Earnings Growth and the Bull Market of the 1990s:

Is There a Case for Rational Exuberance? †

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Abstract

This paper examines whether permanent earnings growth, crucial to stock valuation, increased during the last decade as suggested by proponents of the 'New Economy.' Using S&P 500 earnings for 1951-2000, we do not find strong evidence of either a one-time structural break or gradual change. However, the confidence interval on permanent earnings growth is wide enough to include an increase that is consistent with the bull market of the late 1990s. Thus we cannot reject a rational basis for that exuberance.

Key words: *Irrational exuberance; New Economy; Earnings growth; Bull market*

1. Introduction

No one would object to describing the 1990s as a bull market. Over the decade, the Standard and Poor's 500 index increased in real terms from 408 (January 1991) to 1291 (December 2000), with most of the rise occurring in the second half of the decade (The index was at 522 in January 1995.) While academics and practitioners generally agree that investors' enthusiasm played a key role in the stock price spike, they have diverging views on what caused such enthusiasm.

In his 2000 best-selling book, *Irrational Exuberance*, Robert Shiller argues that investors' irrationality was responsible. Enthusiasm bred more enthusiasm and rising stock prices encouraged investors to bid stocks up even further in a "self-fulfilling prophecy, a feed-back loop". Taking extreme valuation ratios as supporting evidence, he predicts in an article co-authored with John Campbell that stock prices are doomed to fall dramatically with the worst case where the stock market could lose more than three-quarters of its real value; see Campbell and Shiller (2001).

The opposing view is that stocks were rationally valued, reflecting changes in fundamentals occurring as a result of the New Economy. Higher productivity growth enabled by the technological and communications revolution substantially elevated the prospects of earnings growth; see Browne (1999) and Greenwood and Jovanovic (1999). Indeed, financial analysts' forecast of earnings growth over years to come collected in the late 1990s by The Institutional Brokers Estimate System (I/B/E/S) was more than 10%. Such optimism made stocks more attractive and investors were willing to pay higher prices.

Further, earnings growth in the 1990s seemed to justify the optimism: real earnings grew at an average annual rate of 6.6% after the 1990-1991 recession while the average growth rate over the last five decades was only 2.2%. Caution should, however, is advised in interpreting this as a permanent increase since the standard deviation of real earnings growth is large, 20.5% for 1991-2000. Formal tests for a change are thus in order before drawing conclusions.

We consider and test two types of changes in the permanent growth rate. First is one-time permanent structural break in the mean of earnings growth at an unknown point, since the New Economy story suggests a fairly abrupt change. We employ Bayesian model selection rather than classical tests to take advantage of its appealing features; see Koop and Potter (1999). The Bayesian approach produces as a by-product the posterior probability distribution of the unknown break point that provides us with a visual summary of information regarding the break point. The second type of permanent movement under consideration is gradual change since the old era could have made a smooth transition to the new era. Smooth change is assumed to obey a random walk that is the permanent component in an unobserved components model of earning growth. We test if the parameter governing the variance of the permanent movement is zero against small positive, using Stock and Watson's (1998) (SW hereafter) asymptotic median unbiased estimation. Section 2 presents the models and their inference. Section 3 discusses the empirical results. Section 4 concludes the paper.

2. Two Models of Earnings Growth

Gordon's (1962) valuation model motivates the importance of expected earnings growth as a fundamental determinant of stock prices:

$$P_{t-1} = \frac{E_t}{(R - \mu)} \tag{1}$$

where P_{t-1} is stock price at time $t-1$; E_t is earnings at time t; [1] R is a constant discount rate; and μ is a constant rate at which earnings are expected to grow. Clearly, price is very sensitive to a change in μ, an increase causing the price to rise.

To see whether the expected earnings growth rate might plausibly have changed in the 1990s we consider two models which accommodates a permanent change in the mean. In the first model, the mean undergoes one-time permanent structural change at an unknown date. In the second one, the mean is allowed to change gradually over time, following a random walk which captures permanent changes in the growth rate. [2]

2-1. A model with a structural break in the mean growth rate

The model we employ to allow for a possible structural break in the mean of earnings growth is as follows:

$$y_t = \mu_t + e_t \tag{2}$$

[1] The model is, in fact, expressed in terms of dividend instead of earnings. However, there exist potential problems that prevent dividend from reflecting firms' ability to generate profits. Examples include dividend smoothing (Kleidon (1986)), share repurchases by executives with stock options to boost stock prices, and the tendency of firms to pay less dividends (Fama and French (2000)).

[2] Barsky and De Long (1993) drop the assumption of Gordon valuation model that the dividend growth rate has a constant mean known to the agents throughout the sample, and instead postulate an environment in which investors estimate, period by period, a growth rate that is nonstationary. In this framework, they show that long-run movement in dividends drives long-run fluctuations in the U.S. stock market.

$$\mu_t = \mu_0(1 - D_t) + \mu_1 D_t, \qquad \mu_0 < \mu_1 \tag{3}$$

$$D_t = \begin{cases} 0, & \text{if t=1,2, } \cdots, \tau \\ 1, & \text{if t= } \tau \text{ +1 , } \cdots, \text{ T} \end{cases} \tag{4}$$

$$\phi(L)e_t = u_t, \quad u_t \sim i.i.d.N(0, \sigma_u^2) \tag{5}$$

where y_t is the growth rate of earnings at time t. D_t is a variable that determines the regime of the mean of y_t. When D_t=0, the unconditional expectation of y_t is μ_0; when D_t=1, it is μ_1. The roots of $\phi(L) = 0$ lie outside the unit circle. In order to allow for the possibility of one-time permanent but endogenous structural break with unknown change point (τ), we follow Chib (1998) and Kim and Nelson (1999) in treating D_t as a discrete latent variable with the following transition probabilities:

$$Pr(D_t = 0|D_{t-1} = 0) = q, \quad Pr(D_t = 1|D_{t-1} = 1) = 1, \qquad 0 < q < 1 \tag{6}$$

If a structural break has not occurred up to time t, that is, if D_t =0, the probability that a structural break occurs at time t+1 is 1-q. Because we consider one-time break only, once a structural break occurs at $t = \tau$, we have $D_{\tau+j} = 1$ for all $j > 0$. We denote this model Model 1. If $\mu_0 = \mu_1$, Model 1 reduces to a simple autoregressive process, denoted Model 0.

To test for a structural break we take the Bayesian approach. Notice that the unknown break point τ is a nuisance parameter that exists under the alternative hypothesis but not under the null. As indicated in Koop and Potter (1999), Bayesian model selection based on the Bayes Factor has an advantage over classical tests of Andrews and Ploberger (1999) in integrating out τ: The classical tests fail to incorporate information about the break point contained in the data whereas the Bayesian approach does incorporate that information. To carry out Bayesian we employ the Markov Chain Monte Carlo (MCMC) integration method of Gibbs sampling. As suggested in Chib (1995), the simulation method enables us to easily obtain the marginal likelihoods as by-products. These marginal likelihoods are inputs into the Bayes factor, which is defined as the ratio of marginal likelihoods for the models under consideration:

$$B_{10} = \frac{m(\tilde{Y}_T|\mu_0 \neq \mu_1)}{m(\tilde{Y}_T|\mu_0 = \mu_1)} \tag{7}$$

where B_{10} is the Bayes factor in favor of Model 1 over Model 0, $m()$ is a marginal likelihood and $\tilde{Y}_T = [y_{p+1}, \cdots, y_T]$ where p is the lag order of $\phi(L)$. Details of the Gibbs sampling procedure are described in the Appendix A.

For Bayesian model selection, we adopt the criteria of Jeffreys (1961) in which $lnB_{10} \leq 0$ is evidence in support of Model 0 where ln indicates the natural logarithm; $0 < lnB_{10} \leq 1.15$, very slight evidence against Model 0; $1.15 < lnB_{10} \leq 2.3$, slight evidence against Model 0; $2.3 < lnB_{10} \leq 4.6$, strong to very strong evidence against Model 0; $lnB_{10} > 4.6$, decisive evidence against Model 0.

2-2. A model with slowly time-varying mean growth rate

While the New Economy suggests a structural break in the mean of earnings growth, we cannot rule out the possibility that it changes slowly period by period so that structural break tests are not capable of identifying permanent movements in earnings growth. To accommodate and test this possibility, we consider the following model, rewriting equation (2) and (5):

$$y_t = \mu_t + e_t \tag{2}$$

$$\phi(L)e_t = u_t, \quad u_t \sim i.i.d.N(0, \sigma_u^2) \tag{5}$$

$$\mu_t = \mu_{t-1} + v_t, \quad v_t \sim i.i.d.N(0, \sigma_v^2) \tag{8}$$

Equation (8) describes how the mean of earnings growth evolves over time. Basically, we allow μ_t to follow a random walk. Since v_t has a permanent effect on μ_t in the sense that its effect never dies out, μ_t captures permanent movements in earnings growth. In contrast, the shock u_t has a temporary effect on e_t: Its effect dies out. Thus, the model is an unobserved components model in which earnings growth y_t has a stochastic long-run permanent component μ_t around which a cyclical transitory component e_t fluctuates.

To see if there are permanent movements in earnings growth, we test whether or not the variance of v_t (or variance of $\Delta\mu_t$) is zero. If it is zero, no permanent movements in earnings growth exist, which is the null hypothesis. The alternative under consideration is that the variance is very small positive. This is because a unit root in earnings growth could contribute a very small share of earnings growth volatility. Many authors including SW document that the model under this particular alternative is equivalent to a model having a nearly unit MA root where tests for a unit AR root (ADF unit root tests) have a high false-rejection rate of a unit AR root. In fact, ADF unit root tests reject the null hypothesis of a unit AR root in earnings growth.

