Holdup with Subsidized Investment

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Abstract

A holdup model is analyzed in which one party, the seller, has an investment project that the other party, the buyer, can subsidize. The investment project remains the seller's; she cannot transfer her entire control rights to it. In particular, she can always refuse to allow the buyer to subsidize her investment if the subsidy would put the buyer in too strong a bargaining position ex post. Even with the subsidization opportunity, the holdup inefficiency is still present, except in the special cases in which one party has all the bargaining power. The adoption of a contract, however, allows full efficiency to be achieved more generally. In particular, if the seller's investment project imposes a positive externality on the buyer but does not reduce her own production costs, a buyer's option contract exists that achieves full efficiency. This is in contrast to the result of Che and Hausch (1999) that without a subsidization opportunity, contracting has no value in this "purely cooperative" case. If the investment lowers the seller's costs as well as raises the buyer's value, whether full efficiency can be achieved depends on how cooperative the investment is. Full efficiency can be achieved if the cooperativeness of the investment is either sufficiently high or sufficiently low.

1 Introduction

Coase's (1937) celebrated work addresses the key insight that transaction costs play a central role in the theory of the firm and directs researchers to look at sources of transaction costs. As pointed out by Klein, Crawford, and Alchian (1978) and Williamson (1985), one of the most important transaction costs is associated with the so-called *holdup problem*.

The holdup problem occurs if the value of a transaction between two parties depends on an ex ante investment. It arises under two conditions. First, the investment must be

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relationship specific, i.e., the benefit of investment must be substantially reduced if the parties change trading partners. Because of this opportunity cost of changing trading partners, the parties are locked into the relationship once the investment is made. Second, the contract must be *incomplete*, i.e., it must be impossible to specify every relevant contingency in the contract.[1] This leads to ex post bargaining in which the non-investing party can successfully demand a part of the investment benefit, i.e., the non-investing party "holds up" the investing party. Anticipating this ex post opportunistic behavior, the investing party underinvests.[2]

To the best of the author's knowledge, the existing literature on the holdup problem adopts the following common assumption: *the control right to each investment project belongs exclusively to one party.* That is, the investment level in a project is assumed to be determined solely by the party who owns the control right to that project. If a different party is supposed to choose the investment level, the control right has to be transferred, say, through integration.

In reality, however, there are investment projects whose control rights are not so exclusive. Some investment projects entail *a subsidization opportunity* by a party who is *allowed* to invest by the controlling, "owning" party. In Asanuma (1989) such evidence regarding components design can be found in the Japanese automaker-supplier relationships; some automakers, which once bought some simple parts from their suppliers, later purchased parts in "increasingly more assembled forms" (Asanuma (1989, p14)). This fact shows that such investment project as designing is not exclusive, and can be subsidized.[3] Bart (1989) documents that component designing for Xerox products has been shifting from Xerox to its suppliers. Holmström and Roberts (1998) describe the case of human capital investment; "The automakers also assist the suppliers in improving productivity and lowering costs; technical support engineers are a major part of the automakers' purchasing staff, and they spend significant amounts of time at the suppliers' facilities."[4]

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[1] This is based on the empirical observation that, in many cases, there are no written ex ante contracts on details of trade between, say, an automotive assembler and parts suppliers. This has led researchers to simply assuming there is no ex ante contract.

[2] Various ways to remedy this inefficiency problem have been proposed. For the related work, see the literature review in section 2.

[3] He classifies suppliers producing customized parts into two category; one with "drawings supplied" by the automaker, and the other with "drawings approved."

[4] We can also find discussions and cases for subsidization in other literature. Hart (1995, pp.69-70n) points out that physical investment projects are often "alienable": if its owner permits, another party can invest in the project. Coase (2000) provides evidence of subsidization by General Motors (p. 28); "It is evident that what Sloan was referring to was not a dispute over whether the Fisher Body plants should be located near the General Motors assembly plants but one over which organization should put up the capital required to do it. ... The cost was borne by General Motors. No doubt this also happened for many of the

A subsidization opportunity generates new issues regarding the parties' investment behavior. The first issue is to investigate the non-owning party's incentives to invest. Once this party is allowed to invest, the party may wish to contribute toward a more efficient investment level than the owning party wishes to achieve. This new incentives to invest by the non-owning party might be undermined by the owning party's holdup opportunity, which arises if subsidization occurs. The second issue is to investigate whether the owning party actually allows the other party to subsidize. The owning party might find that refusing to allow subsidization is optimal. It is thus not clear whether inefficiency property due to holdup is remedied if subsidization is possible.

This paper incorporates subsidization into a standard holdup model. Specifically, we analyze a holdup model in which one party, the seller, has an investment project that the other party, the buyer, can subsidize.[5] The investment project remains the seller's; she cannot transfer her entire control rights to it. The buyer can subsidize the project by directly making the investment if the seller permits him. We keep the following three typical assumptions in the standard holdup literature. (i) No aspect of investment is contractible; a contract cannot specify how much is invested or who makes the investment. (ii) Trade quantities and payments are contractible. This allows the parties to sign a menu contract from which a pair of quantity and payment is chosen according to the choice rule specified in the contract (such as buyer-option, seller-option, or more complicated rules).[6] (iii) The parties cannot commit not to renegotiate the contract; when they find the chosen trade quantity to be ex post inefficient, the parties revise it to the efficient quantity. This leads to bargaining over the additional surplus generated by renegotiation, with the status quo payoff determined by the original contract.

We restrict attention to the case where the subsidizing party has the same investing technology as the owning party. As a consequence, the gains from investment only depend on the total level of investment but not on how much each party contributes to the total investment. This is clearly a strong restriction, but not entirely unrealistic. In fact, Asanuma (1989) documents that, for such parts as seats, Japanese automakers possess detailed information about processes of producing them. This suggests that both parties have approximately the same cost structure in designing these parts.

The main result of this paper is that a buyer-option contract exists that achieves full

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other Fisher Body plants."

[5]By symmetry, we can treat the case where the buyer owns the project and the seller may subsidize, which corresponds to the above case in Asanuma (1989).

[6]Both parties are assumed to send verifiable messages to the court without costs. A pair of trade and payment can be contingent on those messages in general. An option contract requires a message (or request) from one party; for example, a buyer-option contract allows the buyer to choose a pair among the alternatives.

efficiency if the seller's investment project imposes a positive externality on the buyer but does not reduce her own production costs, i.e., the investment is "purely cooperative." Moreover, full efficiency can be achieved even if the cooperativeness of the investment is not pure, but just sufficiently high. This is in contrast to the result of Che and Hausch (1999) that without a subsidization opportunity, contracting has no value in these highly cooperative cases.

The effect of introducing a subsidization opportunity is twofold. First, it reduces the holdup inefficiency in general. The induced investment level when subsidization is possible is (weakly) more efficient than when subsidization is not possible. This is because the seller can make an additional investment even after the buyer invests and hence a positive level of subsidization occurs if and only if both parties are better off with it.[7] This implies that if full efficiency is achieved when subsidization is not possible, it is still achievable after introducing a subsidization opportunity. In particular, the following result of Edlin and Reichelstein (1996) holds even when subsidization is possible: if the seller's investment only reduces her own production cost but not improve the buyer's valuation, i.e., the investment is "purely selfish", a noncontingent contract exists such that the seller invests at the first-best level.

Second, a subsidization opportunity can cause the seller to behave opportunistically. She can always refuse to allow the buyer to subsidize her investment if the subsidy would put the buyer in too strong a bargaining position ex post. This shows that a subsidization opportunity does not simply flip the role of parties, i.e., it cannot transfer the seller's project to the buyer.

Subsidization creates two new effects of contracting. First, a contract affects the buyer's incentive to subsidize (*the incentive effect on the buyer's subsidization*). If a contract can change the status quo point of ex post bargaining in favor of the buyer, it will improve the incentive. This is a mirror image of the effect of a contract on the seller's incentive to invest. Second, a contract affects the seller's incentive to permit subsidization (*the incentive effect on the seller's permission*). This is due to the new seller opportunism discussed above. An optimal contract must take these two effects into account.

Without the seller opportunism, full efficiency for a purely cooperative investment would be rather trivial. It would be achieved as a corollary of the result of Edlin and Reichelstein (1996); the seller's purely cooperative investment would be just the buyer's purely selfish investment when subsidization is possible, and thus a noncontingent contract would

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[7]The buyer subsidizes if and only if the benefit of subsidizing exceeds that of foregoing the opportunity and letting the seller invest. The seller allows the buyer to invest if and only if the resulting subsidized investment gives her more profit than when she does not allow subsidization.

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induce the buyer to invest efficiently. However, Edlin-Reichelstein's noncontingent contract indeed induces the seller to refuse subsidization, and results in an inefficient investment. To achieve full efficiency in this purely cooperative case, a contract needs to give the right incentive to the buyer to subsidize efficiently, and the right incentive to the seller to allow the buyer to invest. It is shown in the analysis that a buyer-option contract does this but no noncontingent contract does.

The important implication of these results is that the transaction cost associated with holdup disappears if the seller's investment is highly cooperative and can be subsidized by the buyer. It is also shown that the holdup problem is solved by adopting a contract if the seller's investment has a small externality (i.e., highly selfish cases). Holdup therefore causes no inefficiency if the investment imposes either a very small or a very large positive externality. This paper provides a new evaluation of the transaction cost due to holdup when a subsidization opportunity is available.

The structure of this paper is the following. The related literature is reviewed in section 2. The set up of the model is presented in section 3. In section 4, we discuss the holdup problem and underinvestment issues with subsidization. Section 5 provides contractual remedies to the holdup inefficiency in the purely selfish and the purely cooperative cases. Section 6 deals with hybrid cases by giving a specific example. In section 7, a generalization of the results in section 6 follows. Concluding remarks are addressed in section 8.

2   Related Literature

Holdup   The literature on solutions to the holdup problem can be classified into two groups according to the degree of incompleteness of contracts.[8] The first group assumes that no aspect of a transaction is ex ante contractible. The only ex ante contractible variables are property/control rights and hence solutions to holdup take such forms as integration (Klein et al.(1978), Williamson (1985)) and allocation of property rights (Grossman and Hart (1986), Hart and Moore (1990)).

