Tax-Price Competition for Internationalized Public Goods[#]

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Abstract: According to globalization, certain kinds of public goods, which used to be working as the goods just in a national economy, are getting internationalized or released for foreigners as well. In addition, those goods often exhibit differentiated features for a certain degree. The goods as such are therefore quite different by nature from the *international collective or public goods* in the literature. We explore the financing scheme for such kind of *internationalized public goods*, by a simple model of two countries competing each other with taxing and pricing. Our main results are; 1) The more populated the foreign country, the better off the home country if the degree of product differentiation is sufficiently high, or the foreign is sufficiently populated. Otherwise it may not; 2) While it is distorted by the governmental incentives to utilize foreigners as financial resources, the welfare in a Nash equilibrium is always better than that gained by autarky for any relative size of population and for any degree of differentiation; 3) While, in standard theory of game or oligopoly, more limited strategies for players tend to provide better off in equilibrium as a paradoxical consequence, discriminatory-pricing equilibrium here is better off for sufficiently larger country rather than more limited, uniform pricing scheme.

[#] Very preliminary version.

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1.Introduction

It is a salient fact in the last decades that globalization has been proceeding with prodigious speed due to the end of the Cold War, technological progress of transportation, and a Copernican revolution in information technology such as internet. Not only information, but also commodities, humankind, or money move vastly among countries, wherein international boundaries as obstacle would be relatively ineffective these days.

As interdependent relationship among countries thickens, the role of internationally collective or public goods becomes really important. The global environment and military alliance for security are often cited as good examples. Indeed, international institutions work much more than before such that EU coordinates the interests of most European countries to pursue prosperity of Europe as a whole. They are integrating primal functions of nations, even money, into common property, that is the international public goods for them.

In the literature, there is a great stream of researches on international collective or public goods. The assumption on purity with the goods is undoubtedly useful there as for providing a polar case that illuminates the intrinsic characteristics of the goods by simplification. With pure public goods, voluntary contribution by nation-states such as for NATO has been explored as an application of the theory on private provision of public goods. The most celebrating result as such is presented by Warr (1983), which proves the neutrality of income distribution among countries in Nash equilibrium provision of public goods.

Except for some international organizations such as UN or the global environment, however, most international public goods in reality are localized more or less due to geography or ownership/control of the goods. Typical examples are international ports/airports and non-military satellites for communications or weather forecasts. According to globalization, moreover, certain kinds of public goods, which used to be working as the goods just in a national economy, are getting internationalized or released for foreigners as well. Recently, high-tech medical cares are becoming more and more to a kind of international public goods since patients of serious disease often fly to foreign hospitals those are developed for specific purposes subsidized much by the national governments. World Heritage Activity organized by UNESCO is another example, whereby more than 700 valuable sites are spread around so many countries.

Most of them need huge amount of fixed costs rather than marginal ones, which implies necessity of public provision domestically. That is, for some degree, one might say that non-rivalry characteristic works significantly in such goods once provided. Even with some capacity constraint or geographical limitation, therefore, the nation-state could afford to allow foreigners to access her goods.

In addition, one could observe that those goods often have differentiated features for a certain degree. That is, the goods provided by different countries are not perfect substitutes, by which households in any country might enjoy variety of them. Apparently, there exists some differentiation in examples such as medical care with some specialty or World Heritages. Likewise, airport services are differentiated in the departure time of flights, while artificial satellites are in their orbits.

The goods as such are therefore quite different by nature from the collective goods in the international setting, which are often used interchangeably as international public goods in the literature. Accordingly, let us call the former as *internationalized public goods* in the present paper.

On the other hand, due to excludable characteristics of such goods[1], the provider, the national government may charge some fee or specific tax for the access by domestic consumers as well as foreign consumers, in addition to income-tax revenues those should cover the costs mostly if it would be the case of *perfectly domestic* public goods.

In fact, those facilities mentioned above are financed mainly by each national government or some public organization, yet are often open for foreigners as well with certain amount of user fee in most cases. In this case, each national government has a principal responsibility including financial supports, while foreign citizens as well as the own nation often enjoy it by traveling there and paying some exhibition charges or taxes for staying at hotels around there[2].

Unlike the argument by Tiebout (1956) on local public goods in regional setting, however, it may not be plausible to assume free migration between countries. Hence, our purpose here is not for attempting to establish some optimality characteristics with localization in public goods, yet for highlighting some degree of localization of public goods in international setting to reflect the reality, which seem becoming important more and more recently.

Rather, we focus here on the interdependent behavior of nation states those are conjunct in the financial strategies for providing the public goods. That is, in the case we analyze here, there is a possibility to gain the budget by revenues of user/access fee, and thus the strategy of each

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[1] Many of international organizations also exhibit excludability, although they are often sited as examples of international "public goods."

[2] An extreme example is so-called "NHS tourism" in UK, where even illegal immigrants could have free medical treatments through NHS. It attracts many foreigners from countries with expensive services.

government is maximizing welfare of her own citizens by choosing optimal combination of income tax and price of charge. To put it differently, it is an extension of the optimal taxation theory to incorporate the international setting wherein the countries could depend on each other for achieving the optimal taxation.

Unlike the conventional model of fiscal competition where each government has only a single instrument such as capital tax (see, Wildasin (1991) among others), each government has two choice variables in our model, namely income-tax rate and price of charge. For the sake of simplicity, therefore, the quantity of goods provided is given exogenously as a fixed cost and the marginal costs for the provision are assumed out in the subsequent analysis. While the outcome of discriminatory pricing seems obvious here and thus will be discussed later, it is assumed instead that each government should set a uniform price for all beneficiaries or users in the main part of this paper. In other words, the price of charge is assumed the same for foreigners as its own citizens.

On the other hand, the present model differs from the bilateral oligopoly theory of international trade in two folds. First, the income tax on own citizens is available here as a tool for budgeting the provision of goods, which is substitutable to user charge. Second, as mentioned above, the price of charge is uniform for all users in the main body of the analysis.

An important aspect here is to examine the effect of relative size in population on Nash equilibrium. In the case of *pure* international public goods, some results are obtained as so-called *disproportionate burden sharing* or *exploitation of the great by small*, firstly by Olson and Zeckhauser (1966), followed by Sandler (1992) or Boadway and Hayashi (1999), among others. Especially, Boadway and Hayashi (1999) consider the effect of population explicitly and show that, *among contributing countries*, citizens in a larger country are worse off than those in a smaller country, in the Nash equilibrium of the voluntary contribution game between countries. Since international migration is much less than regional, it is a distinguished perspective of international issues on public goods.

While our analysis is focusing the strategy of tax-price combination rather than the quantity of contribution for public goods, one might also expect that more population in the other country should be always better for us because more people would access/visit our public goods with the payment of access charges, which helps our national finance consequently. It is not necessarily true, however, according to our main results as follows.

