Some New Semiparametric Panel Stochastic Frontiers
A Bayesian Penalized Approach

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ABSRACT

Greene (2002, 2004) examines several extensions of the panel stochastic frontier models including what he calls the "true" fixed and random effect stochastic frontier models. In this paper we extend these two models to their semiparametric alternatives where the functional form for production function (or other forms of technology) is assumed to be unknown. To this end we use the Bayesian penalized approach to stochastic frontiers developed in Hajargasht et al. (2003). Finally we illustrate our estimation method with an example using real data.

* I'm indebted to my supervisors, Prof. Prasada Rao and Dr. Chris O'Donnell, for their support. All errors are my own responsibility.

Introduction

Stochastic frontier modeling with panel data has relied primarily on results from traditional fixed and random effects models. These models fail to distinguish between inefficiency and technological differences across firms; they assume all the firms share the same frontier functions and all the differences are due to inefficiency. In practice, firms might face different technologies and assuming the same frontier technology could lead to wrong inefficiency estimates. Recently, there have been some studies to address this problem. Tsionas (2001) proposes a random coefficient approach while Orea and Kumbhakar (2003) use a latent model approach to alleviate this shortcoming. Greene (2002, 2004) examines several extensions of the conventional fixed and random effect models. He extends the fixed effects model to what he calls the "true" fixed effects model by adding an extra one-sided random term to the standard fixed effect stochastic frontier model and shows how to practically estimate such a model. He then considers additional approaches, the "true" random effect and random parameters models. In this paper we extend "true" fixed and random effects models of Greene to their semiparametric alternative.

Standard stochastic frontier models are parametric; the researcher has to assume a particular functional form for technology and inefficiency distribution. If the specified functional form is not correct, the estimates will be inconsistent. Relaxing this assumption is particularly important in the context of the stochastic frontier modeling because inefficiencies, the primary aim of the estimation, measured as deviations from a frontier function. There have been many studies with the purpose of relaxing the parametric assumption in stochastic frontier framework, for example Fan & et al. (1995), Kneip & Simar (1996), Henderson (2002), Adams et al. (1999) and Kumbhakar & Tsionas (2002)[1]. In Hajargasht et al. (2003) we applied the Bayesian penalized approach to semiparametric estimation of stochastic frontier models and explained its advantages. The purpose of this paper is to extend Greene's true fixed and random effect stochastic frontier models to the semiparametric case where the functional form for the

[1] - All these studies have used local nonparametric approaches (particularly kernel method) to estimating of stochastic frontiers.

representation of technology (production or cost function) is assumed to be unknown using the Bayesian penalized spline approach developed in Hajargasht et al. (2003).

The structure of the paper is as follows: In section 1 we introduce the Bayesian penalized spline approach to nonparametric estimation. Section 2 discuses the current literature on fixed and "true" fixed effect stochastic frontier model and we generalize them to semiparametric case in section 3. Section 4 and 5 deal with "true" random effects and its generalization to semiparametric case using the Bayesian penalized spline approach. In the presentation of parametric fixed and random effect we follow Greene (2002). We conclude the paper with empirical application of the proposed models.

1- Bayesian Penalized Approach to Nonparametric Regression

There are several approaches to nonparametric regression modeling including (but not limited to) local approach; wavelets; regression splines; smoothing and penalized splines (Ruppert et al 2003). In this paper we use a variant of smoothing splines called P-spline (i.e. Eilers and Max 1996 and Ruppert et al. 2003). We have explained in detail the advantages of the Bayesian version of this approach particularly in the context of estimation of stochastic frontier in Hajargasht et al. (2003). Here we just have a quick review[2].

