Modelling dependence structure in size-sorted portfolios: A structural multivariate GARCH model

GEORGE MILUNOVICH^{*}

A new model is developed that augments a structural VAR specification with a GARCH covariance matrix. The model is utilised to study time series dependencies between three size-sorted portfolios from the Australian Stock Exchange. Even after accounting for contemporaneous correlations the returns on small and medium firm portfolios are found to lag the large firm portfolio returns. An asymmetric lag structure is also found in the structural variance equations. The evidence is consistent with the Lo and MacKinlay (1990) lead-lag effect and the volatility spill-over hypothesis of Condrad, Gultekin and Kaul (1991).

JEL Classifications: Keywords: C30, C32, G10 Multivariate GARCH models; Size-sorted Portfolios; Simultaneous Equations; Conditional Impulse Response Functions

I would like to thank Minxian Yang, Lance Fisher and one anonymous reader for their helpful comments and suggestions. The first drafts of this paper were completed independently of similar work by Rigobon and Sack (2003).

^{*} School of Economics, University of New South Wales, Sydney, Australia; Telephone: ++ (02) 9385 1346; Email: <u>g.milunovich@unsw.edu.au</u>

1. Introduction

Time variation in the first and second conditional moments has been reported for many financial time series². Concentrating on the second conditional moment in the multivariate framework, Bollerslev, Engle and Wooldridge (1988) have pioneered a multivariate GARCH model. The multivariate GARCH literature has grown and evolved ever since, with authors such as Bollerslev (1990), Engle and Kroner (1995) and Engle (2002) making the greatest impact and setting the course for further research. The main issue that one faces with a multivariate GARCH model is the number of parameters that need to be estimated. The systems are usually large with some of initial formulations containing over 70 parameters in just three variables. Researchers have thus focused on creating parameter restrictions so as to make the systems smaller while maintaining positive definite covariance matrices.

In this paper, I propose a new model of the multivariate GARCH family. The specification derives variance parameter restrictions from the structure imposed on the mean equations. Because the model is derived from a structural VAR specification I refer to it as a structural GARCH. A structural GARCH differs from existing multivariate GARCH models in that it estimates structural variance rather than reduced form parameters. The difference between the structural and the reduced form conditional covariance matrices is analogous to the difference between the structural and the reduced form VAR systems. The usual procedure to estimating a multivariate GARCH model is to fit either a univariate ARMA process to each series in question, as in Conrad, Gultekin and Kaul (1991), or to model the mean equations jointly via a reduced form VAR such as

² See Pagan (1996) for a comprehensive review of the financial econometrics literature.

in Cha and Oh (2000). The covariance matrix is then estimated from the reduced form residual series. Therefore, it can be argued that the above mentioned techniques would produce reduced form parameters in both the mean and variance equations. As the structural GARCH estimates structural mean equations it also produces structural variance parameters. This point is further discussed in Section 3. A traditional analysis toolbox, including the impulse response and forecast error variance decomposition analyses, is easily adapted to accommodate the structural GARCH framework. Further, due to the nonlinearities present in the model, conditional impulse response function (CIRF) and conditional forecast error variance decomposition (CFEVD) are defined. The two are dependent on conditional volatility levels. Conditional impulse response functions are of particular interest as they quantify effects of interactions between different volatility regimes (eg. low vs. high volatility) and innovation shocks.

As an empirical application of the model I examine the dependence structure in weekly returns on three size-sorted portfolios from the Australian stock market. The focus of this study is twofold. Firstly, I test for the lead-lag effect in portfolio returns as initially documented by Lo and McKinlay (1990). This hypothesis posits that small-firm portfolio returns lag large-firm portfolio returns but not the other way around. The approach taken here is innovative in that it specifies and tests structural parameters rather than those that are reduced form. Secondly, tests of the asymmetric volatility spill-over hypothesis of Conrad, Gultakin and Kaul (1991) are conducted. These authors propose and test the hypothesis that volatility spills over from large to medium and medium to small firms and not in any other direction. Although the hypothesis has been studied and tested in various forms and on different data sets the present application is different in

two ways. Utilizing the structural GARCH model I conduct the volatility spill-over tests on structural rather than the commonly tested reduced form variance parameters. Employing the Australian stock market data is interesting in its own right as the effect has been documented previously for several international stock markets.

I find evidence in support of both the Lo and MacKinlay lead-lag effect and Conrad, Gultakin and Kaul's volatility spill-over hypothesis. In particular, I find statistically significant coefficients on lagged large cap index returns in small and medium firm portfolio equations. Similarly, volatility spills over from medium to small firm portfolio returns and not in any other direction.

The rest of the paper is organised as follows. Section 2 presents a literature review. Section 3 develops the structural GARCH model while Section 4 presents empirical results from a study on three size-sorted portfolios. The conclusion is in the final section.

2. Literature Review

2.1 Modelling Time Series Dynamics in Short-Term² Portfolio Returns

The study of security returns has its roots in the seminal work of Louis Bachelier (1900) and his formulation of the random walk hypothesis. Bachelier's hypothesis, nowadays better know as the random walk hypothesis, essentially implies that stock market returns are unpredictable. The theory has been studied and tested ever since.

One of the first studies to question the random walk hypothesis in the context of sizesorted portfolios is Lo and MacKinlay (1988). Using weekly data on five equally weighted size-sorted portfolios from the NYSE and AMEX they strongly reject the

² Daily, weekly and monthly returns are regarded as short-term returns as in Fama (1991), p. 1578.

random walk hypothesis. The authors show that the portfolio returns exhibit strong positive correlations, even though individual returns are on average weakly and negatively autocorrelated. Lo and MacKinlay attribute their findings to crossautocorrelations between individual security returns. In a later study, Lo and MacKinlay (1990) document substantial differences in the behaviour of small and large cap portfolios. Firstly, they demonstrate that small cap portfolio returns are more predictable than the large firm portfolio returns. Secondly, they show that lagged returns on the portfolio of large market capitalisation firms explain a significant portion of the current returns on small stocks, but not the other way around. Thus, they document an asymmetry in the predictability of returns on small and large market cap portfolios. Other authors quickly followed in the steps of Lo and MacKinlay, examining the lead-lag relationships in different stock markets and for different time periods. Fargher and Weigard (1998) investigate the impact of technological and regulatory changes on the lead-lag effect and find that the lead-lag effect has diminished in the more recent past. They explain their findings using the argument of improved market efficiency and better dissemination of information. McQueen, Pinegar and Thorley (1996) study directional asymmetry in response to good and bad news. Small firms appear to respond with a lag to good but not bad news. That is, adverse information appears to be impounded in the price of small firms instantaneously. Evidence is also found in support of the lead-lag effect in Asian markets. Chang, McQueen and Pinegar (1999) find cross-autocorrelations in six Asian markets including Hong Kong, Japan, Singapore, South Korea, Taiwan and Thailand. However they confirm McQueen, Pinegar and Thorly's asymmetric reaction to good news only for Taiwan. Chang, McQueen and Pinegar do not find sufficient evidence to infer that the degree of cross autocorrelation has weakened since 1987.