It is well established that inference of σ_v by Maximum Likelihood estimation tends to have a point mass at zero so that it is readily mistaken for zero when its true value is small. For this reason, after redefining v_t as $\frac{\lambda}{T}\eta_t$ where T is the number of observations, we employ the methodology of SW that yields an estimator of λ with a property of median unbiasedness (MUB hereafter) in large sample. The main idea in deriving the estimators is that under a reasonable normalization $\sigma_\eta = \frac{\sigma_u}{\phi(1)}$, the limiting distributions of test statistics under the null $\lambda = 0$ depend only on λ and so do their medians. [3] Thus, inverting the medians yields the median unbiased estimators of λ if the functions are monotone increasing and continuous in λ.

[3] By construction, the definition raises an issue that the variance of v_t depends on λ and σ_η which are not separately identified. This normalization also enables us to identify the two parameters.

3. Empirical Results

The data examined in this paper are quarterly earnings of the S&P 500 index, 4-quarter total, adjusted to the index obtained from the table entitled "Earnings, Dividends and Price-Earnings Ratios–Quarterly" of Standard and Poor's Statistical Service *Security Price Index Record*. [4] To reflect the feature of 4-quarter total, we choose an order of 4 for the autoregressive process $\phi(L)$ throughout the paper. The nominal earnings are divided by the GDP deflator from the DRI Basics database to produce real earnings, which are transformed to percentage growth at an annual rate, y_t, by setting y_t=400 $\times\Delta$ ln(real earnings). Since we are interested in the possibility of a dramatic increase in the mean of earnings growth in the most recent decade, we use only post-war data starting from 1951. In order to deal with the New Economy issue, we exclude period after 2000 during which the New Economy has become less strident with the collapse of internet-company stock valuations.

3-1. Inference in a model with possible structural break

We employ normal priors for $\tilde{\mu} = [\mu_0, \mu_1]'$ and $\tilde{\phi} = [\phi_1, \phi_2, \phi_3, \phi_4]'$, an inverted gamma distribution for σ_u^2, and a beta distribution for q. In particular, the following three alternative sets of priors are used

Prior #1: $\tilde{\mu} \sim N(0, I_2)$; $\tilde{\phi} \sim N(0, I_4)$; $\frac{1}{\sigma_u^2} \sim Gamma(1, 2)$; $q \sim Beta(8, 0.045)$

Prior #2: $\tilde{\mu} \sim N(0, 2 * I_2)$; $\tilde{\phi} \sim N(0, 2 * I_4)$; $\frac{1}{\sigma_u^2} \sim Gamma(1, 4)$; $q \sim Beta(8, 0.2)$

Prior #3: $\tilde{\mu} \sim N(0, 0.5 * I_2)$; $\tilde{\phi} \sim N(0, 0.5 * I_4)$; $\frac{1}{\sigma_u^2} \sim Gamma(1, 1)$; $q \sim Beta(8, 0.1)$

[4] The data are also available at http://aida.econ.yale.edu/shiller, the website of Robert Shiller.

All inference is based on 10,000 Gibbs simulations, with the initial 2,000 simulations discarded to mitigate the effects of initial conditions. We report results for only Prior #1 since the empirical results turn out to be insensitive to prior specifications.

Table 1 summarizes the Bayesian inference of the parameters for Model 0, no structural break model, and Model 1, structural break model. [5] A comparison of the log of marginal likelihoods clearly indicates that Model 0 dominates Model 1: The log of the Bayes factor is -2.61, evidence against the model with structural break. [6] The posterior probability distribution of the break date (τ) shown in Figure 1 confirms this. Over the entire period considered, the distribution is somewhat diffuse. Even though the posterior mode of the break date is 1991:IV, no substantial mass around exists. Further, the probability of structural break ($Pr(D_t = 1)$) shown in Figure 2 steadily increases while it tends to sharply increase in the case of structural break.

There is substantial evidence in the literature that many macroeconomic and financial time series underwent volatility change in the form of structural break in the mid 1980s; see, among others, Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2002) and *van* Dijk, Osborn and Sensier (2002). As seen in Figure 1, earnings growth seems more volatile in the past two decades (standard deviation of 20.48%) than in the previous three decades (standard deviation of 13.85%). We investigate whether and when volatility change in earnings growth occurred and whether such heteroskedasticity influences inference about structural break in the mean.

From Bayesian inference of the model with structural break only in the variance (Its specification is presented in Appendix B.), we find decisive evidence in favor of one-time volatility increase occurring 1986:III. [7] The log of the Bayes factor in favor of the model

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[5] The posterior means of the parameters are similar to the Maximum Likelihood estimates which are not reported here.

[6] We find similar results from classical tests proposed in Andrews and Ploberger (1994) and Bai and Perron (1998).

[7] The estimated break date 1986:III is calculated from the expected duration of the

with a structural break only in the variance against the model with no structural break in the mean and variance (Model 0) is 6.94. [8] The posterior distribution of the change point shown in Figure 3 is very tightly clustered around its posterior mode. In addition, the probability of structural break displays a sharp increase at the change point (Figure 4). [9]

Given the evidence of structural break in the variance, we re-examine structural break in the mean assuming that the structural break in the variance occurred and the break point for the variance is known to us. The resulting model takes the following form, rewriting equations (2), (3), (4) and (6):

$$y_t = \mu_t + e_t \tag{2}$$

$$\mu_t = \mu_0(1 - D_t) + \mu_1 D_t, \qquad \mu_0 < \mu_1 \tag{3}$$

$$D_t = \begin{cases} 0, & \text{if t=1,2, } \cdots, \tau \\ 1, & \text{if t= } \tau \text{ +1 , } \cdots, \text{ T} \end{cases} \tag{4}$$

$$Pr(D_t = 0|D_{t-1} = 0) = q, \quad Pr(D_t = 1|D_{t-1} = 1) = 1, \qquad 0 < q < 1 \tag{6}$$

$$\phi(L)e_t = u_t, \quad u_t \sim i.i.d.N(0, \sigma^2_{u,S^*_t}) \tag{5$'$}$$

$$\sigma^2_{u,S^*_t} = \sigma^2_{u,0}(1 - S^*_t) + \sigma^2_{u,1} S^*_t, \qquad \sigma^2_{u,0} < \sigma^2_{u,1} \tag{10}$$

$$S^*_t = \begin{cases} 0, & \text{if t=1,2, } \cdots, \tau^* \\ 1, & \text{if t= } \tau^* \text{ +1 , } \cdots, \text{ T} \end{cases} \tag{11}$$

where $\sigma^2_{u,0}$ and $\sigma^2_{u,1}$ are variances of earnings growth before and after the break point known to us as τ^*=1986:III. S^*_t is a dummy variable indicating the regime of variance.

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state before the structural break occurs, $\frac{1}{1-\bar{p}}$, where $\bar{p}$ is the posterior mean of p, which is the probability of a break in the variance occurring at time t given no break up to time $t-1$.

[8] Posterior moments from the model are not reported here but are available upon request.

[9] It seems interesting that aggregate profit becomes more volatile after the mid 1980s while the economy becomes more stable since the mid 1980s. This issue deserves more research.

The model is denoted Model 1A. Likewise, Model 0A with the variance dummy and no break in the mean is extended from Model 0.

Table 2 summarizes the Bayesian inference of the parameters for Model 0A and Model 1A. Despite more efficient use of information contained in the data, we still find no evidence of structural change in the mean. The log of the Bayes factor in favor of Model 1A is -3.31. The posterior probability distribution of the break date (τ) shown in Figure 5 is more diffuse than in Figure 1. This leads us to conclusion that higher earnings growth in the 1990s provides only weak evidence of an abrupt shift in the mean growth of earnings.

Meanwhile, as mentioned earlier, the posterior mode of the change point in the mean is 1991:IV. In addition, the increase in the posterior mean of the growth rate is sizable: μ_1=5.8621% is almost twelve times larger than μ_0=0.4667% (Table 1). Therefore, it seems reasonable to examine the possibility that other type of permanent change in earnings growth occurred in the 1990s.

3-2. Inference in a model with time-varying mean

To test if the mean of earnings growth changes over time following a random walk, we consider four test statistics from which MUB estimator of λ is derived. Nyblom's (1989) L-statistic designed to test the constancy of a mean over time against random walk alternatives is paid a main attention. Additionally, we consider the mean Wald (MW) statistic and exponential Wald (EW) statistic of Andrews and Ploberger (1994), and Quandt's (1960) likelihood ratio (SW) statistic. [10]

Table 3 presents those test statistics under the null hypothesis of no time variation in μ_t ($H_0 : \lambda$=0) against the alternative that it varies over time following a random walk ($H_1 : \lambda \neq 0$). Panel A is for the model with homoskedastic disturbances introduced in

[10] These are test statistics for a break but their limiting distributions yield MUB estimator of λ. SW also use these statistics to make inference about λ in GDP growth.

subsection 2-2. In the preceding subsection 3-1, we found significant volatility increase in earnings growth at 1986:III. Since heteroskedasticity may have an impact on inference, we introduce heteroskedasticity into the transitory component of earnings growth e_t. [11] In particular, we assume that it has GARCH(1,1) disturbances following Harvey, Ruiz and Sentana (1992). [12] Panel B is the results for the heteroskedastic case.

The results clearly show that the null hypothesis of constant mean growth rate cannot be rejected at the conventional significance level. p-values given in the second column range from 0.2 (for SW statistic) to 0.7 (for L statistic) for the homoskedastic case. Taking into account heteroskedasticity does not change the test result: p-values range from 0.4 (for SW statistic) to 0.8 (for L statistic).

Nonetheless, if the tests have insufficient power, λ could be nonzero meaning permanent movements. In fact, MUB estimates of λ given in the third column are small and nonzero: 1.53 (EW) and 4.80 (SW) for the homoskedastic case and 2.35 (SW) for heteroskedastic case. These correspond to point estimates of σ_v 0.17% and 0.54% (Panel A), and 0.30% (Panel B) given in the last column. [13] Moreover, we notice that a lot of uncertainty is involved in λ and in turn σ_v: The confidence intervals of σ_v in some cases are wide enough to include 2%. Taken together, these results suggest that the existence of even highly variable permanent changes in the mean growth rate of earnings are not excluded.

