Average-cost pricing, increasing returns, and optimal output in a model with home and market production

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Abstract

The analysis of economies of specialization at the individual level by Yang & Shi (1992) and Yang & Ng (1993) is combined with the Dixit & Stiglitz (1977) analysis of monopolistic-competitive firms to show that, ignoring administrative costs and indirect effects (such as rent-seeking), even if both the home and the market sectors are produced under conditions of increasing returns and there are no pre-existing taxes, it is still efficient to tax the home sector to finance a subsidy on the market sector to offset the under-production of the latter due to the failure of price-taking consumers to take account of the effects of higher consumption in reducing the average costs and hence prices, through increasing returns or the publicness nature of fixed costs. Within market production, it is efficient to subsidise more the sector with a higher fixed cost, a lower elasticity of substitution between goods, and a lower degree of importance in preference which all increases the degree of increasing returns.

Keywords: increasing returns, average-cost pricing, monopolistic competition, home production, optimal output, fixed costs.

JEL Classifications: D43, H21, L11.

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The issue of increasing returns is one of those that will be raised incessantly as a neat general solution is lacking and many different outcomes are possible. Increasing returns are also prevalent in the real economy. Arrow (1995) explains how the relevance of information and knowledge in production makes increasing returns prevalent. (See also Wilson 1975, Radner & Stiglitz 1984, Arthur 1994. On empirical evidence for increasing returns, see Ades & Glaeser 1999, Antweiler & Trefler 2002.) Recent interest in the topic has also been considerable, as witnessed by a number of collected volumes (Arrow, et al. 1998, Buchanan & Yoon 1994a, Heal 1999.)[1] A model much used in the analysis is one developed by Dixit & Stiglitz (1977) with symmetrical monopolistic competitive firms with free entry and produced with a fixed cost and a constant marginal cost. Though a special case, it reflects much of reality as much of increasing returns may be traced to some big fixed-cost components (a piece of land for farming, a factory for manufacturing production, a shop for retailing, learning costs for many skilled activities, etc.) and the variable costs (raw materials, intermediate goods purchased, stocks ordered, etc.) are largely constant within a wide range. While this model captures much of increasing returns at the firm level, those at the economy level arising from the economies of specialization made possible by the division of labor are analysed by Yang & Ng (1993). This latter framework starts from the most basic level of individual decisions on what activities (home production, trade, employment, consumption, etc.) to undertake to maximize utility and the interaction of the activities of individuals and their implications on economic organization, trade, growth, etc. Though the emergence of firms and other issues (including the choice of the variety of products; see Yang & Shi 1992) are also analysed, the economies of specialization is taken to be confined to the individual level. In principle, one may use this framework to

[1] For a survey of the earlier literature on the economics of product variety which is related to monopolistic competition and increasing returns, see Lancaster (1990).

model the increasing returns at the firm level through the complicated interaction between different individuals within the firm and their interaction with other factors employed by the firm, but the complication involved may raise issues of manageability. There are also models that analyse the increasing returns from employing production methods with more intermediate goods (e.g. Ethier 1979, Romer 1986, Buchanan & Yoon 1994b). However, there are advantages in directly allowing for increasing returns at the firm level from the fixed-cost components as in the Dixit-Stiglitz model. Moreover, the majority of production in most advanced economies is undertaken by firms with increasing returns prevailing over the whole relevant range of production, but also with individual home production still taking place. The present paper combines the analysis of economies of specialization at the individual level by Yang & Shi (1992) and Yang & Ng (1993) with the Dixit & Stiglitz (1977) analysis of the market production by monopolistic-competitive firms. (For an earlier model combining home production with market production by firms emphasizing the role of the number of intermediate goods and different stages of production, see Locay 1990. Here, the complications due to intermediate goods and stages in production are ignored.) It is hoped that this combination moves a step closer to the real economy with both home and firm/market production. The model developed in Section 1 below could be used to analyse problems other than those discussed in this paper.

It is shown in Section 1 that, even if both the home and the market sectors are produced under conditions of increasing returns and there are no pre-existing taxes (including income taxes) on the market sector, it is still efficient to tax the home sector to finance a subsidy on the market sector to offset the under-production of the latter. The home sector is not under-produced because the increasing returns there are fully taken into account by the individuals/households. In the production by firms for the market, as the output is priced at average cost and each consumer takes the price as given, the effect of higher consumption in reducing the price through increasing returns is not taken into account. Viewed differently, the fixed cost of production possesses the publicness characteristic, causing under-production. However, the taxation of the home sector may not be practically feasible. Section 2 allows market production to have two sectors and shows that it is efficient to tax the sector with a lower fixed cost, a higher elasticity of substitution between goods, and a higher degree of importance in preference (as all these factors contribute to a lower degree of increasing returns) and subsidize the other sector. Qualifications on the applicability of these results in the real economy are discussed in the concluding section.

1. A model with home and market production

1.1 The model

Consider an economy with M identical consumers. Each of them has the following decision problem for consumption, working, and home production.

