t$+1.98π_t+0.92i_{t-1}-2.65q_t+0.00q_{t-1}	2.75	1.70	2.16	2.08	22.59	2
i_t=1.89$\tilde{y}_{t-1}$+1.14π_{t-1}+0.72i_{t-1}-0.87q_t-0.74q_{t-1}	3.05	1.99	2.25	2.49	26.91	4

What we can observe is that the best performing simple rule from the baseline model is the best performing rule in this alternative model. Furthermore, the reduction in performance relative to the optimal rule is particularly small — about 1.5%. While the relative performance of contemporaneous rules relative to rules that are restricted to respond to lagged variables is much similar, there appears to a premium on smoothed rules. Rules that respond to the lag of the nominal interest rate perform well under the alternative model.

3.2.3 Alternative Preferences

Table 5 below presents results from a third set of experiments. Specifically an argument on the deviations of the real exchange rate from equilibrium is attributed the same weight as inflation and the output gap in the preferences of the central bank. This implies that the expected present value of the sum of future deviations of goal variables with be higher.

Table 5: Evaluating Simple Rules: Alternative Model

	Variances				*Loss*	*Rank*
	σ_i	$\sigma_{\tilde{y}}$	σ_q	$\sigma_{\bar{\pi}}$	*fn.*	
Optimal discretion	3.65	1.74	5.32	2.28	16.79	1
Closed economy rules						
i_t=0.73$\tilde{y}_t$+1.66π_t	4.23	1.85	5.38	2.55	21.39	9
i_t=1.02$\tilde{y}_{t-1}$+1.48π_{t-1}	4.35	1.98	6.02	2.76	23.24	13
i_t=0.75$\tilde{y}_t$+0.85π_t+0.56i_{t-1}	3.96	1.78	5.86	2.44	18.27	4
i_t=0.90$\tilde{y}_{t-1}$+1.10π_{t-1}+0.27i_{t-1}	4.34	2.01	6.30	2.81	22.44	10
Open economy rules						
i_t=0.87$\tilde{y}_t$+1.65π_t-0.14Δq_t	4.22	1.82	5.46	2.56	21.12	8
i_t=1.00$\tilde{y}_{t-1}$+1.51π_{t-1}+0.08Δq_t	4.26	1.96	5.86	2.71	23.00	12
i_t=0.70$\tilde{y}_t$+0.82π_t+0.58i_{t-1}+0.04Δq_t	3.96	1.78	5.84	2.43	18.23	3
i_t=0.71$\tilde{y}_{t-1}$+0.84π_{t-1}+0.50i_{t-1}+0.24Δq_t	4.05	1.92	6.00	2.62	20.66	7
Some more open economy rules						
i_t=2.33$\tilde{y}_t$+2.56π_t-0.92q_t+0.28q_{t-1}	3.92	1.76	4.96	2.41	19.67	5
i_t=1.27$\tilde{y}_{t-1}$+1.65π_{t-1}-0.05q_t-0.11q_{t-1}	4.12	1.95	5.56	2.68	22.72	11
i_t=2.03$\tilde{y}_t$+1.32π_t+0.92i_{t-1}-0.50q_t-0.05q_{t-1}	3.67	1.73	5.28	2.29	17.07	2
i_t=1.13$\tilde{y}_{t-1}$+0.91π_{t-1}+0.65i_{t-1}+0.10q_t-0.34q_{t-1}	3.73	1.91	5.40	2.56	19.73	6

Turning to the table, this turns out to be true — the loss is much higher than in both the baseline model and for the alternative model.

Again the best performing rule responds to the contemporaneous output gap, inflation and the real exchange rate, and in addition, the lag of the real exchange rate and the nominal interest rate. Indeed, the rules that can respond to both contemporaneous and lagged values of the real exchange rate are the best performing rules when the central bank cares about the deviations of the real exchange rate from equilibrium.

3.2.4 Robust Simple Rules

What these alternative scenarios show is that the performance of different simple rules depends on the environment, including both the model of the economy and the set of central bank preferences.

Given that central banks operate in an uncertain environment, one desirable quality in a simple rule is good performance across alternative environments. Thus researchers have conducted a search for a rule is "robust", yielding good performance across a range of models. The conference volume "Monetary Policy Rules" is an example of a search for such a rule. Typically researchers have found that optimal rules tend to overfit the policy response to the model and that optimal rules perform badly when applied out of context, within alternative models of the economy or for alternative sets of preferences.

The extent to which simple rules are robust in three alternative open economy contexts is tested according to the following method. Simple rules, with parameters optimized to a particular context are applied to an alternative context, variances traced out and the out of context performance evaluated according to the loss. The out of context rankings of the alternative policy rules are presented in the table below. The column to the far right of the table evaluates the overall performance of the simple rules according to the metric with which points are awarded to formula one racing drivers for placing in a particular race.

Table 6: Evaluating Simple Rules: Baseline Model

Modelling Context	M_1		M_2		M_3		Formula 1
Modelling Rules	R_2	R_3	R_1	R_3	R_1	R_2	Rankings
Optimal rule	1	2	12	12	2	4	3=
Closed economy rules							
$i_t=\phi_1\tilde{y}_t + \phi_2\pi_t$	7	7	4	3	6	2	6
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}$	11	12	7	5	12	11	8=
$i_t=\phi_1\tilde{y}_t+\phi_2\pi_t+\phi_3 i_{t-1}$	4	4	1	1	9	6	1=
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}+\phi_3 i_{t-1}$	12	13	6	6	13	12	8=
Open economy rules							
$i_t=\phi_1\tilde{y}_t + \phi_2\pi_t + \phi_3\Delta q_t$	6	8	3	4	7	3	7
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}+\phi_3\Delta q_t$	10	11	9	9	11	8	12
$i_t=\phi_1\tilde{y}_t + \phi_2\pi_t+\phi_3 i_{t-1}+\phi_4\Delta q_t$	3	5	2	2	8	7	5
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}+\phi_3 i_{t-1} + \phi_1\Delta q_t$	9	10	13	13	10	10	13
Some more open economy rules							
$i_t=\phi_1\tilde{y}_t+\phi_2\pi_t+\phi_3 q_t+\phi_4 q_{t-1}$	5	3	5	8	3	1	3=
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}+\phi_3 q_t+\phi_4 q_{t-1}$	13	9	8	11	5	13	11
$i_t=\phi_1\tilde{y}_t+\phi_2\pi_t+\phi_3 i_{t-1}+\phi_4 q_t+\phi_5 q_{t-1}$	2	1	10	7	1	5	1=
$i_t=\phi_1\tilde{y}_{t-1}+\phi_2\pi_{t-1}+\phi_3 i_{t-1}+\phi_4 q_t+\phi_5 q_{t-1}$	8	6	11	10	4	9	7

1. M_1 Baseline model, R_1: Baseline model rules
2. M_2: Alternative transmission mechanism model, R_2: Alternative transmission rules
3. M_3: Alternative preferences model, R_3: Alternative preference rules
4. Ranking denotes rules performance in the alternative context
5. Under the rules of formula one, 10 points are allocated for 1st, 8pts for 2nd, 6pts for 3rd, 5pts for 4th, 4pts for 5th, 3pts for 6th, 2pts for 7 and 1pt for 8th.

Take the first column of the table. This column displays the ranking of the rules optimized on the second model environment (where the model is closer to a closed economy), labelled R_2, evaluated under the baseline model, M_1. The column shows that the rule that responds to all the state variables in the model is the best performing rule, relative to the other simple rules, even though it is optimized for an alternative environment. For this case, the optimal rule does not suffer from overfitting.