1) The more populated the foreign country, the better off the home country if the degree of product differentiation is sufficiently high, or the foreign is sufficiently populated. Otherwise,

yet, it may not.

2) Although it is distorted by the governmental incentives to utilize foreigners as financial resources, the welfare in the Nash equilibrium is always better than that gained by autarky for any relative size of population, and for any degree of differentiation or variety loving. Hence, there should be no incentive for each country to avoid accesses by the other nation.

3) While, in standard theory of game or oligopolistic competition, more limited strategies for players tend to provide better off in equilibrium as a paradoxical consequence, discriminatory pricing equilibrium here is better off for sufficiently larger country rather than more limited, uniform pricing scheme.

The present paper is organized as follows. The basic model is given in Section 2. Main results of our analysis as comparative statics are described in Section 3. Section 4 provides some qualifications and elaborations. Section 5 concludes the paper.

2. The Model

Suppose there are two countries, namely home (*h*) and foreign (*f*). Each of them provides a kind of public goods, which is not perfectly pure for international, namely, for the other nation, but is localized for some extent to the site situated or to its ownership and/or control. Furthermore, the goods provided in each country are differentiated for a certain degree. It is assumed that only a certain amount of fixed cost, q_h and q_f, respectively, is necessary for the provision of the goods at each country. That is, they would be almost pure public goods by nature in the territory, since congestion is assumed out through the present paper. The departure here from the standard public goods is that each government may limit the beneficiaries to people who pay the user/access fee, and moreover may open the goods for the other nation with the charge for the access. On the other hand, unlike the theory of international public/collective goods in the literature, in which the voluntary contribution or the income transfer between governments is often discussed, the other nation would contribute for financing the goods by paying the price of charge here. Let us focus on the scheme of uniform charge first whereby each government does not discriminate each other nation, while the case of discriminatory charge will be discussed in Section 4. Each charge is priced by the government and denoted by p_h and p_f, respectively. Likewise, the governments levy income tax as a rate, t_h and t_f, respectively, on the nation of own for financing the goods.

The countries might be adjacent or at least are close enough to each other in terms of technological distance that concerns here. In the case of international port, for example, they shall be close geographically for being interdependent on the public goods provided by the other country. In the case of communications satellite, say Comsat, on the other hand, geographical distance does not matter so much. Since one of our main concerns is the influence of relative country sizes, the population of each country is denoted by N_h and N_f, respectively.

Households of each country enjoy the consumption of composite good as well as public goods provided and priced by both governments, subject to budget constraint of each nation. Observing the behavior of households in both countries, each government tries to maximize the utility of the own nation by setting the income-tax rate as well as the access charge subject to the budget constraints for financing the fixed costs of the goods. Hence, the model can be interpreted as a two-stage game where the governments maximize own country's welfare given the choice of the other in the first stage. In the second stage, the households maximize utility given the income-tax rate as well as the prices of charges, which are determined as a Nash equilibrium in the first-stage game between the governments. The outcome is given as a Nash equilibrium of both governments' choices for the tax rates and the prices, while the resulting indirect utilities as well as the demands of households follow it. In this sense, one might view this model as an application of optimal tax theory to oligopolistic game between two countries.

Now that the standard backward induction applies to obtain the perfect equilibrium, the second stage of the game is described first. In the second stage, households of each country maximize their utility subject to their budget constraint as the standard consumer theory, given the income-tax rate and charges for access to the quasi-public goods of both countries, which are provided in the first stage as a Nash equilibrium.

The components of utility are the consumption of composite good a la Hicks, which is numerare here, and the number of access to both countries' goods. The number of access might be a derived demand of consumption of the public goods, yet it works as a surrogate measure of the consumption itself.

That is, we postulate utility of each nation as follows.

$$u_h = u\,(z_h, m_{hh}, m_{hf}) \qquad (1)$$

$$u_f = u\,(z_f, m_{fh}, m_{ff}) \qquad (2)$$

where, u_i is the household utility of country $i\,(i = h, f)$; z_i is the consumption of composite good at country *i*; m_{ij} is the number of access of household in *i* to the public goods provided by country $j\,(j = h, f)$.

For households, some access costs for consuming the quasi-public goods would be necessary other than charges levied by governments. In most cases such as medical care, natural/cultural heritage, or port services, for instance, travel costs are inevitable for the access, which might differ between the goods due to geographical and/or technological differential. Each budget constraint for households is therefore given as follows.

$$(1-t_h)y_h = z_h + (p_h+\tau_1)m_{hh} + (p_f+\tau_2)m_{hf} \quad (3)$$

$$(1-t_f)y_f = z_f + (p_h+\tau_2)m_{fh} + (p_f+\tau_1)m_{ff} \quad (4)$$

where, t_i is the income-tax rate of country $i\,(i=h,f)$; y_i is the income or per capita GDP of country *i*; p_i is the price of charge for country *i*'s public goods; τ_1 is the access cost for domestic goods; τ_2 is for foreign goods. It is assumed that $\tau_2 \geq \tau_1$ and they are exogenously given by geographical and/or technological condition.

In the first stage, each government tries to maximize the indirect utility, $V_h\,[V_f]$ i.e., maximized utility in (1) [(2)] by households subject to each budget constraint (3) [(4)], by choosing (t_h, p_h) $[(t_f, p_f)]$, given the choice of the other government in Nash manner and subject to fiscal constraint of each as follows.

$$q_h = N_h \cdot t_h \cdot y_h + p_h(N_h \cdot m_{hh} + N_f \cdot m_{fh}) \quad (5)$$

$$q_f = N_f \cdot t_f \cdot y_f + p_f(N_h \cdot m_{hf} + N_f \cdot m_{ff}) \quad (6)$$

The left hand side of each constraint is the fixed cost of the public goods. The first part in the right is the income-tax revenue and the second the revenue of access charge.

3. Analytical Results and Comparative Statics

Since the governments have *two* choice variables in the first stage, namely income tax and access charge, unlike a la fiscal-competition models where a single tax or the quantity of public goods provided/contributed is chosen as a governmental choice variable, we need to employ a specific type of utility function in order to explore the characteristics of an equilibrium. That is, quadratic-quasi-linear type of utility is assumed here.

While there is a long tradition of using quasi-linear type in public economics or the theory of public-utility pricing to avoid income effects, the quadratic form, often employed in industrial organization or new economic geography as well[3], allows us to introduce product differentiation between public goods provided by two countries.

