Information Aggregation and Efficiency in Agency Contracts with Endogenous Externality *

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Abstract

In this paper we investigate the principal–multi agent relationship with moral hazard where a risk neutral principal contracts with multiple risk averse agents whose actions are unobservable to the principal. We show that the well–known trade–off between incentive and risk sharing can be asymptotically resolved as the number of agents becomes sufficiently large, when an arbitrary fraction of agents can obtain unverifiable perfect signals about the actions of other agents. In particular the contract to attain the asymptotic efficiency has the following features: (i) The wage schemes to some agents are contingent on the task performances of other agents as well as their own performances even though all of them are technologically and statistically independent each other. (ii) The wage scheme specifies only two payment levels for each agent. (iii) The principal does not need to observe all the performances of agents. (iv) The almost first best is uniquely implemented.

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1 Introduction

In this paper we provide a new approach for resolving the trade–off between incentives and risk sharing in the principal–multi agent model where a risk neutral principal contracts with many risk averse agents. In our model all agents are symmetric, and there exist no technological and statistical relations among their task performances at all. Despite such independence structure, we show that *interdependent* wage scheme which makes wages of one agent contingent on the performances of others can attain almost the first best, by identifying the large number effect that contracting with sufficiently many agents has the value of providing appropriate work incentives to them with arbitrary small risk.

The standard principal–agent theory has discovered the result that there exists the basic tension between incentives and risk sharing in contracting environments where trading partners have different risk attitudes and choose privately observed actions after contract is signed: When a risk neutral principal contracts with risk averse agents, she should impose no risk on them but this will undermine their work incentives. [1]

In general efficient risk sharing and the provision of work incentives cannot be compatible each other. However, there are a few papers which show that such trade–off can be resolved in certain situations. One approach to this is that the principal hires multiple agents who can monitor about their chosen actions each other and can report messages about these actions. In this case, it is known that, by using the technique of the implementation theory, the first best can be attained as a unique equilibrium outcome (See Ma (1988)). The other approach is to consider the dynamic structure of principal–agent relationships by allowing renegotiation of initial contracts after agents take actions. Hermalin and Kaz (1991) show in the single–agent model that any implementable action without renegotiation is also implemented at the first best cost when the principal can obtain an unverifiable perfect signal about agent's actions and initial contract is renegotiated after such information is revealed. Ishiguro and Itoh (2001) also show that the principal can attain the first best in the multi–agent model with renegotiation even when agents' actions are still unobservable to the principal at the renegotiation stage.

Our main finding is that the principal can asymptotically attain the first best payoff as a unique equilibrium payoff as the number of agents becomes sufficiently large under certain information structure. The key feature of our model is that there exists some subset of agents, called *monitoring agents*, who can obtain *unverifiable* perfect signals about the actions of other agents by taking monitoring activity. For example, the agents working at upper

[1] See Holmström (1979), Grossman and Hart (1983) and Mirrlees (1979) for pioneer works on the moral hazard problems.

levels of a hierarchy may perform monitoring activities to supervise their subordinates as well as they are engaged in productive activities. First we suppose that such monitoring activity is costless and enforceable, and later extend the model to allow monitoring to be costly and unobservable to the principal.

In contrast to Ma (1988), in our model the principal cannot use the contracts which are contingent on the reports about the monitoring outcomes, because they are assumed to be unverifiable. Thus our result can be also applied to the environments where communication between the principal and the agents are limited so that the latter cannot tell to the former about observed information about their actions because the contents of actions may be very complex and indescribable.

As an alternative way, we may be able to resort to the model that the principal can organize the production structures as the sequential stages in which the agents who act at subsequent stages (followers) can observe the actions taken by the previous agents (leaders). [2] Such sequential production structures are often observed in production lines in the real world.

The main role of the above monitoring structure is to create *endogenous* externality among agents by designing interdependent wage schemes. Specifically, we will construct the mechanism by dividing the set of all agents into two subsets: One is the set of agents (called non–monitoring agents) who choose the actions first and one of whom is randomly selected by the principal to be monitored by other agents (monitoring agents) at subsequent stage of the mechanism. We will call the agent who was selected to be monitored from the first set of agents *the selected agent.* The monitoring agents choose the actions after they obtain unverifiable perfect signals about the action of the selected agent.

Then the wage schemes to monitoring agents can be designed such that they respond to the different actions of the selected agent by choosing different actions. In particular, under the specified schemes, all monitoring agents are induced to react to the first best action of the selected agent by choosing the largest action among all possible actions but react to any lower action than the first best one by choosing some lower action than the largest one. This can endogenously create the externality effect that the action choice of the selected agent affects not only his own task performance but also those of monitoring agents through its influence on their action choices. Thus the task performances of monitoring agents convey the useful information about whether the selected agent works well or not. Then, by aggregating the performances of all monitoring agents, the principal can check the deviation of

[2]Strausz (1999) considers such sequential productions in the deterministic partnership model, and shows that there exists a budget balancing sharing scheme to attain the full efficiency. The current paper differs from Strausz's model: Our focus will be on how the trade–off between risk sharing and incentives can be resolved in the agency setting, while in Strausz (1999) stochastic and risk elements are assumed away from the model.

non–monitoring agents from the first best action.

If the above information aggregation is possible, it suffices to design a simple wage scheme offered to non–monitoring agents, by specifying only two wage levels, bonus and penalty, which will be paid according to the outcomes of the aggregated performances of monitoring agents. Specifically we will use a statistical test contract which checks whether the aggregated performances of monitoring agents can pass some statistical test or not. [3]

The statistical test here estimates whether or not the aggregated performances can be on average close to their expected values conditional on all monitoring agents choosing the largest action within some small positive constant. If the selected agent can pass this test he will be paid a bonus, while if he fails the test he must pay a penalty. If the selected agent shirks, then all monitoring agents will shirk as well at the subsequent stage. As a consequence, the principal can collect many signals about such deviation of the selected agent and detect it with almost probability one. Then any deviation of the selected agent can be heavily punished by taking a sufficiently large penalty. On the other hand, by choosing the first best action, the selected agent can pass the statistical test with almost probability one: By the property of the incentive scheme offered to monitoring agents, they will react to the first best action of the selected agent by choosing the largest action. Then, by the Law of Large Numbers and the property of the statistical test contract offered to the selected agent, the probability to pass the statistical test converges to one as the number of agents goes to infinity. Thus the selected agent can almost surely obtain the bonus.

