Asset Ownership and Asset Values Over Project Lifecycles

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PRELIMINARY

Abstract

This paper develops a theory of outside ownership where such an ownership arrangement mitigates an external finance problem. Part of the gains from outside ownership accrue to asset owners which determines the asset value. The theory provides a context to analyze asset ownership and asset values over project lifecycles. When there are adjustment costs in realizing the full gains from outside ownership, (i) assets take time to peak in value, and (iii) the outsiders's share of asset ownership increases gradually.

Keywords Asset ownership; Asset value; Project lifecycles; Entry and exit

JEL Classification L2, J3, G3, O3

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1 Introduction

In capitalist economies the owners of non-human assets are separate from the owners of human assets in production. The prevalence of this "outside ownership" arrangement suggests there are gains from assigning ownership, or residual controls rights of assets to outsiders. This paper develops a theory of outside ownership, where such an ownership arrangement mitigates an external finance problem. Part of the gains from outside ownership accrue to asset owners which determines the asset value. As a result, asset values are *conditional* on an outside ownership arrangement. I elaborate on this mechanism below.

This theory provides a context to analyze asset ownership and asset values over project lifecycles. I model investment projects which are subject to "adjustment costs" in realizing the gains from outside ownership. In frontier projects which appear every period, agents are constrained in their investment, but as projects are repeated, outside ownership mitigates this underinvestment as assets can gradually implement more gains from outside ownership. Project productivities are assumed to decrease over time, and in equilibrium there is a continuous entry and exit of projects.

Project specific asset values increase then decrease over time. At first, values increase as assets can implement greater gains through outside ownership, afterwards values decrease as the effect of falling productivities dominates. Through rate of return equalization, the endogenous income stream accruing to outside owners is negative when asset values appreciate, and positive when asset values depreciate. The relatively low income stream in early stages of projects is the opportunity cost of creating assets which can implement the gains from outside ownership. These indirect costs of asset creation are distinct from the direct material costs of assets, and the evolution of asset values they imply reflects an evolution of Tobin's q: the excess of asset values over their material cost.

The analysis sheds light on why (i) technology specific assets take time to peak in value, and (ii) why the source of these delays are changes in Tobin's q. Consistent with this observation, Greenwood and Jovanovic

(1999) and Hobijn and Jovanovic (2001) argue that the arrival of the IT revolution in the early 1970s initially depressed the aggregate stock market before causing it to rise in the mid 1980s. Laitner and Stolyarov (2003) document that this delayed peak arose from changes in average Tobin's q, rather than changes in aggregate physical investment. They explain there are adjustment costs in accumulating technology specific "intangible capital" which takes the form of proprietary knowledge. I model a particular form of proprietary knowledge associated with realizing the gains from outside ownership.[1]

In periods when asset values appreciate, production yields a joint product of goods and asset value. The component of assets associated with asset appreciation is initially owned by insiders (providers of human assets in production) then sold to outsiders. This incremental increase in asset value is falling as a share of total value. As a result, the share of asset value under outside ownership increases gradually over the lifecycle. A stylized fact is that while young firms using newer technologies tend to be owner managed, older firms using older technologies are outside owned. Even among publicly listed firms, Mikkelson, Partch and Shah (1997) document an increasing trend in the share of outside ownership over time.

In sum, the twin predictions of (i) increasing outside ownership of assets and (ii) the delayed peak in asset values over project lifecycles are the empirical targets of this paper. These predictions are set in a context where asset values are conditional on the outside ownership of assets.

I now elaborate on the key mechanisms of the model. Consider a two period project where an agent invests in skills in the first period to realize output in the second period together with a project specific asset. The skills acquired in the first period are asset specific: a fixed

[1] Moreover, among publicly listed firms, young firms (who are more likely to use new technologies) are less likely to pay dividends than older firms. For instance, Pastor and Veronesi (2002) document that only 28% of listed firms pay dividends in their first year, and even 10 years after listing only 51% of firms do. This suggests investors anticipate assets associated with new technologies to appreciate more in value.

quantity of output cannot be realized without the asset. In the first period, the agent is credit constrained because he cannot commit to repay loans made against second period output, and the marginal return on his investment is greater than the market interest rate. Assume the scrap value of the asset is zero.

In this setup, assigning ownership of assets to an outsider can improve outcomes. In the second period of the project, just before output is realized, the owner can use her control rights to hold-up the agent, and extract output which cannot be realized without the asset. This second period hold-up is anticipated in the first period. When the asset owner competes to attract the agent to her project in the first period, she must offer transfers to the agent. This transfer is set at a level which would make the agent indifferent between working for the asset owner or pursuing an outside option in period one. The combination of ex post hold-up and ex ante competition to attract agents implements transfers from the owner to agent which resembles a loan. Because agents are credit constrained in their investment in period one, outside ownership is superior to self ownership of assets.[2]

The agent's outside option is at least what he would get from the project under self ownership of the asset. Suppose this is his outside option. Since agents are credit constrained under self ownership, the level of transfer in period one which would make the agent indifferent between working for the asset owner or self ownership, is less than the discounted value of the output extracted through hold-up in the second period. The difference between the period one transfer and discounted output extracted through hold-up is that component of the asset value which is *conditional* on outside asset ownership.

The transfers implemented through outside ownership is an inferior substitute for a loan collateralized by an output level equal to that extracted through hold-up. Competing lenders would offer a loan in period one equal to the discounted output level. However, such a loan cannot

[2]Felli and Roberts (2002) and Acemoglu and Shimer (1999) look at how the combination of ex post hold up and ex ante competition can nullify the incentives to underinvest associated with ex post hold up. In my model the combination of these forces actually improves outcomes.

exist, because agents cannot commit to repay loans after the second period output is realized. Outcomes through such loans and outside asset ownership differ because two or more lenders can compete to offer the loans above, but only one asset can implement the gains from outside ownership described.

In sum, there are gains from outside asset ownership when credit constrained agents can use assets to commit to being held up ex post. When assets which can implement such gains are scarce, they have value conditional on outside ownership.

The second innovation of this paper is to consider the creation and destruction of such assets over project lifecycles. Two period projects are carried out by two period lived agents and projects can be repeated over time. New projects arrive each period and project productivities fall over time. The key to the gains from outside ownership are the level of output which agents can commit to being held up. The evolution of this commitment constraint is set as follows: the execution of a project up to some output level, allows agents to be held up to that output level in future repetitions of the project. This characterizes an adjustment cost of realizing the gains from outside ownership.

In frontier projects, there are no gains from outside ownership and agents are self employed. Over time, agents can commit a greater level of output to hold up, which implements greater investment and output. A by-product of ever greater output is in turn a greater level of commitment to being held up, and consequently a greater asset value. Since productivity is falling over time, the marginal increase in the asset value is falling. This marginal increase in asset value is initially owned by the investing agent or "insider", and sold to outsiders after production. As a result, the insider share of assets is falling over time from full ownership until it reaches zero.

Once this latter stage is reached, outsiders with full asset ownership implement unconstrained levels of investment, and offer agents lifetime earnings equal to their outside option. Asset values decrease over time, and eventually projects become so unproductive they are discontinued.

This paper adopts the Grossman-Hart-Moore definition of asset ownership as conferring the right to control assets in contractually unspecified situations.[3] In an environment of incomplete contracts, the identity of asset owners matters when asset specific investments are being made, and asset owners can hold up other agents who have sunk asset specific investments. A series of closely related models of outside ownership in this context have been developed by Chui (1998), De Meza and Lockwood (1998), and Rajan and Zingales (1998).