This paper belongs to the second group that assumes some aspects of a transaction, such as the quantity to be traded, are contractible. Some authors solve the holdup problem by giving full bargaining power to one party through contracting. For example, Chung

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[8] The definition of incompleteness of contracts varies among researchers. We simply define a contract to be incomplete if variables are noncontractible. Alternatively, a contract "becomes" incomplete if the parties optimally leave contingencies open. That is, specifying more contingencies is valueless in disciplining the parties' behavior, due to the complexity of the state of the world (Segal (1999) and Hart and Moore (1999)) or strategic ambiguity (Bernheim and Whinston (1998)), for example. See also Tirole (1999) for discussion.

(1991) and Aghion, Dewatripont and Rey (1994) assume the ex post bargaining procedure is contractible.[9]

This paper's approach is closest to that of Edlin and Reichelstein (1996) and Che and Hausch (1999). These papers assume that the ex post bargaining procedure is not contractible; bargaining is governed by an exogenous bargaining procedure.[10] In this case, whether full efficiency can be achieved depends on how cooperative the investment is. Edlin and Reichelstein (1996) show that full efficiency can be achieved if the investment is purely selfish. Che and Hausch (1999), on the other hand, prove that contracting has no value to improve efficiency if the investment is highly cooperative.

None of these papers considers investments that can be subsidized.

Delegation Subsidization in our model is a delegated action: the party that has the authority to control the investment is able to allow the other party to invest. Delegation of authority is extensively analyzed by Aghion and Tirole (1997). More related to this paper is the one by Aghion, Dewatripont, and Rey (2002). They consider a *non-contractible but transferable* control; the control of an action can be delegated even when a contract cannot specify this delegation. They analyze a repeated interaction of a principal and an agent in which the agent has private information. They find that in equilibrium, the principal delegates her action to the agent in an earlier period, in order for the agent to build a reputation. Our model makes a similar assumption on control rights, but differs by examining how a contract affects the incentive to subsidize.

Partnership The structure of our model is similar to that of partnership (Holmström (1982), Legros and Matthews (1993)) in which several parties contribute to the same project and share the benefits from it. In those models, the agents' efforts affect a verifiable signal upon which a contract can be written to induce efforts. In our model, there is no such verifiable signal of the exerted effort/investment levels. The contractible variables instead affect the parties' ex post bargaining position which may indirectly induce the desired effort levels. The partnership models also assume that each agent can freely contribute to the common project, whereas our model assumes the investment project is owned by an agent who is able to prohibit others from investing in it.

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[9]These papers solve the holdup problem in a two-sided investment environment. Since giving full bargaining power renders the right incentive to invest to one party, an additional instrument (e.g., a specific performance contract) is required to provide an incentive to the other.

[10]These papers take the bargaining procedure as a black box, and typically assume that the outcome of bargaining is a generalized Nash bargaining solution.

Mechanism Design with Renegotiation This paper applies results from the literature on mechanism design with renegotiation. The fundamental theoretical contribution is made by Maskin and Moore (1999), who give characterization theorems for implementability in a general setting. Segal and Whinston (2002) derive a convenient representation theorem for risk-neutral, quasilinear utilities and apply it to the standard holdup problem. We use some of their results.

3   Model

Primitives. A seller produces an intermediate good for a buyer. In advance of trade, the seller can make a relationship-specific investment. We assume that the buyer can subsidize this investment. The investment may affect the quality of the good or the costs of producing it. Further details of the investment are discussed below. Let (q, t) denote the quantity of the intermediate good to be traded and the monetary transfer from the buyer to the seller, respectively. The investment level x is measured in terms of money. Let $v(q, x)$ and $c(q, x)$ denote the buyer's ex post valuation function and the seller's ex post cost function respectively.[11] Let us assume v and c are C^2-functions defined on $[0, q_{max}] \times [0, x_{max}]$ with $v(0, x) = c(0, x) = 0.$ The buyer's valuation v is strictly increasing and strictly concave in q, and increasing and concave in x. The seller's cost c is strictly increasing and strictly convex in q, and decreasing and convex in x. The parties' payoffs are quasi-linear in the transfer t and investment cost.

The *ex post surplus generated by* (q, x) is $S(q, x) = v(q, x) - c(q, x)$. This surplus $S(q, x)$ is by assumption strictly concave in q, and thus for each x, a unique maximizer $q^*(x)$ of $S(q, x)$ exists. We call it *the ex post efficient trade given investment level* x. To characterize it by a first-order condition, we assume $0 < q^*(x) < q_{max}$. It is continuously differentiable by the implicit function theorem. Let $S^*(x) = S(q^*(x), x)$ denote *the ex post efficient surplus given investment level* x. The envelope theorem implies S^* is increasing, i.e.,

$$S^{*\prime}(x) = v_x(q^*(x), x) - c_x(q^*(x), x) \geq 0.$$

The *first-best* or *efficient* investment level is a solution to

$$\max_x S^*(x) - x.$$

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[11] These v and c are the value functions derived by the buyer's and the seller's optimization problems. The valuation function v, for example, measures the buyer's maximized profits of his final products, given his technological constraint and q units of intermediate good. The level of investment x may also affect the quality of an intermediate good, and thus the buyer's profits.

We assume that $S^*(x)$ is strictly concave in order to have a unique solution to the problem.[12] Let x^* denote the efficient investment and assume $x^* > 0$ (otherwise there is no underinvestment problem).

In addition to these assumptions, let us suppose the following *complementarity conditions* hold: (i) $v_{qx} > 0$ for all (q, x) unless $v_x \equiv 0$, and (ii) $c_{qx} < 0$ for all (q, x) unless $c_x \equiv 0$. Thus, as the investment level increases, the buyer's marginal valuation of trade increases and the seller's marginal cost of production decreases, provided the investment affects the valuation or cost. For example, we must have $v_{qx} \equiv 0$ if the investment has no effect on the buyer's valuation, i.e., $v_x \equiv 0$.

Investment The key assumption is that the seller can allow the buyer to invest. The seller may lend the buyer the key to her facility, or let her workers provide necessary expertise to the buyer. The buyer can then invest in the seller's project. It is natural to assume that the seller can invest more after the buyer invests;[13] the seller can postpone producing the good if she observes the buyer's investment is inadequate, and wants to make an additional investment.

We assume that the buyer has the same investment technology as does the seller, so that the benefit of investment does not depend on who invests. For a concrete example, consider a situation in which the seller can buy and customize a machine to be used in her facility. The buyer may well buy and customize the same machine at the same cost if the seller provides necessary expertise. As another example, consider that the seller can invest in her worker's skill for inspection of her customized products. The buyer can send his worker in the seller's place to acquire the skill at his own cost if the seller permits him to do so. This assumption implies that the total investment level is the sum of the buyer's and the seller's investments.

Subsidization is a delegated *action* in our model. It is not a side payment from the buyer to the seller. Note that a contract cannot force the seller to invest because the investment is unverifiable (discussed below). If the buyer just gives money to the seller, the seller will "take the money and run." The buyer needs to spend money by directly making the investment.

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[12]Strict concavity of S^* is implied if, for example,

$$|v_{xx} - c_{xx}| > \left| \frac{(v_{qx} - c_{qx})^2}{v_{qq} - c_{qq}} \right|,$$

for all (q, x). Thus if $v - c$ is sufficiently concave, S^* is strictly concave.

[13]The results of this paper will not change if the seller can reduce the investment level.
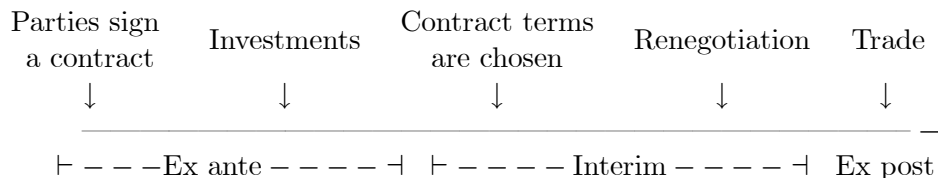
Accordingly, we consider the following extensive game at the investment stage:

1. The seller decides whether or not the buyer is allowed to invest.

2. If allowed, the buyer decides how much to invest, $x^b \geq 0$. If not allowed to invest, $x^b = 0$.

3. After observing x^b, the seller decides how much to invest, $x^s \geq 0$. The total investment level is then $x = x^b + x^s$.

The buyer's and seller's investment costs are x^b and x^s, respectively, since the investment is measured by money. The investment costs are sunk once they are made.

In the subsequent analysis, we will see that the nature of the seller's investment affects the possibility of contractual remedy to the holdup problem. Following Che and Hausch (1999), the investment is called *selfish* if it reduces the seller's cost of production, i.e., $c_x < 0$ for $q > 0$. Alternatively, the investment is called *cooperative* if it enhances the buyer's valuation, i.e., $v_x > 0$ for $q > 0$. Note that it can be both selfish and cooperative at the same time. Given the maintained assumptions, there are three cases to consider: *the purely selfish case*, $v_x = 0$ and $c_x < 0$; *the purely cooperative case*, $v_x > 0$ and $c_x = 0$; and *the hybrid case*, $v_x > 0$ and $c_x < 0$.

Timing   The time line of our model is as follows.

Parties sign a contract	Investments	Contract terms are chosen	Renegotiation	Trade
↓	↓	↓	↓	↓

⊢ − − −Ex ante − − − − ⊣ ⊢ − − − − Interim − − − − ⊣ Ex post

Contracts   Following the holdup literature, we assume that both parties observe the investment, but it is unverifiable to any third party. It is often too costly to verify relationship-specific skills, expertise, or even physical investments. The buyer's valuation and the seller's cost of production are also assumed to be unverifiable. If they are verifiable, the third party can infer the investment from those variables. The quality of the intermediate good is not verifiable either, if the investment enhances it.

A transaction is assumed to be verifiable, and a contract can thus specify a quantity to be traded and a monetary transfer. Following the implementation literature (e.g., Maskin and

Moore (1999)), we allow each party to send arbitrary messages to choose the contract terms. A contract is then generally a message game $\{q\left(m^b, m^s\right), t\left(m^b, m^s\right)\}$ where $m^i \in M^i$, $i = b, s$, is party i's message.[14] One typical message game is the "shoot-the-liar" mechanism, in which the parties have to pay a huge penalty when the messages disagree. This mechanism works well when the parties can commit not to renegotiate, but it does not work when they cannot commit (see Che and Hausch (1999) and Maskin and Tirole (1999)).