Investigating dependencies in the second conditional moment Conrad, Gultekin and Kaul (1991) find evidence of conditional autoregressive heteroscedasticity in size-sorted portfolio returns. They relate their empirical evidence to a theoretical model of Ross' (1989), in which he associates price volatility with the rate of information flow. They also document asymmetric volatility spill-overs, where the volatility shocks of the large cap index affects the conditional volatility of the small cap index but not the other way around. In a later study Kroner and Ng (1998) confirm Conrad, Gultekin and Kaul's findings utilizing several different versions of the multivariate GARCH model.

2.2. Theoretical Propositions

Although researchers seem to agree on the stylised facts relating to size-sorted portfolios, there are substantial differences in the beliefs as to what causes them. Several theories have sprung up in attempts to provide an adequate explanation. Boudoukh, Richardson and Whitelow (1994) categorise the competing theories into three camps which they refer to as heretics, loyalists and revisionists.

Heretics believe that the time series patterns occur because the prices of small firms either over-react or only partially adjust to common market information, while the large firms adjust to new information instantaneously. Thus the cross-autocorrelations are due to differences in the speed of adjustment to shocks in common factors. Amongst heretics Lo and MacKinlay (1990) and Jegadeesh and Titman (1995) are the two most prominent studies. More recent evidence in support of this theory comes from a Richardson and Peterson (1999) article in which they show that the large cap index Granger causes small firm portfolio returns in the US markets.

The second group of theorists, the loyalists, believe that the market processes information rationally and that short horizon correlations are not due to fundamentals but market frictions and microstructure effects. Market imperfections such as non-synchronous trading, bid-ask spread and various trading mechanisms such as market architectures, systematic changes in inventory holdings and information flows are commonly cited. Lo and MacKinlay (1990) examine nonsynchronous trading arguments and conclude that even after assuming excessively high levels of non-trading probabilities it is unlikely that non-synchronous trading causes the lead-lag effect. Mech (1993) is concerned with transaction costs as a cause of the lead-lag effect, a theory supported by his cross-sectional data tests. Boudoukh, Richardson and Whitelow (1994) themselves assume the loyalist position and develop a model that allows for heterogenous non-trading. However they conclude that non-synchronous trading cannot account for all of the lead-lag effect.

Revisionists theorise that markets are efficient and that even in a completely frictionless market short-horizon returns can be autocorrelated. Changing risk premiums can be explained by intertemporal asset pricing models such as conditional versions of the APT and consumption based CAPM. In this literature Conrad and Kaul (1989) and Conrad, Gultekin and Kaul (1991) claim that predictable variations at short-horizons are attributable to variations in expected returns. Connoly and Conrad (1991) use cointegration and simulation tests to compare a time varying factor model to a lagged price adjustment model and find evidence in favour of the time varying factor model. Hameed (1997) uses principal component analysis and the Kalman filter methodology to extract factors and shows that the time varying factor model provides a better fit than a price adjustment model.

Given that each of the three theories is supported by some but not all empirical evidence, it is unclear which hypothesis explains the phenomena best. One could also suspect that not all theories are mutually exclusive.

2.3. Multivariate GARCH Models

Multivariate GARCH models were introduced by Bollerslev, Engle and Wooldridge (1988) in an application of the CAPM with time varying covariances. Since then several competing models have been developed. Some of the most frequently employed ones are: the VECH model of Bollerslev, Engle and Wooldridge (1988), the Constant Correlation model (CCC) of Bollerslev (1990), the BEKK model of Engle and Kroner (1995) and more recently the Dynamic Conditional Correlation model of Engle (2002). The VECH specification can be written as:

$$y_{t} = x_{t}^{'}\beta + u_{t}$$

$$u_{t} = h_{t}\varepsilon_{t}$$

$$\varepsilon_{t} \sim nid(0, I_{n}) \qquad (1)$$

$$H_{t} = h_{t}h_{t}^{'}$$

$$vech(H_{t}) = \varpi + \sum_{i=1}^{q} B_{i}vech(H_{t-i}) + \sum_{i=1}^{p} A_{i}vech(u_{t-i}u_{t-i}^{'})$$

where y_t is a vector of returns, x_t represents a vector of explanatory variables and H_t is the conditional variance matrix. The VECH model has $n(n+1)/2 + (p + q)n^2/4$ parameters, thus a model with three variables (n = 3) and p=q=1 has 78 GARCH parameters. Due to the large number of parameters in (1) Bollerslev (1990) proposed a Constant Conditional

Correlation model. CCC restricts the correlations between variables to be time invariant, thus modelling the conditional covariance structure as:

$$H_{ijt} = \rho_{ij}\sqrt{H_{iit}}\sqrt{H_{jjt}}$$
(2a)

where H_{ijt} is the ij^{th} element of the conditional H_t covariance matrix and $\forall i \neq j$. Conditional variances are defined as:

$$H_{iit} = \sigma_i + B_i H_{iit-1} + A_i \left(u_{it-1} u_{it-1} \right).$$
(2b)

This model has 3n+(n(n+1)/2) parameters, thus 15 variance parameters if n = 3. The CCC model is positive definite if the correlation matrix is positive definite. Another popular formulation is the BEKK model, which represents a solution to the positive definiteness problem. It is defined as:

$$H_{t} = \omega + BH_{t-1}B' + A\varepsilon_{t-1}\varepsilon_{t-1}A' .$$
(3)

As the second and the last term of the above equation are expressed in the quadratic form, given a positive definite ω , the conditional covariance matrix is guaranteed to be positive definite. This model has $5/2n^2 + n/2$ parameters and there are 24 coefficients in a three variable system. As the most recent addition to the growing family of Multivariate GARCH models, Dynamic Conditional Correlation specification is a generalisation of Bollerslev's (1990) CCC model where the constant correlation coefficients are made time varying:

$$H_t = G_t R G_t \tag{5a}$$

is replaced by:

$$H_t = G_t R_t G_t \tag{5b}$$

where H_t is the conditional covariance matrix, $G_t = diag \left\{ \sqrt{h_{i,t}} \right\}$ and R is the correlation matrix. The major advantage of DCC is that it directly parameterises conditional correlation, which is of primary interest to many financial applications.