(1) Max: $u = l^{1-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1}[\sum_{j\in J} x_j^{\rho_2}]^{\beta/\rho_2}$ (utility function)

s.t. $\sum_{r\in R} p_r x_r = w(1-l-\sum_{j\in J} l_j)$ (budget constraint)

$x_j = \dfrac{l_j - a}{c}$ (home production function)

where p_r is the price of good r which are market goods, w is the price of labour, x_r is the amount of good r that is purchased from the market, R is the set of market goods, x_j is the amount of good j which is home good, l_j is the amount of labour used in producing home good j, $a<1$ is the fixed cost of producing a home good, c is the marginal cost in home production, J is the set of home goods, l is leisure, $(1-l-\sum_{j\in J} l_j)$ is the amount of labour hired by firms, $\rho_i \in (0,1)$ (open interval) is the parameter of elasticity of substitution between each pair of consumption goods, α, β are preference parameters, and u is the utility level. It is assumed that each consumer is endowed with one unit of labour, which is the *numeraire*, so $w=1$. Each consumer is a price taker and her decision variables are l, l_j and x_r. It is assumed that the elasticity of substitution $1/(1-\rho_i) > 1$,or$1> \rho_i >0$ for both i = 1, 2.

Denoting the number of home goods as m, by symmetry, the budget constraint can be rewritten as follow:

(2) $$\sum_{r\in R} p_r x_r + m l_h + l = 1$$

where l_h is labor used in the production of each home good.

From the Lagrangean function L (where λ is used as the multiplier) of the above maximization problem, the first-order conditions[2] are:

(3) $$\frac{\partial L}{\partial x_r} = 0 \;\Rightarrow\; \alpha l^{1-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1 - 1} x_r^{\rho_1 - 1} m^{\beta/\rho_2} (\frac{l_h - a}{c})^{\beta} = \lambda p_r$$

(4) $$\frac{\partial L}{\partial m} = 0 \;\Rightarrow\; \frac{\beta}{\rho_2} l^{1-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1} m^{\beta/\rho_2 - 1} (\frac{l_h - a}{c})^{\beta} = \lambda l_h$$

(5) $$\frac{\partial L}{\partial l_h} = 0 \;\Rightarrow\; \beta l^{1-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1} m^{\beta/\rho_2} (\frac{l_h - a}{c})^{\beta - 1} = c\lambda m$$

(6) $$\frac{\partial L}{\partial l} = 0 \;\Rightarrow\; (1-\alpha-\beta) l^{-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1} m^{\beta/\rho_2} (\frac{l_h - a}{c})^{\beta} = \lambda$$

(2) $$\frac{\partial L}{\partial \lambda} = 0 \;\Rightarrow\; \sum_{r\in R} p_r x_r + m l_h + l = 1$$

Using (2)-(6), we can get following solutions

(7) $$l_h = \frac{a}{1-\rho_2}$$

(8) $$l = \frac{\rho_2(1-\alpha-\beta)}{\rho_2 + \beta(1-\rho_2)}$$

(9) $$m = \frac{\beta(1-\rho_2)}{a[\rho_2 + \beta(1-\rho_2)]}$$

(10) $$x_r = \frac{\alpha\rho_2}{[\rho_2 + \beta(1-\rho_2)] p_r^{\frac{1}{1-\rho_1}} (\sum_{s=1}^{n} p_s^{\frac{\rho_1}{\rho_1 - 1}})}$$

where n is the number of market goods.

[2] It may be checked that second-order conditions are satisfied.

Before we consider the behaviour of firms, we first get the own price elasticity of demand for good r, using (10), we have

$$\frac{\partial \ln x_r}{\partial \ln p_r} = \frac{\rho_1 - n}{n(1-\rho_1)} \tag{11}$$

This formula is called Yang-Heijdra formula (Yang and heijdra, 1993).

Next, we consider the firms' decision problems. We assume that the market structure is monopolistic competition. Each firm produces a good under conditions of increasing returns to scale. Because of global increasing returns to scale, only one firm can survive in the market for a good. If there are two firms producing the same good, one of them can always increase output to reduce price by utilizing further economies of scale, thereby driving the other firm out of the market. Therefore, the monopolist can manipulate the interaction between quantity and price to choose a profit maximizing price. Free entry into each sector is however assumed. Free entry will drive the profit of a marginal firm that has the lowest profit to zero. Any positive profit of the marginal firm will invite a potential entrepreneur to set up a new firm to produce a differentiated good. For a symmetric model, this condition implies zero profit for all firms.

Assume that the production function of good r is

$$X_r = (l_r - A)/b$$

so that the labor cost function of good r is

$$l_r = bX_r + A \tag{12}$$

where A is the fixed cost and b the constant marginal cost. The first-order condition for the monopolist to maximize profit with respect to output level or price implies that

$$MR = p_r[1 + 1/(\partial \ln x_r / \partial \ln p_r)] = MC = b \tag{13}$$

where MR and MC stand for marginal revenue and marginal cost, respectively. Inserting the expression for the own price elasticity of demand $\partial(\ln x_r)/\partial(\ln p_r)$ in (11) into (13), we have

$$p_r = \frac{b(n-\rho_1)}{\rho_1(n-1)} \tag{14}$$

The zero profit condition implies

(15) $$p_r X_r = bX_r + A$$

1.2 General equilibrium and comparative statics

Since marked goods are symmetric, so we have $X_r = X_s = X$, $x_r = x_s = x$, $p_r = p_s = p$, $r, s = 1, 2, \cdots, n$. In addition, home goods are also symmetric, so we have $l_j = l_k = l_h$, $x_j = x_k = x_h$, $j, k = 1, 2, \cdots, m$. The general equilibrium is given by (7)-(10), (14), (15) and the market clearing condition *Mx* = *X*, which involve the unknowns *p*, *n*, *m*, *l*, l^h, *x*, *X*. Here, the subscripts of variables are skipped because of symmetry. Hence, the general equilibrium values of the various variables are