That is, we postulate utility function of each nation as follows:

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[3] See Vives (1999) for the application to industrial organization theory, and Ottaviano et al. (2002) for economic geography, among others.

$$u_h = z_h + \alpha(m_{hh} + m_{hf}) - \frac{1}{2}\beta(m_{hh}^2 + m_{hf}^2) - \gamma m_{hh} m_{hf} \quad (7)$$

$$u_f = z_f + \alpha(m_{fh} + m_{ff}) - \frac{1}{2}\beta(m_{fh}^2 + m_{ff}^2) - \gamma m_{fh} m_{ff} \quad (8)$$

where, $\alpha, \beta,$ and γ are positive parameters. α represents the strength of pleasure in consuming the quasi-public goods and is assumed sufficiently large for strictly positive demands. As both public goods here are supposed to be differentiated or subject to love of variety for households, it is assumed $\beta > \gamma > 0$ as usual. The ratio, $\rho = \beta / \gamma \ (> 1)$, represents the magnitude/degree of product differentiation or love of variety.

For the sake of comparative statics, let us redefine several parameters for foreign to the ratio to those for home as follows.

$$N_f = \theta_1 \cdot N_h, \quad (\theta_1 > 0) \quad (9)$$

$$q_f = \theta_2 \cdot q_h, \quad (\theta_2 > 0) \quad (10)$$

$$y_f = \theta_3 \cdot y_h, \quad (\theta_3 > 0) \quad (11)$$

$$\tau_2 = \theta_4 \cdot \tau_1, \quad (\theta_4 \geq 1) \quad (12)$$

The Nash equilibrium strategy of both governments, $\{(t_h^*, p_h^*), (t_f^*, p_f^*)\}$, and thus the other variables, $\{(V_h^*, z_h^*, m_{hh}^*, m_{hf}^*), (V_f^*, z_f^*, m_{fh}^*, m_{ff}^*)\}$, derived by the strategy in equilibrium, are obtained analytically by solving the two-stage game, yet they are too tedious to spread here (see Appendix for the results of equilibrium configuration). We accordingly focus on the comparative statics of the equilibrium. As countries are symmetric at basic parts, the task is pursued just for home variables in the subsequent analysis. The composite goods here are barely working as residual against main variables such as the demands for public goods, due to the characteristics of quasi-linear utility, and thus let us exempt about it. While most results are proved just by calculation, some details are given in Appendix.

Since our main concern is on the effects of relative size of the countries, let us first examine the cases with the marginal increase in relative population of foreign, namely θ_1.

[Effects of population size]

$$\frac{\partial t_h^*}{\partial \theta_1} < 0 \quad (13)$$

Inequality (13) shows that income-tax rate at home decreases according to the increase in population of the other country, which implies that the smaller country could depend more on the larger through the access charges paid by the other nation.

$$\frac{\partial p_h^*}{\partial \theta_1} < 0, \text{ if } \rho < g(\theta_1)$$
$$\frac{\partial p_h^*}{\partial \theta_1} = 0, \text{ if } \rho = g(\theta_1) \qquad (14)$$
$$\frac{\partial p_h^*}{\partial \theta_1} > 0, \text{ if } \rho > g(\theta_1)$$

where,

$$g(\theta_1) = \frac{1}{(\theta_1 + 2)^2}(\theta_1^2 - 1 + \sqrt{2\theta_1^4 + 4\theta_1^3 + 2\theta_1^2 + 1})$$

Since $\rho > 1$ by assumption, $\partial p_h^* / \partial \theta_1 > 0$ for $\theta_1 \leq 3$ ($\because g(3) = 1$). That is, if the other country, foreign, is less than treble of home, the home price of access charge in Nash equilibrium increases for larger population of foreign. Or, if the degree of product differentiation or love of variety is sufficiently large, i.e., $\rho \geq 1 + \sqrt{2} = \lim_{\theta_1 \to \infty} g(\theta_1)$, it also holds. If the other country is more than treble and the degree of differentiation is not so large, however, this comparative static characteristic may be opposite according to inequality (14). It means, when the other is sufficiently large relative to the degree of differentiation or love of variety, the equilibrium price at home decreases if the other becomes larger. See Figure 1 for this relationship.

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Figure 1

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It might be interesting that while the increase in population of the other country always decreases income tax rate at home as in (13), its influence on the price has an "inversion layer" as above. On the other hand, it is readily to verify that the equilibrium price ratio, p_h^* / p_f^*, always increases when the foreign becomes larger, i.e.,

$$\frac{\partial (p_h^* / p_f^*)}{\partial \theta_1} > 0 \qquad (15)$$

That is, the relative price of some country always increases if the other country becomes larger, while the equilibrium price itself may move to both directions. Those results can be summarized as follows.

Lemma 1. When the foreign country is more populated, the home tends to depend on the revenue of access charge rather than income tax. That is, her income-tax rate always decreases and the relative price of home goods always increases, while her equilibrium price itself might decrease if the other country is more than treble and the degree of differentiation is not sufficiently large.

Following this result, we find

$$\frac{\partial m_{hh}^*}{\partial \theta_1} < 0 \qquad (16)$$

$$\frac{\partial m_{hf}^*}{\partial \theta_1} > 0 \qquad (17)$$

Since the relative price of home goods becomes expensive when the other country becomes larger given as (15), the demand for home goods always escape to foreign goods.

The effect on the welfare is summarized as follows.

$$\frac{\partial V_h^*}{\partial \theta_1} > 0, \text{ for } \forall \theta_1 \text{ if } \rho \geq 1+\sqrt{2} \qquad (18)$$

Or,

$$\frac{\partial V_h^*}{\partial \theta_1} > 0, \text{ for } \forall \rho > 1 \text{ if } \theta_1 \geq 0.0756 \qquad (19)$$

That is, the larger foreign country is always welcome, provided that the degree of variety loving is sufficiently large, or the foreign is sufficiently large. Yet, our result shows that the sign might be negative if both sufficient conditions are not satisfied. That is, if households do not mind about the variety or the provided goods in both countries are not so much differentiated, and the foreign is sufficiently small compared with home, the marginal increase of foreign population would hurt home country's welfare. This is a crucial difference of our case with *quasi-public* goods from one with *purely collective or public* goods analyzed by Boadway and Hayashi (1999). They consider the effect of population explicitly and show that, *among contributing countries*, citizens in a larger country are worse off than those in a smaller country, in the Nash equilibrium of the voluntary contribution game between countries, which implies $\partial V_h^* / \partial \theta_1 > 0$ here.

While our analysis is focusing the strategy of tax-price combination rather than the quantity of contribution for public goods, one might also expect that more population in the other country should be always better for us because more people would access/visit our public goods with the payment of access charges, which helps our national finance. It is not necessarily true, however, as seen above. We shall restate this result as follows.

Proposition 1.

The more populated the foreign country, the better off the home country if the degree of product differentiation is sufficiently high (such as the condition in (18)), or the foreign is sufficiently populated (such as in (19)). Otherwise it may not.

Second, the remaining results of comparative statics, namely effects of increases in access costs to public goods, are as follows.

