The effect of exchange rate uncertainty on US imports from the UK: Consistent OLS estimation with volatility measured by an ARCH-type model

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Abstract

This paper investigates the effect of exchange rate volatility on US-UK bilateral trade flows. As part of econometric problems arising from a generated variable, we consider a special case when an ARCH type auxiliary model is used to measure uncertainty in the exchange rate and discuss a procedure for the correct inference of the OLS estimates of the primary equation in the second stage, which includes the generated variable. By applying this two-step approach, we find a statistically significant, negative impact of exchange rate uncertainty on US imports from the United Kingdom.

Keywords: ARCH model; Consistent OLS estimation; Generated regressors; Volatility

JEL classification: C22, C51, F14

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1. Introduction

Since the adoption of the floating exchange rate system in the early 1970s, a number of studies have tried to establish a systematic link between risks in exchange rates and trade volume. While Ethier (1973), Cushman (1986), and Peree and Steinherr (1989) demonstrate theoretically the negative effects of exchange rate uncertainty on trade flows, Franke (1991), Sercu and Vanhulle (1992), and Viaene and de Vries (1992) suggest opposing arguments in favour of a positive relationship between trade and exchange rate volatility. In recent work Barkoulas et al. (2002) take an intermediate position by arguing that the overall effects depend upon the source of uncertainty in exchange rates. Among many others, empirical studies related to this issue include Hooper and Kohlhagen (1978), Kenen and Rodrik (1986), Pozo (1992), Chowdhury (1993), Kroner and Lastrapes (1993), Arize et al. (2000), and De Grauwe and Skudelny (2000). However the empirical evidence is also mixed, depending on the choices of sample period, model specification, proxies for exchange rate volatility, and countries considered.

In investigating the above issue, an econometric difficulty is that the data series of volatility in exchange rates are not directly observed and thus measured in an indirect way. In recent years, many empirical studies in the literature use ARCH type models to generate the volatility and estimate the structural equation in the second stage with the conventional OLS technique, by replacing the unobserved volatility with the measured proxy.[1] A problem with this two-step procedure, however, is that even though the application of the OLS method leads to consistent estimators, the estimators do not have consistent covariance matrix and, as a result, are inefficient (Pagan 1984). That is, the standard errors of the OLS estimators are larger than those of conventional OLS estimators, due to the composite error term involving noise in the auxiliary equation. This implies that the application of the test statistics based on conventional OLS estimation may be misleading, even in a large sample. Thus, to have statistically reliable inferences, the non-spherical covariance matrix of the OLS estimates in the second stage should be adjusted by taking account of time dependence and heteroscedasticity in error terms.

In this paper we revisit the issue of the possible effect of exchange rate risk on US bilateral imports from the United Kingdom. Our particular attention is on an econometric problem arising from a generated variable of volatility in exchange rates. We consider a special case when an ARCH type model is used to measure exchange rate uncertainty and discuss a procedure for the correct inference of OLS estimates in the second stage. More specifically, given that uncertainty in exchange rates is

[1] The popularity of ARCH models in measuring volatility stems from the models' usefulness in capturing non-constant, clustered time varying variance in higher moments, which represents stochastic processes by which risk terms are generated (see Bollerslev et al. (1992) for a comprehensive overview of the literature).

captured by an ARCH class auxiliary model, it is demonstrated that there exists an orthogonal condition between structural parameters (including a measured volatility) and error terms, because the risk variable generated by an ARCH class model has a 'strong property', as defined by Pagan and Ullah (1988). Then, the orthogonality condition is exploited to derive OLS-based GMM estimators, using the Newey and West (1987) method. This method adjusts the non-scalar covariance matrix of OLS estimators in the second stage, mainly due to the generated regressor. By applying this approach, we find a statistically significant, negative impact of exchange rate uncertainty on US imports from the United Kingdom.

The paper is organised as follows. Section 2 discusses the orthogonality condition of OLS estimation for a primary equation, when the equation includes a risk variable measured by an ARCH class auxiliary model, and demonstrates that this condition can be exploited to derive OLS-based GMM estimators. Section 3 describes the data used and how the volatility of the exchange rate is measured. Section 4 presents empirical results. Finally, conclusions are provided in Section 5.

