Bundling Electronic Journals and Competition among Publishers*

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Abstract

Site licensing of e-journals has been revolutionizing the way academic information is distributed. However, many librarians are concerned about the possibility that publishers might abuse site licensing by practicing bundling. In this paper, we analyze how bundling affects journal pricing in the context of STM electronic journal market and offer a novel insight on the bundling of a large number of information goods. We find that (i) when bundling is prohibited, surprisingly, market structure does not affect prices (ii) when bundling is allowed, each publisher finds bundling optimal and bundling increases the industry profit while reducing social welfare and (iii) any asymmetry-increasing merger is profitable but reduces social welfare.

Keywords: Bundling, Journal Pricing, Site Licensing, Merger

JEL numbers: D4, K21, L41, L82

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1 Introduction

Site licensing of electronic journals (e-journals, henceforth) has been revolutionizing the way academic information is distributed. Under site licensing, there is no need to spend time to look for a paper in a library and many people can download, read and print a paper simultaneously from their offices at anytime. Furthermore, e-journals' websites provide additional services such as search tools, hypertext linking, remote access etc. Therefore, it seems that, sooner or later, e-journals will supplant print journals as the norm.

However, many librarians are concerned about the possibility that commercial publishers might abuse site licensing for their advantage. First, commercial publishers have aggressively raised prices at a rate disproportionate to any increase in costs or quality. According to Association of Research Library (ARL) in US,[1] during the period of 1986-2002, the unit cost for journal subscriptions has grown at the rate of 7.7% per year, which is more than twice of the growth rate of the unit cost of monographs, 3.6%. As Figure 1 shows, up to 2000, the increase in the budget of the libraries could not match the increase in journal prices, which resulted in a continuous decrease in the amount of journals purchased during most of the period. High subscription prices charged by commercial publishers even induced some academic societies whose journals had been published by them to start new competing journals as in the case of the launch of *Journal of the European Economic Association* by European Economic Association.[2] Second, site licensing of e-journals allows commercial publishers to employ powerful pricing strategies such as price discrimination based on usage[3] and bundling while, under print journals, neither price discrimination and nor unbundling was the practice.[4] In particular, librarians are concerned about bundling. For instance, according to Kenneth Frazier (2001), director of libraries at University of Wisconsin Madison,

"the content is 'bundled' so that individual journal subscriptions can no longer be cancelled in their electronic format. (The Academic Press IDEAL program and the full ScienceDirect package offered by Elsevier are examples of such licensing agreements)."[5]

[1] See "Monograph and Serial Costs in ARL Libraries 1986-2002" at http://www.arl.org/stats/arlstat/.

[2] See Thodore Bergman's website: http://www.econ.ucsb.edu/~tedb/Journals/alternatives.html

[3] For instance, Derk Haank (2001), the CEO of Elsevier Science, says "What we are basically doing is to say that you pay depending on how useful the publication is for you - estimated by how often you use it." See also Bolman (2002) and Key Perspectives (2002) about price discrimination.

[4] In the case of print journals, arbitrage through resale has to some extent prevented publishers from practicing price discrimination. In contrast, in the case of e-journals, access to a journal is simply leased and hence resale is impossible.

[5] He further argues that "the push to build an all-electronic collection can't be undertaken at the

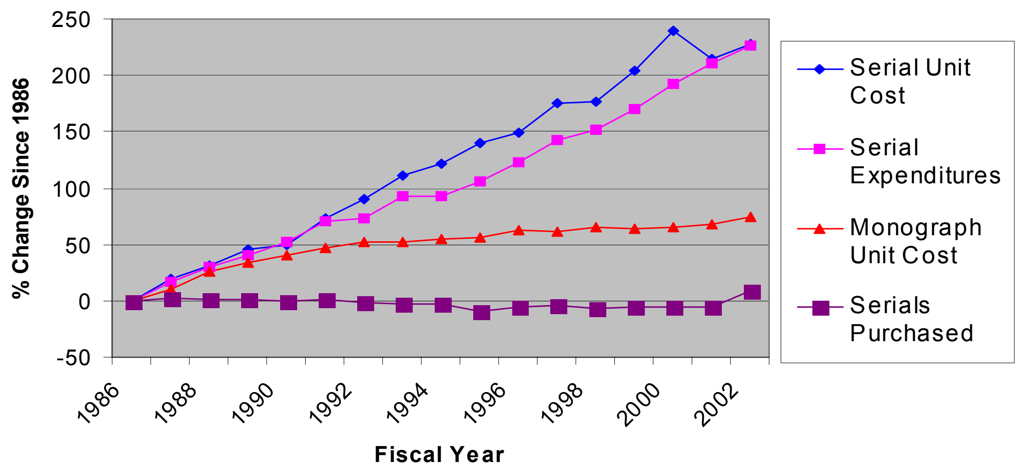

Figure 1: Monograph and Serial Costs in ARL Libraries, 1986-2002 (Source: ARL)

Moreover, U.S. and U.K. competition authorities approved three years ago one of the biggest-ever science publishing mergers between Reed-Elsevier (RE henceforth) and Harcourt in spite of many librarians' protests. Indeed, the report of U.K. Competition Commission (2001) shows concern about potential welfare losses due to the merging publishers' of bundling of their e-journals. Before the merger, RE's ScienceDirect, was the most developed website and offered access to around 1,150 journals and Harcourt's IDEAL offered access to 320 journals.

In this paper, we analyze publishers' incentives to practice bundling and the ensuing effects on social welfare and derive implications on merger analysis. Instead of considering the transition from print journals to e-journals, we consider the situation in which e-journals are the norm and assume that publishers practice price discrimination based on usage.[6] Therefore, we assume away heterogeneity among libraries and build a model in which each competing publisher sells a portfolio of journals to a library which wants to

risk of: (1) weakening that collection with journals we neither need nor want, and (2) increasing our dependence on publishers who have already shown their determination to monopolize the information market place."

[6]This implies that the pricing schemes we study in this paper might not correspond to what we observe now. In fact, the transition implies a change from subscription-based pricing models to usage-based models and since a sudden switch in the pricing models generates a large change in the total prices that allow a library to maintain its subscription to a given collection of journals, publishers are introducing a progressive change (Bolman, 2002).

build a portfolio of journals and monographs under a budget constraint.[7] We analyze how bundling affects journal pricing through its impact on the library's allocation of budget between journals and books. Although we assume that there is no direct substitution among the journals in that the value the library derives from a journal is determined independently of whether or not it buys any other journal, there can be an indirect substitution among journals and among journals and monographs[8] through the budget constraint. The utility that the library derives from spending money on books is assumed to be strictly increasing and strictly concave.

We first consider independent pricing (i.e., no-bundling) and show an irrelevance result that market structure does not affect prices. For instance, in the simple case of homogenous journals in which every journal has the same value, we show that all the journals are sold at the same price for any market structure. The irrelevance result holds also in the general case of heterogenous journals if the equilibrium exists and if the industry profit is lower than the budget: in particular, we show the equivalence between the outcome under the maximum concentration (i.e., the monopolist outcome) and the outcome under the minimum concentration in which each publisher sells only one journal. The irrelevance result is related to the fact that under independent pricing, each journal is priced according to a "marginal opportunity cost pricing rule" in the following sense: when a publisher sells a journal, he expects that his journal is the marginal journal (i.e., the last journal purchased by the library) and chooses a price p to match the library's opportunity cost of using p such that the library is indifferent between buying the journal at p and spending p instead on buying books. A monopolist cannot realize a higher profit than the one under the marginal opportunity cost pricing rule since, in order to realize a higher profit, he has to increase the price of the marginal journal, which induces the library to stop buying the journal.

When bundling is allowed, we show that each publisher has an incentive to bundle all his journals. We identify two effects of bundling. First, bundling has *the direct effect of softening competition from books.* To provide an intuition, let us consider a publisher having two journals of the same value u. Under independent pricing, he expects that each of his journals is the last journal to be purchased and chooses the same price p for them. Suppose now that he bundles the journals and chooses $2p$ as a price for the bundle. If the

[7] Typically, an academic library's material budget is spent on journals and monographs (Gooden et al. (2002)).

[8] Because of journal price increases, many university libraries have been forced to reallocate dollars from monographs to journals (Kyrillidou (1999)).

bundle is the last to be purchased among all the bundles (or journals), the library must be strictly better off by buying the bundle than by spending $2p$ on books: since the marginal utility from spending money on books strictly decreases, the utility from spending $2p$ on books is strictly smaller than twice the utility from spending p on books. Therefore, the publisher can charge $2p + \varepsilon (> 2p)$ for the bundle and still induce the library to buy it. This direct effect of bundling increases with the size of bundle, which implies that a large publisher gains much more than a small publisher in terms of the direct effect.

Second, a publisher's bundling has *an indirect effect of inflicting negative pecuniary externalities on all the other publishers*. The very fact that bundling allows a publisher to increase his profit implies that after the bundling of a publisher, there is less budget left for books and all the other publishers' journals. This in turn implies that for all the other publishers, the competition from books is tougher and therefore they have to lower their prices in order to sell them. In particular, a small publisher which has only a small number of journals does not gain much from the direct effect of bundling while he can lose a lot from the indirect effect if big publishers bundle their journals. Therefore, bundling is a profitable and credible strategy: it not only increases the bundling publisher's profit but also decreases the profits of rivals and can even induce their exit.

The insight based on the direct and indirect effects of bundling suggests that any merger increases the merging publishers' profits because of the direct effect while reducing the rivals' profits because of the indirect effect. We also show that bundling (or any merger) increases the industry profit. This result implies that the library consumes less books after bundling. Since bundling can induce exit of small publishers, we conclude that bundling decreases social welfare by reducing both book and journal consumption. For the same reasons, any asymmetry-increasing merger reduces social welfare. Our finding is consistent with the prediction of Kyrillidou (1999) that if the current trend continues, the budget for monographs will be the resource depleted fastest, as only about 10% of the materials budget will be spent on purchasing monographs by 2019. Finally, when we examine publishers' incentive to acquire a journal from a third-party, we find that in the absence of bundling all the publishers have the same willingness to pay for the journal while under bundling, the largest publisher has always the highest willingness to pay. This suggests that bundling might seriously affect industry dynamics such that the largest publisher becomes even larger through the purchase of the titles sold by small publishers forced to exit the market.

Most of the papers on bundling study bundling of two (physical) goods in the context of second-degree price discrimination and focus on either surplus extraction (Schmalensee

(1984), McAfee et al. (1989), Salinger (1995) and Armstrong (1996, 1999)) or entry deterrence (Whinston (1990) and Nalebuff (2004)). Bakos and Brynjolfsson (1999, 2000)'s papers are an exception in that they study bundling of a large number of information goods while maintaining the second-degree price discrimination framework. Their first paper shows that bundling allows a monopolist to extract more surplus since the law of large numbers reduces the variance of average valuations[9] and the second paper applies this insight to entry deterrence. Although we study bundling of a large number of information goods, our model is quite different from theirs since the law of large numbers cannot be applied in our setting as we assume complete information on the buyer's valuation for each object in sale. Conventional wisdom says that bundling has no effect in such a setting and this is true if the budget constraint is not binding. However, when the constraint is binding, we show that bundling is a profitable and credible strategy both in terms of surplus extraction and entry deterrence since bundling allows a firm to extract more surplus by softening competition from an alternative use of the budget (books in our setting) and, furthermore, reduces the other firms' profits by inflicting on them negative pecuniary externalities.

Our paper is related to McCabe (2002b)'s paper that studies the pricing of print and e-journals.[10] In the case of print journals, he assumes no price discrimination and no bundling while, in the case of e-journals, he assumes prefect price discrimination and bundling. Although his setting is similar to ours, there are important differences. First, he considers the transition from print journals to e-journals while we consider the situation when this transition is over. Second, he does not provide the comparative statics of the transition while we provide the comparative statics of bundling versus no-bundling in the digital world. Furthermore, he assumes bundling in the case of e-journals while we show that in equilibrium all publishers adopt bundling. Last, he does not consider substitution between books and journals.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we consider the simple case of homogenous journals and explain all our main results with minimum technical details. In Section 4, we consider the general case of homogenous journals and provide a theorem covering this case. Section 5 provides concluding remarks. All the proofs which do not appear in the main text are gathered in Appendix.

[9] See also Armstrong (1999).

[10] He has also an empirical paper (2002a), already published, that shows that mergers significantly contributed to journal price increases.

2 Model

As we said in the introduction, we consider the situation in which the transition from print journals to e-journals is completed. Since at the mature stage of site-licensing, journal prices will depend on usage, we assume that publishers have complete information about the value that a library attaches to a journal. Furthermore, this assumption allows us to focus on the effects of bundling which arise when one cannot apply the law of large numbers. Therefore, we assume away heterogeneity of libraries and consider a library with budget $M(> 0)$, which is assumed to be known to all publishers.

2.1 Journals and publishers

There are N number of publishers; publisher j is often denoted simply by j. We only consider profit-maximizing publishers. Let n_j be the number of journals that publisher j publishes ($j = 1, ..., N$) and $n \equiv \sum_{j=1}^{N} n_j (\geq N)$ the total number of journals. Let $u_{ij} > 0$ represent the utility (or the surplus) the library obtains from journal $i = 1, ..., n_j$ of publisher j. Let $U_j \equiv \sum_{i=1}^{n_j} u_{ij}$ and $U \equiv \sum_{j=1}^{N} U_j$. The marginal cost of producing (i.e., providing access to) a (existing) journal is assumed to be zero.

In order to focus on the impact of bundling on journal pricing, we consider a situation in which the number of journals produced by publisher j is exogenously fixed at n_j (and therefore U_j is also given) and the fixed cost of producing them has already been incurred. An important difference between print journals and e-journals is that for print journals, each year a publisher sells only the subscription of that year while, in the case of e-journals, in principle, each year (or each period) a publisher licences the access to both the current issues and the whole previous issues. Therefore, if journals are already well established and have quite a volume of previous issues, at least in a short run it is reasonable to assume that publishers already incurred the fixed cost of producing the journals.

