

# Network Externalities, Demand Inertia, and Dynamic Pricing in an Experimental Oligopoly Market

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## Abstract

This paper analyses dynamic pricing in markets with network externalities. Network externalities imply demand inertia, because the size of a network increases the usefulness of the product for consumers. Since past sales increase current demand firms have an incentive to set low introductory prices in order to increase prices as their networks grow. However, in reality we observe decreasing prices. This could be due to other factors dominating the network effects. We use an experimental duopoly market with demand inertia to isolate the effect of network externalities. We find that experimental prices are rather consistent with real world observations than with theoretical predictions.

Keywords: Network Externalities, Demand Inertia, Experiments, Oligopoly

JEL-Codes: L13, C92

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# 1 Introduction

For many commodities the individual utility of consumption depends on how many other people consume the commodity, too. In their seminal paper Katz and Shapiro (1985) refer to this phenomenon as network externalities. They point out that the reason for this dependence may be due to direct physical effects or due to indirect effects which they refer to as consumption externalities. Examples for the direct physical effect are communication networks like telephone or E-mail where the usefulness of having access obviously increases with the number of people you can communicate with. The classic example for indirect effects are computer operating systems where the number of people using it determines how many applications are written for the system, which in turn determines how useful the OS is for the consumer. Another prominent example is the market for video game consoles, where the number and the quality of games developed for a system determines the usefulness of the system to consumers.

The existence of network externalities has crucial effects on conduct in and performance of markets. Issues such as compatibility, co-ordination to technical standards and effects on pricing and quality of services create challenges for economic theory (Economides, 1996). There is still some discussion about how significant network externalities are in producing market failures.<sup>1</sup> However, the literature that explores adoption of technologies (Belleflamme, 1998; Kristiansen, 1996), standards and lock-in of technologies (Witt, 1997; De Bijl and Goyal, 1995), compatibility issues (Baake and Boom, 2001), and product introduction (Katz and Shapiro, 1992) is very extensive. We focus on another less researched aspect of network externalities: dynamic pricing under demand inertia.

In a market where at least two competing networks coexist, which are substitutes for consumers, current network size is correlated with future demand. The larger a network is today the higher is the demand tomorrow. Consequently, future demand is positively correlated with sales today. Network externalities imply demand inertia. An example for coexisting networks is the market for game consoles. Currently there are three non-compatible competing systems: Sony Playstation2,

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<sup>1</sup>Katz and Shapiro (1994) argue that network externalities have a high significance, while in the same journal and volume Liebowitz and Margolis (1994) make their case for network externalities being hardly a reason for market inefficiencies.

Microsoft Xbox, and Nintendo GameCube (see Schilling 2003 for an analysis of the game console market). Coexistence of at least two standards is the rule in the game console market since the late eighties (Sega Genesis and Nintendo SNES until 1994, Sony Playstation and Nintendo 64 from 1996 to 1999, and the three currently competing systems since 2001).

Demand inertia due to network externalities *ceteris paribus* puts pressure on competing firms to introduce their products with very low prices in order to increase the size of their network quickly. Cabral, Salant and Woroch (1999) explore the conditions necessary for a low introductory price being optimal for a monopolist operating under network externalities. We show that a low introductory price is also optimal in a duopoly with competing networks. For a monopolist it is optimal to increase its price over time (Bensaid and Lesne, 1996). The same is true in our duopoly model. And in fact the introductory price of the Xbox in November 2001 was quite low (US \$299) and exactly matched the price of the Playstation2. Estimates suggest that Microsoft lost between US \$100 and \$125 per unit sold.<sup>2</sup> However, contrary to the theoretical prediction the prices for Xbox and Playstation2 did not increase, but dropped further (Xbox: US \$ 149.99 on March 29, 2004; Playstation2: US \$149 on May 4, 2004). Price cuts by one of the two firms are usually countered by a subsequent equivalent price cuts of the competitor.

There are many reasons why firms in reality may decrease prices over time: Intertemporal price discrimination or reduced costs due to learning by doing or scale economies are examples. The decreasing prices may be easily explained if these forces dominate the incentives created by demand inertia. However, due to the multiple effects at work it not possible to evaluate the effects of networks externalities alone by only looking at observed pricing behaviour. We use a laboratory experiment to separate the effect demand inertia has from other effects. By eliminating all other factors that may play a role in dynamic pricing we can be sure that the observed effect is due to demand inertia or due to idiosyncrasies of oligopoly markets. Experimental oligopoly markets typically show a certain degree of collusion not explained by game theory. In order to separate the network effect from ideosyncratic collusion in a repeated oligopoly we run a control treatment

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<sup>2</sup>See Schilling (2003), p 16.

with an identical market, but without demand inertia.

Reinhard Selten (1965) was the first to develop a comprehensive model of oligopolistic competition with demand inertia. Our model is similar in the way that demand inertia exists. However, in Selten's model there is no direct strategic interaction in any particular period as the period payoff does not depend on the prices of the competitors. The interaction is only indirect as the future demand is influenced by past price differences. Keser (1993, 2000) implements Selten's model in the laboratory and compares the observed play with the equilibrium prediction. She is mainly interested in categorizing different patterns of behaviour. She is less interested in evaluating the effect demand inertia has. This is difficult in her design due to the lack of immediate interaction, which does not allow for a control treatment where firms compete under the same conditions, but without demand inertia.<sup>3</sup> The ability to do so is crucial for our research question, because we want to compare markets with demand inertia created by network externalities with markets that do not have network characteristics, but are identical otherwise.

The remainder of the paper is structured as follows. In the next section we present our model. In the appendix we show how the demand function used can be derived from simple consumption decisions for goods with network externalities. Section 3 derives some equilibrium predictions for the model. Section 4 describes the experimental setup, while section 5 reports the main results. We conclude in section 6.

## 2 The model

In this section we develop a bare-bone model of a market with network externalities. We reduce this market to its essentials and eliminate any other factor that could have an influence on dynamic pricing. We use a multi-period Bertrand duopoly with differentiated products. Market demand in each period is perfectly inelastic with a total market demand of  $a$  per period.<sup>4</sup> The market has a lifetime

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<sup>3</sup>Another problem for the isolation of demand-inertia effects is the inclusion of interest payments on early periods to simulate discounting. The effects of this design and the inertia can not easily be separated.