2. OLS-based GMM estimation with a regressor measured by an ARCH model

This section discusses that if an ARCH class auxiliary model is used to measure the unobserved variable of volatility, there exists an orthogonality condition between explanatory variables and error terms in the second-stage structural equation, and shows that such condition can be exploited to derive OLS-based GMM estimators.

Consider the following structural equation on the basis of some information set $\Im_{t-1}$:

$$y_t = \beta x_t + \gamma h_t^2 + e_t, \tag{1}$$

where x_t is a $(1\times p)$ vector of independent regressors, h_t^2 is a variable of risk due to unexpected fluctuation in the exchange rate ξ_t, and e_t is the disturbance term with the property $E(e_t \mid \Im_{t-1}) = 0$. Given a smoothness assumption that the risk variable h_t^2 depends linearly on some variables z_t, which are the past squared innovations in a standard ARCH model

$$\xi_t = \eta\psi_t + \varepsilon_t, \qquad \varepsilon_t \mid \Im_{t-1} \sim N(0, h_t^2),$$

(1) can be rewritten as:

$$\begin{aligned} y_t &= \beta x_t + \gamma h_t^2 + e_t \\ h_t^2 &= \rho z_t. \end{aligned} \tag{2}$$

If a series $\tilde{h}_t^2$ is available from a correctly specified ARCH-class auxiliary equation with the property that $E(\tilde{h}_t^2) = h_t^2$,[2] the system (2) becomes a typical triangular system as follows:

$$y_t = \beta x_t + \gamma \tilde{h}_t^2 + u_t,$$

[2] Since $\tilde{h}_t^2$ is a conditional function of z_t, $E(\tilde{h}_t^2)$ is itself a random variable.

$$\tilde{h}_t^2 = \rho z_t + v_t, \qquad (3)$$

where $u_t = e_t + \gamma(h_t^2 - \tilde{h}_t^2)$ and $v_t = (\tilde{h}_t^2 - h_t^2)$.

Since one of the fundamental properties of the ARCH model is $E(\varepsilon_t^2) = E(E(\varepsilon_t^2 \mid \Im_{t-1})) = E(\tilde{h}_t^2)$, the series $\tilde{h}_t^2$ has a *strong property* $\tilde{h}_t^2(N) \xrightarrow{a.s} h_t^2 \ \forall t$ as $N \to \infty$, where *N* denotes some index (not dependent on *t*), as defined by Pagan and Ullah (1988). This property leads $\gamma(h_t^2 - \tilde{h}_t^2) \xrightarrow{a.s} 0$ in (3) as $N \to \infty$.[3] Under this framework, a usual two-step procedure is first to obtain a $\sqrt{T}$ consistent estimator of ρ, $\hat{\rho}$, from the system (2), and to regress y_t against x_t and $\hat{h}_t^2 = \hat{\rho} z_t$. Then, an estimable form of equation (1) can be written as

$$y_t = \beta x_t + \gamma \hat{h}_t^2 + u_t, \qquad (4)$$

where $u_t = e_t + \gamma(h_t^2 - \hat{h}_t^2)$.

Note that although the strong property of the unconditional variance of ε_t to its conditional variance in an ARCH class model provides an orthogonality condition $E(\hat{h}_t^2 u_t) = 0$ for the OLS estimators of β and γ in (4) to be consistently estimated, the asymptotic variance of the estimators is not equivalent to the variance of conventional OLS estimators (see Appendix). The standard errors calculated by the latter understate the true standard errors of the two-step estimators, mainly due to the composite error u_t that involves noises in equation (4). This implies that the application of conventional test statistics will be misleading, even in large samples. To correct this problem, OLS-based GMM estimation may be used by directly exploiting the orthogonality condition between errors and the risk variable generated by an ARCH-class auxiliary equation (see Hansen (1982)).

