# A Constrained State-Space Approach to the Prediction of Comparable Real Income across Countries 

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#### Abstract

: Studies on growth performance and catch-up and convergence of countries require and make extensive use of internationally and temporally comparable data on real gross domestic product (GDP) expressed in a common currency unit. The International Comparison Program (ICP), a project supported by the World Bank, OECD and a host of other international bodies, provides estimates of purchasing power parities (PPPs) as the most robust and appropriate converter of nominal GDP into a common currency unit reflecting differences in levels of prices of goods and services in different countries. The coverage of the ICP, however, has been limited to a few benchmark years, roughly every five years since 1970, and to only those countries participating in benchmark comparisons. Over the last two decades, the Penn World Tables (PWT) filled this gap by providing extensive tabulations of real GDP data for a large number of countries and for a 50 -year period covering both participating and non-participating countries and benchmark and non-benchmark years. The PWT figures are essentially extrapolations based on results from benchmark years and country-specific growth rates with particular emphasis on comparisons from specific benchmark years. The main purpose of the paper is to show how a constrained state-space formulation of the problem can be used to generate PPPs, and real GDP data, that is consistent with data generated by the ICP for the benchmark years. Treating the ICP data as an unbalanced panel, the paper presents a suitable spatially autocorrelated econometric model for PPPs which is reformulated as a constrained state-space form to complete the panel. The empirical illustration focuses on 24 OECD countries from 1971 to 2000 and generates a complete set of predicted PPPs for all the countries and years. Unlike the PWTs figures, it is possible to compute standard errors associated with the predicted figures.


JEL classification: C53, C33

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## Introduction

Over the last three decades, studies on growth performance of countries and the issues of catch-up and convergence have been prominent. These studies require and make extensive use of internationally comparable data on real gross domestic product (GDP) and real per capita incomes expressed in a common currency unit which are derived after adjusting for price level differences across countries and price movements over time.

In 1988, the United Nations (UN) embarked on a major project to standardise the way economic aggregates are compiled and assembled in the form of a system of national accounts (SNA) and its recommendations were finalised in leading to the 1993 SNA. But, conceptually and practically, the UN national accounts will only be completed when a common measure of economic aggregates (compiled by different countries and expressed in respective national currency units) can be expressed in a common currency unit. The longstanding recognition of the deficiencies of using nominal exchange rates for these comparisons mainly due to the existence of nontraded goods and services in different countries and due to capital movements and exchange market intervention, gave rise to the International Comparison Program (ICP) which developed purchasing-power parities (PPPs) as the most robust and appropriate converter for accurately reflecting differences in the levels of prices of goods and services in different countries. However, ICP, a project supported by the World Bank, UN, OECD, EU and a host of other international bodies, provides PPPs and real economic aggregates only for specific years when participating countries undertake extensive price surveys and the data collected are used in conjunction with
other national accounts data. The coverage of the ICP, however, has been limited and results have been available only for the benchmark years, 1975, 1980, 1985, 1990, 1993, 1996 and 1999. Country coverage also varies greatly across different benchmark years with coverage limited mostly to developed countries and a small proportion of developing countries. Therefore, the limited temporal and spatial coverage of the ICP has limited the applicability of the resulting PPPs and real aggregates.

The ICP benchmark PPPs have been extrapolated to create a complete panel of internationally comparable GDP and its components for a large number of countries spanning over 40 years. Two alternative data sets have resulted, the Penn World Tables (PWT) (currently Mark 6.1 is available) (see Summers and Heston, 1988 and 1991) and the World Bank's tables published in the World Development Indicators (WDI) (see Ahmad 1996). A third set of available tables is that by Maddison (2001), where very long time series are provided for ICP participating countries. The methodology of construction of these tables is relatively similar in that PWT and WDI first, extrapolate ICP results from a given benchmark to non-ICP participating countries (spatial extrapolation) using regression techniques ${ }^{1}$. Second, national price level's growth rates are used to interpolate temporally. Maddison does not attempt to extrapolate to non-participating countries. A fundamental problem of consistency arises from these methods. The use of price level's growth rates to interpolate between benchmark years for participating countries results in discrepancies between the interpolated values and the benchmark values. As a consequence, the interpolation is based on a single benchmark year (usually the latest available) and all previous

[^1]benchmark observations are discarded. If follows that different benchmark exercises lead to a different data set (ie the data in Marks 5 and 6 of PWT differ).

Our approach is fundamentally different in that the temporal and spatial completion of the panel is achieved jointly. Our econometric method is basically a learning algorithm that moves from one time period to the next incorporating new temporal information (ie new ICP results or country specific growth rates) and spatially predicting incomplete PPPs (non-participating countries during an ICP exercise, all countries for non-benchmark years). The success of the learning algorithm depends on the economic and econometric model formulated to predict PPPs. We review the existing literature on the modeling of national price levels next. Section 3 presents the approached proposed in this study, including both the econometric model and derived state space model. Section 4 discusses the estimation by Kalman filtering techniques. Section 5 presents a small empirical illustration of the method for OECD countries. Section 6 concludes.

## 2. The problem

Purchasing power parities (PPPs) of currencies are compiled by international organizations on a regular basis. The PPP of the currency of a country compared to a reference currency provides a measure of the number of the currency units of the country that have the same purchasing power as one unit of the currency of the reference country. For example, PPP of Au\$ $1.30=$ US\$ 1.00 shows that 1.30 Australian dollars have the same purchasing power as one US dollar at a given point of time.

Let $\operatorname{PPP}_{j, t}$ represent the PPP of currency of country $j$ at a time point $t^{2}$. These PPPs are compiled using extensive price surveys conducted in the countries

[^2]participating in the International Comparison Program. As such surveys and the process involved is very resource intensive, these PPPs are compiled only for selected years, referred to as "benchmark years", and for only those countries that participate in the ICP. The PPPs are computed using specialized index number methodology designed specifically for the purpose of international comparisons. Kravis et al (1983) and Roberts (2004) provide an excellent exposition of the methodology underlying international comparisons. PPPs are used in the place of exchange rates of currencies for purposes of comparing real incomes and various output aggregates. The World Development Indicators of the World Bank and the Human Development Index (HDI) of the United Nations Development Program make use of per capita real incomes derived by converting gross domestic product of different countries, expressed in respective national currency units, into a common currency unit through the use of PPPs. Availability of PPPs on a regular basis is considered very essential by most international organizations.

As these PPPs are available only for a subset of countries and also for only a few years, the main problem is one of meaningfully extrapolate or predict PPPs for countries not participating in the ICP comparison and for all the years in between benchmark years. These are the two issues considered in the present problem.

## Extrapolation of PPPs for the non-participating countries

Suppose there are M participating countries in a given bench mark year. Then we have $\operatorname{PPP}_{\mathrm{j}}(\mathrm{j}=1,2,3, \ldots, \mathrm{M})$ as the output of the ICP exercise. The problem is one of extrapolating these PPPs to countries outside the ICP. The basic approach used in the literature thus far is to formulate a regression relationship between observed PPPs with a set of explanatory variables. This is achieved through the use of the concept of
national price level. If $E R_{j}$ denotes the exchange rate of currency of country $j$, then the national price level, $\mathrm{PL}_{\mathrm{j}}$, for country j is defined as:

$$
\begin{equation*}
P L_{j}=\frac{P P P_{j}}{E R_{j}} \tag{1}
\end{equation*}
$$

For example, if the PPP and ER for Japan are 155 and 80 yen respectively, then the price level in Japan is 1.94 indicating that prices in Japan are roughly double to that in the United States.

There is considerable literature (Kravis and Lipsey 1983 and 1986; Clague, 1988; Bergstrand, 1996 to select a few papers) focusing on the problem of explaining the national price levels. It has been found that for most developed countries the price levels are around unity and for most developing countries these ratios are usually well below unit. Most of the explanations of price levels are based on productivity differences in traded and non-traded goods across developed and developing countries. There is a general consensus that variables like resource abundance, population, size of the agriculture sector in the economy, trade balance, openness, educational attainment and share of exportable services (such as tourism) are the primary drivers of the price levels. In general it is possible to identify a vector of regressor variables and postulated a regression relationship:

$$
\begin{equation*}
P L_{j}=F\left(X_{1}, X_{2}, X_{3}, \ldots X_{k}\right)+e_{j} \tag{2}
\end{equation*}
$$

where $e_{j}$ is a random disturbance with specific distributional characteristics.
Once this regression model is specified properly and estimated, the resulting estimates of the parameters can be used in predicting the "price levels" for nonparticipating countries. Since the exchange rates, $E R_{j}$, are observed it is possible to obtain predictions of the PPPs.

## Extrapolation of PPPs for non-benchmark years

In comparison to the prediction of PPPs for non-participating countries discussed in the previous section, the methodology used in the extrapolation of the PPPs is relatively simple. Since $\mathrm{PPP}_{j, t}$ represents the PPP for country $j$ in period $t$ relative to the United States (or some other reference country), it is possible to obtain a PPP for country $j$ in period $t+1$ by adjusting $\operatorname{PPP}_{j, t}$ for differential movements of prices in country $j$ and the US over the period $t$ to $t+1$. National price movements are measured through the gross domestic product deflator (or the GDP deflator) for period $t+1$ relative to period $t$. This is due to the fact that PPPs from the ICP refer to the whole GDP, GDP deflators are used in this extrapolation process. ${ }^{3}$ The extrapolation of PPP to period $t+1$ is given by the formula:

$$
\begin{equation*}
P P P_{j, t+1}=P P P_{j, t} \times \frac{G D P D e f_{j,[t, t+1]}}{G D P D e f_{U s,[t, t+1]}} \tag{3}
\end{equation*}
$$

The appropriate extrapolation of the price level is then given by

$$
\begin{equation*}
P L_{j, t+1}=\frac{P P P_{j, t+1}}{E R_{j, t+1}}=\frac{P P P_{j, t}}{E R_{j, t}} \times \frac{G D P D e f_{j,[t, t+1]}}{G D P D e f_{U S,[t, t+1]}} \times \frac{E R_{j, t}}{E R_{j t+1}} \tag{4}
\end{equation*}
$$

This extrapolation process can be used in conjunction with the prediction model in equation (2).

The use of equations (2) and (4) in practice can lead to some consistency issues. For example if two different benchmark PPPs, say for years 1985 and 1990, are used in obtaining extrapolations to the year 1997, there will be two different predictions for the PPP in year 1997. Which one should be use? Or is there some way of obtaining a single prediction for 1997 that makes use of all the historical

[^3]information that is available at a given point of time $t$ ? This is the problem that is pursued in the following section.

## 3. A Unified and Consistent Approach

Our approach starts by considering the ICP benchmark data as an unbalanced data set. This is so as it contains observations (roughly every five years from 1975 to 1999) with varying coverage of countries of PPP conversion factors (PPPs). It is also important to note that growth rates in PPPs for the complete panel can be computed from published national account statistics and the definition in (4) (we discuss the observed growth rates in Section 4). These growth rates are used by previous methods to extrapolate PPPs across time for participating countries as already stated.