3. Econometric Specification: Structural GARCH Model

3.1. Model Derivation

Similar to the structural VAR specification, the structural GARCH is derived with an assumption that innovations are uncorrelated. However, the innovations are allowed to follow a GARCH type conditional volatility process. Thus, they are not necessarily independent and can be characterised by variances that may exhibit cross-equation dependences. The model can be derived from a p^{th} order structural vector autoregression in the following way. If y_t is an $(n \times 1)$ matrix of dependent variables:

$$B_{0}y_{t} = C + B_{1}y_{t-1} + B_{2}y_{t-2} + \dots + B_{p}y_{t-p} + u_{t}$$

$$u_{t} = g_{t}\varepsilon_{t}$$

$$\varepsilon_{t} \sim iidn(0, I_{n})$$

$$E_{t-1}(u_{t}) = 0$$

$$Var_{t-1}(u_{t}) = G_{t}$$

$$G_{t} = g_{t}g_{t}^{'}$$
(6)
$$G_{t} = \begin{bmatrix} g_{1t}^{2} & & \\ & \ddots & 0 \\ & & & & g_{kt}^{2} \end{bmatrix}$$

where u_t 's are structural shocks whose variances are modeled in a standard GARCH framework:

$$diag(G_t) = \omega + \alpha_1 (u_{t-1} \circ u_{t-1}) + \dots + \alpha_p (u_{t-p} \circ u_{t-p}) + \beta_1 diag(G_{t-1}) + \dots + \beta_q diag(G_{t-q})$$
(7)

where diag() operator stacks main diagonal elements into a column vector, and ' \circ ' is the element by element multiplication operator. A reduced form model can be obtained as:

$$Y_{t} = B_{0}^{-1}C + B_{0}^{-1}B_{1}Y_{t-1} + B_{0}^{-1}B_{2}Y_{t-2} + \dots + B_{0}^{-1}B_{p}Y_{t-p} + B_{0}^{-1}g_{t}\varepsilon_{t}$$
(8a)

or:

$$Y_{t} = \pi + \phi_{1}Y_{t-1} + \phi_{2}Y_{t-2} + \dots + \phi_{p}Y_{t-p} + \eta_{t}$$

$$\eta_{t} \sim (0, H_{t}) \qquad .$$

$$H_{t} = B_{0}^{-1}G_{t}B_{0}^{-1'}$$
(8b)

From the above specification we can see that the reduced form errors η_t are contemporaneously correlated. Furthermore, the covariance matrix H_t is a linear combination of the structural covariance matrix G_t . This can be illustrated in a trivariate system as:

$$H_{t} = \begin{bmatrix} h_{1t}^{2} & h_{12t} & h_{13t} \\ h_{12t} & h_{2t}^{2} & h_{23t} \\ h_{13t} & h_{23t} & h_{3t}^{2} \end{bmatrix} = \begin{bmatrix} g_{1t}^{2} & g_{21}^{0}g_{1t}^{2} & \left(B_{31}^{0}+B_{21}^{0}B_{32}^{0}\right)g_{1t}^{2} \\ B_{21}^{0}g_{1t}^{2} & g_{21}^{0}g_{1t}^{2}+g_{2t}^{2} & B_{32}^{0}g_{2t}^{2} \\ \left(B_{31}^{0}+B_{21}^{0}B_{32}^{0}\right)g_{1t}^{2} & B_{32}^{0}g_{2t}^{2} & \left(B_{31}^{0}+B_{21}^{0}B_{32}^{0}\right)g_{1t}^{2}+B_{32}^{0}g_{2t}^{2}+g_{3t}^{2} \end{bmatrix}.$$
(9)

In the above equation, the variance matrix H_t is firstly written in its reduced form and secondly in its structural formulation. Should one estimate the variance matrix H_t in terms of the reduced form equations h_{ijt} , one will no longer be able to test structural parameters of G_t . This relationship is thus similar to that of the reduced and structural VAR systems.

Given that the structural variance matrix G_t is correctly specified, the reduced form variance matrix is guaranteed to be positive definite due to the quadratic form of the inverse of B_0 matrix. The number of parameters in the variance covariance matrix is $\frac{n^2 - n}{2} + (n \times k)$, where k is the number of parameters in each variance equation, assuming each variance equation has the same number of parameters. The unconditional structural covariance matrix can be calculated from (7) as:

$$E(G_t) = \left(I - \sum_{i=1}^p \alpha_i - \sum_{i=1}^q \beta_i\right)^{-1} \omega .$$
(10)

3.2. Identification and Estimation Issues

Estimation of (6) follows the standard theory of simultaneous equations. Due to the correlation structure between regressors and errors one cannot estimate (6) directly. A commonly used solution is the recursive approach, which involves imposing triangularity restrictions on the B_0 matrix. In the multivariate GARCH context, the restrictions that (preferably) come from economic theory translate exogeneity in the mean equations into exogeneity in the structural variances. Assuming *iid* normally distributed structural innovations ε_t , a log-likelihood function can be specified as:

$$\ell(\theta) = -\frac{T}{N}\ln(2\pi) - \frac{T}{N}\ln\left(\sum_{p}^{T}\det(B_{0}^{-1}G_{t}B_{0}^{-1'})\right) - \frac{1}{2}\sum_{p}^{T}\left(B_{0}^{-1}g_{t}\varepsilon_{t}\right)'\left(B_{0}^{-1}G_{t}B_{0}^{-1'}\right)^{-1}\left(B_{0}^{-1}g_{t}\varepsilon_{t}\right) \quad (11a)$$

which simplifies to:

$$\ell(\theta) = -\frac{T}{N}\ln(2\pi) - \frac{T}{N}\ln\left(\sum_{p}^{T}\det(B_{0}^{-1}G_{t}B_{0}^{-1'})\right) - \frac{1}{2}\sum_{p}^{T}\varepsilon_{t}^{'}\varepsilon_{t}.$$
(11b)

The likelihood function in (11) can be maximised numerically using the Berndt, Hall, Hall, and Hausman (1974) algorithm. Depending on the parameters of interest from (6), one could employ either a one-stage or a two-stage estimation technique. Should one be primarily interested in the parameters from the variance equation, the model can be consistently estimated in two stages³. In the first stage, specify a reduced form VAR equation (8b) and estimate it via OLS. Subsequently estimate the structural variance

matrix parameters via maximum likelihood from the residuals. Alternatively, if one is interested in testing hypotheses relating to the structural form mean equation parameters, then one could specify the log-likelihood in terms of parameters from (8a). Given that the log-likelihood function is correctly specified, the information matrix can be calculated as:

$$\Im(\theta) = E\left\{ \left(\frac{\partial l_t(\theta)}{\partial \theta} \right) \left(\frac{\partial l_t(\theta)}{\partial \theta} \right)^{\prime} \right\}.$$
(12)