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[3] This characteristic of the risk variable generated by an ARCH class model is contrasted with that of other alternative measures. For example, if h_t^2 is generated by a moving average of variances or standard deviations calculated from the mean value of a random variable, the generated $\tilde{h}_t^2$ has a *weak property* $E(\tilde{h}_t^2 \mid \Im_{t-1}) = h_t^2$ (see, for details, Pagan and Ullah (1988)). In this case, even though $E(x_t u_t) = 0$, $E(\tilde{h}_t^2 u_t) \neq 0$ because it involves $\gamma(h_t^4 - E(\tilde{h}_t^4))$ which is only degenerate (zero variance) when $h_t^4 = E(\tilde{h}_t^4)$. The correlation of the regressor $\tilde{h}_t^2$ with the error u_t causes the OLS estimator, γ, to be biased toward zero - that is, attenuation bias. In this case, to avoid the problem of such an errors-in-variables bias, an instrumental variable τ_t for $\tilde{h}_t^2$ can be used to achieve an orthogonality condition $E(\tau_t u_t) = 0$ (see Pagan (1984)).

Since the regression residuals u_t is uncorrelated with the explanatory variables in (4), a moment condition for the GMM estimation of the equation can be written, by defining $w_t = (x_t, \hat{h}_t^2)$ and $\varphi = (\beta, \gamma)'$:

$$E(f(\varphi, \overline{w}_t)) = E(w_t(y_t - w_t\varphi)) = 0\,, \qquad (5)$$

where φ has an unknown $((p+1)\times 1)$ vector of parameters and $f(\cdot)$ is a differentiable q-dimensional vector-valued function of data $\overline{w}_t = (y_t, x_t, \hat{h}_t^2)$. Then, the GMM estimate $\hat{\varphi}$ is the value of φ that minimizes a criterion function

$$g(\varphi; \overline{y}_t)\hat{\Omega}_T^{-1} g(\varphi; \overline{y}_t)\,, \qquad (6)$$

where $g(\varphi; \overline{y}_t)$ is the sample moment of $f(\varphi, \overline{w}_t)$ with the property $g(\varphi; \overline{y}_t) = (1/T)\sum_{t=1}^{T} w_t(y_t - w_t'\varphi)$ and $\hat{\Omega}_T$ is an estimate of $\Omega = \lim_{T\to\infty}(1/T)\sum_{t=1}^{T}\sum_{v=-\infty}^{\infty} E\{[f(\varphi, \overline{w}_t)][f(\varphi, \overline{w}_{t-v})]'\}$. It should be noted that, in fact, the moment condition (5) has (p+1) orthogonality conditions, of which the number is the same as the number of unknown parameters in φ, such that $q = p + 1$, thus just-identifying the overall system.

Under this just-identification, the objective function (6) can be minimized by setting [4]

$$g(\hat{\varphi}; \overline{y}_t) = (1/T)\sum_{t=1}^{T} w_t(y_t - w_t'\hat{\varphi}) = 0\,. \qquad (7)$$

Solving (6) for $\hat{\varphi}$ becomes

$$\hat{\varphi} = \left(\sum_{t=1}^{T} w_t w_t'\right)^{-1}\left(\sum_{t=1}^{T} w_t y_t\right), \qquad (8)$$

which is the usual OLS estimator. On the other hand, the weighting matrix could be calculated from the following spectral-based method suggested by Newey and West (1987), who extend White's (1984) time domain approach for the capture of time dependence in samples:

$$\hat{\Omega}_T = \hat{S}_0 + \sum_{j=1}^{m}\omega(j,m)(\hat{S}_v + \hat{S}_v')\,, \qquad (9)$$

where $\hat{S}_v = (1/T)\sum_{t=v+1}^{T}(w_t\hat{u}_t\hat{u}_{t-v}w_{t-v}')$, $\hat{u}_t = y_t - \hat{\varphi}w_t$, $\omega(j,m)$ is a weight function to smooth the sample autocorrelation function, and *m* is a bandwidth parameter chosen from a function *m(T)* of sample size.[5]

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[4] In the case of the over-identification where the number of orthogonality conditions exceeds the number of parameters to be estimated, this condition is not held.