[Effects of access costs]

$$\frac{\partial t_h^*}{\partial \tau_1} > 0 \qquad (20)$$

$$\frac{\partial t_h^*}{\partial \theta_4} > 0 \qquad (21)$$

The governments would depend heavily on income tax when either access cost other than the charges becomes greater, as shown by inequalities (20) and (21).

$$\frac{\partial p_h^*}{\partial \tau_1} < 0 \qquad (22)$$

$$\frac{\partial p_h^*}{\partial \theta_4} < 0 \qquad (23)$$

By contrast, if the access costs are greater, the equilibrium price always decreases in order to attract demands both from domestic and from foreign.

Correspondingly, the demands for public goods have characteristics as follows.

$$\frac{\partial m_{hh}}{\partial \tau_1} > 0, \text{ for } \forall \theta_1 \text{ if } \theta_4 \geq \frac{2\rho^2 - 1}{\rho} \qquad (24)$$

$$\frac{\partial m_{hh}}{\partial \tau_1} < 0, \text{ for } \forall \theta_1 \text{ if } \theta_4 \leq \frac{2\rho + 1}{\rho + 2} \qquad (25)$$

This result is summarized as follows.

Lemma 2.

If the relative access cost to the other country's goods, θ_4, is sufficiently high compared with the degree of variety loving(such as the condition in (24)), the proportional increase in access costs always multiply the demands for home goods no matter what the relative country size. On the contrary, if the relative cost to foreign goods is sufficiently small compared with the degree of variety loving(such as in (25)), the proportional increase in access costs to both goods always reduces home good demand. For intermediate region for θ_4 against the degree of variety loving, i.e., $(2\rho+1)/(\rho+2) < \theta_4 < (2\rho^2-1)/\rho$, the sign is ambiguous as it also depends on θ_1.

See Figure 2 about this result.

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Figure 2

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$$\frac{\partial m_{hh}}{\partial \theta_4} > 0 \qquad (26)$$

It seems obvious that home good demand always increase when the relative access cost to the other country's increases, even in equilibrium.

The characteristics of demand for foreign goods are much simpler and plausible as follows.

$$\frac{\partial m_{hf}}{\partial \tau_1} < 0 \qquad (27)$$

$$\frac{\partial m_{hf}}{\partial \theta_4} < 0 \qquad (28)$$

The following results for access costs seem obvious as technological access/transport costs work just as burden for the society here.

$$\frac{\partial V_h}{\partial \tau_1} < 0 \qquad (29)$$

$$\frac{\partial V_h}{\partial \theta_4} < 0 \qquad (30)$$

4. Qualifications and Elaborations

(1) Social optimum

It is not difficult to construct the cooperative situation of both countries in determining the tax rate as well as prices of charge, given the scales of both public goods, i.e., fixed costs. In this case, we might postulate a Benthamite social welfare function as follows.

$$W = N_h \cdot V_h + N_f \cdot V_f \qquad (31)$$

where, $V_i\ (i = h, f)$ is given by the behavior of households, namely the second stage of the game as before. In the first stage, however, both of the two countries together try to maximize the Benthamite welfare, W, defined by (31) subject to the common fiscal constraint as follows.

$$q_h + q_f = N_h \cdot t_h \cdot y_h + N_f \cdot t_f \cdot y_f + p_h(N_h \cdot m_{hh} + N_f \cdot m_{fh}) + p_f(N_h \cdot m_{hf} + N_f \cdot m_{ff}) \quad (32)$$

Their choice variables are (t_h, t_f, p_h, p_f), and we find the optimal solution $(t_h^o, t_f^o, p_h^o, p_f^o)$ as follows.

$t_h^o = \forall t_h \geq 0$ and $t_f^o = \forall t_f \geq 0$, such that $q_h + q_f = N_h \cdot t_h \cdot y_h + N_f \cdot t_f \cdot y_f$ and $p_h^o = o, p_f^o = 0$. (33)

As our formulation of social optimum is subject to budget constraint for public, it might be the second-best solution. Since in the setting there are no marginal costs for public side, preferences are symmetric, and government could utilize income taxation, the solution exhibits standard type of the first best, namely marginal cost pricing wherein the prices of access are zero while the fixed costs are covered by income tax somehow.

A simple example attaining the first best is that each government finances own public goods solely with income tax levied on each nation (and with no access charge), although households of the other country would come and enjoy free ride on it. Alternatively, some side payments or income redistribution satisfying (33) might be allowed. In the non-cooperative situation, however, governments tend to avoid free ride of foreigners and try to utilize them as sources of revenues, which would lead to the overcharges on access to the public goods and thus the distortion on each economy. Although our setting assumes out the marginal costs for public side, similar results are readily obtained in the case with the costs.

Note that, while the technological access costs, τ_1 or τ_2, decreases drastically in modern history, the tendency of the distortion might be getting greater, as shown in (20)~(23).

(2) Autarky

Each country could refuse applications from foreigners or never validate agreements for reciprocal favors over some public goods to be autarky. In this setting, however, we find the proposition as follows.

Proposition 2.

The welfare in the Nash equilibrium is always better than that gained by autarky for any relative sizes of population, and for any degree of differentiation or variety loving. Furthermore, if the other country does not permit the access of the nation, a country can be better off by accepting the foreigners with some charges. Hence, in the non-cooperative situation, there should be no incentive for each country to avoid accesses by the other nation.

(3) Discriminatory pricing

While in the main body of our analysis the uniform pricing has been employed, one might be interested in the case where governments may distinguish home residents from foreigners. In other words, let us analyze the situation with discriminatory pricing here.

As anticipated by results in the social optimum or by our setting in the cost structure of public goods, free access should be chosen for own residents by each government (namely, marginal cost pricing for whom income tax could be levied) while positive charge for foreign due to the incentive for each government to utilize foreign users as financial source, unlike the social optimum. The main results for this case are summarized as follows.

Proposition 3.

(i) The welfare in the uniform pricing is better than that gained by discriminatory case if $\theta_1 \geq (\sqrt{73}-3)/8$, for any $\rho > 1$. (ii) The welfare in the uniform pricing is better than that gained by discriminatory case if $1/2 < \theta_1 < (\sqrt{73}-3)/8$, for any $\rho \in (1, \hat{\rho}]$, where $\hat{\rho} = (2\theta_1^2 + \theta_1 + 2\sqrt{\theta_1(4\theta_1^2 + 8\theta_1 + 3)(\theta_1 - 1)^2} / (-8\theta_1^3 - 10\theta_1^2 + 5\theta_1 + 4)$ (iii) The welfare in the uniform pricing is worse than that gained by discriminatory case if $1/2 < \theta_1 < (\sqrt{73}-3)/8$, for any $\rho > \hat{\rho}$. (iv) The welfare in the uniform pricing is worse than that gained by discriminatory case if $\theta_1 \leq 1/2$, for any $\rho > 1$.