3.3. Conditional Impulse Response Functions

Due to the nonlinear structure of the model, conditional impulse response functions can be defined for both the first and second moment equations. The mean equation impulse response function is calculated with respect to a standard deviation shock to an orthogonal innovation. Because conditional heteroscedasticity is explicitly modelled, the response of the dependent variable will be affected, amongst other things, by volatility levels. Thus, one may be interested to see how different volatility regimes (eg. low vs. high) influence the response functions. The standard, unconditional, impulse response function can also be calculated by evaluating the response function at an unconditional volatility level. In order to see how a structural shock *j* at time period *t* affects variable *i*, *s* periods ahead, at time t+s one can re-write equation (8b) in its MAR form as:

$$y_{t} = (I - \phi_{1} - \dots - \phi_{p})^{-1} \pi + \eta_{t} + \psi_{1} \eta_{t-1} + \dots + \psi_{\infty} \eta_{t-\infty}$$

$$y_{t} = (I - \phi_{1} - \dots - \phi_{p})^{-1} \pi + B_{0}^{-1} g_{t} \varepsilon_{t} + \psi_{1} B_{0}^{-1} g_{t-1} \varepsilon_{t-1} + \dots + \psi_{\infty} B_{0}^{-1} g_{t-\infty} \varepsilon_{t-\infty}$$
(13)

where π is the VAR constant term, ϕ 's are the VAR parameters, ψ 's are MAR parameters and $\eta_t = B_0^{-1} g_t \varepsilon_t$. Assuming a GARCH (1,1) specification in the variance equations, it

³ See Pagan (1986).

can be seen that the total effect on variable *i*, *s* periods after the shock to the orthogonal innovation *j* is given by the following partial derivative:

$$\frac{\partial Y_{it+k}}{\partial \varepsilon_{jt}} = \frac{\partial Y_{it+k}}{\partial \eta_{jt}} \frac{\partial \eta_{jt}}{\partial \varepsilon_{jt}} + \frac{\partial Y_{it+k}}{\partial \eta_{jt+1}} \frac{\partial \eta_{jt+1}}{\partial \varepsilon_{jt+1}} \frac{\partial g_{jt+1}}{\partial \varepsilon_{jt}} .$$
(14)

The impulse response function is defined as the conditional expectation of the above quantity:

$$E_{t}\left[\frac{\partial Y_{it+k}}{\partial \varepsilon_{jt}}\right] = E_{t}\left[\frac{\partial Y_{it+k}}{\partial \eta_{jt}}\frac{\partial \eta_{jt}}{\partial \varepsilon_{jt}} + \frac{\partial Y_{it+k}}{\partial \eta_{jt+1}}\frac{\partial \eta_{jt+1}}{\partial g_{jt+1}}\frac{\partial g_{jt+1}}{\partial \varepsilon_{jt}}\right].$$
(15)

The above impulse response function can be estimated by simulating (8b). Because the first quantity in (15) includes the conditional expectation of g_t , which is known at time period t, the impulse response function depends on conditional volatility as well as the time lag k. The initial response to one standard deviation shock to ε_{1t} is given by:

$$\frac{\partial Y_{1t}}{\partial \varepsilon_{1t}} = g_{1t}$$

$$\frac{\partial Y_{2t}}{\partial \varepsilon_{1t}} = \beta_{21}g_{1t}$$

$$\frac{\partial Y_{3t}}{\partial \varepsilon_{1t}} = \beta_{31}g_{1t}$$
(16)

where one would use the estimated value \hat{g}_{1t} to make the above operational.

Another interesting issue is how a shock to a structural factor j at time t affects conditional variance i at some future time period t+k. The impulse response function for the variance equation of variable i is defined as:

$$E_{t}\left[\frac{\partial g_{it+k}}{\partial \varepsilon_{jt}^{2}}\right].$$
(17)

The expectation is conditional on all information up to the time period *t*, thus the quantity can be estimated though simulation of (7) by setting the initial value of g_{t-1} to its estimated value and shocking one of the squared error terms, while setting the rest to zero.

3.4. Conditional Forecast Error Variance Decomposition

From the MAR representation in (13) it can be seen that a k step forecast error variance can be written as:

$$Var(Y_{t+k} - \hat{Y}_{t+k}) = B_0^{-1}GB_0^{-1'} + \psi_1 B_0^{-1}GB_0^{-1'}\psi_1^{'} + \dots + \psi_k B_0^{-1}GB_0^{-1'}\psi_k^{'}.$$
 (18)

One could make this quantity operational by replacing required quantities G_{t+k} 's by their forecasts made at time t, $\hat{G}_{t+k,t}$. A conditional version of the variance decomposition can be defined as:

$$Var_{t}\left(Y_{t+k}-\hat{Y}_{t+k}\right) = B_{0}^{-1}\hat{G}_{t+1,t}B_{0}^{-1'} + \psi_{1}B_{0}^{-1}\hat{G}_{t+2,t}B_{0}^{-1'}\psi_{1}^{'} + \dots + \psi_{k}B_{0}^{-1}\hat{G}_{t+k,t}B_{0}^{-1'}\psi_{k}^{'}.$$
 (19)

Further, it is possible to calculate two versions of the conditional variance decomposition. Firstly, one could calculate the contribution of the i^{th} orthogonal innovation to the forecast error variance as:

$$Var_{t}\left(Y_{t+k}-\hat{Y}_{t+k}\right) = \hat{B}_{0}^{-1}\hat{g}_{it+1,t}\hat{g}_{it+1,t}\hat{B}_{0}^{-1'} + \dots + \hat{\psi}_{k}\hat{B}_{0}^{-1}\hat{g}_{it+k,t}\hat{g}_{it+k,t}\hat{B}_{0}^{-1'}\hat{\psi}_{k}^{'}$$
(20)

where $\hat{g}_{it+k,t}$ refers to the column *i* of standard deviation matrix g_t . Quantities calculated in such fashion can be thought of as proportions of the forecast error variance due to each of the orthogonal innovations.