Using theoretically based arguments (as discussed in the previous section), an econometric model for the price level ratio $\left(\mathrm{PL}_{t}=\mathrm{PPP}_{t} / \mathrm{ER}_{t}\right)$ (if observable) for the panel of countries can be specified. This model includes autocorrelated errors to recognize the temporal dimension of PL and spatially correlated errors to incorporate cross-country correlations. Following Doran (1996) and Rambaldi, Hill and Doran (2004), we re-write the model in a state-space form, so that the observation equation is specified as a function either the observed PLs or an observable function of them (ie their growth rates). We note PLs are observed during benchmark years for participating countries, and the growth rates in PLs are observed for the complete panel.

## The Econometric Model

Let $\mathbf{Y}_{t}(\mathrm{~N} \times 1)$ be the price level $\left(\mathrm{PPP}_{t} / \mathrm{ER}_{t}\right)$ for the $N$ countries at time $t$ (if observed). The aim is to produce predictions ( $\hat{\mathbf{Y}}_{t}$ ), given benchmark observations for
some countries in some years, and growth rates for all countries in all years. The starting point is a suitable econometric model:

$$
\begin{equation*}
\mathbf{y}_{t}=\mathbf{X}_{\mathrm{t}} \beta+\mathbf{e}_{\mathrm{t}} \tag{5}
\end{equation*}
$$

where,
$\mathbf{y}_{t}$ is the natural $\log$ of $\mathbf{Y}_{t}$
$\mathbf{X}_{t}$ a $\mathrm{N} \times \mathrm{K}$ matrix of observed related economic variables derived from the theoretical literature of price levels (PL), such us trade openness, education levels, resource abundance, etc.
$\beta$ a $\mathrm{K} \times 1$ unknown parameter vector
$\mathbf{e}_{t}$ an $\mathrm{N} \times 1$ unobserved random error, which is specified to account for autocorrelation and spatial autocorrelation, as follows

$$
\begin{align*}
& \mathbf{e}_{\mathrm{t}}=\rho \mathbf{e}_{\mathrm{t}-1}+\mathbf{u}_{\mathrm{t}}  \tag{6}\\
& \mathbf{u}_{\mathrm{t}}=(\mathbf{I}-\phi \mathbf{W})^{-1} \zeta_{\mathrm{t}} \tag{7}
\end{align*}
$$

where,
$\rho$ a scalar autocorrelation parameter.
$\phi$ a scalar spatial autocorrelation parameter,

W a known matrix measuring spatial contiguity of countries.
$\zeta_{t} \sim \operatorname{MN}\left(\mathbf{0}, \sigma_{1}^{2} \mathbf{I}\right)$
$E\left[\mathbf{u}_{t} \mathbf{u}_{t}^{\prime}\right]=\sigma_{1}^{2} \mathbf{\Omega}_{t}=\sigma_{1}^{2}(\mathbf{I}-\phi \mathbf{W})^{-1}\left[(\mathbf{I}-\phi \mathbf{W})^{-1}\right]^{\prime}$
$E\left[\mathbf{u}_{t} \mathbf{u}_{t-s}^{\prime}\right]=\mathbf{0} \quad \forall \mathrm{s} \neq 0$

The state space representation

We define $\boldsymbol{\alpha}_{t}=\left[\mathbf{e}_{t}^{\prime}, \mathbf{e}_{t-1}^{\prime}\right]^{\prime}$ as the unobservable "state vector." It follows from (6)

$$
\begin{equation*}
\alpha_{t}=\mathbf{D} \alpha_{t-1}+\xi_{t} \tag{8}
\end{equation*}
$$

where,
$\mathbf{D}=\rho \mathbf{I}_{\mathbf{2 N}}$
$\xi_{t} \sim \operatorname{MN}\left(\mathbf{0}, \sigma_{1}^{2} \mathbf{Q}_{\mathbf{t}}\right)$ which accounts for spatial autocorrelation arising out of countries' characteristics (this follows from (7)).
$\mathbf{Q}_{\mathrm{t}}=\mathbf{I}_{2} \otimes \boldsymbol{\Omega}_{t}$
$\boldsymbol{\alpha}_{t} \sim \operatorname{MN}\left(\mathbf{0}, \mathbf{P}_{t}\right)$

Furthermore, there exists at all $t$, a vector $\mathbf{y}_{t}^{*}$ of observations. This vector has dimension $\mathrm{N}_{1}=\mathrm{N}$ in non-benchmark years, and $\mathrm{N}_{1}=\mathrm{N}+\mathrm{N}_{t}$ in benchmark years, where $1 \leq \mathrm{N}_{t} \leq \mathrm{N}$ and $\mathrm{N}_{t}$ is the number of countries participating in the benchmark exercise in year $t$.

In benchmark years we observe:

$$
\begin{equation*}
\mathbf{S}_{t} \mathbf{y}_{t}=\mathbf{S}_{t}\left[\mathbf{I}_{N}, \mathbf{0}\right] \boldsymbol{\alpha}_{t}+\mathbf{S}_{t} \mathbf{X}_{t} \boldsymbol{\beta}+\boldsymbol{\eta}_{t} \tag{9}
\end{equation*}
$$

where,
$\mathbf{S}_{t}$ is $\mathrm{N}_{t} \times \mathrm{N}$, a known selection matrix accounting for PL only being available for some countries
$\eta_{t} \sim\left(0, \sigma_{2}^{2} \mathbf{V}_{t}\right)$ is a random error that acknowledges that benchmark exercises can carry some measurement error.
$\mathbf{V}_{t}$ is $\mathrm{N}_{t} \times \mathrm{N}_{t}$, diagonal, with elements assumed to be inversely related to the measurement error in the benchmarking exercise for country $j$.

For all years growth rates in PL are observed from national accounts, $\mathbf{G r}_{t}=\mathbf{y}_{t}{ }^{-}$ $\mathbf{y}_{t-1}$, we discuss the definitions below. Our state-space representation will preserve the integrity of these growth rates. That is, the completed panel of PL will be consistent with observed growth rates.

It follows that for all years we observe:

$$
\begin{equation*}
\mathbf{G r}_{t}=\mathbf{y}_{t}-\mathbf{y}_{t-1}=\left[\mathbf{I}_{N},-\mathbf{I}_{N}\right] \boldsymbol{\alpha}_{t}+\left(\mathbf{X}_{t}-\mathbf{X}_{t-1}\right) \boldsymbol{\beta} \tag{10}
\end{equation*}
$$

Thus,

We then write (9) and (10) as:

$$
\begin{equation*}
\mathbf{y}_{t}^{*}=\mathbf{Z}_{t} \boldsymbol{\alpha}_{t}+\mathbf{G}_{t} \boldsymbol{\beta}+\boldsymbol{\varepsilon}_{t} \tag{11}
\end{equation*}
$$

Where,
$\mathbf{y}^{*}=\binom{\mathbf{S}_{t} \mathbf{y}_{t}}{\mathbf{G} \mathbf{r}_{t}}$ in benchmark years, and $\mathbf{y}^{*}=\mathbf{G r}_{t}$ in non-benchmark years
$\mathbf{Z}_{t}=\binom{\mathbf{S}_{t}\left[\mathbf{I}_{N}, \mathbf{0}\right]}{\left[\mathbf{I}_{N},-\mathbf{I}_{N}\right]}$ in benchmark years, and $\mathbf{Z}_{t}=\left[\mathbf{I}_{N},-\mathbf{I}_{N}\right]$ in non-benchmark years
$\mathbf{G}_{t}=\binom{\mathbf{S}_{t} \mathbf{X}_{t}}{\mathbf{X}_{t}-\mathbf{X}_{t-1}}$ in benchmark years, and $\mathbf{G}_{t}=\left(\mathbf{X}_{t}-\mathbf{X}_{t-1}\right)$ in non-benchmark years
$\varepsilon_{t}$ is random with mean zero and variance-covariance $\mathbf{H}_{t}$,
$\mathbf{H}_{t}=\left[\begin{array}{ll}\mathbf{V}_{t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{array}\right]$ is $\mathrm{N}_{1} \times \mathrm{N}_{1}$ in benchmark years and $\mathbf{H}_{t}=\mathbf{0}_{N \times N}$ in non-benchmark years

Equation (10), and therefore (11) is, in effect, a constraint insuring the predictions of PL's growth rates, $\mathbf{G r}_{t}$, will be identical to observed benchmark values. This
constrain maps the state vector to the known information. The observation equation (11) together with (8), constitute a conventional state-space model (SSM). SSMs cannot be estimated with ordinary regression techniques, however, it is well known that the Kalman Filter (KF) can be used to estimate (or predict) $\boldsymbol{\alpha}_{t}$ optimally from the observations $\mathbf{y}_{1}^{*}, \mathbf{y}_{2}^{*}, \ldots, \mathbf{y}_{t}^{*}$. In addition, the KF produces as outputs the one-step ahead prediction error, $\mathbf{v}_{t}$ and its covariance matrix $\mathbf{F}_{t}$ (required to evaluate the likelihood function) and can be used to simultaneously obtain estimates $\hat{\boldsymbol{\alpha}}_{t}$ and $\hat{\boldsymbol{\beta}}$ with their respective standard errors, as well as the parameters of the covariances $\mathbf{Q}_{t}$ and $\mathbf{H}_{t}$. A full discussion of the KF algorithm and maximum likelihood estimation can be found in Harvey (1990, 100-110 and 130-133).

A fundamental property of the Kalman Filter (see Doran (1992)), guarantees $\hat{\mathbf{G}} \mathbf{r}_{t}=\mathbf{G} \mathbf{r}_{t}$ for all years, ensuring the consistency of the constructed series (see also Doran (1996) and Rambaldi, Hill and Doran (2004)).

Finally a prediction of PL, $\hat{\mathbf{y}}_{t}$, at time $t$ for all countries is given by:

$$
\begin{equation*}
\hat{\mathbf{y}}_{t}=\mathbf{X}_{t} \hat{\boldsymbol{\beta}}+\left[\mathbf{I}_{N}, 0\right] \hat{\boldsymbol{\alpha}}_{t} \tag{12}
\end{equation*}
$$

A note on national Price Level's Growth rates

An expression for the growth rate in $\mathrm{PL}_{j}=\mathrm{Y}_{j t}$ can be obtained from equation (4):

$$
\begin{aligned}
& Y_{j t}=\frac{P P P_{j t}}{E R_{j t}}=\frac{P P P_{j, t-1}}{E R_{j, t-1}} \times \frac{G D P D^{j} f_{j,[t-1, t]}}{G D P D e f_{U S,[t-1, t]}} \times \frac{E R_{j, t-1}}{E R_{j t}} \\
& Y_{j t}=Y_{j t} \times \frac{G D P D e f_{j,[t-1, t]}}{G D P D e f_{U S,[t-1, t]}} \times \frac{E R_{j, t-1}}{E R_{j t}}
\end{aligned}
$$

Then,

$$
G r_{j t}=\frac{Y_{j t}-Y_{j, t-1}}{Y_{j, t-1}}=\left[\frac{G D P D e f_{j,[t-1, t]}}{G D P D e f_{U S,[t-1, t]}} \times \frac{E R_{j, t-1}}{E R_{j t}}\right]-1
$$

An approximation to $G r_{j t}$ is given by the logged differences of $\mathrm{Y}_{j t}$

$$
\begin{equation*}
y_{j t}-y_{j, t-1}=\ln \left[\frac{G^{\prime} D P \operatorname{Def}_{j,[t-1, t]}}{G D P D e f_{U S,[t-1, t]}}\right]+\ln \left[\frac{E R_{j, t-1}}{E R_{j t}}\right] \tag{13}
\end{equation*}
$$

This is the definition used in the state-space model defined in equation (10).

