Optimal Pollution Tax in Cournot Oligopsonistic Oligopoly

Koji Okuguchi
Department of Economics and Information, Gifu Shotoku Gakuen University
1-38 Nakauzura,Gifu-shi,Gifu-ken 500-8288,Japan
E-mail:okuguchi@gifu.shotoku.ac.jp

Abstract

The optimal pollution tax rate which maximizes the total social surplus is derived for Cournot oligopsonistic oligopoly where product and factor markets are assumed to simultaneously imperfectly competitive. The optimal pollution tax rate is compared with the marginal value of the environmental damage. Three special cases are considered to elucidate the economic implications of the optimal rate.

Keywords:oligopsonistic oligopoly,Cournot oligopoly,optimal pollution tax.
JEL Nos:D43,L13

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1. Introduction

Cournot oligopoly has been extensively studied almost exclusively under the implicit assumption of perfectly competitive factor markets(see Okuguchi and Szidarovszky(1999),Vives(1999) and Martin(2000) for the comprehensive surveys).However,oligopolistic firms procure, more often than not, factors of production from imperfectly competitive factor markets. Okuguchi(1998,2000) has analyzed Cournot oligopoly under the condition of imperfectly competitive factor market,i.e.Cournot oligopsonistic oligopoly. Firms produce not only goods but also bads(pollution) which damage environment. Pollution level can be controlled by government by imposing pollution tax on polluting firms.Some economists have analyzed the optimal pollution tax rate which maximizes the net total social surplus for Cournot oligopoly facing perfect competition in factor market(see,for example,Barnett(1980),Baumol and Oates(1980),Levin(1985),Okuguchi and Yamazaki(1994), Simpson(1995) and Okuguchi(2003)).A general finding is that the optimal pollution tax rate maybe higher,lower or equal to the marginal value of the environmental damage,the monetary value of the environmental degradation due to marginal increase in the total pollution level.

In this paper I will derive a general formula for the optimal pollution taxation within Cournot oligopsonistic oligopoly without product differentiation, for which both product and factor markets are assumed to be imperfectly competitive.In Section 2,I will prove that there exists a unique Cournot equilibrium for oligopsonistic oligopoly under general assumptions,given the level of pollution tax rate. In Section 3,I will derive the optimal rate of pollution tax capitalizing on the existence result in Section 2.The economic implications of the general result are ambiguous.I will therefore consider three special cases to elucidate them.Section 4 concludes.

2.Existence of Cournot Equilibrium

Suppose n firms that produce identical goods to sell in an imperfectly competitive market. Suppose also that they buy one kind of factor of production in an imperfectly competitive factor market. I will use the following notation.

t :pollution tax rate per unit effluent.

y_i :output of firm i.

$Y \equiv \Sigma y_j$:industry output.

p :product price.

$p = p(Y)$:inverse demand function.

l_i :factor demand of firm *i*.

$L \equiv \Sigma l_i$:total factor demand.

$w = w(L)$:wage rate.

π_i :profit of firm *i*.

$y_i = f_i(l_i)$:production function of firm *i*.

$e_i = g_i(y_i)$:emission of effluent of firm *i*.

$E = \Sigma e_i$,total emission by all firms.

Throughout this paper I assume differentiability of the relevant functions up to the orders as necessary. I impose the following restrictions on the inverse demand ,wage,production and emission functions

Assumption 1 : $p' < 0, w' \geq 0, f_i^{'} > 0, f_i^{''} \leq 0, g_i^{'} > 0, g_i^{''} \geq 0, i = 1,2,...,n$. (1)

By definition,net profit of firm i

$$\pi_i = p(\Sigma f_j(l_j)) f_i(l_i) - l_i w(\Sigma l_j) - t g_i(f_i(l_i)), i = 1,2,..., n. \quad (2)$$

I assume that all firms form expectations on other firms' factor demands a la Cournot. Assume $l_i > 0$ .Then the first and second order conditions for maximizing (2) with respect to l_i are given by (3) and (4),respectively.

