Delegated Management in Dynamic Oligopolies

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Abstract

This paper studies the strategic value of delegation in dynamic interactions, where principals provide managers with intertemporal incentives in order to obtain a competitive advantage. While direct management offers intertemporal commitment opportunities, the separation of ownership from production decisions allows precommitment within the current period. The solution concept of Markov-perfect equilibrium helps avoid the imposition of exogenous restrictions on the composition and the functional form of compensation contracts. The linear-quadratic game yields a tractable MPE that illustrates the properties of dynamic delegation: i) with low adjustment costs and discount factors delegating principals are able to attain output levels close to those of a Stackelberg leader; ii) managerial utility parameters affect equilibrium wages, but have no impact on production choices.

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1 Introduction

It is a well known fact that the internal organization of the firm and the implied structure of corporate decision making have direct and strategic repercussions for market conduct. The adoption of an internal hierarchy establishes rules that govern complicated interactions among decision makers with different objectives. The synchronization of those objectives with profit maximization is usually achieved via appropriate contract design.

The choice of the firm's internal structure depends on the properties of the operating environment. It is determined by the availability and the allocation of information, as well as by strategic considerations related to market competition. There exists a huge body of work on intra-firm vertical relations under asymmetric information. In classical agency models (e.g. Mirrlees (1976), Harris and Raviv (1979), Holmstrom (1979)) owners delegate decision making to informed agents and devise contracts to mitigate internal incongruences. However, the trade-off between risk insurance and performance incentives gives rise to inefficiencies which cannot be resolved by contracting.

The strategic delegation literature offers a different perspective on the principal-agent relationship. In these models delegation does not arise exogenously, but emerges as an equilibrium phenomenon. When firms operate in a market environment, the separation of ownership from management serves as a commitment tool that provides a strategic advantage over existing competitors. Principals can use contracts to invoke production decisions that mimic Stackelberg leadership and thus enjoy the benefits of precommitment.

A common problem of delegation theory is that the set of potential contracts is too broad, which creates indeterminacies and reduces predictive power. This necessitates the imposition of various constraints on the principals' strategy sets in order to pin down the equilibrium of the delegation game. These restrictions are usually related to the following modelling aspects:

- **The selection of performance measures**. Managerial compensation can be based on a variety of market indicators. The literature considers a number of contract types where remuneration is tied to sales (Sklivas (1987), Fershtman and Judd (1987)), to future profits (Petkov (2003)), to the opponent's performance (Miller and Pazgal (2001)), etc. These models account for somewhat different production incentives and consequently their equilibria diverge.

- **The functional form of compensation contracts**. Usually the literature focuses on contracts that are linear in the adopted performance measures, which greatly simplifies the analysis. However, Basu (1993) demonstrates that this restriction is not innocuous. He relaxes the linearity assumption in a Fershtman-Judd framework and shows that with general contract functions any outcome can be supported as a subgame-perfect equilibrium of the delegation game.

The present paper takes a dynamic approach to resolve the above indeterminacy issues. I consider an infinite-horizon model in which technological specificities (e.g. adjustment costs) create a payoff link between successive periods of interaction. In this environment the principals can i) engage in direct management by choosing their own output levels, or ii) offer management contracts to agents who experience disutility from production-related effort, but expect to be rewarded in the future according to their current performance.

The analysis focuses on the Markov-perfect equilibrium of the delegation game. Thus, the only restriction on the composition of compensation contracts is history independence. Wages are tied to the natural industry state, which eliminates any ambiguities regarding the choice of performance measures. I do not make any ad hoc assumptions about the functional form of compensation contracts. It is determined in equilibrium by payoff maximization and rational expectations. The stationarity of Markov perfect strategies ensures that future renegotiations will not lead to contract alterations. While the structure of the model has to include specification of the agents' utility functions, their parameters do not affect equilibrium output decisions. This affirms the generality of the obtained results.

The dynamic infinite-horizon setup provides an accurate representation of common real-life situations in which managerial pay is based on past performance. It highlights the role of expectations and reputation concerns in managerial decision making. The intertemporal aspect of the principal-agent relationship provides important insights for the time profile of compensation and the impact of market structure on wage dynamics.

The remainder of the paper is organized as follows: Section 2 outlines the structure of the model: payoffs, timing of activities and the equilibrium concept. Next I consider the benchmark case of delegation within a single firm. The equilibrium conditions show that with costless delegation the principal is able to attain his first-best production path. Moreover, when managerial utility is additively separable, the compensation structure does not depend on demand and technology parameters. A closed-form solution for the equilibrium contract is provided for the case of linear-quadratic payoffs.

Section 4 analyzes delegation in duopolistic competition. It compares three alternative regimes of decision making: i) bilateral direct management, ii) unilateral delegation, and iii) bilateral delegation. While direct management offers intertemporal commitment opportunities, the separation of ownership from production decisions allows precommitment within the current period. The paper derives and interprets the optimality conditions that determine the behavior of the market participants. In the case of linear quadratic payoffs I characterize a tractable MPE involving linear strategies. The computations demonstrate the superiority of delegation: with low adjustment costs and discount factors it permits the realization of production levels similar to those of a Stackelberg leader. Thus, during the up-front adoption of corporate internal structure at least one principal will choose to delegate.

The paper concludes with a brief summary of the main results and suggestions for future research.

2 Setup

2.1 Industry Structure

Consider an industry whose structure is characterized by an $(m+n)$-firm dynamic oligopoly. Let $\mathbf{x}^t = (x_1^t, x_2^t, ..., x_{m+n}^t)$ denote the period$-t$ output vector[2]. The production technology entails adjustment costs, thus in period t firm j's profits also depend on $\mathbf{x}^{t-1}$: $\pi_j^t = \pi_j(\mathbf{x}^t, \mathbf{x}^{t-1})$, where π_j is concave in x_j^t. All players have a common discount factor δ.

The firms' owners (also referred to as the principals) care about remaining lifetime profits $\Pi_j^t = \sum\limits_{\tau=t}^{\infty} \delta^{\tau-t}\pi_j^{\tau-t}$. Without loss of generality assume that the first m principals are involved in direct production management. That is, they choose their own output levels. The remaining n principals delegate decision making to proxies (also called agents or managers), who expect to receive future compensation w_j^{t+1} for choosing current output levels x_j^t.