[5] Newey and West (1987) suggest allowing the bandwidth parameter *m* to grow more slowly than $T^{1/4}$, such that $\lim_{T\to\infty} m(T) = +\infty$ and $\lim_{T\to\infty}[m(T)/T^{1/4}] = 0$. Alternatively, a time domain method proposed by White (1984) could be applied with a growing function, $m = o(T^{1/3})$. But this approach does not guarantee that the estimated covariance matrix is positive semi-definite. If $m = 0$ in equation (9), White's (1980) heteroscedasticity standard errors are obtained as a special case. If the weight function

With this application, the OLS-based GMM estimate of equation (4) can be characterized as $\hat{\varphi} \approx N(\varphi, \hat{V}_T / T)$, where $\hat{V}_T = (\hat{Q}_T^{-1} \hat{\Omega}_T \hat{Q}_T^{-1})$ is the GMM approximation for the variance-covariance matrix of $\hat{\varphi}$ and $\hat{Q}_T = \partial g(\varphi; \bar{y}_T) / \partial \varphi' |_{\varphi = \bar{\varphi}_T}$ $= (1/T) \sum_{t=1}^{T} \partial w_t (y_t - w_t' \varphi) / \partial \varphi' |_{\varphi = \bar{\varphi}_T}$ = $(-1/T) \sum_{t=1}^{T} w_t w_t'$. The square root of the diagonal elements of the covariance matrix $\hat{V}_T / T$ is the heteroscedasticity-and autocorrelation-consistent standard error for the OLS estimators with the property $\hat{Q}_T^{-1} \hat{\Omega}_T \hat{Q}_T^{-1} \xrightarrow{p} Q_T^{-1} \Omega_T Q_T^{-1}$ (see White (1984) for the proof), and then used to test linear hypotheses in the usual way.

3. Data descriptions and the measurement of exchange rate volatility

To investigate the impact of volatility in the exchange rate, a demand function for US bilateral imports from the UK could be specified as:[6]

$$IM_t = f(EX_t, UY_t, H_t),$$

(10) where IM_t denotes U.S. real imports; EX_t the real exchange rate to proxy the relative price competitiveness of commodities between two countries; UY_t the real income of the United States; and H_t volatility in the real US dollar/British pound exchange rate. It is expected that an increase of real income has a positive effect on imports, but a rise in the real exchange rate, whether it is due to either a variation in the nominal exchange rate or a different rate of inflation between two countries, negatively affects U.S. imports. However, the effect of exchange rate volatility is ambiguous, depending on traders' attitude to risk. If traders are risk-neutral, uncertainty in exchange rates may be an additional opportunity to increase profits and thereby boosts overall trade flows. On the other hand, if traders are risk-averse, the risk due to exchange rate uncertainty is an additional cost, which will tend to depress overall trade volumes.

In this study, we use monthly data that cover the period of the floating exchange rate system from 1974(1) to 2003(4). US real import value deflated by the consumer price index (CPI) is used as the measure of import trade. The real bilateral exchange rate is derived from the US dollar against the sterling pound by adjusting the nominal rate with the relative inflation as measured by the ratio of UK CPI to US CPI. As a proxy for the income level, US industrial production index is used.[7] Since exchange rate volatility is not directly observable, to quantify the variable we use the conditional standard deviation obtained from a GARCH model. The underlying model is a

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$\omega(j, m)$ is chosen to be one, the Newey-West method, which uses a Bartlett window, is identical to the uniform window used by White (1984).

[6] See, for examples, Kenen and Rodrik (1986) and Pozo (1992).

[7] While the source of UK CPI is *UK National Statistics,* all other data were taken from *FRB St. Louis FRED II*.

GARCH (1,1) based on an autoregressive model of order two (*AR* (2)) of the first difference of the real bilateral exchange rate (ex_t) in logarithm, which takes the following common form:

$$\Delta ex_t = \eta_0 + \eta_1 \Delta ex_{t-1} + \eta_2 \Delta ex_{t-2} + \varepsilon_t, \qquad \varepsilon_t \mid I_{t-1} \sim N(0, h_t^2).$$
$$h_t^2 = \rho_0 + \rho_1 \varepsilon_{t-1}^2 + \rho_2 h_{t-1}^2. \tag{11}$$