Secondly one could calculate percentage contributions of each structural innovation shock to the forecast variance. Contribution of innovation i on variable j's forecast variance is given by:

$$\frac{(\hat{B}_{0}^{-1}\hat{g}_{it+1}\hat{g}_{it+1}^{'}\hat{B}_{0}^{-1'} + \dots + \hat{\psi}_{k}\hat{B}_{0}^{-1}\hat{g}_{it+k}\hat{g}_{it+k}^{'}\hat{B}_{0}^{-1'}\hat{\psi}_{k}^{'})[j][j]}{trace(\hat{B}_{0}^{-1}\hat{g}_{it+1}\hat{g}_{it+1}^{'}\hat{B}_{0}^{-1'} + \dots + \hat{\psi}_{k}\hat{B}_{0}^{-1}\hat{g}_{it+k}\hat{g}_{it+k}^{'}\hat{B}_{0}^{-1'}\hat{\psi}_{k}^{'})} \times 100$$

$$(21)$$

4. Empirical Application: Size-sorted portfolios and the structural GARCH specification

4.1. Motivation

The structural GARCH framework is utilised to model dependencies in three size-sorted portfolios based on an argument of diversification. The portfolios contain securities from three different market capitalisation groups and are labelled as the large, medium and small cap indices. The large cap index consists of the top twenty stocks listed on the Australian Stock Exchange. As such firms tend to be well diversified, both operationally and regionally, authors such as Lo and MacKinlay (1990) show that they exhibit no significant autocorrelations. Thus, these firms tend to adjust to new information instantaneously. Being well diversified and depending only on macro economic news the large cap index is assumed to be driven by a latent factor in the form of the error term. The less diversified medium cap index is assumed to be driven by two factors. Firstly, it depends on the market wide information through contemporaneous dependence on the large cap index. Secondly it is affected by its own latent factor, information irrelevant for the large firm portfolio. The least diversified small cap index depends on three factors. Contemporaneous returns on the large cap index and the medium cap index enter its mean equation. News idiosyncratic to small firms is represented by an error term. Each of the three equations is augmented with lagged values of all three portfolios in order to capture any additional dynamics and test the lead-lag hypothesis of Lo and MacKinlay (1990). The second conditional moments are modelled by assuming the structural

innovations to be uncorrelated but exhibit a GARCH structure, thus justifying the use of the structural GARCH model. In the matrix format the model is as follows:

$$B_{0}Y_{t} = C + B_{1}Y_{t-1} + \dots + B_{p}Y_{t-p} + g_{t}\mathcal{E}_{t}$$

$$C = \begin{vmatrix} C_{1} \\ C_{2} \\ C_{3} \end{vmatrix},$$

$$B_{0} = \begin{vmatrix} 1 & 0 & 0 \\ B_{21}^{0} & 1 & 0 \\ B_{31}^{0} & B_{32}^{0} & 1 \end{vmatrix}, B_{i} = \begin{vmatrix} B_{11}^{i} & B_{12}^{i} & B_{13}^{i} \\ B_{21}^{i} & B_{22}^{i} & B_{23}^{i} \\ B_{31}^{i} & B_{32}^{i} & B_{32}^{i} \end{vmatrix} \forall i > 0$$

$$g_{t} = \begin{vmatrix} g_{1t} & 0 & 0 \\ 0 & g_{2t} & 0 \\ 0 & 0 & g_{3t} \end{vmatrix},$$

$$\varepsilon_{t} \sim iidn(0, I_{3})$$

$$(22)$$

where Y_t is a 3x1 vector containing the large, medium and small firm-size portfolio returns, in that order. GARCH (1,1) specification is used for the three structural variance equations. Furthermore, in order to test for volatility spillover effects each variance equation is augmented with squared lagged volatility shocks from other variables.

$$g_{1t}^{2} = \omega + b_{1}g_{1t-1}^{2} + a_{11}u_{1t-1}^{2} + a_{12}u_{2t-1}^{2} + a_{13}u_{3t-1}^{2}$$

$$g_{2t}^{2} = \omega + b_{2}g_{2t-1}^{2} + a_{21}u_{1t-1}^{2} + a_{22}u_{2t-1}^{2} + a_{23}u_{3t-1}^{2}$$

$$g_{3t}^{2} = \omega + b_{3}g_{3t-1}^{2} + a_{31}u_{1t-1}^{2} + a_{32}u_{2t-1}^{2} + a_{33}u_{3t-1}^{2}$$
(23)

4.2. Data Definitions and Statistical Summary

The data set consists of 466 companies that constituted the All Ordinaries Share Price Index as of 26 October 2002 and spans the time period from December 1987 to April 2003. It includes 3996 daily observations on the following variables: daily closing price, market capitalisation and dividends paid. I use the All Ordinaries Share Price Index because although it accounted for only 28 per cent of the total number of companies listed on the Australian Stock Exchange in October 2002, it represented 99 per cent of the total market capitalisation. Simple percentage weekly return series were created for each company, using Wednesday closing prices and dividends paid ending up with 799 weekly observations. Weekly returns, rather than daily, were constructed in order to lessen market microstructure effects such as large bid-ask spreads, non-synchronous trading and complications arising from seasonality issues such as the day-of-the-week effect.

Lo and MacKinlay (1990) have shown that there are statistically significant serial correlations in equally-weighted but not in value-weighted CRSP-NYSE-AMEX return indices. Given that an equally-weighted index gives more weight to smaller stocks than a value-weighted index does, they conclude that firm-size plays an important role in the stock return predictability. Table 1 presents summary statistics on the equally weighted and value weighted portfolios, constructed from the 466 securities included in the sample.

Table 1.										
Autocorrelation in weekly equally weighted and value weighted portfolio returns										
	Mean (% x 100)	St. Dev (% x 100)	Rho(1)	Rho(2)	Rho(3)	Rho(4)	Rho(5)	Q(5)		
Equally Weighted	0.381	1.928	0.24	0.18	0.09	0.07	0.01	81.29		
Value Weighted	0.351	1.495	-0.02	0.06	0.01	-0.03	0.00	3.55		

Autocorrelation coefficients and Ljung-Box Q-statistics for equally weighted and value weighted portfolios consisting of 466 securities from the All Ordinaries Share Index for the sample period 31 January 1987 to 2 February 2003. Asymptotic standard errors are given by $1/\sqrt{T} = 0.035$, Q-critical (5% level) = 11.07.

As can be seen from the above table, the Ljung & Box Q-statistic for autocorrelation is highly significant for the equally weighted but not the value weighted portfolio. Equally weighted portfolio return autocorrelation coefficients range from 0.24 to 0.01, with the first four being statistically significant at the 5 per cent level. On the other hand, only the second order autocorrelation is statistically significant for the value weighted return series. Proceeding in the steps of Lo and MacKinly (1988), I divide the sample into sizesorted portfolios to further investigate the impact of size classification.