The instantaneous utility of firm j's agent is $u(w_j^t, x_j^t)$, where u is increasing and concave in w_j^t. Managerial payoffs depends on the output vector because of disutility of effort associated with production $\left(\dfrac{\partial u_j}{\partial x_j^t} < 0\right)$. The agent's objective is maximization of his remaining lifetime utility $U_j^t = \sum\limits_{\tau=t}^{\infty} \delta^{\tau-t} u(w_j^{\tau-t}, x_j^{\tau-t})$.

The purpose of this paper is to analyze the strategic value of delegation in market interactions. In order to avoid complications related to profit sharing inside the firm, I assume that agents' compensation is small relative to instantaneous profits. Thus, wages affect the owners' payoffs only indirectly through the managerial production decisions.

Suppose that neither party can precommit to future policies and owners have the freedom to adjust compensation in each period. This implies that managers will internalize the effect of their current decisions on future wages, which in turn makes them responsive to intertemporal production incentives.

[2]Bold fonts signify vector notation

2.2 Timing

In period 0 the principals simultaneously choose between two corporate structures: i) direct management (unintermediated choice of output decisions) and ii) delegated management (production decisions are entrusted to agents).

From period 1 on the timing of activities is as follows:

1. Delegating principals simultaneously choose wage offers w_j^t to maximize their remaining lifetime profits $\Pi_j^t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_j(\mathbf{x}^{\tau-t}, \mathbf{x}_j^{\tau-t-1})$.
2. Agents and managing principals simultaneously choose output levels. The agents' objective is maximization of remaining lifetime utility $U_j^t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(w_j^{\tau-t}, x^{\tau-t})$, while the managing principals maximize lifetime profits $\Pi_j^t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_j(\mathbf{x}^{\tau-t}, \mathbf{x}_j^{\tau-t-1})$.

2.3 Solution Concept

The analysis employs the solution concept of subgame-perfect equilibrium. It accounts for the agents' production incentives: forward-looking managers are willing to incur effort disutility in exchange for higher future rewards. The paper focuses on a particular class of subgame-perfect equilibria, namely the Markov-perfect equilibrium of the delegation game: I restrict strategies to depend only on the current state as perceived by the different players, ignoring past history of interactions.

Consider the events in an arbitrary period t:

- The delegating principals are the first movers. From their viewpoint the state of the industry can be summarized by the previous period's output vector $\mathbf{x}^{t-1} = (\mathbf{y}_m, \mathbf{y}_n) = (y_1, y_2, ..., y_{m+n})$. The Markov-perfect equilibrium strategy of delegating principal $j \in \{m+1, .., m+n\}$ is a contract function h_j which maps the previous period's production vector into managerial compensation w_j: $w_j = h_j(\mathbf{y}_m, \mathbf{y}_n)$. For notational purposes let us define the strategy vectors $\mathbf{h}(\mathbf{y}) = (h_{m+1}(\mathbf{y}), h_{m+2}(\mathbf{y}), ..., h_{m+n}(\mathbf{y}))$ and $\mathbf{h}_j(\mathbf{y}, w_j) = (h_{m+1}(\mathbf{y}), ..., h_{j-1}(\mathbf{y})\ , w_j, h_{j+1}(\mathbf{y}),\ ..., h_{m+n}(\mathbf{y}))$.

- The agents and the managing principals are the second movers. They take the current wage vector $\mathbf{w}_n = (w_{m+1}, w_{m+2}, ..., w_{m+n})$ as given. Thus, their perceived industry state is characterized by the output/wage vector $(\mathbf{y}_m, \mathbf{w}_n)$.
  - Let the Markov-perfect equilibrium strategy of agent j be a function g_j^t which maps the state into a current output decision x_j: $x_j = g_j^t(\mathbf{y}_m, \mathbf{w}_n)$. Let us define the strategy vectors $\mathbf{g}(\mathbf{y}_m, \mathbf{w}_n) = (g_{m+1}(\mathbf{y}_m, \mathbf{w}_n), ..., g_{m+n}(\mathbf{y}_m, \mathbf{w}_n))$ and $\mathbf{g}_j(\mathbf{y}_m, \mathbf{w}_n, x_j) = (g_{m+1}(\mathbf{y}_m, \mathbf{w}_n), ..., g_{j-1}(\mathbf{y}_m, \mathbf{w}_n)\ , x_j, g_{j+1}(\mathbf{y}_m, \mathbf{w}_n), ..., g_{m+n}(\mathbf{y}_m, \mathbf{w}_n))$.
  - Similarly, let the Markov perfect equilibrium strategy of managing principal $i \in \{1, .., m\}$ be a function $f_i : (\mathbf{y}_m, \mathbf{w}_n) \to x_i$. Define the strategy vectors $\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n) = (f_1(\mathbf{y}_m, \mathbf{w}_n), ..., f_n(\mathbf{y}_m, \mathbf{w}_n))$ and $\mathbf{f}_i(\mathbf{y}_m, \mathbf{w}_n) = (f_1(\mathbf{y}_m, \mathbf{w}_n), ..., f_{i-1}(\mathbf{y}_m, \mathbf{w}_n),\ x_i,\ f_{i+1}(\mathbf{y}_m, \mathbf{w}_n), ..., f_n(\mathbf{y}_m, \mathbf{w}_n))$.

First, consider the problem of the delegating principals. They take into account the effect of their choices on managerial behavior later in that period. Thus, in their decision problems they incorporate the agents' equilibrium strategies.

Let $V_j(\mathbf{y})$ be the value function of delegating principal $j \in \{m+1, .., m+n\}$. Since in each period he chooses his strategy optimally, his agent's wage satisfies the Bellman equation:

$$V_j(\mathbf{y}) = \max_{w_j}\{\pi(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), g(\mathbf{y}_m, \mathbf{h}_j(\mathbf{y}, w_j)), \mathbf{y})) + \delta V_j(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{g}(\mathbf{y}_m, \mathbf{h}_j(\mathbf{y}, w_j)\} \quad (1)$$

By assumption h_j is the equilibrium strategy of principal j, therefore it solves

$$h_j(\mathbf{y}) = \arg\max_{w_j}\{\pi(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), g(\mathbf{y}_m, \mathbf{h}_j(\mathbf{y}, w_j)), \mathbf{y})) + \delta V_j(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{g}(\mathbf{y}_m, \mathbf{h}_j(\mathbf{y}, w_j)\} \quad (2)$$

Now adopt the viewpoint of the firm j's manager. Let $W_j(\mathbf{y}_m, \mathbf{w}_n)$ be his value of encountering an output/wage vector $(\mathbf{y}_m, \mathbf{w}_n)$. In equilibrium the agent has correct expectations regarding the future reward for his current effort. Optimality implies that output will satisfy his Bellman equation:

$$W_j(\mathbf{y}_m, \mathbf{w}_n) = \max_{x_j}\{u_j(w_j, x_j) + \delta W_j(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{h}(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{g}_j(\mathbf{y}_m, \mathbf{w}_n, x_j))\} \tag{3}$$