The estimated equation is:

$$\Delta ex_t = \underset{(0.001)}{0.0004} + \underset{(0.06)}{0.43} \Delta ex_{t-1} - \underset{(0.06)}{0.15} \Delta ex_{t-2}$$
$$h_t^2 = \underset{(0.000007)}{0.00005} + \underset{(0.05)}{0.13} \varepsilon_{t-1}^2 + \underset{(0.04),}{0.78} h_{t-1}^2 \tag{12}$$

where the values in the parentheses represent standard errors. Except the constant term in the *AR(2)*, all the coefficients in equation (12) are statistically significant at the 5% significance levels. The coefficients of ρ_0, ρ_1, and ρ_2 exceed zero, and $\rho_1 + \rho_2 = 0.91 < 1$. These results ensure that the conditional variance is strictly positive, thus satisfying the necessary conditions of equation (11). The statistical significance of the GARCH effect in the model is again confirmed by a Wald statistic $\chi^2(2) = 2{,}805.4$ for the test of a joint hypothesis $\rho_1 = \rho_2 = 0$. Figure 1 plots the volatility measured by the model.

4. Effects of exchange rate volatility on trade volumes

For analysis, all variables were transformed into logarithms. Lower case letters denote logs of the corresponding capitals in equation (10). To examine the non-stationarity of the data, we first conducted augmented Dickey-Fuller (ADF) tests (Dickey and Fuller, 1979). With an initial maximum lag of 12, the auxiliary lags were selected on the basis of the Schwarz Bayesian criterion (SBC). For the variables of levels, the testing equation included an intercept and a linear trend, but in the case of differenced variables only an intercept was included. The test results reported in Table 1 indicate that im_t, uy_t, and ex_t are integrated of order one, *I(1)*, at the 5% levels, respectively, but h_t appears to be *I(0)*. Note that the measured volatility, h_t, has been generated by a finite variance stochastic process, which does not accumulate past errors, and so could be expected to be stationary. To examine whether the variables used are cointegrated, the maximum likelihood testing procedure suggested by Johansen (1988) was applied with the treatment of h_t as *I(1)*, even though some caveats may be applied. In an initial 12th –order vector autoregressive (VAR) model with a constant term but no trend, two lags were selected for the test on the basis of the SBC criterion. The intercept was not restricted to lie in the cointegration space. The standard statistics of the Johansen test reported in Table 2 show that both the maximum eigenvalue and trace statistics strongly reject the null of no cointegration in favour of at least one cointegrating relationship, and little evidence exists for more than one.

With these cointegrated variables, one useful econometric model is an error correction model (ECM) (see Engle and Granger (1987)). Initially an unrestricted single equation-based ECM that is equivalent to a third-order autoregressive distributed lag (ADL) model, was estimated with OLS under the assumption that the variables of ex_t, uy_t, and h_t are weakly exogenous:[8]

$$\Delta im_t = c_0 + \sum_{i=1}^{2} c_{1i}\Delta im_{t-i} + \sum_{i=0}^{2} c_{2i}\Delta ex_{t-i} + \sum_{i=0}^{2} c_{3i}\Delta uy_{t-i} + \sum_{i=0}^{2} c_{4i}\Delta h_{t-i} + c_5 im_{t-1} + c_6 ex_{t-1} + c_7 uy_{t-1} + c_8 h_{t-1} + u_t. \tag{13}$$

As shown in Table 3, the results of the initial estimation show that most of the coefficients are not easily interpretable and statistically insignificant. Since the over-parameterisation of the unrestricted model may capture accidental features of the sample, to reduce the sample dependence we sequentially simplified the model by eliminating statistically insignificant parameters.[9] A finally derived model is

$\Delta im_t =$	$0.51\,\Delta im_{t-1}$	$0.29\,\Delta im_{t-2}$	$+1.94\,\Delta uy_{t-1}$	$0.10\,im_{t-1}$	$+0.08\,uy_{t-1}$
	(0.05)	(0.05)	(0.83)	(0.03)	(0.02)
	[0.05]	[0.05]	[0.73]	[0.03]	[0.02]
	$0.05\,ex_{t-1}$	$1.40\,h_{t-1}$			
	(0.04)	(1.16)			(14)
	[0.03]	[0.71]			

$R^2 = 0.30$, $\hat{\sigma} = 0.13$, $DW = 2.00$, $F_{AR}(12{,}329) = 6.97$, $F_{ARCH}(12{,}329) = 1.76$, $\chi^2_N(2) = 6.38$, $F_{RESET}(1{,}340) = 1.29$, $F_H(1{,}346) = 2.76$, ( ) standard errors with the OLS estimation, and [ ] standard errors adjusted by the Newey-West method.

