On the Costs and Benefits of a Mixed Educational Regime

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Abstract

This paper studies the costs and benefits of a mixed educational regime in which tax-financed public schools and tuition-financed private schools coexist. In the model, households are not allowed to borrow for education, but have a choice to educate their children in a public or a private school. The tax rate is determined by majority vote. The future income of children depends on the quality of education and stochastic ability. Using a numerical method, I calibrate the model to the U.S. economy, calculate the long-run outcomes and compare them with those in a purely public or private educational regime, consisting solely public or private schools.

Simulations reveal that in the mixed regime long-run mean income is higher and long-run income inequality is lower than in the private regime. They also reveal that although there is no significant difference in long-run mean income between the mixed and the public regime, the tax rate and the quality of public education are significantly lower and thus long-run income inequality is higher in the mixed regime than in the public regime.

To investigate the determinants of the performance of the mixed regime relative to the public or the private regime, I simulate the model with various parameter values. I find that the mixed regime generates higher long-run mean income than the public regime if the elasticity of future income to education is high. I also find that long-run mean income is higher (lower) in the mixed regime than in the public regime if the variance of stochastic ability is large (small).

I use the model to evaluate the effects of the introduction of private educational vouchers. The results suggest that the introduction of vouchers improves long-run total welfare, and reduces long-run income inequality as long as public education is maintained.

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1 Introduction

Recently, many politicians and researchers discuss educational reforms to achieve efficiency and/or equity in education. On one hand, some argue that government should make use of private education to enhance efficiency in education sectors. For example, the introduction of private educational vouchers (such as the voucher experiments in Milwaukee and New York) has been discussed as a policy to encourage students to enroll in private schools and improve efficiency through competition among schools. On the other hand, some argue that government should help public education to provide equal educational resources for all students, especially for children from poor families. The supporters of this view back the idea of a centralization of an educational regime such as the reform from local-financing to state-financing public schools in California.

To assess these reforms, it is important to notice that the current educational systems are characterized by the presence of tax-financed (public) schools and tuition-financed (private) schools, because the effects of reforms depend heavily on how they affect the share of public (private) schools in the economy. Table 1 shows the share of students in public schools for primary and secondary education in each state of the United States. The maximum share is 96 percent in Wyoming, and the minimum is 79 percent in Delaware. On average, 88 percent of students are in public schools. In fact, *no* state has a purely public or a purely private education system, and the majority of students are in public schools.

Despite being the norm, the mixed regime has received little attention in the literature of education and income inequality, and it is typically assumed that education is provided only publicly or only privately.[1] For instance, using a dynamic political economy model, Glomm and Ravikumar (1992) study the implications for income distribution and growth of different educational regimes by comparing a purely public educational regime, in which only public schools are available, with a purely private regime, in which only private schools are available to households. However, since the current educational regime is the mixed one, the model of the mixed regime is needed for the understanding of the effects of educational reforms.

In this paper, I propose a dynamic political economy model of the mixed regime and analyze the effects of some educational policies for economic outcomes such as mean income and the degree of income inequality. In my model, households are not allowed to borrow for education, but can choose to send their children to a private or a public school. All public schools are financed by taxes and offer the same quality of education for free of charge. Attending a private school requires paying

[1] See Section 2 for the details of the related literature.

tuition, but households can choose the quality of education received by their children. The tax rate and the quality of public education are determined in each period by majority vote. The economy is populated by a sequence of two-period-lived overlapping generations, and each household consists of one parent and one child. Parents have identical preferences over current consumption and the future income of their child, but different endowments of household income. The future income of children depends not only on the quality of education they receive in school but also on stochastic ability.

Using the model, I answer the following questions. Why do we observe mixed regimes everywhere? Which regime is sustainable, the purely public, the purely private, or the mixed regime? What are the effects of privatization or centralization of educational regimes on income distribution? If one were to compare the mixed regime to the public or the private regime can one rank the resulting outcomes? What are the effects of the introduction of private educational vouchers?