<sup>4</sup>This rather restrictive assumption is not crucial for the qualitative results of the model, but will prove very useful for the identification of treatment effects in the experiment.

of  $T$  periods. Network externalities are captured by a state variable  $s_i^t$  - the share of past sales in the industry - which positively influences the individual period demand. The share of past sales is defined as

$$s_i^t := \frac{\sum_{k=1}^{t-1} q_i^k}{[t-1]a} \text{ for } t > 1 \quad (1)$$

where  $i \in \{1, 2\}$  denotes the firm,  $t$  gives the actual period, and  $q_i^k$  is the quantity sold in period  $k$  by firm  $i$ . Note that  $s_i^1$  cannot be defined by the expression above. So we need an initial condition which may reflect initial beliefs of the consumers about the quality of firms' products. Reputation, product reviews, and advertising may play a role.

The period sales of firm  $i$  are defined as

$$q_i^t := \max\{s_i^t a + p_j^t - p_i^t, 0\} \quad i, j \in \{1, 2\}, i \neq j \quad (2)$$

where  $p_j^t$  and  $p_i^t$  are prices. Both firms have the same degree of market power stemming from the consumers' preferences for the different goods varieties. Differences in market power at time  $t$  only arise from different market shares  $s_i^t$ , which only depend on past sales. The market shares are capturing the relative size of the network. We chose to link the benefits from the size of the network rather to the market share than to the absolute past sales for two reasons. Firstly, the marginal benefit of today's sales for tomorrow's market power is decreasing in the past sales. This reflects that the marginal benefit for the consumers from increases in network size is believed to be decreasing. Secondly, new consumers who decide in period  $t$  which brand to buy will put less weight on the nominal differences in network sizes if both networks are already large.

The current and future demand functions are common knowledge. So the two firms simultaneously choose prices  $p_i^t, p_j^t$  in each period  $t$  after having learned the market outcome of the previous period  $t - 1$ . They are fully aware of how the current sales will influence the future market power.

We show in the appendix that the demand functions above can be derived from consumer decisions for goods with network externalities similar to the framework used in Katz and Shapiro (1985).

### 3 Some equilibrium predictions

In this section we will establish some equilibrium predictions. We will see that in spite of the simple structure of the model solving for the full equilibrium path is impractical. Therefore we will establish some qualitative results only. Later on we will use a computer algorithm to solve numerically for the equilibrium prices for the parameters used in the experiment.

This extensive-form game has many Nash Equilibria. However, to rule out equilibria that contain empty threats we add the requirement of subgame-perfectness. We therefore use backward induction and begin with the final period. We have to determine the optimal actions in the final period for both firms and all possible histories. All payoff relevant history is captured by the market share. Therefore, the firms at period  $T$  will maximize the period payoff for a given market share. The payoffs are given:

$$\Pi_i^T := p_i^T [as_i^T - p_i^T + p_j^T] \quad i, j \in \{1, 2\}, i \neq j$$

Then the first-order conditions give the following best response functions:<sup>5</sup>

$$b_i(p_j^T) = \frac{as_i^T + p_j^T}{2} \quad i, j \in \{1, 2\}, i \neq j, \quad (3)$$

which gives the optimal prices:<sup>6</sup>

$$p_i^{T*} = \frac{a[1 + s_i^T]}{3} \quad (4)$$

$$p_j^{T*} = \frac{a[2 - s_i^T]}{3} \quad (5)$$

We now turn to the penultimate period. At period  $T - 1$  the firms foresee what will happen in the last period depending on the prices they set in period  $T - 1$ . Put differently, arriving at period  $T - 1$  the firms know that their prices in  $T - 1$  will cause certain period outputs. They also know how these period outputs will influence their market share in period  $T$ . As they anticipate already how they and

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<sup>5</sup>The second-order conditions are obviously satisfied.

<sup>6</sup>Note that  $s_j^T = 1 - s_i^T$ .

their competitors will behave in the last period for market shares, they can will set their prices such that the sum of the profits in periods  $T - 1$  and  $T$  will be maximized. Firm  $i$ 's aggregate profit is given by

$$\Pi_i^{T-1} + \Pi_i^T := \sum_{l=T-1}^T p_i^l [as_i^l - p_i^l + p_j^l].$$

Using the anticipated equilibrium prices  $p_i^{T*}$  and  $p_j^{T*}$  for the last period we get

$$\Pi_i^{T-1} + \Pi_i^T = p_i^{T-1} [as_i^{T-1} - p_i^{T-1} + p_j^{T-1}] + \left[ \frac{a(1 + s_i^T)}{3} \right]^2. \quad (6)$$

Recall the definition of the market share and write  $s_i^T$  as a function of  $q_i^{T-1}$  and  $s_i^{T-1}$ :

$$s_i^T = \left[ \sum_{l=T-2}^T q_i^l + q_i^{T-1} \right] / a[T-1]$$

Using the demand definition from (2) for  $q_i^{T-1}$  and simplifying leads to

$$s_i^T = s_i^{T-1} + \frac{p_j^{T-1} - p_i^{T-1}}{a[T-1]}$$

Replacing  $s_i^T$  in equation (6) and simplifying gives an expression for the aggregate profit, which includes the anticipated behaviour in the last period and which only depends on the prices in period  $T - 1$ :

$$\begin{aligned} \Pi_i^{T-1} + \Pi_i^T &= p_i^{T-1} [as_i^{T-1} - p_i^{T-1} + p_j^{T-1}] \\ &+ \left[ \frac{p_j^{T-1} - p_i^{T-1} + a [1 + s_i^{T-1}] [T-1]}{3[T-1]} \right]^2 \end{aligned} \quad (7)$$

We see that the anticipated profit for the last period (the second line of 7) depends negatively on the price chosen in period  $T - 1$ . The first order condition is given by

$$-2p_i^{T-1} + p_j^{T-1} + as_i^{T-1} - \frac{2 [p_j^{T-1} - p_i^{T-1} + a [1 + s_i^{T-1}] [T-1]]}{9 [T-1]^2} = 0,$$

which gives to the best response function:

$$b_i(p_j^{T-1}) = \frac{p_j^{T-1} [1 - \omega] + a s_i^{T-1} - a [1 + s_i^{T-1}] [T - 1] \omega}{2 - \omega}, \quad (8)$$

where

$$\omega = \frac{2}{9 [T - 1]^2}.$$

Note that the reaction function taking into account the profit in the last period differs from the reaction function a myopic firm would have by  $\omega$  only. The myopic reaction function can be obtained by setting  $\omega = 0$ . To see this take (8) and set  $\omega = 0$  and compare it to the reaction function for the last period (3) where firms play myopically. They are identical up to the subscript of the market share.

Inspection of (8) shows that firm  $i$  in equilibrium will set a price lower than the myopic price  $p_i^m(s_i^{T-1})$  in period  $T - 1$ .

