Why is on-net traffic cheaper than off-net traffic? Access markup as a collusive device and a barrier to entry[*]

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Abstract

Received literature have shown that if competing networks are restricted to linear and uniform pricing, high access charges can facilitate collusion; a result that breaks down if we allow for non-linear and discriminatory pricing, however. We show that by adding unbalanced calling pattern to the model, incentives for high access charges are restored. High access charges may make the firms collude on high prices. Moreover, when allowing for entry, we show that incumbents can profitably charge high access prices as a device to deter or soften entrants.

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1   Introduction

Telecom network charges typically involve discrimination against off-net traffic, and mobile telephony is a case in point. What matters for how much a person pays for a call to somebody else is often not the physical distance between the two mobile phones involved, but whether they subscribe to the same mobile phone operator or not. Arguably, there might be cost components associated with traffic between networks only, and then the observed pricing pattern may reflect the underlying costs. This answer is not satisfactory, however: a closer inspection reveals that by far the most important cost determinant of a marginal call to another network is the access fee (termination charge) charged by the receiving network, increasing the "economic" distance between subscribers of different networks. From an industry perspective access fees are not real costs, since the access fees paid by one firm are parts of other firms' revenues, and this give rise to another question: Why are the access charges so high?[1]

Armstrong (1998) and Laffont, Rey and Tirole (1998a) have suggested that access charges are high because firms want high prices, and high access charge makes them charge high prices:[2] In a model with linear and uniform pricing, high access charges implies high perceived marginal costs and high prices, but the high costs are then compensated for by correspondingly high access revenues. Consequently, high access charges can be an instrument for collusive pricing. This is not a compelling explanation in markets like the market for mobile telephony, however. Laffont, Rey and Tirole (1998b) demonstrate that if the operators can discriminate between on-net and off-net traffic and have access to two-part tariffs, high access fees can no longer be used to facilitate high prices.[3] On the contrary, high access charges tend

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[1]The same argument applies to ordinary telephony: before the services were automatized, the costs of "producing" a call were an increasing function of the distance between the points of origination and termination of the call, because long-distance calls had to pass more manual switchboards. Today, however, practically all costs of producing telephone calls are fixed costs. The downward trend in prices of international phone traffic is a reflection of this cost structure.

[2]For an exellent survey of the theory of access pricing and interconnection, see Armstrong (2001).

[3]See also Laffont and Tirole (2000, Section 5.5).

to reduce equilibrium profits, as the access charge makes the firms distort consumer prices with an implied welfare loss.[4] With two-part tariffs this means that the consumers are willing to pay a smaller fixed fee.[5] In many Telecom markets, linear and uniform prices are the exception rather than the rule. It thus remains an open question why we do observe high access charges and large price differences between on- and off-net traffic in such markets.

We propose an answer to this puzzle based on the interaction between three different features which we believe characterize the markets in question. The first is the *tariff-mediated network externalities* that arises when firms discriminate against off-net traffic: Subscribing to a large network lowers the average price of calls. Tariff-mediated network externalities are already present in Laffont, Rey and Tirole's (1988b) analysis and are not sufficient to facilitate collusive pricing on their own.

The second is the existence of *exogenous switching costs*: consumers have a relationship with one of the suppliers, and there are certain costs attached to switching supplier. As shown by Klemperer (1987, 1995), such switching costs facilitate collusive pricing, but with two-part tariffs and non-uniform pricing, switching costs do not call for inefficient pricing: Marginal prices (access charges inclusive) should equal marginal costs, and the market power that arises should be used to increase the fixed fee of the two-part tariff. Consequently, the existence of switching costs alone is no reason to set high access charges.[6]

Third, despite the fact that mobile phone owners can reach millions of other persons, they place their calls to a limited number of people, among which friends, family and workmates comprise the bulk of the recipients. The notion of a *calling club* captures the phenomenon that individuals do not place their calls randomly across networks, but have a bias towards calling other members of their calling club

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[4]In fact, recently Gans and King (2001) have shown that in this context access prices should be subsidized, i.e., should be lower than marginal costs of terminating a call.

[5]In an attempt to restore the collusion effect from high access charges Dessein (2003) introduces heterogeneity in volume and subscription demand. However, neither of these features are sufficient to restore the result of high access charges in equilibrium. Moreover, Dessein (2003) does not allow networks to charge different prices for on-net and off-net calls.

[6]See also Gabrielsen and Vagstad (2002).

(their 'friends'). Since these are persons that are called regularly, it is reasonable to assume that their network location is known by all club members.

The combination of calling clubs and tariff-mediated network externalities works as follows: with higher off-net than on-net prices, members of the same calling club would benefit from joining the same network, ceteris paribus. Once they have coordinated on the same network, each member of the calling club has a preference for remaining with that network, giving rise to similar effects as if the products were horizontally differentiated (e.g. in the Hotelling sense). Switching to another network will make it more expensive to reach one's friends in the old network and by that make it more expensive to make an average call, even if both networks charge identical prices and have the same size (i.e. the same number of subscribers).[7] Consequently, also this type of consumer lock-in will reduce competition, albeit at a certain cost: as long as high access charges do not reflect real costs, price discrimination based on call termination is inefficient and will reduce total surplus compared to a situation in which firms set all marginal prices at their marginal costs. Clearly, if consumers are perfectly flexible and the firms' products are perfect substitutes, either firm could poach all of the rival's customers by lowering its off-net price to zero. The undercutting firm would double its customers base and could even charge a higher fixed fee from all consumers due to increased consumer surplus. Since all consumers switch there is no need to worry about a deficit on access charges. Therefore, in order to generate the equilibrium we are looking for, either the firms' products must be differentiated in some sense, or some consumer inflexibility must be assumed. This is the reason why we have incorporated exogenous switching costs in our model.

We present two closely related models. In the first — the duopoly model — two networks that are symmetric in costs and customer bases first jointly set a common access charge and then simultaneously and independently offer consumers two-part

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[7]Apart for the literature on networks discussed above, our model also relate to the literature on network compatibility (see for instance Katz and Shapiro (1985)). However, this literature is more concerned with consumer expectations and the existence of multiple equilibria which is not an issue here.

tariffs, possibly discriminating on the basis of call termination. The population of consumers 'belong' to either network from an unmodelled initial period, and they incur exogenous switching costs if they want to switch supplier.[8] With access charges above the marginal termination cost, firms will price discriminate against off-net calls, implying that members of a calling club should choose to subscribe to the same network. Moreover, if calling clubs are located in the same network, price discrimination will tend to increase individual switching costs which may enable firms to charge higher fixed fees. We demonstrate that there are indeed situations in which the two firms can increase their profits by setting access charges higher than the true cost of access.

In the second model — the entry model — we expose the two duopolists of the first model to an entry threat. It turns out that setting a high access may deter entry, and it may also be beneficial if entry is not deterred: high access charges makes the entrant softer. Interestingly, entry is positively related to the level of exogenous switching costs, a result that is easiest to interpret in terms of the Fudenberg-Tirole (1984) taxonomy of business strategies: high switching costs make the incumbents "fat cats."

The paper is organized as follows. The next section contains the duopoly model and derives the main results from this model. In Section 3 we present and analyze the entry model. In Section 4 we discuss our modelling choices and the robustness of our results, while Section 5 concludes. Proofs are relegated to the appendix.

2   The duopoly model

Consider a market with two firms or networks denoted $i = 1, 2$. Each network shares equally a unit mass of consumers from an unmodelled initial period. Each consumer places one call.[9] We assume that there are exogenous costs s attached to switching

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[8]Although our analysis is couched in a switching cost framework, some of our results is robust to alternative modes of competition. The switching costs in our model can be reinterpreted as the transport cost in a Hotelling type differentiation model.