However, in the second column of the table, the rules optimized on the environment where the central bank is concerned about the volatility, real exchange rate, tell a different story. The rules that responds to the contemporaneous output gap, inflation, the real exchange rate and the lag of the real exchange and the nominal interest actually outperforms the "optimal" rule that responds to all the state variables in the model. In this case, the rule optimized to an alternative environment appears to suffer from overfitting and the simpler rule performs better.

This exercise is repeated for all three sets of rules R_1, R_2 and R_3, across the three alternative modelling environments, M_1,M_2 and M_3. The optimized rules from the baseline model and the environment where the central bank cares about the volatility in the real exchange rate, perform particular badly under the alternative model, where the economy is closer to a closed economy. In fact these rules are almost the worst performing rules. In contrast, the contemporaneous Taylor rule with interest rate smoothing is the best performing out of context rule in the alternative model scenario.

In aggregate, the contemporaneous Taylor rule with interest rate smoothing and the same rules with the addition of a response to the contemporaneous and lagged real exchange rate prove the best rules. However, the optimal rules are the next best set of rules. For the policy experiments considered, using the Taylor rule with interest rate smoothing can improve on optimal rules that are somewhat hampered out of context by overfitting the policy response to the policy environment.

4 Bayesian Rules

As discussed in the introduction, a Bayesian recognizes that their model may be misspecified but unlike the case of Knightian uncertainty, Bayesians can formulate probabilistic statements about the manner in which their model may be misspecified.

This section considers rules optimised in the environment where the policymakers weights the baseline, less open economy and exchange rate preference models equally. That is, the probability associated with the likelihood of each model is one-third.

The algorithm that searches for the optimal rule proceeds as follows. Firstly guess the parameters of the simple rule. Calculate the expectation process conditional on the form of the guess of the simple rule, the reduced form dynamics and the implied loss of the central bank under each model. Construct the expected loss according to the model weights. Update the guess of the optimal parameterization until convergence.

There are at least three criticisms of this exercise. Firstly, the problem is set up as static in the sense that the central bank does not learn in a true Bayesian sense about the relative credence of each model. Secondly, the losses evaluated under each model are not normalized. Thus the Bayesian rule will be slanted toward the model that includes the real exchange rate as an argument in the central bank's loss function because of the higher losses this model naturally incurred. Finally, the magnitude of model variation is relatively small. All three models can be considered generic versions of the same general model.

These points aside, the exercise gives us some insight regarding which rules yield performance robust to some model variation. The table below presents the loss associated with the optimal Bayesian rules for each of the three models and the Bayesian framework where the expected probability of each model is one-third. This model is presented in the fifth column of the table labelled M_0.

Table 7: Evaluating Bayesian Rules

Bayesian Rules	M_1 *Loss*	M_2 *Loss*	M_3 *Loss*	M_0 *Loss*	M_0 *Rank*
Closed economy rules					
$i_t=0.68\tilde{y}_t+1.79\pi_t$	18.08	21.68	29.45	23.07	7
$i_t=0.96\tilde{y}_{t-1}+1.60\pi_{t-1}$	27.28	23.72	35.24	28.74	12
$i_t=0.73\tilde{y}_t+0.90\pi_t+0.58i_{t-1}$	15.69	18.42	21.64	18.58	3
$i_t=0.91\tilde{y}_{t-1}+1.34\pi_{t-1}+0.16i_{t-1}$	27.91	22.98	34.13	28.34	11
Open economy rules					
$i_t=0.75\tilde{y}_t+1.78\pi_t-0.09\Delta q_t$	18.00	21.48	29.44	22.98	6
$i_t=0.91\tilde{y}_{t-1}+1.64\pi_{t-1}+0.14\Delta q_t$	24.71	23.60	33.68	27.33	10
$i_t=0.64\tilde{y}_t+0.82\pi_t+0.62i_{t-1}+0.07\Delta q_t$	15.70	18.39	21.30	18.47	2
$i_t=0.62\tilde{y}_{t-1}+0.86\pi_{t-1}+0.53i_{t-1}+0.32\Delta q_t$	21.89	21.19	26.47	23.19	8
Some more open economy rules					
$i_t=2.75\tilde{y}_t+3.01\pi_t-1.44q_t+0.42q_{t-1}$	14.95	19.81	23.51	19.43	4
$i_t=1.92\tilde{y}_{t-1}+2.13\pi_{t-1}-0.48q_t-0.17q_{t-1}$	21.42	23.88	29.78	25.01	9
$i_t=1.92\tilde{y}_t+1.31\pi_t+0.96i_{t-1}-0.56q_t-0.06q_{t-1}$	13.74	17.29	18.39	16.47	1
$i_t=1.30\tilde{y}_{t-1}+0.99\pi_{t-1}+0.74i_{t-1}+0.03q_t-0.47q_{t-1}$	18.48	20.38	22.99	20.62	5

1. M_0: Bayesian model.
2. M_1: Baseline model.
3. M_2: Alternative model.
4. M_3: Alternative preferences.

Of course the best performing rule is the contemporaneous Taylor rule with interest rate smoothing that also responds to the real exchange rate and its lag,because it represents the most unrestricted rule.

There appears to a premium on allowing the central bank to smooth the rule in the Bayesian context. The next best performing rule allows the central bank to respond to contemporaneous inflation and the output gap, the change in the real exchange rate and the lag of the interest rate. The smoothed Taylor rule is in fact the third best rule and returns a loss about 18% higher than the best performing simple rule.

Furthermore, within the Bayesian context, the coefficient on the lag of the interest rate is uniformly lower than the corresponding coefficient for any of the simple rules for all three models. For example, the coefficient on the lag of the interest rate on the smoothed Taylor rule is 0.58 in the Bayesian framework and takes values of 0.48, 0.56 and 0.21 in the baseline, alternative transmission model and the exchange rate preference models respectively. Given that the parameters on the coefficients are on average, less aggressive in the Bayesian setting, it appears that an open economy inflation targeter may act more conservatively given the type of uncertainty considered within this section.

5 Hansen-Sargent Robust Rules

Knight (1924) describes an alternative form of uncertainty, which can be considered non-parametric in the sense that the policymaker is uncertain about the form the uncertainty takes and is thus unable to form a probability distribution over different possible models. This is the form of uncertainty modelled in Hansen and Sargent's (2002) monograph *Robust Control and Model Uncertainty in Macroeconomics.* Their philosophy is to recognize that policymakers work with models which are regarded as approximations to some true, unknown model (Hansen and Sargent, 2002). The objective for researchers who consider this form of uncertainty (see for example, Onatski and Stock (2002), Tetlow and von zur Muehlen (2001), Hansen *et al.* (1999) or the macroeconomic models in Hansen and Sargent (2002)) is to construct a rule that is robust across a set of models close to the policymaker's best approximation to the true model.

Tetlow and von zur Muehlen (2001) study robust policies within the context of a forward-looking closed economy model, similar to the wage-contracting model of Fuhrer and Moore (1995). They conclude that under unstructured uncertainty, where model misspecification may arise from a range of factors including omitted variables and misspecified dynamics, the implied policy rule is more aggressive than the case where the estimated model is assumed to be the true model. However, when structure is placed on the uncertainty, in particular, when the uncertainty is restricted to parameter uncertainty, the implied policy rule is less aggressive.

This is a similar result to Onatski and Stock (2002) who use the Rudebusch and Svensson (1999) model to compare generalized Taylor-type rules that are robust to specifications of uncertainty. Within this model, the rule that is robust to unstructured uncertainty is more aggressive than the case of no uncertainty and produces a highly volatile economy if the true model is close to the estimated model. Increasing the structure on the form of the uncertainty, generally reduces the aggressiveness of the generalized Taylor rule. Onatski and Stock (2002) state that the model with the most structure is most like the Bayesian, parametric case and that this rule is less aggressive than the case where the estimated model is treated as the true model.