Suppose we are intended in estimation of the following univariate nonparametric regression:

$$y_i = f(x_i) + \varepsilon_i \tag{1}$$

Let $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \ldots\ldots, \beta_{k+2})'$ and consider the following regression spline model

$$f(x, \boldsymbol{\beta}) = \beta_0 + \beta_1 x + \ldots\ldots + \beta_2 x^p + \sum_{k=1}^{K} \beta_{p+k} (x - \kappa_k)_+^p = \mathbf{X}\boldsymbol{\beta} \tag{2}$$

where $(u)_+ = uI(u \geq 0)$ and $\kappa_1 < \ldots\ldots < \kappa_K$ are fixed knots and $\mathbf{X}$ is a matrix with $\mathbf{x}_i = (1, x_i, \ldots, x_i{}^p, (x_i - \kappa_1)_+^p, \ldots, (x_i - \kappa_K)_+^p)$ in it's i-th row. The relation (2) means

[2] - Here we only explain the univariate regression case. For extension to multivariate case see Ruppert et al. or Hajargasht et al. (2003).

that $\boldsymbol{f}$ has been approximated with separate polynomials in the interval between two consecutive knots in a way that it is continuous and has continuous derivatives up to order p-1.

Model (1) can be estimated by replacing $\boldsymbol{f}$ with $\mathbf{X\beta}$ form (2) using ordinary least squares once the knots have been selected. This method is known as the regression spline approach. The problem with this approach is that knot selection procedures are complicated and computationally intensive (Smith and Kohn 1996). Smoothing spline [see Green and Silverman (1994)] is another approach which uses all the observations as knots. Consequently, when the number of observations is large they become computationally impractical. P-splines, as presented in Ruppert & et al. (2003), combine features of smoothing splines and regression splines in such a way that, unlike regression splines, the locations of knots are not crucial, and they have far fewer parameters than smoothing splines. In the P-spline approach we allow K to be large and fixed (i.e. 20 knots on equidistant intervals), but we put a penalty on the $\{\beta_{k+1}\}_{k=1}^{K}$ (the set of jumps in the derivative of $f(x,\boldsymbol{\beta})$) such that $\sum_{k=1}^{K}\beta_{p+k}^2 \le C$. In this case the least square minimization problem can be written as

$$\text{Min}\,(\mathbf{y}-\mathbf{X\beta})'(\mathbf{y}-\mathbf{X\beta}) \text{ Subject to } \boldsymbol{\beta}'\mathbf{K}\boldsymbol{\beta} \le C$$

Where **K** is a diagonal matrix whose first p diagonal elements are 0 and the remaining diagonal elements are 1. It can be shown using a Lagrange multiplier argument, that this is equivalent to choosing $\boldsymbol{\beta}$ to minimize

$$(\mathbf{y}-\mathbf{X\beta})'(\mathbf{y}-\mathbf{X\beta}) + \lambda\boldsymbol{\beta}'\mathbf{K}\boldsymbol{\beta}$$

Simple calculation shows that the penalized least square minimizer $\boldsymbol{\beta}$ will be

$$\boldsymbol{\beta}(\lambda) = (\mathbf{X}'\mathbf{X} + \lambda\mathbf{K})^{-1}\mathbf{X}'\mathbf{y} \qquad (3)$$

***Bayesian Approach*:** In this paper we employ the Bayesian version of the penalized spline. Suppose we are again asked to estimate (1). Based on the above assumptions we know that we can rewrite (1) as regression spline (2) which is a linear regression and can be easily estimated using the Bayesian methodology but we have an extra requirement that $\sum_{k=1}^{K} \beta_{p+k}^2$ must not be big. In the Bayesian approach this requirement can be incorporated using a prior on $\boldsymbol{\beta}$. For example:

$$\boldsymbol{\beta} \sim N(0, \sigma_\beta^2 \mathbf{K}^-) \qquad (4)$$

where $\mathbf{K}$ is a diagonal matrix with the first p diagonal elements equal to zero and $\mathbf{K}^-$ is its generalized inverse. It is not difficult to show that with this prior, the posterior mean for $\boldsymbol{\beta}$ is equal to (3) where $\lambda = \sigma^2 / \sigma_\beta^2$ [Ruppert & et al. 2003]. We can now see the strength of the Bayesian penalized approach for estimation of complex semiparametric models like the semiparametric stochastic frontier models that we are trying to estimate in this paper. This approach enables us to turn the semiparametric model to a parametric counterpart by transforming the nonparametric part of the model to a linear function with particular priors on the coefficients. This together with the powerful Bayesian MCMC simulation methods enables the researcher to estimate more complex semiparametric models which might not be estimable by other nonparametric methods[3]. In section (3) and (5) we show in detail how to specify and estimate the semiparametric versions of Greene's true fixed and random effects using Bayesian penalized approach.