Unlike these models, my theory of outside ownership focuses on its role in overcoming external financing problems. A robust empirical feature of the self employed (who cannot exploit the gains from outside ownership) is that they are credit constrained. In particular, Evans and Jovanovic (1989) and Eakin, Joulfaian and Rosen (1994) find agents endowed with greater wealth are more likely to become self emplyed.

The stationary equilibrium displays project entry and exit as in the canonical model of Hopenhayn (1992). In that paper, new projects incur a fixed entry cost to create an unspecified input, which in equilibrium, earns positive discounted returns equal to the entry cost. The unspecified input must also be serviced by a fixed continuation cost to allow projects to continue. The fixed entry and continuation cost ensure an equilibrium with entry and exit exists. My analysis provides a particular interpretation of this unspecified input: assets which implement the gains of outside ownership. I endogenize the "entry" or creation cost of these assets, while their endogenous "continuation cost" corresponds to the transfers offered to young agents to participate in continuing projects.

A large literature has studied the role of incentive compatible debt contracts in firm dynamics. Albuquerque and Hopenhayn (2003) study limited enforeability and firm dynamics extending models by Thomas and Worral (1994) and Hart and Moore (1994, 1998). Quadrini (2000), DeMarzo and Fishman (2001) and Clementi and Hopenhayn (2002) study asymmetric information and firm dynamics. A wider class of general equilibrium models which study the role financial frictions in the macroeconomy includes Bernanke and Gertler (1989), Kiyotaki and

[3] Two well known papers are Grossman and Hart (1986) and Hart and Moore (1990).

Moore (1997), and Cooley, Marimon and Quadrini (2003). A missing component of this literature is the role of outside asset ownership as a substitute form of external finance, and the interplay between outside ownership and debt in generating observed firm dynamics. This paper attempts to shed some light on these issues.

The next section presents the basic model. Section 3 discusses equilibrium and Section 4 discusses comparative statics. The last section concludes.

2 Model

Consider an overlapping generations economy with a constant population of two period lived agents normalized to 2. Ex ante identical agents have preferences over their young and old period consumption $c_y \geq 0$ and $c_0 \geq 0$ given by,

$$u = c_y + \beta c_0 \qquad \beta \in (0, 1) \tag{1}$$

2.1 Technology

In every period a new set of two period projects arrive exogenously to the economy. Projects can be repeated each period so that period two of a project and period one of its repetition overlap. Let $\tau \in \{0, 1, ...\}$ index the age or vintage of a project relative to a frontier project. A vintage $\tau - 1$ project in the current period becomes a vintage τ project in the next period. There is no uncertainty.

A key feature of the technology is a distinction between repeated and non-repeated projects across vintages. The input-output matrix for a vintage $\tau - 1$ repeated project beginning in period $t - 1$ is:

	Input	Output
Period 1	1 unit labor	$i_{\tau-1,t-1}$ project
	$i_{\tau-1,t-1}$ project	specific skills
	specific investment	
Period 2	1 unit labor	$\delta^{\tau}F(i_{\tau-1,t-1}, n_{\tau,t})$
	$+i_{\tau-1,t-1}$ skills	goods
	$+n_{\tau,t}$ workers	+1 project specific
	+1 project specific	asset seasoned by
	asset *seasoned*	$s_{\tau,t} = \max\{i_{\tau-1,t-1}, s_{\tau-1,t-1}\}$
	by $s_{\tau-1,t-1}$	
or	1 unit labor	$\delta^{\tau}F(i_{\tau-1,t-1}, n_{\tau,t})-$
	$+i_{\tau-1,t-1}$ skills	$\delta^{\tau}F(s_{\tau-1,t-1}, \tilde{n}_{\tau,t})$ goods
	$+(n_{\tau,t} - \tilde{n}_{\tau,t})$ workers	+1 project specific
	+1 *unseasoned* asset	asset seasoned by
	at cost zero	$(i_{\tau-1,t-1} - s_{\tau-1,t-1})$
		if $i_{\tau-1,t-1} \geq s_{\tau-1,t-1}$
		and zero otherwise

A repeated project has a history of production which is summarized by the level of investment undertaken in previous executions of the project. This index of history is embodied in the seasoning of the project specific asset. Every repeated project can use the seasoned asset or a generic unseasoned asset in period two. The use of a generic asset implies only the *marginal* output resulting from investments in excess of the level of seasoning can be realized. To sum, repeated projects must use assets from past executions of the project to realize the full output from the project. The history of a project captured by asset seasoning $s_{\tau-1,t-1}$, determines the degree to which skill investments in the project are asset specific.[4]

In non-repeated projects, there is no history and no seasoned assets.

[4]One way to justify this assumption is that all investment in projects is latently asset specific. The execution of projects up to an output level reveals to potential outside owners the assets upon which specific investments are made.

By construction all frontier $\tau-1=0$ projects are non-repeated. I assume the material cost of assets is zero.[5] Assets must be in place in period one of a project for use in period two, and there is no asset depreciation.

There is a Cobb-Douglas technology,

$$\delta^{\tau} F(i_{\tau-1,t-1}, n_{\tau,t}) = \delta^{\tau} i_{\tau-1,t-1}^{\phi} n_{\tau,t}^{\alpha} \quad \text{where } \phi+\alpha<1,\ \delta \in (0,1) \tag{2}$$

where $\delta \in (0,1)$ implies productivity is falling in vintage. Output is constant returns to scale with respect to the agent acquiring skills, skill level, workers and asset. The technology is Leontieff in that skilled agents and assets are matched one to one.

Workers are hired from competitive labor markets at wage $w_t \geq 0$. Define,

$$\pi_{\tau,t}(i_{\tau-1,t-1}, w_t) \equiv \max_{n_{\tau,t}} \delta^{\tau} i_{\tau-1,t-1}^{\phi} n_{\tau,t}^{\alpha} - n_{\tau,t} w_t \tag{3}$$

This is the maximized income in the second period net of worker wages, when investment $i_{\tau-1,t-1}$ has already been sunk.

Let $V_{\tau,t}(s_{\tau,t}) \geq 0$ denote the asset value of a vintage τ asset seasoned by skill level $s_{\tau,t}$ in period t. Zero material costs of assets means $V_{\tau,t}(0) = 0\ \forall \tau, t$.

The income of a project net of worker wages is:

	Income net of workers
Period 1	$-i_{\tau-1,t-1} - V_{\tau-1,t-1}(s_{\tau-1,t-1})$
Period 2	$\pi_{\tau,t}(i_{\tau-1,t-1}, w_t) + V_{\tau,t}(s_{\tau,t})$
	with seasoned asset
or	$\pi_{\tau,t}(i_{\tau-1,t-1}, w_t) - \pi_{\tau,t}(s_{\tau-1,t-1}, w_t)$
	$+V_{\tau,t}(s_{\tau,t}) - V_{\tau,t}(s_{\tau-1,t-1})$
	with unseasoned asset

In the timing of events, agents produce, then conduct asset transactions, and finally consume.

The contractual environment is as follows. Period two project specific skill and output levels are non-verifiable. Any borrowing in the first

[5]This assumption differentiates the model from existing vintage capital models where new and old vintages coexist because the material costs of old vintage assets have already been sunk.

period of the project must be collateralized by verifiable values. Then, the only source of borrowing available to the economy is that collateralized by the resale value of assets. Market trades verify the value of these assets. Borrowing takes the form of a typical debt contract. If repayment fails to take place *after* production, then lenders have the right to liquidate the collateralized assets.