Two classes of uncertainty can be distinguished dependant on whether the policymaker can place a parametric prior on the form of the uncertainty. If this is true, the uncertainty is Bayesian in nature. Typically in the literature, Bayesian uncertainty implies mitigating the policy response (in the manner of Brainard (1967)), although this finding is not general.

The alternative form of uncertainty is due to Knight (1924) and asserts that it is unreasonable to assume that the policymaker is able to place a prior on the distribution of the model uncertainty. Hansen and Sargent (2002) adapt the optimal linear regulator framework to address this form of uncertainty. This paper focuses on Hansen-Sargent uncertainty and seeks to examine how central banks should change their rule to explicitly address this type of uncertainty.

Assuming the policymaker knows the true form of the economy is an extreme assumption, given the number of models used for policy analysis and the lack of professional agreement regarding a true model of the economy (McCallum, 1997). Given this uncertainty, there may exist disagreement regarding the correct policy setting (Levin, et. al. 1999). The aim of much of the literature on monetary policy rules is to discover a rule that is robust across different models of the economy. This search for a robust policy rule in an uncertain world may also be conducted within a specific model by explicitly addressing the policymaker's uncertainty.

The alternative form of uncertainty is Knightian uncertainty, which may be considered non-parametric in the sense that the policymaker is uncertain about the form the uncertainty takes and is thus unable to form a probability distribution over different possible models. This is the form of uncertainty that Hansen and Sargent (2002) address in their monograph, "Robust Control and Model Uncertainty in Macroeconomics".

Their philosophy is to recognize that policymakers work with models which are regarded as approximations to some true, unknown model (Hansen and Sargent, 2002). The objective for researchers who consider this form of uncertainty (see for example, Onatski and Stock (2002), Tetlow and von zur Muehlen (2001), Hansen *et al.* (1999) or the macroeconomic models in Hansen and Sargent (2002)) is to construct a rule that is robust across a set of models close to the model the policymaker uses as the best approximation to the true model.

The set of models the policymaker wishes to be robust against should be difficult to distinguish from the true model. If this is not true, the rule will be tainted by a desire for robustness against models that are unlikely to occur. How this is achieved is detailed in following sections. However, firstly the optimal linear regulator problem is respecified for a policymaker that desires a robust rule in the sense of Hansen and Sargent (2002).

5.1 The robust control framework

Giordani and Söderlind (2002) provide a convenient exposition of solution methods for the robust control problem under commitment, discretion and simple monetary policy rules. Here we represent an outline of the solution method, assuming that the central bank implements policy under discretion. The specification of the problem hinges on the addition of an evil agent that acts to

$$\min_{\{u\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2x_t' U x_t) \tag{22}$$

$$s.t. \quad x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \upsilon_{t+1}) \tag{23}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{t+1}' \upsilon_{t+1} \leq \eta_0 \tag{24}$$

substituting the constraint on the behaviour of the evil agent into the model of the economy, the constraint on the behaviour of the central bank, gives:

$$\min_{\{u\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2x_t' U x_t - \theta \upsilon_{t+1}' \upsilon_{t+1}) \tag{25}$$

$$s.t. \quad x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \upsilon_{t+1}). \tag{26}$$

Giordani and Söderlind (2002) show that the corresponding Bellman's equation is:

$$\begin{aligned} x_{1t}' V x_{1t} + w_t &= \min_{u_t^*} \max x_{1t}' \tilde{Q} x_{1t} + 2x_{1t}' \tilde{U}_t u_t^* + u_t^{*\prime} \tilde{R}_t u_t^* + \\ &\quad \beta E_t (x_{1t+1}' V_{t+1} x_{1t+1} + w_{t+1})] \qquad (27) \\ s.t. x_{1t+1} &= \tilde{A}_t x_{t+1} + \tilde{B}_t u_t^* + C_1 \varepsilon_{t+1} \qquad (28) \end{aligned}$$

where the initial vector x_{1t} is specified and:

$$\begin{aligned} D_t &= (A_{22} - K_{t+1} A_{12})^{-1} (K_{t+1} A_{11} - A_{21}) \\ G_t &= (A_{22} - K_{t+1} A_{12})^{-1} (K_{t+1} B_1^* - B_2^*) \\ \tilde{A}_t &= A_{11} + A_{12} D_t \\ \tilde{B}_t &= B_1 + A_{12} G_t \\ \tilde{Q}_t &= Q_{11} + Q_{12} D_t + D_t' Q_{21} + D_t' Q_{22} D_t \\ \tilde{U}_t &= Q_{21} G_t + D_t' Q_{22} G_t + U_1^* + D_t' U_2^* \\ \tilde{R}_t &= R^* + G_t' Q_{22} G_t + G_t' U_2^* + U_t^{*\prime} G_t \end{aligned} \tag{29}$$

The first order condition for the associated Bellman's equation is:

$$u_t = -F_{1t} x_{1t} \tag{30}$$

where F_{1t} is:

$$F_{1t} = (\tilde{R}_t + \beta \tilde{B}_t V_{t+1} \tilde{B}_t)^{-1} (\tilde{U}_t' + \beta \tilde{B}_t V_{t+1} \tilde{A}_t). \tag{31}$$

Iterating until convergence, using numerical methods, effectively solves the model backwards in time. Assuming that F_{1t} and K_t converge, the dynamics of the model can be expressed as:

$$x_{1t+1} = Mx_{1t} + C\varepsilon_{t+1} \tag{32}$$

where:

$$M = A_{11} + A_{12}K - B_1^* F_1 \tag{33}$$

and:

$$\begin{bmatrix} x_{2t} \\ u_t^* \end{bmatrix} = Nx_{1t} \tag{34}$$

where:

$$N = \begin{bmatrix} K \\ -F_1 \end{bmatrix}. \tag{35}$$

If the worst case dyanamics do not eventuate, the *approximating* model is:

$$M_a = A_{11} + -B_1^* F_1 \tag{36}$$

5.2 How much robustness?

The previous section established a method for constructing a rule that is robust to a sequence of specification errors. The policymaker's choice of the robustness parameter θ, constrains the evil agent and effectively forms the budget out of which the evil agent constructs a rule for the specification errors for the model.

What is the appropriate choice of θ for the policymaker? The answer is that the policymaker desires a rule that is robust to models that are difficult to distinguish against the model that is used as the best approximation to the economy.

If the alternative model is not sufficiently close to the true model, it is not reasonable to be robust against that alternative model, since it is unlikely that this alternative is the true model. A probability that determines the set of models it is deemed reasonable to be robust against is chosen by the policymaker. This probability is an error detection probability, the probability of making an error in distinguishing the alternative model from the true model based on a sample generated from the true model.

Hansen and Sargent (2002) advocate using log-likelihood ratios of the approximating model against the worst-case model, in order to map a sequence of robustness parameters, to a sequence of probabilities that determine the probability that the worst-case model can be distinguished from the approximating model.