Given such large uncertainty in σ_v, we perform two experiments. First, we examine the dynamics of permanent movements for different values of σ_v within a confidence interval. Basically, Gaussian Maximum Likelihood implemented by the Kalman filter can estimate the parameters of the model, and given the parameter estimates, the time path of the permanent component can be obtained. In doing so, we fix σ_v at a given value, that is,

[11] The corresponding model specification can be found in Appendix C.
[12] We found negligible heteroskedasticity in the permanent component.
[13] Given λ, it is straightforward to obtain σ_v using the relation $\sigma_v = \frac{\lambda}{T}\sigma_\eta$.

σ_v is not estimated while other parameters are freely estimated. Second, we seek the time path of the permanent component that best explains the movement of historical price-earnings ratio and estimate the risk premium based on it. We report only the results for the heteroskedastic case.

To see how the permanent growth behaves when uncertainty in σ_v is accounted for, we choose three values of σ_v=0.26, 0.52, and 0.74. [14] Table 4 reports the parameter estimates from these restricted Maximum Likelihood estimations along with those from an unrestricted estimation in which σ_v is also freely estimated. In both restricted and unrestricted estimations, μ_0 is treated as an unknown constant and estimated following Nyblom (1989). Based on the estimates of the unrestricted and restricted models, the time paths of μ_t are computed using the Kalman smoother and plotted in Figure 6. [15] As predicted, its time path for $\sigma_v = 0.74$ exhibits the biggest variation. The straight line corresponds to the unrestricted MLE which yields the estimate of σ_v nearly close to zero.

A couple of points are worth making. First, starting around 1973, μ_t continues to fall through the 1970s, which suggests that the big retreat in stock prices over the period is in part due to poor performance of the economy. It is of particular interest that the bottom is reached in 1982 when the famous "Death of Equities" cover appeared on *Business Week*.

Second, more importantly, the permanent component of earnings growth increases from the early 1980s through the 1990s. Interestingly, such a trend matches well with the time line of the information technology revolution caused by the New Economy that Hobijn and Jovanovic (2001) argue starts in the early 1980s and persists throughout the 1990s. Figure 7 plots price-earnings ratios implied by the time path of μ_t corresponding

[14] 0.74 is selected because it is the upper bound of the confidence interval from L statistic which is our main focus.

[15] We use the Kalman smoother rather than the Kalman filter to use more information in inference of μ_t. Also, it turns out that the Kalman smooth estimates make more sense. Appendix C presents the state-space representation of the model and details about estimating and smoothing.

to σ_v=0.74, calculated on the basis of the Gordon Valuation model given in Section 2, along with historical price-earnings ratio. 7.5% is used as discount rate R. It is seen that the implied one is able to explain the broad variation of the historical one. Hence, this result supports the rational exuberance hypothesis.

Now we attempt to estimate the risk premium based on the time path of μ_t that best explains the movement of historical price-earnings ratio. Assuming that the discount rate R (the sum of risk free rate R^f and risk premium R^p) is not constant, the Gordon formula implies:

$$\frac{E_t}{P_{t-1}} = (R_t^f + R_t^p) - \mu_t, \tag{12}$$

A good proxy for the risk free rate in practice is yields on 10-year Treasury note in real terms which is obtained by subtracting CPI inflation over the last 12 months from the nominal yields. μ_t can be estimated by the Kalman smoother as described above. Then, the risk premium is measured with error:

$$\hat{R}_t^p = \frac{E_t}{P_{t-1}} - \hat{R}_t^f + \hat{\mu}_t, \tag{13}$$

Since $\hat{\mu}_t$ depends on σ_v, put another way, $\hat{\mu}_t(\sigma_v)$, so does the risk premium.

We search for σ_v over [0.0001 2] that best explains $\frac{E_t}{P_{t-1}}$ in terms of minimizing the variance of resulting residuals. [16] That is,

$$\min_{\sigma_v} \frac{1}{T} \sum_{t=1}^{T} (\hat{R}_t^p - \bar{R}^p)^2, \tag{14}$$

where $\bar{R}^p = \frac{1}{T}\sum_{t=1}^{T} \hat{R}_t^p$. As a result, we obtain the least variable risk premium.

We find that σ_v =0.70 minimizes the variance of the risk premium. This value is within its confidence interval from L statistic. Figure 8 plots the risk premium estimated

[16] The widest 90% confidence interval of σ_v is close to this inverval.

in this manner. We find two observations interesting. First, the implied risk premium continues to rise during the late 1990s. This is at odds with recent studies that find evidence of a decline in the risk premium in an effort to explain the prices run-up of the 1990s; see Blanchard (1993), Claus and Thomas (1999), Siegel (1999), Heaton and Lucas (2000), Jagannathan, McGrattan and Scherbina (2001), and Fama and French (2002).

Second, the risk premium over the period from the mid 1970s through the early 1980s is extraordinarily high, which suggests that the long severe bear market is mainly due to the high risk premium. Modigliani and Cohn (1979) attribute the enormous risk premium to money illusion created by high inflation, arguing that investors used the very high nominal interest rate rather than the real interest rate to discount earnings. [17] Figure 9 appears to verify their argument, where the risk premium is computed with real yields replaced by nominal yields for the period 1973:IV-1980:IV. The magnitude of residuals of the 1970s is now as large as those in other decades.

4. Conclusions

It has been argued that the New Economy, characterized by greater growth in real income and productivity, generated higher permanent earnings growth to which stock prices in the 1990s responded. This paper explores whether there has been a permanent increase in the rate of earnings growth during the last decade. In particular, we consider two types of permanent movements separately: an abrupt shift in the mean at an unknown point and a gradual change in the form of a random walk in an unobserved components

[17] Other inflation-related explanations of the price decline include interactions between inflation and tax codes (see Nelson (1976) and Feldstein (1980)), and the "proxy hypothesis" (see Nelson (1979) and Fama (1981)); for a survey see Sharpe (2001).

model. For earnings growth of the S&P 500 index for the 1951-2000 period, we find that Bayesian model comparison prefers the model without structural break to the structural break model, and that statistical tests based on asymptotic median unbiased estimation do not give strong evidence against the null hypothesis that the mean of earnings growth was unchanged in the 1990s.

However, our Bayesian analysis of a possible structural break does find that the early 1990s was when a structural break in the mean is most likely to have occurred. As always, acceptance of the null hypothesis does not exclude alternative hypotheses within a confidence interval. Accordingly, we can account for a substantial increase in permanent earnings growth, and rise in stock valuation, by allowing for variation in the permanent growth rate that is within the confidence interval.

To sum up, proponents of rational exuberance and the New Economy can take comfort that variation in permanent earnings growth within the confidence interval can explain much of the bull market. Skeptics can fall back on the lack of strong statistical evidence that a change in earnings growth did in fact occur.

Appendix A: Calculation of the Marginal Likelihood based on Gibbs Sampling

In this section, we present a procedure for directly calculating the marginal likelihood for Model 1 by extending Chib (1995). [18] The procedures for other models under consideration are straightforward to modify.

Let's define $\tilde{\theta} = [\tilde{\phi}' \ \tilde{\mu}' \ \sigma_u^2 \ q]$ to be a vector of the parameters of the model, where $\tilde{\phi} = [\phi_1 \ \cdots \ \phi_4]'$ and $\tilde{\mu} = [\mu_0 \ \mu_1]'$. Then, as in Chib (1995) the marginal density of $\tilde{Y}_T = [y_{p+1} \ \cdots \ y_T]'$, by virtue of being the normalizing constant of the posterior density, can be written as:

$$m(\tilde{Y}_T) = \frac{f(\tilde{Y}_T|\tilde{\theta})\pi(\tilde{\theta})}{\pi(\tilde{\theta}|\tilde{Y}_T)}, \tag{A1}$$

where the numerator is the product of the densities of the sample observations and the prior, with all integrating constants included, and the denominator is the posterior density of $\tilde{\theta}$. As the above identity holds for any $\tilde{\theta}$, we may evaluate the marginal density at the posterior mean $\tilde{\theta}^*$. For computational convenience, the preceding equation is taken of the logarithm, resulting:

$$ln\ m(\tilde{Y}_T) = ln\ f(\tilde{Y}_T|\tilde{\theta}^*) + ln\ \pi(\tilde{\theta}^*) - ln\ \pi(\tilde{\theta}^*|\tilde{Y}_T) \tag{A2}$$

The log likelihood function and the log of the prior density at $\tilde{\theta} = \tilde{\theta}^*$ can be evaluated as follows. First, the log likelihood function is given by:

$$ln\ f(\tilde{Y}_T|\tilde{\theta}^*) = \sum_{t=p+1}^{T} ln(\sum_{D_t=0}^{1} p(D_t|\tilde{Y}_{T-1}, \tilde{\theta}^*)\ f(y_t|\tilde{Y}_{T-1}, D_t, \tilde{\theta}^*)), \tag{A3}$$

Second, the log of prior density is given by:

$$ln\ \pi(\tilde{\theta}^*) = ln\ \pi(\tilde{\phi}^*) + ln\ \pi(\tilde{\mu}^*) + ln\ \pi(\sigma_u^{2*}) + ln\ \pi(q^*), \tag{A4}$$

[18] The procedure is a modification from Kim and Nelson (1999) where it is described in the context of a Markov-switching model with an endogenous structural break in the parameters.

where it is a priori assumed that $\tilde{\phi}$, $\tilde{\mu}$, σ_u^2 and q are independent of one another.