[9]Equivalently, each consumer places a unit mass of calls, distributed according to the description below.

supplier. s is uniformly distributed on $[0, \bar{s}]$ with density $g(s) = \frac{1}{\bar{s}}$ and CDF $G(s) = \frac{s}{\bar{s}}$ over the entire support.[10] Hence, $G(s)$ is the proportion of a firm's consumers whose cost of switching to the other firm's product is less than or equal to s.[11]

Utility (gross of payments) of a quantity y of a call equals

$$u(y) = \begin{cases} y - \frac{1}{2}y^2 & \text{if } 0 \leq y \leq 1 \\ \frac{1}{2} & \text{if } y > 1 \end{cases} \qquad (1)$$

This utility function yields rather simple linear demand, and the utility does not depend on who is the recipient of the call.[12] If the price of the call is p per unit, (1) yields the following demand function:

$$y(p) = \begin{cases} 1 - p & \text{if } 0 \leq p \leq 1 \\ 0 & \text{if } p > 1 \end{cases} \qquad (2)$$

and the maximum utility (gross of any fixed fees) from the call is given by the following indirect utility function:

$$v(p) = \begin{cases} \frac{1}{2}(1-p)^2 & \text{if } 0 \leq p \leq 1 \\ 0 & \text{if } p > 1 \end{cases} \qquad (3)$$

We assume that the firms have zero marginal costs and that they can discriminate between on-net and off-net calls.[13] The firms jointly decide the marginal access charge a and then independently and simultaneously offer consumers two-part tariffs. A tariff $\{k, p, q\}$ consists of a fixed fee k, a marginal price p for calls terminated in

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[10] Other distributions yield qualitatively similar results, as long as the distribution is smooth, atomless and has a positive density at $s = 0$ (cf. Klemperer, 1987).

[11] The interpretation of s can either be the traditional switching cost interpretation (assuming that the products are ex-ante homogeneous but ex-post differentiated) or the transportation cost interpretation (with products being both ex-ante and ex-post differentiated, e.g. like in Hotelling type differentiation models).

[12] This latter feature of the specified utility function helps us get rid of a lot of problems associated with price discrimination based on consumer heterogeneity and customer base composition. See Gabrielsen and Vagstad (2001) for an analysis of price discrimination based on customer base composition.

[13] The setup is easily generalized to situations with symmetric constant marginal costs, and it is not difficult to encompass situations in which there are real costs of access as well.

the originating network (internal/on-net price) and a price q for calls terminated in the rival's network (export/off-net price).

Next, we assume that with probability α the call is to a member of one's calling club, and with $(1 - \alpha)$ the call is to an arbitrary person. For tractability, we will assume that $\alpha = \frac{1}{2}$.[14] Moreover, the following assumptions are made about the calling clubs:

A1. Members of the same calling club initially belong to the same network.

A2. There is no overlap between calling clubs.

A3. Members of the same calling club have identical exogenous switching costs.

A4. Members of the same calling club do not coordinate their switching behavior.

The most obvious economic explanation of assumption A1 is perhaps that if it has been common to discriminate against off-net traffic, friends have eventually coordinated on the same network in order to save on calling expenditures[15], but it may also simply be because friends are more likely to have similar preferences and therefore tend to subscribe to similar services.[16] Assumptions A2 and A3 simplifies the technical analysis. Assumption A4 is essential: if friends can coordinate their switching behavior, the existence of calling clubs does not affect the model.[17] To simplify discussion, in what follows we will assume that a calling club consists of

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[14]Most of our results go through with any α. However, some results require that α is above a certain critical value, this citical value being lower than $\frac{1}{2}$. In our setting it seems natural that at least 50% of all calls are to one's own calling club, hence assuming $\alpha = \frac{1}{2}$ is just a simplification.

[15]Having $p < q$ will generate tariff-mediated network externalities and consumers have incentives to sort when choosing their network: friends will coordinate on the same network in local calling clubs. Complete sorting could happen when $p < q$ e.g. if friends enter sequentially and in pairs and that the firms charge identical prices.

[16]Alternatively, friends may have acquired mobile phones at the same times by responding to campaign offers by one of the networks.

[17]Cherdron (2001) has a model that is similar in spirit to our duopoly model. However, he works under the assumption that some members of each calling club has infinite switching costs while the others have no switching costs. He allows them to coordinate, but coordination is no big deal if some of the members cannot move.

only two persons (subsequently called 'friends'), but this is not restrictive as long as each calling club has a negligible mass of consumers.

For a given tariff $\{k, p, q\}$ the equilibrium utility of a representative consumer of network i is given by

$$u_i \;=\; \underbrace{\alpha v(p)}_{\text{friends}} + \underbrace{(1-\alpha)\left(x_i v(p) + (1-x_i)v(q)\right)}_{\text{others}} - \underbrace{k}_{\text{fixed fee}} \tag{4}$$

where x_i is firm i's market share. The corresponding profit of firm i is given by

$$\pi_i = x_i \left(\alpha y(p)p + (1-\alpha)\left(x_i y(p)p + (1-x)y(q)(q-a)\right) + k\right) + T(a, q') \tag{5}$$

where $T(a, q') = (1-\alpha)x_i(1-x_i)ay(q')$ is the access revenues from the other network's consumers, when the other network has set export price q'.

We will, unless explicitly stated, restrict attention to cases in which $\bar{s} \leq \frac{1}{2}$, i.e. the the upper bound on the switching costs is not too high. (If this condition is not satisfied, the two firms will behave like perfect monopolists even without any markup on access, cf. Gabrielsen and Vagstad, 2002). As indicated, our primary interest is in whether it pays for the firms to set a markup on access, i.e., $a > 0$. Thus we have to compare the outcomes for different values of a.

When $a = 0$ Gabrielsen and Vagstad (2002) have shown the following result (which is a relatively straightforward extension to Klemperer (1987)):

Proposition 1 *When $a = 0$, and $\bar{s} \leq \frac{1}{2}$ there exist a unique pure strategy equilibrium involving*

$$p \;=\; q = 0 \tag{6}$$

$$\pi^*(0) \;=\; \frac{1}{2}k = \frac{1}{2g(0)} = \frac{1}{2}\bar{s} \tag{7}$$

The intuition is straightforward: when $a = 0$ there are no reasons for the firms to set inefficient marginal prices, hence $p = q = 0$ for both firms, in order to maximize social surplus. Competition only affects the fixed fees. As usual in switching costs models, whether or not an equilibrium in pure strategies exists depends on the distribution and size of the consumers' switching costs (see Klemperer, 1987). Moreover,

when such an equilibrium exists, the equilibrium fixed fees will depend on the consumers' switching costs. When switching costs are high (i.e., when $\bar{s} \geq \frac{1}{2}$) the firms are able to extract all consumers' surplus through the fixed fees. For lower switching costs ($\bar{s} < \frac{1}{2}$) competition ensures that consumers are left with a strictly positive surplus. The uniform distribution of switching costs turns out to be sufficient to secure the existence and uniqueness of this equilibrium in pure strategies.[18]

Next suppose $a \in (0,1)$. If an equilibrium in pure strategies exists, it must entail $p = 0$ and $q = a$. That is, firms will discriminate between on- and off-net calls because the latter have higher perceived costs. Moreover, this pricing creates tariff-mediated network externalities that will work like a positive switching cost for each individual consumer, who will hesitate to relocate away from his or her friends. Then we can follow the reasoning in Klemperer (1987) to the conclusion that any pure-strategy equilibrium must entail a fixed fee that extracts all consumer's surplus. However, for small access charges this proposed equilibrium is vulnerable to poaching. We will show that for positive, but small values of a there are often no pure-strategy equilibria. There will then be mixed-strategy equilibria, however, but these are complicated to characterize even in a relatively simple model like the present one.[19] Consequently, if a pure-strategy equilibrium exists we must have (each firm having a 50% market share)

$$k = \alpha v(0) + (1 - \alpha)\left(\frac{1}{2}v(0) + \frac{1}{2}v(a)\right) = \frac{1}{4}\left(\frac{1}{2} + 1\right) + \frac{1}{4}\frac{1}{2}(1 - a)^2 \qquad (8)$$

while equilibrium profit equals

$$\pi^*(a) \equiv \frac{1}{2}k + T(a, a) = \frac{1}{4} - \frac{1}{16}a^2 \qquad (9)$$

This will constitute an equilibrium if no firm can make a profitable deviation by undercutting its rival.

There are basically two ways to undercut one's rival: one either tries to poach all of the rival's customers, or one goes after only a fraction of the rival's customers.

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[18] See Gabrielsen and Vagstad (2002) for details.

[19] See Shilony (1977) for an example of how to characterize mixed-strategy equilibria of a model similar to the present one.

In the latter case, the undercutting firm will attract customers with low exogenous switching costs from the other firm.

In principle, it may be more difficult to attract the first of the rival's customers than later customers. Two effects are at play here: The first customers to switch have low switching cost, while later switchers have higher switching costs but an advantage of joining a larger network. A priori it is not clear which effect dominates. The following Lemma establishes that the former effect dominates. (The result turns out to apply to both the duopoly model of this section and the entry model of next.)

Lemma 1 *In both models, in any equilibrium and for any optimal ways to deviate from an equilibrium, if customers with a given exogenous switching cost s' are willing to switch, so are also all consumers with switching costs $s < s'$.*

Proof. See the appendix. ∎

The following two Lemmas describe the profit-maximizing ways to follow each of these strategies. First we consider the case in which a firm poaches all of the rival's customers, and let $\pi^A(\bar{s})$ denote a firm's profit from such a deviation.