The following are relevant particular contracts. *A noncontingent contract*: $q(m^b, m^s) = \bar{q}$, $t(m^b, m^s) = \bar{t}$, $\forall m^b, \forall m^s$. *The null contract*: $q(m^b, m^s) = 0$, $t(m^b, m^s) = 0$, $\forall m^b, \forall m^s$. The null contract is equivalent to not adopting a contract. *A buyer-option contract*: $q(m^b, m^s) = q^b(m^b)$, $t(m^b, m^s) = t^b(m^b)$, $\forall m^b, \forall m^s$. A seller-option contract is defined similarly.

Renegotiation   We suppose that the parties are unable to commit not to renegotiate a contract. If the chosen contract terms $(q^0, t^0) = (q\left(m^b, m^s\right), t\left(m^b, m^s\right))$ are not ex post efficient, i.e., $q^0 \neq q^*(x)$, they renegotiate. The new transfer level is determined so that the renegotiation surplus $S^*(x) - S\left(q, x\right)$ is split according to exogenous bargaining shares (λ^b, λ^s), where $\lambda^b, \lambda^s \geq 0$ and $\lambda^b + \lambda^s = 1$.[15] This renegotiation defines the buyer's and the seller's *interim payoffs* for the pre-renegotiated outcome (q^0, t^0) and the investment level x as follows:

$$v(q^0, x) - t^0 + \lambda^b[S^*(x) - S(q^0, x)] \equiv r^b(q^0, x) - t^0, \tag{1}$$

$$t^0 - c(q^0, x) + \lambda^s[S^*(x) - S(q^0, x)] \equiv r^s(q^0, x) + t^0. \tag{2}$$

The functions r^b and r^s defined above are used in the subsequent analysis.

Ex post efficient renegotiation implies that

$$r^b(q^0, x) + r^s(q^0, x) \equiv S^*(x) \text{ for any } q^0.$$

That is, given the total investment level, the sum of the interim payoffs is the same regardless of the outcome of the message game. This shows that a contract specifies a two-person zero-sum game for any given investment level. By the minimax theorem, each player's payoff in any Nash equilibrium is the minimax value.[16]

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[14] We assume that the messages are sent simultaneously. Extensive games may help achieve uniqueness of equilibrium of the message game, but do not help improve efficiency (see footnote 16).

[15] This assumption on bargaining is as in Che and Hausch (1999) and Segal and Whinston (2002). Edlin and Reichelstein (1996) take (λ^b, λ^s) to be functions of investment and uncertainty realization. Maskin and Moore (1999) assume an arbitrary sharing rule that is efficient and individually rational. Hart and Moore (1988), MacLeod and Malcomson (1993) and Nöldeke and Schmidt (1995) specify the bargaining procedure in a non-cooperative manner.

[16] This gives a justification for the claim in footnote 14 that the use of extensive-form message game

Ex ante Payoffs and Equilibrium Investments Suppose the parties adopt a contract $\{q(m^b, m^s), t(m^b, m^s)\}$. Each party chooses its message given an investment pair, (x^b, x^s). Let $(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s))$ denote an equilibrium of the message game given the investment.[17] Define $R^b(x^b, x^s)$ and $R^s(x^b, x^s)$ by

$$R^b(x^b, x^s) = r^b(q(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s)), x^b + x^s) - t(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s)),$$

$$R^s(x^b, x^s) = r^s(q(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s)), x^b + x^s) + t(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s)).$$

These are the parties' revenues for each investment pair (x^b, x^s), given an equilibrium of the message game. We call them *the buyer's* and *the seller's ex ante revenues* respectively. The choice of equilibrium does not affect the revenues because all equilibria give the same minimax payoffs to the parties.

At the investment stage, the buyer and the seller invest x^b and x^s respectively. Recall that x^b and x^s are measured in terms of money. Therefore the buyer's and the seller's *ex ante payoffs* at the investment stage are, respectively,

$$R^b(x^b, x^s) - x^b,$$
$$R^s(x^b, x^s) - x^s.$$

The *equilibrium investments* are given by a subgame perfect equilibrium in the investment stage.

4 Holdup Problem

In this section, we show that allowing subsidization does not entirely eliminate the underinvestment problem due to holdup except for a special case in which the buyer has full bargaining power. We also show that the equilibrium investment level becomes more efficient by allowing subsidization, given any contract.

If the contract is null, the parties have no obligation to trade. Nevertheless, for any investment level x, trade will be beneficial ex post. The parties then trade and share the ex post efficient surplus. The ex ante payoffs are

$$R^b(x^b, x^s) - x^b = \lambda^b S^*(x^b + x^s) - x^b,$$
$$R^s(x^b, x^s) - x^s = \lambda^s S^*(x^b + x^s) - x^s.$$

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does not help improve efficiency: the normal form representation of the extensive form game has a Nash equilibrium that corresponds to a subgame perfect equilibrium of the extensive form game. The equilibrium payoff is the minimax of the normal form game.

[17] The equilibrium strategies might be mixed strategies and in such cases, we need to modify the notation. The payoffs are also the expected value with respect to the mixed strategies. The following analysis remains valid if the parties are risk-neutral. For notational simplicity, we keep the notation as in the text.

What does an equilibrium look like? Suppose first that the seller's bargaining share λ^s is large. In equilibrium the seller invests and the buyer does not. Even if the buyer is allowed to invest, he finds it beneficial to forgo this opportunity since the seller will make substantial (if not necessarily efficient) investment. The buyer subsidizes the seller's project only if his payoff from investing exceeds his payoff from forgoing the opportunity. Alternatively, suppose that the buyer has full bargaining power ($\lambda^b = 1$). He invests at the first-best level if he is permitted. The seller permits him to invest in equilibrium since she is indifferent between doing and not doing so.

In general, the buyer's payoff from investing is increasing in his bargaining share λ^b whereas his payoff from forgoing is decreasing (since the seller's investment is decreasing). The seller's willingness to permit subsidization grows in λ^b since the buyer's investment is increasing in λ^b. This implies that there is a threshold of λ^b over which the buyer will invest at some positive level in equilibrium.

The underinvestment problem remains unless either party has full bargaining power. To see this, consider first the case in which the seller makes a positive investment. Notice that she always invests at the margin. The investment level is efficient in this case only if she has full bargaining power. Alternatively, consider that the seller does not invest; the investment is entirely subsidized by the buyer. It is clear that the investment is efficient only if he has full bargaining power. This shows that the underinvestment problem disappears by allowing subsidization only when $\lambda^b = 1$.

Inefficiency due to holdup is (weakly) reduced by allowing subsidization. Suppose first that the buyer does not invest in equilibrium. It is straightforward that the investment level must be the same as the one when subsidization is not possible. Alternatively, suppose that the buyer invests in equilibrium. This must imply the buyer's equilibrium payoff must be more than his payoff from forgoing, and the seller's equilibrium payoff must also be more than her payoff from refusing to permit subsidization. In either of the off-path outcomes reached by forgoing or refusing, the seller would invest at the same level as she would when subsidization is not possible. This implies that allowing subsidization must be Pareto-improving and therefore the equilibrium investment must be more efficient. This argument does not rely on not adopting a contract. It holds even when a contract is adopted.

We summarize these results as follows:

Proposition 1 *If a contract is not adopted, the parties underinvest except in the cases where $\lambda^s = 1$ or $\lambda^b = 1$.*

Proposition 2 *Given any contract, an equilibrium investment becomes (weakly) more efficient by allowing subsidization.*

Remark 1 *In the preceding analysis the characteristics of investment do not matter. In contrast, the following analysis shows that the effectiveness of contracting depends on the degree to which the investment is cooperative or selfish.*

Proposition 1 leads to our analysis of contractual remedies to the underinvestment problem.

5 Two Polar Cases

Contracting affects the buyer's and the seller's ex ante revenue functions and therefore alters their incentives to invest. This is because a contract determines the status quo point of the bargaining at the renegotiation stage, which is affected by the parties' investment decision.

The effect of investment on the status quo depends on three things: the primitive functions v and c, the bargaining shares (λ^b, λ^s), and the contract the parties sign. In this section we investigate the purely selfish and the purely cooperative cases separately. The hybrid cases are analyzed in the following sections.

Purely Selfish Case Edlin and Reichelstein (1996) show that in the purely selfish case with no subsidization opportunity, a noncontingent contract achieves full efficiency. Under no uncertainty, the noncontingent contract sets the trade equal to the ex post efficient trade for the first-best investment level, $q^*(x^*)$. The transfer is irrelevant for incentives.

Proposition 2 implies that the same contract also achieves efficiency when subsidization is possible. Thus the following proposition obtains:

Proposition 3 *In the purely selfish case, a noncontingent contract that specifies the ex post efficient trade for the first-best investment level induces the seller to invest efficiently.*

Purely Cooperative Case In this subsection the seller's cost function depends only on q, and we write it simply as $c(q)$.

Che and Hausch (1999) show that if the investment is purely cooperative, contracting cannot improve the investing party's incentives to invest. Therefore, without a subsidization opportunity, the null contract is optimal. This implies that unless the seller has full bargaining power, full efficiency cannot be achieved.

By allowing subsidization, one might conjecture that the seller's purely cooperative investment becomes the buyer's purely selfish investment, and that a noncontingent contract analogous to Proposition 3 thus achieves full efficiency. The following proposition shows that this conjecture is not true.