By assumption g_j is the optimal strategy of agent j, therefore it solves

$$g_j(\mathbf{y}_m, \mathbf{w}_n) = \arg\max_{x_j}\{u_j(w_j, x_j) + \delta W_j(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{h}(\mathbf{f}(\mathbf{y}_m, \mathbf{w}_n), \mathbf{g}_j(\mathbf{y}_m, \mathbf{w}_n, x_j))\} \tag{4}$$

Finally, consider the problem of managing principal $i \in \{1, .., n\}$. Let $\Omega_i(\mathbf{y}_m, \mathbf{w}_n)$ be his value function. His output choice solves the Bellman equation

$$\Omega_i(\mathbf{y}_m, \mathbf{w}_n) = \max_{x_i}\{\pi(\mathbf{x}_n, g(\mathbf{y}_m, \mathbf{w}_n), \mathbf{y})) + \delta\Omega_i(\mathbf{f}_i(\mathbf{y}_m, \mathbf{w}_n, x_i), \mathbf{h}(\mathbf{f_i}(\mathbf{y_m}, \mathbf{w_n}, x_i), \mathbf{g}(\mathbf{y}_m, \mathbf{w}_n))\} \tag{5}$$

Since his equilibrium strategy is f_i, it must be true that

$$f_i(\mathbf{y}_m, \mathbf{w}_n) = \arg\max_{x_i}\{\pi(\mathbf{x}_n, g(\mathbf{y}_m, \mathbf{w}_n), \mathbf{y})) + \delta\Omega_i(\mathbf{f}_i(\mathbf{y}_m, \mathbf{w}_n, x_i), \mathbf{h}(\mathbf{f}_i(\mathbf{y}_m, \mathbf{w}_n, x_i), \mathbf{g}(\mathbf{y}_m, \mathbf{w}_n))\} \tag{6}$$

Definition 1 *The Markov-perfect equilibrium of the delegation game described above consists of i) value functions* $V_j(\mathbf{y}), W_j(\mathbf{y}_m, \mathbf{w}_n)$, $\Omega_i(\mathbf{y}_m, \mathbf{w}_n)$ *which satisfy Bellman equations respectively (1),(3),(5); and ii) strategy functions* $h_j(\mathbf{y}), g_j(\mathbf{y}_m, \mathbf{w}_n), f_i(\mathbf{y}_m, \mathbf{w}_n)$ *which are a fixed point of the mapping defined by (2),(4),(6).*

In order to simplify the notation, I use superscripts to denote partial derivatives: $\psi^j(z_1, z_2, ..., z_k) = \dfrac{\partial \psi}{\partial z_i}$. Also, let w', x' and w'', x'' be the compensation and production vectors respectively one and two periods ahead.

3 Delegation within a Single Firm

First consider wage formation when the firm does not face external competition. Thus, all strategic considerations are internal. The principal designs a compensation contract that motivates the manager to implement the sustainable output sequence which maximizes lifetime profits.

3.1 Equilibrium Conditions

Appendix A derives necessary conditions for the equilibrium strategies of the single-firm delegation game.

Proposition 2 *The Markov-perfect equilibrium of the single-firm delegation game satisfies equations*

$$u^2(w,x) + \delta h^1(x) u^1(w',x') = 0 \tag{7}$$

and

$$\pi^1(x,y) + \delta \pi^2(x',x) = 0 \tag{8}$$

Equation (8) alone determines the output sequence. Given the equilibrium production plan, (7) provides a difference-differential equation for the equilibrium contract function.

Note that condition (8) is identical to the firm's Euler equation in the absence of delegation: a principal who chooses his own output would follow the same production plan. Therefore, delegation still enables the owner to attain his first-best payoff. The manager has no strategic control over future production. This result does not depend on the specification of the instantaneous utility. It follows from the assumption that pay design is costless for the principal and wages affect profits only indirectly through the agent's output decisions.

Condition (7) reflects the manager's intertemporal trade-off: when determining current output, he internalizes the effect on the next period's wage. If utility is additively separable (e.g. $u(w,x) = \eta(w) + \mu(x)$) then (7) is sufficient to pin down the compensation contract $h(x)$.

This implies that the contract structure will depend only the agent's preference parameters and not on demand or the production technology.

3.2 Example: Linear-Quadratic Payoffs

The derivation of the optimal output path is a standard dynamic programming problem and is therefore ignored. The analysis will focus on determining the wage dynamics.

Suppose that the agent's utility is linear-quadratic and additively separable:

$$u(w,x) = Pw - (Q/2)w^2 - Rx + (S/2)x^2$$

We conjecture that the principal's equilibrium strategy is linear in the state variable: $w' = e + dx$. After substitution in (8) we get

$$-R + Sx + \delta d(P - Qw') = 0$$

Solving for w' yields

$$w' = \frac{S}{\delta dQ}x + \frac{-R + \delta fP}{\delta dQ}$$

The conjecture regarding the wage structure is correct if

$$d = \frac{S}{\delta dQ}, \;\; e = \frac{-R + \delta fP}{\delta dQ}$$

I restrict the analysis to paths along which the agent's utility is increasing in his wage, but decreasing in output. Equation (7) implies that d is positive: the manager will produce more today only if he expects a higher reward tomorrow. Thus, in equilibrium

$$d = \sqrt{\frac{S}{\delta Q}}, \;\; e = \frac{-R + \delta P\sqrt{\frac{S}{\delta Q}}}{\sqrt{\frac{S}{\delta Q}}}$$

The piece rate d depends on the discount factor and the ratio of the second derivatives of the utility function. If the marginal disutility of production falls faster than the marginal utility of income, the owner will design a wage contract that is more sensitive to changes in output.

4 Dynamic Duopoly

Now suppose that firms operate in a market environment: their profits depend also on the decisions of their opponents. In such oligopolistic interactions the separation of ownership from decision making can provide principals with a strategic advantage.

If firms compete in strategic substitutes, the ability to credibly commit to aggressive production will discourage opponents and thus will increase profits. Firms endowed with dynamic technologies can use current output as an intertemporal commitment tool to strategically affect future choices. For example, if production involves adjustment costs, managing principals will increase current output to make future downward adjustments costlier.