From this argument, all non–monitoring agents choose the first best action and face almost zero risk. However, monitoring agents choose the largest action, which may not be the first best one, and face the non–trivial risk because their wage schemes must satisfy the standard incentive compatibility constraints. This problem can be resolved, by choosing a sufficiently small fraction of monitoring agents. Therefore, the principal can succeed in eliciting the first best action from almost all agents but imposing sufficiently small risk on them when the number of agents becomes sufficiently large.

Our contract also has the following interesting features: (i) It makes

[3] Al–Najjar (1997) also utilizes the statistical test approach in the two–sided moral hazard model where the principal exerts an unobservable effort as well as many agents do so. Al–Najjar (1997) then shows that the second best optimum, which is attained when the principal can commit herself to her effort choice, can be approximated as the number of agents becomes large enough. The current paper, however, is different from Al–Najjar (1997): First, we address the issue about whether or not the first best optimum can be approximately implemented. Second, we emphasize the role of monitoring among agents, which can be used for creating endogenous externality among them. Third, our mechanism uniquely implements the almost first best, and hence is robust to multiple equilibria. See also Matsushima (2001) for utilizing the statistical test approach in the context of repeated games with imperfect monitoring (the implicit collusion between firms contacting in many markets).

wages of any monitoring agent contingent on the task performances of the selected agent as well as his own performance, even enough all of them are technologically and statistically independent each other. In the standard argument such interdependent schemes become suboptimal when the tasks of agents are technologically and statistically independent. However, we will show that the interdependent wage schemes can asymptotically attain the first best outcome when the number of agents becomes sufficiently large, even in the environments with technological and statistical independence. (ii) It is very simple in that the wage scheme specifies only two wage levels for each agent. Since the contract theory is often criticized on the complexity and reality of the optimal contracts derived from the models, our result may help to fill the gap between the theory and practice, at least in large organizations where many agents participate. (iii) We show the unique implementation result that the principal can attain the almost first best payoff as a unique equilibrium payoff. Thus our result is robust to the problems of multiple equilibria under the proposed mechanism. (iv) The principal does not need to observe all the performances of agents. It is sufficient to obtain verifiable performances of the selected agent and all monitoring agents but not those of all non–monitoring agents.

The remaining sections are organized as follows: In section 2 we will set up the model. In section 3 we will show the main result that the principal can asymptotically attain the first best payoff as a unique equilibrium payoff when the number of agents tends to be sufficiently large. In Section 4 we will extend the model to allow costly and unobservable monitoring and show that the asymptotic efficiency result still holds.

2 The Model

2.1 Contractual Environment

We investigate the principal–multi agent relationship with moral hazard where a risk neutral principal contracts with multiple risk averse agents whose actions are unobservable to the principal. Let $I \equiv \{1, 2, ..., N\}$ denote the set of agents. All agents are identical in that they have the same preference (which will be explained later) and the same production technology. Each agent is assigned a task to be performed and chooses an unobservable action. Let $a_i \in A$ denote the action taken by agent i, where $A \subset \Re$ is a finite set. Let also $a \equiv (a_i)_{i=1}^N \in A^N$ be an action profile of all agents. Let a_{-i} denote a vector $a_{-i} \equiv (a_j)_{j \neq i}$. The (von Neumann and Morgenstern) utility function of agent i is additively separable on his income w_i and action a_i as follows:

$$u(w_i) - C(a_i). \tag{1}$$

The reservation utility of all agents is normalized to zero.

We also make the following standard assumption:

Assumption S: *(i)* $u : [\underline{w}, \infty) \rightarrow \Re$ *is strictly increasing and concave, (ii)* $\lim_{w \rightarrow \underline{w}} u(w) = -\infty$ *and* $\forall\ a_i \in A$, $\exists\ w \in (\underline{w}, \infty)$, $u(w) > C(a_i)$, *and (iii)* C *is strictly increasing.*

Assumption S (i) and (ii) say that each agent is risk averse and we can always find a low payment to punish the agent heavily as well as some payment to ensure he covers his action cost. Assumption S (iii) simply states that any agent dislikes to work hard.

Let denote by $\underline{a} \in \arg\min_{a \in A} C(a)$ the least costly action and define the least cost of action as $\underline{C} \equiv C(\underline{a})$. Let ϕ be the inverse function of u (such inverse exists by Assumption S (i)).

The principal can obtain a benefit $R(a)$ from an action profile $a \in A^N$, by hiring N agents and assigning them to the tasks. [4] One interpretation about this is that R is deterministic but non–verifiable. The other interpretation is that $R(a)$ is the expected value of some random returns generated by an action profile $a \in A^N$. [5] $R(a)$ is assumed to be symmetric and take the form as $R(a) = \sum_{i=1}^{N} r(a_i)$ where $r : A \rightarrow \Re$.

We define the *first best* (FB) solution as the outcome to be attained when the actions of all agents are contractible. Since the agents are symmetric, we will focus on the average payoff of the principal per agent. Let $a^{FB} \in A$ denote the first best action level to maximize the average payoff of the principal. Then, the first best outcome is characterized as the efficient risk sharing and optimal action choice:

$$w_i = \phi(C(a^{FB})), \quad \text{for all } i \in I, \tag{2}$$

and

$$a^{FB} \in \arg\max_{a \in A} r(a) - \phi(C(a)). \tag{3}$$

Let V^{FB} denote the principal's average payoff at the first best solution:

$$V^{FB} \equiv r(a^{FB}) - \phi(C(a^{FB})). \tag{4}$$

To avoid the trivial result, we assume that $a^{FB} > \underline{a}$.

[4]Since task assignment itself is not issue of the paper, we assume that each agent is assigned to a task due to some technological reasons which are exogenously fixed.

[5]In the latter interpretation the random returns may correspond to the task performances we will introduce below. Even when this is not always the case and hence the random returns convey other verifiable information than the task performances to be defined below, our result is not changed because the principal can attain almost the first best even if she simply uses only the latter information by discarding any additional information.

2.2 Information Structure

We assume that the action chosen by each agent is not observable to the principal but she can access to some informative signal about it. Let $y_i \in Y$ be the verifiable performance on the task assigned to agent i. Here $Y \subset \Re$ is the set of all possible performances.