Firstly, I provide an explanation why we observe the mixed regime and why the public or the private regime is not sustainable. Intuitively, if the variance of income distribution is large enough, rich households have the incentive to opt out to private schools. Moreover, if income of the median household is lower than mean income, then public education is supported by the majority. When these two conditions are satisfied in the long run, only steady state with a mixture of public and private schools is sustainable.

Next, I compare the mixed regime with the public or the private regime using a numerical method. I calibrate the model to the U.S. economy, simulate it, and compare the resulting outcomes with those in the public or the private regime. Simulations of the calibrated model reveal that in the mixed regime long-run mean income is higher and long-run income inequality measured by Gini coefficient is lower than in the private regime. In this comparison, one of the benefits of the mixed regime is that it provides public education for poor households, and the cost is that the tax burden weakens the incentives of households to invest in education through private school. In these simulations, the mixed regime generates higher mean income because this benefit dominates its cost.

Regarding the comparison between the mixed and the public regime, simulations reveal that although there is no significant difference in long-run mean income between them, the tax rate and the quality of public education in the mixed regime are significantly lower than in the public regime, and long-run income inequality is higher in the mixed regime. In this comparison, one of the advantages of the mixed regime is that it allows rich households to invest more in education through private schools, and the disadvantage is that in the mixed regime a lower tax rate and a lower quality of public education are chosen by majority to avoid congestion in public schools. In the simulation of the benchmark model, this benefit is cancelled out by the cost, and therefore these two regimes generate same long-run mean income. However, the opting out of rich households to private school increases the long-run income inequality in the mixed regime.

To investigate the determinants of the costs and the benefits of the mixed regime, I simulate the model with different values of the elasticity of future income to education and the variance of stochastic ability. Simulations reveal that the mixed regime always generates higher mean income than the private regime. Moreover, I find that the larger the elasticity (the variance of ability), the larger the difference in mean income between them.

In the comparison between the mixed and the public regime, I find that the mixed regime generates *higher* long-run mean income than the public regime if the elasticity of future income to the quality of education is large. Also, I find that the larger the elasticity, the larger is the difference in mean income between them. Moreover, I find that long-run mean income is higher (lower) in the mixed regime than in the public regime if the variance of stochastic ability is large (small), and the difference in long-run mean income between them is larger the greater the variance of stochastic shocks, because the larger is the variance, the more households opt out to private schools.

Finally, I use the model to analyze the welfare effect of a private educational voucher, defined as a fixed value of tax-financed subsidy to private tuition. The simulations of the model with stochastic ability and vouchers reveal that the introduction of vouchers improves total welfare in the long run. Moreover, it reduces the long-run degree of income inequality *as long as public education is maintained.* However, if the value of a voucher is so large that the decisive household prefers private school, then public education is abandoned and income inequality may rise due to the vouchers.

The remainder of this paper is organized as follows. In Section 2, I review the related literature. In Section 3, I construct the model. In Section 4, I analyze the existence and uniqueness of equilibrium. In Section 5, I characterize steady state. In Section 6, I investigate the costs and the benefits of the mixed regime by comparing it with the public or the private regime. In Section 7, I show that the mixed regime is a consequence of democratic choice of educational regimes in the long run. In Section 8, I study the effect of private educational vouchers. Concluding remarks are in Section 9. All proofs are in Appendix.

2 Related Literature

This paper is related to the literature on income distribution under credit constraints. The relationship between human capital accumulation under *private* educational investment and income inequality has been analyzed by many authors, for instance, Becker and Tomes (1979), Loury (1981), Galor and Zeira (1993), Mookherjee and Ray (2002). The assumption of private education seems inadequate for primary and secondary education. As seen in Table 1, we need to take into account the role of both public and private education explicitly. I also relate to the literature on the political economy of public education. Boldrin (1993) and Saint-Paul and Verdier (1993) study a growth model in which education is provided *only by public schools*. Glomm and Ravikumar (1992) analyze the dynamics of a purely public or a purely private educational regime and compare their long-run outcomes. Gradstein and Justman (1997) compared the dynamics of a purely public educational regime to a purely private educational regime with tax-financed subsidy. Although these papers provide important insights about the differences between a purely public and a purely private educational regime, they do not capture the interaction between the public and private education.