Hansen and Sargent (2002) show how the error detection probabilities can be calculated. Firstly, the approximating model, equation (36), is defined as model A, while the worst-case model, equation (33), is defined as model B. For a fixed sample of observations, Hansen and Sargent (2002) define L_{ij} as the likelihood of that sample for model j under the assumption that model i generates the data. The log likelihood ratio for a given sample can then be expressed as:

$$r_i \equiv \frac{L_{ii}}{L_{ij}}. \tag{37}$$

Consider drawing repeated samples. There are two kind of mistakes that can be made in attempting to determine which model generated the sample data. Firstly, model A could be the true data-generating process yet for a given sample, the log likelihood may be negative. It is possible to calculate the probability of making this mistake in repeated sampling:

$$p_A = \Pr(\mathit{mistake}/A) = \mathit{freq}(r_A \leq 0) \tag{38}$$

i.e., the frequency of generating *negative* log-likelihood ratios is the probability of mistaking model B for model A, when model A is the true data generating process. The corollary is also true for model B so that:

$$p_B = \Pr(\mathit{mistake}/B) = \mathit{freq}(r_B \leq 0). \tag{39}$$

The probabilities of a mistake, p_A and p_B, are functions of the difference between the approximating model, equation (36), and the worst-case model, equation (33), which is a function of the robustness parameter, θ. The probability of detecting a difference between the approximating model and the worst-case model can thus be expressed as:

$$p(\theta) = \frac{1}{2}(p_A + p_B). \tag{40}$$

The next step is to calculate the map between the error detection probabilities and the robustness parameter. Firstly choose an appropriate value for the error detection probability (Hansen and Sargent (2002) use a probability of 0.1 or 0.2 while Giordani and Söderlind (2002) use 0.2). Given the error detection probability, calculate the preference for robustness θ using the map.

Having obtained the error detection probabilities, the next step is to calculate the map between the error detection probabilities and the robustness parameter. Firstly choose an appropriate value for the error detection probability (Hansen and Sargent (2002) use a probability of 0.1 or 0.2 while Giordani and Söderlind (2002) use 0.2). Given the error detection probability, calculate the preference for robustness θ using the map.

Giordani and Söderlind (2002) provide Gauss and Matlab software for calculating the error detection probabilities. This software is used to calculate the map between the error detection probabilities between the approximating and worst-case models for the model calibrated in section 2 for each country. Following Hansen and Sargent (2002) a risk sensitivity parameter is defined where the risk sensitivity parameter $\sigma = -\theta^{-1}$. When $\sigma = 0$ the robustness parameter is infinite and the model conforms to the standard case. When the risk sensitivity parameter is negative, there exists a preference for a robust rule.

5.3 Results

5.3.1 Error Detection

The initial step in evaluating robust rules on the data involves calculating appropriate values of θ. Once suitable error detection probabilities have been chosen, a map from these probabilities to the different values of θ is generated.

Gauss code provided by Giordani and Söderlind (2002) proceeds in the following manner: (i) take the A, B, Q, and R matrices that define the estimated model and simulate the model 10,000 times for sixty quarters;[7] (ii) choose a value of θ; (iii) calculate the log-likelihood ratios for each sample, i.e. calculate the ratio in equation (40) and hence calculate the error detection probability for that value of θ.

This process is repeated over what was considered an appropriate grid for developing a broad picture for the relationships between the robustness parameter θ and the error detection probabilities. A finer grid was used to calculate a value of θ to two decimal places for error detection probabilities of 5%, 10% and 20%. The aim is to find a value of θ such that the probability of making an error in distinguishing the worst-case from the approximating model, is just less than the three error detection probabilities. Models within this detection criterion are models the policymaker wishes to be robust against.

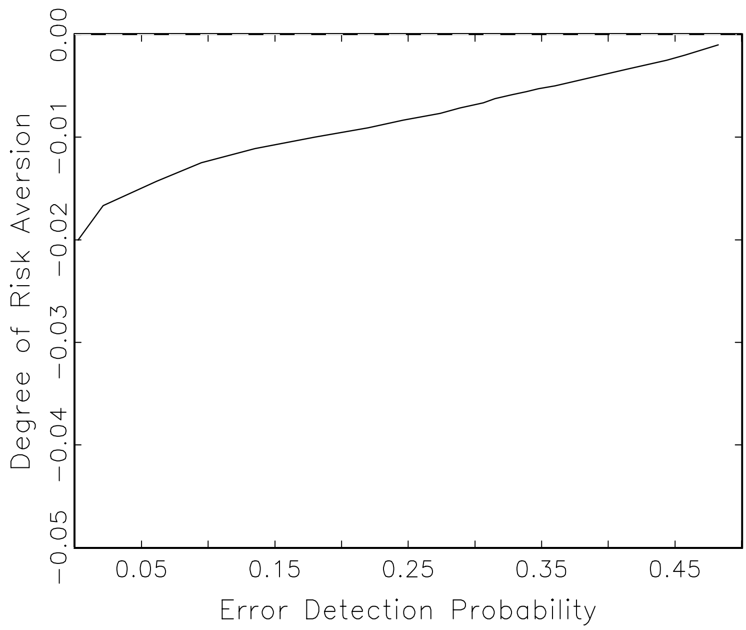
However, little information can be drawn from the robustness parameter *per se.* It is more interesting to evaluate how a preference for robustness slants the optimal rules and affects the dynamics of the model. Outcomes for rules that are robust against alternative models with different error detection probabilities are compared to the outcomes for the optimal rule. In particular, rules that are robust to models where the policymaker has a 5%, 10% and 20% probability of making an error in distinguishing the alternative model from the true model are considered. These error detection probabilities correspond to values of σ, the degree of risk aversion of -0.0147, -0.0124 and -0.0096 which correspond to values of θ of 68.13, 80.60 and 104.06 respectively.

5.3.2 Baseline Model

Table 8 compares the standard discretionary rule, where the policymaker does not fear model misspecification, with three robust rules where the policymaker desires good outcomes but believes that the baseline model may be misspecified.

[7] It is assumed that sixty quarters is an appropriate amount of time to be able to distinguish the true model from the approximating model.

Figure 1: Error Detection Probabilities



Recall that a policymaker that desires a rule that is robust against any model misspecification looks foolish because of extreme pessimism. This extreme pessimism it mitigated by seeking a rule that is robust to models where the policymaker has a 5%, 10% and 20% probability of making an error in distinguishing the alternative model from the true model.

In the table, robust rule (i) corresponds to a value of θ of 104.06, where the policymaker has some preference for robustness. Robust rule (ii) corresponds to a value of θ of 80.60, a moderately robust rule while robust rule (iii) depicts a robust rule that corresponds to a particularly pessimistic view where the policymaker desires a rule that is robust against a worst case model such that the probability of detecting an error between the true and worst case model is only 5%.

Looking at table 8, the baseline rule states that the policymaker should increase the nominal interest rate in response to a positive domestic inflation shock by 70 basis points, increase the nominal interest rate in response to a positive output gap shock by 94 basis points, but should decrease the nominal interest rate by 51 basis points in response to a positive real exchange shock. These type of responses appear intuitive reasonable and this is also true of the response to the lagged variables in the model — the interest rate responds positively to an increase in the lag of the output gap, the lag of domestic inflation, contemporaneous inflation in the foreign good component of consumer inflation and negatively to an appreciation in the lag of the real exchange rate. There is some interest rate smoothing behaviour. According to the baseline rule the nominal interest rate responds by 37 basis points to its own lag.

Table 8: Optimal and Robust Rules: Baseline Model

Rule	$e_{\pi^d t}$	e_{qt}	$e_{\tilde{y}t}$	$\tilde{y}_{t-1}$	π^d_{t-1}	q_{t-1}	i_{t-1}	π^f_t	R_{t-1}
Baseline	0.70	-0.51	0.94	0.89	0.49	-0.29	0.37	0.11	-0.19
Robust (i)	0.98	-0.54	1.11	1.08	0.69	-0.36	0.34	0.15	-0.22
Robust (ii)	1.12	-0.55	1.19	1.18	0.79	-0.40	0.32	0.17	-0.24
Robust (iii)	1.29	-0.56	1.29	1.29	0.90	-0.43	0.31	0.19	-0.26

NB. Under the baseline rule, there is no preference for robustness and $\theta = \infty$.