On the other hand, the separation of ownership from production decisions provides delegating principals with an instantaneous (intratemporal) commitment tool. By leaving decision-making to outside agents whose compensation contracts are designed to motivate aggressive management, owners can affect output choices *within the current period.*

The dynamic nature of the model allows us to avoid imposing ad hoc restrictions on wages which are common in the strategic delegation literature. Instead, contract functions emerge as equilibrium strategies. They are pinned down by the players' payoff maximizing behavior and rational expectations.

4.1 Equilibrium Conditions

4.1.1 Direct Management Duopoly

First consider the benchmark case when owners do not delegate output decisions and compete directly with each other. This is a standard problem of dynamic oligopoly theory. The Markov-perfect strategies $f_1(y_1, y_2), f_2(y_1, y_2)$ satisfy the well-known necessary conditions:

$$\pi_1^1(x,y) + \delta\pi_1^3(x',x) + f_2^1(x)[\delta\pi_1^2(x',x) + \delta^2\pi_1^4(x'',x')] - Z_1\left[\delta\pi_1^1(x',x) + \delta^2\pi_1^3(x'',x')\right] = 0 \quad (9)$$

$$\pi_2^2(x,y) + \delta\pi_2^4(x',x) + f_1^2(x)[\delta\pi_2^1(x',x) + \delta^2\pi_2^3(x'',x')] - Z_2\left[\delta\pi_2^2(x',x) + \delta^2\pi_2^4(x'',x')\right] = 0 \quad (10)$$

where $Z_1 = \dfrac{f_2^2(x')f_2^1(x)}{f_2^1(x')}$, $Z_2 = \dfrac{f_1^1(x')f_1^2(x)}{f_1^2(x')}$

The interpretation of (9), (10) is as follows: a marginal increase in the output of firm $i \in \{1,2\}$ has a direct effect on lifetime payoff through current profits and next period's adjustment costs. The deviation also induces a reaction from the opponent in the subsequent period, which affects payoffs through strategic substitutability. Firm i correctly anticipates the behavior of its competitor and re-adjusts output simultaneously with the opponent's reaction.

4.1.2 Delegated Management Duopoly

Outside management has two opposite effects on current output relative to the no-delegation equilibrium:

- Since the benefit of commitment is enjoyed immediately and not delayed in the future, the value of commitment goes up. This motivates delegating principals to target higher production levels
- Output no longer serves as an intertemporal commitment device, because future production decisions are determined by future wages. If aggressive production cannot

provide a strategic advantage in subsequent interactions, owners will revise downward current output targets.

In principle agents have the potential to affect duopoly production paths by using the opponent firm as a "leverage". However, with linear-quadratic payoffs numerical computations show that manager-induced output effects are negligible.

Unilateral Delegation In order to establish that delegation is an equilibrium phenomenon, it is necessary to show its superiority relative to direct management: when facing conventional competition, principals prefer entrusting output decisions to an outside agent. Thus, first we need to analyze the unilateral delegation game in which only one firm divests ownership from management.

Proposition 3 *Consider the unilateral delegation game in which the principal of firm 2 delegates production decisions to an outside agent. Their Markov perfect equilibrium strategies satisfy respectively*

$$\begin{gathered} g_2^2(y_1,w_2)\{\pi_2^2(x,y)+\delta\pi_2^4(x',x)\}+f_1^2(y_1,w_2)\{\pi_2^1(x,y)+\delta\pi_2^3(x',x)\} \\ -Zf_1^2(y_1,w_2)\{\delta\pi_2^2(x',x)+\delta^2\pi_2^4(x'',x')\}=0 \end{gathered} \tag{11}$$

where

$$Z=\left\{\frac{g_2^2(x_1,w_2')[f_1^2(x_1,w_2')h_2^1(x)+f_1^1(x_1,w_2')]}{f_1^2(x_1,w_2')}-[g_2^2(x_1,w_2')h_2^1(x)+g_2^1(x_1,w_2')]\right\}$$

and

$$\begin{aligned} &\{u_2^2(w_2,x_2)+\delta h_2^2(x)u_2^1(w_2',x_2')\}-\frac{h_2^2(x)f_1^2(x_1,w_2')h_2^1(x')}{h_2^2(x')}\delta u_2^2(w_2',x_2') \\ =\;&\frac{h_2^2(x)f_1^2(x_1,w_2')f_1^1(x_1',w_2'')}{f_1^2(x_1',w_2'')h_2^2(x')}\left\{\delta u_2^2(w_2',x_2')+\delta^2h_2^2(x')u_2^1(w_2'',x_2'')\right\} \end{aligned} \tag{12}$$

The optimality condition of the managing principal of firm 1 is

$$\begin{aligned} &\pi_1^1(x,y)+\delta\pi_1^3(x',x)+[h_2^1(x)g_2^2(x_1,w_2')+g_2^1(x_1,w_2')]\delta\pi_1^2(x',x) \\ = &\frac{[h_2^1(x)g_2^2(x_1,w_2')+g_2^1(x_1,w_2')]h_2^2(x')g_2^2(x_1',w_2'')}{[h_2^1(x')g_2^2(x_1',w_2'')+g_2^1(x_1',w_2'')]}[\delta\pi_1^1(x',x)+\delta^2\pi_1^3(x'',x')] \end{aligned} \tag{13}$$

Proof. See Appendix B ■

Condition (11) incorporates the effects of the ownership/management dichotomy: the principal influences his own output levels indirectly through his wage decisions. A comparison with (10) highlights the role of delegation as an instantaneous commitment device. The first-mover advantage implies that aggressive wage contracts will invoke a reaction from the opponent within the same period, forcing him to surrender market share.

Equation (12) describes the agent's intertemporal trade-off. In addition to the intra-firm strategic considerations, the manager takes into account the effect of his decisions on the opponent firm and its consequences for future wages and output.

Finally, condition (13) characterizes the managing principal's production choices. A marginal deviation from his equilibrium output level will trigger future responses from both the opponent principal and his agent.

Bilateral Delegation If delegation provides strategic benefits, it might be the case that multiple firms will take advantage of it. Suppose that both principals decide to divest ownership from management.