We will assume that the task performance of agent i depends on his own action and some random shock which is not statistically correlated with those of other agents. As in the standard agency model, we will regard y_i itself as a random variable of which probability distribution is affected by action a_i. We then assume that the distribution of each y_i has the full support over Y.

We will also assume that all the task performances $(y_i)_{i=1}^N$ are identically distributed, given all agents choose the same action. In other words the probability distribution of y_i depends only on the action $a_i \in A$ but not on the name of a particular agent.

Let $E[y_i|a_i]$ denote the expected value of the task performance of agent i, conditional on his action a_i.

We will assume that each y_i has a finite variance $\text{Var}(y_i|a_i)$ for any given action $a_i \in A$. Note then that under our assumption $\text{Var}(y_i|a_i)$ depends only on the action level a_i but not on the name of agent. Let also define $\overline{v} \equiv \max_{a \in A} \text{Var}(y|a)$ where $\overline{v} < +\infty$.

Let $F(z|a_i)$ denote the cumulative distribution function of y_i, given an action $a_i \in A$. We will then make the following weak assumption, which states that any lower action than the *largest action*, denoted $\overline{a} \equiv \max A$, or the first best one a^{FB} negatively affects the improvement of the probability distribution of the task performance, as compared to the largest action or the first best one, in the sense of the first order stochastic dominance (FOSD):

Assumption FOSD: *For any $y \in Y$ with $\min Y < y < \max Y$,*

$$F(y|a) > F(y|\overline{a}) \quad \forall\, a < \overline{a}, \text{ and } F(y|a) > F(y|a^{FB}) \;\; \forall\, a < a^{FB}$$

where $\overline{a} \equiv \max A$.

Assumption FOSD requires only "local" conditions on the FOSD improvement of the distribution function. Thus it will be satisfied when the FOSD property "globally" holds, i.e., $F(y|a)$ is decreasing in $a \in A$.

Under Assumption FOSD the expected value of y_i conditional on $a_i \in A$, $E[y_i|a_i]$, has the following property: $E[y_i|\overline{a}] > E[y_i|a_i]$ for all $a_i < \overline{a}$.

We will also define the following function:

$$g(a; z) \equiv \frac{C(\overline{a}) - C(a)}{F(z|a) - F(z|\overline{a})}, \quad \text{for } a < \overline{a} \tag{5}$$

Note that $g(a; z) > 0$ by Assumption FOSD. $g(a; z)$ represents the ratio between the change of action cost and the improvement of the distribution function in the sense of FOSD evaluated at the largest action $\overline{a}$. Note that this function is well-defined because of Assumption S (iii) and Assumption FOSD. We will use $g(a; z)$ later for constructing the mechanism in the proof of our main theorem.

2.3 Monitoring Structure

Although the principal can observe only the realizations of the task performances $y = (y_i)_{i=1}^N$, we will assume that there exists some subset of agents who can obtain unverifiable perfect signals about the actions other agents have chosen by taking some monitoring activity. Specifically we assume that a fraction $\alpha \in (0, 1)$ of all agents can act as such monitoring agents as well as they choose their productive actions. Each of them can obtain an unverifiable perfect signal about any other agent's action. Let $I_m \subseteq I$ denote the set of monitoring agents, where $\# I_m = \alpha N$. We assume that the principal can identify the set of monitoring agents I_m.

For the time being, we will maintain the assumption that the monitoring activity does not cost any monitoring agent and is not subject to the moral hazard problem so that it is enforced by the principal. In Section 4 we will relax this assumption and introduce costly and unobservable monitoring activity.

Alternatively we can resort to the other model in which the principal is allowed to organize the production structures of agents as the sequential stages where the subsequent agents can observe the actions taken by the previous agents (See Strausz (1999) for the similar approach in the partnership model). All we need is that some subset of agents can observe the actions taken by others before they will choose their actions.

As explained in the Introduction, the principal cannot use the revelation mechanisms which are contingent on the reports about the monitoring outcomes as in Ma (1988), because the monitoring outcomes are assumed to be unverifiable.

3 Asymptotic Efficiency

We will now show that the principal can attain the almost first best payoff V^{FB} as a unique equilibrium payoff when the number of agents becomes sufficiently large. Since we will employ the multi–stage mechanism, we will use the subgame perfect equilibrium (SPE) as a solution concept.

Theorem. *Suppose that Assumption S and FOSD are satisfied. Then, for any $\varepsilon > 0$, there exists some $\overline{N}$ such that for all $N \geq \overline{N}$ the principal can obtain $V^{FB} - \varepsilon$ as a unique SPE payoff.*

Proof. See Appendix.

Although the formal proof is relegated to Appendix, we will here explain the mechanism to attain the asymptotic efficiency and discuss its implications.

We will use the following mechanism:

Mechanism

Stage 0 All agents simultaneously decide whether to participate in the mechanism or not as well as they simultaneously announce positive integers $(k_i)_{i=1}^N$ chosen from $\{1, 2, ...\}$. Let denote by M the set of the agents who have decided to participate in the mechanism. Only they can go to the next stage and all others obtain the reservation payoff, zero.

Stage 1-1 If $\#M < N$, all agents of M simultaneously choose the actions. Then the payments to agents are made according to the following wage schemes:

- CASE 1: $\#M < N - 1$. All agents of M obtain the following constant utility payment:

$$u(y_i) \equiv \underline{C} + \delta, \qquad \forall\, y_i \in Y$$

  where $\delta > 0$.

- CASE 2: $\#M = N - 1$. Let $L \subseteq M$ denote the set of the agents who have announced the highest integer at Stage 0. Let also $K \equiv \#L$. Then all agents of L obtain the following constant utility payment:

$$u(y_i) \equiv \underline{C} + \xi^K, \qquad \forall\, y_i \in Y$$

  where $\xi \in (0, 1)$, and all agents of $M \setminus L$ obtain the constant utility payment $\underline{C} + \xi^N$.

CASE 3: If $\#M = N$, the game goes to the following stages (Stage 1-2, 2, 3 and 4).