Proposition 4 *In the purely cooperative case, full efficiency cannot be achieved by a noncontingent contract unless either party has full bargaining power.*

Proof. See Appendix. ∎

To see that the above conjecture is not true, consider that a noncontingent contract $(\overline{q} = q^*(x^*), \overline{t})$ is adopted as in Edlin and Reichelstein (1996). If subsidization transforms seller's purely cooperative investment to a purely selfish investment for the buyer, this contract can induce the buyer to entirely make the first-best investment. The seller must therefore allow the buyer to invest. If the buyer invests at the first-best level, the ex post efficient trade is $q^*(x^*)$ as in the contract. There is no renegotiation in this case. Suppose alternatively the seller refuses to allow the buyer to invest and does not invest afterwards. The resulting investment level is then zero. Under the noncontingent contract, renegotiation must occur and the seller can hold up the buyer through renegotiation; she can successfully demand part of the renegotiation surplus. By pure cooperativeness, i.e., $c(q, x) = c(q)$, the seller's payoff from the status quo is the same for any investment. The seller will therefore be better off if she does not allow the buyer to invest than if she does. This shows that the contract cannot induce the buyer to invest efficiently in equilibrium. Proposition 4 shows that, in the purely cooperative case, any other noncontingent contract would not work to achieve the first-best, unless one party has full bargaining power.

The buyer needs the seller's permission to subsidize. The seller's right to give permission provides extra opportunities for the seller to hold up the buyer. Thus in order to achieve the efficient investment in the purely cooperative case, the parties need to write a contract which avoids this extra holdup problem.

The next proposition shows, in this purely cooperative case, full efficiency is achievable by adopting a buyer-option contract.

Proposition 5 *In the purely cooperative case, the following buyer-option contract induces the buyer to invest entirely at the first-best level in a subgame perfect equilibrium: "The buyer chooses a quantity q and pays c(q) to the seller," or formally,*

$$q(m^b, m^s) = m^b \text{ and } t(m^b, m^s) = c(m^b).$$

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Proof. Given this option contract, if x is the total investment, the buyer's revenue from choosing q is,

$$v(q, x) - c(q) + \lambda^b[S^*(x) - (v(q, x) - c(q))]$$
$$= (1 - \lambda^b)(v(q, x) - c(q)) + \lambda^b S^*(x)$$
$$= \lambda^s S(q, x) + \lambda^b S^*(x).$$

The buyer therefore chooses $q = q^*(x)$, the maximizer of $S(q, x)$. The buyer's ex ante revenue equals the ex post efficient surplus, and correspondingly the seller's ex ante revenue is zero for any investment level. Thus the buyer's and the seller's ex ante payoffs are, respectively,

$$R^b(x^b, x^s) - x^b = S^*(x^b + x^s) - x^b,$$
$$R^s(x^b, x^s) - x^s = -x^s.$$

The seller does not care about the buyer's investment level, and so she will let the buyer invest. The buyer invests at the first-best level since he anticipates that the seller will make no investment afterwards. ∎

Given the buyer-option contract of Proposition 5, the buyer pays the exact cost of whatever output he chooses. This implies that the seller's payoff from the status quo is always zero for any choice of the buyer and the seller's ex ante revenue is thus just the share of the renegotiation surplus. The buyer can always avoid renegotiation by choosing the ex post efficient trade, which leads to no renegotiation and leaves no revenue to the seller. The buyer's ex ante revenue then coincides with the ex post efficient surplus so that the buyer has an incentive to invest efficiently. On the seller's side, the level of investment does not affect her revenue, so she is indifferent between allowing and not allowing the buyer to invest.

The same conclusion obtains even if we modify the contract so that the buyer needs to pay $c(q)$ plus a constant k for any choice of q. This contract achieves the same outcome as would the outright purchasing of the investment project by the seller at price k, *despite* the fact that the contract does not fully transfer the investment control right to the buyer.

Optimal Option Contracts of Finite Alternatives The contract in Proposition 5 has continuum of alternatives. This raises the following question; is it possible to construct a simpler contract that induces the first-best investment? Here we study how to construct a buyer-option contract of finite alternatives that achieves the first-best.

The contract in Proposition 5 induces the following two key equilibrium properties: (i) the buyer will receive the entire surplus if the investment is efficient, and (ii) the seller is indifferent between allowing and not allowing the buyer to invest. The first property obtains since the buyer-option contract contains $(q^*(x^*), c(q^*(x^*)))$; if the efficient investment is made, the buyer chooses this alternative that is ex post efficient and that leaves no surplus to the seller. The second property obtains since the contract contains $(q^*(0), c(q^*(0)))$; the seller would make no investment if she were to refuse, and the buyer would then choose this alternative.

One may then wonder whether a buyer-option contract with two options, $(q^*(0), c(q^*(0)))$ and $(q^*(x^*), c(q^*(x^*)))$, induces the same equilibrium outcome as the original buyer-option contract. This depends on the seller's off-path behavior; if she makes no investment after refusing, the first-best outcome follows. However, she may have an incentive to invest after refusing, as the following argument shows.

The seller's incentives to invest is associated with her revenue function. For the two-alternative buyer-option contract, she receives her share of *the smaller one of two possible renegotiation surpluses*, namely,

$$R^s(x^b, x^s; \lambda^s) = \min\{\lambda^s(S^*(x) - S(q_0, x)), \lambda^s(S^*(x) - S(q_1, x))\}$$

where $x = x^b + x^s$, $q_0 = q^*(0)$ and $q_1 = q^*(x^*)$.[18] We thus identify R^s as a function on the total level of investment, x, and the bargaining share, and write it as $R^s(x; \lambda^s)$ by a notational abuse. The following lemma shows that $S^*(x) - S(q, x)$ is U-shape or monotone in x, and therefore implies that the graph of the seller's revenue function is the lower envelope of two such graphs. In fact, it is depicted as the bold curve in Figure 1.[19]

Lemma 1 *For a given q, $S^*(x) - S(q, x)$ is decreasing at $x < q^{*-1}(q)$, and increasing at $x > q^{*-1}(q)$.*[20]

Proof. See Appendix. ∎

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[18] To see why, note that the seller's status quo payoff is zero regardless of buyer's choice of alternatives, and that the buyer chooses an alternative that minimizes seller's revenue (since the size of the pie is already determined).

[19] Note that renegotiation surplus for a given q is nonnegative, and equals zero iff the ex post efficient trade $q^*(x)$ coincides q. The slope of renegotiation surplus at $x = q^{*-1}(q)$ is zero since it is minimized. This is why $\lambda^s(S^*(x) - S(q_0, x))$ and $\lambda^s(S^*(x) - S(q_1, x))$ have zero value and slope of zero at $x = q^{*-1}(q^*(0)) = 0$ and $x = q^{*-1}(q^*(x^*)) = x^*$, respectively. The buyer chooses an alternative that minimizes the seller's revenue, namely q_0 if the investment level is up to some $\bar{x}$, and q_1 otherwise.

[20] In fact, the ex post efficient trade $q^*(x)$ is strictly increasing (the proof is provided in the proof of this lemma), and thus the inverse is well-defined. In the case where $q^{*-1}(q)$ is empty, $S^*(x) - S(q, x)$ is either strictly increasing or strictly decreasing at all x.
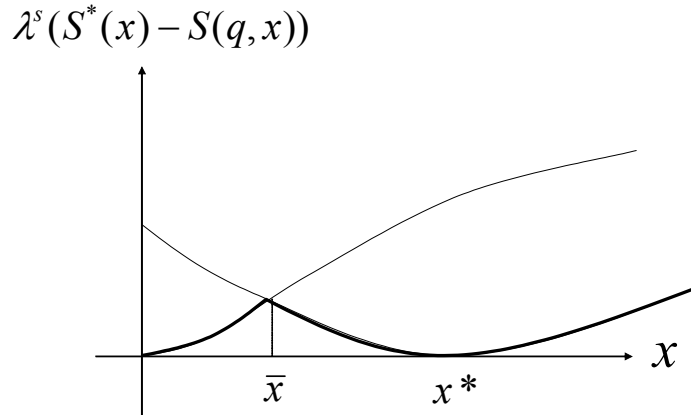
$$\lambda^s(S^*(x) - S(q,x))$$

Figure 1:

The question is whether an investment level x' exists such that making this investment by her own provides positive surplus to her, i.e., $R^s(x'; \lambda^s) - x' > 0$. If so, the seller optimally refuses to allow the buyer to invest. Whether such x' exists depends on the seller's bargaining share λ^s. If the share is close enough to zero, the revenue function is so flat that no such x' exists. Accordingly, define

$$\bar{\lambda}^s = \max\{\lambda^s \in (0,1] | R^s(x^s; \lambda^s) - x^s \leq 0 \text{ for all } x^s \leq x^*\}.^{21}$$

If $\lambda^s \leq \bar{\lambda}^s$, there is no x' such that $R^s(x'; \lambda^s) - x' > 0$. The above buyer-option contract of two alternatives therefore induces the buyer to invest efficiently.

If $\lambda^s > \bar{\lambda}^s$, the contract does not work, but we can still construct another buyer-option contract of finite alternatives that achieves the first-best. Consider adding another alternative $(q = q^*(x), t = c(q^*(x)))$ for some $x \in (0, x^*)$ to the two-alternative contract. The graph of the seller's revenue function becomes the lower envelope of three graphs. Her revenue becomes smaller, and her incentive to invest by her own must therefore be lowered. By adding such alternatives for $x = m/2^n \times x^*$, $m = 1, 2, ..., 2^n - 1$, her revenue function can be arbitrarily close to zero in $[0, x^*]$. Adopting such a buyer-option contract, her incentive to invest after refusing vanishes even when she has the entire bargaining power. We have therefore established the following proposition:

Proposition 6 *In the purely cooperative case, a buyer-option contract of finite alternatives exists which induces the buyer to invest at the first-best. If $\lambda^s \leq \bar{\lambda}^s$, the following buyer-option contract achieves the first-best: $\{(q = q^*(x), t = c(q^*(x))) \text{ for } x \in \{0, x^*\}\}$.*

---

[21]Note that if $x > x^*$, $R^s(x; \lambda^s) - x \leq 0$ for any $\lambda^s \leq 1$. Thus we do not need to look at $x > x^*$.

Uncertainty   Uncertainty is beyond the scope of our analysis, but indeed the buyer-option contract in Proposition 5 works under some types of uncertainty. Suppose that there is a stochastic shock that only affects the buyer's valuation. For example, uncertainty affects the price of final goods that this buyer eventually supplies. The shock is realized after investing but before the mechanism is played. Since the seller's cost function is exactly the same as before, the logic in Proposition 5 remains valid, and the full efficiency result holds as well.