## 4. Estimation Method and Computational Issues

The estimation requires two main steps. First, a numerical optimisation of the likelihood function over the parameters, $\rho, \phi, \sigma_{1}^{2}$ and $\sigma_{2}^{2}$. The likelihood function takes the form:

$$
\begin{equation*}
\operatorname{\ell n} L(\boldsymbol{\beta}, \varphi ; \mathbf{y})=-\frac{1}{2} \ln (2 \pi) \sum_{t=1}^{T} N_{1 t}-\frac{1}{2} \sum_{t=1}^{T} \ln \left|\mathbf{F}_{t}\right|-\frac{1}{2} \sum_{t=1}^{T} \mathbf{v}_{t}^{\prime} \mathbf{F}_{t}^{-1} \mathbf{v}_{t} \tag{14}
\end{equation*}
$$

where, $\varphi=\left[\begin{array}{lll}\rho \phi & \sigma_{1}^{2} & \sigma_{2}^{2}\end{array}\right]$.

A concentrated form of the likelihood function can be shown to be:

$$
\begin{equation*}
\ln L(\boldsymbol{\beta}, \boldsymbol{\gamma} ; \mathbf{y})=-\ln \left(\sigma_{1}^{2}\right)-\left(\sum_{t=1}^{T} N_{1 t}\right)^{-1} \sum_{t=1}^{T} \ln \left|\mathbf{F}_{t}\right| \tag{15}
\end{equation*}
$$

where,
$\gamma=[\rho \phi \lambda]$
$\lambda=\sigma_{1}^{2} / \sigma_{2}^{2}$
$\sigma_{1}^{2}=\sum_{t=1}^{T} \mathbf{v}_{t}^{\prime} \mathbf{F}_{t}^{-1} \mathbf{v}_{t} / \sum_{t=1}^{T} N_{1 t}$

The vector $\beta$ is estimated by a GLS-MLE routine. That is, at every iteration of the numerical optimisation, a new estimate of $\beta$ is obtained and used to continue the
search. When final estimates of $\gamma$ are obtained, they are used to obtain the final GLSMLE estimates of $\beta$.

With $\beta, \mathbf{H}$ and $\mathbf{Q}$ replaced by their estimates, the state space in (11) and (8) is run through the Kalman Filter and smoother to obtain N vectors $\hat{\boldsymbol{\alpha}}_{t}$ of dimension T, and standard errors for $\hat{y}_{i t}$ from the square root of the diagonal elements of the estimated covariance matrix, $\hat{\mathbf{P}}_{t}$ (we discussed the computation of standard errors for PL below). The GLS-MLE estimator of $\beta$ and the computation of the value of the likelihood function, at every iteration, are obtained through a run of the Kalman Filter ${ }^{4}$. The Kalman filter requires starting values for the state-vector and its covariance matrix, $\alpha_{0}$ and $\mathbf{P}_{0}$, respectively. When the transition equation is nonstationary, the unconditional distribution of the state vector is not defined, and the initial distribution of $\alpha_{0}$ must be specified in terms of a diffuse or non-informative prior. $\hat{\mathbf{P}}_{0}=\kappa \mathbf{I}$, where $\kappa$ is a very large positive scalar (see Harvey (1990), Section 3.3.4). A non-stationary transition equation would occur if the autocorrelation parameter, $\rho$, is equal to one. Alternative algorithms have been proposed for the diffuse prior that do not involve $\kappa$ (de Jong $(1988,1989)$ and Koopman $(1997))$. Normally, the effect of a large $\kappa$ disappears within the first few periods, and has no effect on the GLS-MLE estimator ${ }^{5}$.

[^4]Predicted PPP and prediction standard errors

To obtain predicted PPPs we use equation (12) to obtain $\hat{y}_{i t}=\ln \left(\frac{P \hat{P} P_{i t}}{E R_{i t}}\right)$, from which we obtain $P \hat{P} P_{i t}$. The prediction standard errors are computed as:

$$
\begin{equation*}
\operatorname{se}(P \hat{P} P)=\sqrt{\operatorname{var}\left(P \hat{P} P_{i t}\right)}=\sqrt{\operatorname{var}\left(\hat{y}_{i t}\right)} \times P \hat{P} P \tag{16}
\end{equation*}
$$

The expression has been derived from a Taylor's expansion:

$$
\operatorname{Var}(g(\mathrm{x})) \approx \operatorname{Var}\left(\exp \left(\hat{y}_{i t}\right)\right) \approx \operatorname{Var}\left(\hat{y}_{i t}\right)\left[\frac{\partial}{\partial \hat{y}_{i t}} \exp \left(\hat{y}_{i t}\right)\right]^{2}
$$

where $E R_{i t}$ is assumed known and therefore $\operatorname{Var}\left(\ln \left(\mathrm{ER}_{i t}\right)\right)=0$.

## 5. An Illustration

We present a small illustration of the method using OECD data. These data can be easily accessed through the OECD and World Bank sites. Several of the countries in the OECD were participants in the ICP project since its first benchmark year. We use 24 countries for the illustration. They are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, (S.) Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Turkey and the United Kingdom.

The data for the empirical example was for the period 1970 to 2000, annual, and we discuss next the dependent, explanatory, and covariances related variables.

## Dependent Variable

Benchmark PPP information were collected from the OECD site and the World Bank’s Stars data set. Benchmark years were: 1975, 1980, 1985, 1990, 1993, 1996 and 1999. Not all countries in the sample had participated in all the benchmarks (for France, although a participant in all of them, only two benchmarks were available). The results tables presented in the Appendix (see columns 3 and 4 and 9 and 10) show the exchange rate in units of domestic currency per US\$ and PPP data available by country (two countries are shown per table). For the purpose of analyzing the performance of our method we assumed that countries had participated only sporadically in the benchmark exercises. Only benchmark years marked with the superscript "K", Column 4 (and 10), were included in the estimation and prediction of the panel. Growth rates in PL were computed as per formula (13).

## Explanatory Variables

The following variables were included as explanatory variables in the model:

FDI\%: Foreign direct investment, net inflows (\% of GDP)

LE: Life Expectance in years

SERV\%: Services, value added (\% of GDP)

OPEN\%: Trade (\% of GDP)

School enrollment, secondary (\% net), could not be included as the data available from the OECD site were only five-yearly. We believe this variable should be included, and will be in the next iteration of this exercise

## Covariances related Variables

## a) Measuring spatial autocorrelation

A contiguity matrix was constructed were a value of one (1) is given to countries that share a border. For the purpose of this exercise Australia and New Zealand were assumed to share a border, Iceland was assumed to share borders with Denmark and Norway, and Japan with S. Korea. This is the matrix $\mathbf{W}$ in (7).
b) Capturing accuracy of benchmark data collection

We assume that the accuracy of a PPP benchmark is inversely related to a country's GDP per capita. Therefore the matrix $\mathbf{V}_{\boldsymbol{t}}$ is diagonal (see definitions of equation (9)) with values equal to the inverse of each country's GDP per capita measured in constant US\$ of 1995.

## Results

As stated in the previous section the Kalman filter requires staring values for the parameters in $\gamma$. Note that the autocorrelation and spatial autocorrelation parameters are bounded between 0 and 1 . The parameter $\lambda$ is also bounded between zero and one if $\sigma_{1}^{2}>\sigma_{2}^{26}$. Thus a grid search was used first to obtain starting values for the numerical maximization of the log-likelihood function in (15). Grid-search values were used to start a numerical maximization and obtain estimates of $\gamma$. Table 1 presents the results of the grid search and resulting maximized values. The estimates of $\gamma$ were then used to obtain the estimates of $\beta$ (also shown in Table 1).

[^5]Table 1. Maximum Likelihood Estimates

|  | Grid <br> Search | MLE <br> Estimate | Std Err |
| :---: | :---: | :---: | :---: |
| Intercept |  | -0.3076 | 0.5691 |
| FDI\% |  | 0.00012 | 0.0026 |
| LE |  | 0.00014 | 0.00065 |
| SERV\% |  | -0.00056 | 0.00374 |
| OPEN\% |  | $-0.0107^{* * *}$ | 0.00117 |
| $\hat{\lambda}$ | 0.9 | $0.6897^{* * *}$ | 0.2357 |
| $\hat{\phi}$ | 0.2 | $0.1912^{* * *}$ | 0.0162 |
| $\hat{\rho}$ | 0.9 | $0.9812^{* * *}$ | 0.0400 |
| $\hat{\sigma}_{1}^{2}$ |  |  |  |
| $* * * *$ |  |  |  |
| Significant at the 1\% level. |  |  |  |

Significant at the $1 \%$ level.

These estimates were used to run the Kalman Filter and smoother and obtain predictions (and prediction standard errors) of $\mathbf{y}_{t}$ (the natural log of the Price Level ratio) for all years and countries. With these values a prediction of the PPP values and corresponding prediction standard errors for each year and country were computed. The result's tables in the Appendix show the computed values. The model's prediction of the PL's growth rates had to be identical to the actual values for every observation (this is a model's constraint). They were checked to insure the results were time-space consistent.

To gain a visual understanding we have plotted the results for two countries, Australia and Japan, in Figures 1 and 2. In these figures we have included the PPP value published from the ICP benchmark exercises, our model's predictions and 95\% prediction intervals (based on two standard errors), as well as PWT 6.1 predicted PPP values. The circles show when, for the purpose of this exercise, each country was
assumed to have participated in a benchmark exercise. We assumed Australia and Japan only participated in two benchmark exercises 1980, and 1999.


Figure 1. Australia: PPP Predictions, PPP benchmark information available, benchmark information assumed to be available, PWT 6.1 computed PPP AND 95\% Prediction Interval.


Figure 2. Japan: PPP Predictions, PPP benchmark information available, benchmark information assumed to be available, PWT 6.1 computed PPP AND 95\% Prediction Interval.

The complete results of the illustration are in the Appendix tables. These results are very plausible. It is expected that PPPs will be lower than the respective ER for less developed economies. This is clearly observable in the cases of Spain, Portugal, Mexico, S. Korea and Turkey (arguably among the "less" developed economies in the OECD). In Turkey's case, the model has also been able to follow the depreciation of the currency during the periods of high inflation. The results for Australia are as expected since the exchange rate was fixed until 1983 and believed to have been overvalued during this period. This is reflected by the PPPs being considerably higher than the corresponding ER value until 1983.

The prediction of a country's PPP is extremely accurate for years when the benchmark value is used. Overall, prediction errors become generally smaller as more benchmarks have been observed. Thus, in general for any given country, prediction errors are smaller for the later years of the sample. The predictions are
more accurate for countries that are contiguous to others in the sample. This is anticipated as the use of a spatially correlated covariance is expected to improve the spatial prediction and highlights the importance of a wide coverage of the benchmark exercises.

These results are very promising, considering it is just an illustration. There are identified improvements that can be made, namely, the economic model, alternative spatial structure and the inclusion of all available benchmark information can only improve the performance of the method and reduce the size of the standard errors.