(16) $$p = \frac{b(n-\rho_1)}{\rho_1(n-1)},$$

$$X = \frac{\rho_1 A(n-1)}{bn(1-\rho_1)},$$

$$x = \frac{\rho_1 A(n-1)}{bn(1-\rho_1)M},$$

$$l = \frac{\rho_2(1-\alpha-\beta)}{\rho_2+\beta(1-\rho_2)},$$

$$l_h = \frac{a}{1-\rho_2},$$

$$l_r = \frac{A(n-\rho_1)}{n(1-\rho_1)},$$

$$n = \frac{M\alpha\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]} + \rho_1,$$

$$m = \frac{\beta(1-\rho_2)}{a[\rho_2+\beta(1-\rho_2)]}.$$

After obtaining explicit solutions for the general equilibrium values of the variables as functions of the parameters, we may next examine the comparative statics by examining the effects of a change in some parameter on the equilibrium values of the variable, as given below:

(17)
$$\frac{\partial n}{\partial M} = \frac{\alpha\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]} > 0,$$

$$\frac{\partial n}{\partial A} = -\frac{M\alpha\rho_2(1-\rho_1)}{A^2[\rho_2+\beta(1-\rho_2)]} < 0,$$

$$\frac{\partial n}{\partial \alpha} = \frac{M\,\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]} > 0,$$

$$\frac{\partial n}{\partial \beta} = -\frac{M\alpha\rho_2(1-\rho_1)(1-\rho_2)}{A[\rho_2+\beta(1-\rho_2)]^2} < 0,$$

$$\frac{\partial n}{\partial \rho_1} = 1-\frac{M\alpha\rho_2}{A[\rho_2+\beta(1-\rho_2)]} < 0$$

$$\frac{\partial n}{\partial \rho_2} = \frac{M\alpha\beta(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]^2} > 0$$

$$\frac{\partial m}{\partial \beta} = \frac{\rho_2(1-\rho_2)}{a[\rho_2+\beta(1-\rho_2)]^2} > 0,$$

$$\frac{\partial m}{\partial a} = -\frac{\beta(1-\rho_2)}{a^2[\rho_2+\beta(1-\rho_2)]} < 0,$$

$$\frac{\partial m}{\partial \rho_2} = \frac{-\beta}{a[\rho_2+\beta(1-\rho_2)]^2} < 0,.$$

The signs of the above comparative-statics results are all straightforward except that for $\frac{\partial n}{\partial \rho_1}$. It appears to be ambiguous. However, if we substitute the solution for *n* into the solution for *x* in (16), we have the value of *x* as given in (18) below. Since the denominator is positive and $\rho_1 A$ in the numerator is also positive, the remaining part $\{M\alpha\rho_2 - A[\rho_2+\beta(1-\rho_2)]\}$ in the numerator must also be positive for *x* to be positive. [3] As *x* has to be positive for *n* to be meaningful, the sign of $\frac{\partial n}{\partial \rho_1}$ is in fact unambiguously negative.

The comparative-statics results above may be seen to be intuitively agreeable, though not all obvious. For example, an increase in population size M increases the

[3] Economically, the size of the fixed cost of market production A must not be too large in relation to the population size M. Otherwise the economy may not be viable if the labour of all people combined is insufficient to provide for the fixed cost of production, allowing for the necessity of producing some home goods and having some leisure.

number of market goods n as it allows the sharing of the fixed costs over more individuals. An increase in the fixed cost A has the reverse effect of reducing the number of market goods. Similarly, the same applies to the fixed cost of home production a on the number of home goods m. An increase in preference (represented by α) for the market goods increases the number n of market goods and an increase in preference (represented by β) for the home goods decreases the number of market goods. An increase in the elasticity of substitution between different market goods (represented by ρ_1) decreases the number of market goods, as it is less important to have different goods. In contrast, an increase in the elasticity of substitution between different home goods (represented by ρ_2) increases the number of market goods, as it decreases the number of home goods m and hence allows the individual to be able to consume more market goods.

To derive the equilibrium level of utility, first we get the equilibrium values of l, x, x_h, n, m, we have

$$l = \frac{\rho_2(1-\alpha-\beta)}{\rho_2+\beta(1-\rho_2)} \tag{18}$$

$$x = \frac{\rho_1 A[M\alpha\rho_2 - A(\rho_2+\beta(1-\rho_2))]}{bM[M\alpha\rho_2(1-\rho_1)+A\rho_1(\rho_2+\beta(1-\rho_2))]}$$

$$x_h = \frac{a\rho_2}{c(1-\rho_2)}$$

$$n = \frac{M\alpha\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]} + \rho_1$$

$$m = \frac{\beta(1-\rho_2)}{a[\rho_2+\beta(1-\rho_2)]}$$

Insert these values into utility function in (1), we have

$$\begin{aligned}
u_e &= l^{1-\alpha-\beta} n^{\frac{\alpha}{\rho_1}} x^{\alpha} m^{\frac{\beta}{\rho_2}} x_h^{\beta} \\
&= \rho_2^{1-\alpha}\rho_1^{\alpha}\beta^{\frac{\beta}{\rho_2}} b^{-\alpha} M^{-\alpha} a^{\beta-\frac{\beta}{\rho_2}} c^{-\beta} A^{\alpha-\frac{\alpha}{\rho_1}} (1-\rho_2)^{\frac{\beta}{\rho_2}-\beta} \\
&\quad (1-\alpha-\beta)^{(1-\alpha-\beta)} [\rho_2+\beta(1-\rho_2)]^{\alpha-\frac{\alpha}{\rho_1}+\beta-\frac{\beta}{\rho_2}-1} \\
&\quad [M\alpha\rho_2 - A(\rho_2+\beta(1-\rho_2))]^{\alpha} \{M\alpha\rho_2(1-\rho_1)+A\rho_1[\rho_2+\beta(1-\rho_2)]\}^{\frac{\alpha}{\rho_1}-\alpha}
\end{aligned}$$