$$\frac{\partial \pi_i}{\partial l_i} = p'(\Sigma f_j(l_j)) f_i^{'}(l_i) f_i(l_i) + p(\Sigma f_j(l_j)) f_i^{'}(l_i) - (w(\Sigma l_j) + l_i w'(\Sigma l_j)) - t g_i^{'}(f_i(l_i)) f_i^{'}(l_i)$$

$$= 0, i = 1,2,..., n , \quad (3)$$

$$\frac{\partial^2 \pi_i}{\partial l_i^2} = (p' + p'' f_i) f_i^{'2} + (p + p' f_i) f_i^{''} + p' f_i^{''2} - (2w' + l_i w'') - t(g_i^{''} f^{'2} + g_i^{'} f_i^{''}) < 0,.$$

$$i = 1,2,...,n. \qquad (4)$$

Before proceeding further,I introduce the following assumption.
Assumption 2 : For all i

$$p + y_i p' \geq 0,\ p' + y_i p'' \leq 0,\ w' \geq 0,\ w' + l_i w'' \geq 0,\ g_i^{''} f_i^{'2} + g_i^{'} f_i^{''} \geq 0. \qquad (5)$$

It is easy to see that (4),the second order condition, holds under Assumptions 1 and 2. Take into account the definition of L and Y to rewrite (3) as

$$H_i(l_i; L, Y, t) \equiv p'(Y) f_i^{'}(l_i) f_i(l_i) + p(y) f_i^{'}(l_i) - (w(L) + l_i w'(L) - t_i g_i^{'}(f_i(l_i)) f_i^{'}(l_i) = 0,$$

$$i = 1,2,...,n. \qquad (3^{'})$$

This is an implicit function among the variables l_i, L and Y and the parameter t .Solving it with respect to l_i, I obtain

$$l_i \equiv \varphi^i(L, Y, t), i = 1,2,...,n, \qquad (6)$$

where for all $i = 1,2,...,n$,

$$\frac{\partial \varphi^i}{\partial L} = \frac{2w^{'} + l_i w^{''}}{\alpha_i} \leq 0, \qquad (7.1)$$

$$\frac{\partial \varphi^i}{\partial Y} = \frac{-(p' + y_i p'') f_i^{'}}{\alpha_i} \leq 0, \qquad (7.2)$$

$$\frac{\partial \varphi^i}{\partial t} = \frac{g_i^{''} f_i^{'2} + g_i^{'} f_i^{''}}{\alpha_i} < 0. \qquad (7.3)$$

$$\alpha_i \equiv p'(f_i'' f_i + f_i'^2) + pf_i'' - w' - t(g_i'' f_i'^2 + g_i' f_i'').$$

As the right hand side of (6) includes l_i as a component of Y,which is also dependent on l_i,(6) is not a reaction function in the traditional sense of the word.It is introduced for the sake of simplifying the following existence proof.At this stage (7.1-7.3) are true if L,Y and t are all considered to be parameters.Given t, the Cournot equilibrium industry output and factor demand are identifiable as the solution of the following system of two equations.

$$L = \Sigma \varphi^j (L, Y, t) \equiv \varphi(L, Y; t), \text{or}\, F(L, Y; t) \equiv L - \varphi(L, Y, t) = 0 \quad , \tag{8.1}$$

$$Y = \Sigma f_j (\varphi^j (L, Y, t)) \equiv \psi(L, Y; t), \quad \text{or}\, G(L, Y; t) \equiv Y - \psi(L, Y; t) = 0 \, , \tag{8.2}$$

where

$$\varphi_L \leq 0, \varphi_Y \leq 0, \varphi_t < 0 \, , \tag{9.1}$$

$$\psi_L \leq 0, \psi_Y \leq 0, \psi_t < 0. \tag{9.2}$$

I must now introduce an additional assumption.

Assumption 3: $D \equiv (1 - \varphi_L)(1 - \psi_Y) - \varphi_Y \psi_L > 0.$ (10)

As is easily confirmed,this assumption holds if the factor market is perfectly competitive.Suppose t to be given.Since

$$\frac{\partial F}{\partial L} > 0, \frac{\partial G}{\partial Y} > 0 \, ,$$

the Jacobian matrix of F and G becomes a P-matrix under the above assumption.Hence,(8.1) and (8.2) are uniquely solvable with respect to L and Y in the light of the univalence theorem of Gale and Nikaido(1965).Let the solution be

$$L = L(t), Y = Y(t) \, , \tag{11}$$

where

$$\frac{dL}{dt}=\frac{(1-\psi_Y)\varphi_t+\varphi_Y\psi_t}{D}, \tag{12.1}$$

$$\frac{dY}{dt}=\frac{(1-\varphi_L)\psi_t+\psi_L\varphi_t}{D}. \tag{12.2}$$

The denominator of (12.1) and (12.2) is positive by Assumption 3. However, the signs of the numerators are indeterminate,which in turn leads to the indeterminate sign of dl_i/dt .