## Conclusions

This paper shows how a constrained state-space formulation can be used to generate time-space consistent comparable PPPs (therefore GDP) from all the available information, namely, benchmark PPP values obtained by the International Comparisons Program (only covering some countries and some years) and national price levels’ growth rates (available for all years). We treat the information as an unbalanced panel and constrain the results to preserve national price levels' growth rates. Previous methods are based on extrapolating the results of a single benchmark exercise (usually the latest available) to non-participating countries through a regression framework and through time using of national price levels' growth rates. Our results produce some re-adjustment of some of the benchmark values so that the resulting panel of N countries and T years is consistent with national growth rates.

An illustration is presented for 24 OECD countries which, for the purpose of the exercise, are assumed to have only participated in the ICP benchmarking sporadically. A simple econometric model of the log of national price levels (the ratio of PPP to the exchange rate in domestic currency per US\$) is written in a constrained
state-space form accounting for autocorrelation and spatial autocorrelation. The results are consistent with general expectations on the behaviour of the PL ratio.

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## APPENDIX: RESULTS TABLES

## Headings' definitions:

ER: Units of domestic currency per US\$
BPPP: Published Benchmark PPPs values
PLPL: Predicted natural logarithm of PPP/ER
SE: Standard Error of Predicted PPP
${ }^{K}$ Country assumed to have participated in the ICP Benchmark exercise in these years

|  | AUSTRALIA | ER | $\begin{gathered} \hline \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | AUSTRIA | ER | $\begin{gathered} \hline \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 0.883 |  | 0.2355 | 1.1171 | 0.2925 |  | 1.814 |  | 0.8218 | 4.1259 | 1.9943 |
| 1972 |  | 0.839 |  | 0.2587 | 1.0863 | 0.2655 |  | 1.680 |  | 0.8281 | 3.8452 | 1.7880 |
| 1973 |  | 0.704 |  | 0.3340 | 0.9833 | 0.2222 |  | 1.423 |  | 0.8511 | 3.3326 | 1.4885 |
| 1974 |  | 0.697 |  | 0.2458 | 0.8908 | 0.1842 |  | 1.358 |  | 0.7332 | 2.8278 | 1.2108 |
| 1975 |  | 0.764 | 0.943 | 0.1959 | 0.9293 | 0.1736 |  | 1.266 | 1.294 | 0.6865 | 2.5146 | 1.0299 |
| 1976 |  | 0.818 |  | 0.2208 | 1.0205 | 0.1693 |  | 1.304 |  | 0.6083 | 2.3952 | 0.9363 |
| 1977 |  | 0.902 |  | 0.1903 | 1.0908 | 0.1556 |  | 1.201 |  | 0.6089 | 2.2081 | 0.8216 |
| 1978 |  | 0.874 |  | 0.2331 | 1.1030 | 0.1278 |  | 1.055 |  | 0.6053 | 1.9331 | 0.6825 |
| 1979 |  | 0.895 |  | 0.1797 | 1.0707 | 0.0873 |  | 0.972 |  | 0.5130 | 1.6226 | 0.5414 |
| 1980 |  | 0.878 | $1.051^{\mathrm{K}}$ | 0.1800 | 1.0514 | 0.0009 |  | 0.940 | 1.138 | 0.3903 | 1.3891 | 0.4359 |
| 1981 |  | 0.870 |  | 0.1700 | 1.0314 | 0.0808 |  | 1.157 |  | 0.1949 | 1.4064 | 0.4127 |
| 1982 |  | 0.986 |  | 0.1138 | 1.1047 | 0.1188 |  | 1.240 |  | 0.2445 | 1.5830 | 0.4312 |
| 1983 |  | 1.110 |  | 0.1196 | 1.2510 | 0.1597 |  | 1.305 |  | 0.2540 | 1.6829 | 0.4216 |
| 1984 |  | 1.140 |  | 0.1357 | 1.3051 | 0.1861 |  | 1.454 |  | 0.1806 | 1.7419 | 0.3961 |
| 1985 |  | 1.432 | 1.178 | 0.0015 | 1.4340 | 0.2207 |  | 1.504 | 1.079 | 0.1454 | 1.7389 | 0.3525 |
| 1986 |  | 1.496 |  | 0.0934 | 1.6425 | 0.2666 |  | 1.110 |  | 0.3372 | 1.5544 | 0.2839 |
| 1987 |  | 1.428 |  | 0.1452 | 1.6514 | 0.2780 |  | 0.919 |  | 0.3260 | 1.2730 | 0.2028 |
| 1988 |  | 1.280 |  | 0.1863 | 1.5420 | 0.2656 |  | 0.897 |  | 0.2124 | 1.1096 | 0.1452 |
| 1989 |  | 1.265 |  | 0.1290 | 1.4387 | 0.2504 |  | 0.962 |  | 0.1171 | 1.0810 | 0.1011 |
| 1990 |  | 1.281 | 1.387 | 0.0925 | 1.4053 | 0.2445 |  | 0.826 | $1.020^{\mathrm{K}}$ | 0.2109 | 1.0203 | 0.0007 |
| 1991 |  | 1.284 |  | 0.0671 | 1.3729 | 0.2361 |  | 0.849 |  | 0.1043 | 0.9418 | 0.0864 |
| 1992 |  | 1.362 |  | -0.0034 | 1.3570 | 0.2278 |  | 0.799 |  | 0.1545 | 0.9320 | 0.1178 |
| 1993 |  | 1.471 | 1.353 | -0.0568 | 1.3894 | 0.2246 |  | 0.845 | 1.008 | 0.1147 | 0.9480 | 0.1426 |
| 1994 |  | 1.368 |  | -0.0178 | 1.3437 | 0.2056 |  | 0.830 |  | 0.1338 | 0.9489 | 0.1657 |
| 1995 |  | 1.349 |  | -0.0753 | 1.2511 | 0.1769 |  | 0.733 |  | 0.1526 | 0.8535 | 0.1665 |
| 1996 |  | 1.278 | 1.299 | -0.0733 | 1.1876 | 0.1497 |  | 0.769 | 0.987 | 0.0199 | 0.7849 | 0.1671 |
| 1997 |  | 1.347 |  | -0.1619 | 1.1460 | 0.1223 |  | 0.887 |  | -0.1159 | 0.7898 | 0.1806 |
| 1998 |  | 1.592 |  | -0.2512 | 1.2382 | 0.0966 |  | 0.900 |  | -0.1368 | 0.7846 | 0.1903 |
| 1999 |  | 1.550 | $1.297^{\mathrm{K}}$ | -0.1782 | 1.2970 | 0.0009 |  | 0.939 | 0.946 | -0.2150 | 0.7570 | 0.1929 |
| 2000 |  | 1.725 |  | -0.2725 | 1.3134 | 0.1048 |  | 1.085 |  | -0.3664 | 0.7524 | 0.2008 |


|  | BELGIUM | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | CANADA | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 1.211 |  | 0.5634 | 2.1279 | 0.9339 |  | 1.010 |  | 0.5275 | 1.7113 | 0.6884 |
| 1972 |  | 1.091 |  | 0.5943 | 1.9768 | 0.8274 |  | 0.990 |  | 0.5022 | 1.6356 | 0.6336 |
| 1973 |  | 0.966 |  | 0.5224 | 1.6291 | 0.6485 |  | 1.000 |  | 0.4563 | 1.5783 | 0.5879 |
| 1974 |  | 0.966 |  | 0.3163 | 1.3249 | 0.4998 |  | 0.978 |  | 0.4393 | 1.5175 | 0.5427 |
| 1975 |  | 0.912 | 1.111 | 0.3385 | 1.2789 | 0.4553 |  | 1.017 | 1.222 | 0.3779 | 1.4843 | 0.5086 |
| 1976 |  | 0.957 |  | 0.3162 | 1.3129 | 0.4391 |  | 0.986 |  | 0.4272 | 1.5115 | 0.4953 |
| 1977 |  | 0.889 |  | 0.3251 | 1.2299 | 0.3840 |  | 1.064 |  | 0.3514 | 1.5114 | 0.4723 |
| 1978 |  | 0.781 |  | 0.3367 | 1.0932 | 0.3162 |  | 1.141 |  | 0.3028 | 1.5441 | 0.4589 |
| 1979 |  | 0.727 |  | 0.2208 | 0.9064 | 0.2404 |  | 1.171 |  | 0.2725 | 1.5384 | 0.4332 |
| 1980 |  | 0.725 | 0.999 | 0.0684 | 0.7762 | 0.1861 |  | 1.169 | 1.268 | 0.2492 | 1.5001 | 0.3988 |
| 1981 |  | 0.920 |  | -0.1589 | 0.7852 | 0.1667 |  | 1.199 |  | 0.2207 | 1.4950 | 0.3733 |
| 1982 |  | 1.133 |  | -0.2615 | 0.8720 | 0.1588 |  | 1.234 |  | 0.2441 | 1.5747 | 0.3671 |
| 1983 |  | 1.268 |  | -0.2970 | 0.9418 | 0.1388 |  | 1.232 |  | 0.2713 | 1.6164 | 0.3490 |
| 1984 |  | 1.432 |  | -0.3854 | 0.9743 | 0.1005 |  | 1.295 |  | 0.1903 | 1.5665 | 0.3102 |
| 1985 |  | 1.472 | $1.003^{K}$ | -0.3840 | 1.0026 | 0.0007 |  | 1.366 | 1.284 | 0.1322 | 1.5584 | 0.2789 |
| 1986 |  | 1.107 |  | -0.0384 | 1.0657 | 0.0974 |  | 1.390 |  | 0.1260 | 1.5761 | 0.2498 |
| 1987 |  | 0.926 |  | 0.0716 | 0.9942 | 0.1112 |  | 1.326 |  | 0.1555 | 1.5491 | 0.2107 |
| 1988 |  | 0.912 |  | 0.0426 | 0.9512 | 0.1064 |  | 1.231 |  | 0.1658 | 1.4526 | 0.1598 |
| 1989 |  | 0.977 |  | -0.0001 | 0.9767 | 0.0892 |  | 1.184 |  | 0.1381 | 1.3594 | 0.1047 |
| 1990 |  | 0.828 | $0.978^{\mathrm{K}}$ | 0.1659 | 0.9779 | 0.0007 |  | 1.167 | $1.303{ }^{\mathrm{K}}$ | 0.1104 | 1.3030 | 0.0010 |
| 1991 |  | 0.847 |  | 0.1120 | 0.9468 | 0.0934 |  | 1.146 |  | 0.1087 | 1.2772 | 0.0889 |
| 1992 |  | 0.797 |  | 0.2009 | 0.9744 | 0.1310 |  | 1.209 |  | 0.0484 | 1.2686 | 0.1117 |
| 1993 |  | 0.858 | 0.925 | 0.2040 | 1.0517 | 0.1663 |  | 1.290 | 1.263 | -0.0151 | 1.2708 | 0.1186 |
| 1994 |  | 0.829 |  | 0.2332 | 1.0472 | 0.1940 |  | 1.366 |  | -0.0822 | 1.2578 | 0.1107 |
| 1995 |  | 0.731 |  | 0.1945 | 0.8877 | 0.1847 |  | 1.372 |  | -0.1146 | 1.2238 | 0.0852 |
| 1996 |  | 0.768 | 0.913 | 0.0309 | 0.7916 | 0.1804 |  | 1.364 | $1.185^{K}$ | -0.1401 | 1.1853 | 0.0009 |
| 1997 |  | 0.887 |  | -0.1209 | 0.7858 | 0.1926 |  | 1.385 |  | -0.2046 | 1.1284 | 0.0852 |
| 1998 |  | 0.900 |  | -0.1406 | 0.7818 | 0.2033 |  | 1.484 |  | -0.3050 | 1.0936 | 0.1157 |
| 1999 |  | 0.939 | 0.934 | -0.2021 | 0.7668 | 0.2094 |  | 1.486 | 1.191 | -0.3164 | 1.0827 | 0.1390 |
| 2000 |  | 1.085 |  | -0.4174 | 0.7150 | 0.2048 |  | 1.485 |  | -0.3509 | 1.0456 | 0.1536 |