Proposition 4 *In the symmetric bilateral delegation game the Markov-perfect equilibrium strategies of the principal and the agent of firm 1 satisfy respectively*

$$\begin{aligned} &g_1^1(w)\{\pi_1^1(x,y)+\delta\pi_1^3(x',x)-\delta h_2^1(x)[g_1^1(w')-g_1^2(w')](\pi_1^1(x',x)-\pi_1^2(x',x))\}+ \\ &g_2^1(w)\{\pi_1^2(x,y)+\delta\pi_1^4(x',x)-\delta h_1^1(x)[g_1^1(w')-g_1^2(w')](\pi_1^1(x',x)-\pi_1^2(x',x))\}=0 \end{aligned} \tag{14}$$

and

$$u_1^2(w_1, x_1) + \delta h_1^1(x)(u_1^1(w_1', x_1') - \delta[h_1^1(x)g_1^2(w') + h_2^1(x)(g_1^1(w')]u_1^2(w_1', x_1')] = 0 \qquad (15)$$

Similarly, the equilibrium Markov-perfect strategies of the principal and the agent of firm 2 satisfy conditions

$$\begin{aligned} &g_2^2(w)\{\pi_2^2(x,y) + \delta\pi_2^4(x',x) - \delta h_1^2(x)[g_2^2(w') - g_2^1(w')](\pi_2^2(x',x) - \pi_2^1(x',x))]\}+ \\ &g_1^2(w)\{\pi_2^1(x,y) + \delta\pi_2^3(x',x) - \delta h_2^2(x)[g_2^2(w') - g_2^1(w')](\pi_2^2(x',x) - \pi_2^1(x',x))]\} = 0 \end{aligned} \qquad (16)$$

and

$$u_2^2(w_2, x_2) + \delta h_2^2(x)(u_2^1(w_2', x_2') - \delta[h_2^2(x)g_2^1(w') + h_1^2(x)(g_2^2(w')]u_2^2(w_2', x_2')] = 0 \qquad (17)$$

Proof. See Appendix C ■

Each player takes into consideration the effect of his decisions on delegation relations within the firm, as well as on the behavior of the principal/agent of the opponent firm.

Equations (14), (16) show that when principals determine the compensation of their agents, they account for: i) the effect on the current output decisions of both managers and their consequences for lifetime profits (direct and through adjustment costs); and ii) the impact on the opponent principal and the implications for future management.

The agents' equilibrium conditions (15), (17) suggest that output decisions weigh effort disutility against i) the future reward provided by the manager's own principal; and ii) the reaction of the opponent principal in the next period and its consequences for managerial utility.

4.2 Linear-Quadratic Payoffs

Suppose that firms operate in an industry characterized by a linear inverse industry demand $p^t = A - X^t$. The available technology is such that producer i incurs a constant instantaneous unit cost c and a quadratic adjustment cost $\frac{\psi}{2}(x_i^t - x_i^{t-1})^2$. Therefore, his instantaneous profit is given by

$$\pi_i(x_1, x_2, y_1, y_2) = (A - x_1 - x_2 - c)x_i - \frac{\psi}{2}(x_i - y_i)^2$$

Note that $\dfrac{\partial^2 \pi_i}{\partial x_1 \partial x_2} < 0$, thus firms compete in strategic substitutes.

The principals can delegate output decisions to outside agents. There is a pool of potential managers with instantaneous utility functions

$$u(w_i, x_i) = Pw_i - (Q/2)w_i^2 - Rx_i + (S/2)x_i^2$$

Again, the analysis is restricted to equilibrium paths along which agents dislike effort $\left(\dfrac{\partial u_i}{\partial x_i} < 0\right)$ and enjoy monetary rewards $\left(\dfrac{\partial u_i}{\partial w_i} > 0\right)$.

The linear-quadratic specification delivers a tractable Markov-perfect equilibrium involving strategies that are linear in the respective state variables. We conjecture that under direct management output levels are given by $x_i^{t+1} = r^{dm} + s_1^{dm} x_i^t + s_2^{dm} x_{-i}^t$. Similarly, with unilateral delegation the managing principal's strategy function is $x_1^{t+1} = r^{ud} + s_1^{ud} x_1^t + s_2^{ud} w_2^{t+1}$, the delegating principal sets a wage contract $w_2^{t+1} = e^{ud} + d_1^{ud} x_2^t + d_2^{ud} x_1^t$, while the agent chooses output $x_2^{t+1} = a^{ud} + b_2^{ud} x_1^t + b_1^{ud} w_2^{t+1}$. Finally, when both firms engage in delegation the principals set wages according to $w_i^{t+1} = e^{bd} + d_1^{bd} x_i^t + d_2^{bd} x_{-i}^t$ and managers' output strategies are given by $x_i^t = a^{ud} + b_1^{bd} w_i^t + b_2^{bd} w_{-i}^t$.

Substituting those conjectures in the Euler equations give us necessary conditions for the equilibrium strategy parameters. Table 1 ($A - c = 1000$) provides numerical examples of the steady-state values of output, firm profits and wages for duopolies with i) direct management; ii) unilateral delegation; and iii) bilateral delegation.

Table 1: Duopoly Steady State Under Direct Management and Delegated Management

		$\delta = 0.5$ $\psi = 0.1$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$\delta = 0.5$ $\psi = 0.1$ $Q = 0.03$ $S = 0.01$ $P = 80$ $R = 8$	$\delta = 0.8$ $\psi = 0.1$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$\delta = 0.5$ $\psi = 0.4$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$\delta = 0.8$ $\psi = 0.4$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$
No	$\hat{x}^{nd}$	407.00	407.00	455.57	405.34	453.32
Delegation	$\hat{\pi}^{nd}$	75,697	75,697	40,481	76,735	42,316
	$\hat{x}^{d}$	486.92	486.92	486.01	462.70	462.66
Unilateral	$\hat{\pi}^{d}$	123,643	123,643	122,892	120,979	119,665
Delegation	$\hat{x}^{nd}$	259.15	259.15	261.13	275,84	278.69
	$\hat{\pi}^{nd}$	65,807	65,807	66,030	72,122	72,082
	$\hat{w}$	1,976	2,351	1,981	1,921	1,933
Bilateral	$\hat{x}^{d}$	438.09	438.09	436.35	408.23	408.03
Delegation	$\hat{\pi}^{d}$	54,243	54,243	55,546	74,928	75,053
	$\hat{w}^{d}$	1,942	2,358	1,952	1,913	1,929

4.2.1 Firm Output

Direct management competition exhibits the standard properties of dynamic games in strategic substitutes. The use of current output as an intertemporal commitment tool implies that production will exceed the equilibrium levels of a static duopoly with no adjustment costs.

Output is increasing in the player's discount factor, since patient firms assign a higher value to future profits and are willing to incur higher commitment costs. A rise in ψ increases both the effectiveness and the cost of output as a commitment tool for each of the market competitors. The net effect is a fall in steady state output.