**Proposition 1** *In every subgame perfect equilibrium we have  $p_i^{T-1*} < p_i^m(s_i^{T-1})$  and  $p_j^{T-1*} < p_j^m(s_i^{T-1})$ .*

**Proof.** Denote the best response functions for myopic players depending on the current market share in  $T - 1$  as  $b_i^m(p_j^{T-1})$  and  $b_j^m(p_i^{T-1})$ , respectively. If we can show that  $b_i^m(p_j^{T-1}) < b_i(p_j^{T-1})$  and  $b_j^m(p_i^{T-1}) < b_j(p_i^{T-1})$  hold for all  $s_i^{T-1}$  we can conclude that our claim is true, since all best response functions are obviously non-decreasing in the opponents price. As  $b_i^m(p_j^{T-1}) = b_i(p_j^{T-1})$  for  $\omega = 0$  and  $\omega > 0$  we must have  $b_i^m(p_j^{T-1}) > b_i(p_j^{T-1})$  if  $\partial b_i(p_j^{T-1})/\partial \omega < 0$  for all  $\omega$ ,  $s_i^{T-1}$ , and  $p_j^{T-1}$ . Differentiating (8) and simplifying gives

$$\frac{\partial b_i(p_j^{T-1})}{\partial \omega} = \frac{-p_j^{T-1} + a [2 - 2T + s_i^{T-1} [3 - 2T]]}{[2 - \omega]^2} < 0$$

for  $T \geq 2$ . Since the best response function of firm  $j$  is obtained by swapping indices only, the same holds for firm  $j$ . ■

The next step is now to show that the equilibrium prices are smaller in  $T - 1$  than in  $T$  independent of the initial market share and the duration of the market  $T$ . This is conceptually easy but tedious and does not create new insights. Therefore we sketch the proof in the appendix only.



**Proposition 2** *In every subgame perfect equilibrium we have  $p_i^{T-1*} \leq p_i^T$  and  $p_j^{T-1*} \leq p_j^T \forall s_i^{T-1} \in [0, 1]$ .*

**Proof.** See appendix ■

The two propositions above tell us that the price in the penultimate period is a) below the myopic price and b) below the price in the last period. The logic extends naturally to earlier periods, but the increased complexity of the algebra makes it unpractical to solve for the prices in earlier stages. We will do this using a computer algorithm for the parameter values used in the experiment later on.

## 4 Experimental design

We conducted computerized laboratory experiments implementing markets with network externalities in the sense defined above.<sup>7</sup> We also ran some control sessions of comparable markets without network externalities in order to isolate the effect network externalities have on dynamic pricing decisions. We asked students enrolled in the second-year “Microeconomics 2” at the University of Adelaide to participate. All 112 students enrolled in the course had the opportunity to participate. Finally 94 students attended the experimental session. The students were rewarded with a grade bonus on their final mark depending on the performance in the experiment of up to 10 percent. As “Microeconomics 2” is one of the more difficult courses at Adelaide University the subjects were highly motivated. Subjects were trying hard to secure passing the course or to get one of the few distinctions.

Using students from a single course may be viewed as problematic since the sample is definitely not randomly drawn. However, most economic experiments cannot guarantee randomness of the sample. At least we can control for the background of students and their knowledge of economics by using the students of one course.<sup>8</sup> We are aware that this selection gives rise to problems when generalizing the results obtained in the experiment. On the other hand, we believe that using students from an economics course - as opposed to students from different

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<sup>7</sup>We used the computer programme Z-Tree (Fischbacher, 1999) to conduct the experiment. The Z-Tree code for the two treatments can be downloaded from the authors web site.

<sup>8</sup>We exactly know the courses those students have taken. We even know about their performance in the other courses.

courses - does not have too severe drawbacks for our experiment. Since in reality price decisions are taken by people with backgrounds in commerce and economics we don't see a major problem in restricting the subject pool to students of a microeconomics course.<sup>9</sup> However, the usual caveat about using students as subjects applies.

Overall 50 students played the duopoly market with network externalities (treatment NE), and 44 students were assigned to the control treatment, which consisted of a simple duopoly without network externalities (treatment No-NE). In both treatments the subjects played two supergames of ten periods each. The subjects knew that they were paired with the same opponent in both supergames.<sup>10</sup> In every period the subjects had to enter their price choice and a guess what price the opponent might choose. We restricted the valid prices to the range between 0 and 10. After both subjects had chosen their actions they were informed about the market outcome (own price, opponents price, and quantities sold), their period profits. In the NE treatment we additionally displayed the new market share resulting from the actions taken.

In both treatments the subjects were given detailed instructions containing payoff tables and examples how to link choices and payoffs. In the NE treatment, we provided period-payoff tables for different market shares and explained how to extrapolate the payoffs for market shares between tabulated values. Additionally, we carefully explained how the market share evolves depending on previous price choices. The instructions, which were both read aloud and given to the subjects in writing, can be found on the authors web page.

## 4.1 Parametrization and theoretical predictions

In order to implement the underlying model structure in the experiment we had to choose some parameter values. We set the total market demand  $a$  to 10. Additionally, we needed a starting value for the market share for the NE treatment.

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<sup>9</sup>We ran one session with graduate (Masters and Ph.D. students) in order to see if the degree of economic education has an influence. We did not find any striking differences in behaviour. However, the number of observations is not large enough to draw statistical inferences.

<sup>10</sup>This partner treatment design was chosen in order to obtain the maximum number of independent observations. The loss of control due to reputation effects is regarded as not problematic for our research question.

We decided to use a symmetric setting where the market shares are  $s_i^1 = s_j^1 = 1/2$ . Additionally, to avoid that the market share reacts too strongly in the first period we chose to set past sales to 10 units each. We can interpret this in two different ways. Firstly, we could say that there have been two periods of competition before the start of our experiment. Secondly, we could interpret this parameter choice as a reflection of some reputation the firms have due to the customers' experience with other goods this firm has produced.

The baseline duopoly - treatment No-Ne - just consisted of a market where the market shares are constant at  $1/2$  and do not depend on past sales. Note that the strategic situation in all periods of No-NE is identical to the situation in the last period of NE with market shares of  $1/2$ . Consequently, the predicted equilibrium prices for No-NE and all periods are given by equations (4) and (5). For our parameter values the optimal prices  $\bar{p}_1^*, \bar{p}_2^*$  are

$$\bar{p}_1^* = \bar{p}_2^* = 5$$

The derivation of the optimal price path for the NE treatment is much more complex. In order to solve for the equilibrium path we have to conduct backward induction over 10 periods or use a dynamic programming recursive approach in the spirit of Selten (1965).<sup>11</sup> We used a computer algorithm to solve for the equilibrium-price path. The program basically performs backward induction.<sup>12</sup>

Figure 1 shows the predicted equilibrium price path for the NE treatment together with the prediction for No-NE. Note that the symmetric initial market shares lead to a symmetric predicted price path, i.e. the competitors always choose the same price. We see that under NE we expect the price to increase from 0 at the start to 5 in the last period, which is the equilibrium price for the No-NE treatment. As play that deviates from the equilibrium path may lead to market shares different from 0.5 it is necessary to find a way of comparing play after a deviation from equilibrium with the optimal continuation from such a point. Here

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<sup>11</sup>In our model the dynamic programming approach is more complex than in Selten (1965), because we have a duopoly market in each period. In Selten's model the firms are monopolists in each period who have to care for their future demand potentials, which depend on the past sales of all firms.