2- The "True" Fixed Effect Model

Fixed effects stochastic frontier models have been based on Schmidt and Sickles's (1984) treatment of the linear regression model[4]:

$$y_{it} = \alpha_i + \boldsymbol{\beta}\mathbf{x}_{it} + \varepsilon_{it} \qquad (5)$$

[3] - The Bayesian penalized approach has several other attractive properties which have been discussed in Hajargasht (2003).

[4] - For Bayesian approach to fixed effect stochastic frontier see Koop and Steel (2002)

which can be estimated consistently and efficiently by ordinary least squares. The model is reinterpreted by treating α_i as the firm specific inefficiency term. To retain the flavor

of the frontier model, the authors suggest that firms be compared on the basis of

$$\alpha_i^* = \max_i\{\alpha_i\} - \alpha_i \qquad (6)$$

This approach has formed the basis of recently received applications of the fixed effects model in this literature. Some extensions that have been suggested include Cornwell, Schmidt and Sickles proposed time varying effect, $\alpha_{it} = \alpha_{i0} + \alpha_{i1}t + \alpha_{i2}t^2$, and Lee and Schmidt's (1993) formulation $\alpha_{it} = \theta_t\alpha_i$. All these models have a common shortcoming: By interpreting α_i^* as "inefficiency" any other non-efficiency related heterogeneity across firms is ignored. Greene (2002) proposes some alternative models that more explicitly build on the stochastic frontier model instead of reinterpreting the linear regression model. Greene presents the following model as a "true" fixed effects stochastic frontier model

$$y_{it} = \alpha_i + \boldsymbol{\beta}\mathbf{x}_{it} + \varepsilon_{it} - z_{it} \qquad (7)$$

which has been also used in Polachek and Yoon (1996) where α_i is a coefficient representing the heterogeneity across firms and z_{it} is a random error with one sided distribution representing inefficiency. The model can be estimated as a stochastic frontier model simply by creating the dummy variables. The fixed effects model has the virtue that the effects need not be uncorrelated with the included variables. But, there are two problems with the estimation of this model. The first is the practical problem that the model involves many parameters that must be estimated. The second, more difficult problem is the incidental parameters problem. The incidental parameters problem is a persistent bias that arises in nonlinear fixed effects models when the number of periods is small. Greene (2002) addresses both of these problems. He provides a maximum likelihood estimation method which can be practically applied even if the number of firms is very high. Greene also examines the incidental parameter problem in the context of stochastic frontier using a Monte Carlo experiment. Greene has found that the biases in coefficient estimates were surprisingly small and did not appear in the patterns predicted by received results for other models, and, moreover, that there appeared to be no biases transmitted to the estimates of technical inefficiency.

3- Semiparametric True Fixed Effect: The Bayesian Penalized Estimation

Consider following semiparametric fixed effects model for a (possibly unbalanced) panel data

$$y_{it} = \alpha_i + f(\mathbf{x}_{it}) + \varepsilon_{it} - z_{it} \qquad (8)$$
$$t = 1,2,....T_i, i = 1,2,...n$$

where ***f*** is assumed to have an unknown functional form; the only assumption about ***f*** we make is that it satisfies some degree of smoothness. Based on the discussion in section (2) we can rewrite (8) in the following regression spline form

$$y_{it} = \alpha_i + \mathbf{x}_{it}\boldsymbol{\beta} + \varepsilon_{it} - z_{it} \qquad (9)$$

and proceed by assuming $\boldsymbol{\beta}$ has a normal prior distribution of the form $\boldsymbol{\beta} = N(0, \tau^2\mathbf{K}^-)$. z_{it} must have a one sided distribution, here we assume an exponential distribution $z_i \sim Gamma(1, \gamma^{-1})$ [5]. We also make the standard assumption that random error has a normal distribution $\varepsilon_{it} = N(0, \sigma_\varepsilon^2)$. It is also assumed that ε_{it} and z_{it} are independently distributed.

