UNIT ROOT TESTS WITH MARKOV-SWITCHING

---were there bubbles in the property prices of Hong Kong and Seoul?

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Abstract:

Diba and Grossman (1988) and Hamilton and Whiteman (1985) recommended unit root tests for rational bubbles. They argued that if stock prices are not more explosive than dividends, then it can be concluded that rational bubbles are not present.

Evans (1991) demonstrated that these tests will fail to detect the class of rational bubbles which collapse periodically. When such bubbles are present, stock prices will not appear to be more explosive than the dividends on the basis of these tests, even though the bubbles are substantial in magnitude and volatility.

Hall et al. (1999) show that the power of unit root test can be improved substantially when the underlying process of the sample observations is allowed to follow a first-order Markov process.

Our paper applies unit root tests to the property prices of Hong Kong and Seoul, allowing for the data generating process to follow a three states Markov chain. The null hypothesis of unit root is tested against the explosive bubble or stable alternative. Simulation studies are used to generate the critical values for the one-sided test.
The time series used in the tests are the monthly price and rent indices of Seoul's housing (1986:1 to 2003:6) and Hong Kong's retail premise (1980:12 to 2003:1). The investigations show that only one state appears to be highly likely in both cases. The switching unit root tests failed to find explosive bubbles in the price series, which might be due to the fact that the power of test is weak in the presence of heteroscedasticity.

I. Introduction

A rational bubble reflects a self-confirming belief that the price of an asset depends on a variable, or a combination of variables, that is intrinsically irrelevant, or on truly relevant variables in a way that involves parameters that are not part of market fundamentals. A basic difficulty in testing for rational bubbles is that the contribution to asset prices by hypothetical rational bubbles would not be directly distinguishable from that by an unobservable market fundamental.

Diba and Grossman (1988) implements stationarity tests for the existence of explosive rational bubbles without precluding the possible effect of unobservable market fundamentals. They argued that if the first differences of the unobservable variables and the first differences of dividends are stationary in the mean, and if rational bubbles do not exist, then the first differences of stock prices are stationary; or if the levels of the unobservable variables and the first differences of dividends are stationary, and if rational bubbles do not exist, then stock prices and dividends are conintegrated of order (1,1). If, however, stock prices contain a rational bubble, differencing stock prices a finite number of times would not yield a stationary process. Although the finding that the first differences of stock prices are nonstationary, or that stock prices and dividends are not cointegrated do not automatically establish the existence of rational bubbles due to the unobservable variable, the converse inference is however possible. That is, evidence that the first differences of stock prices have a stationary mean or evidence that stock prices are cointegrated with dividends would be evidence against the existence of rational bubbles.

Evans (1991) shows that the stationarity tests, suggested by Diba and Grossman (1984, 1988) and Hamilton and Whiteman (1985), is in fact unable to detect the periodically collapsing bubbles. He demonstrates, using simulations, that when such bubbles are present, stock prices will not appear to be more explosive than dividends on the basis of these tests, even though the bubbles are substantial in magnitude and volotilaty.

Hall, Psaradakis and Sola (1999) argued that testing for collapsing bubbles is essentially one of identifying the expanding phase from the collapsing phases of the bubbles. They proposed a generalized ADF unit root test, which allows for the data generating process to switch parameters in different states. They concluded that, unlike standard unit root tests, which have little power to detect periodically collapsing bubbles, their switching ADF tests are able to give sensible inferences about the DGPs.

This paper intend to apply the switching ADF test, suggested by Hall, Psaradakis and Sola (1999), to the property prices in Hong Kong and Seoul. The remaining paper consists of three sections: the first introduces the literature of unit root tests for rational bubbles; the second give our estimation and test results; and the last section concludes the paper with discussions on our findings.

II. Review

2.1 Diba and Grossman's (1988) tests

In their stationairy test, Diba and Grossman assume that the data generating process can be described by the model consists of equation (2.1.1) to (2.1.5).

$$P_t = (1+r)^{-1} E_t (P_{t+1} + \alpha d_{t+1} + u_{t+1}) \qquad (2.1.1)$$

where P_t: the real stock price at time t;

r : the constant real discount rate.

E_t: the conditional expectations operator;

α : a positive constant that valuates expected dividends relative to expected capital gains.

d_{t+1}: the real dividends payment between time t and t+1;

u_{t+1}: a variable that market participants either observe or construct, but that the researcher does not observe.

The fundamental solution for equation (2.1.1) is

$$F_t = \sum_{j=1}^{\infty} (1+r)^{-j} E_t\left[\alpha d_{t+j} + u_{t+1}\right] \qquad (2.1.2)$$

Whereas the general solution would include a rational bubble component, B_t

$$P_t = F_t + B_t \qquad (2.1.3)$$

and B_t satisfy

$$B_{t+1} = (1+r)B_t + z_{t+1} \qquad (2.1.4)$$

The random variable z_{t+1} is an innovation comprising new information available at date t+1. This information can be intrinsically irrelevant, or it can be related to relevant variables through parameters that are not present in F_{t+1} . The expected future values of z_{t+1} are always zero

$$E_{t-j} z_{t+1} = 0 \text{ for all } j \geq 0 \qquad (2.1.5)$$

Assume that d_t is nonstationary is levels, but the first differences of d_t and u_t are stationary. Then P_t will be nonstationary in levels but stationary in first difference, when rational bubbles do not exist. However, when rational bubbles are in presence, differencing P_t a finite number of times would not yield a stationary process, since B_t would have the generating process

$$\left[1-(1+r)L\right](1-L)B_t = (1-L)z_t \qquad (2.1.6)$$

which is neither stationary nor invertible. [1]

By examining the sample autocorrelations and by applying the standard ADF tests, Diba and Grossman concluded that both real stock prices and dividends are nonstationary in levels but stationary in first differences. They also conducted a cointegration test on the stock prices and dividends. Rearranging equation (2.1.2) and substitute it into equation (2.1.3) yields

$$P_t - \alpha r^{-1} d_t = B_t + \alpha r^{-1} \left[\sum_{j=1}^{\infty} (1+r)^{1-j} E_t \Delta d_{t+j} \right] + \sum_{j=1}^{\infty} (1+r)^{-j} E_t u_{t+j} \qquad (2.1.7)$$

If u_t is stationary in levels, and d_t is stationary in first difference, and if B_t equals zero, then P_t and d_t are cointegrated of order (1,1), with cointegrating vector (1, -αr^{-1}). Their tests, however, show mixed results.

The lack of cointegration in stock prices and dividends could be due to the nonstationarity of the unobservable variable, u_t. They explore this possibility by using the following equation, implied by equation (2.1.1),

$$P_{t+1} + \alpha d_{t=1} - (1+r)P_t = e_{t+1} - u_{t+1}$$

where e_{t+1} is the expectation error. That is

$$e_{t+1} = P_{t+1} + \alpha d_{t+1} + u_{t+1} - E_t(P_{t+1} + \alpha d_{t+1} + u_{t+1})$$

The assumption of rational expectation implies that e_{t+1} are not serially correlated. Thus, if $P_{t+1} + \alpha d_{t+1}$ and P_t are cointegrated of order (1,1) with cointegrating vector (1, -(1+r)),

[1] A similar demonstration is given by Evans (1991) in page 923.

then u_t is stationary in level. Their tests suggests that the null hypothesis of no cointegration can be rejected.