First, the principal divides the set of all agents, I, into two disjoint subsets, denoted I_1 and I_2, where $I_1 \cap I_2 = \emptyset$ and $I_1 \cup I_2 = I$. Moreover, set $I_2 \subseteq I_m$ and let $\beta N \equiv \#I_1$ and $(1 - \beta)N \equiv \#I_2$ where $\beta \in (0, 1)$ and $1 - \beta < \alpha$. Recall here that I_m is the set of monitoring agents and $\alpha \in (0, 1)$ is its fraction relative to all agents.

Let denote $\overline{y}_2 \equiv \sum_{i \in I_2} y_i$ the *aggregated performances* of all monitoring agents of I_2. Let also $\overline{Y}_2$ be the set of all possible $\overline{y}_2$.

Stage 1-2 The agents of I_1 simultaneously choose the actions.

Stage 2 The principal randomly selects one agent from I_1 with equal probability $(1/\beta N)$ and has all agents of I_2 choose the monitoring activity. Let denote by $i^* \in I_1$ the agent selected to be monitored by the agents of I_2. This agent will be called *the selected agent.*

Stage 3 The agents of I_2 simultaneously choose the actions after they have obtained the unverifiable perfect signal about the action the selected agent i^* has chosen at Stage 1-2.

Stage 4 The task performances of all agents are realized and the payments to them are made according to the following wage schemes:

- The utility payment to the agents of I_1: Let define by T the set of the aggregated performances of monitoring agents (I_2), $\overline{y}_2 \equiv \sum_{i \in I_2} y_i$, as follows:

$$T \equiv \left\{ \overline{y}_2 \in \overline{Y}_2 \mid \left| \frac{1}{(1-\beta)N} \overline{y}_2 - E[y|\overline{a}] \right| < \overline{\varepsilon} \right\} \tag{6}$$

  where $\overline{\varepsilon} > 0$ is chosen to satisfy

$$\frac{1}{2} \min_{a < \overline{a}} \left(E[y|\overline{a}] - E[y|a] \right) > \overline{\varepsilon}. \tag{7}$$

  Then the utility payment to the selected agent $i^* \in I_1$ is given by

$$u_{i^*}(\overline{y}_2) \equiv \begin{cases} C(a^{FB}) + \eta & \text{if } \overline{y}_2 \in T, \\ B & \text{if } \overline{y}_2 \notin T \end{cases} \tag{8}$$

  where $C(a) - C(a^{FB}) > \eta > 0$ for all $a > a^{FB}$ and B is chosen to satisfy

$$\gamma < \frac{1}{\beta N} B < \underline{C} - (C(a^{FB}) + \eta) \tag{9}$$

  for some $\gamma \in \Re$.
- Any other agent $k \in I_1$ than i^* obtains the constant utility payment $C(a^{FB}) + \eta$.
- The utility payment to agent $j \in I_2$:

$$u_j(y_j, y_{i^*}) \equiv \begin{cases} \overline{u} & \text{if } y_j \geq \hat{y} \text{ and } y_{i^*} \geq \hat{y}, \\ \underline{u} & \text{otherwise} \end{cases} \tag{10}$$

  where $\min Y < \hat{y} < \max Y$. Here, by defining $\Delta u \equiv \overline{u} - \underline{u}$ and $P(a) \equiv 1 - F(\hat{y}|a)$, $\overline{u}$ and $\underline{u}$ are given so as to satisfy the following inequalities:

$$\frac{1}{\max_{a < a^{FB}} P(a)} \max_{a < \overline{a}} g(a; \hat{y}) > \Delta u > \frac{1}{P(a^{FB})} \max_{a < \overline{a}} g(a; \hat{y}) \tag{11}$$

  and

$$P(\overline{a}) P(a^{FB}) \Delta u + \underline{u} - C(\overline{a}) > 0. \tag{12}$$

The above mechanism essentially consists of two stages: One is the participation stage in which all agents decide whether to participate in the mechanism or not. This corresponds to Stage 0 defined above. The other is the action choice stage in which agents choose their actions, given the wage schemes defined above. The important point at this stage is that the wage schemes are offered to agents, depending on how many agents decided to participate in the mechanism at Stage 0. Except the case that all N agents decide to participate in the mechanism, the utility based payment schemes to all agents are independent of their performances (see CASE 1 and CASE 2 in the mechanism). In the case that all N agents participate in the mechanism (CASE 3), the action choice stages are sequentially designed such that non–monitoring agents (I_1) move first at Stage 1-2 and then monitoring agents (I_2) choose the actions at Stage 3 after one agent, i^*, is randomly selected from the first set of agents (I_1) and his action is monitored by the latter agents at Stage 2.

Furthermore, when CASE 2 is applied (i.e., the number of participating agents is equal to $N-1$), only the agents who have announced the highest integer at Stage 0 can obtain some positive rent ξ^K where K is the number of those agents. This will trigger the integer game so that some agent always has the incentive to break the equilibrium in which CASE 2 is applied by announcing a higher integer. This is because the rent ξ^K is decreasing in the number of the agents who announced the highest integer K. The role of introducing such integer game is simply to eliminate all undesirable equilibria in which CASE 2 is applied, as in the standard implementation theory. We also show that any equilibrium in which CASE 1 is applied can be eliminated. This is simply because any agent can obtain at least positive rents, $\delta > 0$ or $\xi^N > 0$, by participating in the mechanism when CASE 1 is applied. Thus we can ensure that in any SPE all N agents participate in the mechanism. Thus only CASE 3 occurs in any SPE under the proposed mechanism.

In CASE 3, the wage schemes offered to the agents have the following features: First, the wage scheme to the selected agent i^* depends only on the aggregated performances of all monitoring agents (I_2), $\overline{y}_2 = \sum_{i \in I_2} y_i$. More precisely, we define the set of the aggregated performances (see T in (6)) such that their average value ($\overline{y}_2/(1-\beta)N$) is close to their expected value conditional on all the agents of I_2 choosing the largest action $\overline{a}$, i.e., $E[y|\overline{a}]$, within some positive constant $\overline{\varepsilon} > 0$. Then we say that the selected agent can pass the statistical test when the realized aggregated performances $\overline{y}_2$ lie in this set T. The selected agent will be paid a bonus $C(a^{FB}) + \eta$ if he can pass the test but will be paid a penalty B otherwise. This is the wage scheme offered to the selected agent (see (8)), which will work so as to check whether non–monitoring agents have deviated from the first best action or not.