<sup>12</sup>The Mathematica code is available for download from the authors web site.

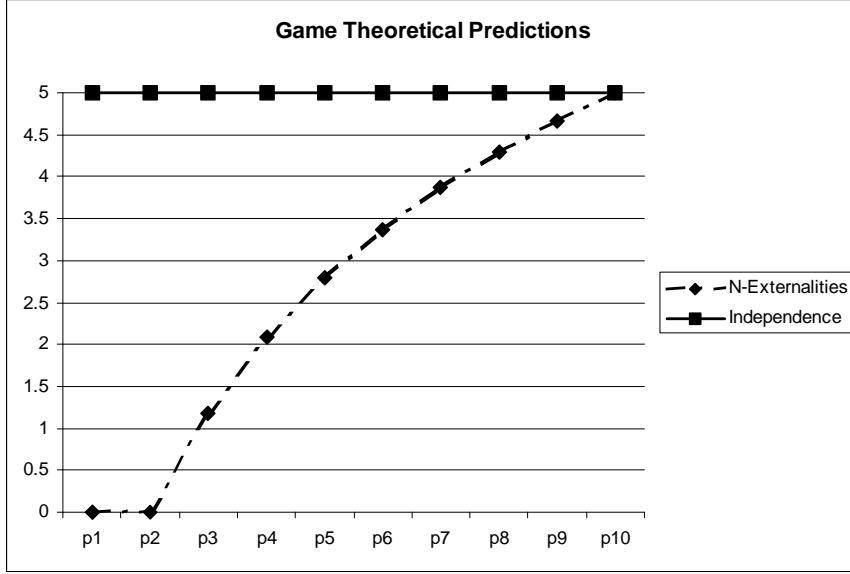


Figure 1: Predicted price paths for the different treatments

the assumption of a perfectly inelastic demand comes into play. Perfectly elastic demand has the implication that the average equilibrium price is independent of the history of play. Observe that equations (4) and (5) can be used to find the average price for the last period

$$\bar{p}^T = \frac{p_i^{T*} + p_j^{T*}}{2} = \frac{5 [1 + s_i^T]}{3} + \frac{5 [2 - s_i^T]}{3} = 5$$

The average price is independent of the history captured by the market share in period  $T$ . This insight tells us that a pair of firms in the NE treatment with an average price below 5 in earlier periods than the final period is fighting for market share. A pair with an average price of 5 is playing myopically, while an average price above 5 can be interpreted as collusion.

The logic of the equilibrium average price being independent of the current market share in the NE treatment extends to earlier periods. This can be seen by investigating the outcomes of the computer algorithm used to solve the supergame or by using a dynamic programming approach, which is contained in the appendix.

**Proposition 3** *The equilibrium prices have the form  $p_i^{t*} = \gamma^t + \phi^t s_i^t$  resulting in*

*an average equilibrium price of  $\bar{p}^t = \gamma^t + \phi^t/2$ , which is independent of the current market share.*

**Proof.** See appendix ■

Consequently, we can say that a pair in the NE treatment with an **average** price higher than the calculated equilibrium price (for equal market shares) do not sufficiently take the future profits into account. This judgment can be made independently of whether the previous play was on the equilibrium path or not. So market shares other than the equilibrium market share firms may have at time  $t$  after off-equilibrium play does not prevent a judgement about how their prices compare to the equilibrium price level.

The independence of the average prices from history of play makes the interpretation of our results possible and gives us the possibility to compare prices between the No-NE treatment, where market shares are fixed at 0.5, and the NE treatment, where market shares other than 0.5 may occur due to off-equilibrium play.

## 5 Results

In what follows we present our main results. The three basic questions will be:

1. How do prices evolve over time compared to the theoretical prediction for different treatments?
2. How do the prices differ among treatments?
3. How competitive do subjects behave under different treatments?

The first question is mainly concerned with the stylized fact that prices for commodities in the real world decrease after they are introduced, while a reduced model of network externalities predicts increasing prices. The second question asks whether network externalities have any influence on pricing behaviour at all, while the third question mainly asks if we can infer from the observed price choices what the impact of network externalities on the degree of competition in a market is.

## 5.1 Evolution of prices

The evolution of chosen prices does not even roughly match the game theoretical prediction in both treatment.<sup>13</sup> While the deviation from Nash Equilibrium in the non-network treatment can be attributed to tacit collusion, it is not clear a priori why the prices under the presence of network externalities do not follow the predicted path. We shortly comment on the evolution of the prices in the No-NE treatment. Then we will discuss the results for the NE treatment in more depth.

### *Treatment No-NE*

Looking at the average prices in the standard Bertrand duopoly with differentiated products (figure 2) we see that as in other experimental studies the average prices are above the Nash Equilibrium prediction for the whole time.<sup>14</sup> However, prices decrease over time. So we observe slowly eroding tacit collusion. Note that although play (particularly in early periods) exhibits considerable tacit collusion, the subjects do by no means come close to the joint profit maximizing outcome, which required both players to choose the maximum price of 10. A remarkable result in our No-NE treatment, which is observed in repeated social dilemma experiments, too, is the existence of a restart effect. As the subject pairs stay the same for both 10-period supergames and the individual periods are independent, the whole experiment is theoretically equivalent to 20 independent periods of duopolistic competition. The subjects do perceive the game differently though. After the first 10 periods of play the announcement that a new game with another 10 periods starts lets the average price return to the level of the first period in game 1.<sup>15</sup> This effect can be interpreted as the restart being a cue for the subjects to newly try to establish co-operation. With the experience from the first game the subjects are more successful in sustaining tacit collusion in game 2. The average prices in game 2 are higher than in game 1. For the first 5 periods the average

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<sup>13</sup>The raw data and the stata programmes for data analysis can be downloaded from the authors web site.

<sup>14</sup>See Huck, Normann and Oechssler (2000) for an example.

<sup>15</sup>There is no statistically significant difference of average prices within pairs between the period 1 prices in game one and two. However pairs increase their prices significantly between the last stage of game 1 and the first stage of game 2 (one-sided Wilcoxon matched-pairs signed-ranks test,  $p < .01$ ).