When each journal is sold independently (i.e., in the absence of bundling), publisher j chooses price $p_{ij} > 0$ for journal i he owns. Let $\mathbf{p} \equiv (p_{11}, ..., p_{n_1 1}, ..., p_{1N}, ..., p_{n_N N}) \in \mathbb{R}^n_{++}$ represent the price vector under independent pricing. Under bundling, publisher j chooses price $P_j > 0$ for the bundle of all his journals; we use Bj to represent the bundle of j. Let $\mathbf{P} \equiv (P_1, ..., P_N) \in \mathbb{R}^N_{++}$ denote the price vector under bundling.

2.2 Library

The library allocates a fixed budget $M > 0$ between buying journals and books (monographs). The library's payoff is given by the sum of three components: the utility it draws from the journals it purchased, the utility it draws from the books it bought and the money left after the purchases. We define a reduced-form utility for books by an indirect utility function $v : [0, +\infty) \to \mathbb{R}_+$ such that $v(m)$ is the library's utility from books when it spends $m \geq 0$ amount of money on buying books; $v(0) = 0$ and $v'(m) > 0 > v''(m)$ for any $m \geq 0$, hence $v(\cdot)$ is strictly increasing and strictly concave. We further assume that $v'(m) > 1$ for all $m \leq M$, therefore the library prefers buying books to keeping money.

When each journal is sold independently, we let $x_{ij} \in \{0, 1\}$ represent the library's choice about journal ij: $x_{ij} = 1$ ($x_{ij} = 0$) means that the library buys (does not buy) this journal; hence, $\mathbf{x} \equiv (x_{11}, ..., x_{n_1 1}, ..., x_{1N}, ..., x_{n_N N})$ belongs to $\{0, 1\}^n$. When all publishers use bundling, $X_j \in \{0, 1\}$ represents the library's choice about Bj: $X_j = 1$ ($X_j = 0$) means that the library buys (does not buy) this bundle; let $\mathbf{X} \equiv (X_1, ..., X_N) \in \{0, 1\}^N$.

Under independent pricing, given $(\mathbf{p}, M)$, the library chooses $\mathbf{x}$ and $m (\geq 0)$ to maximize its payoff[11]

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} u_{ij} x_{ij} + v(m) + \left[M - \sum_{j=1}^{N} \sum_{i=1}^{n_j} p_{ij} x_{ij} - m \right] \tag{1}$$

subject to the budget constraint $\sum_{j=1}^{N} \sum_{i=1}^{n_j} p_{ij} x_{ij} + m \leq M$.[12] Under bundling, given $(\mathbf{P}, M)$, the library chooses $\mathbf{X}$ and $m \geq 0$ to maximize

$$\sum_{j=1}^{N} U_j X_j + v(m) + \left[M - \sum_{j=1}^{N} P_j X_j - m \right]$$

subject to the budget constraint $\sum_{j=1}^{N} P_j X_j + m \leq M$.

2.3 Social welfare

Social welfare is defined as the sum of the payoff of the library, the profits of the journal publishers and the profit of the book industry. The cost of producing a book is composed

[11] As a tie-breaking rule, we assume that if the library is indifferent between buying a journal (or a bundle) and not buying, it buys the journal/bundle. Without this assumption, no equilibrium would exist.

[12] In the timing (described in subsection 2.4), each publisher should make an entry decision before choosing prices. Hence, (1) is correct if all publishers enter. If some publishers do not enter, j runs over the set of the publishers which entered.

of a fixed cost and a marginal cost, about which we make a simplifying assumption: the fixed cost incurred by the book industry is not affected by the library's choice of m and the marginal cost of producing a book is zero.[13] Therefore, social welfare is equal, up to a constant, to the total utility the library draws from journals and books.

2.4 Timing

We consider a three-stage game among publishers. At stage one, each publisher simultaneously decides whether or not to enter the market; entry is costless. For equilibrium selection, we assume that each publisher prefers to stay out and not to engage in competition if his profit upon entry is zero. Let $E \subseteq \{1, ..., N\}$ denote the set of publishers which enter the market; these are called *active publishers*.

At stage two, each publisher simultaneously decides (i) whether to bundle or not his journals and (ii) the price of his bundle or the prices of his journals. Actually, after studying the case of no-bundling (when each journal is sold independently) in subsection 3.1 and the case of bundling (when each publisher bundles all his journals) in subsection 3.2, we examine in subsection 3.3 each publisher's incentive to choose between bundling and no-bundling.

At stage three, the library makes its purchase decision.

We use the concept of Nash equilibrium (NE) to analyze the game starting at stage two and the concept of subgame perfect Nash equilibrium (SPNE) to predict the outcome of the game starting at stage one.

3 The simple case of homogenous journals

In this section we derive all our main results in the simple case of homogenous journals, which means that $u_{ij} = u > 0$ for all ij. In the next section, we consider the general case of heterogeneous journals.

3.1 Independent pricing (no-bundling)

We begin our analysis with the case of independent pricing, which means that at stage two journals are priced independently by active publishers. For expositional facility, we first introduce the concept of marginal bundle of books as follows: given an industry profit

[13] This is only a simplifying assumption. Our social welfare analysis is not qualitatively affected if the (fixed or marginal) cost incurred by the book industry depends on m.

from journals denoted by π $(\leq M)$, define *the marginal bundle of books corresponding to price* p as all the books that the library wishes to buy with budget p after already spending $M-\pi$ on books. Then, the utility from the marginal bundle of books is given by

$$U_{MB}(p,\pi) \equiv v(M-\pi+p) - v(M-\pi) = \int_{M-\pi}^{M-\pi+p} v'(m)dm > 0.$$

It is useful to note some properties of U_{MB} which will be frequently used in this paper; these properties result from the fact that $v(\cdot)$ is strictly concave and strictly increasing: (i) U_{MB} strictly increases with both p and π; (ii) U_{MB} is strictly concave in p.

In order to illustrate the usefulness of $U_{MB}(\cdot)$, we consider the case in which each publisher sells only one journal (i.e., $N=n$) and charges the same price $p(\leq \frac{M}{n})$. Then, the library prefers buying n' number of journals (with $1 \leq n' \leq n$) to buying $n'-1$ number of journals if the following inequality holds:

$$n'u + v(M-n'p) \geq (n'-1)u + v(M-n'p+p)$$

which is equivalent to

$$u \geq U_{MB}(p, n'p).$$

Since $U_{MB}(p,\pi)$ strictly increases with π, $U_{MB}(p,n'p)$ strictly increases with n'. This in turn implies that the library finds it optimal to buy all the journals if and only if it prefers buying n number of journals to buying $n-1$ (i.e., if and only if $u \geq U_{MB}(p,np)$ holds). Furthermore, if $u < U_{MB}(\frac{M}{n}, M)$, p^* satisfying $u = U_{MB}(p^*, np^*)$ is an equilibrium. Suppose that all publishers except j charge p^*. If j charges p^*, then his profit is p^*. Therefore, he has no incentive to choose any price lower than p^*. Suppose that he chooses p_j $(> p^*)$. Then, $u < U_{MB}(p_j, (n-1)p^* + p_j)$ holds and this inequality is equivalent to

$$(n-1)u + v(M-(n-1)p^*) > nu + v(M-(n-1)p^* - p_j),$$

which implies that the library buys all journals except that of j. In case $u \geq U_{MB}(\frac{M}{n}, M) = v(\frac{M}{n})$ holds, $p^* = \frac{M}{n}$ is an equilibrium. Still, publisher j has no incentive to choose a price lower than p^* since he can realize profit p^* by charging p^*. If he chooses $p_j > p^*$, the library cannot afford to buy all the journals and therefore will drop j's one, which is the most expensive journal.

The next proposition states that regardless of the market structure, there exists a unique equilibrium in which all the publishers enter and charge p^*.

Proposition 1 *(irrelevance result) Suppose that all the journals are homogenous (i.e., $u_{ij} = u > 0$ for all ij) and priced independently.*
(i) When $M > nv^{-1}(u)$: There exists a unique SPNE that is the same for any market structure. In the equilibrium, all publishers enter and all the journals are purchased at prices $p_{ij} = p^*$ for any ij where p^* is such that $np^* < M$ and

$$U_{MB}(p^*, np^*) = u \tag{2}$$

(ii) When $M \leq nv^{-1}(u)$: There exists a unique SPNE that is the same for any market structure (except the monopoly case in which the uniqueness result may not hold). In the equilibrium, all publishers enter and all the journals are purchased at prices $p_{ij} = p^*$ for any ij where $p^* = M/n$.

Proof. Conditional on all publishers' entering, we show below that if all publishers except j charge the same price p^*, a best response of publisher j consists in setting $p_{ij} = p^*$ for $i = 1, ..., n_j$; this establishes that $p_{ij} = p^*$ for any ij is a NE of the pricing game. In the proof of theorem 2, we prove that, in a more general setting with heterogenous journals, at most one NE exists under no-bundling; therefore we obtain equilibrium uniqueness in the environment with homogeneous journals. At the unique equilibrium, each publisher makes a positive profit if he enters regardless of how many other publishers are active; hence, in any SPNE each publisher enters.

(i) When $M > nv^{-1}(u)$.
Suppose that all publishers except j choose price p^*. We study the optimal pricing of j having n_j number of journals: the monopoly case is a special case with $n_j = n$. Notice that for any $\mathbf{p}_j \equiv (p_{1j}, ..., p_{n_j j})$, the library will purchase all the journals of which the prices are lower than or equal to p^* because it is willing to buy n number of journals at price p^* from (2). This implies that, in particular, for any $\mathbf{p}_j$ all the journals of publisher k (with $k \neq j$) will be purchased. Publisher j realizes a profit $n_j p^*$ when it chooses $p_{1j} = ... = p_{n_j j} \equiv p^*$. We now prove by contradiction that he cannot realize any profit strictly higher than $n_j p^*$. Suppose that he realizes a profit π_j strictly higher than $n_j p^*$. This must imply that the highest price among all the journals sold by publisher j, denoted by $p_j^{(1)}$, is strictly larger than p^*. Since all the other publishers charge p^*, $p_j^{(1)}$ is the highest price among all the journals sold. However, we have $u = U_{MB}(p^*, np^*) < U_{MB}(p_j^{(1)}, (n - n_j)p^* + \pi_j)$ since $U_{MB}(\cdot, \cdot)$ strictly increases in both arguments. This inequality implies that the library finds it optimal not to buy the highest priced journal: therefore, we get a contradiction. By using a similar argument, we can prove that $p_{1j} = ... = p_{n_j j} \equiv p^*$ is the only way for j to realize the profit $n_j p^*$. Suppose that j realizes the profit $n_j p^*$ by choosing $\mathbf{p}_j$ different

from $(p^*, ..., p^*)$. This implies that the highest price among all the journals sold by j $p_j^{(1)}$ is strictly larger than p^*. But we have $u = U_{MB}(p^*, np^*) < U_{MB}(p_j^{(1)}, np^*)$, which implies that the library does not buy the highest priced journal, leading to a contradiction.

(ii) When $M \leq nv^{-1}(u)$.

Suppose that all publishers different from j charge the same prices $M/n = p^*$ for their journals. If $p_{ij} = M/n$ for $i = 1, ..., n_j$, then all journals are sold because $u \geq U_{MB}(M/n, M)$ is equivalent to $M \leq nv^{-1}(u)$. The inequality $u \geq U_{MB}(M/n, M)$ implies that the library will purchase all the journals with prices equal to M/n or smaller. This implies in particular that for any $\mathbf{p}_j$, all journals of the publishers different from j will be purchased. Therefore, $M - (n - n_j)p^*(= n_j p^*)$ is the maximum amount of money the library will spend on journals of publisher j and this is the profit j achieves by setting $p_{ij} = M/n$ for $i = 1, ..., n_j$. In the monopoly case, selling all the journals at price p^* is an equilibrium: however, it is also possible for a monopolist to realize the same profit M by selling a subset of his journals. ■

As is shown in Figure 2, when the industry profit from journals np^* is smaller than M, p^* is determined by the "marginal opportunity cost pricing" in the following sense: when a publisher chooses a price for each of his journals, he considers each journal the marginal journal (i.e., the last journal purchased by the library) and chooses the price p^* such that after purchasing $n - 1$ number of journals, the library is indifferent between buying an extra journal at p^* and spending p^* instead on buying books: the area of the rectangular ABCD is equal to u.[14] The irrelevance result says that all the journals are sold at the same price p^* for any market structure. In order to give an intuition of the result, we consider when $p^* < \frac{M}{N}$ holds and ask why a monopolist cannot achieve a strictly better outcome than the one under the minimum industry concentration (i.e., each publisher sells only one journal). Note that in the equilibrium, all the journals are sold. In order to increase his profit, a monopolist can employ a strategy of cross-subsidization (i.e., decreasing the prices of some journals and increase the prices of some other journals) or a strategy of selling only a subset of journals or a combination of the two. However, none of the strategies can allow him to achieve a higher profit. First, achieving a higher profit implies that the highest price among all the journals sold is strictly larger than p^*. Second, this implies that the library prefers not buying the most expensive journal

[14] The fact that each publisher regards his journal as the marginal one when choosing its price is similar to what happens in the literature on multilateral bargaining (Stole and Zweibel (1996a,b) and Chemla (2003)). For instance, Chemla studies competition among downstream firms selling to an upstream one and finds that each downstream firm pays the price that the marginal firm would pay to the upstream one. However, none of the papers studies the issue of bundling.