Lemma 2 *When $a > 0$, poaching all involves $p = 0$, $k = \pi^A(\bar{s}) \equiv \frac{1}{2} - \bar{s}$.*

Proof. See the appendix. ∎

This Lemma simply states that in order to poach all of your competitor's customers, it is both sufficient and necessary to offer a mechanism that pays the switching cost for the customer with the highest possible switching cost – that is, s. Next, we investigate whether it can be profitable to poach only a fraction of the rival's customers. Let $\pi^S(\bar{s}, a)$ denote a firm's profit from deviating by poaching some of the rival's customers. Then we can show

Lemma 3 *When $a > 0$, poaching some is profitable when $\frac{1}{6} + \frac{1}{6}a^2 < \bar{s} < \frac{1}{2} - \frac{1}{32}a^2$. If $\bar{s}$ is in this interval, then the optimal poaching strategy involves $p = q = \frac{1}{2}(1-x)a$, where $x = x(\bar{s}, a) \in (\frac{1}{2}, 1)$.*

Proof. See the appendix. ∎

This Lemma is more complicated. First, if $\bar{s}$ is outside the given interval, the optimal market share of the poaching firm is either 1 (if $\bar{s}$ is below the interval) or $\frac{1}{2}$ (if $\bar{s}$ is above the interval). In other words, if switching costs are low it is optimal to poach all, and if switching costs are high it is never optimal to try to poach any of the rival's customers. Second, note that when it is optimal to poach only a fraction of the rival's customers, optimal undercutting entails equal on-net and off-net prices. At first glance this may seem surprising. The reason is that customers come in pairs in this model. The virtue of having $p = q$ is to have the last two customers being equally difficult to attract, and this consideration dominates other effects. The expressions for $x(\bar{s}, a)$ and $\pi^S(\bar{s}, a)$ are given in the appendix.

We are now ready to present the main results of this section.

Proposition 2 *For $a > 0$, the proposed equilibrium is indeed an equilibrium iff*
$$\pi^*(a) \geq \max\left\{\pi^A(\bar{s}), \pi^S(\bar{s}, a)\right\}.$$

Given the definitions in Lemmas 2 and 3, Proposition 2 is obvious and therefore stated without a proof. Its implications, however, deserves some comments, which will be given below.

Proposition 3 *For some values of $\bar{s}$, there exist values of $a \in (0, 1)$ denoted a^* such that i) there exists a pure-strategy equilibrium for $a = a^*$, and ii) $\pi^*(a^*) > \pi^*(0)$.*

Proof. See the appendix. ∎

A perhaps more intriguing question is whether one would expect profitable markup on access to be the rule or the exception. The answer is neither, as illustrated by Figure 1 below.
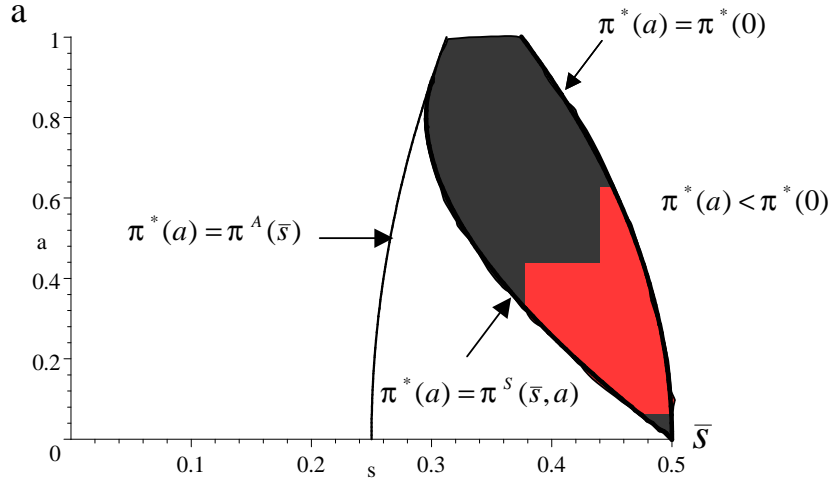
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Figure 1: Pure strategy equilibria with $a > 0$.

On the vertical axis we have $a \in (0,1)$ and on the horizontal axis we have $\bar{s} \in [0, \frac{1}{2}]$. The rightmost curve in the figure shows combinations of a and $\bar{s}$ such that the two equilibria (with $a = 0$ and $a > 0$, respectively) yield the same profit, i.e. $\pi^*(a) = \pi^*(0)$. To the left of this curve, $\pi^*(a) > \pi^*(0)$. The two other curves relate optimal undercutting profits with $\pi^*(a)$. The leftmost curve shows combinations of a and $\bar{s}$ such that a firm is indifferent between its equilibrium profit with $a > 0$ and poaching all of its rival's customers, i.e. $\pi^*(a) = \pi^A(\bar{s})$. To the right of this curve $\pi^*(a) > \pi^A(\bar{s})$. The intermediate curve shows combinations of a and $\bar{s}$ such that $\pi^*(a) = \pi^S(\bar{s}, a)$, i.e. that render the firms indifferent between earning the equilibrium profits with $a > 0$ and poaching only a fraction of the rival's customers. Hence in the shaded area the parameters are such that a pure strategy equilibrium exists, and at the same time the firms earn more profit than if they had rather set $a = 0$. In other words, the shaded area represents instances of profitable markup on access.

First we note that high values of $\bar{s}$ improves the prospects of reaching a stable equilibrium, as usual. Second, for low values of a, $\pi^S(\bar{s}, a) > \pi^A(\bar{s})$, implying that for such low values of a it is most tempting to poach only some of the rival's customers. For high values of a, the reverse is true i.e. $\pi^S(\bar{s}, a) < \pi^A(\bar{s})$ meaning that the

relevant undercutting strategy is to poach all your rival's customers.[20] Finally, note that while the value of a is irrelevant for equilibrium existence if $\bar{s}$ is sufficiently low (no pure strategy equilibrium exists) or high (a pure-strategy equilibrium always exists), there is a region inbetween for which existence of a pure-strategy equilibrium depends on the value of a. It seems like higher values of a promotes the existence, but the reverse can also be true. Finally, since $\bar{s}$ is a given parameter of the two firms' problem while a is a decision variable, we can "maximize out" the latter, thereby concluding that for $\bar{s}$ between (approx.) .3 and .5, a carefully designed markup on access is profitable, while the opposite applies for $\bar{s}$ outside this interval.

To summarize this section, we have demonstrated that high access charges can dampen competition by creating endogenous switching costs for people hesitant to relocate away from their friends. Moreover, for some parameterizations of the model, the benefit from reduced competition is larger than the losses stemming from loss of efficiency associated with marginal prices that are distorted away from their first best level. In the next section we will point at another benefit from the high access charges: they make entry more cumbersome.

3   The entry model

In this section there are two incumbent firms $i = 1, 2$ and one potential entrant denoted by subscript e. As before we assume that each incumbent firm has locked-in one half of the unit mass of consumers with exogenous switching costs uniformly distributed on $(0, \bar{s})$. Thus the share of consumers with switching costs less than or equal to s is $x = G(s) = \frac{s}{\bar{s}}$. The timing is as follows. First the incumbent firms agree on an access price a, $0 \le a < 1$. Then simultaneously each incumbent firm offers contracts $\{p_i, q_i, k_i\}$ and the entrant offers $\{p_e, q_e, k_e\}$. Finally consumers choose where to buy from. To simplify the analysis we assume that there are no fixed costs of entry.[21]

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[20]In the figure the two curves merge for high values of a. Technically, for a higher than about .9, the optimal way to poach some is to poach all.