To complete the description of the technology, let $\mu_{\tau,t}$ denote the period t proportion of old agents who are skilled in vintage τ.

2.2 Outside ownership of assets

Asset ownership confers the right to control assets in situations that are not contractually specified. In the incomplete contractual environment described above, asset ownership structures matter when asset owners threaten to confiscate assets from other agents who provide inputs which are asset specific. The production technology specifies only one type of asset specific input: project specific skills up to the level at which assets are seasoned.

When the agent who embodies these skills is also the asset owner, he simply receives the net income from the project each period.

When the skilled agent and asset owner are separate individuals outcomes become very different. In the second period of the project, just before output is realized, the two parties must bargain over the surplus of the bilateral match between the asset and specifically skilled labor. The bilateral match yields $\pi_{\tau,t}(i_{\tau-1,t-1}, w_t) + V_{\tau,t}(s_{\tau,t})$. The outside option of the asset owner is $V_{\tau,t}(s_{\tau-1,t-1})$, since outside the match, the seasoning of the asset through production is not realized. The outside option of the skilled agent is,

$$\max\left\{\pi_{\tau,t}(i_{\tau-1,t-1}, w_t) - \pi_{\tau,t}(s_{\tau-1,t-1}, w_t) + V_{\tau,t}(s_{\tau,t}) - V_{\tau,t}(s_{\tau-1,t-1}), w_t\right\} \tag{4}$$

The skilled agent has the option of earning the (i) income from the project with an unseasoned asset or (ii) becoming a worker.

The income of the bilateral match minus the outside option of the skilled agent and asset owner equals the match surplus,

$$\min\left\{\pi_{\tau,t}(s_{\tau-1,t-1},w_t),\pi_{\tau,t}(i_{\tau-1,t-1},w_t)-w_t+V_{\tau,t}(s_{\tau,t})-V_{\tau,t}(s_{\tau-1,t-1})\right\} \tag{5}$$

Assume that in bargaining negotiations, the outside owner has full bargaining power and fully extracts the match surplus. Then the period $t-1$ value of assets *conditional on outside ownership* is given by,

$$\begin{aligned} V_{\tau-1,t-1}(s_{\tau-1,t-1}) &= \max\left\{0,-x_{\tau-1,t-1}+\frac{1}{R_t}\left[\pi_{\tau,t}(s_{\tau-1,t-1},w_t)\right]+\frac{1}{R_t}V_{\tau,t}(s_{\tau-1,t-1})\right\} \\ &\quad \text{if agent's outside option producing with unseasoned asset} \quad (6) \\ &= \max\left\{0,\begin{array}{c} -x_{\tau-1,t-1}+\frac{1}{R_t}\left[\begin{array}{c}\pi_{\tau,t}(i_{\tau-1,t-1},w_t)-w_t \\ +V_{\tau,t}(s_{\tau,t})-V_{\tau,t}(s_{\tau-1,t-1})\end{array}\right] \\ +\frac{1}{R_t}V_{\tau,t}(s_{\tau-1,t-1})\end{array}\right\} \\ &\quad \text{if agent's outside option is becoming a worker} \end{aligned}$$

R_t denotes the market interest factor. The terms in the square brackets are simply the match surpluses from (5). The asset equations are no arbitrage conditions in competitive asset markets.

If agents's outside option is using unseasoned assets, the second period match surplus is $\pi_{\tau,t}(s_{\tau-1,t-1},w_t)$. In the first period, the asset owner has to attract a young agent to work with his asset. This involves offering a transfer to the young agent $x_{\tau-1,t-1}\geq 0$. $x_{\tau-1,t-1}\geq s_{\tau-1,t-1}$ since investments up to $s_{\tau-1,t-1}$ are fully appropriated by the owner. Note by construction, agents are necessarily self employed in frontier projects so $x_{0,t-1}=0$.

If agents's outside option is becoming a worker, the second period match surplus is $[\pi_{\tau,t}(i_{\tau-1,t-1},w_t)-w_t+V_{\tau,t}(s_{\tau,t})-V_{\tau,t}(s_{\tau-1,t-1})]$, in the first period, the asset owner also has to attract a young agent to work with his asset. This involves offering transfer to the young agent $x_{\tau-1,t-1}\geq 0$. $x_{\tau-1,t-1}\geq i_{\tau-1,t-1}$ since investments up to $i_{\tau-1,t-1}$ are fully appropriated by the owner. Since the agent has to be made indifferent between working for the owner and becoming a worker, it follows that $x_{\tau-1,t-1}-i_{\tau-1,t-1}=w_{t-1}$: agents have an earnings profile identical to workers. Since outside owners are not credit constrained, they can invest optimally in projects where skilled agents outside option is becoming a worker.

Remark 1 *In projects where skilled agents's outside option is the worker wage, investments are unconstrained. Conversely, constrained investment implies skilled agents's outside option is producing with an unseasoned asset.*

Outside ownership is superior to self ownership. Under self ownership, the investment constraint of agents is,

$$i_{\tau-1,t-1} + V_{\tau-1,t-1}(s_{\tau-1,-1t}) \leq \frac{1}{R_t} V_{\tau,t}(s_{\tau,t}) \tag{7}$$

Self employed agents can only borrow against the resale value of assets. Such agents are better off using unseasoned assets. Given this, self employed agents are best off entering frontier projects since $\delta \in (0,1)$. As long as employed agents are offered lifetime earnings equal to self employed agents in frontier projects, outside ownership is superior to self ownership. This latter condition is ensured by the participation constraint across occupations characterized for an equilibrium below.

At the end of each project the share of assets under outside ownership is given by $\frac{V_{\tau-1,t}(s_{\tau-1,t})}{V_{\tau,t}(s_{\tau,t})} \in [0,1]$. When projects increase the seasoning of assets, the marginal increase in asset values associated with this is initially owned by insiders: agents who acquire project specific skills.

3 Equilibrium

A competitive equilibrium requires in every period (i) an ownership structure of assets and (ii) young agents's choice of occupation, vintage and consumption to maximize lifetime utility subject to the borrowing constraint, earnings across occupations and vintage, the interest factor, and (iii) labor market clearing condition and asset and credit market clearing condition. I restrict the analysis to steady state outcomes where earnings levels, the interest factor, the distribution of labor across occupations and ownership structure of assets are invariant across time: $w_t = w$, $\pi_\tau(i_{\tau-1,t-1}, w_t) = \pi_\tau(i_{\tau-1}, w)$, $V_{\tau,t}(s_{\tau,t}) = V_\tau(s_\tau)$, $\frac{1}{R_t} = \frac{1}{R}$, $\mu_{\tau,t} = \mu_\tau$. Time subscripts are dropped.

Ex ante identical agents enter different occupations as long as their lifetime earnings are equalized across occupations. There are three categories of occupations: (i) workers, (ii) agents entering projects where their outside option in period two is becoming a worker, and (iii) agents entering projects where their outside option is producing with an unseasoned asset. The first two occupation have identical earnings profiles of w each period. Then the participation constraint across occupation and vintage is given by,

$$\begin{aligned} & w + \frac{1}{R}w \\ &= -i_{\tau-1} + x_{\tau-1} + \frac{1}{R}\left[\pi_\tau(i_{\tau-1}, w) - \pi_\tau(s_{\tau-1}, w) + V_\tau(s_\tau) - V_\tau(s_{\tau-1})\right] \\ s.t. i_{\tau-1} &\leq x_{\tau-1} + \frac{1}{R}\left[V_\tau(s_\tau) - V_\tau(s_{\tau-1})\right] \end{aligned} \tag{8}$$

for $\forall\tau - 1$ with positive entry by young agents whose outside option when old is producing with the unseasoned asset. Such agents receive $x_{\tau-1}$ from outside owners in youth, make the investment $i_{\tau-1}$, and enjoy income equal to their outside option in the second period of the project.