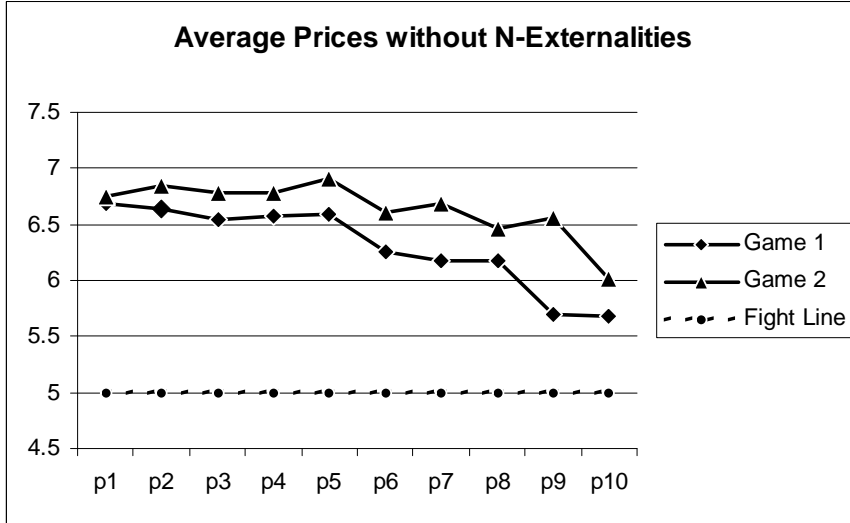


Figure 2: Price paths in the No-NE treatment

prices stay at the restart level (or even slightly above) before the typical erosion of cooperation occurs. The end effect is particularly strong in game 2.

In order to test that the trend of declining prices is not only present in the aggregate, but occurs also within individual pairs we used a Wilcoxon matched-pairs signed-ranks test. For both games the average price per pair is significantly higher in period 1 than in period 6 (one-sided Wilcoxon matched-pairs signed-ranks test,  $p < .01$  for game 1 and  $p < .04$  for game 2). The average prices per pair are significantly higher in period 6 than in period 10 in game 1 ( $p < .03$ ), while the difference in game 2 shows only weak significance ( $p < .09$ ). We now summarize these findings.

**Result 1** *In the Bertrand duopoly without network externality, we find some tacit collusion, which is eroding partially over time. Cooperation is stronger in game 2 and erosion of collusion is weaker and starts later than in game 1.*

#### *Treatment NE*

Prices in the experimental markets with network externalities are far away from the prediction as figure (3) shows. Prices in the early rounds are much higher

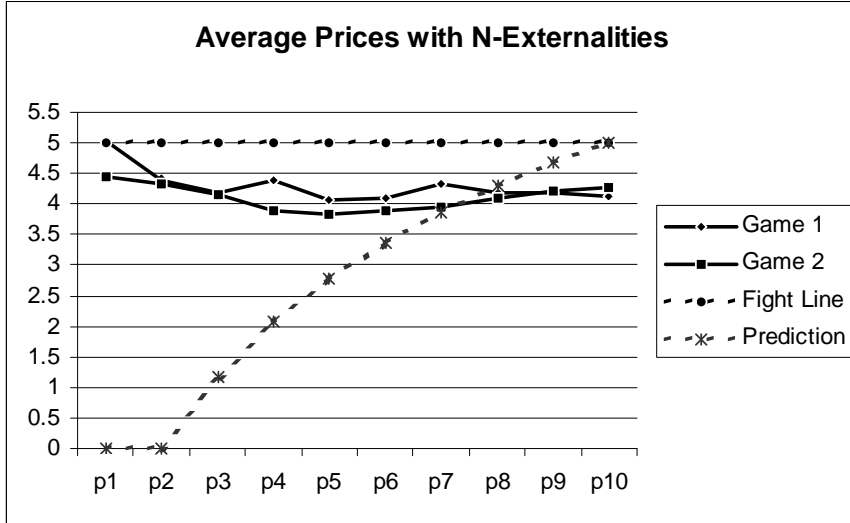


Figure 3: Price paths in the NE treatment

than subgame-perfect equilibrium predicts. However, prices are never above 5, which documents the absence of tacit collusion in the stage games. Additionally, in early periods for both supergames prices decline rather than increase. This is in strong contrast to the prediction. In game 1 the average price of pairs in period 1 is significantly higher than in period 6 ( $p < .01$ ) and in period 10 ( $p < .01$ ). Between periods 6 and 10 there is no statistically significant change. In game 2 the increased experience does not change the decreasing prices in the early periods. Pairs set significantly higher prices in period 1 than in period 6 ( $p < .04$ ). For later periods the competitors seem to increase prices a bit. The difference shows only weak significance, though ( $p < .09$ ).

**Result 2** *In contrast to the theoretical prediction, in NE treatment the average prices per pair are decreasing in the first half of the supergame for both periods. Average prices in early periods are close to the myopic Bertrand Equilibrium.*

Our interpretation of this observed behaviour is the following. As humans are not able to perform backward induction over a many stages (e.g. Selten, 1978 or Brandts and Figueras, 2003) and our supergame is quite complex the subjects in game 1 start off near the stage game equilibrium and use a rule of thumb. This



rule of thumb seems to consist of a heuristic that balances the trade-off between increasing the market share and forgoing short-term profit. The model prediction that a higher present market share should increase the price chosen is turned into the opposite by the subjects. A subject that puts a higher value on the market share in its heuristic will play more aggressively independent of the present market share and will choose a lower price. However, the market share depends negatively on the past prices. So if our statement is true we should observe a negative correlation between current market share and price chosen for a given expected price of the opponent. It is important to take the opponent's expected action into account because without doing this we may miss-interpret a relatively high price as myopic, while - given the expectation that the opponent will set a very high price - it is in fact intended to be very aggressive. In order to test this we created a variable that measures the deviation from the myopic best response to the guessed price of the competitor. This variable captures the intention of a player. The lower this variable is the more aggressive does the subject intend to behave.

Period	2	3	4	5	6	7	8	9	10
Game 1	-**	-	-*	+	-	-*	-***	-*	-**
Game 2	-***	-***	-	-***	-***	-*	-**	-	-

\* 10% level, \*\* 5% level, \*\*\* 1% level

Table 1: Correlation between intention to fight and market share

Table 1 shows that the correlation between the deviation from the myopic best response to the guessed price and the market share is significantly negative for many periods and never significantly positive. Over all the sign of the correlation coefficient is only positive for period 5 in game 1 (highly insignificant with  $p = .39$ ). Note that in the first supergame the motive of fighting for market share is even dominant in the final period, where this cannot be explained by any future profit consideration. This illustrates that subjects rather pursued gaining a high market share as a goal per se than as a way of increasing future profit opportunities.