Corollary 1 *In the purely cooperative case, suppose there is uncertainty that only affects the buyer's valuation. Then, the buyer-option contract of Proposition 5 achieves full efficiency.*

6   Hybrid Cases – An Example

To investigate how contracting improves efficiency in hybrid cases, we first look at a specific example in this section. A generalization follows in the next section.

Let the buyer's valuation and the seller's cost be, respectively,

$$v(q, x) = 4(1 + \alpha x^{\frac{1}{4}})q$$
$$c(q, x) = 4(1 - (1 - \alpha) x^{\frac{1}{4}})q + 2q^2$$

where $\alpha \in [0, 1]$. The parameter α measures cooperativeness of the investment. The condition $\alpha = 0$ corresponds to the purely selfish case, $\alpha = 1$ the purely cooperative case, and $\alpha \in (0, 1)$ a hybrid case. The associated functions and values are

$$S(q, x) = 4x^{\frac{1}{4}}q - 2q^2$$
$$q^*(x) = x^{\frac{1}{4}},$$
$$S^*(x) = 2x^{\frac{1}{2}}$$
$$x^* = 1,$$

The buyer's and the seller's interim payoffs are as follows:

$$r^b(q^0, x) - t^0 = 2\lambda^b x^{\frac{1}{2}} + 4(\alpha - \lambda^b)q^0 x^{\frac{1}{4}} + 2\lambda^b (q^0)^2 - t^0,$$
$$r^s(q^0, x) + t^0 = 2\lambda^s x^{\frac{1}{2}} - 4(\alpha - \lambda^b)q^0 x^{\frac{1}{4}} - 2\lambda^b (q^0)^2 + t^0,$$

where q^0 and t^0 are the chosen terms of a transaction in the message game.

To grasp a rough idea of how contracting affects the parties' incentives to invest, consider a noncontingent contract $(\overline{q}, \overline{t})$. The ex ante revenue functions given this contract are

$$R^b(x^b, x^s) = 2\lambda^b(x^b + x^s)^{\frac{1}{2}} + 4(\alpha - \lambda^b)\overline{q}(x^b + x^s)^{\frac{1}{4}} + 2\lambda^b(\overline{q})^2 - \overline{t}, \tag{3}$$

$$R^s(x^b, x^s) = 2\lambda^s(x^b + x^s)^{\frac{1}{2}} - 4(\alpha - \lambda^b)\overline{q}(x^b + x^s)^{\frac{1}{4}} - 2\lambda^b(\overline{q})^2 + \overline{t}. \tag{4}$$

The contract affects the parties' incentives only through the second terms of the right-hand sides in (3) and (4). The incentive effect of contracting thus depends not only on the degree of cooperativeness, α, but also on the allocation of bargaining shares, λ^b and λ^s. For example, suppose $\alpha - \lambda^b > 0$. The buyer's incentive to invest improves as $\overline{q}$ increases. This observation suggests that the effect of a contract depends on the degree of cooperativeness α *relative to the buyer's bargaining share,* λ^b.

It should be mentioned that the revenue functions (3) and (4) are functions of the total investment $x = x^b + x^s$ for a noncontingent contract. We henceforth use $R^b(x)$ and $R^s(x)$ for the parties' revenue functions for a noncontingent contract.

Efficient Noncontingent Contracts for Highly Selfish Investment Consider the case in which the investment is highly selfish relative to the buyer's bargaining share. In particular, suppose,[22]

$$\alpha - \lambda^b \leq -\frac{\lambda^b}{q_{\max}} \tag{5}$$

$$\Longleftrightarrow \lambda^b \geq \alpha \frac{q_{\max}}{q_{\max} - 1} \equiv \overline{\alpha}.$$

Suppose the parties adopt a noncontingent contract $(\overline{q}, \overline{t})$ for which

$$\overline{q} = -\lambda^b / (\alpha - \lambda^b) \leq q_{\max}.$$

The seller's ex ante revenue given this contract is

$$R^s(x) = 2\lambda^s x^{\frac{1}{2}} + 4\lambda^b x^{\frac{1}{4}} + \text{constant},$$

which is strictly concave in x. The seller's marginal revenue at $x^* = 1$ is

$$R^{s\prime}(1) = \lambda^s (1)^{-1/2} + \lambda^b (1)^{-3/4} = 1$$

so that the seller has the right incentive to invest at the first-best level. This shows that if (5) holds, the noncontingent contract $(\overline{q}, \overline{t})$ achieves full efficiency. Note that for the purely selfish case, $\alpha = 0$, (5) holds for all λ^b. This case corresponds to the result of Edlin and Reichelstein (1996).

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[22] Recall that $q_{\max}$ is the upper bound of quantity traded. In this specific example, $q_{\max} > 1$ must hold in order to have interior solutions for the values such as q^* and x^*.

Efficient Option Contracts for Highly Cooperative Investment   We consider the case in which the investment is highly cooperative and derive a contract that achieves full efficiency. As one might expect from Proposition 4, we need to consider a more complex contract than a noncontingent one. In this subsection, we look at a buyer-option contract $(q(x), t(x))$ where the buyer reports a total investment level, x.

We use an analogy to the standard screening model (see, Mussa and Rosen (1978), Maskin and Riley (1984)). Consider an agent with private information x $(= x^b + x^s)$ and a principal who does not know this agent's type. The agent has a quasilinear utility $r^b(q, x) - t$ where q is the quantity traded and t is the monetary transfer. The following result is well-known: *under the single-crossing property $r^b_{qx} > 0$, a mechanism $(q(x), t(x))$ is incentive compatible (i.e., the agent of type x' optimally chooses the designated pair $(q = q(x'), t = t(x'))$ among the alternatives) if and only if $q(x)$ is nondecreasing and for all x',*

$$R^b(x') \equiv r^b(q(x'), x') - t(x')$$
$$= R^b(0) + \int_0^{x'} r^b_x(q(\tau), \tau) d\tau.$$

The last integral equation is called *the Mirrlees condition.* The transfer function is defined implicitly in the Mirrlees condition.

If we translate the agent into the buyer, and the principal into the court or the enforcer of the contract, the above argument simply shows that the mechanism $(q(x), t(x))$ is a buyer-option contract that induces the buyer's revenue function, $R^b(x)$. Note that it is a function of the total investment $x = x^b + x^s$, not a function of (x^b, x^s).

If $\alpha - \lambda^b > 0$, by (3) the single-crossing property $r^b_{qx} > 0$ holds. Define

$$q(x) = \min \left\{ \frac{1 - \lambda^b}{\alpha - \lambda^b} x^{\frac{1}{4}}, q_{\max} \right\}.$$

This is nondecreasing and thus if $t(x)$ is defined through the Mirrlees condition, a buyer-option contract $(q(x), t(x))$ is incentive compatible.

Suppose the parties adopt the buyer-option contract, $(q(x), t(x))$. The Mirrlees condition gives the derivatives of the buyer's ex ante revenue; for x' such that $q(x') < q_{\max}$,

$$R^{b\prime}(x') = r^b_x(q(x'), x')$$
$$= \lambda^b x'^{-\frac{1}{2}} + (\alpha - \lambda^b) q(x') x'^{-\frac{3}{4}}$$
$$= \lambda^b x'^{-\frac{1}{2}} + (1 - \lambda^b) x'^{-\frac{1}{2}}$$
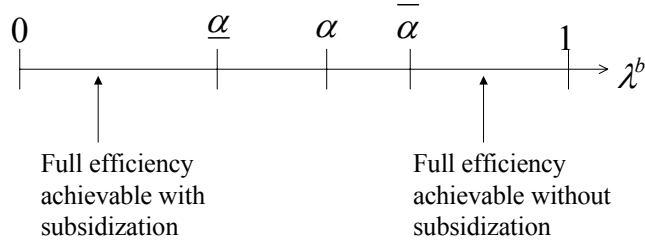$$= S^{*\prime}(x'),$$

Figure 2:

and similarly for x' such that $q(x') = q_{\max}$,

$$R^{b\prime}(x') \leq S^{*\prime}(x').$$

Suppose the investment is highly cooperative relative to λ^b to satisfy

$$\alpha - \lambda^b \geq (1 - \lambda^b)/q_{\max} \tag{6}$$
$$\Longleftrightarrow \lambda^b \leq \frac{\alpha q_{\max} - 1}{q_{\max} - 1} \equiv \underline{\alpha}.$$

Then

$$q(1) = \min\left\{\frac{1 - \lambda^b}{\alpha - \lambda^b}, q_{\max}\right\} \leq q_{\max}$$
$$q(x) < q_{\max} \text{ for } x < x^* = 1.$$

This shows that under (6), the buyer's marginal revenue equals the marginal ex post efficient surplus for $x \in [0, x^*]$. Correspondingly the seller's marginal revenue must equal zero in the same region since the parties' ex ante revenue must sum up to the efficient surplus. Thus the revenue functions in this case are the same as in the ones in Proposition 3 for $x \in [0, x^*]$. The first-best investment must be achievable by this buyer-option contract as in Proposition 3. Note that for the purely cooperative case, $\alpha = 1$, condition (6) holds for any λ^b.

Summary   These results are summarized in Figure 2. If $\lambda^b \geq \overline{\alpha}$ (see (5)), a noncontingent contract achieves efficiency; if $\lambda^b \leq \underline{\alpha}$ (see (6)), an option contract achieves efficiency. Note that $\overline{\alpha} \to 0$ monotonically as $\alpha \downarrow 0$ and $\underline{\alpha} \to 1$ monotonically as $\alpha \uparrow 1$. This implies that for a given λ^b, if the investment is either sufficiently selfish or sufficiently cooperative, full efficiency is achieved by adopting a contract.

If $\underline{\alpha} < \lambda^b < \overline{\alpha}$, i.e., the degree of cooperativeness is intermediate, the two contracts considered above do not achieve efficiency.[23] However, the interval $(\underline{\alpha},\overline{\alpha})$ becomes smaller as $q_{\max}$ increases, and if we allow $q_{\max} = \infty$, there remains only one point, $\alpha = \lambda^b$, at which efficiency is unattainable.[24]

7 General Hybrid Cases

The example of the previous section suggests that in a more general setting, full efficiency should be achievable for a hybrid case that is close to either of the two polar cases. In this section, we give conditions for a hybrid case under which full efficiency can be achieved.