|  | DENMARK | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | FINLAND | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 7.417 |  | 0.8030 | 16.5565 | 9.8954 |  | 0.704 |  | 0.3933 | 1.0430 | 0.2833 |
| 1972 |  | 6.949 |  | 0.8471 | 16.2119 | 9.3841 |  | 0.697 |  | 0.3984 | 1.0387 | 0.2611 |
| 1973 |  | 6.050 |  | 0.8619 | 14.3224 | 8.0223 |  | 0.643 |  | 0.4327 | 0.9907 | 0.2280 |
| 1974 |  | 6.095 |  | 0.7094 | 12.3892 | 6.7094 |  | 0.635 |  | 0.3880 | 0.9355 | 0.1943 |
| 1975 |  | 5.746 | 8.019 | 0.7152 | 11.7491 | 6.1459 |  | 0.619 | 0.777 | 0.3741 | 0.8994 | 0.1649 |
| 1976 |  | 6.045 |  | 0.6806 | 11.9393 | 6.0597 |  | 0.650 |  | 0.3964 | 0.9661 | 0.1616 |
| 1977 |  | 6.003 |  | 0.7075 | 12.1805 | 5.9873 |  | 0.678 |  | 0.3930 | 1.0040 | 0.1484 |
| 1978 |  | 5.515 |  | 0.7668 | 11.8724 | 5.6386 |  | 0.693 |  | 0.3777 | 1.0103 | 0.1242 |
| 1979 |  | 5.261 |  | 0.7268 | 10.8819 | 4.9794 |  | 0.655 |  | 0.3798 | 0.9578 | 0.0847 |
| 1980 |  | 5.636 | 8.383 | 0.6030 | 10.2999 | 4.5264 |  | 0.627 | $0.859^{\text {K }}$ | 0.3140 | 0.8588 | 0.0006 |
| 1981 |  | 7.123 |  | 0.4401 | 11.0612 | 4.6548 |  | 0.726 |  | 0.1770 | 0.8664 | 0.0771 |
| 1982 |  | 8.332 |  | 0.4408 | 12.9475 | 5.2056 |  | 0.811 |  | 0.2105 | 1.0006 | 0.1247 |
| 1983 |  | 9.145 |  | 0.4710 | 14.6470 | 5.6115 |  | 0.937 |  | 0.2123 | 1.1583 | 0.1752 |
| 1984 |  | 10.357 |  | 0.4336 | 15.9784 | 5.8163 |  | 1.011 |  | 0.2522 | 1.3007 | 0.2250 |
| 1985 |  | 10.596 | 9.140 | 0.4470 | 16.5694 | 5.7091 |  | 1.042 | 0.977 | 0.2724 | 1.3688 | 0.2623 |
| 1986 |  | 8.091 |  | 0.6362 | 15.2870 | 4.9875 |  | 0.853 |  | 0.4148 | 1.2909 | 0.2686 |
| 1987 |  | 6.840 |  | 0.6560 | 13.1822 | 4.0515 |  | 0.739 |  | 0.4062 | 1.1098 | 0.2471 |
| 1988 |  | 6.732 |  | 0.5778 | 11.9959 | 3.4506 |  | 0.704 |  | 0.3731 | 1.0217 | 0.2410 |
| 1989 |  | 7.310 |  | 0.4998 | 12.0497 | 3.2157 |  | 0.722 |  | 0.3335 | 1.0073 | 0.2498 |
| 1990 |  | 6.189 | 9.393 | 0.6051 | 11.3338 | 2.7762 |  | 0.643 | 1.074 | 0.4033 | 0.9625 | 0.2493 |
| 1991 |  | 6.397 |  | 0.4760 | 10.2962 | 2.2841 |  | 0.680 |  | 0.3092 | 0.9265 | 0.2495 |
| 1992 |  | 6.036 |  | 0.5072 | 10.0235 | 1.9732 |  | 0.753 |  | 0.2308 | 0.9490 | 0.2646 |
| 1993 |  | 6.484 | 8.786 | 0.4452 | 10.1205 | 1.7087 |  | 0.961 | 1.024 | 0.0588 | 1.0188 | 0.2931 |
| 1994 |  | 6.361 |  | 0.4651 | 10.1271 | 1.3827 |  | 0.879 |  | 0.1375 | 1.0080 | 0.2983 |
| 1995 |  | 5.602 |  | 0.4748 | 9.0066 | 0.8611 |  | 0.734 |  | 0.1389 | 0.8438 | 0.2563 |
| 1996 |  | 5.799 | $8.328^{\mathrm{K}}$ | 0.3620 | 8.3278 | 0.0047 |  | 0.773 | 0.990 | -0.0251 | 0.7535 | 0.2343 |
| 1997 |  | 6.605 |  | 0.2510 | 8.4890 | 0.6574 |  | 0.873 |  | -0.1046 | 0.7864 | 0.2499 |
| 1998 |  | 6.701 |  | 0.2386 | 8.5067 | 0.6588 |  | 0.899 |  | -0.0799 | 0.8298 | 0.2687 |
| 1999 |  | 6.976 | $8.244^{\mathrm{K}}$ | 0.1670 | 8.2441 | 0.0046 |  | 0.939 | 0.996 | -0.1080 | 0.8425 | 0.2774 |
| 2000 |  | 8.083 |  | 0.0370 | 8.3874 | 0.7882 |  | 1.085 |  | -0.2112 | 0.8787 | 0.2944 |


|  | FRANCE | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | GERMANY | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 0.845 |  | 2.2817 | 8.2749 | 3.7501 |  | 1.785 |  | 0.3550 | 2.5454 | 1.9008 |
| 1972 |  | 0.769 |  | 2.2741 | 7.4742 | 3.2304 |  | 1.630 |  | 0.3696 | 2.3592 | 1.7077 |
| 1973 |  | 0.680 |  | 2.2316 | 6.3302 | 2.6015 |  | 1.367 |  | 0.3934 | 2.0252 | 1.4197 |
| 1974 |  | 0.734 |  | 2.0345 | 5.6133 | 2.1866 |  | 1.323 |  | 0.2452 | 1.6907 | 1.1467 |
| 1975 |  | 0.653 |  | 2.0774 | 5.2166 | 1.9180 |  | 1.258 | 1.566 | 0.1928 | 1.5254 | 1.0001 |
| 1976 |  | 0.729 |  | 1.9522 | 5.1325 | 1.7727 |  | 1.287 |  | 0.1323 | 1.4695 | 0.9308 |
| 1977 |  | 0.749 |  | 1.9332 | 5.1772 | 1.6690 |  | 1.187 |  | 0.1492 | 1.3784 | 0.8426 |
| 1978 |  | 0.688 |  | 1.9545 | 4.8574 | 1.4501 |  | 1.027 |  | 0.1729 | 1.2208 | 0.7191 |
| 1979 |  | 0.649 |  | 1.8962 | 4.3200 | 1.1821 |  | 0.937 |  | 0.1090 | 1.0450 | 0.5923 |
| 1980 |  | 0.644 |  | 1.8057 | 3.9196 | 0.9696 |  | 0.929 | 1.309 | 0.0074 | 0.9363 | 0.5096 |
| 1981 |  | 0.829 |  | 1.6109 | 4.1487 | 0.9093 |  | 1.156 |  | -0.1881 | 0.9574 | 0.4995 |
| 1982 |  | 1.002 |  | 1.6079 | 5.0018 | 0.9399 |  | 1.241 |  | -0.1608 | 1.0564 | 0.5272 |
| 1983 |  | 1.162 |  | 1.6079 | 5.8005 | 0.8813 |  | 1.306 |  | -0.1667 | 1.1051 | 0.5262 |
| 1984 |  | 1.332 |  | 1.5704 | 6.4064 | 0.6818 |  | 1.455 |  | -0.2292 | 1.1570 | 0.5242 |
| 1985 |  | 1.370 | $6.617^{\mathrm{K}}$ | 1.5750 | 6.6170 | 0.0047 |  | 1.505 | 1.142 | -0.2389 | 1.1854 | 0.5094 |
| 1986 |  | 1.056 |  | 1.8335 | 6.6056 | 0.6472 |  | 1.110 |  | 0.0274 | 1.1412 | 0.4792 |
| 1987 |  | 0.916 |  | 1.8765 | 5.9838 | 0.7605 |  | 0.919 |  | 0.0862 | 1.0018 | 0.4085 |
| 1988 |  | 0.908 |  | 1.8715 | 5.9011 | 0.8276 |  | 0.898 |  | 0.0755 | 0.9683 | 0.3806 |
| 1989 |  | 0.973 |  | 1.8800 | 6.3737 | 0.9047 |  | 0.961 |  | 0.0782 | 1.0394 | 0.3902 |
| 1990 |  | 0.830 |  | 2.0494 | 6.4444 | 0.8549 |  | 0.826 | 1.068 | 0.2556 | 1.0667 | 0.3781 |
| 1991 |  | 0.860 |  | 1.9772 | 6.2119 | 0.7667 |  | 0.849 |  | 0.2127 | 1.0496 | 0.3506 |
| 1992 |  | 0.807 |  | 2.0543 | 6.2957 | 0.6094 |  | 0.799 |  | 0.3160 | 1.0952 | 0.3404 |
| 1993 |  | 0.863 | $6.570^{\text {K }}$ | 2.0294 | 6.5700 | 0.0043 |  | 0.845 | 1.075 | 0.3246 | 1.1695 | 0.3323 |
| 1994 |  | 0.846 |  | 2.0252 | 6.4139 | 0.6699 |  | 0.830 |  | 0.3584 | 1.1873 | 0.3020 |
| 1995 |  | 0.761 |  | 1.9965 | 5.6028 | 0.8194 |  | 0.733 |  | 0.3717 | 1.0626 | 0.2359 |
| 1996 |  | 0.780 |  | 1.8670 | 5.0450 | 0.8948 |  | 0.769 | 1.037 | 0.2450 | 0.9830 | 0.1825 |
| 1997 |  | 0.890 |  | 1.7259 | 4.9986 | 1.0120 |  | 0.887 |  | 0.1261 | 1.0058 | 0.1564 |
| 1998 |  | 0.899 |  | 1.7098 | 4.9719 | 1.1103 |  | 0.900 |  | 0.1171 | 1.0115 | 0.1140 |
| 1999 |  | 0.939 |  | 1.6310 | 4.7951 | 1.1559 |  | 0.939 | $0.978^{\mathrm{K}}$ | 0.0407 | 0.9776 | 0.0006 |
| 2000 |  | 1.085 |  | 1.4875 | 4.8038 | 1.2422 |  | 1.085 |  | -0.0972 | 0.9849 | 0.1124 |