It is immediately obvious that unilateral delegation offers a strong advantage over opponent managing principals. The delegating firm is able to attain steady state output close to that of a Stackelberg leader in a static duopoly with no adjustment costs. The outcome reflects the relative commitment value of delegation. For example, intertemporal commitment becomes more important if the player is patient. Thus, a rise in the common discount factor will reduce the market share of the delegating firm and increase that of its competitor.

Similarly, a rise ψ increases the effectiveness of intertemporal commitment, which in turn boosts the output of the managing principal.

Bilateral delegation creates instantaneous commitment opportunities for both firms and also generates output levels that exceed equilibrium production of the static game. The immediate nature of this commitment implies that the discount factor does not significantly affect its value. Thus, for high enough δ output is below that of a direct management duopoly.

The computations show that managerial utility parameters have little effect on output levels and profits. Therefore, agents have no strategic control over future production.

4.2.2 Compensation Contracts

Table 2 provides numerical examples of equilibrium contract functions under unilateral and bilateral delegation ($c = 0$).

Table 2: Equilibrium Contract Functions

		$A = 1000$ $\delta = 0.5$ $\psi = 0.1$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$A = 1000$ $\delta = 0.5$ $\psi = 0.1$ $Q = 0.03$ $S = 0.01$ $P = 80$ $R = 8$	$A = 1000$ $\delta = 0.8$ $\psi = 0.1$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$A = 1000$ $\delta = 0.5$ $\psi = 0.4$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$	$A = 1500$ $\delta = 0.5$ $\psi = 0.1$ $Q = 0.05$ $S = 0.02$ $P = 100$ $R = 10$
Unilateral Delegation	e^{ud}	1,823.502	1,930.578	1,863.400	1,834.831	2,003.651
	d_1^{ud}	0.730	0.666	0.576	0.725	0.730
	d_2^{ud}	-0.781	-0.708	-0.624	-0.906	-0.781
Bilateral Delegation	e^{bd}	1,550.854	2,001.014	1,952.157	1,548.028	1,559.68
	d_1^{bd}	0.897	0.819	0.710	0.903	0.897
	d_2^{bd}	-0.004	-0.004	-0.004	-0.008	-0.004

Again, since agents experience effort disutility, they will increase current output only if this raises expected monetary reward in the subsequent period: $d_1^{ud} > 0, d_1^{bd} > 0$. Strategic substitutability implies $d_2^{ud} < 0, d_2^{bd} < 0$.

Wages are more sensitive to changes in competitor's activity under unilateral delegation: $|d_2^{ud}| > |d_2^{bd}|$. The intuition is that output commits the managing principal to a pattern of future behavior. In particular, higher output levels in the past translate into more aggressive current production. Strategic substitutability induces the delegating principal to target a smaller market share by reducing his agent's future wage.

Unlike the monopoly case, competition creates a link between demand and the structure of compensation contracts. For any given state (y_1, y_2) a rise in $A - c$ implies higher wages. A bigger market size increases the benefit of commitment. Thus, delegating principals would want to encourage more aggressive management by raising remuneration.

4.2.3 Delegation as an Equilibrium Choice

During the adoption of corporate structure in period 0 the principals take into account the effect of delegation on market competition. Thus, they will hire managers if this increases lifetime payoffs.

Table 3 shows the lifetime profits of firm 1 conditional on the period-0 delegation decisions for the numerical examples of Table 1 and initial conditions $x_1^0 = x_2^0 = 0$.

Table 3: The Period 0 Game

		2 N	2 D
1	N	75,785	68,220
1	D	121,003	56,367

		2 N	2 D
1	N	75,785	68,220
1	D	121,003	56,367

		2 N	2 D
1	N	163,521	267,942
1	D	487,400	225,422

		2 N	2 D
1	N	76,701	77,256
1	D	113,286	77,418

		2 N	2 D
1	N	174,507	296,711
1	D	463,972	304,166

The analysis demonstrated that the instantaneous commitment opportunities of delegation offer a significant advantage over intertemporal commitment. The delegating principal is able to boost output levels relative to opponents who engage in direct management. Thus, in the above examples at least one principal will decide to delegate production decisions. Moreover, in the last two cases delegation becomes a dominant strategy, so the outcome will involve bilateral delegation.

5 Conclusion

This paper explores the dynamic aspects of the link between competition and the internal organization of corporate decision making. In market interactions where the ability to precommit is beneficial, principals can obtain a strategic advantage by delegating production decisions to motivated outside agents.

The analysis focuses on the delegation of decision making in dynamic monopolistic and duopolistic market environments. While direct management offers means for intertemporal commitment, the separation of ownership from output decisions ensures precommitment within the current period. The intertemporal nature of managerial production incentives prevents the indeterminacies arising in the standard two-period setup: payoff maximization and rational expectations pin down the functional form of Markov-perfect compensation contracts. The MPE of the linear-quadratic game provides numerical examples to illustrate the advantages of delegation, which are particularly strong with low adjustment costs and discount factor. Although managerial preference parameters affect equilibrium compensation, they have no impact on output decisions.

The paper can be extended in several directions. In order to simplify the analysis, I distinguish between two simple forms of internal decision making. A more realistic setup would account for the complicated structure of vertical and horizontal relationships within the corporation, including sub-delegation and divisionalization While the assumption of linear-quadratic payoffs allows a simple closed-form solution, it might be instructive to consider other functional forms in order to verify the generality of the obtained results.

Appendix A. MPE of Delegation in a Single Firm

In each period the agent chooses output optimally. Therefore, he solves the following Bellman equation

$$W(w) = \max_x \{u(w, x) + \delta W(h(x))\}$$

The first-order condition implies that

$$W^1(w') = -\frac{u^2(w, x)}{\delta h^1(x)}$$

Since $g(w)$ is the agent's equilibrium strategy, it must be true that

$$W(w) = u(w, g(w)) + \delta W(h(g(w)))$$

Differentiating with respect to w yields

$$W^1(w) = u^1(w, x) + u^2(w, x)g^1(w) + \delta W^1(w')h^1(x)g^1(w) \tag{18}$$

Substituting the first-order condition in (18) we get (7).

Similarly, the principal's Bellman equation is

$$V(y) = \max_w \{\pi(g(w), y) + \delta V(g(w))\}$$

The optimal wage satisfies the first-order condition

$$V^1(x') = -\frac{\pi^1(x, y)}{\delta}$$

Since the principal's equilibrium strategy is $h(x)$, we have that

$$V(y) = \pi(g(h(y)), y) + \delta V(g(h(y)))$$

Differentiating with respect to y yields

$$V^1(y) = \pi^1(x, y)g^1(w)h^1(x) + \pi^2(x, y) + \delta V^1(x')g^1(w)h^1(x) \tag{19}$$

Substituting the first-order condition in (19) and shifting it one period ahead gives us (8).