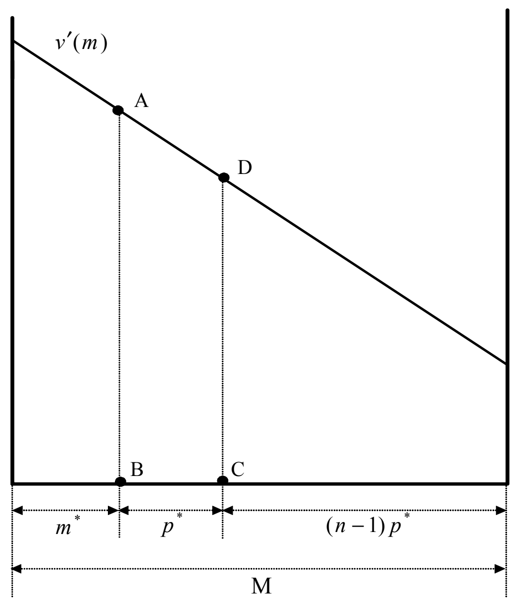

Figure 2: Equilibrium under independent pricing when $M > nv^{-1}(u)$ holds

since it is indifferent between buying the journal at price p^* and not buying it when the industry profit is np^*. Therefore, we get a contradiction.

Example 1 *Suppose $v(m) = 31m - m^2$, $M = 10$, $u = 42$, $n = 3$. Then $U_{MB}(p, \pi) = p(31 - p - 2(M - \pi))$. Under independent pricing, by proposition 1, the equilibrium price p^* of each journal (regardless of market structure) is such that $U_{MB}(p^*, 3p^*) = 42$ since $v(\frac{M}{n}) > u$, implying $p^* = 2$.*

From the irrelevance result, we have the following corollary:

Corollary 1 *Under homogenous journals and independent pricing,*

(i) no merger has an impact on (merging or non-merging) firms' profits and therefore firms have no strict incentive to merger.

(ii) no merger has an impact on social welfare unless the merger creates a monopolist realizing a profit equal to M.

Corollary 1(ii) deserves some explanation. If the industry profit is equal to M, a monopolist can achieve the same profit by selling a subset of journals. Therefore, a merger creating a monopolist can reduce social welfare if the monopolist sells a smaller number of journals than n.

3.2 Bundling

We consider here the case of bundling, which means that at stage two each active publisher bundles his journals and chooses a price for it. Because of homogeneity, we have $U_j = \sum_{i=1}^{n_j} u_{ij} = n_j u$ and, without loss of generality, we suppose that $U_1 \geq U_2 \geq ... \geq U_N$. Let E^* represent the equilibrium set of active publishers, $\mathbf{P}^* \equiv \{P_j^* : j \in E^*\}$ the equilibrium prices charged by the active publishers and $\pi^{B*} \equiv \sum_{j \in E^*} P_j^*$ the equilibrium industry profit under bundling. The analysis we perform is unaffected by whether the journals are homogenous or heterogenous because what matters is the values $U_1, ..., U_N$ of the different bundles; therefore, the results in this subsection apply to the setting of heterogenous journals as well. The next theorem characterizes the unique SPNE in this environment.

Theorem 1 *Suppose that each publisher bundles his journals at stage two. Then, there exists a unique SPNE and it is characterized as follows:*[15]
(i) If $M \leq v^{-1}(U_1 - U_2)$, only the largest publisher enters and realizes profit $P_1^* = M$.
(ii) If M is such that there exists $k \in \{2, ..., N\}$ satisfying $\sum_{j=1}^{k-1} v^{-1}(U_j - U_k) < M \leq \sum_{j=1}^{k} v^{-1}(U_j - U_{k+1})$ (with $U_{N+1} \equiv 0$), only the k largest publishers enter and charge prices $\mathbf{P}^*$ satisfying $\pi^{B*} = \sum_{j=1}^{k} P_j^* = M$ and

$$U_j - U_{MB}(P_j^*, M) = U_{j'} - U_{MB}(P_{j'}^*, M) \geq U_{k+1} \qquad \text{for any } \{j, j'\} \subset E^*. \tag{3}$$

(iii) If $M > \sum_{j=1}^{N} v^{-1}(U_j)$, all the publishers enter and charge prices $\mathbf{P}^*$ satisfying $\pi^{B*} = \sum_{j=1}^{N} P_j^* < M$ and

$$U_{MB}(P_j^*, \pi^{B*}) = U_j \qquad j = 1, ..., N. \tag{4}$$

Proof. See Appendix. ■

We first note that the particular case of proposition 1 in which $n_j = 1$ for any j (i.e., each publisher owns only one journal) is a special case of the parts (ii)-(iii) of this theorem with $U_j = u$ for all j and $N = n$. Note also that all the bundles are sold if and only if $M > \sum_{j=1}^{N-1} v^{-1}(U_j - U_N)$ holds. If $M \leq \sum_{j=1}^{N-1} v^{-1}(U_j - U_N)$ holds, bundling induces the exit of small publishers while, under no-bundling, all journals are sold for any value of M. In what follows, we provide the main intuition about the equilibrium under bundling by examining a special case with two publishers such that $U_1 > U_2$.

[15] Actually, uniqueness of SPNE obtains only along the equilibrium path: there exist several SPNE in this game, but they differ only off the equilibrium path. In the appendix we provide the complete strategy profile for a SPNE immediately after the proof of theorem 1.

Consider first the case in which both publishers enter, sell their bundles and π^{B*} is smaller than M. Then, the equilibrium prices $\mathbf{P}^* = (P_1^*, P_2^*)$ are such that

$$U_{MB}(P_j^*, P_1^* + P_2^*) = U_j, \text{ for } j = 1, 2. \tag{5}$$

Publisher j expects that the other bundle is purchased and chooses his price making the library indifferent between buying his own bundle and not buying it. $\mathbf{P}^*$ constitutes a Nash equilibrium. First, obviously, lowering P_j below P_j^* is not optimal for publisher j. Second, if P_j is increased above P_j^*, then the library prefers dropping Bj since at $\mathbf{P}^*$ the library is indifferent between dropping any single bundle and buying both bundles. A solution to (5) exists if and only if $M \geq v^{-1}(U_1) + v^{-1}(U_2)$. In particular, when $M = v^{-1}(U_1) + v^{-1}(U_2)$, we have $P_j^* = v^{-1}(U_j)$ and $\pi^{B*} = M$ from $U_{MB}(P_j^*, M)(= v(P_j^*)) = U_j$.

Second, consider the case in which all the publishers enter, sell their bundles and $\pi^{B*} = M$. Then, $\mathbf{P}^*$ satisfies

$$U_1 - U_{MB}(P_1^*, M) = U_2 - U_{MB}(P_2^*, M) \geq 0. \tag{6}$$

Publisher j still expects that the other bundle is purchased, but now the library gets a positive surplus with respect to the option of spending P_j^* on books. Notice that there is a kind of Bertrand competition such that this surplus is the same for all the bundles. $\mathbf{P}^*$ constitutes a Nash equilibrium since lowering a price is not optimal and if publisher j chooses a price higher than P_j^*, the library cannot afford to buy both bundles and prefers dropping Bj since at $\mathbf{P}^*$ the library is indifferent between dropping B1 and dropping B2.

Finally, we can obtain the condition under which publisher 1 can sell B1 at price $P_1^* = M$ by inserting $P_1^* = M$ and $P_2^* = 0$ into (6); then we obtain $U_1 - v(M) = U_2$, meaning $v^{-1}(U_1 - U_2) = M$. Indeed, a solution to (6) exists if and only if $v^{-1}(U_1 - U_2) \leq M \leq v^{-1}(U_1) + v^{-1}(U_2)$. Intuitively, when $U_1 \geq v(M) + U_2$ holds, publisher 1 can drive publisher 2 out of the market since for any $P_2 > 0$, the library's payoff from buying B1 at price $P_1 = M$ (i.e., U_1) is larger than $U_2 + v(M - P_2)$, the payoff from buying B2 at $P_2 > 0$ and spending $M - P_2 > 0$ on books.

Example 2 *Suppose $v(m) = 31m - m^2$, $M = 10$, $u = 42$, $n = 3$ as in example 1. Under bundling.*
(i) When $N = 2$ and $n_1 = 2$, $n_2 = 1$, P_1^* and P_2^* satisfy

$$U_{MB}(P_1^*, P_1^* + P_2^*) = 84$$
$$U_{MB}(P_2^*, P_1^* + P_2^*) = 42$$

since $M > v^{-1}(2u)+v^{-1}(u)$*; hence* $P_1^* = 4.36552$ *and* $P_2^* = 1.93812$*. Notice that* $P_1^* > 2p^*$*,* $p^* > P_2^*$ *and* $P_1^* + P_2^* > 3p^*$*.*
(ii) When $N = 1$ *and* $n_1 = 3$*, the monopolist chooses* P^m *satisfying* $U_{MB}(P^m, P^m) = 126$ *since* $M > v^{-1}(3u)$*, hence* $P^m = 7$*. Notice that* $P^m > P_1^* + P_2^* > 3p^*$*.*

Theorem 1 and the discussion following the theorem show that prices under bundling are determined by the marginal opportunity cost pricing as under independent pricing. This is literally true when $\pi^{B*} < M$. When $\pi^{B*} = M$, the equilibrium prices are such that each bundle should give the same extra surplus with respect to the opportunity cost (i.e., the utility that the library gets from the marginal bundle of books). This competition between each bundle of journals and the marginal bundle of books implies that a large publisher (i.e., a publisher with high U_j) has a competitive advantage over a small publisher. Given π^{B*}, since $v'(\cdot)$ is strictly decreasing, as the number of books in the marginal bundle increases, the average surplus of the books in this bundle decreases. Therefore, the marginal bundle of books competing with the bundle of a large publisher has a lower average surplus than the marginal bundle of books competing with the bundle of a small publisher. The next corollary formalizes this intuition in two different, although related, ways. The first result shows that a publisher's profit per value of journal $\frac{P_j^*}{U_j}$ strictly increases with the total value of his bundle U_j; the second result establishes that a large publisher gets a relatively large share of the industry profit.

Corollary 2 [16] *Under bundling we have*
(i) $\{j, h\} \subseteq E^*$ *and* $U_j > U_h$ *imply* $\frac{P_j^*}{U_j} > \frac{P_h^*}{U_h}$*;*
(ii) If $E \subset E^*$ *is such that* $U_1 \geq \sum_{h \in E} U_h$*, then* $P_1^* > \sum_{h \in E} P_h^*$*.*

3.3 Incentive to bundle

In the previous sections, we examined the two different regimes of no-bundling and bundling. In this section, we inquire which of these regimes will emerge endogenously by examining each publisher' incentive to bundle. We have the following result.

Proposition 2 *(i) If publisher* j *realizes profit* $\pi_j > 0$ *under independent pricing, then he can earn the same profit by bundling his journals at price* $P_j = \pi_j$
(ii) in any SPNE in which publisher j *is active, he bundles his journals if* $n_j \geq 2$*.*

[16] The proof of the corollary is straightforward and hence omitted.

Proof. See Appendix. ■

This result says that any profit publisher j can make without bundling his journals can also be obtained by bundling the journals; therefore, bundling is a weakly dominant strategy for each publisher. However, this fact might be consistent with the existence of a SPNE in which one (or more) publisher(s) does not bundle. The second part of the proposition establishes that this is impossible.

We can provide the intuition for the incentive to bundle by examining a simple case. Suppose that publisher j has two journals and, when he does not bundle his journals, the industry profit is $\pi < M$. In this case, the prices chosen by publisher j are the same and this price, denoted by p_j, satisfies the following equation:

$$U_{MB}(p_j, \pi) = u, \tag{7}$$

where, given the industry profit π, p_j makes the utility from the marginal bundle of books equal to u.

Suppose now that publisher j bundles his journals. Consider first the case in which he charges price $2p_j$ for the bundle. Then, from (7) and the strict concavity of $v(\cdot)$, we have

$$2u - U_{MB}(2p_j, \pi) = 2U_{MB}(p_j, \pi) - U_{MB}(2p_j, \pi) = \int_{M-\pi}^{M-\pi+p_j} v'(m)dm - \int_{M-\pi+p_j}^{M-\pi+2p_j} v'(m)dm > 0.$$

Under independent pricing, both journals compete with the same marginal bundle of books having the utility $U_{MB}(p_j, \pi)$. In contrast, under bundling, it is as if the first journal competes with the same marginal bundle of books having utility $U_{MB}(p_j, \pi)$, while the second journal competes with the marginal bundle of books having utility $U_{MB}(p_j, \pi - p_j)$, which is smaller than $U_{MB}(p_j, \pi)$. In other words, given a profit $\pi > 0$, the average surplus of the marginal bundle of books corresponding to price p_j is strictly higher than that of the marginal bundle of books corresponding to $2p_j$. Therefore, *bundling has the direct effect of softening competition from books.* More precisely, there exists an $\varepsilon > 0$ satisfying the inequality

$$U_{MB}(2p_j + \varepsilon, \pi + \varepsilon) \leq 2u.$$

Therefore, if publisher j charges $P_j = 2p_j + \varepsilon$ as the price for bundle, the library will buy it and bundling allows publisher 1 to realize a strictly higher profit.

3.4 Comparative Statics

3.4.1 Industry profit and social welfare

We here study the effect of bundling on industry profit and social welfare. Let π^{I*} denote the industry profit under independent pricing. We have:

Proposition 3 *(i) If $M > nv^{-1}(u)$, bundling strictly increases the industry profit: $\pi^{B*} > \pi^{I*}$. If $M \leq nv^{-1}(u)$, bundling does not affect the industry profit: $\pi^{B*} = \pi^{I*} = M$.*

(ii) Bundling reduces social welfare by reducing book consumption and journal consumption.