These results echo the findings of Tetlow and von zur Muehlen (2001) who find that for a forward-looking model of the US economy, rules that are designed to be robust against unspecified uncertainty, in the sense of Hansen-Sargent robustness, are more aggressive than the standard optimal rule with no preference for robustness on the part of the policymaker.

In general, adopting a rule that is robust in the Hansen-Sargent sense comes at a cost. Under the approximating model, the expected period loss from adopting robust rules is higher. However, for the three cases considered, even a strong preference for robustness does not dramatically affect the expected period loss calculated under the approximating model. Examining the response coefficients shows that adopting a robust rule implies the central bank should respond more aggressively to the state variables in the model. The dynamics of the model under robustness are examined to illustrate how the increased aggressiveness transmits to the economy.

To illuminate the differences in model dynamics when the policymaker slants their rule against unknown misspecification errors, the dynamics of the model are depicted in figure 2 for three alternative scenarios. The standard case, where the policymaker does not in fact slant their rule against misspecification is depicted with a solid black line. The case where the policymaker slants the rule against unknown errors that do not eventuate, such that the underlining model of the constraint is the approximating model, is depicted with a dashed line. The worst case scenario, where the misspecification errors that are feared by the policymaker eventuate and the evil agent's rule for nature is incorporated into the underlying model of the economy, is depicted with the dotted line.

Firstly, turn to the first row of figure 1 and examine the response of the key macroeconomic variables to an output gap shock. For the standard case, domestic inflation increases relatively sharply initially before gradually returning towards zero after several quarters. The initial increase in domestic inflation is more pronounced under the worst case model and domestic inflation remains substantially higher for a number of periods than under the standard case. This is because the evil agent, aiming to maximise the loss of the central bank, delivers dynamics that increase the persistence of both the output gap shock and domestic inflation. To protect against these feared, misspecification dynamics the policymaker slants their rule. If these misspecification errors do not in fact occur, yet the policymaker uses a rule slanted against feared misspecified dynamics a third permutation arises depicted with a dashed line. Under this scenario, domestic inflation increases initially yet is returned towards target rapidly and actually falls below the path of domestic inflation for the standard case. This is because the robust rules in table 2 are more aggressive in responding to the shocks and key state variables in the economy. In the absence of the behaviour of the evil agent that produces destructive misspecification errors, this rule appears overly aggressive and returns domestic inflation to target particularly quickly.

The behaviour of the output gap following the output gap shock is broadly similar across the three alternative scenarios. After the initial output gap shock, the output gap decreases, falling below zero approximately four to six quarters after the shock. Under the worst case scenario, when the feared misspecification errors occur, the output gap remains below zero for an extended period before returning to its mean. Under the approximating scenario, where the worst-case misspecification dynamics do not occur, the output gap is returned to zero slightly more quickly than the standard case.

Turning to the behaviour of the real exchange rate in response to the output gap shock, there is little discernible difference in the behaviour of the real exchange rate across the three alternative scenarios. Recall that the baseline loss function does not include the real exchange rate. This implies that the evil agent, attempting to maximising the loss of the central bank, only manipulates the dynamics associated with the exchange rate to the extent that this leads to bad outcomes for the variables that enter the loss function. Given a limited effective budget to manipulate the model's dynamics, the evil agent focuses their activities on manipulating the persistence of the process that affect the paths of the key macroeconomic variables that enter the central bank's loss function.

The response of the nominal interest rate is revealing about how the aggressiveness of the robust policy rule begins to translate into the three alternative dynamics structures. Under the standard model, the nominal interest rate ticks up approximately 100 basis points in response to the output gap shock before decreasing close to zero after about eight quarters. The initial response of the nominal interest rate under the approximating and worst case model is stronger — an increase of approximately 120 basis points. Note that the initial increase in the nominal interest rate is identical under both the approximating and worst case models because the policy rule is identical and the misspecification dynamics take time to impact on the paths of the variables. After the initial increase, the nominal interest rate decreases particularly rapidly under the approximating model, passing under the path of the standard model between three and four quarters. Under the worst case model, the machinations of the evil agent results in dynamics that force the nominal interest rate to remain about 25 basis points higher than the approximating model until about six quarters when this implies differential in the interest rate path begins to dissipate.

Figure 2 about here...

Turning to the behaviour of the key macroeconomic variables following a domestic inflation shock, we observe a similar pattern in domestic inflation to its behaviour following the output gap shock with slightly more persistence — inflation increases about 100 basis points before decreasing slowly towards zero after about twelve quarters. Again, the path of domestic inflation under the worst case model shows more persistent deviations than the standard case. The path of domestic inflation falls below the path under the standard case after approximately three quarters, reflecting the more aggressive policy stance of the robust rule.

The magnitude of the output gap in response to the domestic inflation shock differs across the three alternative scenarios. Under the standard model, where the policymaker correctly does not fear model misspecification, the policymaker drives output about 0.35% below trend to help reduce domestic inflation. The negative output gap begins to close after about six quarters and is returned close to zero a full sixteen quarters. Under the worst case model, the combination of the more aggressive policy rule and the adverse dynamics imparted on the model by the action of the "evil agent" result in a particularly large recession in response to the domestic inflation — the output falls almost 0.8% below trend, almost double the output gap that occurs under the standard model. That most of this behaviour is attributable to the more aggressive policy rule can be deduced from the path of the output gap under the approximating model which also shows a marked recession. However, this recession is closed relatively more quickly under the approximating model.

Following the domestic inflation shock, the real exchange rate displays a relatively stronger appreciation under both the approximating and worst case models compared with the standard case. This behaviour can be partly attributed to the nominal interest rate which also increases by a larger magnitude for the approximating and worst case model than the path displayed by the standard model.

Finally, the impulse response functions for the variables in response to a real exchange rate shock are particularly similar. Domestic inflation increases sharply initially but falls equally quickly. The output gap falls initially, increases and eventually converges towards zero. The path of the real exchange rate is very similar across all three models because the evil agent finds it unproductive to manipulate the dynamics of the real exchange rate because it does not enter the loss function of the policymaker under the baseline model.

That the worst case dynamics map into a higher loss for the central bank can be seen in table 9. The table depicts the losses under the approximating and worst case models for a range of preferences on the part of the central bank to protect against worst case outcomes.

Firstly, the table shows that the loss the central bank incurs is always higher when the evil agent is able to implement the worst case dynamics. This is unsurprising because the task the metaphor of the evil agent is used to represent the extent to which the nature of misspecified dynamics can impact negatively on the loss the central bank occurs.

Secondly, we can observe an increase in the loss when the central bank begins

Table 9: Loss Comparison under Robust Policy: Baseline Model

	∞	104.06	80.60	68.13
M_a	12.27	14.45	15.73	17.27
M_w	12.27	15.31	16.89	18.68

to slant their rule against misspecified dynamics — even when the worst case dynamics do not eventuate. This can be observed in the first row of table x where the loss increases up to 40% when the central bank slants their rule against misspecified dynamics that have an error detection probability of 5%.