Note that the wage scheme to the selected agent depends only on the performances of monitoring agents (I_2) but not on his own performance.

Second, at Stage 2 all agents of I_2 are forced to monitor what action the selected agent $i^* \in I_1$ has chosen at Stage 1-2. Note here that we are assuming that the monitoring activity is enforceable at no costs. Then, after having observed the unverifiable perfect signal about this action, all agents of I_2 simultaneously choose their actions. In this stage agent $j \in I_2$ will be paid a high utility payment $\overline{u}$ (resp. a low payment $\underline{u}$) if and only if both his own and the selected agent's performances exceed some critical value $\hat{y}$ (resp. otherwise). Note that such payment scheme to the agents of I_2 is well–defined because $P(a^{FB}) > P(a)$ for all $a < a^{FB}$ by Assumption FOSD.

One important implication about the above wage scheme (10) is that any agent of I_2 has the strict incentive to choose the largest action $\overline{a} \equiv \max A$ whenever having obtained the unverifiable perfect signal that the selected agent i^* has chosen the first best action a^{FB} while he has the strict incentive to choose some lower action than $\overline{a}$ whenever having obtained the signal that i^* has chosen a lower action than the first best one. This observation is due to the fact that the definition of Δu implies the following two inequalities:

$$\Delta u > \frac{1}{P(a^{FB})} \max_{a<\overline{a}} g(a;\hat{y}), \tag{13}$$

and

$$\frac{1}{\max_{a<a^{FB}} P(a)} \max_{a<\overline{a}} g(a;\hat{y}) > \Delta u. \tag{14}$$

The first inequality (13) then implies

$$\begin{aligned}\Delta u &> \frac{1}{P(a^{FB})} \max_{a<\overline{a}} g(a;\hat{y}) \\ &\geq \frac{1}{P(a^{FB})} \frac{C(\overline{a}) - C(a)}{P(\overline{a}) - P(a)} \quad \forall\, a \neq \overline{a}\end{aligned}$$

which can be rewritten by

$$P(\overline{a})P(a^{FB})\Delta u - C(\overline{a}) > P(a)P(a^{FB})\Delta u - C(a), \quad \forall\, a \neq \overline{a}. \tag{15}$$

This shows that any agent of I_2 will choose the largest action $\overline{a}$ with certainty when he has obtained the unverifiable perfect signal that the selected agent i^* has chosen the first best action a^{FB}.

On the other hand, the second inequality (14) implies that for any $a' < a^{FB}$ there exists some action $\hat{a} < \overline{a}$ such that

$$P(\hat{a})P(a')\Delta u - C(\hat{a}) > P(\overline{a})P(a')\Delta u - C(\overline{a}). \tag{16}$$

Thus any agent of I_2 never chooses the largest action $\overline{a}$ when he has obtained the unverifiable signal revealing that the selected agent i^* has chosen a lower action than the first best one a^{FB}. Instead, in this case each agent of I_2 will choose some lower action $\hat{a} < \overline{a}$.

Given the above argument, we can then show that any agent of I_1 has the incentive to choose the first best action. If some agent of I_1 deviates from the first action and shirks, [6] by the above argument, all agents of I_2 will react to such deviation by choosing some lower action than the largest one $\overline{a}$ when the deviating agent is selected to be monitored at Stage 2. Thus, shirking by any agent of I_1 creates the large externality effect in the subsequent stage: it affects the task performances of all monitoring agents by changing their action choices at Stage 3. Thus the principal can exploit such externality effect to check whether the agents of I_1 work well or not. In fact, when the number of agents becomes sufficiently large, by the Law of Large Numbers such deviation can be almost perfectly detected and heavily punished. On the other hand, if the selected agent chooses the first best action, then all monitoring agents choose the largest action $\overline{a}$ and hence the selected agent can pass the statistical test with almost probability one, due to the Law of Large Numbers and (6). Anticipating the random selection to be monitored at Stage 2 and fearing a large fine B, all agents of I_1 will choose the first best action. Thus, any agent of I_1 obtains the bonus $C(a^{FB}) + \eta$ with almost probability one and hence is imposed almost zero risk in the equilibrium.

Finally, when the number of agents tends to be large enough ($N \to \infty$), the principal's expected payoff per agent can be given by the average value between the payoffs obtaining from a monitoring agent and a non–monitoring agent:

$$\beta\{r(a^{FB}) - \phi(C(a^{FB}) + \eta)\} + (1 - \beta)\{r(\overline{a}) - W^*\}$$

where W^* denotes the expected wage paid to a monitoring agent (See the Appendix for more precise derivation). Then, by taking a sufficiently small fraction of monitoring agents, $\beta \to 1$, and sufficiently small rent to non–monitoring agents, $\eta \to 0$, the above average payoff converges to the first best one V^{FB}.

The mechanism has several interesting features: First, the wage scheme is simple in that it specifies only two payment levels for each agent. This is desirable property of the mechanism because contract theory has been often criticized on the ground of its reality that derived contracts are more complicated than those observed in practice. Second, the principal does not need to observe all the performances of agents. The wage schemes are contingent only on the performances of the selected agent and all monitoring agents but not on those of all non–monitoring agents. Thus the number of the task performances to be needed for contracting is given by $1 + (1 - \beta)N$. Since we take $\beta \to 1$, the principal is required to observe only the

[6] Any deviation to a higher action than the first best one is not profitable for any agent of I_1 because he can obtain at most $C(a^{FB}) + \eta$ which is, by definition of η, smaller than $C(a)$ when $a > a^{FB}$.

performances of relatively small fraction of agents, $1/N + (1-\beta)$, when $N \to \infty$. Third, the wage scheme to the selected agent is based only on the performances of monitoring agents but not on his own performance. This is because, through the interdependent wage schemes defined by (10), the realized performances of monitoring agents convey sufficient information regarding the action of the selected agent, and hence are utilized for checking whether the selected agent has shirked or not.

4 An Extension to Costly and Unobservable Monitoring

We have so far assumed that the principal can force any monitoring agent to monitor any other agent in the costless way. In this section we will extend the model to allow monitoring to be costly and unobservable to the principal.