The conclusion that Δd_{t+1}, ΔP_{t+1} and $\left(P_{t+1} + \alpha d_{t=1} - (1+r)P_t\right)$ are all stationary would imply that $P_t - \alpha r^{-1} d_t$ is stationary. Therefore, they lamented that the lack of cointegration between P_t and d_t is puzzling. Given these problems, they appealed to an alternative, the Bhargava Tests, to further investigate the stationarity properties of $P_t - \alpha r^{-1} d_t$. Bhargava tests yield the most powerful invariant tests of random-walk hypothesis against the one-sided stationary and explosive alternatives. The existence of explosive rational bubbles would imply that $P_t - \alpha r^{-1} d_t$ has an explosive, rather than a unit, root. The Bhargava tests strongly suggest that stock prices and dividends are cointegrated, and, thus, are consistent with the finding that any unobservable fundamental variables, and the first differences of stock prices and dividends are all stationary.

To verify that their tests would detect explosive bubbles if they were present, they applied the same tests to simulated time-series. Their findings are positive. Hence they concluded in their paper that explosive rational bubbles do not exist in stock prices.

<u>2.2 Evan's (1991) Criticism</u>

Evans argued that, when applied to periodically collapsing rational bubbles, the test procedures suggested by Diba and Grossman can, with high probability, incorrectly lead to the conclusion that these bubbles are not present.

Suppose that the data generation process for stock prices can be adequately represented by the standard present value model given in equation (2.2.1) to (2.2.11)

$$P_t = (1+r)^{-1} E_t(P_{t+1} + d_{t+1}), \quad 0 < (1+r)^{-1} < 1 \tag{2.2.1}$$

variables in the equation have the save interpretations as in equation (2.1.1). This representation ignores the possibility of unobservable fundamentals, since they are not consequential to the point to be made.

The fundamental solution to (2.2.1) is

$$F_t = \sum_{j=1}^{\infty} (1+r)^{-j} E_t\left[d_{t+j}\right] \tag{2.2.2}$$

and the general solution is

$$P_t = F_t + B_t \tag{2.2.3}$$

Where B_t ,the rational bubble, satisfies

$$E_t B_{t+1} = (1+r) B_t \tag{2.2.4}$$

If the first difference the dividends series is a stationary ARMA process and if there are no bubbles, then it can be shown that the first difference of the price series is also a stationary ARMA process, and that P_t and d_t are cointegrated with cointegrating vector $(1, -r^{-1})$. If, instead, Δd_{t+1} is stationary but B_t is not absent, then for some C_t

$$E_t F_t \to C_t + \lambda j \qquad \text{as } j \to \infty \qquad (\quad 2.2.5)$$

where $\lambda = E(\Delta F_t)$

But

$$E_t B_{t+j} = (1+r)^j B_t \qquad (\quad 2.2.6)$$

That is the conditional expectations of the future fundamental price grows linearly in the forecast horizon j, reflecting the unit root in the process, whereas the conditional expectations of future bubbles contains the root (1+r)>1. If B_t is nonzero, as j increases, the conditional expectation P_{t+j} will eventually be dominated by the explosive root (1+r), if a bubble is present. Furthermore, differencing the price will not render the process stationary, since

$$\lim_{j\to\infty} E_t \Delta F_{t+j} = E(\Delta F_t) \text{, a constant} \qquad (2.2.7)$$

but

$$E_t \Delta B_{t+j} = r(1+r)^{j-1} B_t \text{, which is explosive if } B_t \neq 0 \qquad (2.2.8)$$

Hence, the conditional expectation of ΔP_{t+j} will be stable if the bubble is absent, but explosive otherwise.

These considerations are the motivations behind the unit root and cointegration tests by Diba and Grossman (1988).

Evans, however, demonstrated that if the bubbles collapse periodically, such tests have very little power in detecting the presence of bubbles.

Consider the class of rational bubbles that are always positive but collapse periodically

$$B_{t+1} = (1+r)B_t u_{t+1}\ , \qquad B_t \le \alpha \qquad (2.2.9)$$

$$B_{t+1} = [\delta + \pi^{-1}(1+r)\theta_{t+1}(B_t - (1+r)^{-1}\delta)]u_{t+1}\ , \qquad \text{if } B_t > \alpha \qquad (2.2.10)$$

where α and δ are positive parameters with $0 < \delta < (1+r)\alpha$, and

u_t : an exogenous i.i.d positive random variable, with $E_t u_{t+1} = 1$.

θ_{t+1} : an exogenous i.i.d Bernoulli process independent of u , with

$$\Pr(\theta_{t+1} = 1) = \pi$$

$$\Pr(\theta_{t+1} = 0) = 1 - \pi\ , \quad 0 < \pi \le 1$$

Assume

$$u_t = \exp\left(y_t - \frac{\tau^2}{2} \right), \qquad y_t \quad iid, N(0, \tau^2) \qquad (2.2.11)$$

The frequency with which bubbles erupt, the average length of time a bubble expands, and the magnitude of bubble are affected by the process parameters α , δ and π .

When the Bhargava test is applied to the 200 simulated samples of size 100, generated by DGPs described by equations (2.2.1) to (2.2.11), Evans found that the results of tests depend critically on π, the probability per period that the bubble does not collapse. When π is close to one, the tests results are close to those obtained by Diba and Grossman. However, for $\pi \le 0.95$, quit different results are obtained. In fact, when $\pi \le 0.75$, more than 90% of the simulation reject the null hypothesis of a unit root in favor of stable alternatives for both N1 and N2 statistics. These results appear to be robust to moderate changes in the other model parameters.

Evans explains that the maintained hypothesis for the Bhargava test is a first order autoregressive process. When π is close to one, the process for (2.2.10) converges to (2.2.9). But when $\pi \leq 1$, the bubble process in (2.2.10) is a complex nonlinear process, which falls outside the maintained hypothesis. Thus, unless π is close to one, the pattern of periodic collapse generated by (2.2.10) looks more like a stable AR(1) process other than an explosive one, despite of the explosive root in the conditional expectation of the bubble sequence.

Evans also applied the Dickey-Fuller unit root tests and cointegartion tests to the simulated stock prices and dividends, assuming

$$d_t = \mu + d_{t-1} + \varepsilon_t \qquad \varepsilon_t \quad iid, N(0, \sigma_\varepsilon{}^2) \tag{2.2.12}$$

The results clearly show that the DF ϕ_3 statistic is unable to find the bubble when it is present. The cointegration tests, using the Durbin-Watson statistic and the Engle and Granger (1987) ξ_2 and ξ_3 statistics also incorrectly indicate the absence of bubbles in the majority of simulations.

In summary, periodically collapsing bubbles are not detected by standard unit root and cointegration tests.