[21]It is clear that the qualitative results hold also with non–zero costs of entry. Technically, the only effect of a fixed cost of entry is to shift the entrant's profit function by a constant, thereby

3.1 The entrant's problem

Assuming $p_e \leq q_e$ and $p_i \leq q_i$ and supposing that a share x of all consumers switches to the entrant we can formulate the maximization problem of the entrant as follows:

$$
\begin{aligned}
\pi_e(a, \bar{s}) \;=\; \max_{k_e, p_e, q_e} \; & \big\{ x \left[k_e + \alpha(1 - p_e)p_e + (1 - \alpha)\left(x(1 - p_e)p_e \right. \right. \\
& \left. \left. + (1 - x)\, y(q_e)\,(q_e - a) \right) \right] + T_e(a, q_i, x) \big\} \\
\text{s.t.} \quad & \alpha v(p_i) + (1 - \alpha)\left(\tfrac{1-x}{2} v(p_i) + \tfrac{1+x}{2} v(q_i) \right) - k_i + x\bar{s} \\
& \geq \alpha v(q_e) + (1 - \alpha)\left(x v(p_e) + (1 - x)\, v(q_e) \right) - k_e
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
& \alpha v(q_i) + (1 - \alpha)\left(\tfrac{1-x}{2} v(p_i) + \tfrac{1+x}{2} v(q_i) \right) - k_i + x\bar{s} \\
& \geq \alpha v(p_e) + (1 - \alpha)\left(x v(p_e) + (1 - x)\, v(q_e) \right) - k_e
\end{aligned}
\tag{11}
$$

$$
p_e \leq q_e
$$

where $T_e(a, q_i, x) = (1 - \alpha)a(1 - q_i)x(1 - x)$ are the termination revenues that accrue to the entrant.[22]

Lemma 1 demonstrated that when the pair of customers with switching costs $x\bar{s}$ are willing to switch (henceforth called the marginal pair of customers), so are all consumers with lower switching costs. Constraint (10) in the program above secures that one of the marginal pair of consumers will switch and (11) that his friend will switch to the entrant's network. Obviously, if on-net and off-net prices are equal (i.e. $p_i = q_i$ and $p_e = q_e$) the two constraints collapse to one. To decide which of them is binding when on-net prices are lower than off-net prices suppose (10) binds and (11) is slack. Then the difference between the two left-hand sides must be larger than the difference between the two right-hand sides. That is,

$$
v(p_i) - v(q_i) > v(q_e) - v(p_e)
\tag{12}
$$

which always holds when $p_i < q_i$ and $p_e < q_e$. Therefore, for $p_i \leq q_i$ and $p_e \leq q_e$ constraint (10) will be binding and (11) will be slack, or both will collapse to the

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making it easier to deter entry.

[22]With probability $1 - \alpha (= \tfrac{1}{2})$ the call is to an arbitrary person, the revenues from such a call is $a(1 - q_i)$, x is the probability that the call is terminated in the entrant's network and $(1 - x)$ the probability that the call is originated in one of the incumbents' network.

same condition. The intuition is that when on-net prices are lower than off-net prices it is more difficult to attract the marginal consumer than his friend. The reason is that once the marginal consumer has moved, his friend has nothing to gain by staying at the incumbent firm because he will get cheaper calls to his friend when if he moves. Substituting for the indirect utility functions and assuming that (10) binds, yields the condition

$$\frac{3-x}{8}\left(1-p_i\right)^2 + \frac{1+x}{8}\left(1-q_i\right)^2 - k_i + x\bar{s} = \frac{2-x}{4}\left(1-q_e\right)^2 + \frac{x}{4}\left(1-p_e\right)^2 - k_e \quad (13)$$

This constraint implicitly defines k_e as a function of $x, a, k_i, \bar{s}$ and the marginal prices of the firms. It turns out to be more convenient to formulate the entrant's maximization problem as one of choosing x, p_e and q_e. Doing this we can write

$$\begin{aligned}
\pi_e(a,\bar{s}) &= \max_{x,p_e,q_e} \{x\left[k_e + \alpha\left(1-p_e\right)p_e + \left(1-\alpha\right)\left(x\left(1-p_e\right)p_e\right.\right. \\
&\qquad \left.\left. + \left(1-x\right)\left(1-q_e\right)\left(q_e - a\right)\right)\right] + T_e(a, q_i, x)\} \qquad (14) \\
&\text{s.t.} \quad (13) \\
&\qquad p_e \le q_e
\end{aligned}$$

3.2 The incumbents' problem

Each incumbent's maximization problem is similar to the entrant's problem:

$$\begin{aligned}
\pi_i &= \max_{x_i, p_i \le q_i} \left\{ \frac{1-x}{2}\left[k_i + \alpha\left(1-p_i\right)p_i + \left(1-\alpha\right)\left(\frac{1-x}{2}\left(1-p_i\right)p_i\right.\right.\right. \\
&\qquad \left.\left.\left. + \frac{1+x}{2}\left(1-q_i\right)\left(q_i - a\right)\right)\right] + T_i(a, q_j, q_e, x)\right\} \qquad (15) \\
&\text{s.t } (13)
\end{aligned}$$

where $T_i(a, q_j, q_e, x) = a\frac{1}{2}\frac{1-x}{2}\left(\frac{1-x}{2}(1-q_j) + x(1-q_e)\right)$ are the termination revenues that accrues to each incumbent. (As with the entrant we have formulated the maximization problem as one of choosing x instead of k_i.)

3.3 Equilibrium in the entry model

Solving these two maximization problems enables us to give a full characterization of the equilibrium.

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Proposition 4 *Suppose $p_i \leq q_i$ and $p_e \leq q_e$. Then the equilibrium contracts have marginal prices*

$$
\begin{aligned}
p_i &= 0 \\
q_i &= a \\
p_e &= q_e = \frac{1}{2}(1-x)a
\end{aligned}
$$

and fixed fees

$$
\begin{aligned}
k_i &= 11x\bar{s} - 2xa^2 - 3\bar{s} + \frac{3}{2}a \\
k_e &= \frac{1}{2}a + \frac{1}{2}ax + \frac{1}{4}a^2 - \frac{7}{4}xa^2 + 7x\bar{s} - 2\bar{s}
\end{aligned}
$$

where the entrant's equilibrium market share x is given by

$$
x = x(a, \bar{s}) = \frac{1}{a^2}\left(2a^2 - 12\bar{s} + \sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3}\right)
$$

Proof.   See the appendix. ∎

Whereas the incumbents set marginal prices equal to marginal costs, the entrant equate the prices for on- and off-net calls. This rather surprising result deserves some comments.[23] Why does the price structure differ between the two? The key to understand this puzzle is found in exploring the different incentives the firms have. All firms are concerned with efficiency, and this consideration pulls toward pricing according to perceived marginal costs. However, incumbents are concerned with keeping customers whereas the entrant is concerned with attracting customers. As customers come in pairs, we might expect the incumbents to price in a way that make friends reluctant to split, whereas the entrant might be expected to price in a way that make the two members of a pair equally difficult to attract. This is exactly what happens. By equating on- and off-net prices the entrant make the friends equally difficult to attract, and this feature turns out to dominate the efficiency

---

[23]Given that the two prices set by the entrant are equal, it is less of a surprise that the common price is inbetween the marginal cost of internal and external calls. The general expression for the entrant's marginal prices is $p_e = q_e = (1 - \alpha)(1 - x)a$. From this we see that when the probability of a call being to a friend increases or the entrant's market share increases the marginal prices are closer to zero and vice versa when α is small or the entrant has a small market share.

consideration pulling in the direction of setting $p = 0$ and $q = a$. In fact, if the entrant chooses to price efficiently, the efficiency gain goes to the switchers, who get a lower fixed fee in order to make the first in the pair switch. The incumbents, on the other hand, have no such incentives, and stick to marginal cost pricing.

Proposition 5 *The entry equilibrium has the following comparative statics*

$$\frac{\partial x(a, \bar{s})}{\partial a} < 0, \frac{\partial x(a, \bar{s})}{\partial \bar{s}} > 0$$

Proof. See the appendix. ∎

It turns out to be hard to find analytical expressions for how the equilibrium profits of the three firms change as we vary the two underlying parameters a and $\bar{s}$. We will therefore study some examples in more detail.