These agents face the borrowing constraint specified, since they cannot commit to repay against their second period earnings. The only resources available for investment are (i) the transfers from asset owners and (ii) the discounted marginal asset value which these agents can borrow against. Borrowing by such agents against the marginal asset value constitutes the only instance of debt used in the economy. When borrowing constraints bind, the lifetime utility of these agents in given by: $\frac{1}{R}\left[\pi_\tau(i_{\tau-1}, w) - \pi_\tau(s_{\tau-1}, w)\right]$.

Let $i^*_{\tau-1}$ denote the first best or unconstrained level of investment in vintage $\tau - 1$. $i^*_{\tau-1}$ is given by,

$$\frac{d\pi_\tau(i^*_{\tau-1}, w)}{di_{\tau-1}} + \frac{dV_\tau(i^*_{\tau-1})}{di_{\tau-1}} = R \tag{9}$$

From Remark 1, if investment is constrained, the outside option of skilled agents producing with an unseasoned asset.

Investment levels are determined by the investment rule.

Investment Rule: *The investment level for $i_{\tau-1}$, $\forall \tau - 1 \geq 0$ with positive entry by young agents is $i_{\tau-1} = \hat{\imath}_{\tau-1}$ where,*

$$w + \frac{1}{R}w = \frac{1}{R}\left(\pi_\tau(\hat{\imath}_{\tau-1}, w) - \pi_\tau(\hat{\imath}_{\tau-2}, w)\right) \tag{10}$$

if $i^*_{\tau-1} - \hat{\imath}_{\tau-1} > 0$, and $i_{\tau-1} = i^*_{\tau-1}$ otherwise.

The investment rules solve for the investment levels as a function of the worker wage $\hat{\imath}_{\tau-1}(w), i^*_{\tau-1}(w)$. The key to how much *constrained* investment is undertaken is given by the outside option of young agents. This is because asset owners are only willing to provide agents with enough investment funds to make agents indifferent between working for them or pursuing their outside option.

Lemma 1

Let P denote youngest vintage with unconstrained investment, $i_{P-1} = i^*_{P-1}$.

(i) $i_{\tau-1} = \hat{\imath}_{\tau-1}$ $\forall$ $\tau - 1 < P - 1$, $i_{\tau-1} = i^*_{\tau-1}$ $\forall$ $\tau - 1 \geq P - 1$.

(ii) Constrained investments $\hat{\imath}_{\tau-1}$ rising in w and vintage, and $(\hat{\imath}_{\tau-1} - \hat{\imath}_{\tau-2})$ increasing in vintage$\Rightarrow$ $s_\tau = i_{\tau-1}$.

(iii) Unconstrained investments $i^*_{\tau-1}$ falling in w and vintage$\Rightarrow$ $s_\tau = s_{\tau-1}$.

(iv) P falling in w.

Proof. Part (i) follows from $\delta \in (0, 1)$ and the seasoning rule (**??**). From (9) it is clear that $i^*_{\tau-1} = i^*_{\tau-1}(w)$ decreasing in w and vintage since $\delta \in (0, 1)$. $w + \frac{1}{R}w = \frac{1}{R}\pi_1(\hat{\imath}_0, w)$ implies $\hat{\imath}_0(w)$ is increasing in w, $\frac{di_0}{dw} > 0$. $\frac{1}{R}\pi_1(\hat{\imath}_0, w) = \frac{1}{R}\left(\pi_2(\hat{\imath}_1, w) - \pi_2(\hat{\imath}_0, w)\right)$ defines $\hat{\imath}_1 = \hat{\imath}_1(\hat{\imath}_0, w)$. Taking differentials w.r.t. w,

$$\begin{aligned}&\frac{\partial \pi_1(\hat{\imath}_0, w)}{\partial \hat{\imath}_0}\frac{d\hat{\imath}_0}{dw} + \frac{\partial \pi_1(\hat{\imath}_0, w)}{\partial w} \\ &= \left(\frac{\partial \pi_2(\hat{\imath}_1, w)}{\partial \hat{\imath}_1}\frac{d\hat{\imath}_1}{dw} + \frac{\partial \pi_2(\hat{\imath}_1, w)}{\partial w}\right) - \left(\frac{\partial \pi_2(\hat{\imath}_0, w)}{\partial \hat{\imath}_0}\frac{d\hat{\imath}_0}{dw} + \frac{\partial \pi_2(\hat{\imath}_0, w)}{\partial w}\right)\end{aligned}$$

Since the worker share of output is constant from the Cobb Douglas formulation, $\pi_1(\hat{\imath}_0, w)\frac{\phi}{1-\phi} = n_1(\hat{\imath}_0, w)w = -\frac{\partial \pi_1(\hat{\imath}_0, w)}{\partial w}w$; the second equality follows from the envelope theorem. Thus, the differential simplifies to,

$$\frac{\partial \pi_1(\hat{\imath}_0, w)}{\partial \hat{\imath}_0}\frac{d\hat{\imath}_0}{dw} + \frac{\partial \pi_2(\hat{\imath}_0, w)}{\partial \hat{\imath}_0}\frac{d\hat{\imath}_0}{dw} = \frac{\partial \pi_2(\hat{\imath}_1, w)}{\partial \hat{\imath}_1}\frac{d\hat{\imath}_1}{dw}$$

which implies $\frac{di_1}{dw} > 0$. By iteration, all constrained investments are increasing in w. The investment rules even imply constrained investments are accelerating in vintage $(\hat{\imath}_{\tau-1} - \hat{\imath}_{\tau-2}) > (\hat{\imath}_{\tau-2} - \hat{\imath}_{\tau-3})$ due to $\delta \in (0,1)$ and diminishing marginal returns. Finally, P is falling in w, since $i^*_{\tau-1}(w) - \hat{\imath}_{\tau-1}(w)$ is falling in w. ■

Recall the share of assets under outside ownership is given by $\frac{V_\tau(s_{\tau-1})}{V_\tau(s_\tau)}$. In projects with constrained investment $s_\tau = i_{\tau-1} > s_{\tau-1} = i_{\tau-2}$ so there is some increase in asset seasoning and partial inside ownership of assets. In projects with unconstrained investment $s_\tau = s_{\tau-1}$, so there is full outside ownership of assets.

Define project income net of the opportunity cost of input as the dividend,

$$D_{\tau-1}(i_{\tau-1}, w) \equiv \left[-i_{\tau-1} + \frac{1}{R}\pi_\tau(i_{\tau-1}, w)\right] - \left[w + \frac{1}{R}w\right] \tag{11}$$

Combining the no arbitrage asset price conditions (4) with the participation constraint (8) and rearranging implies the following equilibrium asset pricing equation,

$$\begin{aligned} V_{\tau-1}(s_{\tau-1}) &= D_{\tau-1}(i_{\tau-1}, w) + \frac{1}{R}V_\tau(s_\tau) \\ &= \sum_{a-1=\tau-1}^{T-1} \frac{1}{R^{(a-1)-(\tau-1)}} D_{a-1}(i_{a-1}, w) \end{aligned} \tag{12}$$

for $\forall \tau - 1$ with positive entry by young agents. This is a familiar relationship which says that the asset price is equal to the discounted sum of project dividends. The level of asset seasoning affects the asset value through $i_{\tau-1} = i_{\tau-1}(s_{\tau-1})$.