1.3 Optimal output

To analyse the welfare properties, we introduce the government to the model. We let the government tax home production and subsidize market production. This is not restrictive as the tax rate and the subsidy rate may either be positive or negative. Assume that the tax rate of per unit home labor is τ, then consumer's problem is

(20) $$\text{Max:} \quad u = l^{1-\alpha-\beta}[\sum_{r\in R} x_r^{\rho_1}]^{\alpha/\rho_1}[\sum_{j\in J} x_j^{\rho_2}]^{\beta/\rho_2} \qquad \text{(utility function)}$$

$$\text{s.t.} \quad \sum_{r\in R} p_r x_r + \sum_{j\in J} \tau l_j = w(1-l-\sum_{j\in J} l_j) \qquad \text{(budget constraint)}$$

$$x_j = \frac{l_j - a}{c} \qquad \text{(production function)}$$

The equilibrium values for above problem are:

(21) $$l_h = \frac{a}{1-\rho_2},$$

$$l = \frac{\rho_2(1-\alpha-\beta)}{\rho_2+\beta(1-\rho_2)},$$

$$m = \frac{\beta(1-\rho_2)}{a(\tau+1)[\rho_2+\beta(1-\rho_2)]},$$

$$x_r = \frac{\alpha\rho_2}{[\rho_2+\beta(1-\rho_2)]p_r^{\frac{1}{1-\rho_1}}(\sum_{s=1}^{n} p_s^{\frac{\rho_1}{\rho_1-1}})}.$$

In addition, denoting the subsidy rate per unit of market product as σ, the zero-profit condition for each firm is

(22) $$p_r X_r = (b-\sigma)X_r + A$$

We recalculate the equilibrium values of the various variables to obtain:

(23) $$p = \frac{(b-\sigma)(n-\rho_1)}{\rho_1(n-1)},$$

$$X = \frac{\rho_1 A(n-1)}{(b-\sigma)n(1-\rho_1)},$$

$$x = \frac{\rho_1 A(n-1)}{(b-\sigma)n(1-\rho_1)M},$$

$$x_h = \frac{a\rho_2}{c(1-\rho_2)},$$

$$l = \frac{\rho_2(1-\alpha-\beta)}{\rho_2+\beta(1-\rho_2)},$$

$$l_h = \frac{a}{1-\rho_2},$$

$$l_r = \frac{A(n-\rho_1)}{n(1-\rho_1)},$$

$$n = \frac{M\alpha\rho_2(1-\rho_1)}{A[\rho_2+\beta(1-\rho_2)]}+\rho_1,$$

$$m = \frac{\beta(1-\rho_2)}{a(\tau+1)[\rho_2+\beta(1-\rho_2)]}.$$

Finally, by requiring a balanced budget for the government, we have

(24) $$Mm\tau l_h = n\sigma X$$

Using above information, we can get the equilibrium level of utility as

(25)

$$\begin{aligned}
u_e &= l^{1-\alpha-\beta} n^{\frac{\alpha}{\rho_1}} x^{\alpha} m^{\frac{\beta}{\rho_2}} x_h^{\beta} \\
&= \rho_2^{1-\alpha}\rho_1^{\alpha}\beta^{\frac{\beta}{\rho_2}}(b-\sigma)^{-\alpha}(\tau+1)^{-\frac{\beta}{\rho_2}} M^{-\alpha} a^{\beta-\frac{\beta}{\rho_2}} c^{-\beta} A^{\alpha-\frac{\alpha}{\rho_1}}(1-\rho_2)^{\frac{\beta}{\rho_2}-\beta} \\
&\quad (1-\alpha-\beta)^{(1-\alpha-\beta)}[\rho_2+\beta(1-\rho_2)]^{\alpha-\frac{\alpha}{\rho_1}+\beta-\frac{\beta}{\rho_2}-1} \\
&\quad [M\alpha\rho_2 - A(\rho_2+\beta(1-\rho_2))]^{\alpha}\{M\alpha\rho_2(1-\rho_1)+A\rho_1[\rho_2+\beta(1-\rho_2)]\}^{\frac{\alpha}{\rho_1}-\alpha} \\
&= B(b-\sigma)^{-\alpha}(\tau+1)^{-\frac{\beta}{\rho_2}}
\end{aligned}$$

where

$$B \equiv \rho_2^{1-\alpha}\rho_1^{\alpha}\beta^{\frac{\beta}{\rho_2}}M^{-\alpha}a^{\beta-\frac{\beta}{\rho_2}}c^{-\beta}A^{\alpha-\frac{\alpha}{\rho_1}}(1-\rho_2)^{\frac{\beta}{\rho_2}-\beta}(1-\alpha-\beta)^{(1-\alpha-\beta)}$$
$$[\rho_2+\beta(1-\rho_2)]^{\alpha-\frac{\alpha}{\rho_1}+\beta-\frac{\beta}{\rho_2}-1}[M\alpha\rho_2-A(\rho_2+\beta(1-\rho_2))]^{\alpha}$$
$$\{M\alpha\rho_2(1-\rho_1)+A\rho_1[\rho_2+\beta(1-\rho_2)]\}^{\frac{\alpha}{\rho_1}-\alpha}$$

isindependentofthetaxandsubsidyrates τ and σ. The effect of a change in tax rate τ on the equilibrium value of utility with respect to the tax rate, and with the subsidy rate at whatever level that is allowed by the government budget constraint (24) as τ varies, evaluated at $\tau = 0$, is given by

$$\text{(26)} \quad \left.\frac{du_e}{d\tau}\right|_{\tau=0} = \frac{Bb^{-\alpha}\beta[\alpha M\rho_2(1-\rho_1)+A\rho_1(\rho_2+\beta(1-\rho_2)]}{\rho_1\rho_2[M\alpha\rho_2-A(\rho_2+\beta(1-\rho_2))]} > 0$$