In the maximum likelihood approach to fixed effects we don't make any distributional assumption about α_i we consider it as a fixed parameter and the estimation proceeds using dummy variables. The Bayesian analog to this is to use flat non-informative priors for α_i. Note that we can rewrite (9) in the following matrix form

$$\mathbf{Y} = \mathbf{X}^*\boldsymbol{\beta}^* + \boldsymbol{\varepsilon} - \mathbf{z}$$

where

[5] - The analysis can be easily to modify to include other distributions like half-normal.

$$\mathbf{X}^* = \begin{pmatrix} D_1 & 0 & \cdots & 0 & \mathbf{x}_1 \\ 0 & D_2 & \cdots & \cdots & \mathbf{x}_2 \\ \vdots & 0 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & 0 & \vdots \\ 0 & \cdots & \cdots & D_n & \mathbf{x}_n \end{pmatrix} \qquad \boldsymbol{\beta}^* = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ \boldsymbol{\beta} \end{pmatrix}$$

and D_i is a vector of ones with the size of T_i.

In the Bayesian approach to above estimation problem our goal is to find and analyze following posterior

$$p(\boldsymbol{\beta}^*, \mathbf{z}, \sigma^{-2}, \sigma_\beta{}^{-2}, \gamma^{-1} \,|\, \mathbf{y}, \mathbf{x}) \qquad (10)$$

According to the Bayes theorem and considering the independence assumption that we have made we can write:

$$p(\boldsymbol{\beta}^*, \mathbf{z}, \sigma^{-2}, \tau^{-2}, \gamma^{-1} \,|\, \mathbf{y}, \mathbf{x}) \propto p(\mathbf{y} \,|\, \boldsymbol{\beta}^*, \mathbf{z}, \sigma^{-2}) p(\boldsymbol{\beta}^* \,|\, \tau^{-2}) p(\mathbf{z} \,|\, \gamma^{-1}) p(\tau^{-2}) p(\sigma^{-2}) p(\gamma^{-1}) \quad (11)$$

The first term in the right hand side of the above relation is the likelihood function. Using (9) and the normality assumption about the error term it is not difficult to show that

$$p(\mathbf{y} \,|\, \boldsymbol{\beta}^*, \mathbf{z}, \sigma^{-2}) \propto \sigma^{-T} \exp\left\{ -\frac{\sum_{i=1}^{n} \sum_{i=1}^{T_i} \{y_{it} + z_{it} - \mathbf{x}_{it}{}^* \boldsymbol{\beta}^*\}^2}{2\sigma^2} \right\} \qquad (12)$$

To obtain the posterior we need to specify priors on variances and γ. We accept the inverse gamma priors which is standard in Bayesian stochastic frontier analysis. Putting all the information together we obtain

$$p(\boldsymbol{\beta}^*, \mathbf{z}, \sigma^{-2}, \tau^{-2}, \gamma^{-1} \,|\, \mathbf{Y}, \mathbf{X}) \propto \sigma^{-2(T/2+a-1)} \exp\left\{ -\frac{2b + \sum_{t=1}^{T_i} \sum_{i=1}^{n} \{y_{it} + z_{it} - \mathrm{x}_{it}{}^* \boldsymbol{\beta}^*)\}^2}{2\sigma^2} \right\}$$
$$+ \tau^{-2[(m-p)/2+a_\beta-1]} \exp\left\{ -\frac{2b_\beta + \boldsymbol{\beta}^{*\prime} \mathbf{K}^* \boldsymbol{\beta}^*}{2\tau^2} \right\} \gamma^{-(a_\gamma+T-1)} \exp\left\{ -\gamma^{-1} (b_\gamma + \sum_{t=1}^{T_i} \sum_{i=1}^{n} z_{it}) \right\}$$