Three size-sorted portfolios, each consisting of twenty stocks of small, medium and large market capitalisation are obtained for every half-year (end of June and end of December) period within the sample timeframe. The same number of securities was included in each portfolio in order to maintain the same signal to noise ratios. The large firm portfolio consists of the top twenty stocks that account for about 60 per cent of the total sample market value. The medium cap portfolio includes the first twenty stocks above 11 per cent of the cumulative sample market value while the smallest capitalisation portfolio is comprised of the twenty stocks above 2.5 per cent of cumulative value. While the large stock portfolio mirrors the ASX's Large Cap Index, the cutoff points for the medium and small stock portfolios roughly coincide with median cumulative percentage market values of the Medium Cap and Small Cap Market indexes published by the Australian Stock Exchange⁴. Portfolios are re-balanced every half-year in order to monitor the firm-size in each portfolio. Weekly portfolio returns are created by equally weighting individual security returns, using Wednesday closing prices and dividends paid. Caution was exercised to alleviate the non-synchronous trading problem, as first mentioned in Fisher (1966). According to this argument, autocorrelation in portfolio returns is due to the use of stale prices, that is, prices of securities that did not trade during a trading session. Such prices do not accurately reflect all available market

⁴ The Small cap index includes about 200 smallest securities with the market value of about 6 per cent of the total market capitalisation. The Mid Cap market index consists of 50 middle size companies comprising about 10per cent market value, while the Large Cap index consist of the 20 largest companies (Source: ASX, <u>http://www.asx.com.au</u>; last viewed: October 2003)

information and will appear to lag the rest of the market in trading time as their price adjusts to the new information. To prevent the spurious autocorrelations due to nonsynchronous trading, only the securities that have traded in two most recent sessions are used in the calculation of portfolio returns. A similar procedure was originally applied in Mech (1993). Table 2 reports autocorrelations for each of the size-sorted portfolios.

Table 2.									
Summary stat	tistics of weekly	realised retur	ns and squ	ared returns	s of three si	ze portfolios			
	Mean Return (% x 100)	St. Dev. (% x 100)	Rho(1)	Rho(2)	Rho(3)	Rho(4)	Q(4)	JB	
Small	0.27	2.29	0.236	0.135	0.038	-0.036	61.34 (0.00)	64.77 (0.00)	
Medium	0.23	1.79	0.153	0.078	0.018	0.009	23.95 (0.00)	33.13 (0.00)	
Large	0.25	2.05	0.018	0.025	0.022	-0.025	1.62 (0.80)	45.99 (0.00)	
Sqr(Small)	5.33	9.69	0.167	0.022	0.017	0.021	23.23 (0.00)		
Sqr(Medium)	3.25	5.40	0.104	0.055	0.146	0.07	32.11 (0.00)		
Sqr(Large)	4.24	7.56	0.133	0.124	0.09	0.069	37.05 (0.00)		

Small, Medium and Large refer to weekly portfolio returns on the small, medium and large firm portfolios. The Q-statistics tests the hypothesis that all autocorrelations up to lag 4 are jointly zero. The JB-statistic is the Jarque-Bera test for normality, with p values in brackets. Asymptotic standard errors are given by $1/\sqrt{T} = 0.035$.

Medium and small firm portfolios exhibit statistically significant serial correlations as evidenced by large Q-statistics. For the small firm-size portfolio, the first order autocorrelation is 0.236, indicating that a little over 5 per cent of the variation in this portfolio is explained by its first lag. The large cap index, on the other hand, shows no discernable serial correlation pattern. After squaring portfolio returns all three series appear to be strongly autocorrelated. This finding can be interpreted as evidence of time varying volatility.

Time-series properties of the size-sorted portfolios can also be characterised by their cross correlations as in Table 3. The cross-autocorrelations exhibit familiar monotonically decreasing pattern. This structure implies that lagged returns on small

firms are more strongly correlated with current returns on large firms than the other way around. For example, the third entry in column one shows the correlation between the small firm portfolio return and the lagged value of the large firm portfolio return is 20.8 per cent. We can compare this value with the first element of the last column from the same table (–1 per cent), which represents the correlation between the return on the large firm portfolio and the lagged return on the small firm portfolio. The above mentioned asymmetries exist throughout Table 3.

Table 3.									
Cross-autocorrelations in size-sorted portfolio returns									
	Small(t)	Medium(t)	Large(t)						
Small(t-1)	0.236	0.136	-0.010						
Medium(t-1)	0.245	0.153	0.004						
Large(t-1)	0.208	0.165	0.018						
	Small(t)	Medium(t)	Large(t)						
Small(t-2)	0.135	0.022	-0.093						
Medium(t-2)	0.195	0.078	0.032						
Large(t-2)	0.205	0.065	0.025						
	Small(t)	Medium(t)	Large(t)						
Small(t-3)	0.038	0.063	0.011						
Medium(t-3)	0.072	0.017	-0.029						
Large(t-3)	0.137	0.041	0.022						

Small, Medium and Large refer to weekly portfolio returns on the small, medium and large firm portfolios.

4.3. Mean Equation Estimates

Table 4 contains estimated coefficients from the mean equations as specified by (22) and (23). A second order VAR system was chosen according to the AIC and BIC selection criteria.

Large cap index mean equation			Medium cap index mean equation				Small cap inde	x mean equ	nean equation			
	Coef.	t-stat	Prob.		Coef.	t-stat	Prob.		Coef.	t-stat	Prob.	
Constant	0.237	3.445	0.001	Constant	0.09	1.74	0.082	Constant	0.011	0.156	0.876	
Large(t-1)	0.01	0.231	0.817	Large(t-1)	0.081	2.517	0.012	Large(t-1)	0.075	1.607	0.108	
Medium(t-1)	-0.027	-0.54	0.588	Medium(t-1)	0.069	1.779	0.075	Medium(t-1)	0.138	2.626	0.009	
Small(t-1)	0.009	0.227	0.821	Small(t-1)	0.046	1.696	0.11	Small(t-1)	0.072	1.874	0.061	
Large(t-2)	0.026	0.568	0.57	Large(t-2)	-0.002	-0.059	0.953	Large(t-2)	0.1	2.042	0.041	
Medium(t-2)	0.073	1.467	0.143	Medium(t-2)	0.049	1.228	0.22	Medium(t-2)	0.084	1.673	0.094	
Small(t-2)	-0.084	-2.21	0.027	Small(t-2)	0.021	0.75	0.453	Small(t-2)	0.06	1.662	0.097	
				Large(t)	0.423	17.09	0	Large(t)	0.304	7.602	0	
								Medium(t)	0.281	6.031	0	

Table 4. Structural GARCH estimates - mean equations

All contemporaneous beta coefficients are highly significant giving support to the structural GARCH model hypothesis. For every 1 per cent increase (decrease) in the large portfolio return, the medium cap index return increases by 0.42 per cent and the small firm portfolio return by 0.3 per cent. For every 1 per cent increase in the medium firm portfolio return, the small stock portfolio return increases by 0.28 per cent in the same period. The lagged coefficients in the large cap index equation are all statistically insignificant with the exception of the small cap index return lagged two periods. This confirms a low level of predictability in portfolio returns on large firms. Large and medium firm portfolio returns lagged one period are statistically significant at the 10 per cent level in the medium cap index equation. Small firm portfolio returns appear to be predictable from all lagged returns at the 10 per cent significance level. Overall, empirical evidence is consistent with the international findings and suggests that the leadlag effect is present in the Australian market. Given the fact that the structural GARCH model allows us to control for contemporaneous effects and that all lagged statistically significant coefficients in the second and third equations are in fact positive, it appears that the medium and small cap indices initially under-react to information shocks. Using Boudoukh, Richardson and Whitelow's terminology, this finding can be interpreted as evidence in support for the lead-lag hypothesis or heretic explanation.