2.3 Markov-Switching Unit Root Test

Hall, Psaradakis and Sola (1999) argued that, when rational bubbles exist, the dynamics of asset prices are driven by the dynamics of the bubbles. If the bubbles collapse periodically, the values taken by the parameters of the price generating process in the bubble expansion state will differ from that in the bubble collapsing state. That is the model governing the price behavior experiences structural break. When the model has structural breaks, ADF tests have little power. In such cases, allowing for the ADF regression parameters to take on different values in different states will improve the power of the tests. In particular, the authors suggested to make use of the class of dynamic Markov-switching models explored in Hamilton (1989, 1990), and base the unit root test on the following regression model

$$\Delta y_t = \mu_0(1-s_t)+\mu_1 s_t+[\phi_0(1-s_t)+\phi_1 s_t]y_{t-1}+\sum_{j=1}^{k}[\psi_{0j}(1-s_t)+\psi_{1j}s_t]\Delta y_{t-j}+\sigma_e e_t \tag{2.3.1}$$

where $e_t \quad iid, N(0,1)$

and s_t is a state variable independent of e_m for all t and m, and follows first-order Markov chain on the state space $\{0,1\}$ with transition probabilities

$$\Pr(S_t = 1|S_{t-1} = 1) = p$$

$$\Pr(S_t = 0|S_{t-1} = 1) = 1-p$$

$$\Pr(S_t = 0|S_{t-1} = 0) = q$$

$$\Pr(S_t = 1|S_{t-1} = 0) = 1-q \tag{2.3.2}$$

The coefficient on y_{t-1} provides the basis for testing. For example, existence of an explosive rational bubble in prices is consistent with $\phi_0 > 0$ or $\phi_1 > 0$. On the other hand, when $\phi_0 = \phi_1 = 0$ for both price and dividends, there is no rational bubble. A test of the unit root null hypothesis may be based on the asymptotic t-ratios associated with the ML estimates of ϕ_0 or ϕ_1.

The authors conducted a simulation study based on 500 independent realizations of $\{P_t\}$ from DGPs identical to those used by Evans (1991). Two alternative assumptions about the generating mechanism of real dividends are used, namely,

$$d_t = \mu + d_{t-1} + \varepsilon_t \qquad (2.3.3)$$

and

$$\ln d_t = \mu + \ln d_{t-1} + \varepsilon_t \qquad (2.3.4)$$

where

$$\varepsilon_t \quad iid, N(0, \sigma_\varepsilon^{\ 2})$$

Their results show that, unlike the conventional ADF test, Markov-switching ADF procedure has considerable power to detect the presence of bubbles in $\{P_t\}$. They cautioned, however, these results do not imply that switching ADF tests would successfully detect all types of periodically collapsing bubbles. For example, if the contribution of the bubble to the volatility of the prices is not substantial or the probability of the bubble collapse $1-\pi$ is relatively large, it would be difficult for any tests to confirm the presence of the bubble.

The authors then went on to apply the test procedure to investigate the integration properties of consumer prices in Argentina. As argued in Diba and Grossman, whether or not the nonstationarity in prices reflects a rational bubble depends on the time-series properties of the economic fundamentals driving the prices. One known and observable economics fundamental to consumer prices is the money supply. The nonstantionariy in prices could also be caused by the nonstationarity of other unobserved economic fundamentals, rather than a rational bubble. Hence, the authors included two other time series in their tests, the monetary base and exchange rate in Argentina. Since both consumer prices and exchange rate are likely to be driven by common fundamentals, evidence of simultaneously change in these two series would suggest that the nonstationarity in prices is attributable to their market fundamentals. On the other hand, asynchronous changes across the two series may be explained by the presence of a rational bubble. For example, if both series switch simultaneously to the explosive regime, represented by $s_t = 1$, while the money process remains in the no-explosive regime ($s_t = 0$), one can infer that the event is driven by some unobservable economic fundamental common to price and exchange rate, rather than by explosive rational bubbles. Conversely, when price switch to explosive regime whereas the two other series remains in the non-explosive regime, one can conclude there is a rational bubble in the price.

Again, the authors were able to identify rational bubbles presented in the consumer prices and exchange rates of Argentina.

III. Application of Switching ADF to Property Prices

The property markets in many Asian countries boomed in the early 1990s, but busted following the South-East Asian financial crisis in late 1997. Markets reflecting this general trend are Hong Kong, Singapore, Malaysia, etc. An exception to this trend is Korea property market. The large up-swing in property prices during early 1990s in many of the Asian countries, and the recent rise in Korea property prices are generally taken as reflecting speculative bubbles in the popular press.

This paper intends to inquire into the possibility of existence of rational bubbles in the property markets of Hong Kong and Korea, using the switching ADF procedure.

3.1 Data

A price and its associated rent series are selected for each market under consideration from CEIC database. In Hong Kong, these are the retail premise price and rent indices deflated by CPI. Each series make use of two data sets of different frequencies---the first set is quarterly data running from December 1980 to September 2000, the second monthly data stretching from January 1993 to January 2003. In order to combine them, we convert the first set into monthly data by cubic spline. Thus the first half of our data set, running from December 1980 to December 1992, consists of the splined output from the first data set, whereas the remaining half from the second data set. The raw data has 266 observations for both price and rent series. The series for Korea are CPI deflated monthly housing price and housing rent indices between January 1986 and June 2003, with a total of 210 observations of raw data.

3.2 Estimation of the Switching ADF Regression Model

A casual examination of figure one suggests that there might be three regimes governing the movement of each series. In the first state, price is more or less stable or its movement is coupled by rent (e.g. Hong Kong after August 1998; Seoul between September 1987 and February 1990). We call such a state, preliminarily, one in which bubble is dormant. In the second state, price is rising sharply, with little or no corresponding movement in rent (e.g. Hong Kong between October 1993 and July 1994, and between December 1996 and September 1997; Seoul between February 1990 and May 1991). We call such a state one in which bubble is expanding. In the third state, price plunges, with little or no co-movement in rent (e.g. Hong Kong between July 1994 and June 1996, and between September 1997 and August 1998; Seoul between January 1986 and September 1987). We call such a state one in which the bubble is collapsing.

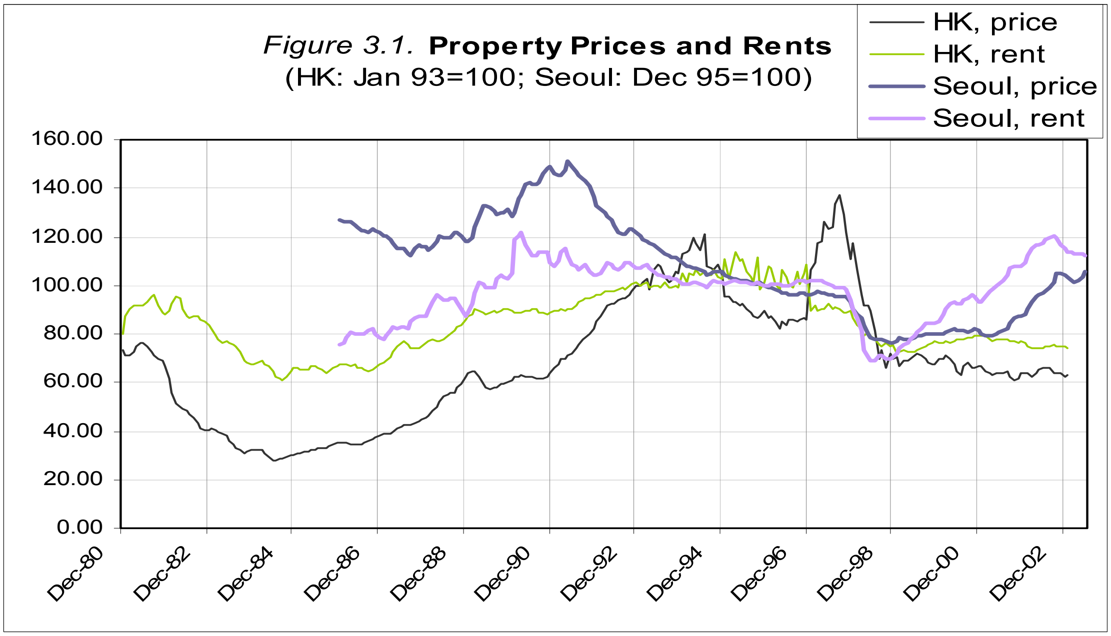


Figure 3.1. **Property Prices and Rents**
(HK: Jan 93=100; Seoul: Dec 95=100)