3.Optimal Pollution Taxation

I am now in a position to determine the optimal pollution tax rate which maximizes the total social surplus W. If both product and factor markets are imperfectly competitive,the net total social surplus is equal to the sum of the firms' profits, the consumer surplus,the surplus accruing to the factor suppliers and the governmental revenue from the pollution tax minus the total value of the environmental damage caused by the total pollution, $D = D(E)$,namely,

$$W=\Sigma\pi_j+\int_0^Y p(\tau)d\tau-p(Y)Y+w(L)L-\int_0^L w(s)ds+t\Sigma g_j(f_j(l_j))-D(\Sigma g_j(f_j(l_j)))$$

$$=\int_0^Y p(\tau)d\tau.-\int_0^L w(s)ds-D(\Sigma g_j(f_j(l_j))). \tag{13}$$

Since all variables appearing on the right hand side of (13) are to be evaluated at the equilibrium,the total social surplus becomes a function of the tax rate in view of (11) and (6). Taking into account the first order condition (3) , I get the first order condition for maximum W as

$$\frac{dW}{dt}=p(Y)\frac{dY}{dt}-w(L)\frac{dL}{dt}-D'\Sigma g_j' f_j' \frac{dl_j}{dt}=0 \quad . \tag{14}$$

A little calculation then leads to the following formula for the optimal pollution taxation.

$$t = D' + \frac{\Sigma(p' f_j^{'} y_j - l_j w') \frac{dl_j}{dt}}{\frac{dE}{dt}}, \tag{15}$$

where D' is the marginal value of the environmental damage. As I have noted already, the sign dl_i/dt is indeterminate,so the optimal tax rate may be higher,lower than or equal to the marginal value of the environmental damage.

I now consider three special in order to clarify economic implications of (15).

Case 1 :Cournot oligopoly.In this case, $w' = 0$,hence

$$t = D' + \frac{p' \Sigma f_j^{'} y_j \frac{dl_j}{dt}}{\Sigma g_j^{'} f_j^{'} \frac{dl_j}{dt}}, \tag{16}$$

which has indeterminate sign. Suppose that all firms be symmetric,that is, $f_i \equiv f$ and $g_i = g$ for all i. Then $l_i \equiv l$ for all i.As the pollution tax must reduce the value of E , $dE/dt < 0$, which leads to $dl_i/dt \equiv dl/dt < 0$. Therefore, the second term on the right hand side of (16) becomes negative,showing that $t < D'$.

Case 2:Cournot oligopsony.In this case $p' = 0$. I get from (15)

$$t = D' - \frac{w' \Sigma l_j \frac{dl_j}{dt}}{\Sigma g_j^{'} f_j^{'} \frac{dl_j}{dt}}, \tag{17}$$

which has ambiguous sign. However, if all firms are symmetric, $dl_i/dt \equiv dl/dt < 0$ .This leads to $t < D'$ as in the case of Cournot oligopoly.

Case 3:Symmetric Cournot oligopsonistic oligopoly

If all firms are symmetric and if,in addition, both product and factor markets are imperfectly competitive, $dl_i/dt \equiv dl/dt < 0$, from which I get $t < D'$ in the light of $p' f_j^{'} y_j - l_j w < 0$.

4.Conclusion

In Section 2, I have formulated Cournot oligopsonistic oligopoly with polluting firms facing imperfectly competitive product and factor markets.I have shown that given the level of the pollution tax rate, there exists a unique equilibrium under general assumptions. Capitalizing on this fact I have derived in Section 3 the optimal pollution tax rate which maximizes the total social surplus in relation to the marginal value of the environmental damage.In general, the optimal tax rate may be higher,lower or equal to the marginal value of the environmental damage. I have elucidated economic implications of the optimal taxation formula for three special cases,that is, Cournot oligopoly,Cournot oligopsony and symmetric Cournot oligopsonistic oligopoly. Finally,I note that if both product and factor markets are simultaneously perfectly competitive,the optimal pollution tax rate is,as is well known, equal to the marginal value of the environmental damage and that if there is only one firm,the optimal tax rate becomes lower than the marginal value of the environmental damage.

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