3.4   Examples

In what follows we will fix the upper bound on the exogenous switching costs, $\bar{s}$, in order to isolate the effects of changing the other parameter a. this section we will work with three different levels of the exogenous switching costs, $\bar{s} \in \left\{ \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \right\}$. Let us first look at the equilibrium market share of the entrant. Figure 2 below plots $x(a, \bar{s})$ for $\bar{s} \in \left\{ \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \right\}$. We have that $x(a, \bar{s}) \geq 0$ in our examples when

$$
\begin{aligned}
x(a, \frac{1}{4}) &\geq 0 \Longrightarrow a \leq 2 - \sqrt{2} = 0.585\,79 \\
x(a, \frac{1}{3}) &\geq 0 \Longrightarrow a \leq 2 - \frac{2}{3}\sqrt{3} = 0.845\,30 \\
x(a, \frac{1}{2}) &\geq 0 \Longrightarrow \text{holds for all } x \in [0, 1]
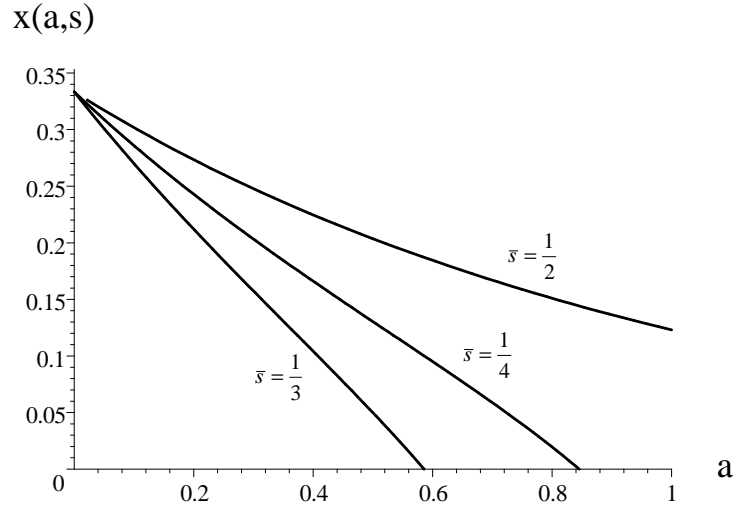\end{aligned}
$$

Figure 2: Equilibrium market share of the entrant.

The lowest line represents $\bar{s} = \frac{1}{4}$, the middle line $\bar{s} = \frac{1}{3}$ and the upper line is for $\bar{s} = \frac{1}{2}$. We see that the entrant's market share is decreasing in the access charge; a sufficiently high access charge may deter entry if switching costs are not too high. Note also that the entrant's market share is increasing in the switching costs, meaning that higher switching costs makes entry more difficult to deter. At first sight it may seem surprising that higher switching costs induces more entry. The key to understand this is that high switching costs turn incumbents into "fat cats" charging high fixed fees and thus creating leeway for the entrant to capture a relatively large fraction of the consumers with the lowest switching costs (cf. Fudenberg and Tirole, 1984).

Similarly, Figure 3 below plots the entrant's profit as a function of access charge for different levels of switching costs.[24] We have that $\pi_e(a, \bar{s}) \geq 0$ when

---

[24]Clearly, if we add a fixed cost of entry, the entrant's profit would be reduced by the same amount, reducing the value of a that is needed to make the entrant's profit negative.

$$\pi_e(a, \frac{1}{4}) \geq 0 \implies a \leq 2 - \sqrt{2} = 0.585\,79$$

$$\pi_e(a, \frac{1}{3}) \geq 0 \implies a \leq 2 - \frac{2}{3}\sqrt{3} = 0.845\,30$$

$$\pi_e(a, \frac{1}{2}) \geq 0 \implies a \leq 1$$
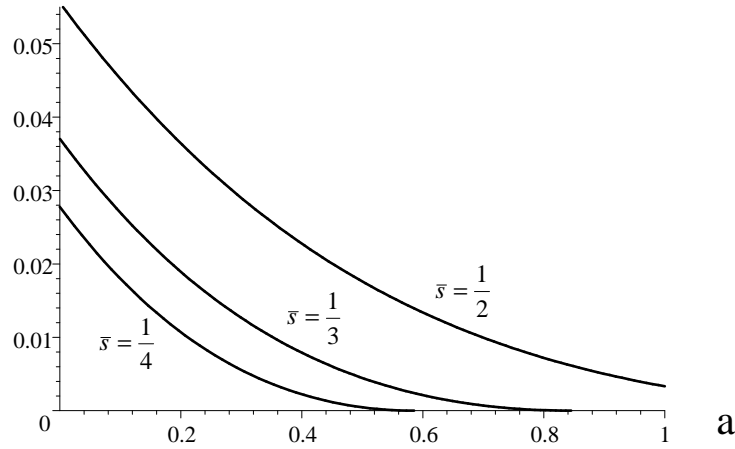
Entrant's profit

Figure 3: The entrant's profit as a function of the access charge.

The lowest line represents $\bar{s} = \frac{1}{4}$, the middle line $\bar{s} = \frac{1}{3}$ and the upper line is for $\bar{s} = \frac{1}{2}$. When a is low enough, entry will occur, and in this case the entrant's profit is increasing in consumers' switching costs. The intuition is as indicated above: when entry occurs, higher switching costs make the incumbents charge high fixed fees, resulting in higher market share and profit for the entrant.

Then finally, look at the incumbent's profit as a function of a. As noted above, for sufficiently high a the entrant may be deterred. For such a high a we are back in the equilibrium in the duopoly model where each incumbent earns $\pi^*(a) = \frac{1}{4} - \frac{1}{16}a^2$. For sufficiently high $\bar{s}$ this duopoly equilibrium is stable. In order to avoid having to deal with situations involving mixed-strategy equilibria, we will restrict attention

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to the following two cases: $\bar{s} = \frac{1}{3}$ and $\bar{s} = \frac{1}{2}$.[25]

When $\bar{s} = \frac{1}{3}$ we know that when $a \geq 0.4926$ the duopoly equilibria are stable (see Proposition 3). Moreover, we have that when $a \leq 2 - \frac{2}{3}\sqrt{3} = 0.845\,30$ the entrant will enter with a non-negative market share. Thus for $a \leq 2 - \frac{2}{3}\sqrt{3}$ the incumbents' profits are increasing in a, and at $a = a \leq 2 - \frac{2}{3}\sqrt{3}$ the equilibrium switches to the duopoly equilibrium which is stable. This is illustrated in Figure 4 below.
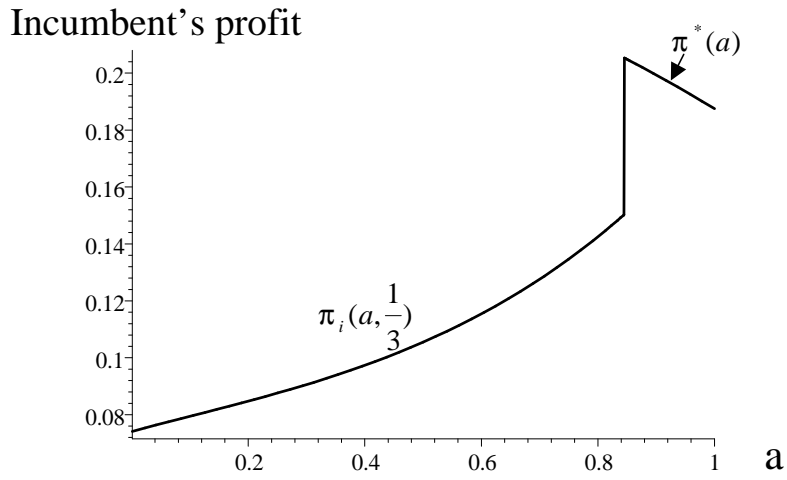

Figure 4: Incumbents' profit as a function of a when $\bar{s} = \frac{1}{3}$.

For intermediate switching costs our model predicts high mark-up on access and entry deterrence.

Finally, when $\bar{s} = \frac{1}{2}$ we know that all duopoly equilibria with $a > 0$ are stable, but that absent the threat from entry, incumbents will prefer the duopoly equilibrium with $a = 0$. However, no $a \leq 1$ can deter entry (cf. Fig. 3), and the optimal entry equilibrium for the incumbents is the one with $a = 1$: The incumbents agree on an access charge that effectively disconnect the networks, in order to reduce entry to a

_____

[25] When $\bar{s} = \frac{1}{4}$, the entry equilibrium has the normal properties, with the incumbent's profit being increasing in a. However, when a is high enough, entry does not take place, and the resulting duopoly has no pure-strategy equilibrium.

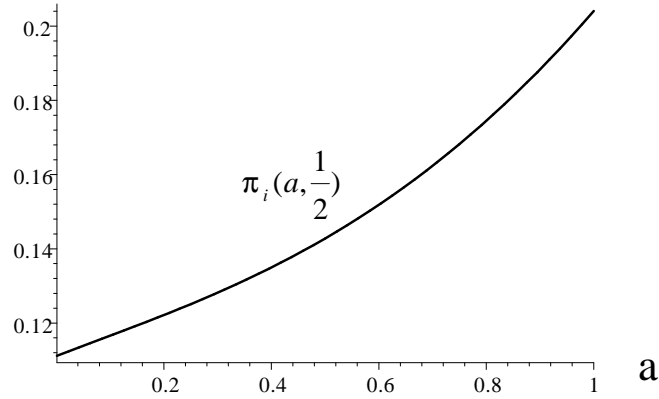minimum. This is illustrated in Figure 5 below.