Proof. (i) From proposition 1 we know that $\pi^{I*} = M$ if $M \leq nv^{-1}(u)$. In contrast, theorem 1 shows that $\pi^{B*} = M$ when $M \leq \sum_{j=1}^{N} v^{-1}(U_j)$. Since $v^{-1}(0) = 0$ and v^{-1} is strictly convex, $U_j = n_j u$ implies $n_j v^{-1}(u) < v^{-1}(U_j)$ and in turn $nv^{-1}(u) < \sum_{j=1}^{N} v^{-1}(U_j)$; this proves the second part of the proposition (i).
Suppose now $M > nv^{-1}(u)$, so that $\pi^{I*} < M$. If $M \leq \sum_{j=1}^{N} v^{-1}(U_j)$ holds, then $\pi^{B*} = M$ from theorem 1 and the proposition (i) trivially holds. Suppose in contrast that $M > \sum_{j=1}^{N} v^{-1}(U_j)$, so that $\pi^{B*} < M$ by theorem 1. In order to prove that $\pi^{B*} > \pi^{I*}$, we notice that for each publisher j we have

$$n_j U_{MB}(p^*, \pi^{I*}) = n_j u = U_j = U_{MB}(P_j^*, \pi^{B*}). \tag{8}$$

Define $P_j(\pi)$ by $U_j \equiv U_{MB}(P_j(\pi), \pi)$ and observe that $P_j(.)$ is strictly decreasing since $U_{MB}(\cdot, \cdot)$ is strictly increasing in both arguments. Since U_{MB} is concave in the first argument, the first two equalities in (8) imply $P_j(\pi^{I*}) > n_j p^*$ for any $n_j \geq 2$. We now prove $\pi^{B*} > \pi^{I*}$ by contradiction. Suppose $\pi^{B*} \leq \pi^{I*}$. Since $P_j(.)$ is strictly decreasing, we must have $P_j^* = P_j(\pi^{B*}) \geq P_j(\pi^{I*})$, which implies $\pi^{B*} \equiv \sum_{j=1}^{N} P_j(\pi^{B*}) \geq \sum_{j=1}^{N} P_j(\pi^{I*}) > \sum_{j=1}^{N} n_j p^* = \pi^{I*}$, which is a contradiction.

(ii) The fact that bundling increases the industry profit implies that the library consumes fewer books under bundling than under no-bundling. Furthermore, all the journals are sold under no-bundling while bundling can induce the exit of small publishers from theorem 1. ■

The intuition for proposition 3 (i) is simple. We have seen in Section 3.3 that, for each publisher, bundling has a direct effect of softening the competition he faces from books. Suppose $\pi^{I*} < M$ and bundling does not increase the industry profit (i.e., $\pi^{B*} \leq \pi^{I*}$). Then, the marginal bundle of books corresponding to a given price p has a lower value under bundling than under independent pricing (i.e., $U_{MB}(p, \pi^{B*}) \leq U_{MB}(p, \pi^{I*})$).

Therefore, each publisher must be able to strictly increase his profit from the direct effect and hence we get a contradiction.

If the publishers are symmetric in the sense that $U_1 = ... = U_N$, then bundling increases the profit of each publisher. If instead publishers are asymmetric, the fact that bundling increases the industry profit is a bad news for small publishers who cannot benefit much from the direct effect of bundling. To provide an intuition, let us consider the competition between a big publisher with $U_1 = (n - N + 1)u$ (and $n > N$) and $N - 1$ number of small publishers with $U_2 = ... = U_N = u$. Suppose that $p^* < \frac{M}{n}$ without bundling and then consider bundling. Obviously, no small publisher can benefit from bundling since it has only one journal. However, by proposition 3, the big publisher's bundling increases the industry profit: $\pi^{B*} > np^*$. This has a negative indirect effect on all the small publishers' profits through *pecuniary externalities* since the marginal bundle of books corresponding to a given price of journal has a higher surplus after the bundling than before the bundling. For instance, if $\pi^{B*} < M$, each small publisher's profit under bundling is P_2^* with $U_{MB}(P_2^*, \pi^{B*}) = u = U_{MB}(p^*, np^*)$, which implies that P_2^* is smaller than p^*. Furthermore, as we have seen in theorem 1, these pecuniary externalities can induce the exit of all small publishers if U_1 is large enough to satisfy $M \leq v^{-1}(U_1 - U_2)$. Since the fact that bundling increases the industry profit implies that the library consumes fewer books under bundling than under no-bundling, bundling reduces social welfare by reducing book and journal consumption.

Remark (independent budget for journals): When there is an independent budget for journals, we have $v(m) = m$. Since most of the effects of bundling are based on the strict concavity of $v(m)$, one can expect that bundling has no effect in this setting. In fact, this is true as long as $U \leq M$: then each publisher is indifferent between bundling and no-bundling since the equilibrium price of a journal or a bundle is simply equal to its value. However, when $U > M$, although bundling does not affect the industry profit which is equal to M, it can reduce social welfare by inducing exit of small publishers. As theorem 1 on bundling shows, when the industry profit is equal to M, there is a kind of Bertrand competition among bundles which makes the extra surplus that the library gets from a bundle with respect to its option of keeping money constant across all the bundles sold. Therefore, U_j/P_j^* decreases with U_j, creating advantage to large publishers.

3.4.2 Mergers

We have seen that no incentive to merger exists under independent pricing. We here study how bundling affects this incentive. Let π^{AM*} (π^{BM*}) represent the industry profit after the merger (before the merger) under bundling and E^{BM*} denote the set of active publishers before the merger.

Proposition 4 *Consider a merger of any two publishers j and k such that $\{j,k\} \subset E^{BM*}$ and $\pi_j^{BM*} + \pi_k^{BM*} < M$. The merger*
(i) strictly increases the joint profit of the merging publishers and strictly decreases the profit of any other publisher;
(ii) strictly increases the industry profit if $\pi^{BM*} < M$, otherwise $\pi^{AM*} = \pi^{BM*} = M$.
(iii) Any asymmetry-increasing merger reduces social welfare.

Proof. See Appendix. ■

This proposition says that a merger between any two active firms is strictly profitable unless the two firms already monopolize the market. As we mentioned before, under bundling each Bj competes with the marginal bundle of books and the average surplus of this bundle decreases as the number of books increases. Therefore, a large bundle of journals faces a relatively soft competition from books. In this way, merger increases the profit of the merged publishers and the industry profit. However, the fact that the library spends more money on the journals of the merging publishers imposes negative pecuniary externalities on all the other publishers, which therefore suffer a loss in profit because of the merger. In particular, an asymmetry-increasing merger can induce the exit of small publishers. Since any merger decreases book consumption by increasing the industry profit, we conclude that any asymmetry-increasing merger reduces social welfare by reducing both journal and book consumption.

3.5 Bundling and incentive to acquire a journal

The previous results have shown that bundling can induce exit of small publishers.[17] Small publishers who are forced to exit the market will attempt to sell their journals to other publishers. In this section, we study how bundling affects publishers' incentive to acquire a journal sold by a third party (such as a publisher exiting the market). We assume that a third-party sells a journal with value u through a second-price auction to publishers and we focus on the unique undominated equilibrium of this auction, in which each publisher bids his willingness to pay for the journal. In order for the auction to make sense, we

[17] Actually, some publishers think that if they are below number five in the shopping list of libraries, there is no guarantee that there will be any money left in the budget of the libraries (Key Perspectives (2002)).

suppose that there are at least two publishers making a positive profit before the auction in the two regimes of no-bundling and bundling and $U_1 > U_2$. We study how bundling affects the winner of the auction.

3.5.1 Independent pricing (no-bundling)

Let $p^*(n)$ denote the equilibrium price described by proposition 1 as a function of the number of journals. If publisher j wins the journal, we know from Proposition 1 that he will sell all his $n_j + 1$ journals at the uniform price $p^*(n+1)$, thus realizing profit $(n_j+1)p^*(n+1)$. If instead publisher j loses the auction, another publisher will win the journal but the equilibrium price will still be $p^*(n+1)$; j's profit will be $n_j p^*(n+1)$. Therefore, the increase in publisher j's profit from winning the auction with respect to losing it is $p^*(n+1)$ for $j \in \{1, ..., N\}$, regardless of the identity of the winner. Hence, all the publishers have the same willingness to pay and make the same bid $b_j = p^*(n+1)$ for all $j \in \{1, ..., N\}$.

3.5.2 Bundling

Under bundling, the industry profit depends on who wins the auction, unlike in the previous case of no-bundling. Therefore, a loser's profit depends on the identity of the winner through pecuniary externalities and some care is needed to evaluate a publisher's willingness to pay for the journal. We obtain the following proposition, in which $\mathbf{b} = (b_1, ..., b_N)$ denotes the equilibrium profile of bids.

Proposition 5 *Suppose a third-party sells a journal through the second-price auction. Then, in the unique undominated equilibrium,*
(i) Under independent pricing, all the publishers make the same bid.
(ii) Under bundling, if there are only two publishers in the auction, then $b_1 \geq b_2$; if there are at least three publishers, then $b_1 > b_j$ for any $j \neq 1$.

Proof. (i) The result under independent pricing is proved in subsection 3.5.1
(ii) Let P_j^k denote the equilibrium price that publisher j charges to his bundle in the case in which publisher k wins the auction and bundles the new journal with all his existing journals. Let $\pi^k \equiv \sum_{j=1}^{N} P_j^k$ and notice that since the industry profit increases (weakly) more as the new journal is integrated to a larger bundle, we have $\pi^j \geq \pi^k$ for any $j \leq k$.[18]

[18] This is due to the strict concavity of $v(\cdot)$. If the inequality $\pi^j \geq \pi^k$ does not hold, we get easily a contradiction as in the proof of proposition 3.

Furthermore, recall that in a second price auction, for a bidder it is a weakly dominant strategy to bid the own willingness to pay for the object. A publisher's willingness to pay is the difference between his profit when he wins the auction and his profit when he loses.

Consider first the simple case of $N = 2$. Then, publisher j's bid is $b_j = P_j^j - P_j^k$ for $j = 1, 2$, $k \neq j$. Hence

$$b_1 - b_2 = \pi^1 - \pi^2$$

We know that $\pi^1 \geq \pi^2$ and, in particular, $b_1 > b_2$ if $\pi^1 > \pi^2$ while $b_1 = b_2$ if $\pi^1 = \pi^2$.

Consider now the general case of $N \geq 3$. Let k^* be such that $P_{k^*}^{k^*} - P_{k^*}^1 = \max_{k \geq 2} \left\{ P_k^k - P_k^1 \right\}$. For simplicity, we assume that k^* is uniquely defined (a property which holds generically) but our proof can be adapted to the case in which there is more than one k^*. Then, we have the following equilibrium

$$b_1 = P_1^1 - P_1^{k^*}, b_j = P_j^j - P_j^1 \text{ for } j \geq 2. \tag{9}$$

Notice that b_1 is publisher 1's willingness to pay for the journal since, by definition $b_{k^*} \geq b_j$ for any $j \geq 2$. We now show that $b_1 > b_{k^*}$; this inequality establishes that b_j in (9) is j's willingness to pay and that publisher 1 wins the auction. If we have $\pi^1 > \pi^{k^*}$, then pecuniary externalities imply $P_j^1 < P_j^{k^*}$ for all $j \notin \{1, k^*\}$. Hence, using again $\pi^1 > \pi^{k^*}$ yields

$$P_1^1 + P_{k^*}^1 > P_1^{k^*} + P_{k^*}^{k^*}, \tag{10}$$

which is equivalent to $b_1 > b_{k^*}$. If instead $\pi^1 = \pi^{k^*}$, then we have necessarily $\pi^1 = \pi^{k^*} = M$ and from (15) in the proof of theorem 1 we still find that $P_j^1 < P_j^{k^*}$ for all $j \notin \{1, k^*\}$, hence (10) still holds.

Furthermore, we can show that there exists no undominated equilibrium in which publisher k ($k \neq 1$) bids $b_k \geq b_1$. Suppose that such an equilibrium exists. Then, there are two cases. First, publisher 1 makes the second highest bid (this includes the case of $b_k = b_1$). In this case, we get a contradiction since we have $b_1 = P_1^1 - P_1^k > b_k = P_k^k - P_k^1$ from (10) and the definition of k^*. Second, the second highest bid is made by publisher $h \neq 1$. Then, we still get a contradiction since we have $b_1 = P_1^1 - P_1^k > P_k^k - P_k^1 > P_k^k - P_k^h = b_k$. ■

This proposition implies that bundling could have a serious impact on the evolution of market structure. In the absence of bundling, publishers have the same willingness to pay for a journal. In contrast, under bundling, the largest publisher has always the highest willingness to pay for the journal. Although a more careful analysis needs to be undertaken to make a prediction on the industry dynamics, our result suggests that bundling might create a vicious circle through which big publishers induce exit of small publishers and become even bigger by purchasing their titles.

4 The general case of heterogenous journals

In this section we consider the general case in which journals can have different values. We first study independent pricing and show that the irrelevance result holds at least between the two extreme cases: the case of maximum industry concentration (the monopolist) and the case of minimum industry concentration (each publisher sells only one journal). For the intermediate setting of oligopoly, we show that the equilibrium prices under the case of minimum industry concentration are the only possible equilibrium prices but that the pricing game may have no (pure-strategy) NE if journals are not sufficiently homogeneous. We later on illustrate equilibrium non-existence through an example.

The outcome of the case in which each publisher sells only one journal ($N = n$) can be obtained from theorem 1 by replacing U_j with u_{1j}, where u_{1j} represents the value of the unique journal owned by publisher j.