For further inference we must be able to analyse the above posterior but this posterior is not of any standard form and it seems unlikely to even randomly draw from it directly. However, we can derive the following conditional distributions:

$$\boldsymbol{\beta}^* \mid \mathbf{z}, \sigma^{-2}, \sigma_\beta^{-2} \gamma^{-1} \sim N\{\mathbf{S}.\mathbf{X}^{*'}.(\mathbf{y}+\mathbf{z}), \sigma^2 \mathbf{S}\} \text{ where } \mathbf{S} = (\mathbf{X}^{*'}\mathbf{X}^* + \frac{\sigma^2}{\tau^2}\mathbf{K}^*)^{-1}$$

$$\sigma^{-2} \mid \mathbf{z}, \boldsymbol{\beta}, \gamma^{-1} \sim G\left\{ a + n/2, b + \frac{(\mathbf{y}+\mathbf{z}-\mathbf{X}^{*'}\boldsymbol{\beta}^*)'(\mathbf{y}+\mathbf{z}-\mathbf{X}^{*'}\boldsymbol{\beta}^*)}{2} \right\}$$

$$\tau^{-2} \mid \mathbf{X}, \mathbf{z}, \sigma^{-2}, \gamma^{-1} \sim G\left\{ a_\beta + (n+m-p)/2, b_\beta + \frac{\boldsymbol{\beta}^{*'}\mathbf{K}^*\boldsymbol{\beta}^*}{2} \right\}$$

$$z_{it} \mid \sigma^{-2}, \boldsymbol{\beta}^*, \gamma^{-1} \sim N\{\mathbf{x}_{it}\boldsymbol{\beta}^* - y_{it} - \gamma^{-1}\sigma^2, \sigma^2\}, z_i \geq 0$$

$$\gamma^{-1} \mid \mathbf{z}, \sigma^{-2}, \boldsymbol{\beta} \sim G(T + a_\gamma, b_\gamma + \sum_{i=1}^{n}\sum_{t=1}^{T_i} z_{it})$$

All the above distributions are of standard forms (normal, truncated normal and gamma) and drawing random numbers from them is fairly easy[6]. So a Gibbs sampler with data augmentation can be set up by sequentially drawing from the above conditional distributions. These draws can be used to obtain posterior means and standard errors.

4- The "True Random Effect"

The random effects model is also motivated by the linear model with the assumption that firm specific inefficiency is constant over time. Thus, the model becomes

$$y_{it} = \boldsymbol{\beta}\mathbf{x}_{it} + \varepsilon_{it} - z_i$$

Where

$$\varepsilon_{it} = N[0, \sigma^2]$$

$$z_i \sim |N[0, \sigma_z^2]| \quad \text{or} \quad z_i \sim \lambda^{-1}\exp(-\lambda^{-1}z_i)$$

This model, proposed by Pitt and Lee (1981) can be estimated by maximum likelihood. It maintains the spirit of the stochastic frontier model and satisfies the condition that the

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6 - Drawing random numbers from a truncated distribution has been discussed in Tsionas (2002)

inefficiency be positive. The random effects model is an attractive specification. But, it has three noteworthy shortcomings. The first is its implicit assumption that the effects are not correlated with the included variables. This problem could be reduced through the inclusion of those effects in the mean and/or variance of the distribution of z_i however. The second problem with the random effects model as proposed here is its implicit assumption that the inefficiency is constant over time and this might be a strong assumption for a long time series of data, There has been efforts to remedy this shortcoming, notably that of Battese and Coelli (1992, 1995) and Kumbhakar (1990). The third shortcoming of this model is the same as characterized the fixed effects regression model. Regardless of how it is formulated, in this model, z_i carries both the inefficiency and, in addition, any time invariant firm specific heterogeneity.