Source: CEIC database

Thus, the following ADF model is fitted to each of the four series

$$\Delta y_t = \mu^{st} + \phi^{st} y_{t-1} + \sum_{k=1}^{K} \psi_k^{st} \Delta y_{t-k} + \varepsilon_t \quad \varepsilon_t \quad iid, N(0, \sigma^2) \qquad (3.2.1)$$

where $s_t \in \{1,2,3\}$, a state variable following the first order Markov Chain

$$\begin{aligned} &\Pr(s_{t+1} = j | s_t = i, s_{t-1} = i_1, \ldots, \zeta_t) \\ &= \Pr(s_{t+1} = j | s_t = i) \\ &\equiv p_{ij} \end{aligned} \qquad (3.2.2)$$

with $\zeta_t = (y_t, y_{t-1}, \ldots, y_1)$, the information set available at time t, and p_{ij} the state transition probability. Equation (3.2.2) says that the probability distribution of s_{t+1} depends on past events only through the value of s_t.

The state variable s_t is not observed, but can be inferred using the discrete Kalman filter described by Hamilton (1994), and summarized below.

Rewrite equation (3.2.1) as

$$y_t = x_t{}'\beta_{st} + \varepsilon_t \qquad \varepsilon_t \quad iid, N(0,\sigma^2) \qquad (3.2.3)$$

Assume $\mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}$ and σ are known with certainty. If the Markov chain is stantionary and ergodic, the iteration to evaluate $\Pr(s_t = i | \zeta_{t-1})$ ($i = 1,2,3$ and $\sum_{i=1}^{3} \Pr(s_t = i | \zeta_{t-1}) = 1$), can start at date $t = 1$ with the unconditional probabilities $\boldsymbol{\pi}$, where $\boldsymbol{\pi}' = (\pi_1 \quad \pi_2 \quad \pi_3)$, and $\pi_i \equiv \Pr(s_t = i)$.

We can evaluate π by solving the system of two equations

$$\begin{aligned} \boldsymbol{\pi} &= F\boldsymbol{\pi} \\ \mathbf{1}`\boldsymbol{\pi} &= 1 \end{aligned} \qquad (3.2.4)$$

where F is the matrix of state transition probabilities

$$F = \begin{pmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{pmatrix} \qquad (3.2.5)$$

and

$$\mathbf{1}` = (1 \quad 1 \quad 1)$$

At step t, the inputs are $\{\Pr(s_t = i|\zeta_{t-1})\}_{i=1}^{3}$ and the outputs are $\{\Pr(s_{t+1} = j|\zeta_t)\}_{j=1}^{3}$, with $\Pr(s_1 = i|\zeta_0) = \pi_i$. Given $\Pr(s_t = i|\zeta_{t-1})$ and given the normality assumption, the conditional density function for y_t is

$$f(y_t|s_t = i, x_t, \zeta_{t-1}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_t - x_t'\beta_i)^2}{2\sigma^2}\right) \tag{3.2.6}$$

Since x_t is predetermined

$$\Pr(s_t = i|x_t, \zeta_{t-1}) = \Pr(s_t = i|\zeta_{t-1})$$

Hence the joint density of y_t and $s_t = i$, given x_t and ζ_{t-1}

$$f(y_t, s_t = i|x_t, \zeta_{t-1}) = f(y_t|s_t = i, x_t, \zeta_{t-1}) \Pr(s_t = i|\zeta_{t-1}),$$

$$i = 1,2,3 \tag{3.2.7}$$

Thus the density of y_t conditional on x_t and ζ_{t-1}

$$f(y_t|x_t, \zeta_{t-1}) = \sum_{i=1}^{3} f(y_t, s_t = i|x_t, \zeta_{t-1}) \tag{3.2.8}$$

By Bayse Rule, the optimal filter of s_t given ζ_t, the information set available at time t, is

$$\Pr(s_t = i|\zeta_t) = \frac{f(y_t, s_t = i|x_t, \zeta_{t-1})}{f(y_t|x_t, \zeta_{t-1})} \tag{3.2.9}$$

and the prediction of s_{t+1}

$$\Pr(s_{t+1} = j|\zeta_t) = \sum_{i=1}^{3} p_{ij} \Pr(s_t = i|\zeta_t) \tag{3.2.10}$$

A more efficient inference about s_t can be obtained by using the entire set of information available to the researcher, ζ_T

$$\Pr\left(s_t = i \middle| \zeta_T\right) = \sum_{i=1}^{3} \Pr\left(s_t = i, s_{t+1} = j \middle| \zeta_T\right) \tag{3.2.11}$$

where

$$\Pr\left(s_t = i, s_{t+1} = j \middle| \zeta_T\right) = \Pr\left(s_{t+1} = j \middle| \zeta_T\right) \frac{\Pr\left(s_t = i \middle| \zeta_t\right) \Pr\left(s_{t+1} = j \middle| s_t = i\right)}{\Pr\left(s_{t+1} = j \middle| \zeta_t\right)}$$

$$i, j = 1,2,3 \tag{3.2.12}$$

$\Pr\left(s_t = i \middle| \zeta_T\right)$ is called the smoothed inference of the state variable. This smoothed probability sequence $\left\{\Pr\left(s_t = i \middle| \zeta_T\right)\right\}_{t=1}^{T}$ can be computed by backwards iteration. The iteration starts with $\Pr\left(s_T = i \middle| \zeta_T\right)$ obtained from the filtering process using equation (3.2.9).

So far we have assumed that $\mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}$ and σ are known to us, where $s_t = \{1,2,3\}$. But in fact these parameters need to be estimated. We can estimate them by maximizing the log likelihood function of the observed data using EM algorithm, since EM algorithm is an efficient approach (Hamilton, 1994). The log-likelihood function to be maximized is $LL = \sum_{t=1}^{T} \log f\left(y_t \middle| x_t, \zeta_{t-1}\right)$, with $f\left(y_t \middle| x_t, \zeta_{t-1}\right)$ given by (3.2.8), The steps of the estimation are given below.

Step one: make an arbitrary initial guess on $\mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}$ and σ;

Step two: calculate the smoothed probabilities of s_t using (3.2.3) to (3.2.12);

Step three: OLS regress $y_t \sqrt{\Pr\left(s_t = i \middle| \zeta_T\right)}$ on $x_t \sqrt{\Pr\left(s_t = i \middle| \zeta_T\right)}$, $i = 1,2,3$, which gives ML estimates $\tilde{\mu}^{st}, \tilde{\phi}^{st}, \tilde{\psi}_k^{st}$, $(k = 1,2,...K)$.

Step four: update σ^2 using the OLS residuals;

$$\tilde{\sigma}^2 = \frac{\sum_{st=1}^{3} \left(y_t - x_t'\tilde{\beta}_{st}\right)\left(y_t - x_t'\tilde{\beta}_{st}\right)}{3 \times (T - N)} \tag{3.2.13}$$

where N: the number of parameters estimated.