Several papers in the literature on educational finance with multiple districts are related to this paper. Benabou (1996) and Fernández and Rogerson (1997,1998) analyze the implications of education-finance reforms. In these models, education is provided only by public schools. Glomm and Lagunoff (1999) study a model of majority vote with two districts, one where public education is financed by taxes, another where it is financed by voluntary donation. However, in their model, since all the students in the same region receive the same quality of education regardless of the amount of donation each household pays, education in both districts can be considered as a public good. In contrast, in my model, households are allowed to opt out to a private school, and the households with private education do *not* benefit from public education even though they pay taxes.

Several authors, for instance, Fernández and Rogerson (1995), Epple and Romano (1996), Glomm and Ravikumar (1998), Glomm and Patterson (2002) and Nechyba (2003), have studied majority-voting equilibrium when opting out is possible.[2] However, these papers analyze equilibrium in a *static* framework, and hence they are inadequate for the analysis of the *dynamic* effects of educational policy.

Three papers are closely related to this paper. Gradstein and Justman (1996) analyze the

[2]The paper by Fernández and Rogerson analyzes a similar type of opting-out, but in their model the *poor* households opt out. Also, Tanaka (2003) analyzes this type of opting out in the context of child labor using a model of majority vote.

dynamics of political economy model with endogenous taxes through majority vote, in which the coexistence of public and private education is allowed. In their model, each agent can *supplement* public education by purchasing private one. In contrast, my model treats private education as an exclusive alternative to public education. This assumption is appropriate to study school choice by households in primary education where majority of students stay in one school after the choice of school.

Bearse, Glomm and Ravikumar (2001) and Tanaka (2001) analyze a *deterministic,* dynamic economy where households have the choice to opt out to private schools. Since the analyses in these papers are limited to the deterministic case, we cannot analyze the role of public schools as social insurance against uncertainty. Moreover, the implications for intergenerational income mobility are missing there. In contrast, this paper extends my previous paper in order to study the implications of school choice for income distribution and income mobility in a *stochastic* environment.

3 The Model

In this section, I set up the model and show how income distribution evolves under given public education policy. Then I describe how public education policy is endogenously determined.

3.1 Preferences

The economy is populated by a sequence of two-period-lived overlapping generations. A continuum of agents with total mass equal to unity is born in each period. Each individual belongs to a household consisting of one old person (the parent) and one young (the child).

All decisions are made by parents, each of whom has identical preferences. The parent in each household decides how to allocate the after-tax income between current consumption and expenditure on education of the child. The parent chooses a school (either private or public), education expenditure (e_t) when she chooses a private school, and current consumption (c_t) to maximize her utility from current consumption and the (pre-tax) income of her child subject to her budget constraint. Let $u(.)$ be the utility function from current consumption which is strictly increasing, strictly concave, continuously differentiable, and $u'(0) = \infty$. Similarly, let $v(.)$ be the utility function from the human capital of her child which is also strictly increasing, concave and continuously differentiable. The utility of the parent at time t can be described with the form;

$$u(c_t) + v(y_{t+1}),$$

a standard formulation of household preferences with warm-glow altruism. I assume that if the parent chooses a private school, she needs to pay the cost of private education from her own after-tax income because of credit constraints against human capital investment.[3] In the absence of credit markets, the budget constraint of the household at time t is given by;

$$c_t + e_t = (1 - \tau_t) y_t.$$

3.2 Technology and Education Finance

In this economy, there are two types of schools (private and public), and agents need to receive formal schooling in *either* a public school *or* a private school to acquire human capital. All schools have the common education technology $h(.)$, which is continuously differentiable, strictly increasing, strictly concave in education expenditure *per student* and satisfies $h'(0) = \infty$ and $h'(\infty) = 0$. Note that I assume no technological difference between public and private schools.