This means that, starting from the original position without any tax/subsidy, a tax on home production which finances for a subsidy on market production increases utility, ignoring administrative costs and any possible side effects, such as rent-seeking activities triggered by the subsidy. Since all firms just break-even in equilibrium, we may base our welfare comparisons simply on the utility levels alone. We thus have,

Proposition 1: In our model with both home and market production under the conditions of increasing returns and average-cost pricing, a subsidy, if not excessive, on market production financed by a tax on home production improves efficiency even if the initial position involves no tax distortion, ignoring administrative costs and any possible side effects.

The possibility for efficiency improvements through some tax/subsidy means that the original equilibrium is not perfectly efficient. What is the source of this imperfect efficiency? We view the imperfect efficiency as a result of the combination of increasing returns and average-cost pricing in market goods. In the model, there are also increasing returns in home production. However, since the individual/household concerned makes the decision to produce/consume, the implications of increasing returns are taken into account and hence the optimizing choice does not result in any inefficiency. On the other hand, market goods produced by firms are sold to individuals at average costs. Since each consumer take the price of each of this good as given, the demand functions for these market goods do not take the implications of increasing returns into account. Each consumer assumes that, no matter how much she buys, the price will not be affected. However, if all consumers buy more of a market good, the fixed cost component of producing this good will be spread over a larger number of units, result in a lower average cost and hence lower price for every consumer. This effect is not taken into account and hence we have the under-production of the market goods. Subsidizing market goods financed by taxing home production may thus be utility increasing. Put it differently, the fixed cost component of a market good may be viewed as possessing a publicness characteristic since it is shared by all consumers. The under-consumption/production of market goods may be said to be related to the public-good nature of the fixed-cost components.

The taxing of home production may not be practically feasible and a lump-sum tax or poll tax may not be politically feasible or distributionally desirable. The subsidy on market production may thus be impracticable. However, if we allow for different degrees of increasing returns between different market goods, it may be feasible to tax market goods with lower degrees of increasing returns and subsidize market goods with higher degrees of increasing returns, as the next section shows.

2. A model with home and differentiated market production

In this section, the model of the previous section is extended to allow for different sectors of market goods that may have different degrees of elasticity of substitution and different degrees of increasing returns (through different values of the fixed cost and marginal cost). Instead of (1), we now have

(1') Max: $$u = l^{1-\alpha_1-\alpha_2-\beta}[\sum_{r\in R_1} x_r^{\rho_1}]^{\alpha_1/\rho_1}[\sum_{k\in R_2} x_k^{\rho_2}]^{\alpha_2/\rho_2}[\sum_{j\in J} x_j^{\rho}]^{\beta/\rho} \qquad \text{(utility function)}$$

$$\text{s.t.} \sum_{r\in R_1} p_r x_r + \sum_{k\in R_2} p_k x_k = w(1-l-\sum_{j\in J} l_j) \qquad \text{(budget constraint)}$$

$$x_j = \frac{l_j - a}{c} \qquad \text{(home production function)}$$

where two types of market goods are allowed, with R_1 and R_2 as the sets of the first and second types and p_r, p_k their prices and x_r, x_k their quantities consumed/demanded by the representative individual respectively. Other aspects and variables and parameters in (2) are similar to those in (1). For example, $\rho_i \in (0,1)$ is the parameter of elasticity of substitution between each pair of the same type market goods, $\rho \in (0,1)$ is the parameter of elasticity of substitution between each pair of home goods. Each consumer is a price taker and her decision variables are l, l_j and x_r, x_k

For the budget constraint, instead of (2), we have:

(2') $$\sum_{r \in R_1} p_r x_r + \sum_{k \in R_2} p_k x_k + m l_h + l = 1$$

For the new maximization problem, we can get the following solutions

(7') $$l_h = \frac{a}{1-\rho}$$

(8') $$l = \frac{\rho(1-\alpha_1-\alpha_2-\beta)}{\rho+\beta(1-\rho)}$$

(9') $$m = \frac{\beta(1-\rho)}{a[\rho+\beta(1-\rho)]}$$

(10') $$x_r = \frac{\alpha_1 \rho}{[\rho+\beta(1-\rho)] p_r^{\frac{1}{1-\rho_1}} (\sum_{s=1}^{n_1} p_s^{\frac{\rho_1}{\rho_1 - 1}})}$$

(10'') $$x_k = \frac{\alpha_2 \rho}{[\rho+\beta(1-\rho)] p_k^{\frac{1}{1-\rho_2}} (\sum_{s=1}^{n_2} p_s^{\frac{\rho_2}{\rho_2 - 1}})}$$

where n_1, n_2 are the numbers of the two sets of market goods.