4.4. Variance Equation Estimates and Volatility Spillover Tests

Structural GARCH is particularly suitable for volatility spill-over tests as it directly identifies orthogonal error terms in the presence of conditional heteroscedasticity. Volatility spill-overs are defined as transmissions of volatility between variables within a system. The simplest way of testing for the effect is by including lagged squared errors for security *j* as an exogenous variable in the conditional variance equation of security *i*. As can be seen from Table 5, a large number of lagged coefficients in volatility equations are statistically significant. This implies that beside their own lagged shocks, volatilities in size-sorted portfolios are also predictable from other portfolios' volatility innovations.

Table 5.

Structural GARCH estimates	 variance 	equations
----------------------------	------------------------------	-----------

Large cap index variance equation				Medium cap index variance equation				Small cap index variance equation			
	Coef.	t-stat	Prob.		Coef.	t-stat	Prob.		Coef.	t-stat	Prob.
Constant	0.278	1.759	0.079	Constant	0.111	2.039	0.041	Constant	0.872	2.235	0.025
Arch Large(t-1)	0.089	3.3	0.001	Arch Large(t-1)	-0.005	-0.873	0.383	Arch Large(t-1)	0.077	2.419	0.193
Arch Medium(t-1)	-0.007	-0.41	0.681	Arch Medium(t-1)	0.074	3.264	0.001	Arch Medium(t-1)	0.078	1.301	0.016
Arch Small(t-1)	0.017	1	0.318	Arch Small(t-1)	0.004	0.497	0.619	Arch Small(t-1)	0.047	1.251	0.211
Garch(t-1)	0.831	14.62	0	Garch(t-1)	0.879	23.33	0	Garch(t-1)	0.52	4.319	0

In the above table ARCH terms refer to volatility shocks whereas GARCH quantities relate to coefficients on the lagged volatility series. In the large firm portfolio volatility equation its own ARCH and GARCH coefficients are highly significant at the 1 per cent significance level. This indicates that the large cap index volatility is affected by both its own history as well as by its lagged volatility shock. Lagged volatility shocks of medium and small firm portfolios have no statistically significant impact on the large firm portfolio variance. The second equation tells a similar story: the ARCH and GARCH

coefficients are significant at the 1 per cent significance level whereas other lagged terms are not significant at any conventional level. This time however, large and small firm portfolio volatility shocks contribute no additional information to the variance equation. Lastly, the small firm portfolio volatility appears to be dependent on its own history but also on lagged volatility shocks from the medium cap index. All three conditional variances are stationary as evidenced by Table 5 and Figure 1 below.



4.5. Conditional Impulse Response Functions in Returns

Although Table 5 above gives an indication as to how portfolio returns are affected by their lagged values, it does not tell us how a shock to any of the structural errors transmit through the system. For example, a shock to the orthogonal innovation in the large cap index equation will affect all three portfolios contemporaneously, however it will produce perturbations of different magnitudes and durations for each of the portfolios. As explained in section 3.3., impulse response functions in a structural GARCH model are dependent on volatility levels. Thus Figures 2 to 4 present impulse responses at three different levels of volatility: high, medium and low. High and low levels are dated from

Figure 1 as 12 November 1997 and 7 February 2001 respectively. I have chosen to use the unconditional variance matrix as a proxy for the medium level of market volatility. In order to emphasise the impact of volatility on impulse response functions the next three sets of graphs are ordered according to volatility levels.

Figure 2.

Conditional Impulse Response Functions to a shock in the Large Cap Index evaluated at high, medium and low volatility levels.



The above figure presents conditional impulse response fuctions for large, medium and small capitalisation index returns to one standard deviation shock in the large cap index structural innovation. The three graphs present impulse responses at high, medium and

low levels of volatility. Volatility plays a crucial role in the impulse response analysis as evidenced by the above graphs. As expected, due to the low order of dependence in the return equations (two lags), most of the response in the return series disappears after a short period of time, five weeks in the above case. However, volatility levels affect the magnitudes and signs of the responses. A one standard deviation shock to innovation 1 or market wide news has the greatest impact during times of high volatility. The response of the large firm portfolio return is equal to its estimated standard deviation at the time of initial shock (see (16)), which is above 3 per cent. For the periods of medium and low levels of volatility the initial responses of Portfolio One are about 2 per cent and 1.5 per cent. Similar patterns are found in the other two portfolios' response functions.

Figure 3.

Conditional Impulse Response Functions to a shock in the Medium Cap Index evaluated at high, medium and low volatility levels.





Figure 3 presents impulse responses due to a shock in orthogonal innovation 2, that is the medium firm portfolio innovation. This error was interpreted as the market information relevant to less diversified medium size firms but not the large firms. A shock to this innovation has the biggest effect on the large firm portfolio return during the period of medium volatility. It produces an increase in the large portfolio returns of just over 0.5 per cent. Its effect on the large firm portfolio is however negative during times of low volatility one week after the initial shock. This could be caused by investors moving into medium size firms following a positive news announcement. It is also interesting to see that during the medium and low levels of volatility medium and small firm portfolio returns respond in exactly opposite directions to a shock in this innovation.

Figure 4.

Conditional Impulse Response Functions to a shock in the Small Cap Index evaluated at high, medium and low volatility levels.





A news shock to the small cap index structural innovation has a positive impact on the portfolio of small firms during periods of large volatility and a slightly negative effect at times of medium and low volatilities a week after the shock. The observation of positive versus negative responses could possibly be explained by an overreaction hypothesis reported in Lehmann (1990). Firm prices adjust to an initial overreaction, however, during times of high volatility there is enough momentum in the market to lessen the adjustment and still produce a positive response.

4.6. Conditional Impulse Response Functions in Volatilities

Given the small order of dependence in the variance equations (one lag) one would expect monotonically decreasing impulse response functions in the variance equations. Figure 5 shows impulse response functions during a medium level of volatility.

Figure 5. Conditional impulse response functions in variance equations



The above figures presents conditional impulse response fuctions in variance equations of the large, medium and small cap indexes measured in standard deviation units due to shocks in large, medium and small cap index innovations. An interesting observation that

can be made from the above graphs is the length of time it takes for a shock in one structural factor to transmit through the system. It takes about 150 time periods, that is 150 weeks or just under three years for a shock to lose its effect. Thus, although stationary, structural variances exhibit strongly persistent dynamics. Furthermore, it appears that a shock transmits though the system more quickly in the portfolio of small firms than in any other portfolio.