Appendix B. Unilateral Delegation

Suppose that the principal of firm 1 makes his own production decisions, while the principal of firm 2 delegates decision-making to an outside agent.

The Delegating Principal

The Bellman equation of the delegating principal of firm 2 is as follows:

$$V_2(y_1, y_2) = \max_{w2}\{\pi_2(f_1(y_1, w_2), g_2(y_1, w_2), y_1, y_2) + \delta V_2(f_1(y_1, w_2), g_2(y_1, w_2))\}$$

The corresponding first-order condition is

$$\pi_2^1(x, y)f_1^2(y_1, w_2) + \pi_2^2(x, y)g_2^2(y_1, w_2) + \delta f_1^2(y_1, w_2)V_2^1(x) + \delta g_2^2(y_1, w_2)V_2^2(x) = 0$$

The envelope condition with respect to w_2 can be written as

$$\begin{aligned} V_2^2(y) &= h_2^2(y)[\pi_2^1(x, y)f_1^2(y_1, w_2) + \pi_2^2(x, y)g_2^2(y_1, w_2)] + \pi_2^4(x, y) + \\ &\quad h_2^2(y)\delta[f_1^2(y_1, w_2)V_2^1(x) + g_2^2(y_1, w_2)V_2^2(x)] \end{aligned}$$

Substitution of the first-order condition yields one of the partial derivatives of the value function

$$V_2^2(y) = \pi_2^4(x, y) \tag{20}$$

The other partial derivative is given by

$$V_2^1(x) = -\frac{1}{\delta f_1^2(y_1, w_2)}\{\pi_2^1(x, y) f_1^2(y_1, w_2) + \pi_2^2(x, y) g_2^2(y_1, w_2) + \delta g_2^2(y_1, w_2)\pi_2^4(x', x)\} \tag{21}$$

The envelope condition with respect to y_1 is

$$\begin{aligned} V_2^1(y) &= \pi_2^1(x, y)[f_1^2(y_1, w_2)h_2^1(y) + f_1^1(y_1, w_2)] + \pi_2^2(x, y)[g_2^2(y_1, w_2)h_2^1(y) + g_2^1(y_1, w_2)] + \pi_2^3(x, y) \\ & \quad \delta[f_1^2(y_1, w_2)h_2^1(y) + f_1^1(y_1, w_2)]V_2^1(x) + \delta[g_2^2(y_1, w_2)h_2^1(y) + g_2^1(y_1, w_2)]V_2^2(x) \end{aligned}$$

Substituting (20) and (21) yields the Euler equation (11).

The Agent

The agent of firm 2 takes the output/wage vector (y_1, w_2) as given. His Bellman equation is as follows:

$$W_2(y_1, w_2) = \max_{x_2}\{u_2(w_2, x_2) + \delta W_2(f_1(y_1, w_2), h_2(f_1(y_1, w_2), x_2))\}$$

The corresponding first-order condition can be written as

$$W_2^2(x_1, w_2') = -\frac{u_2^2(w_2, x_2)}{\delta h_2^2(x)}$$

The envelope condition for w_2 is

$$\begin{aligned} W_2^2(y_1, w_2) &= u_2^1(w_2, x_2) + g_2^2(y_1, w_2) u_2^2(w_2, x_2) + \\ &\quad \delta f_1^2(w_1, y_2) W_2^1(x_1, w_2') + \delta[h_2^2(x) g_1^1(y_1, w_2) + h_2^1(x) f_1^2(y_1, w_2)] W_2^2(x_1, w_2') \end{aligned}$$

Substituting $W_2^2(x_1, w_2')$ gives us

$$W_2^1(x_1', w_2'') = -\frac{1}{\delta f_1^2(x_1, w_2')} \left\{ \frac{u_2^2(w_2, x_2)}{\delta h_2^2(x)} + u_2^1(w_2', x_2') \right\} + \frac{h_2^1(x')}{\delta h_2^2(x')} u_2^2(w_2', x_2')$$

The envelope condition for y_1 is

$$\begin{aligned} W_2^1(y_1, w_2) &= g_2^1(y_1, w_2) u_2^2(w_2, x_2) + \\ &\quad \delta f_1^1(y_1, w_2)] W_2^1(x_1, w_2') + \delta[h_2^2(x) g_2^1(y_1, w_2) + h_2^1(x) f_1^1(y_1, w_2)] W_2^2(x_1, w_2') \end{aligned}$$

Substituting $W_2^1(x_1, w_2')$ and $W_2^2(x_1, w_2')$ yields the Euler equation (12) of the agent.

The Managing Principal

Now consider the problem of the managing principal of firm 1. He solves the following Bellman equation

$$\Omega_1(y_1, w_2) = \max_{x_1} \{\pi_2(x_1, g_2(w_1, y_2), y_1, y_2) + \delta V_2(x_1, h_2(x_1, g_2(w_1, y_2)))\}$$

The first-order condition implies that

$$\Omega_1^1(x_1, w_2') + h_2^1(x) \Omega_1^2(x_1, w_2') = -\frac{\pi_1^1(x, y)}{\delta}$$

From the envelope condition for y_2 and the first order condition, we get

$$\Omega_1^1(y_1, w_2) = g_2^1(y_1, w_2)\pi_1^2(x, y) + \pi_1^3(x, y) + \delta h_2^2(x) g_2^1(y_1, w_2)\Omega_1^2(x_1, w_2') \tag{22}$$

Similarly, using the envelope condition for w_2 and the first order condition gives us

$$\Omega_1^2(y_1, w_2) = g_2^2(y_1, w_2)\pi_1^2(x, y) + \delta h_2^2(x) g_2^2(y_1, w_2)\Omega_1^2(x_1, w_2') \tag{23}$$

After multiplying (23) by $h_1^2(y)$ and adding it to (22) we get

$$\Omega_1^2(x_1', w_2'') = -\frac{\pi_1^1(x, y) + \delta\pi_1^3(x', x)}{\delta^2[h_2^1(x)g_2^2(x_1, w_2') + g_2^1(x_1, w_2')]h_2^2(x')} - \frac{\pi_1^2(x', x)}{\delta h_2^2(x')}$$

Substituting in the first-order condition, we get

$$\Omega_1^1(x_1', w_2'') = -\frac{\pi_1^1(x', x)}{\delta} + \frac{h_2^1(x')[\pi_1^1(x, y) + \delta\pi_1^3(x', x)]}{\delta^2[h_2^1(x)g_2^2(x_1, w_2') + g_2^1(x_1, w_2')]h_2^2(x')} + \frac{h_2^1(x')\pi_1^2(x', x)}{\delta h_2^2(x')}$$

Plugging $\Omega_1^2(x_1', w_2'')$ in (23) yields the Euler equation (13) of the managing principal..