Evaluation of the posterior density at $\tilde{\theta} = \tilde{\theta}^*$ is more demanding, but we can take advantage of the approach proposed by Chib (1995). For this purpose, consider the following decomposition of the posterior density:

$$\pi(\tilde{\theta}^*|\tilde{Y}_T) = \pi(\tilde{\phi}^*|\tilde{Y}_T)\pi(\tilde{\mu}^*|\tilde{\phi}^*, \sigma_u^{2^*}, q^*, \tilde{Y}_T)\pi(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, q^*, \tilde{Y}_T)\pi(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{Y}_T) \quad (A5)$$

where

$$\pi(\tilde{\phi}^*|\tilde{Y}_T) = \int \pi(\tilde{\phi}^*|\tilde{\mu}, \tilde{D}_T, \sigma_u^2, q, \tilde{Y}_T)\pi(\tilde{\mu}, \tilde{D}_T, \sigma_u^2, q|\tilde{Y}_T)\ d\tilde{\mu}\ d\tilde{D}_T\ d\sigma_u^2\ dq \quad (A6)$$

$$\pi(\tilde{\mu}^*|\tilde{\phi}^*, \tilde{Y}_T) = \int \pi(\tilde{\mu}^*|\tilde{\phi}^*, \tilde{D}_T, \sigma_u^2, q, \tilde{Y}_T)\pi(\tilde{D}_T, \sigma_u^2, q|\tilde{\phi}^*, Y_T)d\tilde{D}_T\ d\sigma_u^2\ dq \quad (A7)$$

$$\pi(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, \tilde{Y}_T) = \int \pi(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, \tilde{D}_T, q, \tilde{Y}_T)\pi(\tilde{D}_T, q|\tilde{\phi}^*, \tilde{\mu}^*, Y_T)d\tilde{D}_T\ dq \quad (A8)$$

and

$$\pi(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{Y}_T) = \int \pi(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{D}_T, \tilde{Y}_T)\pi(\tilde{D}_T|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, Y_T)d\tilde{D}_T \quad (A9)$$

where $\tilde{D}_T = [D_1\ \cdots\ D_T]'$.

The above decomposition of the posterior density suggests that $\pi(\tilde{\phi}^*|\tilde{Y}_T)$ can be calculated based on the full Gibbs run, and $\pi(\tilde{\mu}^*|\tilde{\phi}^*, \tilde{Y}_T)$, $\pi(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, \tilde{Y}_T)$, and $\pi(q^*\ |\tilde{\phi}^*\ , \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{Y}_T)$ can be calculated based on draws from the reduced Gibbs runs. The following explains how each of these can be calculated based on output from appropriate Gibbs runs:

$$\hat{\pi}(\tilde{\phi}^*|\tilde{Y}_T) = \frac{1}{G}\sum_{g=1}^{G} \pi(\tilde{\phi}^*|\tilde{\mu}^g, \sigma_u^{2g}, \tilde{D}_T^g, q^g, \tilde{Y}_T), \quad (A10)$$

$$\hat{\pi}(\tilde{\mu}^*|\tilde{\phi}^*, \tilde{Y}_T) = \frac{1}{G}\sum_{g_1=1}^{G} \pi(\tilde{\mu}^*|\tilde{\phi}^*, \sigma_u^{2g_1}, \tilde{D}_T^{g_1}, q^{g_1}, \tilde{Y}_T), \quad (A11)$$

$$\hat{\pi}(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, \tilde{Y}_T) = \frac{1}{G}\sum_{g_2=1}^{G} \pi(\sigma_u^{2^*}|\tilde{\phi}^*, \tilde{\mu}^*, \tilde{D}_T^{g_2}, q^{g_2}, \tilde{Y}_T), \quad (A12)$$

$$\hat{\pi}(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{Y}_T) = \frac{1}{G}\sum_{g_3=1}^{G} \pi(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{D}_T^{g_3}, \tilde{Y}_T), \qquad (A13)$$

where the superscript g refers to the g-th draw of the full Gibbs run and the superscript g_i, $i = 1, 2, 3$ refers to the g_i-th draw from the appropriate reduced Gibbs runs. Thus, apart from the usual G iterations for the Gibbs run, we need additional $3 \times G$ iterations for the appropriate reduced Gibbs run. In order to calculate $\pi(q^*|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{Y}_T)$, for example, we need output from an additional G iterations for the following reduced Gibbs run:

i) Generate q from $p(q|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, \tilde{D}_T, \tilde{Y}_T)$;

ii) Generate $\tilde{D}_T$ from $p(\tilde{D}_T|\tilde{\phi}^*, \tilde{\mu}^*, \sigma_u^{2^*}, q, \tilde{Y}_T)$;

Notice that throughout the reduced Gibbs run, $\tilde{\phi}$, $\tilde{\mu}$ and σ_u^2 are not generated but set equal to $\tilde{\phi}^*$, $\tilde{\mu}^*$ and $\sigma_u^{2^*}$, respectively.

Appendix B: A model with a structural break in the variance only

This Appendix presents a model with structural break in the variance only, which is of the form:

$$y_t = \mu_t + e_t, \qquad (1)$$

$$\mu_t = \mu \qquad (B1)$$

$$\phi(L)e_t = u_t, \quad u_t \sim i.i.d.N(0, \sigma^2_{u,S_t}) \qquad (5)$$

$$\sigma^2_{u,S_t} = \sigma^2_{u,0}(1 - S_t) + \sigma^2_{u,1}S_t, \qquad \sigma^2_{u,0} < \sigma^2_{u,1} \qquad (B2)$$

$$S_t = \begin{cases} 0, & \text{if t=1,2, } \cdots\text{, } \tau_v \\ 1, & \text{if t= } \tau_v \text{ +1 , } \cdots\text{, T} \end{cases} \qquad (B3)$$

where S_t is a variable that determines the regime of variance; τ_v is the date of break in the variance. The corresponding transition probabilities are:

$$Pr(S_t = 0|S_{t-1} = 0) = p, \quad Pr(S_t = 1|S_{t-1} = 1) = 1, \quad 0 < p < 1 \qquad (B4)$$

Appendix C: The model with a slowly varying mean with GARCH disturbances and its estimation

The model with GARCH disturbances is as follows:

$$y_t = \mu_t + e_t \tag{2}$$

$$\mu_t = \mu_{t-1} + v_t, \tag{8}$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \phi_3 e_{t-3} + \phi_4 e_{t-4} + u_t + u_t^* \tag{5'}$$

where the GARCH effect is introduced via u_t^*, which is defined as:

$$u_t^* | \psi_{t-1} \ \sim N(0, h_t) \tag{C1}$$

$$h_t = \gamma_0 + \gamma_1 u_{t-1}^{*2} + \gamma_2 h_{t-1} \tag{C2}$$

where ψ_{t-1} is information up to time $t-1$. Dropping u_t^* reduces to the case with homoskedasticity.

The model can be represented as a State-space form which can be estimated by the Kalman filter. In setting up the State-space model, we treat the permanent and cyclical components as well as the GARCH disturbances as state variables. Then, we have the measurement and transition equations of the form:

<u>*Measurement Equation*</u>

$$y_t \equiv \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu_t \\ e_t \\ e_{t-1} \\ e_{t-2} \\ e_{t-3} \\ u_t^* \end{pmatrix}, \tag{C3}$$

or

$$y_t \equiv H\beta_t,$$

<u>*Transition Equation*</u>

$$\begin{pmatrix} \mu_t \\ e_t \\ e_{t-1} \\ e_{t-2} \\ e_{t-3} \\ u_t^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & \phi_3 & \phi_4 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu_{t-1} \\ e_{t-1} \\ e_{t-2} \\ e_{t-3} \\ e_{t-4} \\ u_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_t \\ u_t \\ 0 \\ 0 \\ 0 \\ u_t^* \end{pmatrix} \tag{C4}$$

or

$$\beta_t = F\beta_{t-1} + G\xi_t,$$

$$Q_t \equiv E[\xi_t \xi_t' | \psi_{t-1}] = \begin{pmatrix} \sigma_v^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_u^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_t \end{pmatrix}, \tag{C5}$$

The Kalman filter is the basic tool to estimate the state variable at time t based on available information at time t. It is given by the following six equations:

$$\beta_{t|t-1} = F\beta_{t-1|t-1}, \tag{C6}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q_t, \tag{C7}$$

$$y_t - y_{t|t-1} = y_t - H\beta_{t|t-1}, \tag{C8}$$

$$f_{t|t-1} = HP_{t-1|t-1}H', \tag{C9}$$

$$\beta_{t|t} = \beta_{t|t-1} + K_t(y_t - y_{t|t-1}), \tag{C10}$$

$$P_{t|t} = P_{t|t-1} - K_tHP_{t|t-1}, \tag{C11}$$

where$_{t|t-1}$ indicates the expectation of variable of interest conditional on information up to time t-1. $P_{t|t-1}$ is the variance of $\beta_{t|t-1}$. $f_{t|t-1}$ is the variance of the prediction error $y_t - y_{t|t-1}$. $K_t \equiv P_{t|t-1}H'f^{-1}_{t|t-1}$ is the Kalman gain.

The Kalman filter in this case is not operable as well indicated in Harvey, Ruiz and Sentana (1992), because u^*_{t-1} are unobservable and thus h_t cannot be calculated. Hence, as proposed by them, an approximation $u^*_{t-1}|\psi_{t-1}$ is used. That is,

$$h_t = \alpha_0 + \alpha_1 E[u^{*2}_{t-1}|\psi_{t-1}] + \alpha_2 h_{t-1}, \tag{C12}$$

where using the property of variance of a random variable X

$$E[X^2] = E[X]^2 - E[X - E[X]]^2, \tag{C13}$$

we have

$$E[u^{*2}_{t-1}|\psi_{t-1}] = E[u^*_{t-1}|\psi_{t-1}]^2 + E[u^*_{t-1} - E[u^*_{t-1}|\psi_{t-1}]]^2, \tag{C14}$$

where $E[u^*_{t-1}|\psi_{t-1}]$ is obtained from the last element of $\beta_{t-1|t-1}$ and its mean squared error $E[u^*_{t-1} - E[u^*_{t-1}|\psi_{t-1}]]^2$ is obtained from the last diagonal element of $P_{t-1|t-1}$. $\beta_{t-1|t-1}$ and $P_{t-1|t-1}$ are computed at each iteration of the Kalman filter.