The parameter restrictions on equation (13) are accepted at the 5% significant level by a Wald statistic of $\chi^2(8) = 4.49$ [0.81]. Even though there is evidence of serial autocorrelation, the tests of non-normality and heteroscedasticity are rejected at the 1% and 5% levels, respectively.

Note that all the estimated coefficients are statistically significant at the 5% levels, except the ones of the real exchange rate and exchange rate volatility. The failure of these variables to meet the standard statistical criteria would be caused by the direct application of conventional test statistics with the generated regressor of volatility. As discussed in Section 2, since a GARCH model was used to measure the variable, the estimated OLS parameters reported in (14) would be consistent, but has large standard errors, because of the composite error that involves noises in the GARCH auxiliary equation. To correct the latter problem, the Newey-West method was applied with a

[8] See Engle et al. (1983) for the definition of weak exogeneity. Johansen (1992) alternatively discusses the testing procedure in the framework of a cointegrated VAR system.

[9] Mizon (1995) provides an excellent review of the general-to-specific modelling approach.

Bartlett weight for the lag window.[10] Initially, 9 bandwidth parameters were used. The adjusted standard errors reported in equation (14) show that the statistical efficiency of the OLS estimators is much improved. In particular, the cases of ex_{t-1} and h_{t-1} are dramatic. The coefficients are statistically significant at the 10% and 5% levels, respectively. For a robust result, 14 and 20 bandwidths were used, but no difference was found. Alternatively, a Parzen kernel was also applied with 9 bandwidths, but the result is not too much different from the case of the Bartlett kernel used in this study.[11] With this adjustment that is robust to a variety of heteroscedasticity and autocorrelation in the residuals, equation (14) may be used as a benchmark model for the empirical investigation of risk in the real exchange rate on U.S. imports from Britain.

The estimated results indicate that the short-run change in U.S. imports from the UK is mainly affected by its own lags and by a change of the previous income level. There are no impact effects from the real exchange rate and its volatility in the short-run. This may reflect the stickiness of trade contracts between countries. The measured feedback coefficient, –0.10, has an expected sign with statistical significance, but indicates a slow adjustment to the past disequilibrium in import trade volumes. Although the short-run parameters are the subject of economic theory considerations in relation to the time form responses and speed of adjustment, these may not be the major concern related to the theoretical hypotheses characterizing the long-run responses in equilibrium. By setting all changes in the short-run to zero, a long-run static-state equilibrium of (14) is obtained:

$$im_t = 0.8uy_t - 0.5ex_t - 14.00h_t \ .$$

As would be expected, an increase in U.S. real income has a positive effect on imports in the long run, whilst an increase in the real exchange rate (U.S dollar depreciation) results in a smaller volume of real imports from UK. The long-run coefficient of the volatility measure, which is our major concern in this study, shows a negative sign. The magnitudes of the estimated coefficients are compared with 1.19, -0.81, and -14.90 for uy_t , ex_t , and h_t , respectively, in Kenen and Rodrik (1986).

5. Conclusions

In this paper, we have investigated the possible effects of risk in exchange rates on US bilateral imports from Britain. For statistically valid inferences with a generated volatility variable, we discussed a special case when an ARCH class model is used to measure uncertainty in the real exchange rate. Specifically, we demonstrated that there exists an orthogonality condition between the measured volatility and error terms for the consistent OLS estimators of a structural equation in the second stage

[10] Since this spectral based approach is a large sample property, in small samples the method is very sensitive to the chosen window and its truncation points. For details, see Andrews (1991) and Newey and West (1994) on the selection of an optimal value of the lag truncation for different weights.