Specifically we will assume that any monitoring agent must incur some cost $\rho > 0$ when he monitors any other agent's action. Furthermore, the principal cannot directly observe whether each monitoring agent has performed the monitoring activity or not. Let $m \in \{1, 0\}$ denote the action of a monitoring agent representing whether he takes the monitoring activity ($m = 1$) or not ($m = 0$). The principal is assumed to obtain a verifiable informative signal about the monitoring activity of each monitoring agent. Let $s \in \{s_1, s_2\}$ denote this signal and assume that $s = s_i$ occurs with probability $q(s_i|m) \in (0, 1)$ and $q(s_1|1) > q(s_1|0)$. Thus, obtaining the signal s_1 more accurately reveals the fact that a monitoring agent has taken the monitoring activity ($m = 1$) rather than he has not taken it ($m = 0$).

Since the signal s is verifiable, contract can be contingent on this. Let $(\overline{v}, \underline{v})$ be the utility payment scheme offered to any monitoring agent, where $\overline{v}$ (resp. $\underline{v}$) denotes the utility payment made when the signal s_1 is obtained (resp. the signal s_2 is obtained). We add this scheme $(\overline{v}, \underline{v})$ to the original payment scheme $(\overline{u}, \underline{u})$.

To specify $(\overline{v}, \underline{v})$, we define the following function which represents the expected payoff of the monitoring agent without adding the new scheme $(\overline{v}, \underline{v})$:

$$U(a_j, a_{i^*}) \equiv P(a_j)P(a_{i^*})\Delta u - C(a_j). \tag{17}$$

Then, since $q(s_1|1) > q(s_1|0)$, we can choose $\Delta v \equiv \overline{v} - \underline{v}$ to satisfy the following inequality:

$$\begin{aligned}
&\min_{a_{i^*}} \left(\max_{a_j} U(a_j, a_{i^*}) \right) + q(s_1|1)\Delta v - \rho \\
&> \max_{a_{i^*}} \left(\max_{a_j} U(a_j, a_{i^*}) \right) + q(s_1|0)\Delta v.
\end{aligned} \tag{18}$$

Here the left hand side of the above inequality is the minimum payoff each monitoring agent can obtain when he takes the monitoring activity

($m = 1$) while its right hand side is the maximum payoff he can obtain when he does not take the monitoring activity ($m = 0$). Since the action of the selected agent i^* is not observable to any monitoring agent who does not take the monitoring activity, his expected payoff off the equilibrium path when he does not perform the monitoring activity depends on his belief about what action the selected agent has chosen at Stage 1-2. However, the above strict inequality implies that any monitoring agent has the strict incentive to monitor the action of the selected agent, *whatever* beliefs about the selected agent's action he has off the equilibrium path after he does not take the monitoring activity. Thus, under the above payment scheme $(\overline{v}, \underline{v})$, any monitoring agent will monitor the selected agent at Stage 2 with certainty.

Then the base payments $\underline{v}$ and $\underline{u}$ can be freely chosen to satisfy the individual rationality constraint of monitoring agents:

$$P(\overline{a})P(a^{FB})\Delta u + \underline{u} + q(s_1|1)\Delta v + \underline{v} - C(\overline{a}) - \rho > 0. \tag{19}$$

Although the additional incentive compatibility constraint (18) imposes further risk on monitoring agents, such efficiency loss can be taken as small as possible by choosing a sufficiently small fraction of monitoring agents ($\beta \to 1$). Therefore, the principal can still attain the asymptotic efficiency, even when the monitoring activity is costly and subject to the moral hazard problem.

5 Concluding Remarks

In this paper we have investigated the principal–multi agent relationship with moral hazard where a risk neutral principal contracts with multiple risk averse agents whose actions are unobservable to the principal. Our main finding is that the standard trade-off between incentives and risk sharing can be asymptotically resolved as the number of agents becomes sufficiently large, when a fraction of them acts as monitoring players who can obtain unverifiable perfect signals about the actions of other agents.

From our result we can derive several implications about organizational designs. First, as Al–Najjar (1997) also discussed in the different context, the informational economy of scale works in large organizations: It is beneficial to organize many production tasks which are technologically and statistically unrelated each other. This creates the source of information to be used for checking whether agents take appropriate actions or not. Second, our result may also explain the fact that large organizations are often formed as the hierarchical structure where relatively small fraction of agents act as monitoring players who supervise their subordinates. The role of monitoring performed by the agents at upper levels of a hierarchy is to encourage the subordinates to work well by linking the wage schemes of the former with the performances of the latter.

6 Appendix: Proof of the Theorem

Under the mechanism defined in the text, we will first show the following series of claims.

Claim 1. *All agents of M choose the least costly action $\underline{a}$ with certainty in the subgame at Stage 1-1.*

Proof. This follows from the utility payment schemes defined in the mechanism: The utility payment scheme to any agent of M who has decided to participate at Stage 0 is independent of the realizations of all the task performances, when $\#M \neq N$. Thus all agents of M surely choose the least costly action $\underline{a}$ in the subgame at Stage 1-1. $\#M$. Q.E.D.

Next we will consider the subgame at Stage 3 where monitoring agents of I_2 choose the actions simultaneously. As we discussed in the text, the expected payoff of each agent $j \in I_2$ depends only on his own action as well as the action of the selected agent, a_{i^*}, which has been already fixed at Stage 1-2:

$$P(a_j)P(a_{i^*})\Delta u + \underline{u} - C(a_j).$$

Let $\mu_j(a_{i^*})$ denote the mixed action strategy of agent $j \in I_2$ which is used at Stage 3 contingent on the observed action of the selected agent a_{i^*}, where $\mu_j : A \to \Delta(A)$ is a mapping from A to the set of probability distributions over A, $\Delta(A)$.

Claim 2. *In the subgame at Stage 3, agent j of I_2 chooses $\mu_j(a_{i^*})$ which has the support over $\Sigma(a_i^*)$ as follows:*

$$\Sigma(a_{i^*}) \equiv \arg\max_{a \in A} P(a)P(a_{i^*})\Delta u - C(a),$$

where $\Sigma(a^{FB}) = \{\overline{a}\}$ and $\overline{a} \notin \Sigma(a_{i^*})$ for any $a_{i^*} < a^{FB}$.