Given $\beta_{T|T}$ at the last iteration of the Kalman filter, the following can be iterated for $t = T-1, T-2, \cdots, 1$ to get the smoothed inferences about β_t conditional on information up to time T.

$$\beta_{t|T} = \beta_{t|t} + P_{t|t}F'P^{-1}_{t+1|t}(\beta_{t+1|T} - F\beta_{t|t}), \qquad (C15)$$

$$P_{t|T} = P_{t|t} + P_{t|t}F'P^{-1}_{t+1|t}(P_{t+1|T} - P_{t+1|t}){P^{-1}_{t+1|t}}'FP'_{t|t}, \qquad (C16)$$

References

[1] Andrews, David and Werner Ploberger, 1994, "Optimal changepoint tests for normal linear regression", *Journal of Econometrics*, Vol. 70, pp. 9-38.

[2] Bai, Jushan and Pierre Perron, 1998, "Estimating and Testing Linear Models with Multiple Structural Changes", *Econometrica*, Vol. 66, pp. 47-78.

[3] Barsky, Robert B. and J. Bradford De Long, 1993, "Why does the stock market fluctuate?", *Quarterly Journal of Economics*, Vol. 108, No. 2, pp. 291-311.

[4] Browne, Lynn E., 1999, "U.S. Economic Performance: Good Fortune, Bubble, or New Era", *New England Economic Review*, Federal Reserve Bank of Boston, May/June, pp. 4-20.

[5] Campbell, John Y. and Robert J. Shiller, 2001, "Valuation Ratios and the Long-run Stock Market Outlook: an Update", Working paper 8221, National Bureau of Economic Research.

[6] Chib, Siddhartha, 1995, "Marginal likelihood from the Gibbs output", *Journal of the American Statistical Association*, Vol. 90, pp. 1313-1321.

[7] Chib, Siddhartha, 1998, "Estimation and Comparison of Multiple Change-point Models", *Journal of Econometrics*, Vol. 86, pp. 221-241.

[8] *van* Dijk, Dick, Denise R. Osborn and Marianne Sensier, 2002, "Changes in Variability of the Business Cycle in the G7 Countries", Centre for Growth and Business Cycle Research Discussion Paper No. 8, University of Manchester.

[9] Fama, Eugene, 1981, "Stock Returns, real activity, inflation, and money", *American Economic Review*, Vol. 71, No. 4, pp. 545-565.

[10] Fama, Eugene F. and Kenneth R. French, 2000, "Disappearing dividends: changing firm characteristics or lower propensity to pay?", *Journal of Financial Economics*, Vol. 60, pp. 3-43.

[11] Fama, Eugene F. and Kenneth R. French, 2002, "The Equity Premium", *Journal of*

Finance, Vol. 57, pp. 637-659.

[12] Feldstein, Martin, 1980, "Inflation and the Stock Market", *American Economic Review*, Vol. 70, No. 5, pp. 839-847.

[13] Gordon, Myron, 1962, *The Investment, Financing, and Valuation of the Corporation*, Irwin: Homewood IL..

[14] Greenwood, Jeremy and Boyan Jovanovic, 1999, "The information-Technology Revolution and the Stock Market", *American Economic Review*, Vol. 89, pp. 116-122.

[15] Harvey, Andrew C., Esther Ruiz, and Enrique Sentana, 1992, "Unobserved Components Models with ARCH Disturbances", *Journal of Econometrics*, Vol. 52, pp. 129-157.

[16] Heaton, John, and Deborah J. Lucas, 2000, "Stock Prices and Fundamentals", *NBER Macroeconomics Annual 1999,* Ben Bernanke and Julio Rotemberg (eds.), MIT Press: Cambridge MA., Vol. 14, pp. 213-242.

[17] Hobijn, Bart and Boyan Jovanovic, 2001, "The Information-Technology Revolution and the Stock Market: Evidence", *American Economic Review*, Vol. 91, pp. 1203-1220.

[18] Jagannathan, Ravi, Ellen R. McGrattan, and Anna Scherbina, 2001, "The Declining Equity Premium", Working paper 8172, National Bureau of Economic Research

[19] Jeffreys, Harold, 1961, *Theory of Probability*, 3rd ed., Clarendon Press: Oxford.

[20] Kim, Chang-Jin and Charles R. Nelson, 1999, "Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle.", *Review of Economics and Statistics*, Vol. 81, No. 4, pp. 608-616.

[21] Kleidon, Allan W., 1986, "Variance Bounds Tests and Stock Price Valuation Models", *Journal of Political Economy*, Vol. 94, No. 5, pp. 953-1001.

[22] Koop, Gary and Simon M. Potter, 1999, "Bayes factors and nonlinearity: Evidence from economic time series", *Journal of Econometrics*, Vol. 88, pp. 251-281.

[23] McConnell, Margaret M. and Gabriel Perez-Quiros, 2000, "Output fluctuations in the United States: What has changed since the early 1980s?", *American Economic Review*, Vol. 90, pp. 1464-1476.

[24] Modigliani, Franco, and Richard A. Cohn, 1979, "Inflation and the stock market", *Financial Analysts Journal*, Vol. 35, No.2, pp. 24-44.

[25] Nelson, Charles R., 1976, "Inflation and Capital Budgeting", *Journal of Finance*, Vol. 31, 923-31.

[26] Nelson, Charles R., 1979, "Recursive Structure in U.S. Income, Prices, and Output", *Journal of Political Economy*, Vol. 87, 1307-27.

[27] Nyblom, Jukka, 1989, "Testing for the Constancy of Parameters Over Time", *Journal of the American Statistical Association*, Vol. 84, pp. 223-230.

[28] Quandt, Richard E., 1960, "Tests of the Hypothesis That a Linear Regression System Obeys Two Separate Regimes", *Journal of the American Statistical Association*, Vol. 55, pp. 324-330.

[29] Sharpe, Steven A., 2001, "Reexamining Stock Valuation and Inflation: The Implications of Analysts' Earnings Forecasts", Working paper in Finance and Economics Discussion Series, Federal Reserve Board.

[30] Shiller, Robert J., 2000, "Irrational Exuberance", Princeton University Press: Princeton NJ..

[31] Siegel, Jeremy J., 1999, "The Shrinking Equity Premium.", *Journal of Portfolio Management*, Vol. 26, Iss. 1, pp. 10-17.

[32] Stock, James H. and Mark W. Watson, 1998, "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model", *Journal of the American Statistical Association*, Vol.93, pp. 349-358.

[33] Stock, James H. and Mark W. Watson, 2002, "Has the Business Cycle Changed and Why?", Manuscript.

Table 1. Posterior moments from Model 0 and Model 1

	Model 0		Model 1	
Parameters	Mean	SD	Mean	SD
μ_0	2.4123	1.6536	0.4667	2.3234
μ_1			5.8621	4.0441
ϕ_1	0.6138	0.0701	0.6138	0.0716
ϕ_2	0.1380	0.0833	0.1358	0.0831
ϕ_3	0.0004	0.0844	-0.0024	0.0837
ϕ_4	-0.2828	0.0710	-0.2824	0.0714
σ_u	8.6472	0.8897	8.5800	0.8813
q			0.9870	0.0176
$ln\ \ m(\tilde{Y}_T)$	-225.2583		-227.8713	
Log of Bayes factor in favor of Model 1 over Model 0: -2.61				

Model 1 indicates a model with a structural break in the mean of earnings growth ($\mu_0 \neq \mu_1$). Model 0 is a model without a structural break (μ_0=μ_1). SD refers to standard deviation. $ln\ m(\tilde{Y}_T)$ refers to the log of marginal likelihood.

Table 2. Posterior moments from Model 0A and Model 1A

	Model 0A		Model 1A	
Parameters	Mean	SD	Mean	SD
μ_0	1.7157	1.4471	0.6044	1.9860
μ_1			5.9931	4.8515
ϕ_1	0.6958	0.0715	0.6946	0.0726
ϕ_2	0.0288	0.0889	0.0297	0.0860
ϕ_3	0.0081	0.0860	0.0094	0.0856
ϕ_4	-0.2546	0.0694	-0.2567	0.0707
$\sigma_{u,0}$	5.3720	0.6765	5.4156	0.6833
$\sigma_{u,1}$	16.9116	3.2870	16.8193	3.2601
q			0.9883	0.0167
$ln\ \ m(\tilde{Y}_T)$	-217.3026		-220.6106	
Log of Bayes factor in favor of Model 1A over Model 0A: -3.31				

Model 1A indicates a structural break model in the mean of earnings growth ($\mu_0 \neq \mu_1$) with a variance dummy. Model 0A is a model with no structural break in the mean (μ_0=μ_1) but with a variance dummy. SD refers to standard deviation. $ln\ m(\tilde{Y}_T)$ refers to the log of marginal likelihood.

Table 3. Tests of $H_0 : \lambda$=0 (Constant mean growth rate), median unbiased estimates and 90% confidence intervals

Test Statistic	p-value	$\hat{\lambda}$	λ 90% CI	$\hat{\sigma}_v$	σ_v 90% CI
			Panel A: Homoskedastic Case		
L	0.72	0.00	(0.00 - 8.83)	0.00	(0.00 - 1.00)
MW	0.61	0.00	(0.00 - 11.12)	0.00	(0.00 - 1.23)
EW	0.44	1.53	(0.00 - 14.37)	0.17	(0.00 - 1.63)
SW	0.20	4.80	(0.00 - 20.18)	0.54	(0.00 - 2.29)
			Panel B: Heteroskedastic Case		
L	0.85	0.00	(0.00 - 5.78)	0.00	(0.00 - 0.74)
MW	0.78	0.00	(0.00 - 7.54)	0.00	(0.00 - 0.97)
EW	0.70	0.00	(0.00 - 8.90)	0.00	(0.00 - 1.14)
SW	0.38	2.35	(0.00 - 14.88)	0.30	(0.00 - 1.91)

The first column is test statistics from which the median unbiased estimator of λ is derived. L is Nyblom's (1989) statistic; MW and EW are the mean Wald and exponential Wald statistics of Andrews and Ploberger (1994); SW is the Quandt (1960) likelihood ratio statistic.