Public education is financed by taxes levied on all households in the economy and equally distributed for free of charge to all students in public schools. Let $F_t(y)$ be the cumulative distribution function (CDF) of household income in the economy at date $t = 0, 1, 2, \ldots$. Given $F_t(y)$ and a uniform tax rate on household income at time t ($\tau_t \in [0, 1]$), total tax revenue at time t is given by;

$$\int \tau_t y dF_t(y) \equiv \tau_t Y_t.$$

Let N_t and E_t be the amount of students and the education expenditure *per student* in public schools at time t, respectively. Assuming that public education is the only public good of the economy, N_t and E_t need to satisfy the government budget constraint;

$$N_t E_t = \tau_t Y_t. \tag{1}$$

The future income of children depends both on the quality of education, measured by education expenditure per student in school, and individual-specific stochastic ability.[4] Let $\gamma_{t+1}^i \in \Gamma \subseteq R_+$

[3] Heckman (2000) and Keane and Wolpin (2001) point to the empirical importance of family background including family income when they are in the early phase of human capital (ability) formation for the final educational attainment of children. These observations are consistent with the view of credit constraints in the early phase of education. Moreover, this crucial assumption is justified by the impossibility of taking future human capital as collateral. See Loury (1981) for more discussions on this assumption.

[4] The "quality" of education is measured by education expenditure *per student*. This positive relationship between education expenditure and outcome of education (e.g., lifetime earnings) is empirically supported by, for instance, Card and Krueger (1996). Moreover, since I assume that public education is publicly-provided private goods, there is congestion in public schools. This congestion effect is consistent with empirical facts that school choice raises educational resources per student in public schools in Hoxby (2001) and Lankford and Wyckoff (2001).

be the ability of the child in date t (thus the parent in the next period) of household i.[5] I assume that the ability of each individual is i.i.d. and drawn from a probability distribution with a CDF $G(\gamma)$ and the density function $g(\gamma)$.

If a child with ability γ_{t+1} attends a public school with E_t of expenditure per student, her income at time $t+1$ is given by;

$$y_{t+1} = \gamma_{t+1} h(E_t).$$

Every private school is financed by private tuition paid by the household whose child is in the private school. If the child with ability γ_{t+1} attends a private school with private tuition e_t, her income at time $t+1$ is given by;

$$y_{t+1} = \gamma_{t+1} h(e_t).$$

I assume that parents can choose the quality of education in the private school by changing education expenditure in the school. Alternative interpretation of this assumption is that there is a list of private schools differentiated by the amount of tuition and the parent chooses one school from the list for her child.

3.3 Parent's School Choice

Let $V^{Pub}(y_t; \tau_t, E_t)$ be the indirect utility function of the parent with the pre-tax income y_t when the parent sends her child to a public school under τ_t and E_t. Since public education is provided for free of charge, she pays no education cost from her after-tax income and consumes all the disposable income $(1-\tau_t)y_t$. The future income of her child with ability γ_{t+1} is given by $\gamma_{t+1}h(E_t)$. I assume that parents do not observe the ability of their child when they make the educational choice. Hence, the indirect utility of the household with public education is given by;

$$V^{Pub}(y_t; \tau_t, E_t) = u((1-\tau_t)y_t) + \int v(\gamma h(E_t)) dG(\gamma).$$

Similarly, let $V^{Pri}(y_t; \tau_t)$ be the indirect expected utility function of the household with income y_t when the parent sends her child to a private school under a tax rate τ_t. Contrary to the case with public education, the parent chooses the quality of private school to maximize her utility, and pays tuition from her after-tax income. Hence, the indirect utility of the household with private education is given by;

$$V^{Pri}(y_t; \tau_t) = \max_{0 \leq e_t \leq (1-\tau_t)y_t} u((1-\tau_t)y_t - e_t) + \int v(\gamma h(e_t)) dG(\gamma).$$

[5]In the following exposition, I omit the household index i.