Two new issues arise in a general setting. First we need a measure of cooperativeness of the investment analogous to α, $\overline{\alpha}$, and $\underline{\alpha}$ in the example. Second, a noncontingent or option contract may not achieve efficiency and we may need to use a general message game for an optimal contract.

The second issue involves two analytical problems. The first problem is to specify the set of feasible contracts. However, the revelation principle applies so that without loss of generality, we restrict attention to direct mechanisms. The second problem is to derive the ex ante revenues from a direct mechanism. It is extremely cumbersome to derive them directly. However, Segal and Whinston (2002) provide a characterization result for feasible ex ante revenues and we use this result.

We discuss these two problems in detail before analyzing the general hybrid cases.

"The Revelation Principle" Recall that the state space of the message game is the set of investment pairs, $\{(x^b, x^s)\}$. The revelation principle implies that we can take each party's message to be the investment pair that s/he has observed. However, in the present context, a stronger result holds. The next proposition shows that each party's message just needs to be a report of the total investment level, $x = x^b + x^s$.

Proposition 7 *Given any message game, the induced buyer's and seller's ex ante revenue functions satisfy* $R^b(x^b, x^s) = \tilde{R}^b(x)$ *and* $R^s(x^b, x^s) = \tilde{R}^s(x)$. *Moreover, there exists a message game in which each party truthfully reports the total investment level, x, such that*

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[23] In fact, we can show that full efficiency is not achievable by any contract in this case.

[24] Sufficiency of the Mirrlees condition does not require compactness of the domain of the primitive functions.

the same revenue functions $\tilde{R}^b(x)$ and $\tilde{R}^s(x)$ are induced.[25],[26]

Proof. Consider a message game $\{q(m^b, m^s), t(m^b, m^s)\}$ with arbitrary message spaces, M^i, $i = b, s$. Given an investment pair (x^b, x^s), the payoff functions of this message game are (see (1) and (2))

$$r^b(q(m^b, m^s), x^b + x^s) - t(m^b, m^s) \text{ for the buyer,}$$
$$r^s(q(m^b, m^s), x^b + x^s) + t(m^b, m^s) \text{ for the seller.}$$

Let $(m^{b*}(x^b, x^s), m^{s*}(x^b, x^s))$ denote a Nash equilibrium of the message game given the investment pair (x^b, x^s). Recall that the message game is a two-person zero-sum game. The parties' equilibrium payoffs are thus the minimax values.

Consider another investment pair $(x^{b\prime}, x^{s\prime})$ such that $x^b + x^s = x^{b\prime} + x^{s\prime}$. The payoff functions given this new investment pair $(x^{b\prime}, x^{s\prime})$ are clearly the same as those given the old investment pair (x^b, x^s). The resulting equilibrium payoffs must be the same minimax values even if the equilibrium strategy profile might be different. This shows that if $x^b + x^s = x^{b\prime} + x^{s\prime} = x$,

$$R^b(x^b, x^s) = R^b(x^{b\prime}, x^{s\prime}) \equiv \tilde{R}^b(x),$$
$$R^s(x^b, x^s) = R^s(x^{b\prime}, x^{s\prime}) \equiv \tilde{R}^s(x).$$

In order to construct the desired mechanism, it is convenient to define the following mapping: $x \mapsto \psi(x) = (x^b, x^s)$ for some (x^b, x^s) such that $x^b + x^s = x$. This $\psi(x)$ gives a representative investment pair, (x^b, x^s), for each total investment level, x.

Consider the following message game $\{\tilde{q}(\tilde{m}^b, \tilde{m}^s), \tilde{t}(\tilde{m}^b, \tilde{m}^s)\}$ with a message space $\tilde{M}^i = [0, x_{\max}]$, $i = b, s$ such that

$$\tilde{q}(x, x') = q(m^{b*}(\psi(x)), m^{s*}(\psi(x'))),$$
$$\tilde{t}(x, x') = t(m^{b*}(\psi(x)), m^{s*}(\psi(x'))).$$

If both parties report the true total investment level, each party receives its minimax payoff of the original game. Suppose one party deviates from reporting the true level. By construction of this message game and the definition of the minimax value, the deviating party must receive less than the minimax payoff of the original game. Truth-reporting is

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[25] This proposition can be generalized. Suppose that the payoff-relevant variable for the message game is some aggregate value $\phi(x_1, ..., x_n)$ for a state $(x_1, ..., x_n)$. In this case we can restrict attention to the message game in which each party announces ϕ.

[26] Segal and Whinston (2002, p. 19) make a similar observation.

therefore a Nash equilibrium of this new message game. The induced revenue functions must be $\tilde{R}^b(x)$ and $\tilde{R}^s(x)$. ∎

We henceforth assume that the messages are reports of x, and that the contract is incentive compatible. The parties can anticipate the equilibrium outcome $\{q(x,x), t(x,x)\}$ for each x, and the buyer's and the seller's ex ante revenues are thus redefined as

$$R^b(x) = r^b(q(x,x), x) - t(x,x),$$
$$R^s(x) = r^s(q(x,x), x) + t(x,x).$$

We call arbitrary revenue functions $R^b(x)$ and $R^s(x)$ *implementable* if there is an incentive compatible contract that generates them.

Segal-Whinston's Characterization   Segal and Whinston (2002) derive a characterization result for implementable utility mappings in mechanism design with renegotiation. It can be applied to characterize implementable revenue functions in our model.

Definition 1 *The buyer's interim payoff r^b satisfies the single-crossing property (SCP) if either $r^b_{qx} \geq 0$ or $r^b_{qx} \leq 0$ in the entire domain.*

If r^b satisfies SCP, the seller's interim payoff, r^s, also satisfies the corresponding SCP. This is because, $r^b(q,x) + r^s(q,x) = S^*(x)$ for all (q,x) by ex post efficient renegotiation, which implies $r^b_{qx} = -r^s_{qx}$ for all (q,x). We henceforth omit the labels "the buyer's" or "the seller's" when we refer the single-crossing property.

Lemma 2 *(Segal-Whinston) Under the single-crossing property, ex ante revenue functions $R^b(x)$ and $R^s(x)$ are implementable iff, there is a function $g : [0, x_{\max}] \to [0, q_{\max}]$ such that for any x',*

$$R^b(x') = R^b(0) + \int_0^{x'} \frac{\partial r^b(g(\tau), \tau)}{\partial x} d\tau,$$
$$R^s(x') = R^s(0) + \int_0^{x'} \frac{\partial r^s(g(\tau), \tau)}{\partial x} d\tau.$$

*We call this $g(x)$ **a generating decision rule**.*[27,28]

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[27]In their original version, a condition that is weaker than the single-crossing property, "Condition $\pm$," suffices to establish the result. From the maintained assumptions in our model, it is harder to check if Condition $\pm$ holds than the single-crossing property. The single-crossing property holds in highly selfish and cooperative cases, as we show below.

[28]Two differences between this lemma and the result for screening should be mentioned. The first one

This lemma says that for any contract, there is a corresponding decision rule that generates the ex ante revenue functions induced by the contract. Conversely, for any decision rule that generates R^b and R^s, there is a corresponding contract that induces the revenue functions.

An advantage of using this lemma is in taking the derivatives of the revenue functions, which represent the parties' incentives to invest. Consider a generating decision rule $g(x)$. The Mirrlees condition implies

$$R^{b\prime}(x) = \frac{\partial r^b(g(x),x)}{\partial x} = \lambda^b S^{*\prime}(x) + \lambda^s v_x(g(x),x) + \lambda^b c_x(g(x),x) \ \ a.e.$$

$$R^{s\prime}(x) = \frac{\partial r^s(g(x),x)}{\partial x} = \lambda^s S^{*\prime}(x) - \lambda^s v_x(g(x),x) - \lambda^b c_x(g(x),x) \ \ a.e.$$

The last two terms in each line capture the effect of a contract on the party's incentive to invest. Note that the null contract corresponds to $g(x) \equiv 0$ and with it these two terms are zero.

The interpretation of the single-crossing property, $r^b_{qx} > 0$, is as follows: consider a noncontingent contract $(\overline{q}, \overline{t})$. The buyer's ex ante revenue function is then

$$\begin{aligned} R^b(x) &= r^b(\overline{q}, x) \\ &= v(\overline{q}, x) - \overline{t} + \lambda^b[S^*(x) - S(\overline{q}, x)] \\ &= \lambda^b S^*(x) + \lambda^s v(\overline{q}, x) + \lambda^b c(\overline{q}, x) - \overline{t}. \end{aligned}$$

The marginal revenue of investment is

$$R^{b\prime}(x) = \lambda^b S^{*\prime}(x) + \lambda^s v_x(\overline{q}, x) + \lambda^b c_x(\overline{q}, x)$$

Suppose the single-crossing property $r^b_{qx} > 0$ holds, i.e.,

$$\lambda^s v_{qx}(q, x) + \lambda^b c_{qx}(q, x) > 0.$$

Then increasing the trade quantity $\overline{q}$ will improve the buyer's incentive to invest.

lies in the restriction to the generating decision rule. In screening, the generating rule must be monotonic whereas it is arbitrary in mechanism design with renegotiation. This is explained by the fact that the information extracted from the message game is richer than that from the option contract. In screening, only one agent with private information reports a message. This is an option contract. In mechanism design with renegotiation, two parties who share information report messages.

Another difference is in the role of the generating decision rule. In screening, the rule coincides with the equilibrium decision rule. In mechanism design with renegotiation, it need not, due to renegotiation. See Segal and Whinston (2002, p.11).

Two further remarks should be mentioned. First, it is typically complicated to construct a contract that corresponds to some generating decision rule g. See Segal and Whinston (2002) for a construction. The contract has a simple form only in special cases. For example, it is a noncontingent contract $(\overline{q}, \overline{t})$ if and only if $g(x) = \overline{q}$. If $r^b_{qx} > 0$, it is a buyer-option contract if and only if $g(x)$ is nondecreasing. Second, the single-crossing condition may not hold in the general case. It depends on the primitive functions and the allocation of bargaining shares. We will find conditions under which it holds.