Corollary 3 *In the n-publisher-n-journal setting, there exists a unique SPNE. The equilibrium prices* $\mathbf{p}^*$ *are such that:*
(i) If $M \leq v^{-1}(u_{11} - u_{12})$, $p^*_{11} = M$ *with* $E^* = \{1\}$.
(ii) If M *is such that there exists* $k \in \{2, ..., n\}$ *satisfying* $\sum_{j=1}^{k-1} v^{-1}(u_{1j} - u_{1k}) < M \leq \sum_{j=1}^{k} v^{-1}(u_{1j} - u_{1k+1})$ *(with* $u_{1n+1} \equiv 0$*), then* $E^* = \{1, ..., k\}$ *and the equilibrium price vector* $\mathbf{p}^*$ *is such that* $\sum_{j=1}^{k} p^*_{1j} = M$ *and*

$$u_{1j} - v(p^*_{1j}) = u_{1j'} - v(p^*_{1j'}) \geq u_{1k+1} \qquad \textit{for any } \{j, j'\} \subset E^*.$$

(iii) If $M > \sum_{j=1}^{n} v^{-1}(u_{1j})$*, then* $E^* = \{1, ..., n\}$ *and* $\mathbf{p}^*$ *is such that* $\pi^* < M$ *and*

$$U_{MB}(p^*_{1j}, \pi^*) = u_{1j} \qquad j = 1, ..., n.$$

The next theorem describes the outcome under no-bundling. In part (a) of the theorem, we have $N = 1$. In part (b), $u^{(h)}$ represents the h-th highest value among all the values of journals and $\bar{p}^*_{-j}$ is the price of the most expensive journal among all the journals excluding publisher j's ones.

Theorem 2 *Consider independent pricing.*
(a) In the monopoly environment, the monopolist enters and his profit is smaller than M *if* $M > \sum_{i=1}^{n} v^{-1}(u_{i1})$ *and equal to* M *otherwise. In the former case, the equilibrium prices in the n-publisher-n-journal setting are the unique profit maximizing prices. In the latter case, the equilibrium prices in the n-publisher-n-journal setting are optimal for the monopolist.*

(b) In a setting of oligopoly, assume that $\sum_{h=1}^{n-1} v^{-1}(u^{(h)} - u^{(n)}) < M$ *– therefore all journals are sold in the n-publisher-n-journal setting. There exists a unique candidate SPNE and it is characterized as follows: all publishers enter and prices are the same as* $\mathbf{p}^*$ *(i.e., those in the n-publisher-n-journal setting);* $\mathbf{p}^*$ *is a NE of the pricing game if journals are homogeneous enough, but not necessarily otherwise. The following condition is necessary for* $\mathbf{p}^*$ *to be a NE of the pricing game: for* $j = 1, ..., N$

$$v'(M - \pi^* + \bar{p}^*_{-j}) \geq \frac{(n_j - 1) \prod_{h=1}^{n_j} v'(M - \pi^* + p^*_{hj})}{\sum_{s=1}^{n_j} \prod_{h \neq s} v'(M - \pi^* + p^*_{hj})} \tag{11}$$

Theorem 2(a) says that the industry structure does not affect the outcome in the two extreme environments if the industry profit is lower than M.[19] Furthermore, theorem 2(b) says that if all the journals are sold in the n-publisher-n-journal setting, the outcome is the same for any market structure with $N \geq 2$ as long as the pricing game has a NE. Since in the case of homogenous journals, all the journals are sold and the equilibrium exists, proposition 1 is a special case of theorem 2. Equilibrium (in pure strategy) may not exist since a multi-journal publisher may have an incentive to deviate from the candidate equilibrium by altering several prices at the same time, which is impossible for a publisher having only one journal. In the appendix we provide a sufficient condition for existence, which is satisfied if journals are approximately homogenous. The next example gives an idea of how the candidate NE may fail to be an equilibrium.

Example 3 *Consider a setting with* $v(m) = 5\sqrt{m}$, $M = 12.5$, $N = 2$, $n_1 = 2$, $n_2 = 1$, $u_{11} = u_{21} = 2$ *and* $u_{12} = 10$. *Since* $v^{-1}(y) = \frac{y^2}{25}$ *and* $M > \sum_{h=1}^{3} v^{-1}(u^{(h)})$, *the only candidate equilibrium under independent pricing is such that* $p_{11} + p_{21} + p_{12} < M$, $2 = U_{MB}(p_{11}, \pi) = U_{MB}(p_{21}, \pi)$ *and* $10 = U_{MB}(p_{12}, \pi)$; *this yields* $p^*_{11} = p^*_{21} = 1.122$ *and* $p^*_{12} = 8.81$. *Condition (11) is* $v'(M - \pi + p^*_{12})(v'(M - \pi + p^*_{11}) + v'(M - \pi + p^*_{21})) \geq v'(M - \pi + p^*_{11})v'(M - \pi + p^*_{21})$ *and reduces to* $\frac{5}{\sqrt{1.446+8.81}} \geq \frac{5}{2\sqrt{1.446+1.122}}$, *which is (strictly) satisfied. However, there exists a profitable deviation for publisher 1: if he increases the prices of each of his journals by* 0.06, *then the library's payoff is maximized by purchasing only the two journals of publisher 1.*[20]

[19] As in the case of homogenous goods, when the monopolist's profit is equal to M, it might be possible for him to realize $\pi = M$ by selling a strict subset of his journals. Therefore, the irrelevance result does not hold when the industry profit is equal to M.

[20] The payoff from buying journals 11 and 21 is $4 + 5\sqrt{12.5 - 2 * 1.182} = 19.918543$; the payoff from buying 11 and 12 (or 21 and 12) is $12 + 5\sqrt{12.5 - 1.182 - 8.81} = 19.918333$; the payoff from buying all the journals is $14 + 5\sqrt{12.5 - 2 * 1.182 - 8.81} = 19.7576$.

This example might appear counterintuitive since the library buys the three journals under prices $\mathbf{p}^*$ while after the increase in p_{11} and p_{21}, the library buys journals 11 and 21 but not journal 12 of which the price has not increased. Note first that conditional on that journals 11 and 21 are purchased at prices $p_{11} = p_{11}^* + \varepsilon_1 > p_{11}^*$ and $p_{21} = p_{21}^* + \varepsilon_2 > p_{21}^*$, the effect of pecuniary externalities implies that journal 12 is not purchased because $u_{12} = U_{MB}(p_{12}^*, p_{11}^* + p_{21}^* + p_{12}^*)$ but $u_{12} < U_{MB}(p_{12}^*, p_{11}^* + \varepsilon_1 + p_{21}^* + \varepsilon_2 + p_{12}^*)$. However, given that it is suboptimal to buy all journals, it is puzzling that the dropped journal is the one whose price is unchanged. The direct comparisons of payoffs among different alternatives sheds light on this issue; consider $\varepsilon_1 = \varepsilon_2 = \varepsilon$. If journals 11 and 21 are purchased, the library's payoff is reduced by $v(M - \pi + p_{12}^*) - v(M - \pi + p_{12}^* - 2\varepsilon)$ with respect to the payoff before the changes in prices; if journals 11 and 12 (or 21 and 12) are purchased, the library's payoff decreases by $v(M - \pi + p_{11}^*) - v(M - \pi + p_{11}^* - \varepsilon)$. Therefore, journal 12 is eliminated if $v(M - \pi + p_{12}^*) - v(M - \pi + p_{12}^* - 2\varepsilon) < v(M - \pi + p_{11}^*) - v(M - \pi + p_{11}^* - \varepsilon)$. Even though $2\varepsilon > \varepsilon > 0$, if $p_{12}^* > p_{11}^*$ it is possible that the inequality holds for some ε because of the strict concavity of v: in the previous example, it holds for $\varepsilon = 0.06$. There is much more money left for books when an expensive journal is dropped than when a cheap one is dropped. Therefore, the utility loss from spending 2ε less money on books in the former case can be smaller than the utility loss from spending ε less money in the latter case.

Our previous result on bundling (theorem 1) is valid regardless of whether the journals are homogenous or heterogenous since the relevant parameters are the values $(U_1, ..., U_N)$ of the different bundles. Conditional on equilibrium existence under independent pricing, all the other results that we obtained in the case of homogenous journals hold in the general case of heterogenous journals as well: the results regarding the incentive to bundle, the effect of bundling on profits, social welfare, mergers and the incentive to acquire a journal hold in the general case. We also emphasize that in the three stage game in which each publisher chooses between bundling and no-bundling, there always exists a SPNE in which every active publisher bundles his journals and E^* and $\mathbf{P}^*$ are determined by theorem 1. Furthermore, any SPNE in which a publisher does not bundle his journals requires the use of a weakly dominated strategy from proposition 2(i).

5 Concluding remarks

Our analysis reveals that there is a strong conflict between the private and social incentives of bundling e-journals: each publisher wants to bundle his journals and bundling increases the industry profit but reduces social welfare. In particular, big publishers' bundling not only reduces consumption of monographs but also can make small publishers unable to sell their journals even though they own high-quality journals. In this respect, it is noteworthy that Wolters Kluwer, which is the sixth-largest player in the industry by revenues, recently opted to exit scientific publishing and to focus solely on medical publishing citing lack of scale as the reason for the exit.[21]

We found that bundling has two other important effects. First, bundling creates incentives for mergers. However, any asymmetry-increasing merger reduces social welfare by reducing book and journal consumption. In contrast, mergers among small publishers who would not be able to sell their journals because of their lack of size might increase social welfare. Alternatively, it would be desirable for small publishers having high-quality journals to sell their journals through a common agency as in the case of JSTOR. Second, bundling can have a serious impact on the evolution of market structure by changing the incentive to acquire other journals. We have shown that in the absence of bundling, publishers have the same willingness to pay for a journal while, under bundling, the largest publisher has always the highest willingness to pay. Hence, bundling might create a vicious cycle through which big publishers induce exit of small publishers and become even bigger by purchasing their titles.

Finally, our framework can be applied to other economic situations such as bundling (block booking) in distribution of movies, TV or radio programs[22]. Our analysis suggests that block booking of high-quality movies with low-quality ones would make it difficult for small producers to get their movies into theaters even though they have high-quality and therefore provides a rationale for the *per se* illegal status of block booking in U.S. under section 1 of the Sherman Act.[23]

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[21] See Gooden et al. (2002).

[22] Since we assume price discrimination based on usage, our explanation of block booking is very different from the one based on second-degree price discrimination given by Stigler (1968).

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Appendix

Lemma 1 *Let A denote a set of items with $P_A = \sum_{a \in A} p_a \leq M$. If only the items in A are available for purchase, then all of them are purchased if and only if $u_a + v(M - P_A) \geq v(M - P_A + p_a)$ for all $a \in A$.*

Proof. Let $U_A \equiv \sum_{a\in A} u_a$.

($\Rightarrow$) If buying all the items in A is optimal, then buying all must give a higher utility than buying all except an item a for all $a \in A$. Therefore, we get the following inequality:

$$U_A + v(M - P_A) \geq U_A - u_a + v(M - P_A + p_a), \quad \text{for all } a \in A,$$

which is equivalent to $u_a + v(M - P_A) \geq v(M - P_A + p_a)$ for all $a \in A$.

($\Leftarrow$) Let $B \subseteq A$ denote a subset of A with $P_B = \sum_{a\in B} p_a$ and $U_B \equiv \sum_{a\in B} u_a$. Suppose that $u_a + v(M - P_A) \geq v(M - P_A + p_a)$ holds for all $a \in A$, which implies

$$\sum_{a\in B} u_a \,(= U_B) \geq \sum_{a\in B} U_{MB}(p_a, P_A).$$

Furthermore, since $U_{MB}(p_a, P_A)$ is strictly concave in the first argument and $P_B = \sum_{a\in B} p_a$, we have:

$$\sum_{a\in B} U_{MB}(p_a, P_A) \geq U_{MB}(P_B, P_A),$$

implying $U_B \geq U_{MB}(P_B, P_A)$. If this inequality holds, buying all items in A gives a higher utility than buying all except a subset B for any $B \subseteq A$ since

$$U_A + v(M - P_A) \geq U_A - U_B + v(M - P_A + P_B) \Leftrightarrow U_B \geq U_{MB}(P_B, P_A).$$

Therefore, buying all the items in A is optimal. ■

Proof of Theorem 1

We first prove two lemmas which allow us to easily prove theorem 1. We first analyze the game starting at stage two. In the following lemma we consider the case in which all the publishers entered at stage one. This is without loss of generality since all our arguments in the proof of the lemma hold true even if a strict subset of $\{1, ..., N\}$ entered at stage one.

Lemma 2 *Suppose that all publishers enter.*
(i) There exists a (unique) NE $\boldsymbol{P}^*$ *in which each publisher makes a strictly positive profit with* $\pi^{B*} = M$ *if and only if* $\sum_{j=1}^{N-1} v^{-1}(U_j - U_N) < M \leq \sum_{j=1}^{N} v^{-1}(U_j)$. *In the equilibrium, we have:*

$$U_j - v(P_j^*) = U_{j'} - v(P_{j'}^*) \geq 0 \qquad \text{for any } \{j, j'\} \subset \{1, ..., N\}. \tag{12}$$

(ii) There exists a (unique) NE $\boldsymbol{P}^*$ *in which each publisher makes a strictly positive profit with* $\pi^{B*} < M$ *if and only if* $M > \sum_{j=1}^{N} v^{-1}(U_j)$. $\boldsymbol{P}^*$ *satisfies (4).*

Proof of (i) We split the proof into two claims.

Claim 1 $\mathbf{P}^*$ such that $P_j^* > 0$ for any j and $\pi^{B*} = M$ is a NE if and only if it satisfies (12).

($\Leftarrow$) Suppose $P_j = P_j^* > 0$ for any j with $\pi^{B*} = M$ and $\mathbf{P}^*$.satisfies (12). Consider now a deviation of publisher j. For any $P_j \leq P_j^*$, all bundles are sold by lemma 1 and the profit of j is P_j. If publisher j sets $P_j > P_j^*$, then the library can afford at most $N-1$ bundles and $U - U_k + v(P_k^*) < U - U_j + v(P_j)$ for any $k \neq j$. Thus, the library buys all the bundles except Bj and j's profit is 0.