It might be interesting that uniform pricing configuration is not always better off than discriminatory pricing such as the cases (iii), (iv). In the literature of game theory, paradoxically enough, Nash equilibria with *limited* strategies/actions tend to be better off for players (see a seminal work of Schelling (1960). See Anderson et al. (1995) for a typical example of international trade and a recent example in oligopoly theory by Kaneda and Matsui (2003), among others). Our result shows, however, that unrestrained pricing policy is advantageous for larger countries if the population is more than double or if the population is a bit less than twofold yet with strong love of variety.

Given the other country's strategy on the charge including pricing policy, namely uniform or discriminatory one here, a country has a strong incentive to employ discriminatory pricing since it provides a fine tuning scheme for unilateral pricing. Unlike the standard story where both players could be better off with some binding contract for narrowing the strategy space of each (namely, a ban on discrimination in pricing against foreigners) even though they are not globally cooperative, the larger country here might not have any incentive to agree for limiting their pricing policy to the uniform. Rather, as seen above, the larger country might be more likely to agree with the accord that forbids both countries charging for accesses to public goods, depending on the allocation of tax burden for providing the goods.

5. Concluding Remarks

In the present paper, we construct a simple model of tax-price competition over the financial scheme for public goods, which are primarily pure domestically and thus provided by each nation-state, yet foreigners are also allowed by paying the price of charge. The importance of such kinds of public goods increases rapidly according to the progress of globalization, while the characteristics of the goods are quite different from those of the conventional international public goods in the literature. By introducing the differentiated features of such kind goods, we investigate mainly the effects of population difference between countries in terms of the degree of product differentiation as well as several kinds of pricing schemes.

While our results might be interesting for being different from those with conventional goods, one could extend the analysis in several ways. First, the quantity or capacity is given exogenously as the fixed cost. Parallel to the contribution game in the literature of international public goods, it should be better to incorporate the game over the capacity itself with our tax-price competition.

Second, although we could obtain the results analytically (i.e., without any numerical simulation), it seems apparent that quadratic quasi-linear utility employed here brings some limitation into the analysis. In this regard, the generality of our results should be examined further with other types of utility functions.

Third, as usual, our analysis has a limitation as a partial equilibrium analysis. While it might be justified traditionally that the expenditure for the goods analyzed seems relatively small in total consumption, general equilibrium framework would be better for corresponding with our shrinking world wherein the opportunity for accessing the public goods provided by other countries is getting immense.

Appendix

1. Nash equilibrium configuration

With uniform pricing, the Nash equilibrium price and tax for home country are obtained as follows.

$$p_h^* = \frac{\theta_1(2\beta + \theta_1 \cdot \beta + \gamma)(\alpha\beta - \alpha\gamma - \theta_4 \cdot \tau_1 \cdot \beta + \tau_1 \cdot \gamma)}{\beta^2(2\theta_1^2 + 5\theta_1 + 2) - \theta_1 \cdot \gamma^2} \tag{A.1}$$

$$t_h^* = \frac{q_h}{y_h \cdot N_h} + D_h^* \tag{A.2}$$

where, D_h^* is the distorted component of income-tax rate due to the oligopolistic setting as follows.

$$\begin{aligned} D_h^* = & -\theta_1(2\beta + \theta_1 \cdot \beta + \gamma)(\alpha\beta - \alpha\gamma - \theta_4 \cdot \tau_1 \cdot \beta + \tau_1 \cdot \gamma)[\{(\theta_1^3 \\ & + 4\theta_1^2 + 5\theta_1 + 2)\beta^3 - (\theta_1^3 + 3\theta_1^2 + 3\theta_1 + 1)\beta^2\gamma - (\theta_1^2 \\ & + 2\theta_1 + 1)\beta\gamma^2\}\alpha - (\theta_4\theta_1^3 + 2\theta_4\theta_1^2 + 2\theta_1^2 + 5\theta_1 + 2)\tau_1\beta^3 \\ & + (\theta_1^3 + \theta_4\theta_1^2 + 2\theta_1^2 + 3\theta_4\theta_1 + \theta_4)\tau_1\beta^2\gamma + (\theta_1^2 - \theta_4\theta_1 \\ & + 3\theta_1 + 1)\tau_1\beta\gamma^2 - (\theta_4 - 1)\theta_1\tau_1\gamma^3]/(\beta - \gamma)(\beta \\ & + \gamma)\{\beta^2(2\theta_1^2 + 5\theta_1 + 2) - \theta_1 \cdot \gamma^2\}^2 \cdot y_h \end{aligned} \tag{A.3}$$

In addition, the equilibrium price ratio is found as follows.

$$\frac{p_h^*}{p_f^*} = \frac{(\theta_1\beta + 2\beta + \gamma)\theta_1}{2\theta_1\beta + \beta + \theta_1\gamma} \tag{A.4}$$

Accordingly, the equilibrium demands for public goods are obtained as follows.

$$\begin{aligned} m_{hh}^* = & [\{(\theta_1^2 + 3\theta_1 + 2)\beta^2 - (\theta_1^2 + 2\theta_1 + 1)\beta\gamma - (\theta_1 + 1)\gamma^2\}\alpha\beta \\ & + \{(\theta_4 - 2)\theta_1^2 + (2\theta_4 - 5)\theta_1 - 2\}\tau_1\beta^3 + \{(2\theta_4 - 1)\theta_1^2 \\ & + (4\theta_4 - 2)\theta_1 + \theta_4\}\tau_1\beta^2\gamma + \{(-\theta_4 + 2)\theta_1 + 1\}\tau_1\beta\gamma^2 \\ & + (-\theta_4 + 1)\tau_1\theta_1\gamma^3]/(\beta - \gamma)(\beta + \gamma)\{\beta^2(2\theta_1^2 + 5\theta_1 + 2) - \theta_1 \cdot \gamma^2\} \end{aligned} \tag{A.5}$$

$$m_{hf}^* = \frac{\beta(\theta_1 + 1)(2\theta_1\beta + \beta + \theta_1\gamma)(\alpha\beta - \alpha\gamma - \theta_4\tau_1\beta + \tau_1\gamma)}{(\beta - \gamma)(\beta + \gamma)\{\beta^2(2\theta_1^2 + 5\theta_1 + 2) - \theta_1 \cdot \gamma^2\}} \tag{A.6}$$

The (per capita) welfare of home country in equilibrium is given as follows.

$$V_h^* = y_h - \frac{q_h}{N_h} + S_h^* \tag{A.7}$$

That, gross income net of *potential* share of the burden for public goods plus “the *gross* surplus” gained by the consumption of public goods. In other words, $S_h^* - q_h/N_h$ means the *net* surplus by that, where S_h^* is obtained as follows.