Proof. Since the action set A is finite and the expected payoff of any agent of I_2 depends only on his own action as well as that of the selected agent i^* which has been already fixed at Stage 3, any agent of I_2 has the optimal action choice in any subgame at Stage 3. Thus, $\Sigma(a_{i^*}) \neq \emptyset$ for all $a_{i^*} \in A$. In particular, as we have argued (see inequalities (15) and (16) in the main text), any agent of I_2 will choose the largest action $\overline{a}$ with certainty when the selected agent i^* has chosen the first best action a^{FB} but choose a lower action than $\overline{a}$ with certainty when he has not done so. Thus we have

$\Sigma(a^{FB}) = \{\overline{a}\}$ and $\overline{a} \notin \Sigma(a_{i^*})$ for any $a_{i^*} < a^{FB}$. Q.E.D.

Then we will turn to the subgame at Stage 1-2 in which agents of I_1 choose the actions.

Claim 3. *All agents of I_1 choose the first best action a^{FB} with certainty in the subgame at Stage 1-2, when the number of agents N becomes sufficiently large.*

Proof. Take any agent $l \in I_1$ and suppose that he chooses a lower action than the first best one a^{FB}, i.e., $a_l < a^{FB}$. Suppose also that he is selected to be monitored at Stage 2, i.e., $i^* = l$. Then, by Claim 2, after having obtained the unverifiable perfect signal about the action a_l, all agents of I_2 never choose the largest action $\overline{a}$, i.e., $\overline{a} \notin \Sigma(a_l)$. Thus $\overline{a} \notin \text{supp}\mu_j(a_l)$ when $a_l < a^{FB}$ where $\text{supp}\mu_j(\cdot)$ denotes the support of $\mu_j(\cdot)$.

Take any action profile $\hat{a} \in \prod_{j \in I_2} \text{supp}\mu_j(a_l)$.

We will define by $P(\overline{y}_2 \in T; \hat{a})$ the probability that the shirking agent l can pass the statistical test ($\overline{y}_2 \in T$), given the action profile $\hat{a}$. Then we can obtain the following:

$$
\begin{aligned}
& P(\overline{y}_2 \in T; \hat{a}) \\
&= P\left(\left|\frac{1}{(1-\beta)N}\overline{y}_2 - E[y|\overline{a}]\right| < \overline{\varepsilon}; \hat{a}\right) \\
&= P\left(\frac{1}{(1-\beta)N}\left|\overline{y}_2 - \sum_{j \in I_2} E[y_j|\hat{a}_j] + \sum_{j \in I_2} E[y_j|\hat{a}_j] - N(1-\beta)E[y|\overline{a}]\right| < \overline{\varepsilon}; \hat{a}\right) \\
&\leq P\left(\frac{1}{(1-\beta)N}\left|\sum_{j \in I_2} E[y_j|\hat{a}_j] - N(1-\beta)E[y|\overline{a}]\right| < 2\overline{\varepsilon}; \hat{a}\right) \\
&\quad + P\left(\frac{1}{(1-\beta)N}\left|\overline{y}_2 - \sum_{j \in I_2} E[y_j|\hat{a}_j]\right| > \overline{\varepsilon}; \hat{a}\right).
\end{aligned}
$$

Here, by Chebyshev's inequality, the second term appeared in the last expression can be bounded above by

$$
\frac{\sum_{j \in I_2} \text{Var}(y_j|\hat{a}_j)}{\overline{\varepsilon}^2(1-\beta)^2N^2} \leq \frac{\overline{v}}{\overline{\varepsilon}^2(1-\beta)N}
$$

which converges to zero as $N \to \infty$ (Recall here that $\overline{v} = \max_{a \in A} \text{Var}(y|a)$).

The first term in the last expression can be also written by

$$
P\left(\frac{1}{(1-\beta)N}\left|\sum_{j \in I_2} \{E[y_j|\hat{a}_j] - E[y_j|\overline{a}]\}\right| < 2\overline{\varepsilon}; \hat{a}\right)
$$

$$
\begin{aligned}
&= P\left(\frac{1}{(1-\beta)N}\sum_{j\in I_2}\{E[y_j|\overline{a}]-E[y_j|\hat{a}_j]\}<2\overline{\varepsilon};\hat{a}\right)\\
&\leq P\left(\min_{a<\overline{a}}\left(E[y|\overline{a}]-E[y|a]\right)<2\overline{\varepsilon};\hat{a}\right)
\end{aligned}
$$

which becomes zero because by definition of $\overline{\varepsilon}$:

$$\min_{a<\overline{a}}\left(E[y|\overline{a}]-E[y|a]\right)/2>\overline{\varepsilon}.$$

Thus $P(\overline{y}_2\in T;\hat{a})\to 0$ as $N\to\infty$. This convergence result holds for any action profile of agents of I_2 belonging to the support of their action strategies, i.e., any $\hat{a}\in\prod_{j\in I_2}\text{supp}\mu_j(a_l)$. In other words, the shirking agent l would fail the statistical test with almost probability one as $N\to\infty$, whatever action profiles of agents of I_2 are considered from the support of their action strategies. Hence the shirking agent l would be made a low payment B with almost probability one if he were selected to be monitored at Stage 2.

More precisely, the shirking agent $l\in I_1$ will obtain the following expected payoff:

$$\frac{1}{\beta N}\Big\{P(\overline{y}_2\in T;\hat{a})(C(a^{FB})+\eta)+P(\overline{y}_2\notin T;\hat{a})B\Big\}+\left(1-\frac{1}{\beta N}\right)(C(a^{FB})+\eta)-C(a_l)$$

because he will be selected from I_1 to be monitored at Stage 2 with probability $1/\beta N$ and obtain the expected payment $P(\overline{y}_2\in T;\hat{a})(C(a^{FB})+\eta)+P(\overline{y}_2\notin T;\hat{a})B$ while he can obtain $C(a^{FB})+\eta$ when he will not be selected, which occurs with probability $1-1/\beta N$. By $\lim_{N\to\infty}P(\overline{y}_2\in T;\hat{a})=0$ and the definition of B (see (9)), the above expected payoff can be negative when $N\to\infty$.

Next suppose that the same agent $l\in I_1$ chooses the first best action a^{FB} at Stage 1-2 and that he was selected to be monitored at Stage 2. By Claim 2, after having obtained the unverifiable perfect signal about a_l, all agents of I_2 will choose the largest action $\overline{a}$ with certainty at Stage 3.