($\Rightarrow$) Assume that a NE $\mathbf{P}^*$ exists such that $P_j^* > 0$ for any j and $\pi^{B*} = M$. By lemma 1, all the bundles are sold if and only if

$$U_j \geq v(P_j^*) \qquad \text{for } j = 1, ..., N. \tag{13}$$

In order for publisher j to have no incentive to increase P_j above P_j^*, the following condition – which jointly with (13) reduces to (12) – must be satisfied:

$$U - U_k + v(P_k^*) \leq U - U_j + v(P_j^*) \qquad \text{for any} \quad j, k \tag{14}$$

To see why, suppose $U - U_k + v(P_k^*) > U - U_j + v(P_j^*)$ for some j, k and publisher j slightly increases P_j above P_j^*. Then, the library cannot afford to buy all the bundles and $U - U_k + v(P_k^*) > U - U_j + v(P_j)$ implies that it prefers to drop Bk rather than Bj. Therefore, j's profit is higher after the price increase.

Claim 2 A (unique) $\mathbf{P}$ exists which satisfies $P_j > 0$ for any j, $\pi^B = M$ and (12) if and only if $\sum_{j=1}^{N-1} v^{-1}(U_j - U_N) < M \leq \sum_{j=1}^{N} v^{-1}(U_j)$ holds.

Write $(P_1, ..., P_{N-1})$ as a function of P_N by using (12): $P_j = v^{-1}[U_j - U_N + v(P_N)]$, for $j = 1, ..., N-1$. Combining this with $\pi^B = M$, we obtain:

$$F(P_N) \equiv \sum_{j=1}^{N-1} v^{-1}[U_j - U_N + v(P_N)] + P_N - M = 0 \tag{15}$$

F is strictly increasing in P_N and $v^{-1}(U_N)$ is the highest value of P_N consistent with (13). We find $F(0) = \sum_{j=1}^{N-1} v^{-1}(U_j - U_N) - M$ and $F[v^{-1}(U_N)] = \sum_{j=1}^{N} v^{-1}(U_j) - M$. Thus, a (unique) solution $P_N^* > 0$ to (15) exists if and only if $\sum_{j=1}^{N-1} v^{-1}(U_j - U_N) < M \leq \sum_{j=1}^{N} v^{-1}(U_j)$ is satisfied. This proves statement (i) of the lemma.

Proof of (ii) The proof of this statement requires proving three claims.

Claim 1 $\mathbf{P}^*$ such that $P_j^* > 0$ for any j and $\pi^{B*} < M$ is a NE if and only if it satisfies (4).

($\Leftarrow$) Assume that $\mathbf{P}$ such that $P_j > 0$ for any j and $\pi^B < M$ satisfies (4). Then publisher j has no incentive to increase the price of Bj above P_j because that would induce the library not to buy Bj.
($\Rightarrow$) Assume a NE $\mathbf{P}^*$ exists such that $P_j^* > 0$ for any j and $\pi^{B*} < M$. Then the inequality $U + v(M - \pi^{B*}) \geq U - U_j + v(M - \pi^{B*} + P_j^*)$ must hold for any j, otherwise the library does not buy all the bundles. However, if the above inequality is satisfied strictly for a particular j, then publisher j (given $\mathbf{P}_{-j} = \mathbf{P}^*_{-j}$) has the incentive to deviate by choosing $P_j > P_j^*$ such that $U + v(M - \pi^{B*}_{-j} - P_j) > U - U_j + v(M - \pi^{B*}_{-j})$ and in this case the library still buys all the bundles, including Bj. Therefore, (4) needs to be satisfied by $\mathbf{P}^*$.
Claim 2 If (4) has a solution such that $\pi^B < M$, then $M > \sum_{j=1}^N v^{-1}(U_j)$.
Let $\varepsilon \equiv M - \pi^B > 0$. Then (4) is written as $v(\varepsilon + P_j) - v(\varepsilon) = U_j$ for any j. Therefore, we have $P_j = v^{-1}[U_j + v(\varepsilon)] - \varepsilon$ and P_j is strictly increasing in ε. By adding up over j we find $M > \sum_{j=1}^N P_j = \sum_{j=1}^N v^{-1}[U_j + v(\varepsilon)] - n\varepsilon > \sum_{j=1}^N v^{-1}(U_j)$.
Claim 3 If $M > \sum_{j=1}^N v^{-1}(U_j)$, then (4) has a solution such that $\pi^B < M$; moreover, such a solution is unique.
From (4) we obtain $P_j = \pi - M + v^{-1}[U_j + v(M - \pi)]$ for all j, and adding up over j yields

$$\sum_j v^{-1}[U_j + v(M - \pi)] + (N-1)\pi = NM \tag{16}$$

At $\pi = 0$ the left hand side is larger than NM since $U_j > 0$ for any j. At $\pi = M$ the left hand side is $\sum_{j=1}^N v^{-1}(U_j) + (N-1)M$ which is smaller by assumption than NM. Thus, there exists a $\pi^* \in (0, M)$ which solves (16) and prices are obtained as $P_j = \pi^* - M + v^{-1}[U_j + v(M - \pi^*)]$. Furthermore, the left hand side of (16) is monotonically decreasing in π, hence π^* is unique and so equilibrium prices.

We now consider the three stage game and present the next lemma which focuses on entry:

Lemma 3 *(i) If $\sum_{j=1}^{k-1} v^{-1}(U_j - U_k) < M$ for some $k \in \{2, ..., N\}$, $\{1, ..., k\} \subseteq E^*$ in any SPNE. For any $M > 0$, publisher 1 enters the market in any SPNE.*
(ii) If $M \leq v^{-1}(U_1 - U_2)$, in any SPNE $j \notin E^*$ for any $j > 1$. If $\sum_{j=1}^{k-1} v^{-1}(U_j - U_k) < M \leq \sum_{j=1}^{k} v^{-1}(U_j - U_{k+1})$ for some $k \in \{2, ..., N\}$ where $U_{N+1} \equiv 0$, in any SPNE $j \notin E^*$ for any $j > k$.

Proof of (i): Suppose that $\sum_{j=1}^{k-1} v^{-1}(U_j - U_k) < M$ for some $k \in \{2, ..., N\}$. We prove that if $j \notin E^*$ for some $j \leq k$, then publisher j has an incentive to enter since $P_j^* > 0$ in any NE of the game which starts at stage two when the set of active publishers is $E^* \cup \{j\}$.

Consider an arbitrary NE $\mathbf{P}^*$ of this game and let T denote the subset of publishers in $E^* \cup \{j\}$ which obtain a positive profit in this NE. Arguing by contradiction, suppose that $j \notin T$. This fact implies (i) $\sum_{h\in T} P_h^* = M$, otherwise publisher j may earn a positive profit by setting P_j smaller than $M - \sum_{h\in T} P_h^*$ and close to 0; (ii) $h \notin T$ for any $h > j$ since, otherwise, j would gain $P_h^* > 0$ by setting $P_j = P_h^*$ for some $h > j$ such that $h \in T$ (in this case, the library would prefer to buy Bj rather than Bh); therefore $T \subset \{1, ..., k\}$. Let $z \in \arg\min_{h\in T}[U_h - v(P_h^*)]$; thus, $U_h - v(P_h^*) \geq U_z - v(P_z^*)$ for any $h \in T$ or, equivalently, $v^{-1}[U_h - U_z + v(P_z^*)] \geq P_h^*$. Hence, $M = \sum_{h\in T} P_h^* \leq \sum_{h\in T\backslash\{z\}} v^{-1}[U_h - U_z + v(P_z^*)] + P_z^*$. This right hand side is increasing in P_z^* and $v^{-1}(U_z - U_j)$ is the highest possible value for P_z^*: if $P_z^* > v^{-1}(U_z - U_j)$, then publisher j can realize a positive profit by setting P_j close to 0 since, after buying the bundles Bh for $h \in T\backslash\{z\}$, the library would prefer to buy Bj rather than Bz. At $P_z^* = v^{-1}(U_z - U_j)$, $M \leq \sum_{h\in T\backslash\{z\}} v^{-1}[U_h - U_z + v(P_z^*)] + P_z^*$ reduces to

$$M \leq \sum_{h\in T} v^{-1}(U_h - U_j) \tag{17}$$

However, from $T \subset \{1, ..., k\}$ and $U_j \geq U_k$ we obtain $\sum_{h\in T} v^{-1}(U_h - U_j) \leq \sum_{h=1}^{k-1} v^{-1}(U_h - U_k)$ and the latter term is smaller than M by assumption. Therefore, (17) is violated and we obtain a contradiction.

The argument in the first paragraph of the proof taken with $k = 1$ leads to the contradiction that the set T is such that $\sum_{h\in T} P_h^* = M$ and $h \notin T$ for any $h \geq 1$. Thus, $1 \in E^*$ in any SPNE.

Proof of (ii): In this proof, let $k = 1$ if $M < v^{-1}(U_1 - U_2)$; otherwise k is defined as in the statement of lemma 3(ii). We know $\{1, ..., k\} \subseteqq E^*$. Suppose that $j \in E^*$ for some $j > k$ and, to fix the ideas (without loss of generality), that $h \notin E^*$ for any $h > j$. We below prove that no NE of the game starting at stage two exists such that $P_h^* > 0$ for any $h \in E^*$. From lemma 2 above, the existence of such NE requires $\sum_{h\in E^*\backslash\{j\}} v^{-1}(U_h - U_j) < M$, while by assumption we have $\sum_{h\in E^*\backslash\{j\}} v^{-1}(U_h - U_j) \geq \sum_{h=1}^{k} v^{-1}(U_h - U_{k+1}) \geq M$, where the first inequality follows from $\{1, ..., k\} \subseteq E^*$. Therefore, we obtain a contradiction. ■

♣ **Proof of theorem 1**

We note that lemma 2 can be adapted to the setting in which only the k largest publishers entered at stage one by replacing N with k.

Proof of (i): Lemma 3 implies $E^* = \{1\}$; lemma 2(i) implies $P_1 = M$.

Proof of (ii): Lemma 3 implies $E^* = \{1, ..., k\}$. Lemma 2(i) written with k instead of N implies that a (unique) NE $\mathbf{P}^*$ exists in which each publisher makes a strictly positive profit with $\pi^{B*} = M$. In order to prove that (3) holds, we need to show that

$U_j - v(P_j^*) \geq U_{k+1}$ for any $j \in E^*$. Adapting the proof of lemma 2(i) to our setting with k active publishers, we infer that $\mathbf{P}^*$ satisfies (12), hence let $P_j = v^{-1}[U_j - U_k + v(P_k)]$, for $j = 1, ..., k-1$ and P_k solve

$$G(P_k) \equiv \sum_{j=1}^{k-1} v^{-1}[U_j - U_k + v(P_k)] + P_k - M = 0 \tag{18}$$

Since G is strictly increasing and $\sum_{j=1}^{k-1} v^{-1}(U_j - U_k) < M \leq \sum_{j=1}^{k} v^{-1}(U_j - U_{k+1})$, there exists a (unique) solution P_k^* to (18) and $0 < P_k^* \leq v^{-1}(U_k - U_{k+1})$; hence $U_k - v(P_k^*) \geq U_{k+1}$ and $U_j - v(P_j^*) \geq U_{k+1}$ for any $j = 1, ..., k-1$ with $P_j^* = v^{-1}[U_j - U_k + v(P_k^*)]$.

Proof of (iii): From lemma 3(i), each firm makes a strictly positive profit upon entry; Therefore, all the firms enter. The rest of the result follows from lemma 2(ii). ■

Here we provide the complete strategies for a SPNE. When the set of active publishers is $E \neq \emptyset$, let $U^{(j)}$ denote the j-highest value in $\{U_h : h \in E\}$. If $k \in \{1, ..., \#E\}$ ($\#E$ is the number of elements in E) is such that $\sum_{j=1}^{k-1} v^{-1}(U^{(j)} - U^{(k)}) < M \leq \sum_{j=1}^{k} v^{-1}(U^{(j)} - U^{(k+1)})$ [with $U^{(k+1)} \equiv 0$ if $k = \#E$], the profile of prices $\mathbf{P}= \{P_h : h \in E\}$ characterized as follows is a NE of the game starting at stage two: $P_j = M$ if $U_j < U^{(k)}$; $U_j - v(P_j) = U_h - v(P_h) \geq U^{(k+1)}$ if $U_j \geq U^{(k)}$ and $U_h \geq U^{(k)}$; $\sum_{j:U_j \geq U^{(k)}} P_j = M$ [in this NE, Bj is sold if and only if $U_j \geq U^{(k)}$]. If instead $M > \sum_{j=1}^{\#E} v^{-1}(U^{(j)})$, then $\mathbf{P}$ is a NE if and only if it satisfies $v(M - \sum_{h \in E \setminus \{j\}} P_h) - v(M - \sum_{h \in E} P_h) = U_j$ for any $j \in E$. ■

Proof of Proposition 2

We use $I \subseteq E^*$ to represent the set of publishers which enter and sell their journals independently; $B \equiv E^* \setminus I$ is the set of publishers which enter and bundle their journals. Let π_j denote the profit of publisher j, $j \in E^*$; $\pi \equiv \sum_{j \in E^*} \pi_j$ is the industry profit. In any SPNE we have $\pi_j > 0$ for any $j \in E^*$, otherwise j would not enter; therefore, Bj is sold for any $j \in B$ and each $j \in I$ sells at least one of his journals. Without loss of generality, we assume $p_{1j} \geq ... \geq p_{n_j j}$ for any $j \in I$. For any $j \in I$, let T_j denote the set of journals not sold and S_i the set of sold journals; $T \equiv \cup_{i \in I} T_i$ and $S \equiv \cup_{i \in I} S_i$. Homogeneity implies that the h cheapest journals are sold, where h is determined endogenously.

Proof of (i) Suppose that $j \in I$ and the library optimally spends π_j in buying some journals of publisher j. Then it is still optimal for the library to buy Bj at price π_j.