The terminal vintage T, is defined as the youngest non-frontier vintage such that $V_T(s_T) = 0$. Substituting into (12), the following inequalities must hold for T,

$$D_{T-1}(i_{T-1}, w) \geq 0 \tag{13}$$
$$D_T(i_T, w) < 0$$

The dividend must be non-negative for the penultimate vintage, and negative for the terminal vintage.

From (12), the free entry of assets $V_0(0) = 0$, implies the following condition must hold in equilibrium,

$$0 = \sum_{\tau-1=0}^{T-1} \frac{1}{R^{\tau-1}} D_{\tau-1}(i_{\tau-1}, w) \tag{14}$$

The discounted value of net incomes over project lifetimes sum to zero. This condition, the T investment equations and the terminal vintage conditions solve for the T investment levels $\{i_{\tau-1}\}_{\tau-1=0}^{T-1}$, terminal vintage T and worker wage w.

Lemma 2 In an equilibrium where $w > 0$,

(i) Skilled agents must coexist in vintages 1 to T.

(ii) The terminal vintage is finite $T < \infty$.

(iii) Investment in the frontier vintage must be constrained

$i_0 = \hat{i}_0 \Rightarrow T \geq 2$.

Proof. (i) By construction, $V_\tau(s_\tau) > 0 \ \forall 1 \leq \tau \leq T-1$, so it is worthwhile to implement projects from (12). (ii) For $w > 0$, there must exist a $T < \infty$.

(iii) Suppose not, so these agents can borrow $\frac{1}{R}V_1(i_0) > 0$ to finance investment $i_0 \leq \frac{1}{R}V_1(i_0)$, such that $i_0 = i_0^*$. Since $\delta \in (0, 1)$, the participation constraint (8) and terminal vintage conditions (13) imply that $T = 1$ and $V_1(i_0) = 0$ which is a contradiction. ■

Given values for $\{i_{\tau-1}\}_{\tau-1=0}^{T-1}, T, w$, the equilibrium $V_\tau(s_\tau)$ values are determined from (12). The equilibrium values of $V_\tau(s_{\tau-1})$ are determined by modifying the constrained investment rules (9) for the lower level of seasoning. The level of transfers to young agents x_τ, is determined from (8) given $V_\tau(s_\tau), V_\tau(s_{\tau-1}), \{i_{\tau-1}\}_{\tau-1=0}^{T-1}, T, w$.

Since unseasoned assets are only used in frontier projects, the density of skilled agents across coexisting vintages must be uniform, $\mu_\tau = \mu \, \forall 1 \leq \tau \leq T$. Given values for $\{i_{\tau-1}\}_{\tau-1=0}^{T-1}, T, w$, the labor market clearing condition solves for the equilibrium distribution of agents across vintages and occupations,

$$\frac{\mu}{2}\sum_{\tau=1}^{T} n_\tau(i_{\tau-1}, w) = 1 - \mu T \tag{15}$$

On the left hand side is the demand for workers summed across vintage divided by 2 since only half of the workers are old. On the right hand side is the population of old minus the population of non-workers.

Finally in the credit market, the linear preferences of agents from (1) imply $R = \frac{1}{\beta}$ as long as the young as a group are not constrained in their lending and asset transactions. I assume throughout that this holds (that is, the population of workers is large in the economy).[6]

Proposition 1

(i) A non-degenerate equilibrium exists where
$w > 0, \{i_{\tau-1}\}_{\tau-1=0}^{T-1} > 0$ and $T < \infty$.

(ii) A degenerate equilibrium exists where
$w = \{i_{\tau-1}\}_{\tau-1=0}^{\infty} = 0$ and $T = \infty$.

(iii) If young agents are born with endowment $\varepsilon > 0$,
the non-degenerate equilibrium is unique.

Proof in Appendix.

In the analysis which follows outcomes for the non-degenerate equilibrium are discussed.

3.1 Properties of equilibrium

Proposition 2

(i) $\exists$ a $Q \leq T$ such that, the dividend $D_{\tau-1}(i_{\tau-1}, w)$ is increasing in vintage $\forall \; \tau - 1 \leq Q$, and decreasing in vintage $\forall \; \tau - 1 > Q$.

[6] Define $w_{P-1} \equiv x_{P-1} + \frac{1}{R}\left[V_P(i_{P-1}) - V_\tau(i_{P-2})\right] - i_{P-1}$, the earnings net of investment of an agent entering the youngest vintage with unconstrained investment. Given linear preferences, as long as the young as a group are not credit constrained,

$$\mu w \sum_{\tau=1}^{T} \frac{n_\tau(w)}{2} \geq \mu \left[\sum_{\tau=1}^{T} V_{\tau-1}(s_{\tau-1}) + \sum_{\tau=1}^{T} i_{\tau-1} + \sum_{\tau=P}^{T} w_{\tau-1}\right]$$

the equilibrium interest factor is $R = \frac{1}{\beta}$.

(ii) When investment is constrained, $D_{\tau-1}(i_{\tau-1}, w)$ is decelerating in vintage. Let P denote the youngest vintage such that investment is unconstrained. Then, $D_{\tau}(i_{\tau}, w) - D_{\tau-1}(i_{\tau-1}, w)$
is falling in τ for $\forall\ \tau \leq P$.
(iii) When investment is not constrained, $D_{\tau-1}(i_{\tau-1}, w)$ is
falling and accelerating in vintage. That is,
$D_{\tau}(i_{\tau}, w) - D_{\tau-1}(i_{\tau-1}, w) < 0$
is increasing in τ for $\forall\ \tau > P \geq Q$.
Proof in Appendix.

3.1.1 Asset values over the lifecycle

[Figure 1] summarizes the lifecycle of vintage specific asset values and net incomes. From the asset price equations (12), the growth of asset prices is,

$$V_{\tau}(s_{\tau}) - V_{\tau-1}(s_{\tau-1}) = -\left[-i_{\tau-1} + \beta\pi_{\tau}(i_{\tau-1}, w) - (w + \beta w)\right] + (1-\beta)\, V_{\tau}(s_{\tau}) \tag{16}$$

This combined with Proposition 2 implies asset prices first increase then decrease over the lifecycle of projects. The relatively low and negative net incomes when assets appreciate in value can be interpreted as the cost of creating assets which can implement the gains from outside ownership.

Successive repetitions of young projects increase the extent to which agents acquiring project skills can expose themselves to being held up, which in turn implements higher levels of constrained investment. Projects with unconstrained investment are continued as long as the net income from projects under outside ownership is positive. Note the project specific asset value must peak before net income but after the vintage at which net income becomes positive.

3.1.2 Asset ownership over the lifecycle

The share of assets under outside ownership is given by $\frac{V_{\tau}(s_{\tau-1})}{V_{\tau}(s_{\tau})}$.

Lemma 3

(i) Agents are self employed in frontier projects, $\frac{V_1(0)}{V_1(i_0)} = 0$.
(ii) Outsiders own all assets once investment is unconstrained,
$\frac{V_{\tau}(s_{\tau-1})}{V_{\tau}(s_{\tau})} = 1$ for all $\tau \geq P$.