|  | GREECE | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | HUNGARY | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 0.088 |  | 0.0676 | 0.0942 | 0.0479 |  | 59.822 |  | 4.1320 | 0.9602 | 0.3361 |
| 1972 |  | 0.088 |  | 0.0216 | 0.0899 | 0.0443 |  | 55.260 |  | 4.0957 | 0.9198 | 0.3071 |
| 1973 |  | 0.087 |  | 0.0087 | 0.0877 | 0.0418 |  | 48.966 |  | 4.0779 | 0.8296 | 0.2635 |
| 1974 |  | 0.088 |  | -0.0458 | 0.0841 | 0.0387 |  | 46.752 |  | 4.2164 | 0.6897 | 0.2076 |
| 1975 |  | 0.094 | 0.086 | -0.1151 | 0.0839 | 0.0373 |  | 43.971 |  | 4.2907 | 0.6022 | 0.1711 |
| 1976 |  | 0.107 |  | -0.1316 | 0.0940 | 0.0404 |  | 41.575 |  | 4.2175 | 0.6126 | 0.1634 |
| 1977 |  | 0.108 |  | -0.0668 | 0.1011 | 0.0418 |  | 40.961 |  | 4.1902 | 0.6203 | 0.1545 |
| 1978 |  | 0.108 |  | -0.0548 | 0.1021 | 0.0406 |  | 37.911 |  | 4.1381 | 0.6048 | 0.1395 |
| 1979 |  | 0.109 |  | -0.0435 | 0.1041 | 0.0398 |  | 35.578 |  | 4.1299 | 0.5723 | 0.1210 |
| 1980 |  | 0.125 | 0.121 | -0.1717 | 0.1054 | 0.0386 |  | 32.532 |  | 4.0602 | 0.5611 | 0.1072 |
| 1981 |  | 0.163 |  | -0.2807 | 0.1228 | 0.0431 |  | 34.314 |  | 4.0766 | 0.5821 | 0.0986 |
| 1982 |  | 0.196 |  | -0.1694 | 0.1655 | 0.0554 |  | 36.631 |  | 4.0171 | 0.6595 | 0.0958 |
| 1983 |  | 0.258 |  | -0.1316 | 0.2265 | 0.0722 |  | 42.671 |  | 4.0032 | 0.7790 | 0.0916 |
| 1984 |  | 0.331 |  | -0.0606 | 0.3113 | 0.0941 |  | 48.042 |  | 3.9590 | 0.9168 | 0.0756 |
| 1985 |  | 0.405 | 0.231 | 0.0100 | 0.4094 | 0.1169 |  | 50.119 | $1.020^{\mathrm{K}}$ | 3.8946 | 1.0200 | 0.0016 |
| 1986 |  | 0.411 |  | 0.1404 | 0.4727 | 0.1270 |  | 45.832 |  | 3.8124 | 1.0127 | 0.0771 |
| 1987 |  | 0.397 |  | 0.1783 | 0.4750 | 0.1193 |  | 46.971 |  | 3.7896 | 1.0618 | 0.1058 |
| 1988 |  | 0.416 |  | 0.1903 | 0.5036 | 0.1174 |  | 50.413 |  | 3.6828 | 1.2680 | 0.1411 |
| 1989 |  | 0.477 |  | 0.1770 | 0.5689 | 0.1220 |  | 59.066 |  | 3.6039 | 1.6077 | 0.1848 |
| 1990 |  | 0.465 | 0.413 | 0.3032 | 0.6300 | 0.1226 |  | 63.206 |  | 3.3919 | 2.1265 | 0.2365 |
| 1991 |  | 0.535 |  | 0.1866 | 0.6446 | 0.1148 |  | 74.735 |  | 3.2903 | 2.7832 | 0.2771 |
| 1992 |  | 0.559 |  | 0.1783 | 0.6686 | 0.1067 |  | 78.988 |  | 3.1631 | 3.3409 | 0.2542 |
| 1993 |  | 0.673 | 0.541 | 0.0551 | 0.7109 | 0.0985 |  | 91.933 | $4.160^{\mathrm{K}}$ | 3.0955 | 4.1601 | 0.0069 |
| 1994 |  | 0.712 |  | 0.0588 | 0.7551 | 0.0856 |  | 105.160 |  | 2.9784 | 5.3498 | 0.4314 |
| 1995 |  | 0.680 |  | 0.0264 | 0.6981 | 0.0562 |  | 125.681 |  | 2.9304 | 6.7086 | 0.7579 |
| 1996 |  | 0.706 | $0.628^{\mathrm{K}}$ | -0.1181 | 0.6277 | 0.0006 |  | 152.647 |  | 2.8971 | 8.4231 | 1.1546 |
| 1997 |  | 0.801 |  | -0.1714 | 0.6751 | 0.0539 |  | 186.789 |  | 2.8721 | 10.5686 | 1.6572 |
| 1998 |  | 0.867 |  | -0.1683 | 0.7329 | 0.0820 |  | 214.402 |  | 2.8852 | 11.9729 | 2.0793 |
| 1999 |  | 0.897 | 0.677 | -0.1582 | 0.7658 | 0.1039 |  | 237.146 |  | 2.8966 | 13.0928 | 2.4675 |
| 2000 |  | 1.072 |  | -0.2915 | 0.8012 | 0.1244 |  | 282.179 |  | 2.9869 | 14.2336 | 2.8714 |




|  | $\begin{aligned} & \hline \text { KOREA } \\ & \text { (S.) } \end{aligned}$ | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | $\begin{gathered} \text { Predicted } \\ \text { PPP } \end{gathered}$ | SE | MEXICO | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 347.150 |  | -0.8128 | 154.0045 | 52.9666 |  | 0.013 |  | -3.4439 | 0.0004 | 0.0002 |
| 1972 |  | 392.890 |  | -0.8062 | 175.4384 | 57.5059 |  | 0.013 |  | -3.4261 | 0.0004 | 0.0002 |
| 1973 |  | 398.320 |  | -0.8310 | 173.5177 | 54.0971 |  | 0.013 |  | -3.3987 | 0.0004 | 0.0002 |
| 1974 |  | 404.470 |  | -0.8745 | 168.7002 | 49.8359 |  | 0.013 |  | -3.3488 | 0.0004 | 0.0002 |
| 1975 |  | 484.000 | 222.853 | -0.9091 | 194.9995 | 54.3265 |  | 0.013 | 0.0097 | -3.3156 | 0.0005 | 0.0002 |
| 1976 |  | 484.000 |  | -0.7384 | 231.3011 | 60.4866 |  | 0.015 |  | -3.3519 | 0.0005 | 0.0002 |
| 1977 |  | 484.000 |  | -0.7119 | 237.5036 | 57.9091 |  | 0.023 |  | -3.3579 | 0.0008 | 0.0004 |
| 1978 |  | 484.000 |  | -0.6438 | 254.2371 | 57.3578 |  | 0.023 |  | -3.1491 | 0.0010 | 0.0004 |
| 1979 |  | 484.000 |  | -0.5972 | 266.3633 | 55.0131 |  | 0.023 |  | -3.1104 | 0.0010 | 0.0004 |
| 1980 |  | 607.430 | 413.376 | -0.7241 | 294.4500 | 54.8452 |  | 0.023 | 0.0188 | -3.0268 | 0.0011 | 0.0004 |
| 1981 |  | 681.030 |  | -0.7132 | 333.7489 | 55.2058 |  | 0.025 |  | -2.9799 | 0.0012 | 0.0005 |
| 1982 |  | 731.080 |  | -0.6799 | 370.4131 | 52.6620 |  | 0.056 |  | -3.1702 | 0.0024 | 0.0009 |
| 1983 |  | 775.750 |  | -0.6319 | 412.3770 | 47.5372 |  | 0.120 |  | -2.8515 | 0.0069 | 0.0025 |
| 1984 |  | 805.980 |  | -0.6089 | 438.4306 | 35.5105 |  | 0.168 |  | -2.4321 | 0.0147 | 0.0051 |
| 1985 |  | 870.020 | $472.565^{\text {K }}$ | -0.6103 | 472.5720 | 0.6921 |  | 0.257 | 0.1364 | -2.2567 | 0.0269 | 0.0088 |
| 1986 |  | 881.450 |  | -0.5185 | 524.8127 | 37.8221 |  | 0.612 |  | -2.2315 | 0.0657 | 0.0205 |
| 1987 |  | 822.570 |  | -0.4587 | 519.9593 | 45.8851 |  | 1.378 |  | -1.8187 | 0.2236 | 0.0665 |
| 1988 |  | 731.470 |  | -0.3727 | 503.8799 | 44.4658 |  | 2.273 |  | -1.3460 | 0.5917 | 0.1667 |
| 1989 |  | 671.460 |  | -0.2844 | 505.2344 | 36.4096 |  | 2.462 |  | -1.0667 | 0.8471 | 0.2253 |
| 1990 |  | 707.764 | $562.172^{\mathrm{K}}$ | -0.2303 | 562.1658 | 0.6727 |  | 2.813 | 1.533 | -0.9869 | 1.0483 | 0.2619 |
| 1991 |  | 733.353 |  | -0.1748 | 615.7412 | 48.9713 |  | 3.018 |  | -0.8568 | 1.2813 | 0.2988 |
| 1992 |  | 780.651 |  | -0.1572 | 667.0685 | 74.0305 |  | 3.095 |  | -0.7596 | 1.4479 | 0.3127 |
| 1993 |  | 802.671 | 660.590 | -0.1134 | 716.5917 | 96.0949 |  | 3.116 | 2.122 | -0.7137 | 1.5261 | 0.3021 |
| 1994 |  | 803.446 |  | -0.0800 | 741.6905 | 113.2943 |  | 3.375 |  | -0.7382 | 1.6132 | 0.2887 |
| 1995 |  | 771.273 |  | -0.0797 | 712.1561 | 119.9611 |  | 6.419 |  | -0.9929 | 2.3783 | 0.3769 |
| 1996 |  | 804.453 | 744.399 | -0.1522 | 690.8894 | 125.7248 |  | 7.600 | 3.789 | -0.7587 | 3.5587 | 0.4838 |
| 1997 |  | 951.289 |  | -0.2634 | 730.9838 | 141.6690 |  | 7.919 |  | -0.6381 | 4.1833 | 0.4599 |
| 1998 |  | 1401.440 |  | -0.4853 | 862.5749 | 176.1817 |  | 9.136 |  | -0.6291 | 4.8700 | 0.3751 |
| 1999 |  | 1188.820 | 754.893 | -0.2707 | 906.9243 | 193.6531 |  | 9.560 | $5.634^{\mathrm{K}}$ | -0.5289 | 5.6337 | 0.0100 |
| 2000 |  | 1130.960 |  | -0.3630 | 786.6546 | 176.3754 |  | 9.456 |  | -0.4566 | 5.9892 | 0.4525 |