[11] To save space, we do not report the results. They can be provided on request.

and that this condition can be exploited to derive OLS-based GMM estimators. The empirical results applied this approach indicate that volatility in the real US/UK exchange rate has a negative impact on US imports from the United Kingdom. The overall application seems to well illustrate our discussions in Section 2 and, in particular, provides statistically significant OLS estimators, which include a measured variable as a proxy of exchange rate volatility. This result is statistically reliable and compares to most previous studies that fail to seriously consider econometric problems of OLS estimation involving a regressor generated from an auxiliary equation.

With a generated variable, the econometric literature in general recommends to use instrumental variables taken from underlying information set in order to avoid the problem of an errors-in-variables bias (see Pagan (1984)). The role of the instrumental variable is to establish an orthogonal condition between errors and measured independent regressors. However, this approach suffers from the problems of searching proper exogenous instruments, which have no correlations with innovations in dependent variables, and of weak instruments, which can arise when the instruments used are weakly correlated with included endogenous variables or when the number of instruments are large (see Podivinsky (1999) and Stock et al. (2002) for recent surveys on this issue). Considering these weaknesses of the IV method, if an ARCH class model is used to measure an unobserved volatility, the procedure discussed in this study would have an advantage in deriving robust evidence to the alternative IV method.

Appendix

Assume that $\mathrm{cov}(e_t^{'}\varepsilon_t) = 0$, $T^{-1}\sum w_t^{'}e_t \xrightarrow{p} 0$, $T^{-1}\sum z_t^{'}e_t \xrightarrow{p} 0$, $\hat{\rho} \xrightarrow{p} \rho$, and $\sqrt{T}(\hat{\rho}-\rho)$ has a limiting normal distribution with a covariance matrix D_ρ and is independent of e_t. Under this construction, the asymptotic properties of the OLS estimators of equation (4) can be written as

$$(\hat{\varphi}-\varphi) = (\sum w_t^{'}w_t)^{-1}(\sum w_t^{'}u_t) = (T^{-1}\sum w_t^{'}w_t)^{-1}(T^{-1}\sum w_t^{'}u_t). \qquad \text{(A-1)}$$

From Assumption 3, the first term converges to $(T^{-1}\sum w_t^{'}w_t)^{-1} \xrightarrow{p} Q_T^{-1}$, but the second term is

$$T^{-1}\sum w_t^{'}u_t = T^{-1}\sum w_t^{'}(e_t + \gamma(h_t^2 - \hat{h}_t^2))$$
$$= T^{-1}\sum w_t^{'}e_t + T^{-1}\sum w_t^{'}\gamma(h_t^2 - \hat{h}_t^2).$$

Since $T^{-1}\sum w_t^{'}e_t \xrightarrow{p} 0$ and $\hat{h}_t^2$ has a strong property such that $\gamma(h_t^2 - \hat{h}_t^2) \xrightarrow{a.s} 0$ as $N \to \infty$ (as shown Section 2), the second term becomes $T^{-1}\sum w_t^{'}u_t \xrightarrow{p} 0$. Hence, (A-1) leads to $(\hat{\varphi}-\varphi) \xrightarrow{p} Q_T^{-1} \cdot 0 = 0$, verifying the consistency of the OLS estimators.

The asymptotic distribution of φ can be written as

$$\sqrt{T}(\hat{\varphi}-\varphi)=(T^{-1}\sum w_t^{'}w_t)^{-1}(T^{-1/2}\sum w_t^{'}u_t)\,. \tag{A-2}$$

Since the first term converges in probability to Q_T^{-1}, the proof of the distribution of $\sqrt{T}(\hat{\varphi}-\varphi)$ mainly depends on the distribution of $T^{-1/2}\sum w_t^{'}u_t$. Using equations (2) and (3), the second term can be rewritten as

$$\begin{aligned} T^{-1/2}\sum w_t^{'}u_t &= T^{-1/2}\sum w_t^{'}(e_t+\gamma z_t(\rho-\hat{\rho})) \\ &= T^{-1/2}\sum w_t^{'}e_t + T^{-1/2}\gamma\sum w_t^{'}z_t(\rho-\hat{\rho}) \\ &= T^{-1/2}\sum w_t^{'}e_t + \gamma(T^{-1}\sum w_t^{'}z_t)T^{1/2}(\rho-\hat{\rho})\,. \end{aligned}$$