It's informative to consider the how the dynamics of the model are tweaked by the evil agent to produce worst case dynamics. The reduced form dynamics of the approximating model are depicted below.[8]

$$\begin{bmatrix} \varepsilon_{\pi^d t+1} \\ \varepsilon_{qt+1} \\ \varepsilon_{\tilde{y}t+1} \\ \pi_t^d \\ q_t \\ i_t \\ \pi_{t+1}^f \\ \pi_t^f \\ R_t \end{bmatrix} \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.750 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ -0.065 & -0.019 & 1.102 & 0.869 & -0.045 & -0.224 & -0.015 & 0.003 & -0.220 \\ 1.505 & -0.025 & 0.236 & 0.490 & 1.053 & -0.063 & -0.024 & 0.018 & -0.047 \\ 1.059 & 0.350 & 2.128 & 1.914 & 0.741 & -0.405 & 0.202 & -0.011 & -0.426 \\ 1.124 & -0.549 & 1.194 & 1.180 & 0.787 & -0.395 & 0.323 & 0.168 & -0.239 \\ 0.529 & 0.175 & 1.064 & 0.957 & 0.371 & -0.702 & 0.101 & 0.495 & -0.213 \\ 0.026 & -0.082 & 0.050 & 0.045 & 0.0180 & -0.009 & 0.005 & 0.000 & -0.010 \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi^d t} \\ \varepsilon_{qt} \\ \varepsilon_{\tilde{y}t} \\ \pi_{t-1}^d \\ q_{t-1} \\ i_{t-1} \\ \pi_t^f \\ \pi_{t-1}^f \\ R_{t-1} \end{bmatrix}.$$

Compare these dynamics to the worst case dynamics:

$$\begin{bmatrix} \varepsilon_{\pi^d t+1} \\ \varepsilon_{qt+1} \\ \varepsilon_{\tilde{y}t+1} \\ \pi_t^d \\ q_t \\ i_t \\ \pi_{t+1}^f \\ \pi_t^f \\ R_t \end{bmatrix} \begin{bmatrix} 0.084 & -0.009 & 0.052 & 0.059 & 0.059 & -0.021 & -0.010 & 0.013 & -0.010 \\ -0.004 & 0.752 & -0.004 & -0.004 & -0.003 & 0.001 & 0.002 & -0.001 & 0.001 \\ 0.027 & -0.007 & 0.033 & 0.031 & 0.019 & -0.012 & -0.007 & 0.005 & -0.007 \\ -0.027 & -0.026 & 1.139 & 0.906 & -0.019 & -0.237 & -0.023 & 0.010 & -0.228 \\ 1.564 & -0.031 & 0.273 & 0.531 & 1.095 & -0.078 & -0.031 & 0.027 & -0.055 \\ 1.047 & 0.353 & 2.117 & 1.903 & 0.733 & -0.400 & 0.205 & -0.013 & -0.423 \\ 1.121 & -0.547 & 1.191 & 1.177 & 0.785 & -0.394 & 0.326 & 0.168 & -0.238 \\ 0.542 & 0.174 & 1.074 & 0.968 & 0.380 & -0.707 & 0.100 & 0.497 & -0.215 \\ 0.020 & -0.081 & 0.044 & 0.039 & 0.014 & -0.007 & 0.006 & -0.001 & -0.009 \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi^d t} \\ \varepsilon_{qt} \\ \varepsilon_{\tilde{y}t} \\ \pi_{t-1}^d \\ q_{t-1} \\ i_{t-1} \\ \pi_t^f \\ \pi_{t-1}^f \\ R_{t-1} \end{bmatrix}.$$

The diagonal elements show the autoregressive component of the dynamics. What we observe is increased persistence in the model shocks and increased persistence in the other state variables, domestic inflation, foreign inflation and the real exchange rate. That the evil agent alters the dynamics to produce more persistence in the model shocks and key state variables is a typical finding within the literature.

5.3.3 Alternative Model

Under the alternative model, the signs of the implied response to the state variables and shocks in the model are the same as for the baseline model. There is, however, some marked differences in the magnitude of the responses relative to the baseline model. These differences in magnitude are broadly intuitive — the policymaker responds less to the lag of the real exchange rate and the real exchange rate shock because the affect of the real exchange on the other state variables has been mitigated. Instead, the policymaker responds more aggressively to the lag of domestic inflation, the lag of the output gap and the

[8]These dynamics will differ from the standard model only to the extent that the rule under the approximating model differs from the standard case. These rules are shown in table 8.

domestic inflation and output gap shocks. There is now slightly more interest rate smoothing relative to the baseline model.

Table 10: Optimal and Robust Rules: Alternative Model

Rule	$e_{\pi^d t}$	e_{qt}	$e_{\tilde{y}t}$	$\tilde{y}_{t-1}$	π^d_{t-1}	q_{t-1}	i_{t-1}	π^f_t	R_{t-1}
Baseline	0.90	-0.41	1.15	1.10	0.63	-0.17	0.44	0.06	-0.23
Robust (i)	1.25	-0.44	1.40	1.37	0.87	-0.22	0.42	0.09	-0.28
Robust (ii)	1.43	-0.45	1.52	1.50	1.00	-0.25	0.40	0.10	-0.30
Robust (iii)	1.59	-0.46	1.62	1.62	1.11	-0.27	0.38	0.11	-0.32

NB. Under the baseline rule, there is no preference for robustness and $\theta = \infty$.

As the preference for rules that are robust to model misspecification increases, the policymaker responds more aggressively to the key variables in the standard transmission mechanism, the output gap and domestic inflation, and in addition, responds more aggressively to the real exchange rate and its shock.

That the model of the economy with a reduced exchange rate channel makes policy more difficult can be observed from table x which depicts the losses under the alternative exchange rate channel model. The loss under the standard case is 17.47 — about 40% higher than the baseline model. Similar increases in the loss can be observed when the policymaker slants their rule when they fear, to different degrees, model misspecification.

5.3.4 Alternative Preferences

This subsection examines robust policy rules when then the central bank desires to minimise the volatility of the real exchange rate.

A desire to minimize exchange rate deviations allows the evil agent to engineer worst case outcomes particularly easily by increasing the persistence of the real exchange, inflation and the output gap within the model. Thus the appropriate choices of theta, θ, that correspond to error detection probabilities of 5%, 10% and 20% must be recalibrated by conducting the same trial and error detection exercise outlined earlier in this section. These values are 206.0, 156.2 and 134.5 and the rules associated with these error detection probabilities are presented below.

The baseline rule with a preference for exchange rate stability, where the policymaker does not slant the rule against feared model misspecification, is broadly similar to the baseline rule for the baseline model. The rule suggests that the nominal interest rate should be increased in response to domestic inflation

Table 11: Loss Comparison under Robust Policy: Alternative Model

	∞	156.8	117	100.2
M$_a$	17.47	20.38	22.15	23.74
M$_w$	17.47	21.79	24.12	26.09

Table 12: Optimal and Robust Rules: Alternative Preferences

Rule	$e_{\pi^d t}$	e_{qt}	$e_{\tilde{y}t}$	$\tilde{y}_{t-1}$	π^d_{t-1}	q_{t-1}	i_{t-1}	π^f_t	R_{t-1}
Baseline	0.71	-0.69	0.79	0.77	0.50	-0.32	0.23	0.16	-0.16
Robust (i)	0.87	-0.68	1.03	1.00	0.61	-0.38	0.22	0.18	-0.21
Robust (ii)	0.96	-0.68	1.16	1.12	0.67	-0.40	0.22	0.18	-0.23
Robust (iii)	1.03	-0.67	1.27	1.23	0.72	-0.44	0.22	0.19	-0.25

NB. Under the baseline rule, there is no preference for robustness and $\theta = \infty$.

shocks, output gap shocks, the lag of domestic inflation, the lag of the output gap, foreign good inflation in the consumer price index and the nominal interest. The interest rate should decrease in response to exchange rate shocks, the lag of the real exchange rate and the lag of the real long interest rate. Furthermore, the magnitude of the responses when the policymaker cares about exchange rate volatility are in the same order of magnitude relative to the baseline case. For example, in table 12, according to the baseline rule the implied response to domestic inflation shocks is 0.70 while the comparable coefficient in table x is 0.71. In addition, the response to the output gap shock is 0.94 in the baseline rule and 0.79 for the case of exchange rate stability. Broadly, stressing similarities rather than differences would seem appropriate. That the two rules are so similar underlines the inability of the policymaker to simultaneously reduce volatility in both inflation and the exchange rate and emphasizes the trade-off that is involved.