The period 10 average prices in the NE treatment are for both games (4.12 in game 1 and 4.28 in game 2) below the myopic average price of 5. In sharp contrast

to this the average prices in the No-NE treatment lie above 5 (5.58 in game 1 and 6.02 in game 2). The picture becomes even clearer when we look at the intended play. In the No-NE treatment subjects choose prices close to the best response to the guessed price of the opponent in period 10. On average the chosen prices are .11 below the best response in game 1 and only .05 below the best response in game 2. In the NE treatment the intended play shows that subjects were still fighting for market share in the final period. In game 1 the chosen prices were on average .64 below the best response to the guessed price of the opponent. The difference in game 2 was smaller, but still substantial with average prices being .46 below the best response.

**Result 3** *Subjects use a heuristic that puts certain weights on short-term profit and on market share rather than backward induction as a behavioural rule. The price differences to the myopic best responses to the expected price of the opponent are negatively correlated with the current market share. This is consistent with a heuristic and incompatible to backward induction.*

## 5.2 Network externalities and competitiveness

Figures 4 and 5 compare the average prices in the treatments for game 1 and 2. It is obvious that the prices in the treatment with network externalities are consistently lower than in the No-NE treatment ( $p < .01$  for periods 1 to 18 and  $p < .025$  for the remaining two periods, one-sided Mann-Whitney U-test). We observe that the price differences are higher in game 1 (roughly 1.7) than in game 2 (between 1.8 and 3), which is no surprise as we observed that the average prices in the No-NE treatment are generally higher in game 2, while in most periods in the NE treatment prices are higher in game 1.

**Result 4** *Average prices in the No-Ne treatment are significantly higher than in the NE treatment for both games. The prices differ more strongly in game 2.*

The setting in the No-NE treatment is relatively collusion friendly, while in the NE treatment the network externalities introduce an additional competitive element - the struggle for market share. So the result that in a market with modest

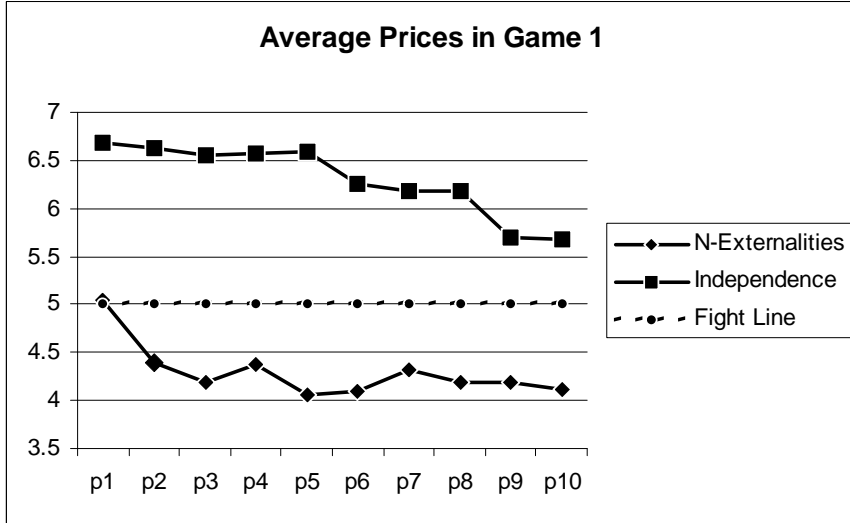


Figure 4: Price differences game 1

network externalities competition is higher than in a market without network externalities is not surprising. More surprising is that the price difference in the market does not strongly decrease the closer we get to the end of the product lifetime.<sup>16</sup>

An interesting question is to compare the predicted average effects network externalities have on distribution in theory and experiment. Are the consumers getting a relatively better deal out of the additional competition due to network externalities in theory or in the experiment? A measure is the relative benefit of the network externalities in the experimental sessions compared with the theoretical benefit. As we used a perfectly inelastic demand we cannot use consumer surplus as a measure.<sup>17</sup> However, we can compare the profits the firms make in theory and practice. Since the quantities in theory and in the experiment are constant for all rounds, we can use the average price per round as an indication how much potential consumer surplus the firms were able to transform into profits. Table 2 shows the average prices over all rounds and firms. We see that the absolute

<sup>16</sup>There is an end effect in game two. The price difference shrinks in the last round. However, in early periods where we expect the gap to narrow with a high rate, the gap even increases.

<sup>17</sup>Note that in our model due to the perfectly inelastic demand collusion does not cause any efficiency loss.

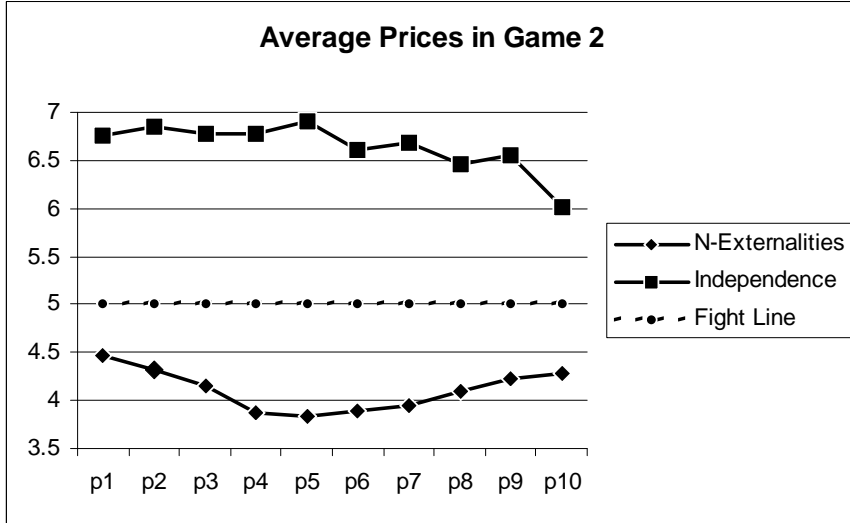


Figure 5: Price differences in games 2

benefit for consumers (the average price difference between No-NE and NE) is in the same range for theory and the two experimental games (2.27 versus 2.0 and 2.53, respectively); but due to tacit collusion in the No-NE treatment leading to high average prices the relative benefit of increased competition is smaller in the experimental NE market than theory predicts.