Proof of (ii) We first prove in three steps that in any SPNE, all journals of all active publishers are sold and use this property to prove proposition 2(ii).

Step 1 If $\#I \geq 2$, then each journal sold independently has the same price $\tilde{p}$: $p_{ij} = \tilde{p}$ for any $ij \in S$.

Proof. For each $j \in I$, let $p_j^{(1)} \equiv \max_{ij \in S_j}\{p_{ij}\}$ be the highest price for the journals sold by publisher j. Without loss of generality we assume $\{1,2\} \subseteq I$ and then prove that $p_1^{(1)} = p_2^{(1)}$. If $p_1^{(1)} < p_2^{(1)}$ let publisher 1 increase $p_1^{(1)}$ by $\varepsilon > 0$ and small; we show that all his journals in S_1 are still sold, hence π_1 increases. Indeed, if at least one journal of publisher 2 with price $p_2^{(1)}$ is still purchased, then all the journals in S_1 are so since they are cheaper; if instead all journals of 2 with price $p_2^{(1)}$ are not purchased, then certainly no journal in T is bought ($p_{ij} > p_2^{(1)}$ for any $ij \in T$) and the new industry profit is at most $\pi - p_2^{(1)} + \varepsilon < \pi$; $u \geq U_{MB}(p_1^{(1)}, \pi)$ implies $u \geq U_{MB}(p_1^{(1)} + \varepsilon, \pi - p_2^{(1)} + \varepsilon)$ for a small ε. With a similar argument we prove that p_{i1} is constant for any $i1 \in S_1$. If $p_1^{(1)} > p_{i1}$ for some $i1 \in S_1$, then 1 can charge a uniform price for his journals in S_1 such that his profit slightly increases if all journals in S_1 are sold. This is certainly the case since (i) if at least one journal of publisher 2 with price $p_2^{(1)}$ is still purchased, then all the journals in S_1 are so since they are cheaper; (ii) if instead all journals of 2 with price $p_2^{(1)}$ are not purchased, then no journal in T is bought and the positive pecuniary externality effect is at work. ■

Step 2 If $\#I \geq 2$, then all the journals are sold.

Proof. If $i1 \in T_1$ for some i, let publisher 1 set $p_{i1} = \varepsilon$ and reduce all prices of journals in S_1 by $\frac{\varepsilon}{1+n_1}$. All journals in $S_1 \cup \{i1\}$ are purchased both if some journal of 2 is so (because the journals in $S_1 \cup \{i1\}$ are cheaper then the journals in S_2) and also if no journal of 2 is bought, in view of positive pecuniary externality. ■

Step 3 In no SPNE we have $I = \{1\}$ and $T_1 \neq \emptyset$.

Proof. If $I = \{1\}$, $T_1 \neq \emptyset$ and $\pi < M$, let 1 charge uniform price $p_1 = \frac{\pi_1 + \varepsilon}{n_1}$ for all of his journals. Then all his journals are purchased since $u \geq U_{MB}(p_1^{(1)}, \pi)$ and $p_1^{(1)} > p_1$ imply $u \geq U_{MB}(p_1, \pi + \varepsilon)$ if ε is small.

If $I = \{1\}$, $T_1 \neq \emptyset$ and $\pi = M$, let $k = \#T_1$ and $k' = \#S_1$. Consider the deviation of $j \in B$ which consists in increasing the price of Bj by $\varepsilon > 0$. A necessary condition for this deviation to be not profitable is that there exists k_j such that

$$U - U_j + k_j u + v(P_j - k_j \bar{p}_j) \geq U - u + v(p_1^{(1)} - \varepsilon) \tag{19}$$

where $\bar{p}_j$ is the average price of the k_j cheapest journals in T_1 Suppose 1 bundles all journals at price $P_1 = \pi_1 + \delta$. Then the payoff from dropping Bj is $U - U_j + v(P_j - \delta)$, while the payoff from eliminating B1 is $U - U_1 + v(\pi_1)$; we prove that $U_1 + v(P_j - \delta) > U_j + v(\pi_1)$. Since $U_1 = (k' + k)u$ and (19) hold, it suffices to prove that $k'u + U_j + v(p_1^{(1)} - \varepsilon) - u - v(P_j - k_j \bar{p}_j) + v(P_j - \delta) > U_j + v(\pi_1)$, which holds since $\delta < k_j \bar{p}_j$, $u \geq v(p_1^{(1)})$ and $\pi_1 \leq k' p_1^{(1)}$. ■

Therefore, in the rest of the proof we can assume that all journals of all active publishers are sold.

Consider first the case in which $\pi < M$. In view of the analysis carried out in the proof of theorem 2[24], we have $u_a = U_{MB}(p_a, \pi)$ for any item a (a journal or a bundle); suppose that publisher j bundles his $n_j \geq 2$ journals at price $P_j = \pi_j + \varepsilon$ with $\varepsilon > 0$ and small. We prove that Bj is going to be sold, hence the profit of publisher j increases with respect to individual sale. Denote by Z the set of items which the library does not buy after the bundling by publisher j; $P_Z \equiv \sum_{a \in Z} P_a$; we now derive a contradiction under the assumption that B1 belongs to Z. The library's payoff is then $\sum_{a \notin Z} u_a + v(M - \pi + P_Z)$, but if it adds $B1$ to the items it purchases its payoff becomes $n_j u + \sum_{a \notin Z} u_a + v(M - \pi + P_Z - \pi_1 - \varepsilon)$ and it is larger than $\sum_{a \notin Z} u_a + v(M - \pi + P_Z)$ if and only if $n_j u + v(M - \pi + P_Z - \pi_1 - \varepsilon) > v(M - \pi + P_Z)$, which is equivalent to

$$n_j u > U_{MB}(\pi_1 + \varepsilon, \pi - P_Z + \pi_1 + \varepsilon) \tag{20}$$

From $u = U_{MB}(\tilde{p}, \pi)$ we find $n_j u > U_{MB}(\pi_1, \pi)$ and since $\pi_1 \leq P_T$, (20) holds at $\varepsilon = 0$ and by continuity it holds for a small $\varepsilon > 0$ as well.

Consider now the case of $\pi = M$. Then we have $u_a - v(M - \pi + p_a) = z \geq 0$ for any item a, in view of the proof of theorem 2 If publisher j bundles his n_j journals at price $\pi_j + \varepsilon$ with $\varepsilon > 0$ and small, the library has not enough money to buy all available items, but it will purchase all except one by lemma 1. The payoff from excluding Bj is $U - un_j + v(\pi_j)$ and the payoff from dropping a different item a is $U - u_a + v(p_a - \varepsilon)$. The latter payoff is larger than the former since $\pi_j = n_j \tilde{p}$ and $n_j u - v(n_j \tilde{p}) > n_j z > z = u_a - v(p_a)$, hence the inequality $n_j u - v(\pi_j) > u_a - v(p_a - \varepsilon)$ holds for a small ε. ■

Proof of Proposition 4

Without loss of generality, we suppose that publisher 1 merges with publisher 2. Let P_j^{BM*} and P_j^{AM*} denote the prices before the merger and after the merger, respectively, of Bj, $j = 1, ..., N$; $P_{1\&2}^{*}$ is the price charged by publisher 1&2 after the merger. Consider first the case in which $\sum_{j=1}^{N} v^{-1}(U_j) < M$, so that $\pi^{BM*} < M$ and assume that $v^{-1}(U_1 + U_2) + \sum_{j=3}^{N} v^{-1}(U_j) < M$, otherwise it is obvious that $\pi^{AM*} > \pi^{BM*}$. To prove the result by contradiction, we suppose $\pi^{AM*} \leq \pi^{BM*}$. Condition (4) implies

$$U_1 + U_2 = U_{MB}(P_1^{BM*}, \pi^{BM*}) + U_{MB}(P_2^{BM*}, \pi^{BM*}) \tag{21}$$

$$U_1 + U_2 = U_{MB}(P_{1\&2}^{*}, \pi^{AM*}) \tag{22}$$

[24] Since we must consider the case in which some journals are bundled while the others are not, we need to use the result from theorem 2 which covers the no-bundling in the general case of heterogenous journals.

Since $U_{MB}(P_1^{BM*}, \pi^{BM*}) + U_{MB}(P_2^{BM*}, \pi^{BM*}) > U_{MB}(P_1^{BM*} + P_2^{BM*}, \pi^{BM*})$, we have $U_{MB}(P_{1\&2}^*, \pi^{AM*}) > U_{MB}(P_1^{BM*} + P_2^{BM*}, \pi^{BM*})$. This inequality with $\pi^{AM*} \leq \pi^{BM*}$ implies $P_{1\&2}^* > P_1^{BM*} + P_2^{BM*}$. Furthermore, (4) for $j = 3, ..., N$ implies

$$\frac{dP_j}{d\pi} = 1 - \frac{v'(M - \pi)}{v'(M - \pi + P_j)} < 0 \tag{23}$$

which says that the merger (weakly) increases the profit of any non-merged publisher. Since the merger increases each publisher's profit, it contradicts the assumption $\pi^{AM*} \leq \pi^{BM*}$. Therefore, we must have $\pi^{AM*} > \pi^{BM*}$; this implies that the profit of publisher j ($j = 3, .., N$) is reduced because of (23) and hence the profit of publisher 1&2 is larger than $P_1^{BM*} + P_2^{BM*}$.

The result can be similarly proved for the case in which $\sum_{j=1}^{k-1} v^{-1}(u_j - u_k) < M \leq \sum_{j=1}^{k} v^{-1}(u_j - u_{k+1})$ for some $k \geq 3$, so that $E^* = \{1, ..., k\}$ and $\pi^{BM*} = M$.

Proof of Theorem 2

Proof of theorem 2(a): Consider the case $M > \sum_{i=1}^{n} v^{-1}(u_{i1})$. Let π^{I*} denote the industry profit under the n-publisher-n-journal setting. We first prove by contradiction that the monopolist cannot achieve a profit higher than π^{I*}. Suppose that the monopolist can realize a profit $\pi > \pi^{I*}$. This implies that among the journals sold, there must be at least a journal $i1$ of which the price p_{i1} is strictly higher than the price under the n-publisher-n-journal setting p_{1i}^*. Then, we have:

$$u_{i1} = u_{1i} = U_{MB}(p_{1i}^*, \pi^{I*}) < U_{MB}(p_{i1}, \pi).$$

This implies that the journal is not purchased and we get a contradiction.

With a similar argument, we can prove by contradiction that selling all the journals at prices $\mathbf{p}^*$ is the only way to realize π^{I*}. If there is any other way to achieve the profit π^{I*}, then among the journals sold, there must be at least a journal $i1$ of which the price p_{i1} is strictly higher than p_{1i}^* and this journal cannot be sold as we have seen above.
If $M \leq \sum_{i=1}^{n} v^{-1}(u_{i1})$, the monopolist can obtain profit M by choosing the prices $\mathbf{p}^*$ as under the n-publisher-n-journal setting because they induce the library to buy all the journals $11, ..., k1$ from lemma 1.

Proof of theorem 2(b): Steps 1 and 2 below show that if $\sum_{h=1}^{n-1} v^{-1}(u^{(h)} - u^{(n)}) < M$, the only possible NE prices in the pricing game, conditional on all publishers entering, are the one in the n-publisher-n-journal setting. Step 3 proves that (11) is a necessary condition for NE.

Step 1 If $\sum_{j=1}^{N}\sum_{i=1}^{n_j} v^{-1}(u_{ij}) < M$, then in any NE of the pricing game the condition $u_{ij} = U_{MB}(p_{ij}, \pi)$ is satisfied for any ij.

Proof. By using lemma 1 as in the proof to part (a) above we find that $\pi < M$ in any NE. Lemma 1 also implies

$$u_{ij} \geq U_{MB}(p_{ij}, \pi) \tag{24}$$

for any ij. We now prove by contradiction that any NE prices satisfy $u_{ij} = U_{MB}(p_{ij}, \pi)$. Suppose that (24) is satisfied strictly for some journal of publisher j; to fix ideas, suppose without loss of generality that $j = 1$ and that (24) is slack for $i = 1, ..., n'_1$ and binds for $i = n'_1 + 1, ..., n_1$. Let publisher 1 increase slightly p_{i1} for $i = 1, ..., n'_1$ and reduce p_{i1} slightly for $i = n'_1 + 1, ..., n_1$, without changing the sum of prices of his journals. Now, (24) is slack for all journals of 1 and all journals are still purchased. We now want to prove that the profit of publisher 1 increases if p_{11} is increased by a small $\varepsilon > 0$ because all his journals are still purchased. At the new prices $\mathbf{p}'$, suppose it is optimal for the library to buy all the journals but the ones in the set T and $i1 \in T$ for some i; $P'_T \equiv \sum_{ij \in T} p'_{ij}$. By purchasing also journal $i1$ the library's payoff increases by $u_{i1} - U_{MB}(p_{i1}, \pi - P'_T)$. The increase in payoff is strictly positive for all $\varepsilon > 0$ small enough since $u_{i1} > U_{MB}(p'_{i1}, \pi)$. ■

Step 2 If $\sum_{h=1}^{n-1} v^{-1}(u^{(h)} - u^{(n)}) < M < \sum_{j=1}^{N}\sum_{i=1}^{n_j} v^{-1}(u_{ij})$, then in any NE $u_{ij} - U_{MB}(p_{ij}, \pi)$ is non-negative and constant over ij.