4.7. Conditional Forecast Error Variance Decomposition

Figure 6 shows a percentage contribution of each structural innovation to the forecast variances as given by (21). About 70 per cent of the total impact of structural innovation one transfers to the forecast variance of the large firm portfolio, while the remaining 30 per cent is split between the medium and small cap indices.



Figure 6.

Conditional forecast error variance decompositions



The above graphs represent percentage contributions of one standard deviation shocks to structural innovations in the Large, Medium and Small Cap Indices to forecast error variances of the three portfolio.

Medium cap index innovation two has its largest impact on its own portfolio forecast error variance, accounting for more than 90 per cent of the one step ahead MSE, with its contribution decreasing to about 87 per cent for the nine step ahead forecast. Its effect on the MSE of the small cap portfolio increases from about 3 per cent to more than 10 per cent. Its effect on the forecast error variance of the portfolio of large firms is marginal at about 1.5 per cent relative to other portfolio variances. Small cap index innovation transmits most uncertainty to its own index with its effect ranging from about 100 per cent for a one week ahead MSE to just under 93 per cent for a nine step ahead forecast error variance.

5. Conclusion

This paper has introduced a new model of the multivariate GARCH type, namely the structural GARCH model. Structural GARCH is derived from a structural VAR model, where it is assumed that structural shocks have time dependent variance structures. Conditional impulse response and conditional variance decomposition analyses are derived. They are both dependent on volatility levels. Conditional forecast variance analysis is innovative in that it is calculated using forecasted covariance matrices.

A triangular version of structural GARCH is fitted to weekly returns on three sizesorted portfolios from the Australian Stock Exchange from 1988 to 2003. Evidence is found in support of the lead-lag hypothesis of Lo and MacKinlay (1990). In particular, it is shown that small firm portfolio returns depend on lagged large and medium size firm portfolio returns, even after controlling for contemporaneous effects. The medium firm portfolio returns are only dependent on their own past and the part returns of large firm portfolio returns. The large cap index appears to exhibit no autocorrelation and only a small proportion of its returns is explained. These findings are generally consistent with international evidence. Conrad, Gultakin and Kaul's (1991) volatility spill-over hypothesis is also tested. The volatility spill-over hypothesis is conducted on structural rather than the reduced form variances. Although no volatility spill-over is found from the large to medium firm portfolios, evidence is found for volatility spill-over from medium and small cap indices.

References

- Bachelier, L., 1900, Theory of speculation, reprinted in P. Cootner (ed), The random character of stock market prices (MIT Press, Cambridge, 1964).
- Bollerslev, T., 1990, Modelling the coherence in short-run nominal exchange rates: A generalised ARCH model, Review of Economic Statistics 72, 498-505.
- Bollerslev, T.R., F. Engle and J. Wooldridge, 1988, A capital asset pricing model with time varying covariances, Journal of Political Economy 96, 116-131.
- Boudoukh, J., M. Richardson and R. Whitelaw, 1994, A tale of three schools: insights on autocorrelations of short-horizon stock returns, Review of Financial Studies 7, 530-573.
- Cha, B. and S. Oh, 2000, The relationship between developed equity markets and the Pacific Basin's emerging equity markets, International Review of Economics and Finance 9, 299-322.
- Chang, E., G. McQueen and J. Pinegar, 1999, Cross-autocorrelation in Asian stock markets, Pacific-Basin Finance Journal 7, 471-493.
- Conrad, J., and G. Kaul, 1989, Mean reversion in short-horizon expected returns, Review of Financial Studies 2, 225-240.
- Conrad, J., M. Gultekin, G. Kaul, 1991, Asymmetric predictability of conditional variances, Review of Financial Studies 4, 597-622.
- Engle, R.F., 2002, Dynamic conditional correlation A simple class of multivariate GARCH models, Journal of Business and Economic Statistics 20.
- Engle, R. F., T. Ito and W-L. Lin, 1990a, Meteor showers or heat waves? Heteroskedastic intra daily volatility in the Foreign Exchange Market, Econometrica 58, 525-542.
- Engle, R.F.and K. Kroner, 1995, Multivariate simultaneous GARCH, Econometric Theory 11, 122-150.
- Fama, E. F., 1991, Efficient Capital Markets: II, The Journal of Finance 46, 1575-1617.
- Fargher, N., and R. Weigand, 1998, Changes in the stock price reaction of small firms to common information, The Journal of Financial Research 21, 105 121.
- Fisher, L., 1966, Some New Stock Market Indices, Journal of Business, 39, 191-225
- Hameed, A., 1997, Time varying factors and cross-autocorrelations in short-horizon stock returns, Journal of Financial Research 20, 435-458.
- Jegadeesh, N. and S. Titman, 1995, Short-horizon return reversals and the bid-ask spread, Journal of Financial Intermediation 4, 116-132.
- Kroner, K. and V. Ng, 1998, Modeling asymmetric comovements of asset returns, The Review of Financial Studies Vol. 11, No. 4., 817 844.
- Lehman, B., 1990, Fads, martingales, and market efficiency, Quarterly Journal of Economics 105, 1-28.
- Lo, A., and C. MacKinlay, 1988, Stock market prices do not follow random walks: Evidence from a simple specification test, Review of Financial Studies 1, 175-205.
- Lo, A., and C. MacKinlay, 1990, When are contrarian profits due to stock market overreaction?, Review of Financial Studies 3, 175-205.
- Lo, A., and C. MacKinlay, 1990a, An econometric analysis of non-synchronous trading, Journal of Econometrics 45, 181-211.
- McQueen, G., M. Pinegar and S. Thorley, 1996, Delayed reaction to good news and the cross autocorrelation of portfolio returns, Journal of Finance 51, 889-919.
- Mech, T., 1993, Portfolio return autocorrelation, Journal of Financial Economics 34, 307-344.
- Pagan, A., 1986, Two stage and related estimators and their applications, Review of Economic Studies 53, 517-538.
- Pagan, A., 1996, The econometrics of financial markets, Journal of Empirical Finance 3, 15-102.
- Richardson, T. and D. Peterson, 1999, The cross-autocorrelation of size-based portfolio returns in not and artifact of portfolio autocorrelation, The Journal of Financial Research 22, 1 13.

- Rigobon, R. and B. Sack, 2003, Spillovers Across U.S. Financial Markets, NBER Working Paper No. 9640
- Ross, S., 1989, Information and volatility: The no-arbitrage martingale approach to timing and resolution irrelevancy, The Journal of Finance Vol. 44, No. 1, 1-17.