	Theory	Game 1	Game 2
No-NE	5.00	6.30	6.64
NE	2.73	4.30	4.11
Benefit of NE absolute	2.27	2.00	2.53
Benefit of NE relative	45.5%	31.8%	38.1%

Table 2: Average prices over all rounds

**Result 5** *Increased competition due to network externalities reduces average prices approximately by the amount the theory predicts. The relative price-reduction is smaller in the experimental markets, though.*

## 6 Conclusion

Markets with network externalities are characterized by demand inertia. This demand inertia creates an incentive for firms producing competing products to set low introductory prices for their products as they seek to increase the size of their network. Optimal prices increase when the market matures as the incentive to fight for market share gets weaker the closer the market gets to the end of the product cycle. In reality we usually do not observe increasing prices when markets with network externalities mature. However, this could be due to other countervailing effects dominating. These effects could be intertemporal price discrimination, increasing returns to scale due to learning by doing, or decreasing demand. In order to be able to determine the effect of demand inertia created by network externalities on dynamic prices we conducted a laboratory experiment. In the experimental setup we excluded all possible factors that may play a role in dynamic pricing, but demand inertia. Additionally, we ran a control treatment where network externalities were absent. This gave us the opportunity to isolate the effect of demand inertia on dynamic pricing.

We found that as theory predicts that the average prices are lower if network externalities are present. However, average prices under network externalities in accordance with the real world decrease if subjects are not experienced. Theory predicts increasing prices. Even if subjects gain some experience prices still decrease over time in young markets. They only increase slightly when markets mature. We attribute this deviation from the theoretical prediction to the inability of subjects to conduct backward induction over a long and rather complicated supergame. We suggest that subjects instead use a rule of thumb that mitigates the trade-off between current profit and future profit potential depending on the market share. This rule of thumb seems to be surprisingly stable over time. Intended aggressiveness of play is positively correlated with the market share throughout the supergame. This means that people that play aggressively at the beginning of the game do not cash in on their obtained high market share in later periods, but stick to their rule of thumb and continue to play aggressively. This suggests that people are not only not able to perform backward induction, but have also problems to at least intuitively follow the logic of intertemporal profit maximization.

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## A Derivation of the inverse demand function

In this appendix we demonstrate how the specific demand function used in the text can be derived from simple (specific) preferences for differentiated goods and network sizes. For comparable preferences we get similar results for the inverse demand functions. We chose this specific setting to keep the inverse demand functions as simple as possible.

Assume that consumers purchase one unit of the good per period. Every period  $a$  consumers are active in the market. They decide which brand to buy by comparing the net surplus the goods are creating. As the goods are not homogeneous the consumers *ceteris paribus* prefer one of the brands. Denote the surplus a certain consumer  $k$  derives from consuming the product of firm  $i$  as  $\theta_i^k$ . Define the additional surplus  $\nu$  for consumer from the network of product  $i$  as half the average sales of good  $i$  per past period:

$$\nu_i := \sum_{l=1}^{t-1} q_i^l / 2(t-1)$$

This can be interpreted in two ways:

1. The consumer learns some quality aspects of the good from the number of previous sales.
2. The consumer cares only for number of sales in the actual period (valued at 1/2 monetary unit each) and forms expectations according to the average past sales.<sup>18</sup>

Then the total net surplus of consuming good  $i$  is given by:

$$CS_i^k := \nu_i^k + \theta_i^k - p_i$$

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<sup>18</sup>In the equilibrium of a symmetric duopoly these expectations are rational and we have a rational expectation equilibrium as both firms sell  $a/2$  in every period.



So consumer  $k$  will purchase good  $i$  whenever  $CS_i^k > CS_j^k$  or

$$\Delta\theta^k > p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1]$$

where  $\Delta\theta^k$  is given by  $\theta_i^k - \theta_j^k$ . Suppose that the differences between values  $\Delta\theta^k$  for the differentiated products is distributed uniformly on the interval  $[-a/2, a/2]$ . Then for given prices and network sizes the number of consumers buying good  $j$  is given by

$$\begin{aligned} q_j^t &= aF\left(p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1]\right) = \\ &= -1/2a + p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1] \end{aligned}$$

Recall that  $\sum_{l=1}^{t-1} q_i^l = at - \sum_{l=1}^{t-1} q_j^l$ , since past total sales of all brands are equal to  $ta$ . Replacing  $\sum_{l=1}^{t-1} q_i^l$  gives

$$\begin{aligned} q_j^t &= p_i - p_j + \frac{1}{t-1} \sum_{l=1}^{t-1} q_j^l = \\ &= as_j^t + p_i - p_j, \end{aligned}$$

which is the inverse demand function we use. The demand for firm  $i$  is just  $a - q_j^t = as_1^t + p_j - p_i$ .

## B Proof of proposition 1

This proof is conceptually easy, but quite tedious. We only sketch the proof and omit some intermediate calculations.

**Proof.** We use the best response functions (2) in order to compute the equilibrium prices in the penultimate stage. This gives the following equilibrium price for player  $i$ :

$$p_i^{T-1*} = a \frac{-41 + 3s_i^{T-1} [T-1]^2 [9T-11] + 3T [41 + T [9T-35]]}{3 [T-1] [23 + 27T [T-2]]}$$

Note that  $p_j^{T-1*}$  is found by just replacing the index. Using those equilibrium prices and the law of motion for the market share we can compute the equilibrium price in the final period as a function of the market share in period  $T - 1$  and subtract the equilibrium price obtained above:

$$p_i^{T-1*} - p_i^{T*} = \frac{a}{3} \left[ \frac{1}{T-1} - \frac{6 [2s_i^{T-1} - 1] [T-1]}{23 + 27T(T-2)} \right]$$

Further inspection shows that this converges to 0 when  $T$  approaches infinity. Additionally, the roots for  $T$  are all smaller than 2 ( $T_{1/2} = 1 \pm 2/\sqrt{33 - 12s_i^{T-1}}$ ). As  $T \in [2, \infty)$  we know that  $p_i^{T-1*} - p_i^{T*} \geq 0$  for all  $s_i^{T-1}$  if we find a valid  $T$  such that  $p_i^{T-1*} - p_i^{T*} > 0$  holds. For  $T = 2$  we get  $p_i^{T-1*} - p_i^{T*} = a [29 - 12s_i^{T-1}] / 69 > 0$ .

■

## C Dynamic programming approach to prove proposition 2

In this section we outline the dynamic programming solution of the NE-game. We assume functional forms for prices and continuation payoff and show that these assumptions are correct. The derivation of the average price in the main text uses these functional forms.