Under the initial conditions that $(\rho-\hat{\rho})$ and e are independently distributed, $T^{-1}\sum w_t^{'}e_t \xrightarrow{p} 0$, and $\sqrt{T}(\hat{\rho}-\rho)$ has a limiting normal distribution with a covariance matrix D_ρ, the limiting distribution of $T^{-1/2}W^{'}u$ then becomes

$$T^{-1/2}W^{'}u \xrightarrow{d} N(0,\sigma_e^2 p\lim T^{-1}W^{'}W+\gamma^2(p\lim T^{-1}W^{'}Z)D_\rho(p\lim T^{-1}Z^{'}W)\,.$$

Hence, the asymptotic distribution of φ is

$$\sqrt{T}(\hat{\varphi}-\varphi) \xrightarrow{d} N(0,Q_T^{-1}\Omega_T Q_T^{-1})\,, \tag{A-3}$$

where the positive (semi) definite symmetric matrix Ω_T is $\Omega_T=\sigma_e^2(p\lim T^{-1}\sum w_t^{'}w_t)+\gamma^2(p\lim T^{-1}\sum w_t^{'}z_t)D_\rho(p\lim T^{-1}\sum z_t^{'}w_t)$. This shows that the variance of conventional OLS estimators, $\sigma_e^2(p\lim T^{-1}\sum w_t^{'}w_t)$, understates the true standard errors of the estimators in the two-step procedure. □

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TABLES

Table 1: Augmented Dickey-Fuller test statistics

Variables	im_t	rp_t	uy_t	h_t
t-value	- 3.39 (4)	- 2.73 (3)	- 2.05 (2)	- 4.17 (1)
Variables	$\Delta\ im_t$	$\Delta\ rp_t$	$\Delta\ uy_t$	
t-value	- 10.34 (10)	- 12.18 (1)	- 8.99 (2)	

Notes: (1) The critical values of the ADF test are –3.42 for the level variables and –2.87 for the differenced ones, at the 5% levels, respectively. (2) The selected lags are in parentheses.

Table 2: Cointegration analysis

Eigenvalues	0.11	0.05	0.03	0.00
Hypothesis	$\gamma = 0$	$\gamma \leq 1$	$\gamma \leq 2$	$\gamma \leq 3$
Max statistic	39.27	18.54	10.65	0.05
95% c.v.	27.42	21.12	14.88	8.07
Trace statistic	68.51	29.24	10.70	0.05
95% c.v.	48.88	31.54	17.86	8.07

Note: γ indicates the number of cointegrating vectors.

Table 3. The estimated results of equation (13)

Lags	0	1	2
Δim_t	-1.00 (-)	- 0.51 (0.06)	- 0.30 (0.05)
Δrp_t	- 0.09 (0.26)	0.31 (0.28)	- 0.28 (0.26)
Δuy_t	0.51 (0.94)	1.98 (0.98)	- 0.50 (0.94)
Δh_t	0.97 (2.77)	3.46 (2.76)	- 2.25 (2.76)
im_t		- 0.10 (0.03)	
rp_t		- 0.05 (0.05)	
uy_t		0.08 (0.03)	
h_t		- 1.49 (1.26)	

T=1974(7) - 2003(4), $R^2 = 0.31$, $\hat{\sigma} = 0.13$, $DW = 2.01$, $F_{AR}(12{,}319) = 6.33$, $F_{ARCH}(12{,}319) = 1.63$, $\chi^2_N(2) = 5.14$, $F_{RESET}(1{,}330) = 1.26$, $F_H(1{,}344) = 3.50$,
where F_{AR} denotes the Lagrange multiplier (LM) test for twelve-order autocorrelation; F_{ARCH} the Engle (1982) tests for twelve-order autoregressive conditional heteroscedasticity; χ^2_N the test for normality; F_{RESET} Ramsey's (1969) test for omitted variables and incorrect functional form; F_H the White (1980) test for heteroscedasticity; and ( ) standard errors.

Figure 1. Plot of the volatility measured by the GARCH model