Incumbent's profit

Figure 5: Incumbents' profits as a function of a when
$\bar{s} = \frac{1}{2}$.

4   Concluding remarks

Previous literature have shown that high access charges between competing Tele-com networks can be used as a device for facilitating collusion, but only as long as attention is restricted to uniform, linear pricing: If firms can offer two-part tar-iffs and can discriminate between on- and off-net calls, high access charges are no longer profitable; it will only induce inefficient prices and thereby loss of revenues for the networks. An apparent puzzle therefore is why networks typically charge a substantial markup on access, leading to inefficient prices of off-net calls.

The contribution of this paper is to demonstrate that by introducing exogenous switching costs in combination with local calling clubs, incentives to charge a markup on access are restored. Higher off-net than on-net prices gives consumers an incentive to locate on the same network as friends. Once there, consumers will be loyal to this network in so far as – for equal prices – there are switching costs associated with relocating away from friends. For firms, increased loyalty give room for higher prices and thereby higher profit. This must of course be balanced against the distortions

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costs of high export prices – a cost that tends to be born by the firms if consumers are relatively homogeneous and two-part tariffs can be used. In our duopoly model we have shown that the benefits from access markup may dominate the losses. Alas, the opposite may also be true, perhaps suggesting that this cannot explain the almost universal practice of charging a markup on access.

However, we have also identified an alternative motive for charging a markup on access: The consumer lock-in associated with high access will make it harder for entrants to get consumers sign up for their services — high access charges may deter or at least soften potential entrants to the market. In our entry model we find that a markup on access is *always* profitable: the value of reduced or eliminated entry always dominates the profit loss stemming from less efficient consumption. Clearly, as an explanation of the observed practice this is more satisfactory.

We conclude with a comment about the relationship between the model presented here and Laffont et al. (1998b). The fundamental difference between their model and ours is that we assume the existence of local calling clubs where friends may coordinate on the same network. We find this assumption especially compelling when it comes to exploring equilibria where firms discriminate against off-net calls. The existence of coordinated calling clubs tend to bias the calling pattern in favor of on-net calls, a feature not present in Laffont et al (1998b). Whether consumers are 'locked-in' to a network with switching costs or by transportation costs (as in Laffont et al.) this calling bias will increase the individual costs of switching supplier and therefore support higher fixed fees. The simple reason is that when considering to switch, a larger fraction of your calls will have to be off-net calls, or reversely, staying with your original network creates a high consumers' surplus because a large fraction of your calls are on-net calls at a low marginal price. As it turns out, this feature may be sufficient to tilt the equilibrium in favor of high access charges and discrimination based on call termination.

5   Appendix

Proof of Lemma 1.

The proof is in two parts. First consider the duopoly model. Let $R_D(x)$ denote the net gain in utility of a customer with switching costs $x\bar{s}$ when he switches from his present supplier i (who gives him zero net utility) to the undercutting opponent j with market share x:

$$R_D(x) = \frac{1}{2}v(q_j) + \frac{1}{2}\left(xv(p_j) + (1-x)v(q_j)\right) - k_j - x\bar{s} \tag{16}$$

Differentiating with respect to x yields

$$R'_D(x) = \frac{1}{2}\left(v(p_j) - v(q_j)\right) - \bar{s} < 0 \tag{17}$$

since $\bar{s} \geq \frac{1}{4}$ by assumption, $v(p) - v(q) \leq \frac{1}{2}$.

Next consider the entry model. Let $R_E(x)$ denote the net gain in utility of a customer with switching costs $x\bar{s}$ who switches from one of the incumbents to the entrant with market share x:

$$R_E(x) = \left[\frac{1}{2}v(q_e) + \frac{1}{2}\left(xv(p_e) + (1-x)v(q_e)\right) - k_e\right] - x\bar{s}$$
$$- \left[\frac{1}{2}v(p_i) + \frac{1}{2}\left(\frac{1-x}{2}v(p_i) + \frac{1+x}{2}v(q_i)\right) - k_i\right] \tag{18}$$

Differentiating with respect to x yields

$$R'_E(x) = \frac{1}{2}\left(v(p_e) - v(q_e) + \frac{1}{2}v(p_i) - \frac{1}{2}v(q_i)\right) - \bar{s} < 0 \tag{19}$$

which holds in equilibrium, since then $v(p_e) - v(q_e) = 0$, $\frac{1}{2}v(p_i) - \frac{1}{2}v(q_i) \leq \frac{1}{4}$, and $\bar{s} \geq \frac{1}{4}$.

$R'_D(x) < 0$ and $R'_E(x) < 0$ means that in both models the net gains from switching is decreasing with the amounts of switching, that is, the discouragement that lies in the increased switching cost is stronger than the encouragement that comes with the larger network. ∎

Proof of Lemma 2.

With $a < 1$ there are several ways to undercut. If poaching all, the undercutting firm will set $q = 0$. (This eases the switching constraints without giving rise to any access deficit, since there will eventually be no external calls.) Then the after

switching utility of any of the poached customer equals $v(0) - k - \bar{s}$. A sufficient condition to make all switch is to make the pair with the highest switching costs switch. That is, to have

$$v(0) - k - \bar{s} \geq 0 \text{ or } k \leq v(0) - \bar{s} = \frac{1}{2} - \bar{s} \equiv \pi^A(\bar{s}) \tag{20}$$

Stability then requires (necessary condition)

$$\frac{1}{2} - \bar{s} \leq \frac{1}{4} - \frac{1}{8}a^2 \frac{1}{2} \Leftrightarrow \bar{s} \geq \frac{1}{4} + \frac{1}{16}a^2 \tag{21}$$

In order to stop poaching all, the switching costs must be large enough. Reducing the markup on access relaxes this constraint and thereby requires less. The intuition is clear: high a reduces equilibrium profit without affecting undercutting profit. Therefore, higher switching costs are needed to deter this form of undercutting. ∎

Proof of Lemma 3.

When a firm tries to attract only a fraction of his competitor's customers, the market share after undercutting is $x \in \left(\frac{1}{2}, 1\right)$. Assume that a share $t = G(z) \in (0,1)$ of the consumers switch to the undercutting network, meaning that consumers with switching costs less than or equal to z switch. The share t will depend on the undercutter's p, q and k in a continuous way. Moreover, t is functionally related to x:

$$x = \frac{1+t}{2} = \frac{1+G(z)}{2} \Leftrightarrow G(z) = \frac{z}{\bar{s}} = 2x - 1 \Leftrightarrow z = (2x-1)\bar{s} \tag{22}$$

The undercutting firm solves the following program:

$$\max_{p,q,k,x} \left\{ x \left(\frac{1}{2}y(p)p + \frac{1}{2}(xy(p)p + (1-x)y(q)(q-a)) + k \right) + T \right\}$$

$$\text{s.t.} \quad \frac{1}{2}v(q) + \frac{1}{2}(xv(p) + (1-x)v(q)) - k \geq z \tag{23}$$

$$\frac{1}{2}v(p) + \frac{1}{2}(xv(p) + (1-x)v(q)) - k \geq z \tag{24}$$

$$z = (2x-1)\bar{s}$$

where (23) secures that the consumer with switching cost of exactly z will switch, and (24) that his friend will switch. Moreover, the access revenues can be written

$$T = x(1-x)\frac{1}{2}y(a)a$$

24

We will first show that optimal undercutting entails $p = q$. Suppose $p = 0$ and $q = a$ are part of a solution to this problem. Then it is easy to see that the only relevant constraint is constraint (23). Suppose next that we change p and k but keeps entry unchanged. The effect on profit is then

$$
\begin{aligned}
\frac{\partial \pi}{\partial p} &= x \left(\left(\frac{1}{2} + \frac{1}{2}x \right) (y(p) + py'(p)) + \frac{1}{2}xv'(p) \right) \\
&= x \left(\frac{1}{2} - \left(2\frac{1}{2} + x - x\frac{1}{2} \right) p \right) > 0 \text{ for } p = 0
\end{aligned}
$$

Hence the optimal undercutting p is positive. Its exact value (assuming that constraint (23) is still the only binding constraint) is

$$
\frac{1}{2} - \left(2\frac{1}{2} + x - x\frac{1}{2} \right) p = 0 \Leftrightarrow p = \frac{1}{x+2}
$$

Differentiating with respect to q yields

$$
\begin{aligned}
\frac{\partial \pi}{\partial q} &= x \left(\frac{1}{2}(1-x)y(q)(q-a) + \frac{1}{2}v(q) + \frac{1}{2}(1-x)v(q) \right) \\
&= x \left(\frac{1}{2}(1-x)(a-q) - \frac{1}{2}(1-q) \right) < 0 \text{ for } q \geq a \\
\frac{\partial^2 \pi}{\partial q^2} &= x \left(\frac{1}{2}(1-x)(-1) + \frac{1}{2} \right) = \frac{1}{2}x^2 > 0
\end{aligned}
$$

Hence it is clear that $\frac{\partial \pi}{\partial q} < 0$ for all q and therefore that optimal undercutting cannot involve $q > p$.