As a first pass at extending the model, Greene considers the following true random effects specification:

$$y_{it} = \boldsymbol{\beta}\mathbf{x}_{it} + \alpha_i + \varepsilon_{it} - z_{it}$$

where α_i is the random firm specific effect, ε_{it} and z_{it} are the symmetric and one sided components specified earlier. In essence, this would appear to be a regression model with a three part disturbance, which immediately raises questions of identification. However, that interpretation would be misleading, as the model actually has a two part composed error;

$$y_{it} = \boldsymbol{\beta}\mathbf{x}_{it} + \alpha_i + v_{it}$$

which is an ordinary random effects model, albeit one in which the time varying component has an asymmetric distribution. The conditional (on α_i) density is that of the compound disturbance in the stochastic frontier model,

$$f(v_{it}) = \frac{\Phi(-v_{it}\lambda/\sigma)}{\Phi(0)}\frac{1}{\sigma}\phi(\frac{v_{it}}{\sigma})$$

Thus, this is actually a random effects model in which the time varying component does not have a normal distribution, though α_i may. In order to estimate this random effects model by maximum likelihood, as usual, it is necessary to integrate the common term out of the likelihood function. There is no closed form for the density of the compound disturbance in this model. However, Greene proposes a simulation based approach to estimate the model. This model can be written equivalently as a stochastic frontier with a firm specific random constant term,

$$y_{it} = (a + \alpha_i) + \boldsymbol{\beta}\mathbf{x}_{it} + \varepsilon_{it} - z_{it}$$

This look likes a random parameter model in which only the intercept is random. Greene and Tsionas (2002) have extended the model to following random parameter model

$$y_{it} = (\boldsymbol{\beta} + \boldsymbol{\alpha}_i)\mathbf{x}_{it} + \varepsilon_{it} - z_{it}$$

Here in addition to the intercept, the slope parameters are also random. Greene proposes a simulated maximum approach to estimate the above model while Tsionas uses a Bayesian approach. In the next section we extend our Bayesian semiparametric approach to estimate semiparametric version of the true random effect model where the function form for the representation of technology is assumed to be unknown but all the other specifications will be the same.

5- Semiparametric "True" Random Effect: Bayesian penalized estimation

Consider following semiparametric random effects model for a (possibly unbalanced) panel data

$$y_{it} = \alpha_i + f(\mathbf{x}_{it}) + \varepsilon_{it} - z_{it}$$
$$t = 1, 2, \ldots T_i, i = 1, 2, \ldots n$$

In the classical approach to fixed effects models, α_i is treated as a fixed parameter while it is treated as a random variable when dealing with a random effects model. In the Bayesian approach to both fixed and random effect α_i is treated as random and the distinction is the priors that we specify for the individual effects. For a fixed effect model e put a non-hierarchical prior like the non-informative priors we used in section (4). For a random effect model we specify a hierarchical prior on α_i like[7]

$$\alpha_i \sim N(\mu, {\sigma_\alpha}^2)$$

where the hyperparameters is also given the following priors

$$p({\sigma_\alpha}^{-2}) \sim G(a_\alpha, b_\alpha)$$
$$\mu \sim N(\mu_0, {\sigma_0}^2)$$

All the other assumptions and priors are same as the fixed effect model. With a similar argument it is not difficult to show that the posterior will be

$$p(\boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, {\sigma_\alpha}^{-2}, \tau^{-2}, \mu, \gamma^{-1} \mid \mathbf{Y}, \mathbf{X}) \propto \sigma^{-2(T/2+a-1)} \exp\left\{ -\frac{2b + \sum_{t=1}^{T_i}\sum_{i=1}^{n} \{y_{it} - \alpha_i + z_{it} - \boldsymbol{\beta}\mathrm{x}_{it})\}^2}{2\sigma^2} \right\} {\sigma_\alpha}^{-2(n/2+a_\alpha-1)} \exp\left\{ -\frac{2b_\alpha + \sum_{i=1}^{n}\{\alpha_i - \mu\}^2}{2{\sigma_\alpha}^2} \right\}$$

$$+ {\sigma_\beta}^{-2[(m-p)/2+a_\beta-1]} \exp\left\{ -\frac{2b_\beta + \boldsymbol{\beta}'\mathbf{K}\boldsymbol{\beta}}{2{\sigma_\beta}^2} \right\} \gamma^{-(a_\gamma+T-1)} \exp\left\{ -\gamma^{-1}(b_\gamma + \sum_{t=1}^{T_i}\sum_{i=1}^{n} z_{it}) \right\} {\sigma_0}^{-1} \exp\left\{ -\frac{(\mu-\mu_0)}{2{\sigma_0}^2} \right\}$$