Proof. If $M \leq \sum_{j=1}^{N}\sum_{i=1}^{n_j} v^{-1}(u_{ij})$, then $\pi = M$ in any NE because, by arguing as in the proof of theorem 1, we see that the system $u_{ij} = U_{MB}(p_{ij}, \pi)$ has no solutions. Let $z_{ij} = u_{ij} - v(p_{ij})$ and $z = \min_{ij}\{z_{ij}\}$; we prove that if $z_{ij} > z$ for some ij, then there exist a profitable deviation for publisher j; to fix ideas, suppose without loss of generality that $j = 1$ and that $z_{i1} > z$ for $i = 1, ..., n'_1$ and $z_{i1} = z$ for $i = n'_1 + 1, ..., n_1$. Let publisher 1 increase slightly p_{i1} for $i = 1, ..., n'_1$ and reduce p_{i1} slightly for $i = n'_1 + 1, ..., n_1$, without changing the sum of prices of his journals; let $\mathbf{p}'_{i1}$ be the new price vector for journals of 1. Now $z'_{i1} > z$ for all i and all journals are still purchased. We want to prove that the profit of publisher 1 increases if p_{11} is increased by a small $\varepsilon > 0$ because all his journals are still purchased. Now the library cannot afford to buy all the journals, but it can afford to (and is willing to) purchase $n - 1$ journals by lemma 1. The library's payoff if it drops journal $i1$ is $U - u_{i1} + v(p'_{i1} - \eta)$ ($\eta = 0$ if $i = 1$, $\eta = \varepsilon$ otherwise) and it is $U - u_{ij} + v(p_{ij}) = U - z$ if it eliminates a journal ij ($j \neq 1$) such that $z_{ij} = z$. Since $u_{i1} - v(p'_{i1}) > z$ for any i and ε is small, we have $U - z > U - u_{i1} + v(p_{i1} - \eta)$ for any i. ■

Step 3 (11) is a necessary condition for $\mathbf{p}^*$ to be a NE of the pricing game.

Proof. Suppose publisher 1 modifies the prices of journals 11 to $n_1 1$ by $\varepsilon_1, ..., \varepsilon_{n_1}$, respectively, with $\varepsilon = \varepsilon_1 + ... + \varepsilon_{n_1} > 0$;[25] the new prices for journals of 1 are $p_{i1} = p^*_{i1} + \varepsilon_i$, $i = 1, ..., n_1$. After these changes in prices, not all journals will be purchased; this is obvious if $\pi = M$, otherwise use lemma 1 and the equalities $u_{ij} = U_{MB}(p^*_{ij}, \pi)$ for all ij. However, if ε is small then only one journal will be dropped and the remaining $n-1$ will be purchased.

The library's payoff if it drops a journal ij $(j \neq 1)$ is $U - u_{ij} + v(M - \pi + p^*_{ij} - \varepsilon)$, reduced by $v(M - \pi + p^*_{ij}) - v(M - \pi + p^*_{ij} - \varepsilon) + k$ ($k = 0$ if $\pi < M$ and $k \geq 0$ if $\pi = M$) with respect to the payoff $U + v(M - \pi)$ before the change in prices by 1. Since v is strictly concave, this reduction in payoff is minimized for the journal ij $(j \neq 1)$ with the highest price; let $\bar{p}^*_{-1}$ denote the highest price for a journal not owned by publisher 1. If instead the library does not buy a journal $i1$, its payoff is $U - u_{i1} + v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i)$, reduced by $v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i) + k$. Therefore, the library prefers to drop the highest priced journal of the other publishers rather than one journal of 1 if $v(M - \pi + \bar{p}^*_{-1}) - v(M - \pi + \bar{p}^*_{-1} - \varepsilon) < v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i)$ for $i = 1, ..., n_1$, a set of conditions equivalent to $v(M - \pi + \bar{p}^*_{-1}) - v(M - \pi + \bar{p}^*_{-1} - \varepsilon) < \min\{v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i),\ i = 1, ..., n_1\}$.[26] Therefore, given ε, the library is interested in choosing $\varepsilon_1, ..., \varepsilon_{n_1}$ such that $\min\{v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i),\ i = 1, ..., n_1\}$ is maximized. The optimal values of $\varepsilon_1, ..., \varepsilon_{n_1}$, given ε, are denoted by $\bar{\varepsilon}_1, ..., \bar{\varepsilon}_{n_1}$ and satisfy

$$\begin{gathered} v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i) = \\ v(M - \pi + p^*_{n_1 1}) - v(M - \pi + p^*_{n_1 1} - \varepsilon + \varepsilon_{n_1}) \quad \text{for } i = 1, ..., n_1 - 1 \\ \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_{n_1} = \varepsilon \end{gathered} \tag{25}$$

if there exists a solution to (25). We prove in the following that (25) has a solution if ε is close to 0. Therefore, a profitable deviation for publisher 1 exists if, for a small value of ε, the following inequality holds

$$v(M - \pi + \bar{p}^*_{-1}) - v(M - \pi + \bar{p}^*_{-1} - \varepsilon) < v(M - \pi + p^*_{11}) - v(M - \pi + p^*_{11} - \varepsilon + \bar{\varepsilon}_1) \tag{26}$$

Clearly, (26) is an equality at $\varepsilon = 0$; we apply the implicit function theorem to (25) to obtain[27] $\frac{d\bar{\varepsilon}_i}{d\varepsilon} = 1 - \frac{(n_1-1)\prod_{h\neq i} v'(M-\pi+p^*_{h1}-\varepsilon+\bar{\varepsilon}_h)}{\sum_{s=1}^{n_1}\prod_{h\neq s} v'(M-\pi+p^*_{h1}-\varepsilon+\bar{\varepsilon}_h)}$ and $\frac{d\bar{\varepsilon}_i}{d\varepsilon} = 1 - \frac{(n_1-1)\prod_{h\neq i} v'(M-\pi+p^*_{h1})}{\sum_{s=1}^{n_1}\prod_{h\neq s} v'(M-\pi+p^*_{h1})}$ at $\varepsilon = 0$. Therefore, the derivative of the right hand side of (26) at $\varepsilon = 0$ is $-v'(M - \pi + p^*_{11})(-1 +$

[25] We allow that the prices of some journals are not changed.

[26] With $n_1 = 1$ this inequality fails because $\varepsilon_1 = \varepsilon$ and the right hand side is 0.

[27] By totally differentiating we find

$$v'(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i)(d\varepsilon - d\varepsilon_i) \quad = \quad v'(M - \pi + p^*_{n_1 1} - \varepsilon + \varepsilon_{n_1})(d\varepsilon - d\varepsilon_{n_1}),\ i = 1, ..., n_1 - 1$$

$\frac{d\bar{\varepsilon}_1}{d\varepsilon}) = \frac{(n_1-1)\prod_h v'(M-\pi+p_{h1})}{\sum_{s=1}^{n_1}\prod_{h\neq s} v'(M-\pi+p_{h1})}$ and the derivative of the left hand side of (26) at $\varepsilon = 0$ is $v'(M - \pi + \bar{p}^*_{-1})$; this proves that (11) with $j = 1$ is satisfied if no profitable deviation for publisher 1 exists. The example after theorem 2 shows that (11) is not a sufficient condition. ■

A sufficient condition for p^* to be a NE of the pricing game

Step 1 A profitable deviation for publisher 1 exists if and only if there exists a profitable deviation which induces the library to drop only one journal of other publishers.

Proof. Let π^{new} denote the industry profit after the change in prices by 1. Suppose that after the change in prices by 1, the library stops purchasing two journals a and b of publishers different from 1, with prices p^*_a and $p^*_b \geq p^*_a$. This implies $\varepsilon > p^*_b$, otherwise we have a contradiction since the industry profit after buying all journals except b is smaller than the previous profit ($\pi^{new} \leq \pi$) and therefore $U_{MB}(p_a, \pi^{new}) \leq a$ from $U_{MB}(p_a, \pi) \leq a$. Notice that $u_{i1} \geq U_{MB}(p_{i1}, \pi^{new})$ for $i = 1, ..., n_1$. Now let all the prices of journals of 1 be multiplied by $\lambda \in (0, 1)$ such that $\lambda \sum_{i=1}^{n_1} p_{i1} = \varepsilon - p^*_b + \sum_{i=1}^{n_1} p^*_{i1} > \sum_{i=1}^{n_1} p^*_{i1}$; we prove that all journals of 1 are purchased at the new prices, implying that if 1 can increase his profit by ε by inducing the library to drop two journals of the other publishers, then he can also deviate by inducing the library to drop only one other journal.

In case a and b are not purchased, then all journals of 1 are purchased since $u_{i1} > U_{MB}(\lambda p_{i1}, \pi^{new} - p^*_b)$ for $i = 1, ..., n_1$. In case that only one journal between a and b is purchased, with price $p^*_c \in \{p^*_a, p^*_b\}$, then the library's maximum expense is $\pi' = \pi^{new} - p^*_b + p^*_c \leq \pi^{new}$ and we have $u_{i1} > U_{MB}(\lambda p_{i1}, \pi')$ for $i = 1, ..., n_1$. ■

Step 2 No profitable deviation for publisher 1 exists if $\hat{\varepsilon} \equiv \frac{n_1}{n_1-1}(M - \pi + p^*_{n_1 1}) > \bar{p}^*_{-1}$ and the following inequality holds:

$$v(M-\pi+\bar{p}^*_{-1})-v(M-\pi+\bar{p}^*_{-1}-\varepsilon) > v(M-\pi+p^*_{i1})-v(M-\pi+p^*_{i1}-\varepsilon+\bar{\varepsilon}_1) \text{ for } \varepsilon \in (0, \bar{p}^*_{-1}) \tag{27}$$

Proof. We start by showing that a solution to (25) exists for any $\varepsilon < \hat{\varepsilon}$. Without loss of generality, assume $u_{11} \geq u_{21} \geq ... \geq u_{n_1 1}$, which implies $p^*_{11} \geq p^*_{21} \geq ... \geq p^*_{n_1 1}$. From $v(M - \pi + p^*_{i1}) - v(M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i) = v(M - \pi + p^*_{n_1 1}) - v(M - \pi + p^*_{n_1 1} - \varepsilon + \varepsilon_{n_1})$ the concavity of v implies that, given $\varepsilon_{n_1} \leq \varepsilon$, the left hand side is larger than the right hand side if $\varepsilon_i = p^*_{n_1 1} - p^*_{i1} + \varepsilon_{n_1}$ and (weakly) smaller if $\varepsilon_i = \varepsilon_{n_1}$; therefore,

$$\varepsilon_i \in (p^*_{n_1 1} - p^*_{i1} + \varepsilon_{n_1}, \varepsilon_{n_1}] \text{ for } i = 1, ..., n_1 \tag{28}$$

$$d\varepsilon_1 + d\varepsilon_2 + ... + d\varepsilon_{n_1} = d\varepsilon$$

with solution $\frac{d\varepsilon_i}{d\varepsilon} = 1 - \frac{(n_1-1)\prod_{h\neq i} v'(M-\pi+p^*_{h1}-\varepsilon+\varepsilon_h)}{D}$, where $D = \sum_{s=1}^{n_1}\prod_{h\neq s} v'(M-\pi+p^*_{h1}-\varepsilon+\varepsilon_h) > 0$.

Furthermore, from the same equality we find

$$\varepsilon_i = \pi - M + \varepsilon - p^*_{i1} + v^{-1}[v(M-\pi+p^*_{i1}) + v(M-\pi+p^*_{n_1 1}-\varepsilon+\varepsilon_{n_1}) - v(M-\pi+p^*_{n_1 1})],\ i = 1, ..., n_1 - 1 \tag{29}$$

By setting $\varepsilon_{n_1} = \varepsilon$ we obtain $\varepsilon_i = \varepsilon$ for all i and $\varepsilon_1 + ... + \varepsilon_{n_1} = n_1\varepsilon > \varepsilon$; by setting $\varepsilon_{n_1} = \frac{\varepsilon}{n_1}$ we obtain $\varepsilon_i < \frac{\varepsilon}{n_1}$ for $i = 1, ..., n_1 - 1$ because of (28), hence $\varepsilon_1 + ... + \varepsilon_{n_1} < \varepsilon$. Therefore, there exists $\varepsilon_{n_1} \in (\frac{\varepsilon}{n_1}, \varepsilon)$ which, together with the values found by using (29), solves (25). However, we can set $\varepsilon_{n_1} = \frac{\varepsilon}{n_1}$ only as long as $M - \pi + p^*_{n_1 1} - \varepsilon + \frac{\varepsilon}{n_1} \geq 0$, which is equivalent to $\varepsilon \leq \hat{\varepsilon}$. Notice that with $\varepsilon_{n_1} = \frac{\varepsilon}{n_1}$ we have $\varepsilon_i > p^*_{n_1 1} - p^*_{i1} + \frac{\varepsilon}{n_1}$ and $M - \pi + p^*_{i1} - \varepsilon + \varepsilon_i > M - \pi + p^*_{n_1 1} - \varepsilon + \frac{\varepsilon}{n_1}$.

We have proved above that we can restrict attention to deviations of publisher 1 which induce the library to drop only one journal of the other publishers. Hence, it suffices to consider the values of ε in $(0, \bar{p}^*_{-1})$. Since $\hat{\varepsilon} > \bar{p}^*_{-1}$, for every $\varepsilon < \bar{p}^*_{-1}$ there exists a solution to (25) and (27) implies that dropping one journal of publisher 1 is better for the library than dropping any journal of other publishers.[28]

Since $\hat{\varepsilon} \geq \frac{n_1}{n_1 - 1} p^*_{n_1 1}$, the inequality $\hat{\varepsilon} > \bar{p}^*_{-1}$ is satisfied if $\bar{p}^*_{-1}$ is not much larger than $p^*_{n_1 1}$, the price for the lowest priced journal of publisher 1; this fact reduces to saying that journals are approximately homogeneous. Likewise, if journals of publisher 1 are nearly homogeneous then $\bar{\varepsilon}_1$ is about equal to $\frac{\varepsilon}{n_1}$ and (27) holds since $\bar{p}^*_{-1}$ is close to p^*_{11}. ■

[28] We remark that for values of ε not small the library may wish to drop two or more journals of 1, even though (27) is violated.