Table 4. Maximum Likelihood Estimates of the Model with slowly-varying mean

Parameter	MLE	Estimates with fixed σ_v		
σ_v	5.7663^{-6} (0.0933)	0.2600	0.5200	0.7400
σ_u	0.0085 (0.1107)	0.0059 (0.1116)	0.0003 (0.0813)	0.0051 (0.2135)
γ_0	10.2640 (6.2465)	10.2430 (6.3392)	10.0725 (6.3974)	9.9515 (6.3157)
γ_1	0.2475 (0.0852)	0.2490 (0.0872)	0.2477 (0.0883)	0.2468 (0.0894)
γ_2	0.6958 (0.0940)	0.6960 (0.0960)	0.6984 (0.0971)	0.6996 (0.0973)
ϕ_1	0.6946 (0.0743)	0.6969 (0.0746)	0.6954 (0.0754)	0.6925 (0.0757)
ϕ_2	0.1159 (0.0956)	0.1194 (0.1004)	0.1230 (0.0956)	0.1251 (0.0996)
ϕ_3	-0.0380 (0.0852)	-0.0358 (0.1080)	-0.0347 (0.0909)	-0.0335 (0.0979)
ϕ_4	-0.2599 (0.0672)	-0.2555 (0.0758)	-0.2559 (0.0705)	-0.2584 (0.0722)
μ_0	1.3969 (1.4564)	1.4114 (2.2772)	1.0727 (3.6076)	0.5517 (7.3876)

MLE is an unrestricted MLE. The last three columns are estimates by restricted MLEs with σ_v set to the values indicated in the row for σ_v. In all estimations, μ_0 is treated as an unknown constant to be estimated. Parentheses contain standard errors.

Figure 1. Posterior probability distribution of date of change in the mean of earnings growth

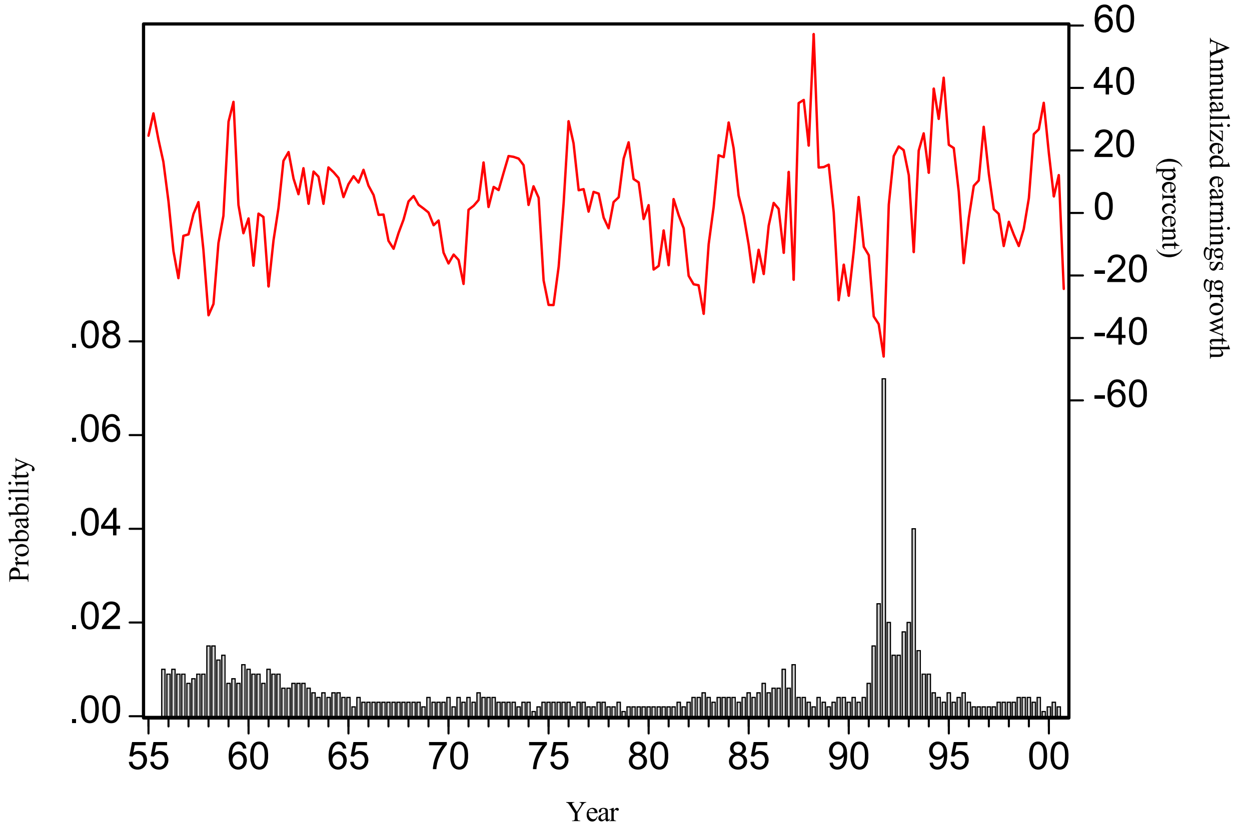

Figure 2. Probability of structural change in the mean of earnings growth

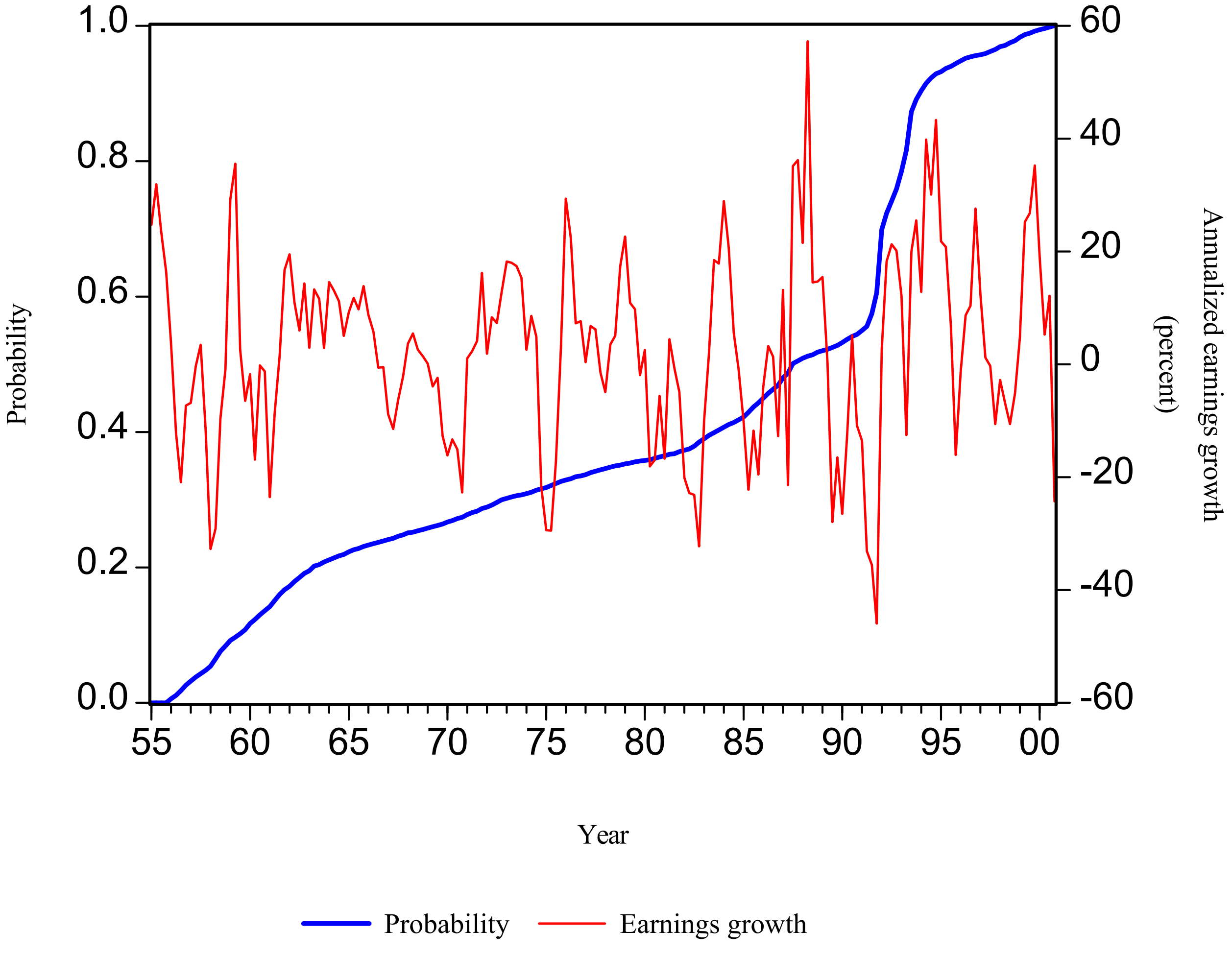

Figure 3. Posterior probability distribution of date of change in the variance of earnings growth