Let $\overline{a}\in A^{(1-\beta)N}$ denote the action profile of agents of I_2 where all of them choose $\overline{a}$. Then, if agent l is selected by the principal to be monitored at Stage 2, he will face the following probability to fail the test:

$$
\begin{aligned}
P(\overline{y}_2\notin T;\overline{a}) &= P\left(\left|\frac{1}{(1-\beta)N}\overline{y}_2-E[y|\overline{a}]\right|\geq\overline{\varepsilon};\overline{a}\right)\\
&\leq \frac{\text{Var}(y|\overline{a})}{\overline{\varepsilon}^2(1-\beta)N}\\
&\to 0 \quad (N\to\infty)
\end{aligned}
$$

where the inequality follows from the Chebyshev's inequality.

Thus, by choosing a^{FB}, agent l will obtain the following expected payoff:

$$\frac{1}{\beta N}\left\{P(\overline{y}_2 \in T;\overline{a})(C(a^{FB})+\eta)+P(\overline{y}_2 \notin T;\overline{a})B\right\}+\left(1-\frac{1}{\beta N}\right)(C(a^{FB})+\eta)-C(a^{FB})$$

which converges to $\eta > 0$ as $N \to \infty$, because $B/\beta N$ is bounded (see (9)) and $P(\overline{y}_2 \notin T;\overline{a}) \to 0$ when $N \to \infty$.

Finally suppose that agent $l \in I_1$ chooses a higher action than the first best one, i.e., $a_l > a^{FB}$. By the property of the wage scheme (8), any agent of I_1 cannot obtain higher payoffs than $C(a^{FB}) + \eta - C(a)$, which is then negative for any action $a > a^{FB}$ due to the definition of η. Thus any agent of I_1 would obtain a negative payoff if he chose a higher action than the first best one.

The above arguments then show that any agent of I_1 has no incentives to choose other actions than a^{FB} at Stage 1-2, when the number of agents N becomes sufficiently large, because those actions give him negative expected payoffs while choosing a^{FB} yields a positive rent $\eta > 0$ with almost probability one. Q.E.D.

Finally, we will show that all agents participate in the mechanism at Stage 0.

Claim 4. *When the number of agents N is sufficiently large, there exists a SPE in which all agents decide to participate in the mechanism at Stage 0 with probability one (i.e. Pr$[\#M = N] = 1$) and other possibilities never become SPEs (i.e. Pr$[\#M \neq N] = 0$ in any SPE).*

Proof. First, suppose that all but one of I decide to participate in the mechanism with certainty at Stage 0 and consider the incentive of the remaining agent. If such agent does not participate in the mechanism, he will obtain the reservation payoff, zero. However, if he participates in the mechanism, he will obtain some positive payoff, regardless of being a monitoring or non–monitoring agent: If $\#M \neq N$, all participating agents will obtain at least positive rents $\delta > 0$ or ξ^N. If $\#M = N$, by Claim 2 and 3, any monitoring agent will obtain a positive rent by (12) and any non–monitoring agent will obtain the payoff $\eta > 0$ with almost probability one. Thus we have a SPE having all agents participating in the mechanism with certainty at Stage 0.

Next we will show that $\Pr[\#M \neq N] = 0$ in all SPEs.

By Claim 1, in the subgame at Stage 1-1 with the outcome of Stage 0 being $\#M \leq N-1$, any agent surely chooses the least costly action $\underline{a}$.

We first show that $\Pr[\#M = N-1] = 0$ in any SPE. Suppose contrary to the claim that $\Pr[\#M = N-1] > 0$ in some SPE. Then some agent i who has decided to participate with positive probability at Stage 0 has the incentive to raise an announced integer and become a unique “winner” in the integer game to obtain the prize $\xi > 0$. This is explained as follows: Note that announced integers affect only the equilibrium outcomes in which CASE 2 is applied but not others where CASE 1 and 3 are applied. Furthermore, the prize ξ^K obtained by the winners who have announced the highest integer is decreasing in the number of them K and all losers who have not announced the highest integer obtain a smaller rent ξ^N. Thus, the deviation to become a unique winner in the integer game can increase the expected payoffs in all the states when CASE 2 is applied but not those in other states. The states when CASE 2 is applied have positive measures because $\Pr[\#M = N-1] > 0$. Thus we must have $\Pr[\#M = N-1] = 0$ in any SPE.

Next note that $\Pr[\#M < N-1] = 1$ never happens in any SPE: If such case occurs, some agent must not participate in the mechanism with certainty at Stage 0 but then he would deviate to choose "participation" and the highest integer with certainty. This deviation gives him some positive payoff (Note that such deviation can increase the number $\#M$ at most by $N-1$). Thus $\Pr[\#M < N-1] < 1$ in any SPE.

Then we must have $\Pr[\#M = N] > 0$ because $\Pr[\#M = N-1] = 0$ by the above argument. Then $\Pr[\#M = N] > 0$ implies that all agents must choose "participation" with strictly positive probabilities at Stage 0. However, this can be satisfied only when $\Pr[\#M = N] = 1$ so that all agents choose "participation" with probability one because if $0 < \Pr[\#M = N] < 1$ some agent must choose "not participation" with strictly positive probability but this contradicts the fact $\Pr[\#M = N-1] = 0$.

Therefore, we must have $\Pr[\#M = N] = 1$ in all SPEs. Q.E.D.

From Claim 1–4, we have established the result that a SPE exists and all SPEs must have the following equilibrium properties when $N \to \infty$: (i) All agents of I_1 choose the first best action a^{FB} while all agents of I_2 choose the largest action $\overline{a}$. (ii) Any agent of I_1 is paid $C(a^{FB}) + \eta$ with almost probability one. (iii) The expected wage paid to any agent of I_2 is given by

$$W^* \equiv P(\overline{a})P(a^{FB})\phi(\overline{u}) + (1 - P(\overline{a})P(a^{FB}))\phi(\underline{u}).$$

Thus, when N is large enough, the (average) expected payoff of the principal per agent can be unique and given by

$$\beta\{r(a^{FB}) - \phi(C(a^{FB}) + \eta)\} + (1-\beta)\{r(\overline{a}) - W^*\}.$$

Then, by taking $\eta \to 0$ and $\beta \to 1$ along with $N \to \infty$, [7] we show that the average payoff of the principal converges to the first best one V^{FB}.

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