Step five: update p_{ij}

$$p_{ij} = \frac{\sum_{t=2}^{T} \Pr\left(s_t = j, s_{t-1} = i \middle| \zeta_T\right)}{\sum_{t=2}^{T} \Pr\left(s_{t-1} = i \middle| \zeta_T\right)} \tag{3.2.14}$$

Step six: update π

$$\pi_i = \Pr\left(s_1 = i \middle| \zeta_T\right) \tag{3.2.15}$$

Step seven: repeat step two to six until the parameters and the likelihood converge.

3.3. Asymptotic Properties of ML Estimators

Suppose $\tilde{\theta}$ is the ML estimator of θ, and θ_0 the true value of θ, where θ`={ $\mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}, \sigma$ }. Subject to certain regularity conditions (Caines, 1988, Ch7), $\tilde{\theta}$ is consistent and asymptotically normal, with limiting distribution

$$\sqrt{T}\varphi_{2D,T}^{\frac{1}{2}}(\tilde{\theta} - \theta_0) \xrightarrow{d} N(0, I) \tag{3.3.1}$$

i.e.

$$\tilde{\theta} \xrightarrow{d} N(\theta_0, T^{-1}\varphi_{2D,T}^{-1}) \tag{3.3.1'}$$

where $\varphi_{2D,T}$ is the information matrix from the sample of size T

$$\varphi_{2D,T} = -\frac{1}{T} E\left(\sum_{t=1}^{T} \frac{\partial^2 LogL_t}{\partial\theta\partial\theta`} \middle| \theta = \theta_0 \right) \quad (3.3.2)$$

$$\lim_{T\to\infty} \varphi_{2D,T} \xrightarrow{p} \hat{\varphi} = -\frac{1}{T}\left(\sum_{t=1}^{T} \frac{\partial^2 LogL_t}{\partial\theta\partial\theta`} \middle| \theta = \tilde{\theta} \right) \quad (3.3.3)$$

The reported standard errors for $\tilde{\theta}$ are the square roots of the diagonal elements of

$$(T\hat{\varphi})^{-1} = -\left(\sum_{t=1}^{T} \frac{\partial^2 LogL_t}{\partial\theta\partial\theta`} \middle| \theta = \tilde{\theta} \right)^{-1}$$

3.4 Computing Hessian by Numerical Method (Wheatley, 2004)

Consider a continuous function f , and the value $f(x+\Delta x)$. By Taylor's expansion

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x + \frac{f''(x)}{2}(\Delta x)^2 \quad (3.4.1)$$

rearrange

$$f''(x) \approx \frac{f(x+\Delta x) - f(x) - f'(x)\Delta x}{(\Delta x)^2} \times 2 \quad (3.4.1')$$

Also consider the value $f(x-\Delta x)$. By Taylor's expansion

$$f(x-\Delta x) \approx f(x) - f'(x)\Delta x + \frac{f''(x)}{2}(\Delta x)^2 \quad (3.4.2)$$

which gives

$$f''(x) \approx \frac{f(x-\Delta x) - f(x) + f'(x)\Delta x}{(\Delta x)^2} \times 2 \quad (3.4.2')$$

Combining (3.4.1') and 3.4.2')

$$f''(x) \approx \frac{f(x-\Delta x) - 2f(x) + f(x+\Delta x)}{(\Delta x)^2} \tag{3.4.3}$$

Let $f = LL$, then

$$\frac{\partial^2 LL}{\partial\theta\partial\theta`} \approx \frac{LL(\theta+\Delta) - 2\times LL(\theta) + LL(\theta-\Delta)}{\Delta^2} \tag{3.4.4}$$

In our experiment, a range of values of Δ, from 10^{-1} to 10^{-5}, are tried out to allow for variation in the values of log-likelihood function. The results are fairly stable under different choices of Δ.

3.5 Lag Selection in ADF Regression

Consider

$$\Delta y_t = \alpha + \gamma y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + \varepsilon_t$$

where p is to be determined.

Taking the general-to-specific procedure, we start by setting $p = k$, where $k = k\max \approx \sqrt{T}$, and T is the size of the sample. Estimate the above equation by OLS, and test:

$H0 : k = k\max - 1$
$H1 : k = k\max$

if H0 is rejected, set $k = k\max$. Otherwise, test:

$$H0: k = k\max - 2$$
$$H1: k = k\max - 1$$

…

Stop when H0 is rejected.

The test results are presented in the table below.

Table 3.1. **Lag Selection**

	Seoul Housing		Hong Kong Retail premise	
	Price	Rent	Price	Rent
No. of lags	3	1	10	8
Original T	210		266	
T after lag and difference	206	208	255	257

3.6 Smoothed State Probabilities

The smoothed probabilities computed using method described in section 3.2 suggests that only the first state, i.e. one in which bubble is dormant, is highly likely throughout the sample period for both price and rent series of Hong Kong and Seoul[2]. Refer to figure 3.2 through figure 3.5. This is consistent with the values of state transition probabilities, which show that there is a tendency for the DGPs to switch into the first state from other

[2] The estimation of a 2-state switching model also suggests only the first state is highly likely.

states, and that there is a lack of tendency to switch into other states from the first state. Refer to table 3.2.