Appendix C. Bilateral Delegation

Consider the problems of the principal and the agent of firm 1. Symmetry implies that reciprocal conditions will hold for firm 2.

The Agents' Problem

The agent's Bellman equation is as follows:

$$W_1(w_1, w_2) = \max_{x_1}\{u_1(w_1, x_1) + \delta W_1(h_1(x_1, g(w_1, w_2)), h_2(x_1, g(w_1, w_2)))\}$$

The corresponding first-order condition is

$$u_1^2(w_1, x) + \delta W_1^1(w')h_1^1(x_1, x_2) + \delta W_1^2(w')h_2^1(x_1, x_2) = 0$$

Therefore,

$$\delta W_1^1(w')h_1^1(x) + \delta W_1^2(w')h_2^1(x) = -u_1^2(w_1, x)$$

Since $g_1(w_1, w_2)$ and $g_2(w_1, w_2)$ are equilibrium strategies, they satisfy

$$\begin{aligned} W_1(w_1, w_2) &= u_1(w_1, g_1(w_1, w_2)) + \\ &+\delta W_1(h_1(g_1(w_1, w_2), g_2(w_1, w_2)), h_2(g_1(w_1, w_2), g_2(w_1, w_2))) \end{aligned} \tag{24}$$

Differentiation with respect to w_1 gives us

$$\begin{aligned} W_1^1(w) &= u_1^1(w_1, x) + u_1^2(w_1, x)g_1^1(w) \\ &+\delta W_1^1(w')[h_1^1(x')g_1^1(w) + h_1^2(x')g_2^1(w)] + \delta W_1^2(w')[h_2^1(x')g_1^1(w) + h_2^2(x')g_2^1(w)] \end{aligned} \tag{25}$$

Differentiation with respect to w_2 yields

$$\begin{aligned} W_1^2(w) &= u_1^2(w_1, x)g_1^2(w) \\ &+\delta W_1^1(w')[h_1^1(x')g_1^2(w) + h_1^2(x')g_2^2(w)] + \delta W_1^2(w')[h_2^1(x')g_1^2(w) + h_2^2(x')g_2^2(w)] \end{aligned} \tag{26}$$

Since the game is symmetric, it must be true that $h_1^2(x) = h_2^1(x)$, $h_1^1(x) = h_2^2(x), g_1^2(x) = g_2^1(x)$, $g_1^1(x) = g_2^2(x)$. Substituting the first-order condition in (25) and (26) gives us respectively

$$W_1^1(w) = -g_1^2(w)u^2(w_1, x) + u^1(w_1, x)$$

and

$$W_1^2(w) = -g_2^2(w)u^2(w_1, x)$$

After shifting those expressions one period ahead and plugging them in the first order condition, we get (15)

The Principals' Problem

The principal's Bellman equation is

$$V_1(y_1, y_2) = \max_{w_1}\{\pi_1(g_1(w_1, w_2), g_2(w_1, w_2), y_1, y_2) + \delta V_1(g_1(w_1, w_2), g_2(w_1, w_2))\}$$

The corresponding first-order condition is

$$\pi_1^1(x_1, y)g_1^1(w) + \pi_1^2(x, y)g_2^1(w) + \delta V_1^1(x)g_1^1(w) + \delta V_1^2(x)g_2^1(w) = 0$$

Thus,

$$\delta V_1^1(x)g_1^1(w) + \delta V_1^2(x)g_2^1(w) = -(\pi_1^1(x, y)g_1^1(w) + \pi_1^2(x, y)g_2^1(w))$$

Since $h_1(y_1, y_2), h_1(y_1, y_2)$ are the equilibrium strategies, we can write the Bellman equation as

$$\begin{aligned} V_1(y_1, y_2) \quad &= \quad \pi_1(g_1(h_1(y_1, y_2), h_2(y_1, y_2)), g_2(h_1(y_1, y_2), h_2(y_1, y_2))), y_1, y_2) + \\ &\quad +\delta V_1(g_1(h_1(y_1, y_2), h_2(y_1, y_2)), g_2(h_1(y_1, y_2), h_2(y_1, y_2))) \end{aligned}$$

Differentiating with respect to y_1 gives us

$$\begin{aligned} V_1^1(y) \quad &= \quad \pi_1^1(x, y)[g_1^1(w)h_1^1(y) + g_1^2(w)h_2^1(y)] + \pi_1^2(x, y)[g_2^1(w)h_1^1(y) + g_2^2(w)h_2^1(y)] + \pi_1^3(x, y) \\ &\quad +\delta V_1^1(x)[g_1^1(w)h_1^1(y) + g_1^2(w)h_2^1(y)] + \delta V_1^2(x')[g_2^1(w)h_1^1(y) + g_2^2(w)h_2^1(y)] \end{aligned}$$

Similarly, differentiating with respect to y_2 gives us

$$\begin{aligned} V_1^2(y) &= \pi_1^1(x,y)[g_1^1(w)h_1^2(y) + g_1^2(w)h_2^2(y)] + \pi_1^2(x,y)[g_2^1(w)h_1^2(y) + g_2^2(w)h_2^2(y)] + \pi_1^4(x,y) \\ &+\delta V_1^1(x)[g_1^1(w)h_1^2(y) + g_1^2(w)h_2^2(y)] + \delta V_1^2(x)[g_2^1(w)h_1^2(y) + g_2^2(w)h_2^2(y)] \end{aligned}$$

Symmetry and substitution of the first-order condition yield respectively

$$V_1^1(y) = -h_2^1(y)[g_1^1(w) - g_1^2(w)](\pi_1^1(x,y) - \pi_1^2(x,y)) + \pi_1^3(x,y)$$

and

$$V_1^2(y) = -h_2^2(y)[g_1^1(w) - g_1^2(w)](\pi_1^1(x,y) - \pi_1^2(x,y)) + \pi_1^4(x,y)$$

Finally, substituting the derivatives of the principal's value function in the first-order condition gives us (14).

Applying the same methodology to the problems of the principal and the agent of firm 2 yields conditions (17) and (16).

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