$$
\begin{aligned}
Sh := \frac{1}{2}(& -12\,\theta1^3\,\gamma^2\,\tau1^2\,\beta^3\,\theta4 + 8\,\theta1^2\,\gamma^3\,\alpha^2\,\beta^2 - 10\,\theta1^4\,\gamma^2\,\tau1\,\beta^3\,\alpha - 10\,\theta1^2\,\gamma^4\,\alpha\,\beta\,\tau1 \\
& - 40\,\theta1^3\,\tau1\,\beta^5\,\alpha - 4\,\theta1^2\,\gamma^4\,\theta4\,\tau1^2\,\beta - 26\,\theta1^2\,\gamma^2\,\tau1^2\,\beta^3 + 4\,\theta1\,\gamma^3\,\beta^2\,\theta4\,\tau1^2 \\
& - 8\,\gamma^2\,\theta4\,\tau1^2\,\beta^3\,\theta1^4 + \gamma^4\,\theta4^2\,\tau1^2\,\theta1^2\,\beta - 26\,\theta1^4\,\alpha\,\beta^5\,\theta4\,\tau1 + 36\,\theta1^3\,\gamma\,\alpha\,\beta^4\,\theta4\,\tau1 \\
& - 40\,\theta1\,\tau1\,\beta^5\,\alpha - 48\,\theta1^3\,\gamma\,\tau1^2\,\beta^4\,\theta4 + 6\,\gamma\,\beta^4\,\theta1^3\,\theta4^2\,\tau1^2 - 4\,\beta^3\,\tau1\,\theta1^5\,\alpha\,\gamma^2 \\
& + 2\,\gamma^5\,\theta4\,\tau1\,\theta1^2\,\alpha - 48\,\theta1^3\,\alpha\,\beta^5\,\theta4\,\tau1 + 4\,\theta1^2\,\gamma^2\,\beta^3\,\theta4\,\tau1\,\alpha - 2\,\theta1\,\gamma\,\theta4^2\,\tau1^2\,\beta^4 \\
& + 4\,\gamma\,\theta4^2\,\tau1^2\,\beta^4\,\theta1^4 - 8\,\tau1\,\beta^5\,\theta1^4\,\alpha + 2\,\beta\,\theta1^3\,\gamma^4\,\tau1^2 - 12\,\gamma^3\,\beta^2\,\theta1^2\,\theta4\,\tau1\,\alpha - 2\,\gamma\,\alpha^2\,\beta^4 \\
& + 17\,\theta1^2\,\theta4^2\,\tau1^2\,\beta^5 - 2\,\theta1\,\gamma^3\,\tau1^2\,\beta^2 + 2\,\theta1^2\,\gamma^5\,\tau1^2 + 9\,\theta1^4\,\gamma^2\,\tau1^2\,\beta^3 + 6\,\gamma^2\,\tau1\,\beta^3\,\alpha \\
& + 20\,\beta^5\,\tau1^2\,\theta1 + 48\,\theta1^3\,\gamma\,\tau1\,\beta^4\,\alpha - 2\,\alpha\,\beta^5\,\theta4\,\tau1 + 2\,\gamma\,\tau1\,\beta^4\,\alpha + 17\,\theta1^4\,\alpha^2\,\beta^5 \\
& - 18\,\theta1\,\gamma^2\,\tau1^2\,\beta^3 - 34\,\theta1^2\,\gamma\,\tau1^2\,\beta^4\,\theta4 + 2\,\beta^5\,\theta1^5\,\theta4^2\,\tau1^2 + \beta^5\,\theta4^2\,\tau1^2 + 2\,\beta^3\,\tau1^2\,\theta1^5\,\gamma^2 \\
& - 4\,\gamma^2\,\beta^3\,\theta1^2\,\theta4^2\,\tau1^2 + 32\,\theta1\,\gamma^2\,\tau1\,\beta^3\,\alpha + 13\,\theta1^4\,\theta4^2\,\tau1^2\,\beta^5 + 8\,\gamma^2\,\theta4\,\tau1\,\beta^3\,\theta1^4\,\alpha + 4\,\beta^5\,\tau1^2 \\
& + 7\,\theta1^2\,\gamma^4\,\beta\,\tau1^2 + 24\,\theta1^3\,\theta4^2\,\tau1^2\,\beta^5 + 6\,\theta1^3\,\gamma^3\,\tau1^2\,\beta^2 + 26\,\theta1^4\,\gamma\,\tau1\,\beta^4\,\alpha - 8\,\gamma^3\,\alpha\,\beta^2\,\theta1^4\,\tau1 \\
& - 2\,\theta1\,\gamma^2\,\beta^3\,\theta4^2\,\tau1^2 + 12\,\theta1^3\,\gamma^2\,\tau1\,\beta^3\,\alpha - 12\,\theta1\,\gamma\,\tau1^2\,\beta^4\,\theta4 - 16\,\theta1^3\,\gamma^3\,\alpha\,\beta^2\,\tau1 \\
& - 2\,\theta1^2\,\gamma^5\,\tau1\,\alpha + 18\,\theta1^4\,\gamma\,\theta4\,\tau1\,\beta^4\,\alpha - 2\,\gamma\,\tau1^2\,\beta^4\,\theta4 - 26\,\theta1^4\,\gamma\,\tau1^2\,\beta^4\,\theta4 \\
& + 2\,\theta1^2\,\gamma^4\,\alpha\,\beta\,\theta4\,\tau1 + 4\,\gamma^3\,\beta^2\,\theta1^3\,\theta4\,\tau1^2 + 44\,\theta1^3\,\alpha^2\,\beta^5 - 2\,\gamma^2\,\beta^3\,\theta1^3\,\theta4^2\,\tau1^2 \\
& + 12\,\theta1\,\gamma\,\tau1\,\beta^4\,\alpha - 4\,\theta1\,\gamma^4\,\beta\,\tau1\,\alpha - 12\,\theta1\,\alpha\,\beta^5\,\theta4\,\tau1 + 4\,\theta1^2\,\gamma^2\,\tau1^2\,\beta^3\,\theta4 \\
& - 2\,\theta1^2\,\gamma\,\theta4^2\,\tau1^2\,\beta^4 - 4\,\alpha\,\beta\,\gamma^4\,\theta1^3\,\tau1 + 2\,\theta1\,\gamma^4\,\beta\,\tau1^2 + 4\,\beta^4\,\tau1\,\theta1^5\,\alpha\,\gamma + 2\,\alpha^2\,\beta\,\gamma^4\,\theta1^3 \\
& - 8\,\tau1\,\beta^5\,\alpha + 4\,\gamma^2\,\beta^3\,\theta1\,\theta4\,\tau1^2 - 4\,\beta^5\,\theta4\,\tau1\,\alpha\,\theta1^5 + 48\,\tau1\,\beta^3\,\theta1^2\,\gamma^2\,\alpha - 2\,\gamma^5\,\theta4\,\tau1^2\,\theta1^2 \\
& - 2\,\theta1^2\,\gamma^3\,\tau1^2\,\beta^2 + 26\,\theta1\,\alpha^2\,\beta^5 + 38\,\theta1^2\,\gamma\,\alpha\,\beta^4\,\theta4\,\tau1 + \theta1^4\,\gamma^2\,\alpha^2\,\beta^3 + 4\,\theta1^2\,\gamma^4\,\alpha^2\,\beta \\
& - 14\,\theta1\,\gamma\,\alpha^2\,\beta^4 + 16\,\theta1\,\gamma\,\theta4\,\tau1\,\beta^4\,\alpha + 4\,\gamma^3\,\alpha^2\,\beta^2\,\theta1^4 - 4\,\beta^4\,\alpha^2\,\theta1^5\,\gamma - 4\,\theta1\,\gamma^3\,\beta^2\,\theta4\,\tau1\,\alpha \\
& + 2\,\beta^5\,\alpha^2\,\theta1^5 - 4\,\beta^4\,\tau1^2\,\theta1^5\,\theta4\,\gamma + 2\,\theta1^2\,\gamma^3\,\beta^2\,\theta4^2\,\tau1^2 - 42\,\theta1^3\,\gamma\,\alpha^2\,\beta^4 + 4\,\theta1^4\,\beta^2\,\gamma^3\,\tau1^2 \\
& - 34\,\theta1^2\,\alpha\,\beta^5\,\theta4\,\tau1 + 8\,\theta1^2\,\gamma^3\,\beta^2\,\theta4\,\tau1^2 + 16\,\theta1^3\,\gamma^2\,\alpha\,\beta^3\,\theta4\,\tau1 - 4\,\gamma^3\,\beta^2\,\theta1^3\,\theta4\,\tau1\,\alpha \\
& + 50\,\theta1^2\,\alpha^2\,\beta^5 + 4\,\theta1^4\,\tau1^2\,\beta^5 - 36\,\theta1^2\,\gamma\,\alpha^2\,\beta^4 + 2\,\gamma\,\theta4\,\tau1\,\beta^4\,\alpha - 66\,\theta1^2\,\tau1\,\beta^5\,\alpha + 5\,\alpha^2\,\beta^5 \\
& + 34\,\theta1^2\,\gamma\,\tau1\,\beta^4\,\alpha - 4\,\theta1^2\,\gamma^3\,\alpha\,\beta^2\,\tau1 - 3\,\gamma^2\,\tau1^2\,\beta^3 + 2\,\theta1\,\gamma^4\,\alpha^2\,\beta + 4\,\beta^4\,\theta1^5\,\gamma\,\theta4\,\tau1\,\alpha \\
& + 10\,\theta1^3\,\gamma^3\,\alpha^2\,\beta^2 + 2\,\theta1\,\gamma^3\,\alpha^2\,\beta^2 - 26\,\theta1^2\,\gamma^2\,\alpha^2\,\beta^3 - 22\,\theta1^4\,\gamma\,\alpha^2\,\beta^4 + 2\,\beta^3\,\alpha^2\,\theta1^5\,\gamma^2 \\
& - 14\,\theta1^3\,\gamma^2\,\alpha^2\,\beta^3 + 6\,\theta1\,\beta^5\,\theta4^2\,\tau1^2 - 16\,\theta1\,\gamma^2\,\alpha^2\,\beta^3 + 20\,\theta1^3\,\tau1^2\,\beta^5 + 33\,\theta1^2\,\tau1^2\,\beta^5 \\
& - 3\,\gamma^2\,\alpha^2\,\beta^3) \Big/ ((\beta-\gamma)\,(\beta+\gamma)\,(2\,\beta^2 + 5\,\beta^2\,\theta1 + 2\,\beta^2\,\theta1^2 - \gamma^2\,\theta1)^2)
\end{aligned}
\tag{A.8}
$$