Again this posterior is not of any standard form and there is no easy way to analyse it directly. However, we can derive following conditional distributions:

$$\boldsymbol{\beta} \mid \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, {\sigma_\alpha}^{-2}, \tau^{-2}, \mu, \gamma^{-1} \sim N\{\mathbf{S}(\mathbf{Y} + \mathbf{z} - \boldsymbol{\alpha}), \sigma^2\mathbf{S}\} \text{ where } \mathbf{S} = (\mathbf{X}^{*\prime}\mathbf{X}^* + \frac{\sigma^2}{\tau^2}\mathbf{K}^*)^{-1}$$

$$\alpha_i \mid \boldsymbol{\beta}, \mathbf{z}, \sigma^{-2}, {\sigma_\alpha}^{-2}, \tau^{-2}, \mu, \gamma^{-1} \sim N\left\{ \frac{\sum_{t=1}^{T_i}[y_{it} - \mathbf{x_{it}}\boldsymbol{\beta} - z_{it}]}{T_i\sigma^{-2} + {\sigma_\alpha}^{-2}}, \frac{1}{T_i\sigma^{-2} + {\sigma_\alpha}^{-2}} \right\}$$

$$z_{it} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \sigma^{-2}, {\sigma_\alpha}^{-2}, \tau^{-2}, \mu, \gamma^{-1} \sim N\{[\mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i - y_{it} - \gamma^{-1}\sigma^2], \sigma^2\}, z_{it} \geq 0$$

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[7] - The standard assumption about α_i is normal distribution but the analysis can proceed by assuming a more flexible approach: mixture of normal distributions.

$$\sigma^{-2} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma_{\alpha}{}^{-2}, \tau^{-2}, \mu, \gamma^{-1} \sim G\left\{ a + nT/2, b + \frac{\sum_{t=1}^{T_i}\sum_{i=1}^{n}\{y_{it} + z_{it} - \alpha_i - \mathbf{x}_{it}\boldsymbol{\beta}\}^2}{2} \right\}$$

$$\mu \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, \sigma_{\alpha}{}^{-2}, \tau^{-2}, \gamma^{-1} \sim N\left\{ \frac{\sigma_{\alpha}^2 \mu_0 + \sigma_0^2 \sum \alpha_i}{\sigma_{\alpha}^2 + n\sigma_0^2}, \frac{\sigma_{\alpha}^2 \sigma_0^2}{\sigma_{\alpha}^2 + n\sigma_0^2} \right\}$$

$$\sigma_{\alpha}{}^{-2} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, \tau^{-2}, \mu, \gamma^{-1} \sim G\left\{ a_{\alpha} + n/2, b_{\alpha} + \frac{\sum_{i=1}^{n}(\alpha_i - \mu)^2}{2} \right\}$$

$$\tau^{-2} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, \sigma_{\alpha}{}^{-2}, \mu, \gamma^{-1} \sim G\left\{ a_{\tau} + (m-2)/2, b_{\tau} + \frac{\boldsymbol{\beta}'\mathbf{K}\boldsymbol{\beta}}{2} \right\}$$

$$\gamma^{-1} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{z}, \sigma^{-2}, \sigma_{\alpha}{}^{-2}, \tau^{-2}, \mu \sim G(T + a_{\gamma}, b_{\gamma} + \sum_{t=1}^{T_i}\sum_{i=1}^{n} z_{it})$$

As we see all these distributions are of one of the normal, inverse gamma or truncated normal distributions and drawing random numbers from them is fairly easy. So a Gibbs sampler can be easily setup by sequentially drawing from theses distributions. After a burning period these draws can be used for further analysis.

6- Empirical Application

This part is not complete yet

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