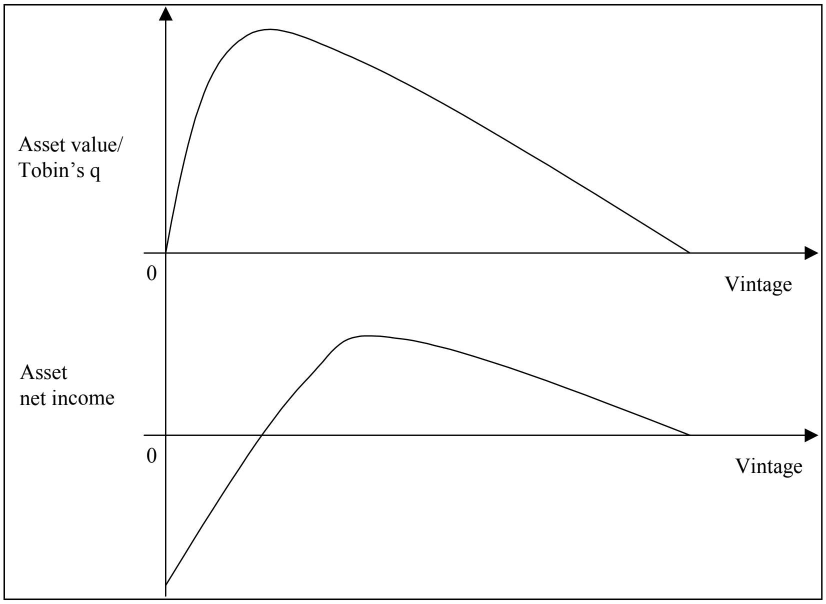

Figure 1: Asset values and net incomes over project lifecycles

(iii) The share of assets under outside ownership is increasing in vintage $\forall \tau < P$.

Proof. Parts (i) and (ii) are straightforward. For constrained investments $\frac{V_\tau(s_{\tau-1})}{V_\tau(s_\tau)} = \frac{V_\tau(i_{\tau-2})}{V_\tau(i_{\tau-1})}$. Part (iii) is true iff $\frac{V_{\tau+1}(i_\tau)}{V_\tau(i_{\tau-1})} < \frac{V_{\tau+1}(i_{\tau-1})}{V_\tau(i_{\tau-2})}$. For constrained investments, net incomes are increasing in investment levels, and from the constrained investment rules, constrained investments are increasing in the seasoning levels of assets used. These imply the gap $V_\tau(i_{\tau-1}) - V_\tau(i_{\tau-2})$ must be falling in vintage. The latter in turn implies $V_{\tau+1}(i_\tau) - V_\tau(i_{\tau-1}) < V_{\tau+1}(i_{\tau-1}) - V_\tau(i_{\tau-2})$. Since $V_\tau(i_{\tau-1}) > V_\tau(i_{\tau-2})$ it follows that $\frac{V_{\tau+1}(i_\tau)}{V_\tau(i_{\tau-1})} < \frac{V_{\tau+1}(i_{\tau-1})}{V_\tau(i_{\tau-2})}$. ■

The transition of asset ownership from inside to outside ownership mirrors a transition of skills from general to asset specific skills as defined by Becker (1964). The share of acquired skills which are asset specific is rising in vintage until skills become fully specific when there is full outside asset ownership.

3.1.3 Debt versus outside ownership

While both debt and outside ownership of assets mitigate an external financing problem, outside ownership is an inferior substitute for debt.

Under debt the borrower appropriates all the gains from trade whereas under outside ownership, the gains from trade are divided between borrower and asset owner, which results in a less effective mitigation of underinvestment.

In equilibrium, debt and outside ownership coexist in the model since they implement transfers in period one of projects that are backed up by different components of period two project income. Outside ownership allows agents to commit to make transfers of asset specific period two output $\pi_\tau(i_{\tau-1}, w)$ through hold-up, which cannot be implemented through debt. Meanwhile, agents in investment constrained projects use the value of the newly seasoned component of assets $[V_\tau(s_\tau) - V_{\tau-1}(s_{\tau-1})]$ as collateral for loans. This last observation implies that at the end of each project, the share of debt in total asset value is exactly equal to the share of assets which are inside owned.

3.1.4 The role of adjustment costs

In the analysis, asset seasoning is limited by the extent to which projects have been implemented in the past. Suppose there is no such adjustment cost to asset seasoning so that agents can commit any level of period two output to hold up. Given the assumption of zero asset material costs, the resulting outcome is straightforward. Only frontier projects are undertaken, with unconstrained levels of investment $i_0 = i_0^*$, and the value of assets owned by outsiders is zero, $V_0 = 0$. The latter reflects that with zero material costs and no adjustment costs to asset seasoning, none of the gains from outside ownership accrue to asset owners.

Suppose assets now carry a material cost $V_0 = K > 0$. Then from the logic of vintage physical capital models, non-frontier projects can coexist with frontier projects. Asset owners will command a discounted income stream which in equilibrium is equal to the material cost of assets. Thus, once positive material costs are introduced, some of the gains from outside ownership accrue to asset owners. However this framework cannot generate a delayed peak in project specific asset values, nor explain how asset values can exceed their material costs. Since all assets are owned by outsiders, such an analysis cannot explain the gradual transition of asset ownership from insiders to outsiders either.

4 Comparative statics

4.1 Imperfect seasoning

The main analysis assumed that this period's output determined the level of output extracted through hold up in next period's repetition of the project. More generally, the level of asset seasoning may not have this one to one mapping. Here I compare economies with different levels of asset seasoning. The asset seasoning rule is modified to,

$$s_{\tau,t} = \max\left\{\theta i_{\tau-1,t-1}, s_{\tau-1}\right\} \text{ where } \theta \in [0,1] \tag{17}$$

The equilibrium conditions affected are the constrained investment rules,

$$w + \beta w = \beta\left(\pi_\tau(\hat{\imath}_{\tau-1}, w) - \pi_\tau(\theta \hat{\imath}_{\tau-2}, w)\right) \tag{18}$$

Lowering θ acts as a subsidy to investment constrained agents relative to workers. When young, such agents receive a lower level of transfers from asset owners which in turn implies they implement a lower level of investment.

For a given w, lower θ implies lower levels of $\hat{\imath}_{\tau-1}$ $\forall 1 \leq \tau - 1 < P$. The level of output which agents can be held up is lower so outside owners offer less funds for investment to attract young agents to their project. From (14) this implies equilibrium w must be lower to satisfy the free entry constraint. Lowering w lowers the outside option of agents, which leads to a further round of reductions in constrained investments $\hat{\imath}_{\tau-1}$, and increases in unconstrained investments $i^*_{\tau-1}$. These results are summarized in the following Proposition.

Proposition 3 Consider two economies with different degrees of asset seasoning $\theta > \theta'$. In the weak seasoning economy θ',
(i) All constrained investments are lower $\hat{\imath}_{\tau-1} > \hat{\imath}'_{\tau-1}$ for $0 \leq \tau-1 < P$ and the youngest unconstrained vintage is older $P \leq P'$.
(ii) The worker wage is lower $w > w'$, and terminal vintage older $T < T'$.

Proof. Given $w > w'$ and $\hat{\imath}_{\tau-1} > \hat{\imath}'_{\tau-1}$ implies $P \leq P'$. From the terminal vintage conditions $w > w'$ implies $T < T'$. ■

Since w is falling in θ, welfare is falling in θ. In particular, when $\theta = 0$, the degenerate equilibrium is unique, and robust to the introduction of an endowment $\varepsilon > 0$ when the young are born.