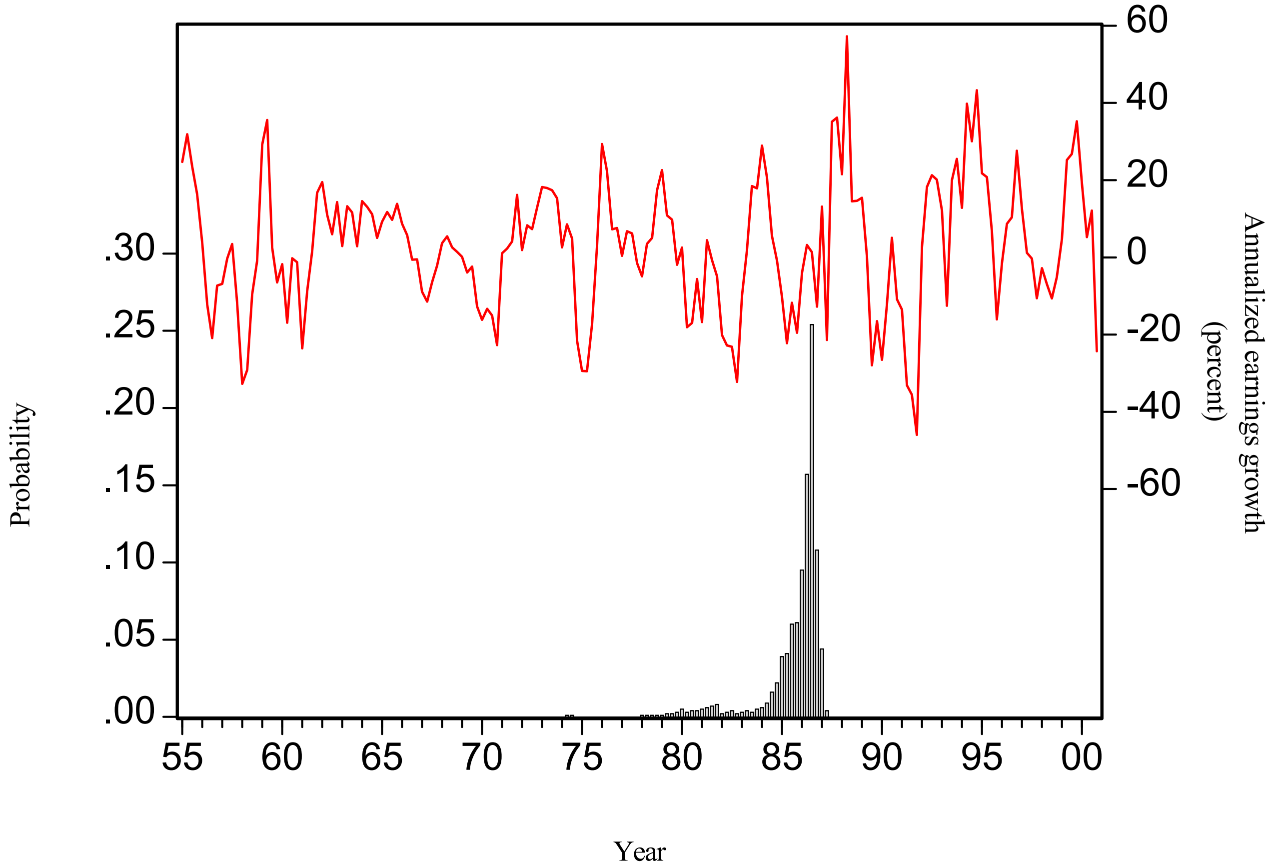

Figure 4. Probability of structural change in the variance of earnings growth

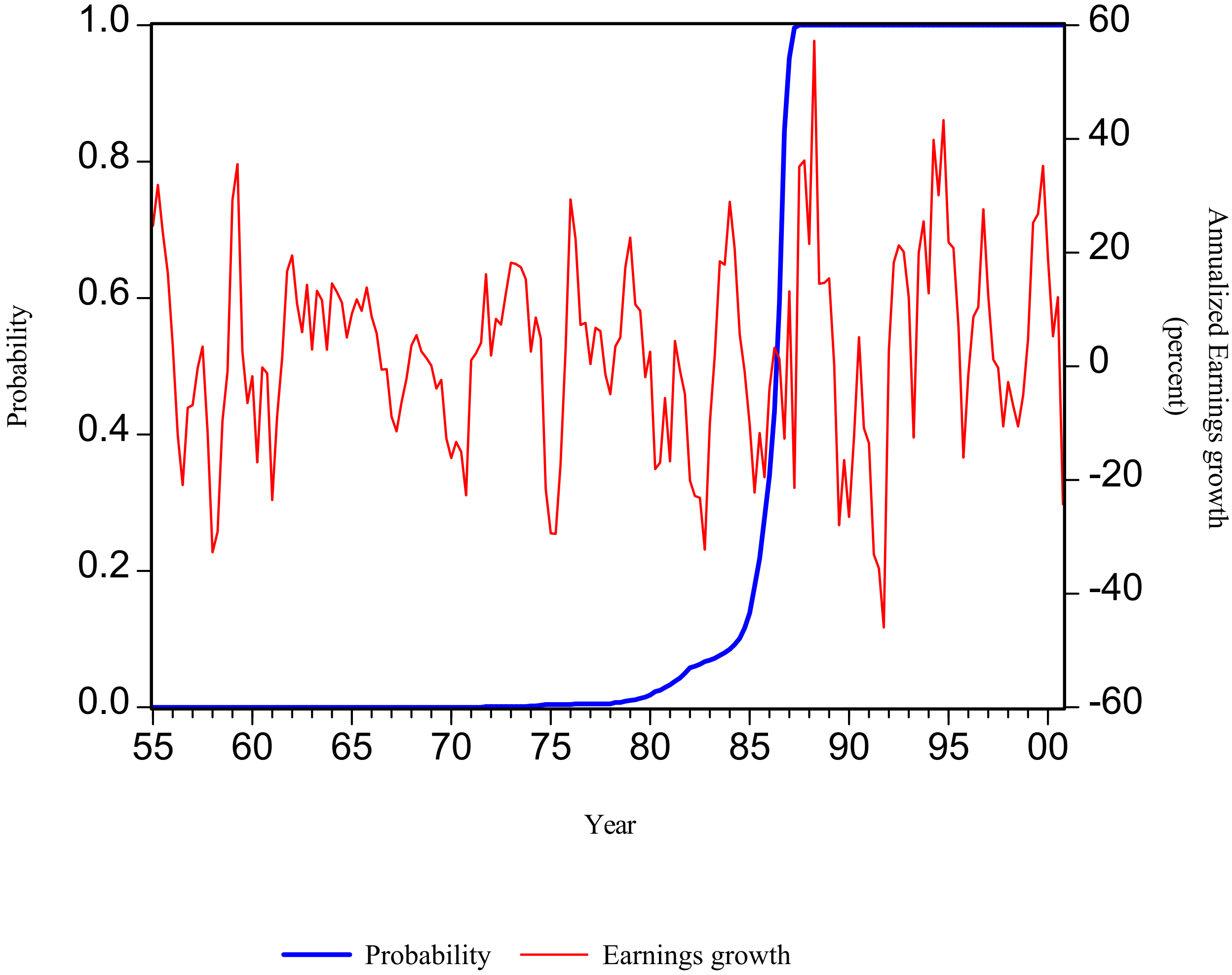

Figure 5. Posterior probability distribution of date of change in the mean of earnings growth when a variance dummy isused