2. Comparative statics

Most proofs of comparative static results are given just by calculation based on the equilibrium configuration described above. We provide here some sketches of them.

(13): It is readily to prove that $\partial t_h^* / \partial \theta_1$ is always negative for any parameters satisfying the assumptions.

(14): It turns out that the sign of $\partial p_h^* / \partial \theta_1$ totally depends on the sign of the equation as,

$$f(\rho|\theta_1) = (\theta_1^2 + 4\theta_1 + 4)\rho^2 - 2(\theta_1^2 - 1)\rho - \theta_1^2 \tag{A.9}$$

Since the derivative of *f* with respect to ρ is always positive for $\rho > 1$, it is enough to examine the larger solution of *f=0* with respect to ρ, namely $g(\theta_1)$ in (14), by which we could judge the sign of *f* with respect to the combination of (ρ, θ_1).

(15): By calculation with (A.4).

(16): By calculation with (A.5).

(17): By calculation with (A.6).

(18): It turns out that the sign of $\partial V_h^* / \partial \theta_1$ totally depends on the sign of the equation as,

$$\begin{aligned} f(\theta_1, \rho) = {} & 2\rho^4\theta_1^5 + (13\rho^2 - 3)\rho^2\theta_1^4 + (38\rho^3 + 13\rho^2 - 8\rho - 4)\rho\theta_1^3 \\ & + (40\rho^4 + 21\rho^3 - \rho^2 - 3\rho - 1)\theta_1^2 + (14\rho^4 + 3\rho^3 + \rho^2 \\ & + 3\rho + 1)\theta_1 + (\rho^2 - 2\rho - 1)\rho^2 \end{aligned} \tag{A.10}$$

It is apparent that all terms in the left hand side except for the last are positive for any $\rho > 1$, and $\theta_1 > 0$. Hence, the sufficient condition in (18) is derived straightforwardly. On the other hand, even if the degree of variety loving does not satisfy that condition, sufficiently large θ_1 would make *f* positive. Since $\partial f / \partial \theta_1 > 0$, for any $\rho > 1$ and $\theta_1 > 0$, it is readily to verify the other sufficient condition described in (19), while the minimum value of θ_1 is an approximation.

(20), (21): By calculation with (A.2).

(22), (23): By calculation with (A.1).

(24), (25): Similarly to (18), we can derive the sufficient conditions for positive and negative cases. In the intermediate area, however, the sign is ambiguous.

(26): By calculation with (A.5).

(27), (28): By calculation with (A.6).

(29), (30): By calculation with (A.7) and (A.8).

3. Autarky

In this case, the maximum welfare of home country is given as follows.

$$V_h^a = \frac{\alpha^2 - 2\tau_1\alpha + \tau_1^2}{2\beta} \tag{A.11}$$

By taking the difference between (A.7) and (A.11), it is readily to verify the first half of Proposition 2.

Since each country does not choose prohibitive prices of charge even in the game with discriminatory pricing below, the second half of the proposition is apparent.