4.2 Owner protection

The main analysis assumed full bargaining power of outside owners over the match surplus. More generally, suppose their bargaining share is given by $\lambda \in [0, 1]$. After substituting in the share of match surplus accruing to agents acquiring skills, the participation constraint is modified to,

$$
\begin{aligned}
& w + \beta w \qquad\qquad (19) \\
&= w_{\tau-1} + \beta \left((1-\lambda)\, \pi_\tau(i_{\tau-1}, w) + \lambda w \right) \\
&= -i_{\tau-1} + x_{\tau-1} + \\
&\quad \beta \left[\pi_\tau(i_{\tau-1}, w) - \lambda \pi_\tau(s_{\tau-1}, w) + V_\tau(s_\tau) - V_\tau(s_{\tau-1}) \right] \\
s.t. i_{\tau-1} &\leq x_{\tau-1} + \beta \left[V_\tau(s_\tau) - V_\tau(s_{\tau-1}) \right]
\end{aligned}
$$

Previously, setting $\lambda = 1$ meant that skilled agents, whose outside option is becoming a worker, experience lifetime earnings identical to workers. When $\lambda \in [0, 1)$, such agents earn a vintage specific wage $w_{\tau-1} < w$, given the anticipated sharing of the match surplus with the owner.

The equilibrium conditions affected are the constrained investment rules,

$$w + \beta w = \beta \left(\pi_\tau(\hat{\imath}_{\tau-1}, w) - \lambda \pi_\tau(\hat{\imath}_{\tau-2}, w) \right) \tag{20}$$

Lowering λ acts as a subsidy to investment constrained agents relative to workers. When young, such agents receive a lower level of transfers from asset owners which in turn implies they implement a lower level of investment.

Proposition 4 Consider two economies with different degrees of owner bargaining power $\lambda > \lambda'$. In economy λ',

(i) All constrained investments are lower $\hat{\imath}_{\tau-1} > \hat{\imath}'_{\tau-1}$ for $0 \leq \tau-1 < P$ and the youngest unconstrained vintage is older $P \leq P'$.

(ii) The worker wage is lower $w > w'$, and terminal vintage older $T < T'$.

Proof. Same as Proposition 3. ■

[Figure 2] summarizes the lifecycle of net incomes across the two economies.

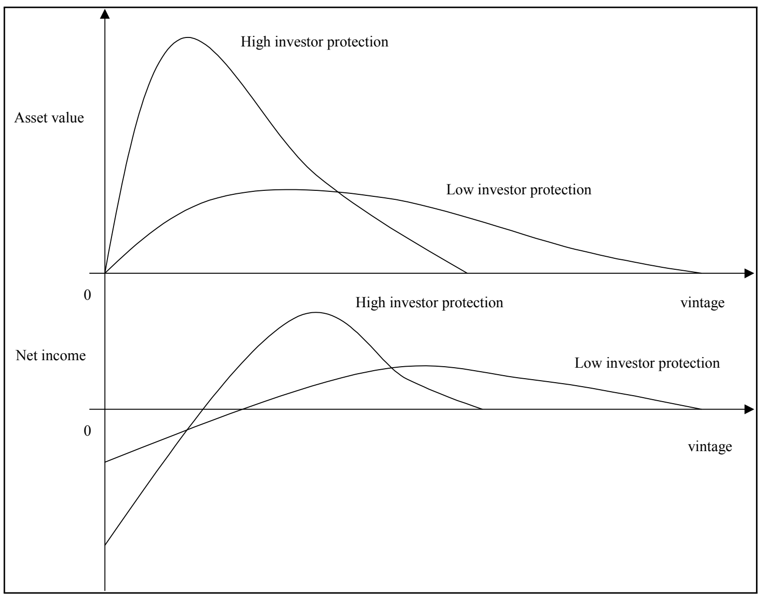

Figure 2: Asset values and net incomes across vintage with different degrees of investor protection

In the context of models of debt, Cooley, Marimon and Quadrini (2003) identify a general equilibrium mechanism where the arrival of more productive technologies increases the outside option of entrepreneurs and thereby implements higher investments in credit constrained projects. In my model this effect is captured by the investment decision rules for constrained investments. Cooley, Marimon and Quadrini then show that countries with lower degrees of contract enforceability will exhibit higher macroeconomic instability since a greater share of projects are investment constrained. In my model lower owner protection expands the number of investment constrained vintages and would lead to a similar prediction.

5 Conclusion

The literature on asset lifecycles, and Laitner and Stolyarov (2003) in particular, has argued that adjustment costs to realizing asset specific proprietary gains can explain the delay between the arrival of technologies and the peak in Tobin q values of technology specific assets. The form in which these proprietary gains and adjustment costs take shape remains a black box. My paper shows when agents are credit constrained, the asset specificity of skills combined with outside asset ownership can mitigate underinvestment, and generate asset specific proprietary gains. In the context of this framework, the adjustment cost which causes a delay in the peaking of such proprietary gains is the gradual process through which the technology specific skills become asset specific.

By marrying the literature on asset lifecycles with my theory of outside ownership, the analysis generated a new prediction: the gradual transition of asset ownership from inside to outside ownership.

References

[1] Acemoglu, D. and R. Shimer (1999), "Holdups and Efficiency with Search Frictions," *International Economic Review*, 40, pp. 827-851.

[2] Albuquerque, Rui and Hugo Hopenahayn (2003), "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, forthcoming.

[3] Becker, G.S. (1964). "*Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*," Chicago IL: University of Chicago Press.

[4] Bernanke, Ben and Mark Gertler (1989), "Agency Costs, Net Worth and Business Fluctuations," *American Economic Review*, 79:14-31.

[5] Chiu Y.S. (1998), "Noncooperative Bargaining, Hostages and Optimal Asset Ownership," *American Economic Review*, 88(4):882-901.

[6] Clementi, Gian Luca and Hugo Hopenhayn, "A Theory of Financing Constraints and Firm Dynamics," Carnegie-Mellon working paper.

[7] Cooley T., R. Marimon, V. Quadrini (2003), "Aggregate Consequences of Limited Contract Enforceability," NYU Stern Working Paper.

[8] DeMarzo, Peter and Michael Fishman (2001), "Agency and Optimal Investment Dynamics," Northwestern University working paper.

[9] de Meza D. and B. Lockwood (1998), "Does Asset Ownership Always Motivate Managers? The Property Rights Theory of the Firm with Alternating Offers Bargaining." *Quarterly Journal of Economics*, 113(2): 361-86.

[10] Evans, D.S. and B. Jovanovic (1989), "An Estimated Model of Entrepreneurial Choice Under Liquidity Constraints," *Journal of Political Economy*, 97, pp. 808-827.

[11] Felli L. and K. Roberts (2002), "Does Competition Solve the Holdup Problem?" C.E.P.R. Discussion paper No. 3535.

[12] Greenwood J. and B. Jovanovic (1999), "The Information-Technology Revolution and the Stock Market," *American Economic Review Papers and Proceedings*, pp. 116-121.

[13] Grossman, S. and O. Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, vol. 94, pp. 691-719.

[14] Hart, O. and J. Moore (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy*, vol. 98, pp.1119-1158.

[15] Hobijn B. and B. Jovanovic (2001), "The Information-Technology Revolution and the Stock Market: Evidence," *American Economic Review*, pp. 1203-1220.