From (10') and (10''), we have

(11') $$\frac{\partial \ln x_r}{\partial \ln p_r} = \frac{\rho_1 - n_1}{n_1(1-\rho_1)}, \quad r \in R_1$$

(11'') $$\frac{\partial \ln x_k}{\partial \ln p_k} = \frac{\rho_2 - n_2}{n_2(1-\rho_2)}, \quad k \in R_2$$

Let the production functions of market goods be

$$X_r = (l_r - A_1)/b_1, \quad r \in R_1$$

$$X_k = (l_k - A_2)/b_2, \quad k \in R_2$$

so that the labor cost functions of market goods are

(12') $$l_r = b_1 X_r + A_1, \quad r \in R_1$$

(12") $$l_k = b_2 X_k + A_2, \quad k \in R_2$$

The zero profit condition gives

(15') $$p_r X_r = b_1 X_r + A_1, \quad r \in R_1$$

(15") $$p_k X_k = b_2 X_k + A_2, \quad k \in R_2$$

Similarly to the derivation of (16), we may derive

(16') $$p_1 = \frac{b_1(n_1 - \rho_1)}{\rho_1(n_1 - 1)},$$

$$p_2 = \frac{b_2(n_2 - \rho_2)}{\rho_2(n_2 - 1)},$$

$$X_1 = \frac{\rho_1 A_1(n_1 - 1)}{b_1 n_1(1 - \rho_1)},$$

$$X_2 = \frac{\rho_2 A_2(n_2 - 1)}{b_2 n_2(1 - \rho_2)},$$

$$x_1 = \frac{\rho_1 A_1(n_1 - 1)}{b_1 n_1(1 - \rho_1)M},$$

$$x_2 = \frac{\rho_2 A_2(n_2 - 1)}{b_2 n_2(1 - \rho_2)M},$$

$$l = \frac{\rho(1 - \alpha_1 - \alpha_2 - \beta)}{\rho + \beta(1 - \rho)},$$

$$l_h = \frac{a}{1 - \rho},$$

$$x_h = \frac{a\rho}{c(1 - \rho)}$$

$$l_1 = \frac{A_1(n_1 - \rho_1)}{n_1(1 - \rho_1)},$$

$$l_2 = \frac{A_2(n_2 - \rho_2)}{n_2(1-\rho_2)},$$

$$n_1 = \frac{M\alpha_1\rho(1-\rho_1)}{A_1[\rho + \beta(1-\rho)]} + \rho_1,$$

$$n_2 = \frac{M\alpha_2\rho(1-\rho_2)}{A_2[\rho + \beta(1-\rho)]} + \rho_2,$$

$$m = \frac{\beta(1-\rho)}{a[\rho + \beta(1-\rho)]}.$$

Their comparative statics are:

(17')
$$\frac{\partial n_1}{\partial M} = \frac{\alpha_1\rho(1-\rho_1)}{A[\rho + \beta(1-\rho)]} > 0,$$

$$\frac{\partial n_1}{\partial A_1} = -\frac{M\alpha_1\rho(1-\rho_1)}{A_1^2[\rho + \beta(1-\rho)]} < 0,$$

$$\frac{\partial n_1}{\partial \alpha_1} = \frac{M\rho(1-\rho_1)}{A[\rho + \beta(1-\rho)]} > 0,$$

$$\frac{\partial n_1}{\partial \beta} = -\frac{M\alpha_1\rho(1-\rho_1)(1-\rho)}{A_1[\rho + \beta(1-\rho)]^2} < 0,$$

$$\frac{\partial n_1}{\partial \rho_1} = -\frac{M\alpha_1\rho}{A_1[\rho + \beta(1-\rho)]} + 1 < 0$$

$$\frac{\partial n_1}{\partial \rho} = \frac{M\alpha_1\beta(1-\rho_1)}{A_1[\rho + \beta(1-\rho)]^2} > 0$$

Similarly, $\frac{\partial n_2}{\partial M} > 0$, $\frac{\partial n_2}{\partial A_2} < 0$, $\frac{\partial n_2}{\partial \alpha_2} > 0$, $\frac{\partial n_2}{\partial \rho_2} < 0$, $\frac{\partial n_2}{\partial \rho} > 0$

$$\frac{\partial m}{\partial \beta} = \frac{\rho(1-\rho)}{a[\rho + \beta(1-\rho)]^2} > 0,$$

$$\frac{\partial m}{\partial a} = -\frac{\beta(1-\rho)}{a^2[\rho + \beta(1-\rho)]} < 0,$$

$$\frac{\partial m}{\partial \rho} = \frac{-\beta}{a[\rho + \beta(1-\rho)]^2} < 0.$$

The qualitative results of the comparative statics are again consistent with intuition.

To analyse the welfare properties, we introduce the government to the model and allow the government to tax or subsidize market production but not home production. Denote the tax/subsidiy rate of per unit of each market good in set one as τ_1 (positive if tax; negative if subsidy) and that on set two as τ_2. Then, the zero profit conditions for firms are

(22') $$p_r X_r = (b_1 + \tau_1) X_r + A_1, \quad r \in R_1$$

(22") $$p_k X_k = (b_2 + \tau_2) X_k + A_2, \quad k \in R_2$$

The equilibrium values of the various variables are given by:

(23') $$p_1 = \frac{(b_1 + \tau_1)(n_1 - \rho_1)}{\rho_1 (n_1 - 1)},$$

$$p_2 = \frac{(b_2 + \tau_2)(n_2 - \rho_2)}{\rho_2 (n_2 - 1)},$$

$$X_1 = \frac{\rho_1 A_1 (n_1 - 1)}{(b_1 + \tau_1) n_1 (1 - \rho_1)},$$

$$X_2 = \frac{\rho_2 A_2 (n_2 - 1)}{(b_2 + \tau_2) n_2 (1 - \rho_2)},$$

$$x_1 = \frac{\rho_1 A_1 (n_1 - 1)}{(b_1 + \tau_1) n_1 (1 - \rho_1) M},$$

$$x_2 = \frac{\rho_2 A_2 (n_2 - 1)}{(b_2 + \tau_2) n_2 (1 - \rho_2) M},$$

$$l = \frac{\rho (1 - \alpha_1 - \alpha_2 - \beta)}{\rho + \beta (1 - \rho)},$$

$$l_h = \frac{a}{1 - \rho},$$

$$x_h = \frac{a\rho}{c(1 - \rho)}$$

$$l_1 = \frac{A_1 (n_1 - \rho_1)}{n_1 (1 - \rho_1)},$$

$$l_2 = \frac{A_2 (n_2 - \rho_2)}{n_2 (1 - \rho_2)},$$

$$n_1 = \frac{M\alpha_1\rho(1-\rho_1)}{A_1[\rho+\beta(1-\rho)]} + \rho_1,$$

$$n_2 = \frac{M\alpha_2\rho(1-\rho_2)}{A_2[\rho+\beta(1-\rho)]} + \rho_2,$$

$$m = \frac{\beta(1-\rho)}{a[\rho+\beta(1-\rho)]}.$$

The balanced budget requirement for the government gives

(24') $$n_1\tau_1 X_1 + n_2\tau_2 X_2 = 0$$

The equilibrium utility value is given by

(25')

$$\begin{aligned}
u_e &= l^{1-\alpha_1-\alpha_2-\beta} n_1^{\frac{\alpha_1}{\rho_1}} x_1^{\alpha_1} n_2^{\frac{\alpha_2}{\rho_2}} x_2^{\alpha_2} m^{\frac{\beta}{\rho}} x_h^{\beta} \\
&= \left(\frac{\rho(1-\alpha_1-\alpha_2-\beta)}{\rho+\beta(1-\rho)}\right)^{1-\alpha_1-\alpha_2-\beta} \left\{\frac{M\alpha_1\rho(1-\rho_1)}{A_1[\rho+\beta(1-\rho)]} + \rho_1\right\}^{\frac{\alpha_1}{\rho_1}} \\
&\quad \left\{\frac{\rho_1 A_1[M\alpha_1\rho - A_1(\rho+\beta(1-\rho))]}{M[M\alpha_1\rho(1-\rho_1) + A_1\rho_1(\rho+\beta(1-\rho))]}\right\}^{\alpha_1} \left\{\frac{M\alpha_2\rho(1-\rho_2)}{A_2[\rho+\beta(1-\rho)]} + \rho_2\right\}^{\frac{\alpha_2}{\rho_2}} \\
&\quad \left\{\frac{\rho_2 A_2[M\alpha_2\rho - A_2(\rho+\beta(1-\rho))]}{M[M\alpha_2\rho(1-\rho_2) + A_2\rho_2(\rho+\beta(1-\rho))]}\right\}^{\alpha_2} \\
&\quad \left\{\frac{\beta(1-\rho)}{a[\rho+\beta(1-\rho)]}\right\}^{\frac{\beta}{\rho}} \left[\frac{a\rho}{c(1-\rho)}\right]^{\beta} (b_1+\tau_1)^{-\alpha_1}(b_2+\tau_2)^{-\alpha_2} \\
&= B(b_1+\tau_1)^{-\alpha_1}(b_2+\tau_2)^{-\alpha_2}
\end{aligned}$$

where

$$\begin{aligned}
B \equiv{}& \left(\frac{\rho(1-\alpha_1-\alpha_2-\beta)}{\rho+\beta(1-\rho)}\right)^{1-\alpha_1-\alpha_2-\beta} \left\{\frac{M\alpha_1\rho(1-\rho_1)}{A_1[\rho+\beta(1-\rho)]} + \rho_1\right\}^{\frac{\alpha_1}{\rho_1}} \\
& \left\{\frac{\rho_1 A_1[M\alpha_1\rho - A_1(\rho+\beta(1-\rho))]}{M[M\alpha_1\rho(1-\rho_1) + A_1\rho_1(\rho+\beta(1-\rho))]}\right\}^{\alpha_1} \left\{\frac{M\alpha_2\rho(1-\rho_2)}{A_2[\rho+\beta(1-\rho)]} + \rho_2\right\}^{\frac{\alpha_2}{\rho_2}} \\
& \left\{\frac{\rho_2 A_2[M\alpha_2\rho - A_2(\rho+\beta(1-\rho))]}{M[M\alpha_2\rho(1-\rho_2) + A_2\rho_2(\rho+\beta(1-\rho))]}\right\}^{\alpha_2} \left\{\frac{\beta(1-\rho)}{a[\rho+\beta(1-\rho)]}\right\}^{\frac{\beta}{\rho}} \left[\frac{a\rho}{c(1-\rho)}\right]^{\beta}
\end{aligned}$$

is independent of the tax/subsidy rates. We calculate the derivative of equilibrium utility level with respect to the tax rate τ_1, with the value of τ_2 given by the government budget constraint (24') as τ_1 varies, evaluated at $\tau_1 = 0$, $\tau_2 = 0$, yielding

(26')

$$\left.\frac{du_e}{d\tau_1}\right|_{\tau_1=0} = \frac{B\{\rho_1\alpha_2\{M\alpha_1\rho - A_1[\rho+\beta(1-\rho)]\} - \rho_2\alpha_1\{M\alpha_2\rho - A_2[\rho+\beta(1-\rho)]\}}{b_1^{\alpha_1+1}b_2^{\alpha_2}\rho_2\{M\alpha_2\rho - A_2[\rho+\beta(1-\rho)]\}}$$