4. Discriminatory pricing

The "gross surplus" of this case is obtained as follows.

$$\begin{aligned} Sh := \frac{1}{8}(& 5\,\alpha^2\,\beta^2 - 8\,\tau1\,\alpha\,\beta^2 + 4\,\tau1^2\,\beta^2 + \theta4^2\,\tau1^2\,\beta^2 - 2\,\alpha\,\beta^2\,\theta4\,\tau1 + 2\,\beta^2\,\theta1\,\theta4^2\,\tau1^2 \\ & + 2\,\beta^2\,\theta1\,\alpha^2 - 4\,\beta^2\,\theta1\,\alpha\,\theta4\,\tau1 - 4\,\beta\,\theta1\,\gamma\,\tau1^2\,\theta4 - 2\,\alpha^2\,\beta\,\gamma - 4\,\beta\,\theta1\,\alpha^2\,\gamma + 4\,\beta\,\theta1\,\gamma\,\alpha\,\theta4\,\tau1 \\ & + 2\,\gamma\,\alpha\,\theta4\,\tau1\,\beta - 2\,\gamma\,\tau1^2\,\theta4\,\beta + 2\,\alpha\,\beta\,\gamma\,\tau1 + 4\,\beta\,\theta1\,\alpha\,\gamma\,\tau1 - 3\,\gamma^2\,\alpha^2 - 4\,\theta1\,\gamma^2\,\alpha\,\tau1 - 3\,\gamma^2\,\tau1^2 \\ & + 6\,\gamma^2\,\alpha\,\tau1 + 2\,\theta1\,\gamma^2\,\tau1^2 + 2\,\gamma^2\,\theta1\,\alpha^2) / (\beta\,(\beta - \gamma)\,(\beta + \gamma)) \end{aligned} \tag{A.12}$$

By taking the difference between (A.8) and (A.12), we can compare those welfare levels. It turns out that the sign of the difference depends on the following.

$$f(\rho|\theta_1) = (8\theta_1^3 + 10\theta_1^2 - 5\theta_1 - 4)\rho^2 + 2(2\theta_1 + 1)\theta_1\rho + (3 - 2\theta_1)\theta_1 \quad \text{(A.13)}$$

The solutions of f =0 can be depicted as Figure 3. Hence, Proposition 3 is readily seen in the figure.

============

Figure 3

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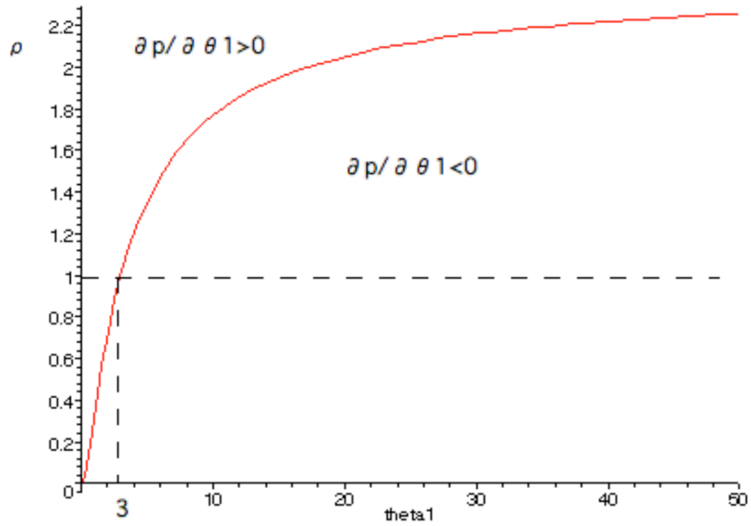

Figure 1. The region of inequality (14)

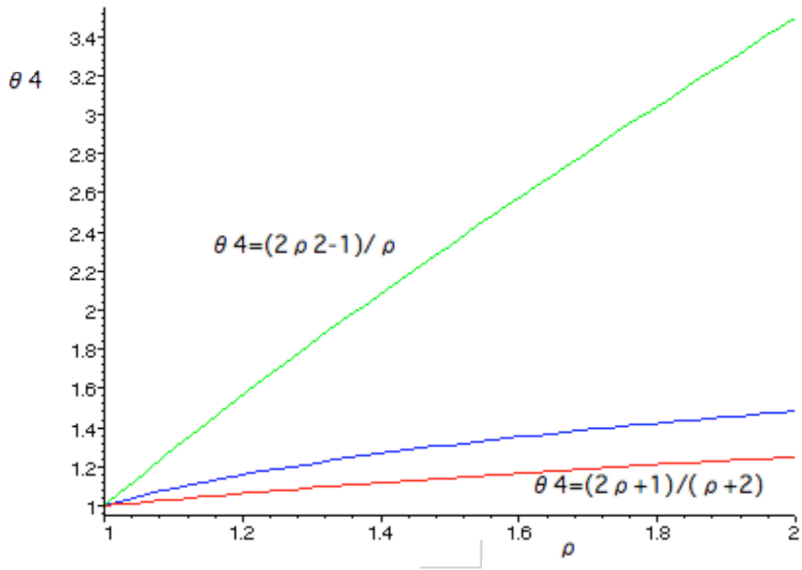

Figure 2. The region of inequalities (24) and (25)

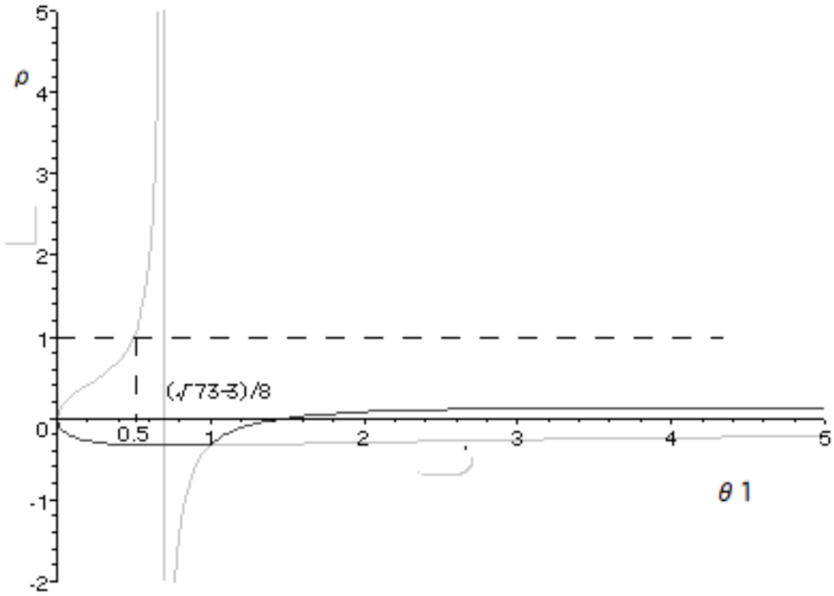

Figure 3. Two solutions of ρ as functions of θ_1