|  | NETHERLAND | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | $\begin{gathered} \text { NEW } \\ \text { ZEALAND } \end{gathered}$ | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 1.5893 |  | 0.7590 | 3.3949 | 1.0302 |  | 0.8806 |  | 0.0538 | 0.9293 | 0.1557 |
| 1972 |  | 1.4564 |  | 0.7968 | 3.2309 | 0.9148 |  | 0.8368 |  | 0.0615 | 0.8899 | 0.1278 |
| 1973 |  | 1.2686 |  | 0.8007 | 2.8253 | 0.7409 |  | 0.7368 |  | 0.0675 | 0.7883 | 0.0913 |
| 1974 |  | 1.2199 |  | 0.6223 | 2.2730 | 0.5462 |  | 0.7154 |  | -0.0775 | 0.6621 | 0.0538 |
| 1975 |  | 1.1476 | 1.3889 | 0.5778 | 2.0452 | 0.4440 |  | 0.8323 | $0.6840^{K}$ | -0.1962 | 0.6840 | 0.0006 |
| 1976 |  | 1.1998 |  | 0.5412 | 2.0613 | 0.3966 |  | 1.0049 |  | -0.1743 | 0.8442 | 0.0662 |
| 1977 |  | 1.1137 |  | 0.5832 | 1.9954 | 0.3292 |  | 1.0303 |  | -0.0782 | 0.9528 | 0.1027 |
| 1978 |  | 0.9818 |  | 0.6144 | 1.8148 | 0.2422 |  | 0.9644 |  | -0.0035 | 0.9610 | 0.1231 |
| 1979 |  | 0.9103 |  | 0.5107 | 1.5170 | 0.1418 |  | 0.9785 |  | -0.0476 | 0.9330 | 0.1337 |
| 1980 |  | 0.9022 | $1.2784^{\mathrm{K}}$ | 0.3485 | 1.2784 | 0.0009 |  | 1.0267 | 0.9643 | -0.0868 | 0.9414 | 0.1459 |
| 1981 |  | 1.1323 |  | 0.1098 | 1.2637 | 0.1161 |  | 1.1528 |  | -0.1043 | 1.0386 | 0.1710 |
| 1982 |  | 1.2117 |  | 0.1316 | 1.3822 | 0.1779 |  | 1.3326 |  | -0.1281 | 1.1724 | 0.2016 |
| 1983 |  | 1.2951 |  | 0.1079 | 1.4426 | 0.2252 |  | 1.4968 |  | -0.1082 | 1.3433 | 0.2382 |
| 1984 |  | 1.4560 |  | 0.0056 | 1.4641 | 0.2615 |  | 1.7640 |  | -0.1709 | 1.4869 | 0.2689 |
| 1985 |  | 1.5072 | 1.1290 | -0.0432 | 1.4435 | 0.2856 |  | 2.0234 | 1.2359 | -0.1353 | 1.7673 | 0.3227 |
| 1986 |  | 1.1118 |  | 0.2372 | 1.4094 | 0.3012 |  | 1.9132 |  | 0.0967 | 2.1075 | 0.3851 |
| 1987 |  | 0.9192 |  | 0.2952 | 1.2349 | 0.2789 |  | 1.6946 |  | 0.2184 | 2.1081 | 0.3819 |
| 1988 |  | 0.8969 |  | 0.2138 | 1.1107 | 0.2609 |  | 1.5264 |  | 0.2402 | 1.9409 | 0.3453 |
| 1989 |  | 0.9623 |  | 0.1328 | 1.0989 | 0.2651 |  | 1.6722 |  | 0.1250 | 1.8948 | 0.3274 |
| 1990 |  | 0.8263 | 0.9824 | 0.2488 | 1.0598 | 0.2597 |  | 1.6762 | 1.6090 | 0.1247 | 1.8988 | 0.3147 |
| 1991 |  | 0.8484 |  | 0.1612 | 0.9968 | 0.2561 |  | 1.7335 |  | 0.0688 | 1.8569 | 0.2904 |
| 1992 |  | 0.7980 |  | 0.2146 | 0.9890 | 0.2642 |  | 1.8618 |  | -0.0003 | 1.8613 | 0.2686 |
| 1993 |  | 0.8428 | 0.9685 | 0.1742 | 1.0032 | 0.2770 |  | 1.8505 | 1.5117 | 0.0087 | 1.8666 | 0.2403 |
| 1994 |  | 0.8259 |  | 0.1674 | 0.9764 | 0.2794 |  | 1.6865 |  | 0.0429 | 1.7604 | 0.1903 |
| 1995 |  | 0.7286 |  | 0.1376 | 0.8360 | 0.2470 |  | 1.5239 |  | 0.0393 | 1.5850 | 0.1245 |
| 1996 |  | 0.7650 | 0.9278 | -0.0169 | 0.7522 | 0.2287 |  | 1.4549 | $1.4779^{\mathrm{K}}$ | 0.0157 | 1.4779 | 0.0012 |
| 1997 |  | 0.8854 |  | -0.1380 | 0.7713 | 0.2406 |  | 1.5124 |  | -0.0486 | 1.4406 | 0.1144 |
| 1998 |  | 0.9002 |  | -0.1370 | 0.7849 | 0.2505 |  | 1.8683 |  | -0.1766 | 1.5659 | 0.1733 |
| 1999 |  | 0.9386 | 0.8923 | -0.1826 | 0.7819 | 0.2546 |  | 1.8896 | 1.4347 | -0.1366 | 1.6484 | 0.2202 |
| 2000 |  | 1.0854 |  | -0.3244 | 0.7847 | 0.2608 |  | 2.2012 |  | -0.2854 | 1.6546 | 0.2539 |


|  | NORWAY | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | PORTUGAL | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 7.0418 |  | 0.7532 | 14.9557 | 4.2965 |  | 0.1412 |  | -1.2273 | 0.0414 | 0.0144 |
| 1972 |  | 6.5883 |  | 0.7699 | 14.2274 | 3.7770 |  | 0.1349 |  | -1.2245 | 0.0396 | 0.0132 |
| 1973 |  | 5.7658 |  | 0.7605 | 12.3351 | 2.9944 |  | 0.1223 |  | -1.2085 | 0.0365 | 0.0115 |
| 1974 |  | 5.5397 |  | 0.6159 | 10.2556 | 2.2403 |  | 0.1267 |  | -1.2921 | 0.0348 | 0.0104 |
| 1975 |  | 5.2269 | 8.3615 | 0.5740 | 9.2792 | 1.7836 |  | 0.1275 | 0.0919 | -1.2263 | 0.0374 | 0.0106 |
| 1976 |  | 5.4565 |  | 0.5331 | 9.2987 | 1.6360 |  | 0.1508 |  | -1.1781 | 0.0464 | 0.0123 |
| 1977 |  | 5.3235 |  | 0.5625 | 9.3427 | 1.4536 |  | 0.1909 |  | -1.1298 | 0.0617 | 0.0153 |
| 1978 |  | 5.2423 |  | 0.5856 | 9.4158 | 1.2221 |  | 0.2192 |  | -1.0484 | 0.0768 | 0.0176 |
| 1979 |  | 5.0641 |  | 0.5791 | 9.0364 | 0.8478 |  | 0.2440 |  | -1.0633 | 0.0843 | 0.0177 |
| 1980 |  | 4.9392 | $8.4734^{K}$ | 0.5397 | 8.4734 | 0.0059 |  | 0.2497 | 0.1634 | -1.0595 | 0.0866 | 0.0164 |
| 1981 |  | 5.7395 |  | 0.4245 | 8.7743 | 0.8304 |  | 0.3070 |  | -1.1551 | 0.0967 | 0.0163 |
| 1982 |  | 6.4540 |  | 0.4396 | 10.0169 | 1.3280 |  | 0.3964 |  | -1.1358 | 0.1273 | 0.0184 |
| 1983 |  | 7.2964 |  | 0.4367 | 11.2923 | 1.8166 |  | 0.5526 |  | -1.1127 | 0.1816 | 0.0212 |
| 1984 |  | 8.1615 |  | 0.4273 | 12.5129 | 2.3029 |  | 0.7302 |  | -1.0618 | 0.2525 | 0.0206 |
| 1985 |  | 8.5972 | 9.5428 | 0.4237 | 13.1332 | 2.6776 |  | 0.8499 | $0.3313^{K}$ | -0.9421 | 0.3313 | 0.0004 |
| 1986 |  | 7.3947 |  | 0.5054 | 12.2577 | 2.7126 |  | 0.7461 |  | -0.6474 | 0.3905 | 0.0304 |
| 1987 |  | 6.7375 |  | 0.5271 | 11.4136 | 2.7034 |  | 0.7027 |  | -0.6264 | 0.3756 | 0.0397 |
| 1988 |  | 6.5170 |  | 0.5086 | 10.8374 | 2.7194 |  | 0.7180 |  | -0.6720 | 0.3667 | 0.0454 |
| 1989 |  | 6.9045 |  | 0.4164 | 10.4707 | 2.7616 |  | 0.7854 |  | -0.6950 | 0.3920 | 0.0534 |
| 1990 |  | 6.2597 | 9.7310 | 0.4316 | 9.6379 | 2.6555 |  | 0.7111 | 0.5173 | -0.5566 | 0.4076 | 0.0589 |
| 1991 |  | 6.4829 |  | 0.3361 | 9.0726 | 2.5986 |  | 0.7207 |  | -0.5463 | 0.4173 | 0.0623 |
| 1992 |  | 6.2145 |  | 0.3629 | 8.9331 | 2.6490 |  | 0.6734 |  | -0.4028 | 0.4501 | 0.0678 |
| 1993 |  | 7.0941 | 8.9309 | 0.2621 | 9.2203 | 2.8210 |  | 0.8021 | 0.5834 | -0.4536 | 0.5096 | 0.0759 |
| 1994 |  | 7.0576 |  | 0.2884 | 9.4172 | 2.9643 |  | 0.8280 |  | -0.3592 | 0.5781 | 0.0835 |
| 1995 |  | 6.3352 |  | 0.3185 | 8.7109 | 2.8139 |  | 0.7537 |  | -0.3200 | 0.5473 | 0.0746 |
| 1996 |  | 6.4498 | 9.1140 | 0.2331 | 8.1430 | 2.6937 |  | 0.7694 | 0.6105 | -0.3821 | 0.5251 | 0.0650 |
| 1997 |  | 7.0734 |  | 0.1467 | 8.1911 | 2.7670 |  | 0.8745 |  | -0.4329 | 0.5672 | 0.0600 |
| 1998 |  | 7.5451 |  | 0.1172 | 8.4837 | 2.9170 |  | 0.8984 |  | -0.3895 | 0.6086 | 0.0474 |
| 1999 |  | 7.7992 | 9.2462 | 0.1352 | 8.9287 | 3.1157 |  | 0.9386 | $0.6348^{\text {K }}$ | -0.3911 | 0.6348 | 0.0006 |
| 2000 |  | 8.8018 |  | 0.1113 | 9.8382 | 3.4949 |  | 1.0854 |  | -0.4782 | 0.6728 | 0.0540 |