From (26'), it can be seen that

(27)
$$\left.\frac{du_e}{d\tau_1}\right|_{\tau_1=0} >/< 0$$

$$\text{if } \frac{\rho_1[M\alpha_1\rho - A_1(\rho+\beta(1-\rho))]}{\alpha_1} >/< \frac{\rho_2[M\alpha_2\rho - A_2(\rho+\beta(1-\rho))]}{\alpha_2}$$

To see the meaning of this condition, consider the following three simple cases:

1. The case of $\rho_1 = \rho_2$ and $\alpha_1 = \alpha_2$ when the condition collapses into $-A_1 > -A_2$, or $A_1 < A_2$. This means that, *ceteris paribus,* if the fixed cost component in the production of market goods of sector one is smaller than that of sector two, it is efficient to tax sector one and subsidize sector two.
2. The case of $\alpha_1 = \alpha_2$ and $A_1 = A_2$ when the condition collapses into $\rho_1 > \rho_2$. This means that, *ceteris paribus,* if the elasticity of substitution between goods within sector one is larger than that within sector two, it is efficient to tax sector one and subsidize sector two.
3. The case of $\rho_1 = \rho_2$ and $A_1 = A_2$ when the condition collapses into $\alpha_1 > \alpha_2$. This means that, *ceteris paribus,* if the preference of individuals is such that goods in sector one is regarded as more important than those in sector two, it is efficient to tax sector one and subsidize sector two.

The intuitive reasons for the three separate points above may be briefly explained. The first point relates to the degree of increasing returns; the higher the fixed cost, the higher the degree of increasing returns and the larger is the publicness characteristic. The elasticity of substitution between goods (second point) is also relevant. The more substitutable are goods within a sector, *ceteris paribus*, the less number of goods of that sector will be produced and more of each good will be produced. (This point is confirmed below.) Then, given that the fixed cost is the same, the degree of increasing returns is lower at higher output. (Defining the degree of increasing returns as the negative of the elasticity of average cost with respect to output, this point may be verified simply by differentiation.) Thus, the sector with a higher elasticity of substitution between goods within that sector has lower degree of increasing returns. A tax on the sector with higher elasticity of substitution and a subsidy on the sector with lower elasticity of substitution is thus taxing the sector with lower degree of increasing returns and subsidising the sector with a higher degree of increasing returns.

The point that, *ceteris paribus*, higher elasticity of substitution leads to a lower number of goods and higher output for each good may be verified. The effect on the number of goods is given by $\frac{\partial n_i}{\partial \rho_i}$ being negative in (17'). The effect on output can be obtained by first substitute the solution for, say, n_1 into that for X_1 from (23'), obtaining

$$X_1 = \frac{\rho_1 A_1\{M\alpha_1\rho - A_1[\rho + \beta(1-\rho)]\}}{b_1\{M\alpha_1\rho(1-\rho_1) + A_1\rho_1[\rho + \beta(1-\rho)]\}}$$

Partial differentiation gives

$$\frac{\partial X_1}{\partial \rho_1} = b_1 M\alpha_1\rho\{M\alpha_1\rho - A_1[\rho + \beta(1-\rho)]\} > 0$$

where the positivity follows from the numerator in the expression for X_1 above.

We may now consider the third point above on the effect of the degree of preference. The more important is a sector regarded by individuals(thehigheris α_i), the more of the goods in that sector are consumed. This again results in a higher output level for goods in that sector, i.e. a higher X_i. This again leads to a lower degree of increasing returns. Thus, *ceteris paribus*, a tax on this sector and a subsidy on a sector with lower importance will be efficient. (This is in line with the result of Heal 1980 that large markets are over-served and small markets are under-served.) Thus, all the three elements of fixed cost, elasticity of substitution, and preference importance parameter are relevant and they all relates to the degree of increasing returns. When more than one of these three elements differ, their effects intertwine and the net effects are as given in (27). Our results may be summarized as

Proposition 2: In our model with both home and market production under the conditions of increasing returns and average-cost pricing with two sectors of different fixed costs, elasticities of substitution, and degree of importance in preference, it is efficient to tax the sector with lower fixed costs, higher elasticity of substitution and/or higher degree of importance in preference and subsidize the other in accordance to (27).

3. Concluding Remarks

Despite the straightforward nature of our results as summarized in the two propositions, the applicability to the real economy is subject to important qualifications. First, the government may not have the information to differentiate which goods should be taxed (and by how much) and which subsidized. Allowing differential tax/subsidies may open a flood gate of rent-seeking activities causing more waste than the efficient gain that could be obtained. Secondly, we have not considered other factors causing imperfect efficiency in the real economy. One important factor is environmental disruption of many production and consumption activities. If the degrees of such disruption are not related to whether a good is home produced or produced for the market, our conclusions may not be much affected. However, there may be some presumption that home production activities are generally less environmentally disruptive than market production. Most of home production consists of home cooking, cleaning, washing, gardening, and childcare, which are largely non-disruptive except for the detergents used. Thus, on the environmental issue, market production should be taxed instead. However, this consideration is to a large extent at least offset by the pre-existence of general taxation including income taxes and value-added or goods and services taxes. These taxes do not fall on home production (the intermediate goods used in home production are produced for the market). If we view these general taxes as largely offsetting to the higher degrees of environmental disruption of most market goods, then the differential degrees of disruption between home and market goods no longer affect our conclusion on the efficiency of subsidizing market goods with higher degrees of increasing returns. The desirability of doing so is then mainly qualified by the first consideration on the lack of information and the promotion of rent-seeking activities.

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