7.1 Efficiency in Highly Cooperative Cases

We prove that full efficiency can be achieved if the investment is sufficiently cooperative. We use Segal-Whinston's lemma to find a contract to be adopted. Since the single-crossing property may not always hold, we need to give a condition for it. To state and prove the result, we define a measure of cooperativeness.

To give a condition for the single-crossing property, define the following measure:

$$\alpha_c = \sup\{k \in [0,1] | (1-k)v_{qx}(q,x) + kc_{qx}(q,x) > 0 \ \forall(q,x)\},$$

and $\alpha_c = 0$ if there is no such k. It is straightforward that the single-crossing property holds if $\lambda^b < \alpha_c$. This shows that Segal-Whinston's lemma is more likely to apply as α_c increases. For a purely cooperative investment, the cross derivative $c_{qx} \equiv 0$ so that $\alpha_c = 1$. For the example of section 6, $\alpha_c = \alpha$.

We measure cooperativeness of the investment as follows:

$$\alpha^*_c = \sup \left\{ k \in [0, \alpha_c] \left| \begin{array}{c} (1-k)v_x(q_{\max}, x) + kc_x(q_{\max}, x) \\ \geq (1-k)S^{*\prime}(x) \ \forall x \in [0, x^*] \end{array} \right. \right\}, \tag{7}$$

and $\alpha^*_c = 0$ if there is no such k.[29] Note that by construction ($k \in [0, \alpha_c]$) $\alpha^*_c \leq \alpha_c$.[30] For the example of section 6, $\alpha^*_c = \alpha$.

To justify this α^*_c as a measure of cooperativeness, consider a purely cooperative investment first, i.e., $c_x \equiv 0$. For all x,

$$(1-k)v_x(q_{\max}, x) + kc_x(q_{\max}, x)$$
$$= (1-k)v_x(q_{\max}, x) \text{ by pure cooperativeness}$$
$$\geq (1-k)v_x(q^*(x), x) \text{by complementarity and } q_{\max} \geq q^*(x)$$
$$= (1-k)S^{*\prime}(x) \text{ by the envelope theorem.}$$

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[29] Che and Hausch (1999) similarly define measures of cooperativeness.

[30] The inequality in (7) can be satisfied for $k > \alpha_c$ for some v and c. We restrict our attention to $\alpha_c > k = \lambda^b$ since otherwise we cannot use Segal-Whinston's characterization lemma.

This shows $\alpha_c^* = 1$. For a hybrid case, if α_c^* is high, $|c_x|$ must be small relative to $|v_x|$ so that the investment must be highly cooperative. For a purely selfish investment, it is clear that $\alpha_c^* = 0$.

To get an intuition for the inequality condition in (7), consider a noncontingent contract. Recall that $\lambda^b v_x + \lambda^s c_x$ measures the incentive effect of the contract. Under the single-crossing condition (assured by $\lambda^b < \alpha_c$), the left-hand side in (7) measures the maximum effect of a noncontingent contract on the buyer's incentive to invest. The right-hand side is the buyer's incentive loss due to hold up given the null contract. Thus this condition says that a noncontingent contract can adequately make up for the incentive loss due to holdup, for the buyer.

With this measure, we prove the following result.

Proposition 8 *Suppose $\alpha_c^* > \lambda^b$ holds. Then a contract exists which induces the buyer to entirely invest at the first-best investment level in a subgame perfect equilibrium.*

Proof. See Appendix. ∎

As in Proposition 5, one way to achieve full efficiency is to solve the following two problems simultaneously: giving the right incentive to invest to the buyer and giving the right incentive to permit subsidization to the seller. If, as depicted in Figure 3, buyer's marginal revenue of investment equals the marginal ex post efficient surplus by adopting a contract, corresponding seller's marginal revenue must be zero, and the two problems are therefore solved. Under the assumption in Proposition 8, we have seen that a noncontingent contract can adequately make up for the incentive loss due to holdup. However, a noncontingent contract might be too blunt to pinpoint the marginal revenues. In such a case, the proposition assures that a more complex contract works to adjust the revenues.

Che and Hausch (1999) show that when investment is highly cooperative, the null contract is optimal. Using the next proposition, we can compare the result of Che and Hausch with Proposition 8.

Proposition 9 *Suppose $\alpha_c > \lambda^b$ holds. Then, without a subsidization opportunity, the null contract is optimal (hence contracting has no value).*

Proof: See Appendix. ∎

This proposition implies the following: if the investment is cooperative enough to achieve efficiency ($\alpha_c^* > \lambda^b$, by Proposition 8), contracting indeed has no value without a subsidization opportunity ($\alpha_c \geq \alpha_c^* > \lambda^b$, by Proposition 9).
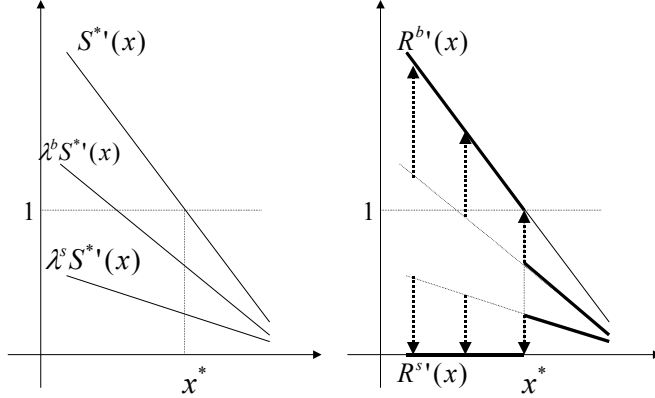
27

Figure 3:

The intuition of Proposition 9 is as follows. Recall that when the single-crossing property holds, every noncontingent contract improves only the buyer's incentive to invest. Since the message game is zero-sum, contracting only impairs the seller's incentive to invest. When subsidization is not possible, the seller's incentive to invest should be maximized so that the null contract is optimal.

7.2 Efficiency in Highly Selfish Cases

We prove that full efficiency can be achieved if the investment is sufficiently selfish. The logic is parallel to that in the last subsection. Subsidization does not occur in equilibrium, and thus this analysis also applies to the case when subsidization is not possible. We compare this result with existing results at the end of this subsection.

To give a condition for the other single-crossing property, $r_{qz}^b < 0$, define the following measure:

$$\alpha_s = \inf\{k \in [0,1]|(1-k)v_{qx}(q,x) + kc_{qx}(q,x) < 0 \ \forall(q,x)\},$$

and $\alpha_s = 1$ if there is no such k. The single-crossing property, $r_{qx}^b < 0$, holds if $\alpha_s < \lambda^b$. Note that for a purely selfish investment, $\alpha_s = 0$, and also that $\alpha_c \leq \alpha_s$ by definition. In the previous example, $\alpha_s = \alpha$.

We measure cooperativeness of the investment in an alternative way:

$$\alpha_s^* = \inf\left\{k \in [\alpha_s, 1] \left| \begin{array}{c} -(1-k)v_x(q_{\max},x) - kc_x(q_{\max},x) \\ \geq kS^{*\prime}(x) \ \forall x \in [0,x^*] \end{array} \right.\right\},$$

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and $\alpha_s^* = 1$ if there is no such k. As in the last subsection, we can show that if the investment is purely selfish, $\alpha_s^* = 0$. For a hybrid case, if α_s^* is low, $|c_x|$ must be large relative to $|v_x|$ so the investment must be highly selfish. For a purely cooperative investment, it is clear that $\alpha_c^* = 1$. These arguments justify that α_s^* measures cooperativeness of the investment. In the previous example, $\alpha_s^* = \overline{\alpha}$. Note that $\alpha_s^* \geq \alpha_s$ by definition.

Following the same logic as in the highly cooperative case, we can derive a contract such that full efficiency is achieved for a sufficiently highly selfish investment. The derived contract is a message game $\{q(\cdot,\cdot), t(\cdot,\cdot)\}$. Applying Proposition 9 in Segal and Whinston (2002), we can derive a buyer-option contract that achieves the equilibrium outcome of the original message game.[31]

Proposition 10 *Suppose $\alpha_s^* < \lambda^b$. Then a buyer-option contract exists such that the seller invests entirely at the first-best level in a subgame perfect equilibrium. Subsidization does not occur in equilibrium.*

Che and Hausch (1999) and Segal and Whinston (2002) derive a first-best *noncontingent* contract for highly selfish cases if the seller's ex ante payoff $R^s(x) - x^s$ is strictly concave for *any* noncontingent contract. This condition may not hold in general and hence in our setting, a more complex contract may be necessary. This result does not contradict Che-Hausch-Segal-Whinston's result since a buyer-option contract can be a noncontingent contract.

7.3   Summary

For general hybrid cases, the level of each party's bargaining power affects whether adopting a contract can achieve full efficiency. Given a hybrid investment, the weaker the buyer's bargaining power, the more likely full efficiency via subsidization is available. Similarly, the stronger the buyer's bargaining power, the more likely full efficiency via non-subsidization is available. Which of those efficiency results are more available depends on the measures of cooperativeness (Figure 4).

When $\alpha_c < \lambda^b < \alpha_s$, we can not say much about optimal contracts, since the single-crossing property does not hold within this region, and we can not use Segal-Whinston's

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[31]The buyer-option contract is constructed as follows: for a given message game, create a new contract by restricting the seller's message only to the equilibrium message when the efficient investment is made. The new contract is a buyer-option contract. By adopting this contract, the seller invests efficiently; her payoff remains the same if she invests efficiently but (weakly) decreases if she invests at a different level because of the zero-sum nature of the game.
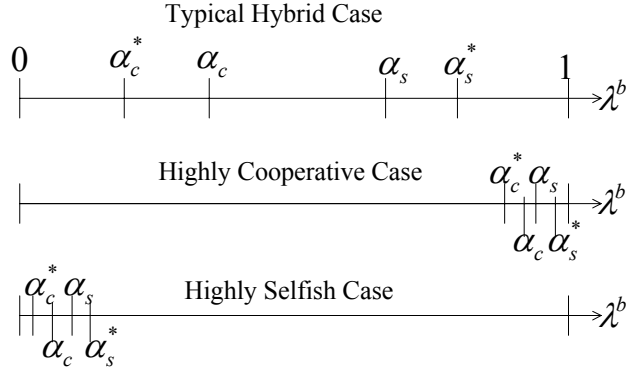
Figure 4:

lemma.[32]

8   Concluding Remarks

We have studied how contracting affects efficiency in a holdup model with a subsidization opportunity. The main finding is that the holdup inefficiency can be avoided when the investment is sufficiently cooperative. In particular, if the investment is purely cooperative, a buyer-option contract achieves full efficiency and no noncontingent contract can achieve full efficiency (except in the special cases where one party has full bargaining power). These results are in stark contrast to the result of Che and Hausch (1999).