|  | SPAIN | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | SWEDEN | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 0.418 |  | -0.4315 | 0.2712 | 0.1430 |  | 5.117 |  | 0.6008 | 9.3307 | 1.8644 |
| 1972 |  | 0.386 |  | -0.3995 | 0.2591 | 0.1322 |  | 4.762 |  | 0.6270 | 8.9153 | 1.5285 |
| 1973 |  | 0.350 |  | -0.3864 | 0.2379 | 0.1173 |  | 4.367 |  | 0.5802 | 7.8016 | 1.0805 |
| 1974 |  | 0.347 |  | -0.4403 | 0.2232 | 0.1063 |  | 4.439 |  | 0.4095 | 6.6861 | 0.6465 |
| 1975 |  | 0.345 | 0.270 | -0.4303 | 0.2244 | 0.1031 |  | 4.152 | $6.303^{\mathrm{K}}$ | 0.4174 | 6.3033 | 0.0046 |
| 1976 |  | 0.402 |  | -0.4660 | 0.2523 | 0.1117 |  | 4.356 |  | 0.4437 | 6.7885 | 0.5826 |
| 1977 |  | 0.457 |  | -0.3938 | 0.3079 | 0.1311 |  | 4.482 |  | 0.4827 | 7.2622 | 0.7632 |
| 1978 |  | 0.461 |  | -0.2785 | 0.3488 | 0.1426 |  | 4.519 |  | 0.5245 | 7.6346 | 0.8023 |
| 1979 |  | 0.403 |  | -0.1757 | 0.3384 | 0.1326 |  | 4.287 |  | 0.5373 | 7.3367 | 0.6296 |
| 1980 |  | 0.431 | 0.424 | -0.2901 | 0.3224 | 0.1208 |  | 4.230 | $7.052^{K}$ | 0.5112 | 7.0518 | 0.0050 |
| 1981 |  | 0.555 |  | -0.4244 | 0.3630 | 0.1298 |  | 5.063 |  | 0.3873 | 7.4586 | 0.7091 |
| 1982 |  | 0.660 |  | -0.3931 | 0.4457 | 0.1515 |  | 6.283 |  | 0.3193 | 8.6462 | 1.1516 |
| 1983 |  | 0.862 |  | -0.4390 | 0.5557 | 0.1791 |  | 7.667 |  | 0.2791 | 10.1355 | 1.6380 |
| 1984 |  | 0.966 |  | -0.3705 | 0.6670 | 0.2030 |  | 8.272 |  | 0.3091 | 11.2673 | 2.0831 |
| 1985 |  | 1.022 | 0.552 | -0.3321 | 0.7332 | 0.2097 |  | 8.604 | 7.973 | 0.3167 | 11.8095 | 2.4187 |
| 1986 |  | 0.842 |  | -0.1069 | 0.7564 | 0.2069 |  | 7.124 |  | 0.4672 | 11.3654 | 2.5265 |
| 1987 |  | 0.742 |  | -0.0719 | 0.6906 | 0.1798 |  | 6.340 |  | 0.4501 | 9.9449 | 2.3659 |
| 1988 |  | 0.700 |  | -0.0737 | 0.6503 | 0.1601 |  | 6.127 |  | 0.4043 | 9.1801 | 2.3132 |
| 1989 |  | 0.712 |  | -0.0729 | 0.6615 | 0.1527 |  | 6.447 |  | 0.3569 | 9.2118 | 2.4393 |
| 1990 |  | 0.613 | 0.658 | 0.0608 | 0.6510 | 0.1394 |  | 5.919 | 9.336 | 0.4485 | 9.2683 | 2.5632 |
| 1991 |  | 0.625 |  | 0.0193 | 0.6367 | 0.1249 |  | 6.048 |  | 0.4323 | 9.3175 | 2.6788 |
| 1992 |  | 0.615 |  | 0.0705 | 0.6603 | 0.1161 |  | 5.824 |  | 0.4580 | 9.2069 | 2.7403 |
| 1993 |  | 0.765 | 0.703 | -0.0283 | 0.7435 | 0.1129 |  | 7.783 | 9.833 | 0.2313 | 9.8089 | 3.0120 |
| 1994 |  | 0.805 |  | 0.0223 | 0.8233 | 0.1018 |  | 7.716 |  | 0.2851 | 10.2610 | 3.2419 |
| 1995 |  | 0.749 |  | 0.0455 | 0.7843 | 0.0683 |  | 7.133 |  | 0.2354 | 9.0269 | 2.9271 |
| 1996 |  | 0.761 | $0.743^{K}$ | -0.0239 | 0.7433 | 0.0006 |  | 6.706 | 9.678 | 0.1865 | 8.0806 | 2.6833 |
| 1997 |  | 0.880 |  | -0.1363 | 0.7678 | 0.0656 |  | 7.635 |  | 0.0286 | 7.8565 | 2.6603 |
| 1998 |  | 0.898 |  | -0.1191 | 0.7971 | 0.0948 |  | 7.950 |  | -0.0152 | 7.8297 | 2.6870 |
| 1999 |  | 0.939 | 0.749 | -0.1544 | 0.8044 | 0.1152 |  | 8.262 | 9.640 | -0.0668 | 7.7286 | 2.6729 |
| 2000 |  | 1.085 |  | -0.2547 | 0.8414 | 0.1386 |  | 9.162 |  | -0.1703 | 7.7274 | 2.7233 |


|  | TURKEY | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE | UK | ER | $\begin{gathered} \text { B } \\ \text { PPP } \end{gathered}$ | PLPL | Predicted PPP | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 |  | 14.92 |  | -3.3507 | 0.5230 | 0.2224 |  | 0.411 |  | 0.1807 | 0.4923 | 0.1299 |
| 1972 |  | 14.15 |  | -3.1638 | 0.5980 | 0.2449 |  | 0.400 |  | 0.1868 | 0.4826 | 0.1188 |
| 1973 |  | 14.15 |  | -3.1160 | 0.6273 | 0.2470 |  | 0.408 |  | 0.1178 | 0.4592 | 0.1047 |
| 1974 |  | 13.93 |  | -3.0278 | 0.6744 | 0.2550 |  | 0.428 |  | 0.0123 | 0.4331 | 0.0906 |
| 1975 |  | 14.44 | 11.06 | -2.9800 | 0.7336 | 0.2658 |  | 0.452 | 0.380 | 0.0379 | 0.4695 | 0.0888 |
| 1976 |  | 16.05 |  | -2.9527 | 0.8379 | 0.2904 |  | 0.557 |  | -0.0082 | 0.5520 | 0.0925 |
| 1977 |  | 18.00 |  | -2.8580 | 1.0331 | 0.3414 |  | 0.573 |  | 0.0588 | 0.6080 | 0.0874 |
| 1978 |  | 24.28 |  | -2.7629 | 1.5325 | 0.4816 |  | 0.522 |  | 0.1282 | 0.5928 | 0.0689 |
| 1979 |  | 31.08 |  | -2.4510 | 2.6790 | 0.7980 |  | 0.472 |  | 0.1535 | 0.5506 | 0.0448 |
| 1980 |  | 76.04 | 52.27 | -2.5202 | 6.1169 | 1.7197 |  | 0.430 | $0.521^{\text {K }}$ | 0.1918 | 0.5213 | 0.0005 |
| 1981 |  | 111.22 |  | -2.1779 | 12.5991 | 3.3275 |  | 0.498 |  | 0.0902 | 0.5446 | 0.0425 |
| 1982 |  | 162.55 |  | -2.1268 | 19.3784 | 4.7784 |  | 0.572 |  | 0.0894 | 0.6260 | 0.0666 |
| 1983 |  | 225.46 |  | -2.0312 | 29.5749 | 6.7539 |  | 0.660 |  | 0.0734 | 0.7100 | 0.0891 |
| 1984 |  | 366.68 |  | -1.9633 | 51.4799 | 10.7763 |  | 0.752 |  | 0.0327 | 0.7768 | 0.1081 |
| 1985 |  | 521.98 | 208.84 | -1.7180 | 93.6551 | 17.7240 |  | 0.779 | 0.551 | 0.0541 | 0.8225 | 0.1225 |
| 1986 |  | 674.51 |  | -1.4804 | 153.4899 | 25.7224 |  | 0.682 |  | 0.1547 | 0.7963 | 0.1238 |
| 1987 |  | 857.22 |  | -1.3213 | 228.6965 | 32.8760 |  | 0.612 |  | 0.1644 | 0.7212 | 0.1149 |
| 1988 |  | 1422.35 |  | -1.2331 | 414.4509 | 48.2405 |  | 0.562 |  | 0.1645 | 0.6627 | 0.1064 |
| 1989 |  | 2121.68 |  | -0.9079 | 855.8468 | 69.7851 |  | 0.611 |  | 0.0881 | 0.6675 | 0.1063 |
| 1990 |  | 2608.64 | $1491.00^{\text {K }}$ | -0.5596 | 1490.7421 | 3.1351 |  | 0.563 | 0.602 | 0.1692 | 0.6670 | 0.1037 |
| 1991 |  | 4171.82 |  | -0.7040 | 2063.310 | 151.779 |  | 0.567 |  | 0.1480 | 0.6574 | 0.0979 |
| 1992 |  | 6872.42 |  | -0.7321 | 3304.989 | 307.419 |  | 0.570 |  | 0.1551 | 0.6654 | 0.0926 |
| 1993 |  | 10984.60 | 5989.79 | -0.7288 | 5300.125 | 522.859 |  | 0.667 | 0.637 | 0.0437 | 0.6966 | 0.0874 |
| 1994 |  | 29608.70 |  | -0.9304 | 11677.471 | 1086.192 |  | 0.653 |  | 0.0898 | 0.7148 | 0.0761 |
| 1995 |  | 45845.10 |  | -0.6546 | 23824.503 | 1752.502 |  | 0.634 |  | 0.0595 | 0.6726 | 0.0524 |
| 1996 |  | 81404.90 | $39274.65^{K}$ | -0.7287 | 39281.468 | 78.188 |  | 0.641 | $0.644^{\mathrm{K}}$ | 0.0047 | 0.6440 | 0.0005 |
| 1997 |  | 151865.0 |  | -0.4881 | 93213.437 | 7442.962 |  | 0.611 |  | 0.0208 | 0.6236 | 0.0498 |
| 1998 |  | 260724.0 |  | -0.1527 | 223802.211 | 25032.468 |  | 0.604 |  | 0.0176 | 0.6145 | 0.0687 |
| 1999 |  | 418783.0 | 197156.6 | 0.1505 | 486800.812 | 66063.773 |  | 0.618 | 0.651 | 0.0038 | 0.6204 | 0.0842 |
| 2000 |  | 625218.0 |  | 0.3952 | 928207.950 | 144108.93 |  | 0.661 |  | -0.0572 | 0.6242 | 0.0969 |


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[^1]:    ${ }^{1}$ The PWT series uses a slightly more detailed method that extrapolates consumption, investment and government separately and then aggregates them to form real GDP series.

[^2]:    ${ }^{2}$ Strictly speaking it is necessary to identify the reference currency used in defining the PPP for the country. Without loss of generality, it is assumed that the US dollar is used as the reference currency.

[^3]:    ${ }^{3}$ It is necessary to choose the type of deflator to use for purposes of extrapolation. The choice is intricately connected to the scope and coverage implicit in the PPP that is being extrapolated.

[^4]:    ${ }^{4}$ This is since the likelihood function in (15) is written as a function of the one-step ahead prediction error which is obtained from the Kalman filter.
    ${ }^{5}$ An exception is in the case of completing very sparse panels, where observations on the dependent variable for some cross-sections do not eventuate until close of the end of the sample. For a discussion see Rambaldi, Hill and Knight (2003).

[^5]:    ${ }^{6}$ This can always be achieved by a simple redefinition of $\lambda$