Table 13: Loss Comparison under Robust Policy: Alternative Preferences

	∞	206.0	156.2	134.5
M$_a$	22.37	27.17	30.31	36.66
M$_w$	22.37	29.33	33.19	33.40

However, there are some differences in comparison of the rules that appear relatively intuitive. The response to the exchange rate shock is more aggressive under the model with a concern for real exchange rate volatility — -0.69 compared to -0.51. In addition there is more interest rate smoothing behaviour under the baseline model. When the policymaker desires less exchange rate volatility the policymaker is forced to move the interest rate in response to shocks and variables that affect the path of the exchange rate.

The robust rules in table 12 suggest that the policymaker should respond 40-60% more aggressively to domestic inflation and output gap shocks and the lags of these variables, if the policymaker is concerned about model misspecification. Interestingly, the response to the exchange rate shock, and the degree of interest rate smoothing remain relatively constant across the range of fears of model misspecification.

Of course, implementing different policy rules implies different model dynamics. Figure 3 shows the dynamics of the model under the standard, approximating and worst case models. For the approximating and worst case models it is assumed that $\theta = 156.2$ so that the policymaker wishes to be robust against models that have a less than 10% probability of error detection.

Firstly turning to the output gap shock, there is more persistence in domestic inflation under the exchange rate concern model because the policymaker cannot lean against domestic inflation pressure so strongly because this affects the real exchange rate. The behaviour of the output gap appear relatively similar under both cases but the increase in the real exchange rate is more pronounced when the policymaker has a concern for the volatility of the real exchange rate. This is particularly true under the approximating and worst case models. These effects are partly driven by the interest rate which shows more persistence under the exchange rate concern model relative to the baseline case. Under the real exchange rate concern model, the persistence in the path of the interest rate is substantially greater under the worst case dynamics than the approximating model. This effect is enhanced in the exchange rate concern model because the evil agent recognizes that persistent deviations of the interest rate from it's mean drive dynamics that affect the additional real exchange rate argument within the loss function.

Secondly, the shock to domestic inflation displays increased persistence (relative to the baseline case) when the policymaker cares about the volatility of the real exchange rate. This is true of the standard, approximating and worst case models, although the persistence if reduced under the approximating model because the worst case misspecification errors do not eventuate. The output gap shows a similar shape to the baseline model across the standard, approximating and worst case models. However, the magnitude of the recession — induced to return inflation to target — is smaller because the persistence displayed in domestic inflation shows the policymaker is less aggressive in returning inflation to target. Because the policymaker is less aggressive with the interest rate response to the domestic inflation shock, the initial increase in the interest rate is not as pronounced relative to the baseline case and the initial appreciation in the real exchange rate is smaller, but more persistent.

The final row of the table depicts the impulse response functions for a real exchange rate shock. The three alternative modelling scenarios, the standard, approximating and worst case models, are tightly bunched for across all four variables. The paths of the variables show slightly smaller movements in a response to the real exchange rate than their counterparts in the baseline model.

Figure 3 about here...

5.4 Robust Simple Rules

Examining robust optimal simple rules present several practical difficulties within the modelling framework in this paper. For each guess of the simple rule we need to solve for the associated rule for the evil agent, the matrix K, that alters the dynamics of the model under the worst-case scenario. This matrix contains 100 parameters, slowing the estimation algorithm considerably.

This in itself is not problematic. However, recall that θ, that parameterized the degree of robustness is determined according to error detection — the worse-case model should not be so different from the baseline model as to be implausible. When the central bank is restricted to use a simple rule, the evil agent is less restricted in terms of the dynamics produced. For a given θ, the worst-case model will be more damaging when the central bank uses a restricted or simple rule. It is a computationally costly exercise for each guess of the simple rule simultaneously: (i) generate the appropriate value of θ;(ii) calculate the dynamics induced by the evil agent; and (iii) calculate the expectation formation process of the private agents in the model; and (iv) calculate the loss under the worst-case model.

For these reasons this section compares an example, a single simple rule (the smoothed Taylor rule that responds to the real exchange and its lag) and its robust counterpart under a very simple assumption about the nature of the model misspecification induced by the evil agent. Specifically, we approximate the misspecification by using the rule the evil agent uses to generate model misspecification for the baseline model under the case of discretion. This represents suboptimal behaviour on the part of the evil agent, using the algorithm to develop results under an optimal response remains for future work. However, restricting the nature of the worst-case specification to be the same under discretion and for optimal rules places should illuminate a simple robust rule and how it operates.

The robust simple rule attempts to minimize the loss under the assumption that the worst case dynamics eventuate. Table 14 below shows that this rule has much less interest rate smoothing than its standard part. Although the response coefficients on the state variables are actually lower in the robust rule, the lack of interest rate smoothing implies the central bank reacts more quickly and more aggressively to the state variables.

This can be observed in the dynamics of the model under the simple robust rule. The rightmost panel in figure 4 depict the response of the nominal interest rate. The interest rate responds more aggressively to inflation and the output gap. The response to the real exchange rate is initially higher under the standard simple rule but the robust simple rule implies a more sustained response to the exchange rate. The responsiveness of the interest rate drives the state variables towards equilibrium particularly rapidly. This is most evidence in the second column of panels which show that the output gap is returned towards zero more quickly under the robust rule.

However, the aggressiveness of the robust rule comes at a price. The loss under the robust rule is about 8% higher if the central bank's fear about model misspecification are unfounded and the baseline model of the economy is in fact the true model of the economy. Thus protection against worst case outcomes generates second-best outcomes if the desire for robust policy is unnecessary. This echoes the findings of the discretionary robust rules.

Figure 4 about here...

Table 14: Simple vs. Simple Robust rule Comparison

	Variances				*Loss*	*worst*
	σ_i	$\sigma_{\tilde{y}}$	σ_q	$\sigma_{\bar{\pi}}$	*fn.*	*case*
Simple rule						
i_t=2.36$\tilde{y}_t$+1.57π_t+0.99i_{t-1}-1.20q_t-0.03q_{t-1}	2.68	1.60	2.74	1.79	12.39	
Robust rule (baseline model, $\theta = 80.6$)						
i_t=1.66$\tilde{y}_t$+1.37π_t+0.46i_{t-1}-0.50q_t-0.15q_{t-1}	2.75	1.70	2.16	2.08	13.40	17.71

6 Conclusion

This paper has analysed two alternative perspectives on model uncertainty — parametric uncertainty and nonparametric uncertainty.

Firstly, a search for a simple rule, robust to a variation in the model of the economy (a less open economy) and robust to a variation in the preferences of the central bank (a concern for exchange rate volatility), was conducted. A smoothed Taylor rule and a smoothed Taylor rule that responds to both the real exchange rate and its lag, proved to yield robust performance across the model and preference variations considered in the paper.

These simple rules also performed well in a Bayesian, parametric framework where the central bank weights each alternative model and preference environment equally. These Bayesian rules showed some policy attenuation relative to the simple rules where the true model is know with certainty. For a small open economy model, calibrated to broadly represent the experiences of three small inflation targeters (Australia, Canada and New Zealand) over the 1990s, Brainard (1967) policy attenuation dominates any uncertainty associated with the persistence of the state variables in the model.