Next, suppose that optimal undercutting entails $q < p$. Then only constraint (24) is relevant. Following the steps of the above analysis then reveals that optimal undercutting involves $q > p$, a contradiction again. (The proof is omitted.) There is only one remaining possibility: optimal undercutting must involve $p = q$.

With $q = p$ the undercutting firm's problem simplifies a lot:

$$
\max_{p,q,k,x} \left\{ x \left(y(p)p - \frac{1}{2}(1-x)y(q)a + k + (1-x)\frac{1}{2}y(a)a \right) \right\}
$$

$$
\text{s.t.} \quad v(p) - k = z = \bar{s}(2x - 1)
$$

or, after using the constraint to eliminate k and substituting for $y(\ )$,

$$
\max_{p,x} \left\{ x \left((1-p)p - \frac{1}{2}(1-x)(a-p)a + \frac{1}{2}(1-p)^2 - \bar{s}(2x-1) \right) \right\}
$$

Letting $I(p,x) \equiv x\left((1-p)p - \frac{1}{2}(1-x)(a-p)a + \frac{1}{2}(1-p)^2 - \bar{s}(2x-1)\right)$ denote the expression to be maximized, differentiating w.r.t. p yields

$$\frac{\partial I}{\partial p} = x\left(-p + a - ax - a\frac{1}{2} + a\frac{1}{2}x\right) = 0 \Leftrightarrow p = \frac{1}{2}(1-x)a \in (0,a)$$

$$\frac{\partial^2 I}{\partial p^2} = -x < 0$$

Substituting the optimal price yields

$$J(x) \equiv I\left(\frac{1}{2}(1-x)a, x\right) = \frac{1}{2}x\left(1 - 2\bar{s}(2x-1) - a^2(1-x)\frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}x + x + 1\right)\right)$$

and differentiating w.r.t. x yields:

$$\frac{\partial J}{\partial x} = \frac{1}{2} - 4\bar{s}x + \bar{s} + \frac{1}{2}a^2\left(-\frac{1}{2}\right)\left(\frac{1}{2} - 4\frac{1}{2}x + 3\frac{1}{2}x^2 - 3x^2 + 1\right) = 0$$

$$\frac{\partial^2 J}{\partial x^2} = -4\bar{s} + a^2\frac{1}{2}\left(2\frac{1}{2} - 3\frac{1}{2}x + 3x\right) = \frac{5}{4}a^2 - 4s - \frac{1}{4}a^2x$$

Interior solutions require

$$\frac{\partial J}{\partial x} = \frac{1}{2} - 4\bar{s}x + \bar{s} - \frac{3}{8}a^2 + \frac{3}{8}a^2x^2 + \frac{1}{2}a^2x = 0$$

$$\Updownarrow$$

$$x = \frac{1}{3a^2}\left(-2a^2 + 16\bar{s} - \sqrt{13a^4 - 88\bar{s}a^2 + 256\bar{s}^2 - 12a^2}\right)$$

This can be a valid solution only if $x \in (\frac{1}{2}, 1)$:

$$\frac{1}{2} < \frac{1}{3a^2}\left(-2a^2 + 16\bar{s} - \sqrt{13a^4 - 88\bar{s}a^2 + 256\bar{s}^2 - 12a^2}\right) < 1$$

$$\Updownarrow$$

$$\frac{1}{6}a^2 + \frac{1}{6} < \bar{s} < \frac{1}{2} - \frac{1}{32}a^2$$

For subsequent reference, substituting the solution for x into the expression for the undercutting firm's profit yields the following profit expression:

$$\pi^S(\bar{s}, a) = -\frac{1}{108}\left(-2a^2 + 16\bar{s} - \sqrt{13a^4 - 88\bar{s}a^2 + 256\bar{s}^2 - 12a^2}\right) \times$$
$$\frac{-12a^2 + 11a^4 - 56\bar{s}a^2 + 128\bar{s}^2 - (8\bar{s} - a^2)\sqrt{13a^4 - 88\bar{s}a^2 + 256\bar{s}^2 - 12a^2}}{a^4}$$

■

Proof of Proposition 3.

This result can be proved by an example. If e.g. $\bar{s} = .4$, then it is easily verified that $a = a^* = .5$ yields a pure-strategy equilibrium with higher payoff than does the equilibrium resulting from $a = 0$. ■

Proof of Proposition 4.

Consider the effect on the entrant's profit in (14) of a marginal increase in p_e.

$$
\begin{aligned}
\frac{\partial \pi_e}{\partial p_e} &= x\left(\frac{\partial k_e}{\partial p_e} + \frac{1}{2}\left(1 - 2p_e\right) + \frac{1}{2}x\left(1 - 2p_e\right)\right) \\
&= x\left(-\frac{1}{2}x\left(1 - p_e\right) + \frac{1}{2}\left(1 - 2p_e\right) + \frac{1}{2}x\left(1 - 2p_e\right)\right) \\
&= x\left(\frac{1}{2}\left(1 - 2p_e\right) - \frac{1}{2}xp_e\right)
\end{aligned}
$$

>From this we have that $\left.\frac{\partial \pi_e}{\partial p_e}\right|_{p_e=0} = x\frac{1}{2} > 0$, hence optimal $p_e > 0$. Now, look at the effect on the entrant's profits from a marginal increase in q_e.

$$
\begin{aligned}
\frac{\partial \pi_e}{\partial q_e} &= x\left(\frac{\partial k_e}{\partial q_e} + \frac{1}{2}\left(1 - x\right)\left(1 - 2q_e + a\right)\right) \\
&= x\left(-\frac{1}{2}\left(1 - q_e\right) - \frac{1}{2}\left(1 - x\right)\left(1 - q_e\right) + \frac{1}{2}\left(1 - q_e\right)\left(1 - 2q_e + a\right)\right) \\
&= x\left(\frac{1}{2}\left(1 - x\right)\left(a - q_e\right) - \frac{1}{2}\left(1 - q_e\right)\right)
\end{aligned}
$$

We have that $\left.\frac{\partial \pi_e}{\partial q_e}\right|_{q_e=a} = x\left(-\frac{1}{2}\left(1 - a\right)\right) < 0$, hence optimal $q_e < a$. Moreover, $\frac{\partial \pi_e}{\partial q_e}$ is negative when

$$
x\left(\frac{1}{2}\left(1 - x\right)\left(a - q_e\right) - \frac{1}{2}\left(1 - q_e\right)\right) < 0 \iff a < \frac{1 - q_e x}{1 - x}
$$

which holds as $q_e < 1$. Hence, $\frac{\partial \pi_e}{\partial q_e} < 0$ for all $a \in [0, 1]$. Consequently, it pays to reduce q_e, and then no equilibrium may involve $p_e < q_e$. Hence we have established that $p_e = q_e$ under the conditions in the proposition.

Next, consider the effect on an incumbent's profit in (15) of a marginal increase

in the on-net price, i.e.

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= \frac{1-x}{2}\left(\frac{\partial k_i}{\partial p_i} + \frac{1}{2}\left(1-2p_i\right) + \frac{1}{2}\frac{1-x}{2}\left(1-2p_i\right)\right) \\
&= \frac{1-x}{2}\left(-\frac{1}{2}\left(1-p_i\right) - \frac{1}{2}\frac{1-x}{2}\left(1-p_i\right) + \frac{1}{2}\left(1-2p_i\right) + \frac{1}{2}\frac{1-x}{2}\left(1-2p_i\right)\right) \\
&= \frac{1-x}{2}\left(\frac{1}{2}\left(1-2p_i-1+p_i\right) + \frac{1}{2}\frac{1-x}{2}\left(1-2p_i-1+p_i\right)\right) \\
&= -\frac{1}{2}p_i\left(\frac{1}{2}+1-x\frac{1}{2}\right) = 0
\end{aligned}
$$

which establishes $p_i = 0$ as the unique solution (it is easily verified that $\partial^2\pi_i/\partial p_i^2 < 0$). Similarly,

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial q_i} &= \frac{1-x}{2}\left(\frac{\partial k_i}{\partial q_i} + \frac{1}{2}\frac{1+x}{2}\left(1-2q_i+a\right)\right) \\
&= \frac{1-x}{2}\left(-\frac{1}{2}\frac{1+x}{2}\left(1-q_i\right) + \frac{1}{2}\frac{1+x}{2}\left(1-2q_i+a\right)\right) \\
&= \frac{1-x}{2}\frac{1}{2}\frac{1+x}{2}\left(a-q_i\right) = 0
\end{aligned}
$$

establishing $q_i = a$ as the unique solution (it is easily verified that $\partial^2\pi_i/\partial q_i^2 < 0$).