Figure 3.2. **State Probabilities of Price (HONG KONG)**

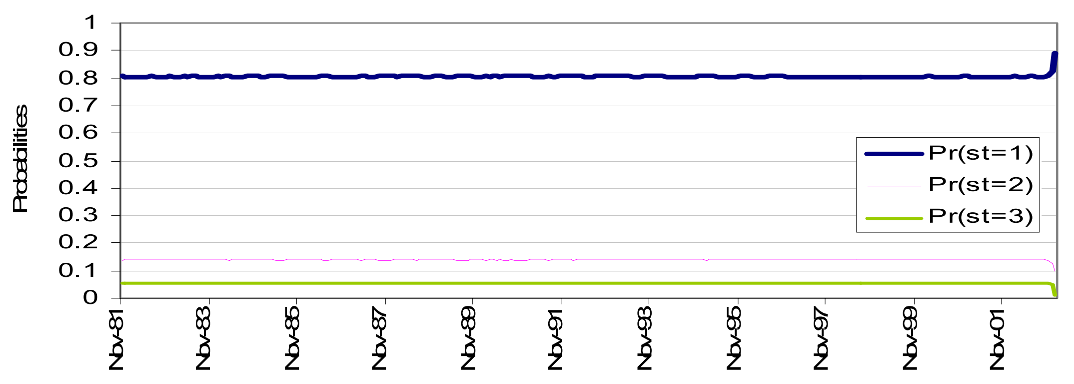


Figure 3.3. **State Probabilities of Rent (HONG KONG)**

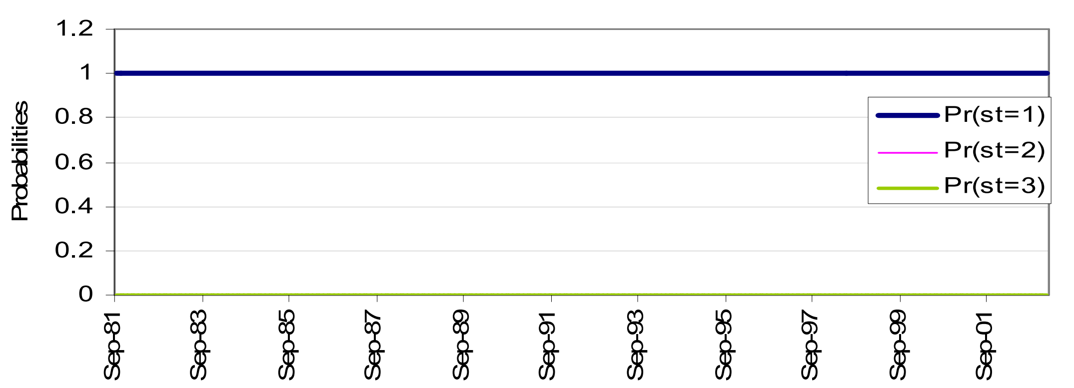
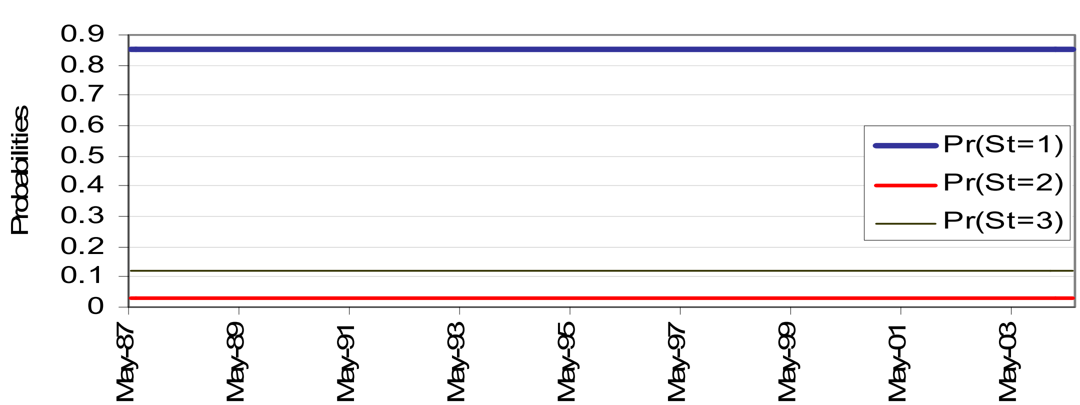


Figure 3.4. **State Probabilities of Price (SEOUL)**



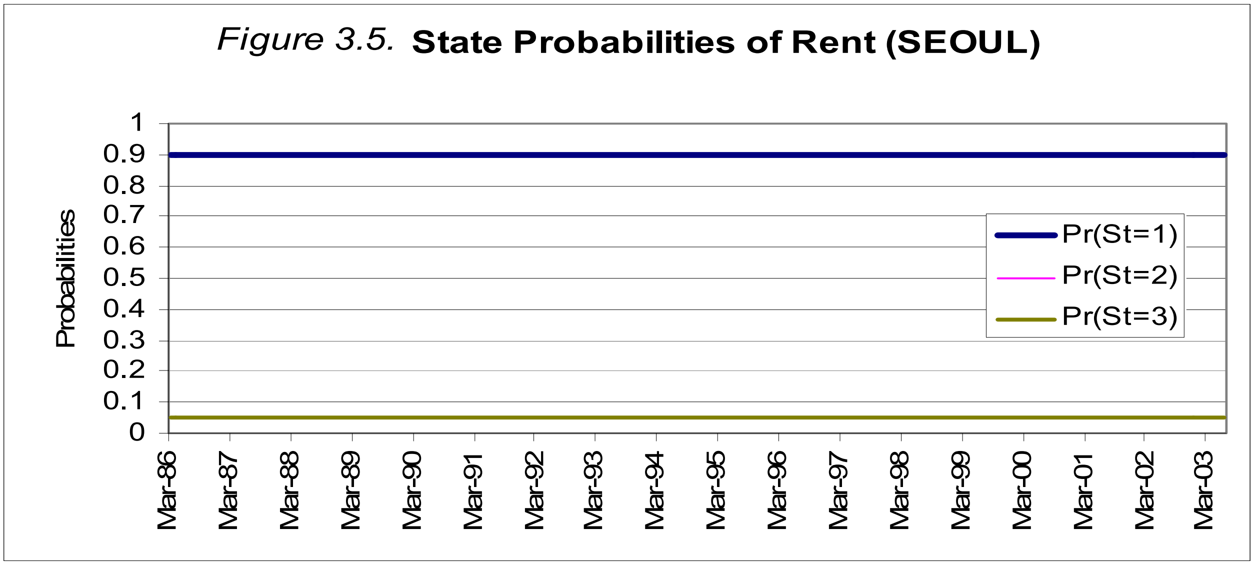


Table 3.2. **State Transition Probabilities**

		p_{11} [3]	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
Hong Kong	Price	0.85	0.11	0.04	0.67	0.23	0.10	0.57	0.28	0.15
	t-ratio	6.83	2.74	0.96	2.74	1.81	0.16	0.95	0.17	0.82
	Rent	0.95	0.05	0.00	0.92	0.08	0.00	0.89	0.10	0.01
	t-ratio	13.57	1.23	0.10	1.22	0.38	0.01	0.10	0.01	0.02
Seoul	Price	0.84	0.03	0.13	0.95	0.01	0.04	0.91	0.02	0.07
	t-ratio	8.36	0.69	2.65	0.73	0.05	0.03	2.61	0.07	0.40
	Rent	0.90	0.05	0.05	0.95	0.02	0.03	0.80	0.15	0.05
	t-ratio	10.20	1.09	1.15	1.28	0.09	0.02	0.97	0.20	0.22

The results for Hong Kong could have been affected by the extension to the series which combines two data sets of different frequencies. But we will not pursue this point further for the time being.

[3] p_{ij} : probability of switch to state j at time $t+1$ if the time t state is i .

3.7 Estimates of Model Parameters

In this section, we list the maximum likelihood estimates of model parameters.

Table 3.3. **Parameter Estimates of Price (HONG KONG)**

Model Parameters									***DGP Parameters***		
St=1			***St=2***			***St=3***					
	Estimates	***se***[4]		***Estimates***	***se***		***Estimates***	***se***		***Estimates***	***se***
ϕ^1	-0.01	0.00	ϕ^2	-0.01	0.01	ϕ^3	-0.01	0.01	σ^2	7.95	1.15
ψ_1^1	0.12	0.08	ψ_1^2	0.12	0.18	ψ_1^3	0.12	0.26	π_1	0.89	1.00
ψ_2^1	0.17	0.08	ψ_2^2	0.17	0.21	ψ_2^3	0.17	0.30	π_2	0.10	1.00
ψ_3^1	-0.03	0.08	ψ_3^2	-0.03	0.16	ψ_3^3	-0.03	0.23	π_3	0.01	1.00
ψ_4^1	0.19	0.08	ψ_4^2	0.19	0.20	ψ_4^3	0.19	0.29			
ψ_5^1	0.19	0.10	ψ_5^2	0.19	0.12	ψ_5^3	0.19	0.18			
ψ_6^1	-0.03	0.07	ψ_6^2	-0.03	0.32	ψ_6^3	-0.03	0.41			
ψ_7^1	-0.03	0.13	ψ_7^2	-0.03	0.10	ψ_7^3	-0.03	0.15			
ψ_8^1	0.21	0.08	ψ_8^2	0.21	0.20	ψ_8^3	0.21	0.28			
ψ_9^1	-0.25	0.09	ψ_9^2	-0.25	0.15	ψ_9^3	-0.26	0.21			
ψ_{10}^1	-0.05	0.08	ψ_{10}^2	-0.05	0.21	ψ_{10}^3	-0.05	0.29			