A second approach to model uncertainty, Hansen-Sargent robust control, where the policymaker fears local model uncertainty in the vicinity of a model considered the best estimate of the economy, and slants policy in the face of this uncertainty, was outlined. It transpires that the robust discretionary rules are more aggressive than their counterpart discretionary rules from the case of no model uncertainty. This suggests that the policymaker should accentuate the policy response under the parametric view of uncertainty. When the policymaker desires good rather than optimal outcomes, and wishes to protect against plausible worst case model misspecification, the worst case model misspecification is associated with the persistence of the state variables in the model.

While calculation of optimal robust rules are computationally demanding, a single example appears to suggest accentuating the policy response in a similar manner to the robust rules under discretion. Obtaining internally consistent simple robust rules and comparing these rules to their simple and Bayesian counterparts remains for future work. However, understanding the nature of the uncertainty policymakers face, and fear, appears to be crucial to the question of whether policy should be accentuated or attenuated under uncertainty.

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Figure 2: Impulse Responses: Baseline Model

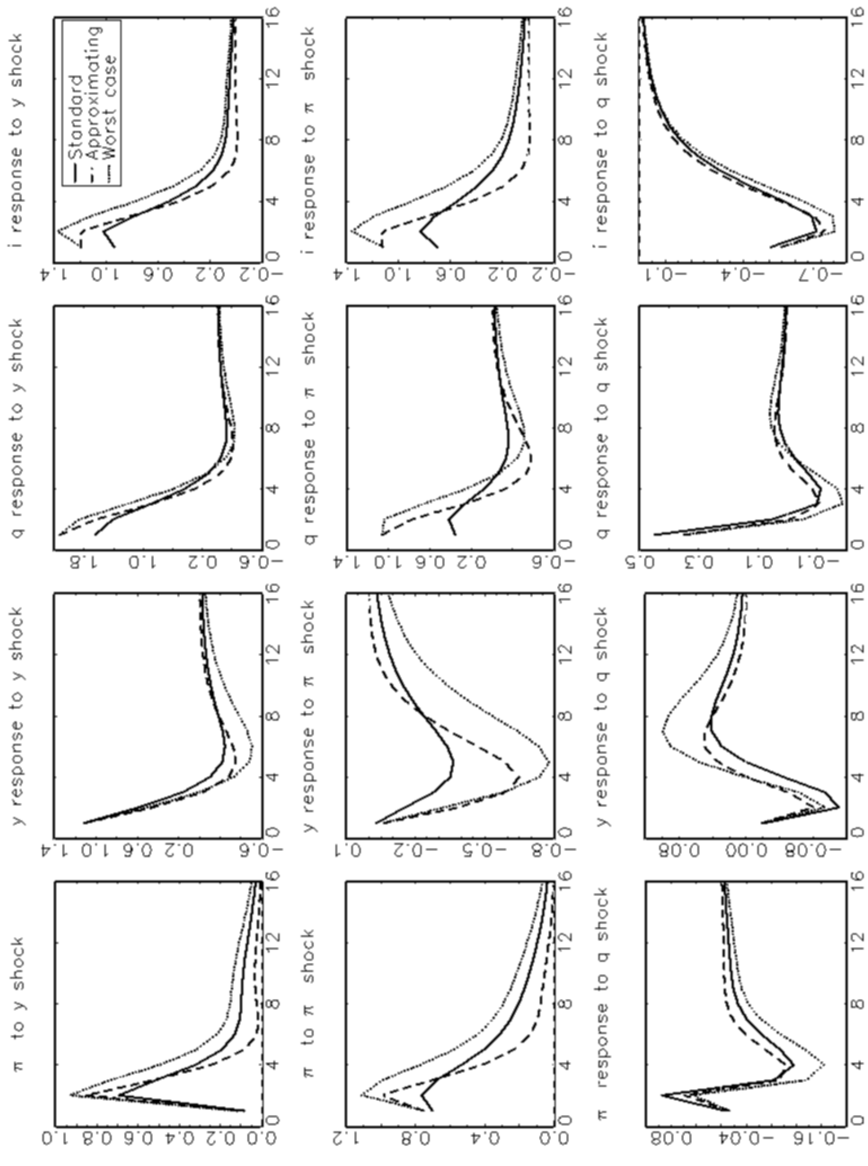

Figure 3: Impulse Responses: Concern for exchange rate volatility

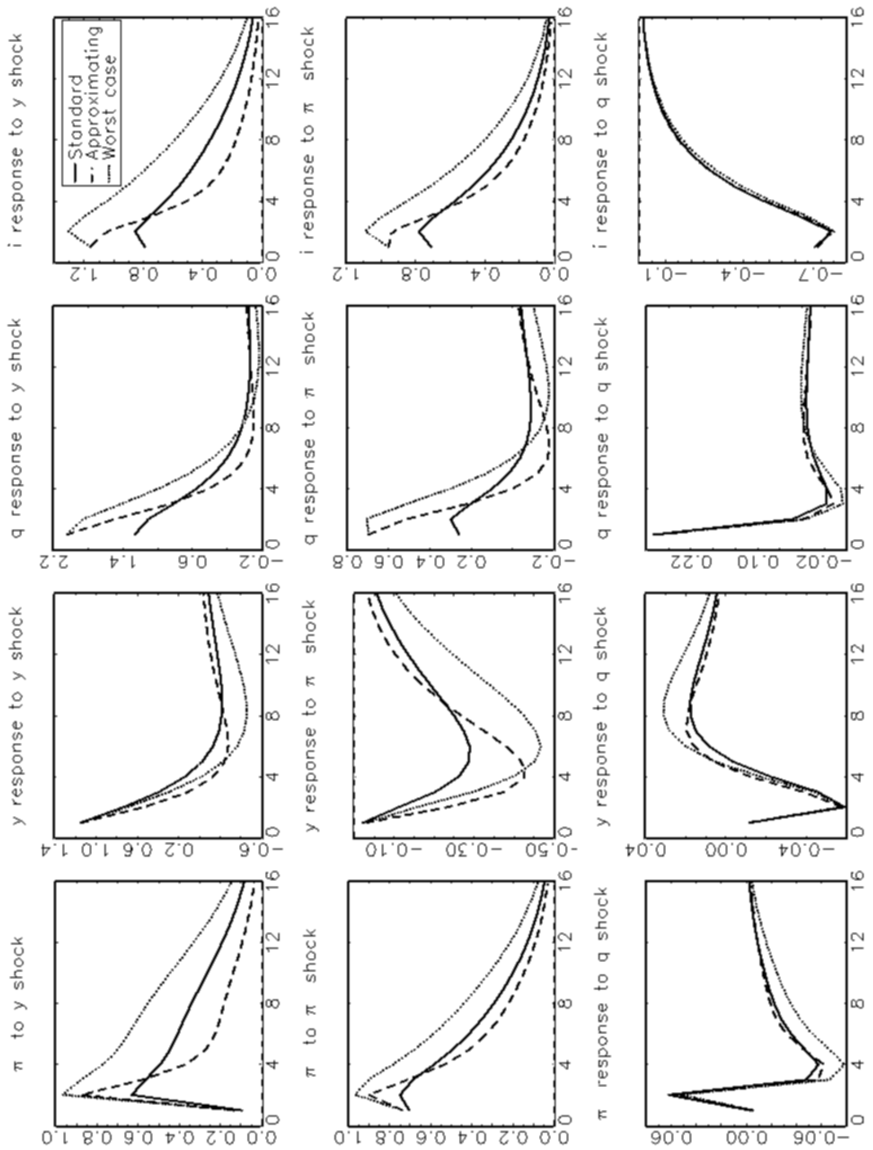

Figure 4: Impulse Response Functions: Simple Rule vs. Robust Simple Rule