When $p_i = 0$, $q_i = a$ and $q_e = p_e$ the maximization problem of the entrant reduces to

$$
\max_{x,p_e=q_e}\left\{x\left(k_e + \left(1-p_e\right)p_e - \frac{1}{2}\left(1-x\right)\left(1-p_e\right)a\right) + a(1-a)\frac{1}{2}x(1-x)\right\}
$$

$$
\text{s.t.}\quad k_e = \frac{1}{2}\left(\frac{1}{2}\left(1-p_e\right)^2 - \frac{1}{2}\right) + k_i - x\bar{s} + \frac{1}{2}\left(\left(1-p_e\right)^2 - \frac{1-x}{4} - \frac{1+x}{4}\left(1-a\right)^2\right)
$$

Maximizing with respect to p_e yields the first-order condition (it is easily verified that $\partial^2\pi_e/\partial p_e^2 < 0$)

$$
\begin{aligned}
\frac{\partial \pi_e}{\partial p_e} &= x\left(\frac{\partial k_e}{\partial p_e} + \left(1-2p_e\right) + \frac{1}{2}\left(1-x\right)a\right) \\
&= x\left(-\left(1-p_e\right) + \left(1-2p_e\right) + \frac{1}{2}\left(1-x\right)a\right) \\
&= x\left(-p_e + \frac{1}{2}\left(1-x\right)a\right) = 0 \Leftrightarrow p_e = \frac{1}{2}\left(1-x\right)a
\end{aligned}
$$

The final step in our derivation of an equilibrium consists in finding the equilibrium fixed fees for the firms. With the marginal prices from above, the problem of

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the entrant reduces to

$$\max_x \left\{ x \left(k_e + (1 - p_e)\, p_e - \frac{1}{2}(1 - x)a(a - p_e) \right) \right\}$$

$$s.t. \quad k_e = \tfrac{1}{2}\left(1 - p_e\right)^2 - \tfrac{1}{2} + k_i - x\bar{s}$$

$$p_e = \tfrac{1}{2}\left(1 - x\right)a$$

or, when inserting for the fixed fee,

$$\max_x \left\{ x \left(\frac{1}{2}\left(1 - p_e\right)^2 - \frac{1}{2} + k_i - x\bar{s} + (1 - p_e)\, p_e \right) - x\frac{1}{2}(1 - x)a(a - p_e) \right\} \tag{25}$$

$$s.t. \ p_e = \tfrac{1}{2}\left(1 - x\right)a$$

Under the same assumptions, the incumbents' maximization problems reduce to

$$\max_x \left\{ \frac{1 - x}{2} \left(k_i + a\frac{1}{2} \left(\frac{1 - x}{2}(1 - a) + x(1 - p_e) \right) \right) \right\}$$

$$s.t. \quad k_i = k_e + \tfrac{1}{2} - \tfrac{1}{2}\left(1 - p_e\right)^2 + x\bar{s} \tag{26}$$

$$p_e = \tfrac{1}{2}\left(1 - x\right)a \tag{27}$$

with first order conditions

$$-\frac{3}{16}x^2 a^2 - \frac{1}{8}xa^2 - \frac{1}{2}a + \frac{1}{4}xa + \frac{5}{16}a^2 - x\bar{s} - \frac{1}{2}k_e + \frac{1}{2}\bar{s} = 0 \tag{28}$$

Similarly, (25) yields first order conditions

$$-\frac{3}{8}a^2 + \frac{1}{2}xa^2 + \frac{3}{8}x^2 a^2 + k_i - 2x\bar{s} = 0 \tag{29}$$

Solving (26), (28) and (29) for $x, k_i,$ and k_e (using (27)) we get

$$x = x(a, \bar{s}) = \frac{1}{a^2}\left(2a^2 - 12\bar{s} + \gamma(a, \bar{s})\right)$$

$$k_i = 11x\bar{s} - 2xa^2 - 3\bar{s} + \frac{3}{2}a$$

$$k_e = \frac{1}{2}a + \frac{1}{2}ax + \frac{1}{4}a^2 - \frac{7}{4}xa^2 + 7x\bar{s} - 2\bar{s}$$

$$\text{where } \gamma(a, \bar{s}) = \sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3}$$

∎

Proof of Proposition 5.

>From Proposition 4 we have that the equilibrium outcome of the entry model can be described as follows:

$$
\begin{aligned}
x(a, \bar{s}) &= \frac{1}{a^2}\left(2a^2 - 12\bar{s} + \gamma(a, \bar{s})\right) \geq 0 \\
\pi_i(a, \bar{s}) &= \frac{1}{8}\left(a^2 - 12\bar{s} + \gamma(a, \bar{s})\right) \times \\
&\qquad \frac{-5a^3 + 8a^4 + 240\bar{s}^2 - 84\bar{s}a^2 + \left(-20\bar{s} - a + 4a^2\right)\gamma(a, \bar{s}) + 12\bar{s}a}{a^4} \\
\pi_e(a, \bar{s}) &= -\frac{1}{4}\frac{-12\bar{s} + 2a^2 + \gamma(a, \bar{s})}{a^4} \times \\
&\qquad \left(-4a^3 + \left(5a^2 - 28\bar{s}\right)\gamma(a, \bar{s}) + 11a^4 - 108\bar{s}a^2 + 336\bar{s}^2\right) \\
\gamma(a, \bar{s}) &= \sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3} \geq 0
\end{aligned}
$$

First consider the entrant's market share. The market share must be non-negative, hence we must have that $x(a, \bar{s}) = \frac{1}{a^2}\left(2a^2 - 12\bar{s} + \sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3}\right) \geq 0$. This holds when

$$
2a^2 - 12\bar{s} + \sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3} \geq 0
$$

which can, after some steps of tedious calculus, be rewritten as

$$
a \leq 2 - 2\sqrt{(1 - 2\bar{s})} \text{ or } \bar{s} \geq \frac{1}{8}a\left(4 - a\right)
$$

Furthermore, $x(a, \bar{s})$ is decreasing in a when

$$
\frac{\partial x(a, \bar{s})}{\partial a} = 2\frac{12\bar{s}\gamma(a, \bar{s}) - 144\bar{s}^2 + 20\bar{s}a^2 + a^3}{a^3\gamma(a, \bar{s})} < 0
$$

This holds whenever

$$
\begin{aligned}
144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3 &< \left(\frac{144\bar{s}^2 - 20\bar{s}a^2 - a^3}{12\bar{s}}\right)^2 \\
&\Updownarrow \\
0 &< 8\bar{s}\left(4\bar{s}\left(9 - 10a\right) + 5a^2\right) + a^3 \qquad (30)
\end{aligned}
$$

We see that a sufficient condition for this to hold is that $a \leq \frac{9}{10}$. For $a \in (\frac{9}{10}, 1]$ the right hand side inequality (30) above is strictly decreasing in $\bar{s}$. Hence, if it holds for $\bar{s} = 0$ it will hold for every $\bar{s} \in (0, \frac{1}{2}]$. When $\bar{s} = 0$ the condition obviously holds, hence the condition holds for every $a \in [0, 1]$ and $\bar{s} \in [0, \frac{1}{2}]$.

Next look at the effect of a marginal increase in consumer switching costs on the entrant's market share:

$$
\begin{aligned}
\frac{\partial x(a, \bar{s})}{\partial \bar{s}} &= -4 \frac{3\gamma(a, \bar{s}) - 36\bar{s} + 5a^2}{a^2 \gamma(a, \bar{s})} > 0 \\
&\Updownarrow \\
\sqrt{144\bar{s}^2 - 40\bar{s}a^2 + 5a^4 - 4a^3} &< \frac{36\bar{s} - 5a^2}{3} \\
&\Updownarrow \\
\frac{20}{9}a &< 4 \Longleftrightarrow a < \frac{9}{5}
\end{aligned}
$$

which is always true, hence increased switching costs increases the market share of the entrant. ∎

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