[4] se: Standard Error

μ^1	0.74	0.22	μ^2	0.73	1.10	μ^3	0.73	2.34			

Table 3.4. **Parameter Estimates of Rent (HONG KONG)**

Model Parameters									***DGP Parameters***		
St=1			***St=2***			***St=3***					
	Estimates	***se***		***Estimates***	***se***		***Estimates***	***se***		***Estimates***	***se***
ϕ^1	-0.02	0.00	ϕ^2	-0.02	0.01	ϕ^3	-0.02	0.04	σ^2	5.71	1.82
ψ_1^1	-0.27	0.07	ψ_1^2	-0.27	0.19	ψ_1^3	-0.27	0.63	π_1	0.99	1.00
ψ_2^1	-0.09	0.07	ψ_2^2	-0.09	0.19	ψ_2^3	-0.09	0.63	π_2	0.01	1.00
ψ_3^1	0.07	0.07	ψ_3^2	0.07	0.22	ψ_3^3	0.07	0.73	π_3	0.00	1.00
ψ_4^1	0.02	0.07	ψ_4^2	0.02	0.19	ψ_4^3	0.02	0.64			
ψ_5^1	-0.02	0.06	ψ_5^2	-0.02	0.32	ψ_5^3	-0.02	1.05			
ψ_6^1	0.06	0.07	ψ_6^2	0.07	0.22	ψ_6^3	0.07	0.73			
ψ_7^1	0.19	0.07	ψ_7^2	0.19	0.23	ψ_7^3	0.19	0.77			
ψ_8^1	-0.03	0.07	ψ_8^2	-0.03	0.16	ψ_8^3	-0.03	0.54			
μ^1	1.23	0.16	μ^2	1.25	2.12	μ^3	1.26	9.46			

Table 3.5. **Parameter Estimates of Price (SEOUL)**

Model Parameters									***DGP Parameters***		
St=1			***St=2***			***St=3***					
	Estimates	***se***		***Estimates***	***se***		***Estimates***	***se***		***Estimates***	***se***
μ^1	0.45	0.09	μ^2	0.45	1.20	μ^3	0.45	1.26			

ϕ^1	-0.005	0.00	ϕ^2	-0.005	0.00	ϕ^3	-0.005	0.00	σ^2	1.29	0.24
ψ_1^1	0.48	0.07	ψ_1^2	0.48	0.25	ψ_1^3	0.48	0.14	π_1	0.85	1.0
ψ_2^1	0.20	0.07	ψ_2^2	0.20	0.28	ψ_2^3	0.20	0.16	π_2	0.03	1.0
ψ_3^1	-0.17	0.07	ψ_3^2	-0.17	0.28	ψ_3^3	-0.17	0.16	π_3	0.12	1.0

Table 3.6. **Parameter Estimates of Rent (SEOUL)**

Model Parameters									***DGP Parameters***		
St=1			***St=2***			***St=3***					
	Estimates	***se***		***Estimates***	***se***		***Estimates***	***se***		***Estimates***	***se***
μ^1	2.12	0.12	μ^2	2.12	1.54	μ^3	2.12	1.51	σ^2	1.29	0.43
ϕ^1	-0.02	0.00	ϕ^2	-0.02	0.01	ϕ^3	-0.02	0.01	π_1	0.90	1.00
ψ_1^1	0.50	0.08	ψ_1^2	0.50	0.13	ψ_1^3	0.50	0.14	π_2	0.05	1.00
									π_3	0.05	1.00

3.8 Unit Root Hypothesis Testing

The unit root test statistic is the t-ratio associated with ϕ, under the null hypothesis of $\phi = 0$. The null distribution of this statistic is unknown but can be generated by bootstrapping. Since only the first state is likely for both Hong Kong and Seoul, we will bootstrap only the parameters associated with that state.

The steps of bootstrapping are described below.

Step one: save the ML parameter estimates $\tilde{\theta}$ and residuals $\{\tilde{\varepsilon}_t\}_{t=1}^{T}$;

Step two: construct an artificial random variable $u \quad i.i.d.N\left(0,\tilde{\sigma}^2\right)$, where $\tilde{\sigma}^2$ the ML estimates of σ^2

Step three: take a random draw from u, denote as $u_1^{(1)}$, and set

$$\Delta y_1^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{-k} + u_1^{(1)}$$

$$\Delta y_2^{(1)} = \tilde{\mu} + \tilde{\psi}_1 \Delta y_1^{(1)} + \sum_{k=2}^{K} \tilde{\psi}_k \Delta y_{-k} + u_1^{(1)}$$

…

$$\Delta y_T^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{T-k}^{(1)} + u_1^{(1)}$$

where: $\Delta y_t^{(1)}$: simulated values of Δy_t;

Δy_{-k}: actual observed values of Δy_t;

$\tilde{\mu}, \tilde{\psi}_k$: ML estimates.

This gives a full sample $\left\{y_t^{(1)}\right\}_{t=1}^{T}$.[5]

Step four: fit the artificial sample to equation (3.2.1), producing estimates of model parameters, $\tilde{\theta}^{(1)}$, and their associated t-ratios.

Step five: repeat step three and four 520 times, gives $\left\{\tilde{\theta}^{(i)}\right\}_{i=1}^{520}$ and their associated t-ratios. The 95% confidence interval for ML estimates $\tilde{\theta}$ and its t-ratio constructed under the null hypothesis include 95% of the values of $\tilde{\theta}^{(i)}$ and their associated t-ratios respectively.

[5] T=206 for Seoul and 255 for Hong Kong.

The resulting critical values are given in table 3.6 alongside the t-ratio for ϕ constructed under the null hypothesis of $\phi = 0$. These results show that, for all series tested, the null hypothesis can be accepted when the alternative is $\phi > 0$, but rejected when the alternative is $\phi < 0$. That is the tests indicate that these series are stationary.

Table 3.7. **Parameter T-Ratio and Critical Valuess**

		t-ratio	Critical values				
			Upper 5%	*Upper 10%*	*Lower 10%*	*Lower 5%*	*Lowest*
Hong Kong	ϕ (*Price*)	-3.22948	1.648	1.341	-1.481	-1.928	-3.467
	ϕ (*Rent*)	-8.37033	1.484	1.184	-1.67	-2.159	-3.231
Seoul	ϕ (*Price*)	-4.9249	1.59	1.23	-1.27	-1.7	-3.32
	ϕ (*Rent*)	-16.2778	1.55	1.26	-1.27	-1.73	-2.66

4. Conclusion and Discussion

The tests show that only one state prevails in each series and none of the four series under consideration has an explosive root, thus one can infer from this result that there are no explosive bubbles in Hong Kong retail premise price and Seoul housing price. However, the unit root tests results, when the alternative is stationarity, run contrary to the standard ADF and Phillips Perron tests which suggest both series are nonstationary in levels but stationary in their first differences. Thus the power of the test is questionable.

A possible contributing factor is the presence of heteroscedasticity in the ML residuals. In (3.2.1), it is assumed that ε_t has a spherical distribution, whereas in fact the ML residuals displays ARCH pattern. Refer to figure 4.1 and 4.2. The heteroscedasticity may also have affected the estimates of state probabilities. To investigate this possibility, one can conduct a simulation study, using DGPs consistent with (3.2.1) but allowing for non-spherical disturbances.