**Step 1: solve the last stage.** Optimal prices are

$$p_i^{T*} = \frac{a [1 + s_i^T]}{3} \tag{9}$$

$$p_j^{T*} = \frac{a [1 + s_j^T]}{3} \tag{10}$$

Stage payoffs are

$$\Pi_i^{T*} = [p_i^{T*}]^2 = \left[ \frac{a [1 + s_i^T]}{3} \right]^2 \quad (11)$$

$$\Pi_i^{T*} = [p_j^{T*}]^2 = \left[ \frac{a [1 + s_j^T]}{3} \right]^2 \quad (12)$$

**Step 2: The law of motion for the state variable**

$$s_i^{t+1} = s_i^t + \frac{p_j^t - p_i^t}{a [1 + t]} \quad (13)$$

**Step 3: Guessing functional forms for the recursion**

$$p_i^{t*} = \phi^t s_i^t + \gamma^t \quad (14)$$

$$\hat{\Pi}_i^t = \Phi^t [s_i^t]^2 + \Gamma^t s_i^t + \Lambda^t \quad (15)$$

The functional forms proposed do definitively work for the last round:

$$\begin{aligned} \phi^T &= \frac{a}{3} \\ \gamma^T &= \frac{a}{3} \\ \Phi^T &= \left[ \frac{a}{3} \right]^2 \\ \Gamma^T &= \frac{2a}{3} \\ \Lambda^T &= \left[ \frac{a}{3} \right]^2 \end{aligned}$$

**Step 4: Bellman equation**

$$\hat{\Pi}_i^t = \pi_i^t + \hat{\Pi}_i^{t+1} \quad (16)$$

Differentiating (16) with respect to  $p_i$  and using (13) gives:

$$\begin{aligned}
\frac{\partial \hat{\Pi}_i^t}{\partial p_i^t} &= -2p_i^t + p_j^t + as_i^t + \frac{\partial s_i^{t+1}}{\partial p_i^t} \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} = \\
&= -2p_i^t + p_j^t + as_i^t - \frac{1}{a[1+t]} \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}}
\end{aligned} \tag{17}$$

We can use the recursion relation from (15) to write the first-order conditions (17) as

$$\frac{\partial \hat{\Pi}_i^t}{\partial p_i^t} = -2p_i^t + p_j^t + as_i^t - \frac{2\Phi^{t+1} \left[ s_i^t + \frac{p_j^t - p_i^t}{a[1+t]} \right] + \Gamma^{t+1}}{a[1+t]} = 0$$

**Step 5: Solving for the prices** By solving for the parameters of the optimal prices we find:

$$\begin{aligned}
p_i^{t*} &= s_i^t \frac{Y^{t+1} [aY^{t+1} - 2\Phi^{t+1}]}{3Y^{t+1} - 4\Phi^{t+1}} + \\
&\frac{2a\Phi^{t+1}Y^{t+1} + [2\Phi^{t+1} + 3\Gamma^{t+1}] [Y^{t+1}]^2}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3} \\
&+ \frac{-a[Y^{t+1}]^3 - 4\Phi^{t+1}[\Phi^{t+1} + \Gamma^{t+1}]}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3}
\end{aligned} \tag{18}$$

where  $Y^t = at$  and recursively

$$Y^t = Y^{t+1} - a. \tag{19}$$

The proposed functional form is correct, since  $p_i^{t*}$  is an affine function of the market share. We can get the coefficient for the optimal price from (18):

$$\phi^t = \frac{Y^{t+1} [aY^{t+1} - 2\Phi^{t+1}]}{3Y^{t+1} - 4\Phi^{t+1}} \tag{20}$$

$$\begin{aligned}
\gamma^t &= \frac{2a\Phi^{t+1}Y^{t+1} + [2\Phi^{t+1} + 3\Gamma^{t+1}] [Y^{t+1}]^2}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3} \\
&+ \frac{-a[Y^{t+1}]^3 - 4\Phi^{t+1}[\Phi^{t+1} + \Gamma^{t+1}]}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3}
\end{aligned} \tag{21}$$

Note that by definition from equation (14) the average price  $\bar{p}^t$  is independent from the current market share if our functional forms are correct:

$$\bar{p}^t = \phi^t/2 + \gamma^t \quad (22)$$

**Step 6: The equilibrium motion for the market share** from equations (13) and (14) we can derive the equilibrium market share recursively:

$$s_i^{t+1} = s_i^t + \frac{[1 - 2s_i^t] \phi^t}{Y^{t+1}} \quad (23)$$

This tells us that the game has a steady state at a market share of 1/2.

**Step 7: Check the functional form for the profits** Note that the future profit only depends on the market share. So we have to find out how the future profit varies with the market share. Differentiating the total future profit at time  $t$  from (16) with respect to  $s_i^t$  gives:

$$\frac{\partial \hat{\Pi}_i^t}{\partial s_i^t} = \frac{\partial \pi_i^t}{\partial s_i^t} + \frac{\partial \pi_i^t}{\partial p_i^t} \frac{\partial p_i^t}{\partial s_i^t} + \frac{\partial \pi_i^t}{\partial p_j^t} \frac{\partial p_j^t}{\partial s_i^t} + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} \frac{\partial s_i^{t+1}}{\partial s_i^t}$$

Using our previous results and assumptions about functional forms from (14), (15), and (23) makes it possible to develop the previous equation

$$\begin{aligned} \frac{\partial \hat{\Pi}_i^t}{\partial s_i^t} &= ap_i^t + [-3p_i^t + p_j^t + as_i^t] \phi^t + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} [1 - 2\phi^t] = \\ &= ap_i^t + [-4\phi^t s_i^t + \phi^t - 2\gamma^t + as_i^t] \phi^t + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} [1 - 2\phi^t] = \\ &= [-4\phi^t s_i^t + \phi^t - 2\gamma^t + 2as_i^t] \phi^t + a\gamma^t + \\ &\quad [1 - 2\phi^t] [2\Phi^{t+1} s_i^{t+1} + \Gamma^{t+1}] \\ &= [-4\phi^t s_i^t + \phi^t - 2\gamma^t + 2as_i^t] \phi^t + a\gamma^t + \\ &\quad [1 - 2\phi^t] [2\Phi^{t+1} [s_i^t + [1 - 2s_i^t] \phi^t] + \Gamma^{t+1}] \end{aligned} \quad (24)$$

Our result from (24) is linear in  $s_i^t$ . So we can see that integration leads to the form we proposed. We get the following recursive relationships:

$$\Phi^t = [1 + 4[\phi^t - 1]\phi^t] \Phi^{t+1} + a\phi^t - 2[\phi^t]^2 \quad (25)$$

$$\Gamma^t = [1 - 2\phi^t] \Gamma^{t+1} + a\gamma^t + \phi^t [\phi^t + 2\Phi^{t+1} - 4\phi^t\Phi^{t+1} - 2\gamma^t] \quad (26)$$

With the recursive relations (20), (21), (19), (23), (25), and (26) we have defined the subgame-perfect equilibrium-behaviour of the firms. Furthermore, we have shown that the functional forms assumed are correct and the average price (defined in 22) is independent from the current market share.