[16] Holtz-Eakin D., D. Joulfaian and H.S. Rosen (1994), "Sticking It Out: Entrepreneurial Survival and Liquidity Constraints," *Journal of Political Economy*, 102, pp. 53-75.
[17] Hopenhayn, H.A. (1992), "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, pp. 1127-1150.
[18] Kiyotaki, N. and J. Moore (1997), "Credit Cycles," *Journal of Political Economy*, 105: 211-248.
[19] Laitner and Stolyarov (2003), "Technological Change and the Stock Market," *American Economic Review*, 93, pp. 1240-1267.
[20] Mikkelson, W., M, Partch and K. Shah (1997), "Ownership and Operating Performance of Companies that Go Public," *Journal of Financial Economics*, 44, pp. 281-307.
[21] Pastor, Lubos and Pietro Veronesi (2002), "Stock Valuation and Learning about Profitability," *Journal of Finance* forthcoming.
[22] Quadrini, Vincenzo (2000), "Investment and Default in Optimal Financial Contracts with Repeated Moral Hazard," NYU Stern working paper.
[23] Rajan R. and L. Zingales (1998), "Power in a Theory of the Firm," *Quarterly Journal of Economics*," 113:387-432.

6 Appendix

Proof of Proposition 1. [INCOMPLETE] (i) Straightforward to check. (ii) Consider the equation for the discounted stream of net incomes, where the investment levels are expressed as functions of w from (9) and (10), as w increases from zero.

$$\sum_{\tau-1=0}^{T-1} \beta^{\tau-1} \left([-i_{\tau-1}(w) + \beta\pi_\tau(i_{\tau-1}(w), w)] - [w + \beta w] \right)$$

In an equilibrium, this discounted sum equals zero from (14).

For investment constrained vintages, the net income can be rewritten as $[-\hat{\imath}_{\tau-1} + \beta\pi_\tau(\hat{\imath}_{\tau-1}, w)] - [w + \beta w] = [-\hat{\imath}_{\tau-1} + \beta\pi_\tau(\hat{\imath}_{\tau-2}, w)]$ from (10). The change in net income resulting from an increase in w is,

$$-\frac{d\hat{\imath}_{\tau-1}}{dw} + \beta\frac{\partial\pi_\tau(\hat{\imath}_{\tau-2}, w)}{\partial i_{\tau-2}}\frac{d\hat{\imath}_{\tau-2}}{dw} - \beta n_\tau(\hat{\imath}_{\tau-2}, w)$$

From Lemma 2 $\frac{d\hat{\imath}_{\tau-2}}{dw} > 0$, and $\frac{d\hat{\imath}_{\tau-1}}{dw} > \frac{d\hat{\imath}_{\tau-2}}{dw}$ by a factor related to the ratios of marginal productivity of investments across constrained vintages. Thus, when w and constrained investments are small, the marginal productivity of investment is very large but the ratio of marginal productivity of investments is small, so net incomes are increasing in vintage for constrained vintages. Also, when w is small, the youngest unconstrained vintage is very old. Thus, the discounted stream of net incomes is increasing in w in the neighborhood of $w = 0$.

Eventually, the discounted stream of net incomes must be falling in w as (i) the youngest unconstrained vintage P is falling in w and (ii) the marginal productivity of investment becomes smaller relative to the ratio of marginal productivity of investments. There exists a unique $w > 0$, which equates the discounted stream of net incomes to zero. The discounted net income is graphed as a function of w in [Figure 4].

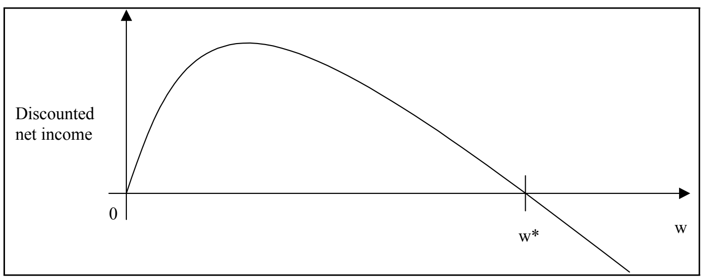

Figure 3: Discounted net incomes as a function of w.

(iii) If an endowment $\varepsilon > 0$ is given to the young upon birth, the discounted stream of net incomes is positive for $w = 0$. Then the degenerate outcome is not an equilibrium. ■

Proof of Proposition 2. (i) Begin by showing that a sequence of net income falling in vintage must be followed by a similar sequence, $[-i_{\tau-1} + \beta\pi_\tau(i_{\tau-1}, w)] \leq [-i_{\tau-2} + \beta\pi_{\tau-1}(i_{\tau-2}, w)] \Rightarrow [-i_\tau + \beta\pi_{\tau+1}(i_\tau, w)] \leq [-i_{\tau-1} + \beta\pi_\tau(i_{\tau-1}, w)]$. Suppose not, then $[-i_{\tau-1} + \beta\pi_\tau(i_{\tau-1}, w)] \leq [-i_{\tau-2} + \beta\pi_{\tau-1}(i_{\tau-2}, w)]$ and $[-i_\tau + \beta\pi_{\tau+1}(i_\tau, w)] > [-i_{\tau-1} + \beta\pi_\tau(i_{\tau-1}, w)]$. The last inequality can only be true if investment $i_{\tau-1}$ is constrained.

Case 1: First suppose investment $i_\tau = \hat{\imath}_\tau$ is also constrained. The relation implies,

$$\begin{aligned}&[-\hat{\imath}_{\tau-1} + \beta\pi_\tau(\hat{\imath}_{\tau-1}, w)] - [-\hat{\imath}_{\tau-2} + \beta\pi_{\tau-1}(\hat{\imath}_{\tau-2}, w)] \\ &< [-\hat{\imath}_\tau + \beta\pi_{\tau+1}(\hat{\imath}_\tau, w)] - [-\hat{\imath}_{\tau-1} + \beta\pi_\tau(\hat{\imath}_{\tau-1}, w)]\end{aligned}$$

From Lemma 1, $(\hat{\imath}_{\tau-1} - \hat{\imath}_{\tau-2}) < (\hat{\imath}_\tau - \hat{\imath}_{\tau-1})$, so the relation implies $\pi_\tau(\hat{\imath}_{\tau-1}, w) - \pi_{\tau-1}(\hat{\imath}_{\tau-2}, w) < \pi_{\tau+1}(\hat{\imath}_\tau, w) - \pi_\tau(\hat{\imath}_{\tau-1}, w)$. From the participation constraint among constrained agents it is known that $\pi_{\tau+1}(\hat{\imath}_\tau, w) - \pi_{\tau+1}(\hat{\imath}_{\tau-1}, w) = \pi_\tau(\hat{\imath}_{\tau-1}, w) - \pi_\tau(\hat{\imath}_{\tau-2}, w)$. Substituting in implies a contradiction given $\delta \in (0, 1)$.

Case 2: Now suppose $i_\tau = i_\tau^*$ is not constrained. This means $\pi_{\tau+1}(i_\tau^*, w) - \pi_{\tau+1}(\hat{\imath}_{\tau-1}, w) \leq \pi_\tau(\hat{\imath}_{\tau-1}, w) - \pi_\tau(\hat{\imath}_{\tau-2}, w)$ which again implies a contradiction.

The proof is completed by observing that (i) net income is negative in the frontier vintage from (14) (ii) positive in the terminal vintage, and (iii) asset values are positive for intermediate vintages. This means that a continued sequence of rising net incomes is followed by a continued sequence of falling net incomes.

(ii) Case 1: First suppose investment $i_\tau = \hat{\imath}_\tau$ is also constrained. The argument used in part (i) can be directly used for the proof.

Case 2: Now suppose $i_\tau = i_\tau^*$ is not constrained. The argument used in part (i) can be directly used for the proof.

(iii) When investment is unconstrained, net incomes must be falling in vintage since $\delta \in (0, 1)$. ■