We have also showed that the subsidization opportunity in our model does not create additional inefficiency given any contract. Thus, thanks to the result of Edlin and Reichelstein (1996), if the investment is purely selfish, full efficiency is achievable by a noncontingent contract without inducing subsidization. In addition, we have derived that a buyer-option contract achieves efficiency if the investment is sufficiently selfish.

The holdup inefficiency seems to remain in intermediate hybrid cases. However, we have not analyzed general hybrid cases where the single-crossing property fails to hold since we cannot apply Segal-Whinston's lemma. In those cases the incentive compatibility condition for a general message game should give a more stringent restriction on implementable revenue functions. It seems more difficult to adjust incentives of the parties to achieve efficiency.

We have not considered uncertainty either, except in a special case for a purely co-

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[32]It seems hard to check if their original weaker condition (Condition ±) holds.

operative investment. Uncertainty creates another holdup problem that complicates the investment incentives. Our approach separates the holdup problem due to uncertainty from the one due to relationship-specific investment. Including uncertainty causes a big analytical problem: Segal-Whinston's lemma cannot apply unless information about the multidimensional state space is aggregated into a single number. Future research should explore how to characterize an optimal or efficient contract in cases with a multidimensional state space.

Despite restricting assumptions, we have seen that a subsidization opportunity improves contractual solutions to the holdup problem in the cases that have been thought of as impossible to remedy (e.g., highly cooperative cases). Introducing subsidization would shed light on the firm's cooperative behavior in making relationship-specific investments and on the firm's decision about integration. To combine repeated interaction and reputation with subsidization might be fruitful to understand the firms' dynamic decision making regarding their behavioral and organizational choice. This issue is left for future research.

A   Appendix

Proof of Proposition 4. By Proposition 1, when either party has full bargaining power, the first-best investment is achieved. Suppose then each party has a positive bargaining share and there is a noncontingent contract $(\overline{q}, \overline{t})$ that will induce x^*.

First, we show that this contract must induce the buyer to entirely invest x^*. Suppose on the contrary the seller invests $x^s \in (0, x^*]$. The seller's revenue is

$$R^s(x^b, x^s) = \overline{t} - c(\overline{q}) + \lambda^s[S^*(x^b + x^s) - S\left(\overline{q}, x^b + x^s\right)]$$
$$\equiv \tilde{R}^s(x^b + x^s).$$

(Note that for a noncontingent contract, the revenues only depend on the total investment given x^b and x^s). Since the seller invests after the buyer,

$$\frac{dR^s}{dx^s}(x^b, x^s) = \frac{\partial R^s}{\partial x^s}(x^b, x^s)$$
$$= \tilde{R}^{s\prime}(x^*)$$
$$= \lambda^s[S^{*\prime}(x^*) - S_x(\overline{q}, x^*)]$$
$$= \lambda^s[1 - v_x(\overline{q}, x^*)]$$
$$< 1.$$

This is a contradiction since the seller's marginal revenue must equal to 1. Thus the buyer invests all the efficient investment.

Next, we show that $\overline{q} = q^*(x^*)$. Recall first that the buyer's revenue given (x^b, x^s) is

$$R^b(x^b, x^s) = v(\overline{q}, x^b + x^s) - \overline{t} + \lambda^b[S^*(x^b + x^s) - S\left(\overline{q}, x^b + x^s\right)]$$
$$\equiv \tilde{R}^b(x^b + x^s).$$

Since the buyer chooses $x^b = x^*$ in equilibrium, his marginal revenue dR^b/dx^b at $(x^*, 0)$ must be 1, the marginal cost. To derive the expression of dR^b/dx^b, he needs to anticipate the seller's response $x^s(x^b)$. However, we have seen that

$$\frac{dR^s}{dx^s}(x^*, 0) < 1,$$

and by continuity, $\dfrac{dR^s}{dx^s}(x^* + \varepsilon, 0) < 1$ for small $\varepsilon > 0$. This shows $x^{s\prime}(x^*) = 0$. Thus,

$$\frac{dR^b}{dx^b}(x^*, x^s(x^*)) = \frac{\partial R^b}{\partial x^b}(x^*, 0)$$
$$= \tilde{R}^{b\prime}(x^*).$$

This implies $\tilde{R}^{b\prime}(x^*) = S^{*\prime}(x^*) = 1$. Notice that $\tilde{R}^b(x) + \tilde{R}^s(x) \equiv S^*(x)$. This implies

$$\tilde{R}^{s\prime}(x^*) = S^{*\prime}(x^*) - \tilde{R}^{b\prime}(x^*) = 0$$
$$\implies \lambda^s[S^{*\prime}(x^*) - S_x(\overline{q}, x^*)] = 0$$
$$\implies \lambda^s[S_x'(q^*(x^*), x^*) - S_x(\overline{q}, x^*)] = 0 \text{ by the envelope theorem}$$
$$\implies \overline{q} = q^*(x^*) \text{ by } S_{qx} = v_{qx} > 0.$$

This shows $\overline{q} = q^*(x^*)$.

Therefore, this noncontingent contract induces no renegotiation on the equilibrium path.

Now suppose that the seller refuses to allow the buyer to invest and does not invest afterwards. The resulting total investment level is zero and the seller's payoff is

$$\overline{t} - c(\overline{q}) + \lambda^s[S^*(0) - S(\overline{q}, 0)]. \tag{8}$$

The renegotiation surplus must be positive since $q^*(0) \neq q^*(x^*) = \overline{q}$.[33] However, the seller's payoff on the equilibrium path equals to the status quo payoff with no renegotiation surplus, which is strictly less than the one in (8). This is a contradiction. ∎

Proof of Lemma 1. First, we show that q^* is strictly increasing. The ex-post efficient trade $q^*(x)$ is defined implicitly by

$$v_q(q^*(x), x) - c_q(q^*(x), x) = 0.$$

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[33] Strict monotonicity of q^* is proved in the proof of Lemma 1.

By the implicit function theorem,

$$q^{*\prime}(x) = -\frac{v_{qx}(q^*(x), x) - c_{qx}(q^*(x), x)}{v_{qq}(q^*(x), x) - c_{qq}(q^*(x), x)} > 0.$$

Recall that the ex post efficient surplus $S^*(x)$ is differentiable, and by the envelope theorem, $S^{*\prime}(x) = v_x(q^*(x), x) - c_x(q^*(x), x)$. Now,

$$
\begin{aligned}
S^{*\prime}(x) - S_x(q, x) &= [v_x(q^*(x), x) - c_x(q^*(x), x)] - [v_x(q, x) - c_x(q, x)] \\
&= [v_{qx}(\tilde{q}, x) - c_{qx}(\tilde{q}, x)](q^*(x) - q)
\end{aligned}
$$

for some $\tilde{q}$ between $q^*(x)$ and q (by the mean value theorem). The term in square brackets in the second line is positive by assumption. Hence

$$S^{*\prime}(x) - S_x(q, x) \gtreqless 0 \text{ iff } q^*(x) \gtreqless q \text{ iff } x \gtreqless q^{*-1}(q).$$

In the case where $q^{*-1}(q)$ is not well-defined, replace it with 0 if $q < q^*(0)$ or with $+\infty$ if $q > q^*(x)$ for all x. ∎

Proof of Proposition 8. Since $\alpha_c > \lambda^b$, the single crossing property holds, and Lemma 2 thus applies. Note the following condition is always available by setting $g(x) = 0$;

$$\lambda^s v_x(0, x) + \lambda^b c_x(0, x) = 0.$$

The condition $\alpha_c^* > \lambda^b$ implies

$$\lambda^s v_x(q_{\max}, x) + \lambda^b c_x(q_{\max}, x) \geq \lambda^s S^{*\prime}(x) \text{ for } \forall x \in [0, x^*].$$

By the intermediate value theorem, there is $q^e(x)$ for $x \in [0, x^*]$ such that

$$\lambda^s v_x(q^e(x), x) + \lambda^b c_x(q^e(x), x) = \lambda^s S^{*\prime}(x) \text{ for } \forall x \in [0, x^*].$$

Define a generating decision rule as

$$g(x) = \begin{cases} q^e(x) & \text{for } x \in [0, x^*], \\ 0 & \text{otherwise.} \end{cases}$$

Then by adopting a contract corresponding to g, the buyer's revenue satisfies

$$R^{b\prime}(x) = \begin{cases} S^{*\prime}(x) & \text{for } x \in [0, x^*], \\ \lambda^b S^{*\prime}(x) & \text{otherwise.} \end{cases}$$

The seller's revenue satisfies

$$R^{s\prime}(x) = \begin{cases} 0 & \text{for } x \in [0, x^*], \\ \lambda^s S^{*\prime}(x) & \text{otherwise.} \end{cases}$$

Therefore, the buyer's revenue function coincides with the ex post efficient surplus in $[0, x^*]$, and the seller's revenue function is zero in the same region. By construction, the buyer's revenue is concave, and the seller will never have incentive to invest (i.e., $R^{s\prime}(x) < 1$ for all x). With the same logic as in Proposition 5, the seller allows the buyer to invest, and the buyer invests efficiently. ∎

Proof of Proposition 9. Since $\alpha_c > \lambda^b$, the single-crossing property $r_{qx}^b > 0$ holds. This is equivalent to $r_{qx}^s < 0$. The seller's marginal revenue given a contract is, for the corresponding $g(x)$,

$$R^{s\prime}(x) = \lambda^s S^{*\prime}(x) - \lambda^s v_x(g(x), x) - \lambda^b c_x(g(x), x) \ a.e.$$

Recall that the null contract gives $R^{s\prime}(x) = \lambda^s S^{*\prime}(x)$, and by the single-crossing property the seller's incentive decreases when $g(x)$ is nonzero. Therefore, the null contract is optimal when there is no subsidization opportunity. ∎

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