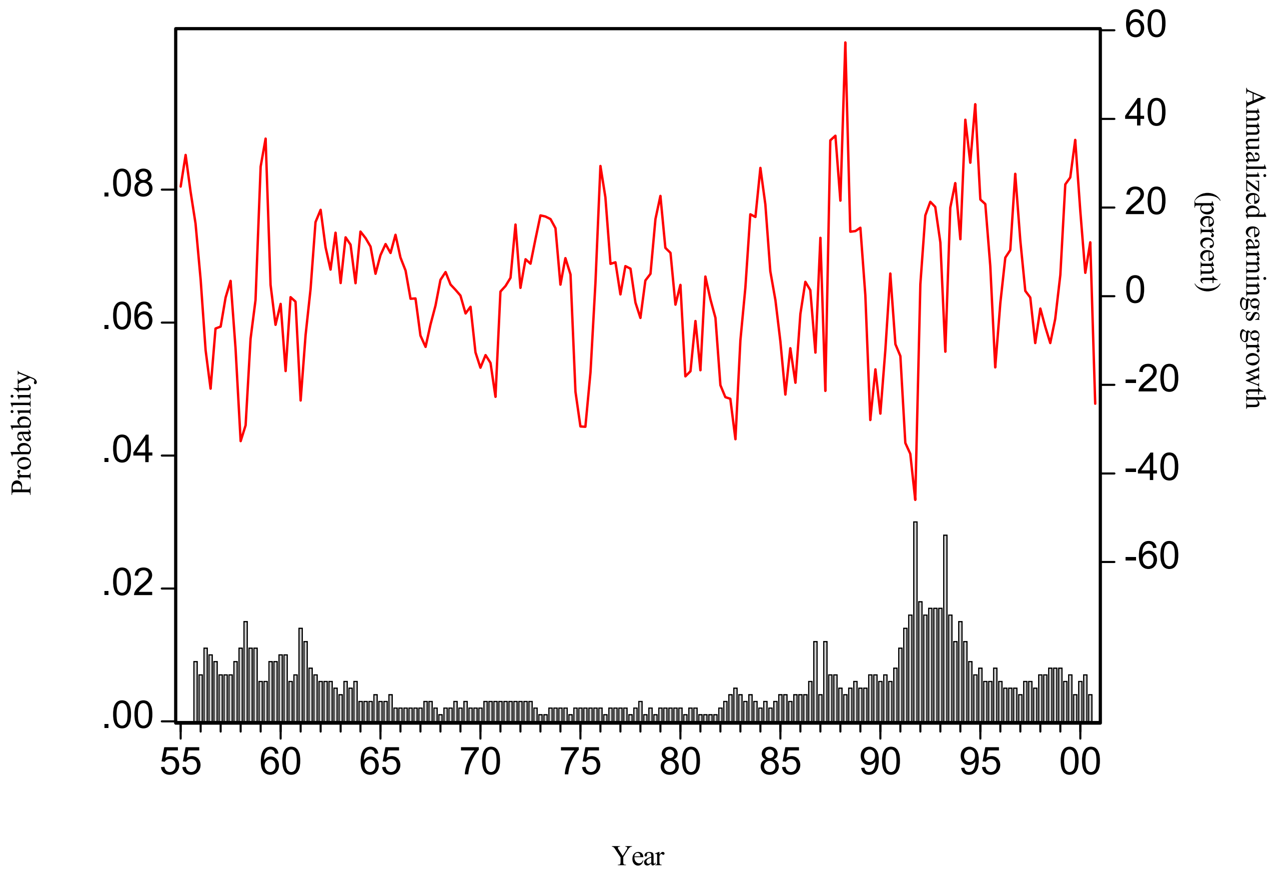

Figure 6. Estimated trends of earnings growth based on four models in Table 4

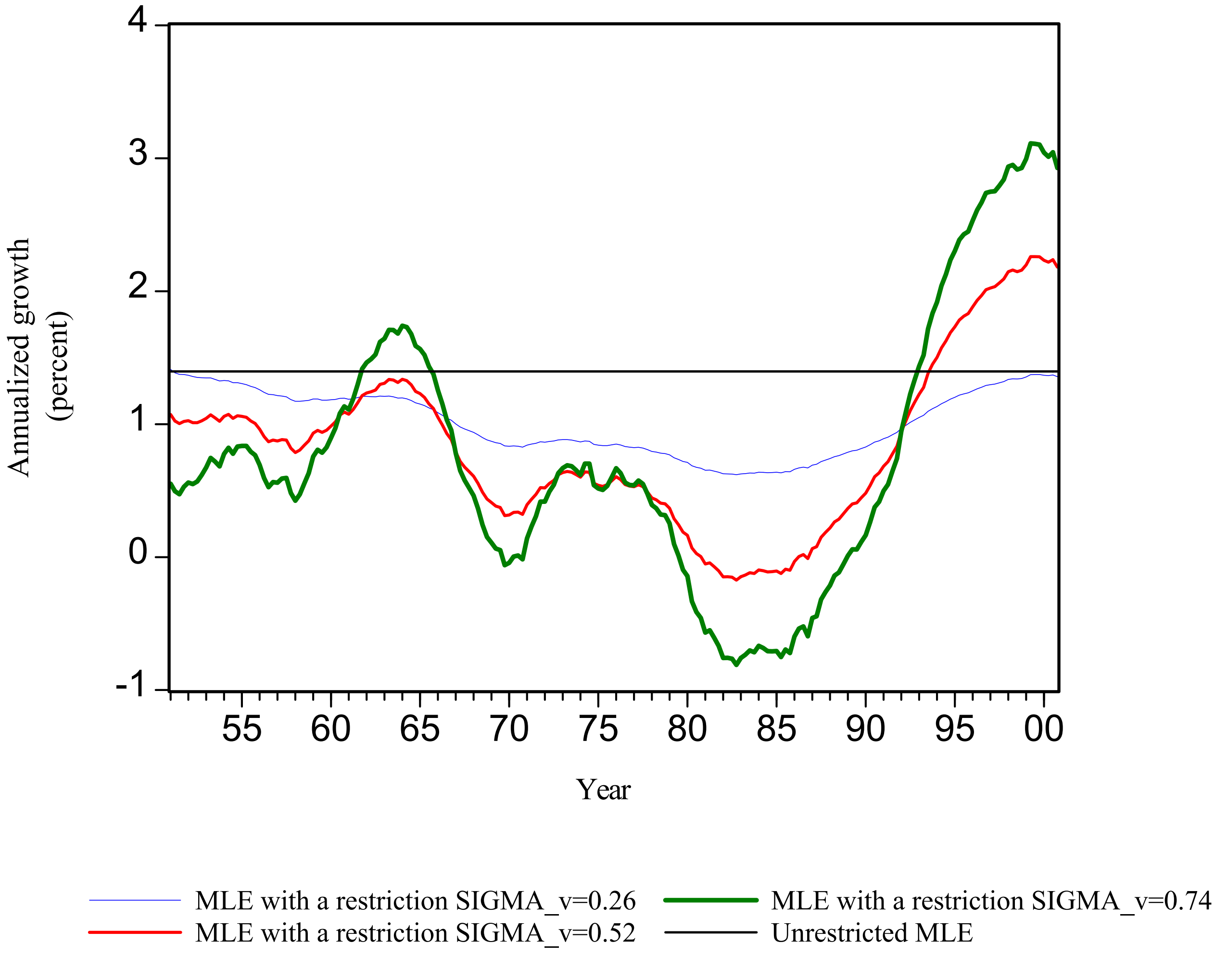

Figure 7. Historical and implied price earnings ratios

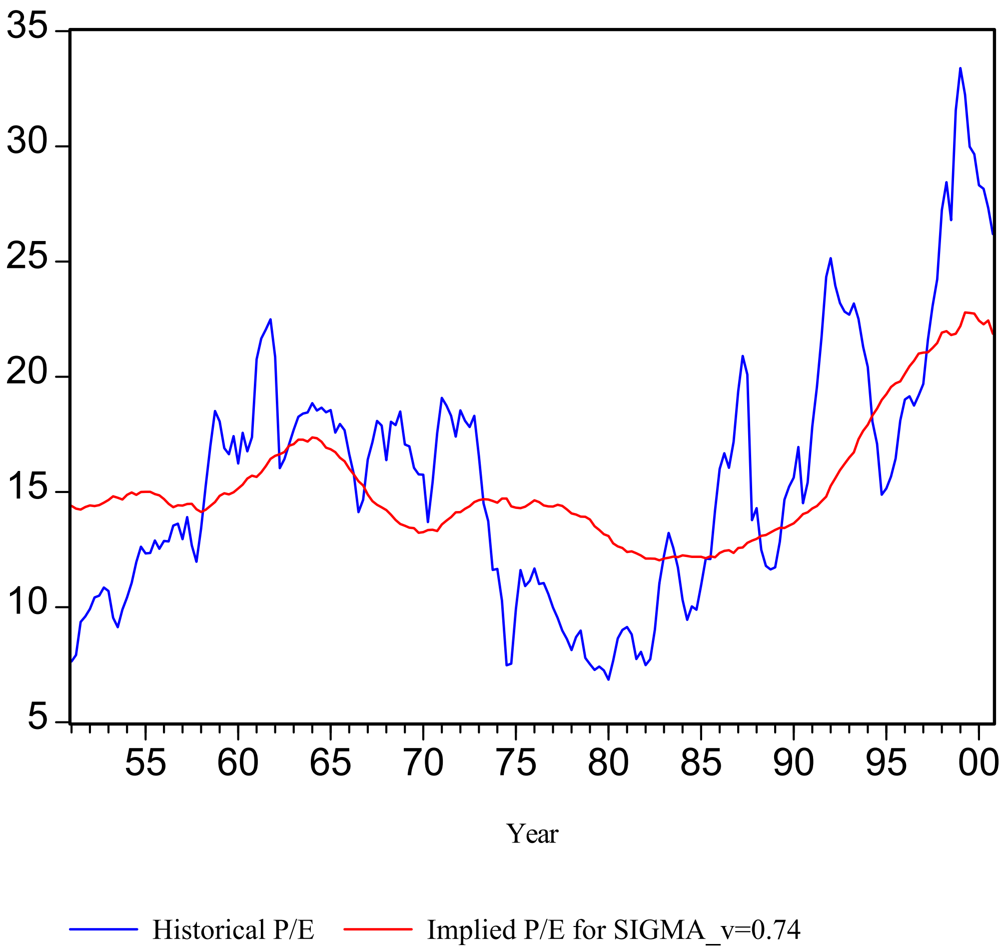

Figure 8. Risk premium with the smallest variation

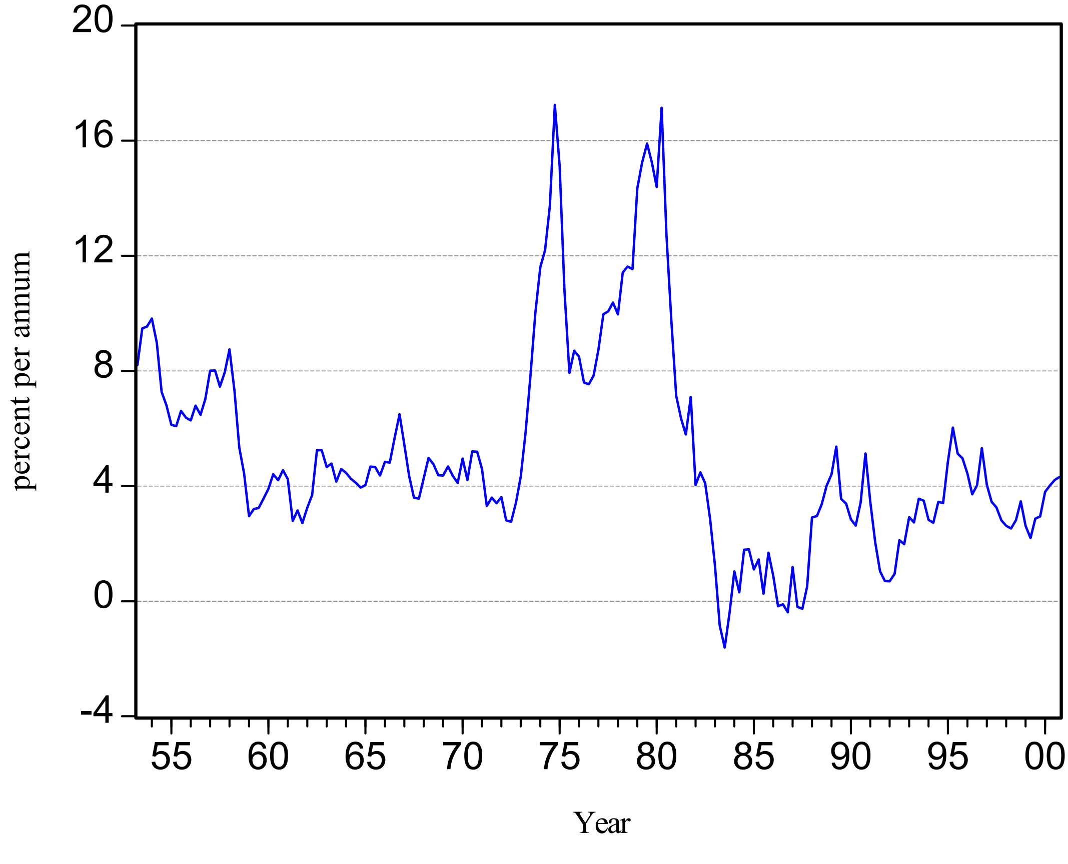

Figure 9. Risk premium shown in Figure 8 when nominal interest rates are used for 1973:IV-1980:IV