Figure 4.1. **ML Estimation Residuals (HONG KONG)**

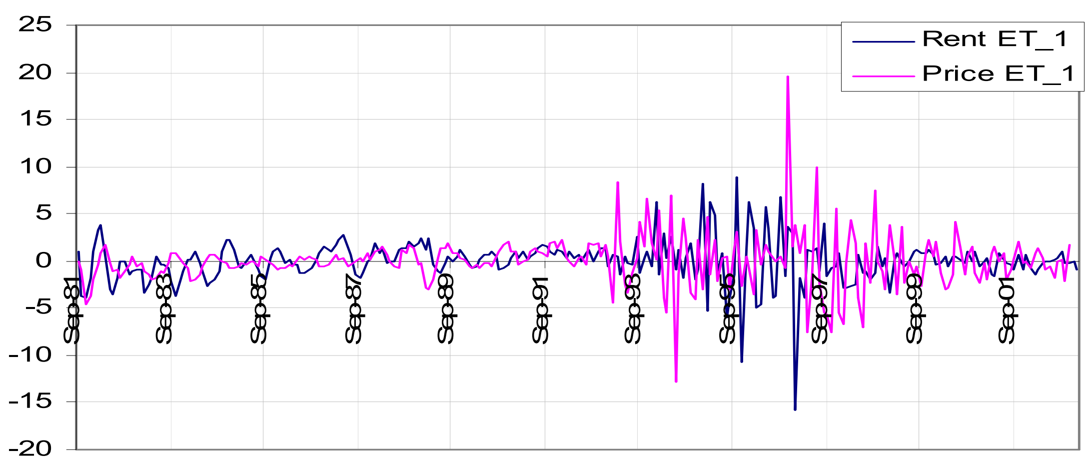
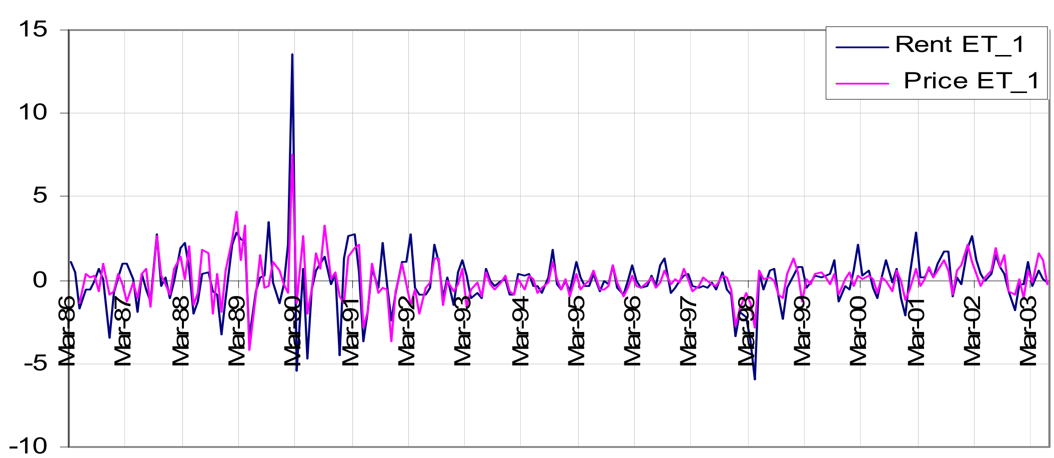


Figure 4.2. **ML Estimation Residuals (SEOUL)**



Other contributing factors could be the way Hessian is calculated or the way the critical values are generated. In using Taylor expansion, the higher order derivative terms are omitted, which may cause non-negligible errors. Thus one may wish to use analytical Hessian is lieu of the numeric Hessian. In bootstrapping, one assumes that the ML parameter estimates are the true values of these parameters, which is unlikely to be true. How much this assumption affects the results is a question mark. Hence, one may wish to use the asymptotic distribution of the model parameters instead (Cavaliere, 2003).

Before completing the paper, notice figure 4.3 and 4.4 show that the extreme volatilities in the ML residuals of price is associated with large price swings. The observation may have something to do with the so-called *critical phenomena* in a complex system, which has its wide applications in natural sciences and is recently employed to explain stock market crashes. The increasing volatility before a price reversal may bear the *log-periodicity* signature which indicates that the death of a bubble is looming up.

Figure 4.3. **Price and ML residuls (HONG KONG)**

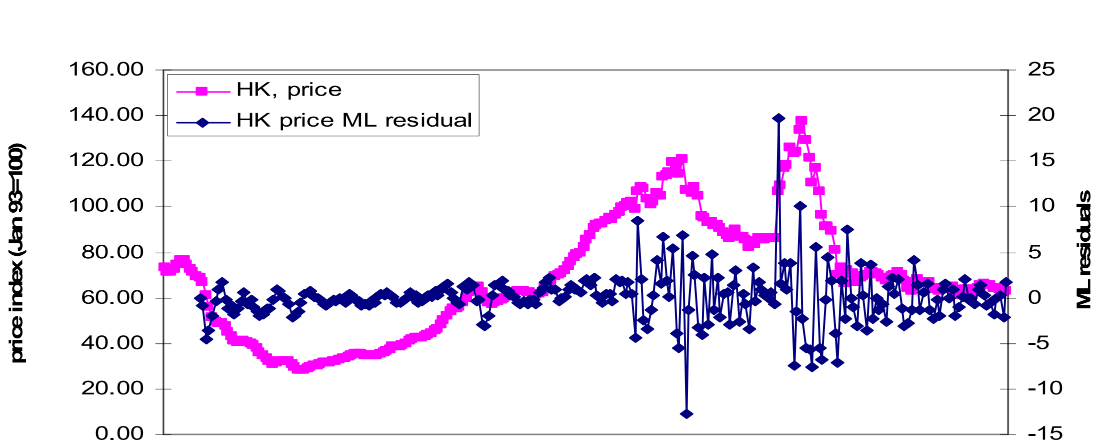
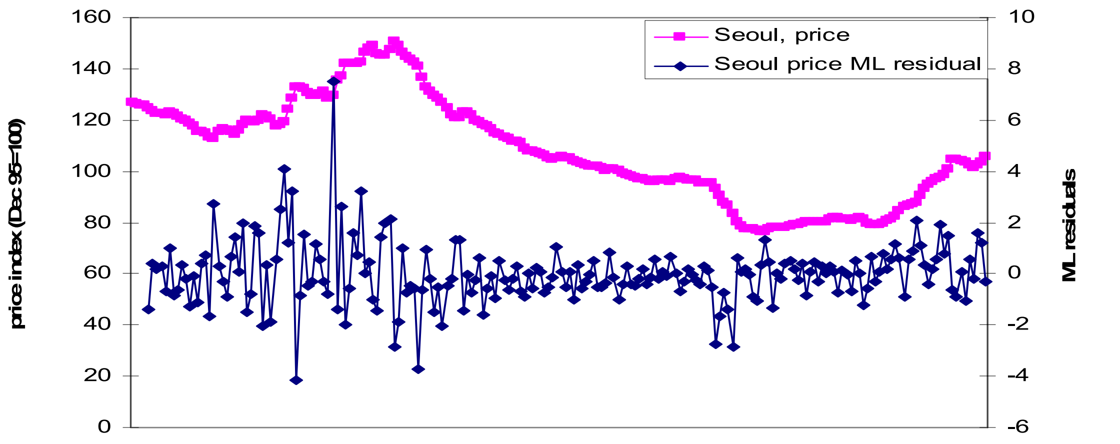


Figure 4.4. **Prices and ML Residuals (SEOUL)**



List of Figures

List of Tables

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