Define $e((1-\tau_t)y_t)$ as the optimal investment in private education of the parent with after-tax income $(1-\tau_t)y_t$ when she chooses private education. Notice that under the above configuration of the utility functions and the production function of human capital, education is normal good, that is, $e((1-\tau_t)y_t)$ is increasing in disposable income $(1-\tau_t)y_t$. Moreover, the Theorem of Maximum guarantees that $e(.)$ is *continuous*. The continuity of $e(.)$ implies that the parent *can* invest *any* amount in education within her budget when she prefers a private school. Hence, there is no indivisibility of education cost, in contrast to what is typically assumed in the literature of income distribution and growth under imperfect credit markets.[6]

Using these indirect utility functions, the problem of school choice for the household with income y_t at time t is given by;

$$V(y_t;\tau_t,E_t) = \max\{V^{Pri}(y_t;\tau_t), V^{Pub}(y_t;\tau_t,E_t)\}, \tag{2}$$

where $V(y_t;\tau_t,E_t)$ is the indirect utility of the household with income y_t at time t *after* school choice. Note that, under a public education policy (τ_t,E_t), the parent with pre-tax income y_t prefers a public school to a private school if $V^{Pub}(y;\tau,E) > V^{Pri}(y;\tau)$, and vice versa.

Parent's school choice can be summarized with a threshold level of income. Let $y^*(\tau_t,E_t)$ be the income level at which a parent is indifferent between a public school and a private school under (τ_t,E_t). All households whose income is higher (lower) than $y^*(\tau_t,E_t)$ strictly prefer a private (public) school.[7]

[*Figure* 1]

Figure 1 describes the two indirect utility functions $V^{Pub}(y;\tau,E)$ and $V^{Pri}(y;\tau)$ for fixed $\tau \in (0,1)$ and $E > 0$. These indirect utility functions are increasing in the pre-tax income of household. Notice that the utility of a household with zero income from a public school is higher than that from a private school ($V^{Pub}(0;\tau,E) > V^{Pri}(0;\tau)$), because public school provides positive quality of schooling, while the household with zero income can afford to send her child to a private school with "zero quality." Moreover, the slope of the indirect utility function with private schooling is always larger than that with public schooling at every household income level y. This is because under private schooling, the parent can allocate a marginal increase in household income to current

[6] See, e.g., Galor and Zeira (1993), Banerjee and Newman (1993), and Aghion and Bolton (1997).

[7] This positive relation between household income and private school attendance is supported empirically by Long and Toma (1988). The formal proof of this claim is in Appendix as Lemma 1.

consumption *and* education of the child, but all the marginal increase in income is spent on current consumption under public schooling. Hence, the marginal utility gain from an increase in income under private schooling is higher than that under public schooling.

An immediate implication of the above observation is that for given τ and E, there exists *at most* one level of income $y^*(\tau, E)$ at which a household is indifferent between public and private schooling. Moreover, as long as the threshold is interior (i.e., $y^*(\tau, E) \in (0, \infty)$), $y^*(\tau, E)$ is strictly increasing in τ and E. On one hand, for a given τ, a higher expenditure per student in public schools shifts up the utility from public education, and attracts more students to public school (the dashed line in Figure 1 is the indirect utility function $V^{Pub}(y; \tau, E')$ with $E' > E$). On the other hand, for a given E, a high tax rate reduces disposable income and discourages the incentive to choose private schooling. As a result, more households prefer public schools under a high tax rate.

3.4 Dynamics of Household Income for given Policies

From the school choice problem, the households with higher income than the threshold level prefer private school, while the households with lower income prefer public school. If a household is indifferent between a public school and a private school, they may choose either of them. Hence, for given (τ, E), the dynamics of household income is given by;

$$y_{t+1} = \begin{cases} \gamma_{t+1} h(E_t) & \text{if } y_t < y^*(\tau_t, E_t), \\ \gamma_{t+1} h(e((1-\tau_t)y_t)) \text{ or } \gamma_{t+1} h(E_t) & \text{if } y_t = y^*(\tau_t, E_t), \\ \gamma_{t+1} h(e((1-\tau_t)y_t)) & \text{if } y_t > y^*(\tau_t, E_t). \end{cases}$$

The distribution of household income in the next period is obtained by aggregating the household income dynamics described above. In period t, the distribution of household income is conceived as a probability measure on the support of y, R_+. Define $\lambda_t(Z)$ as such a measure for $Z \subseteq R_+$. Define the Borel sets of R_+ by $\mathfrak{R}$ and the Lebesgue measure on $\mathfrak{R}$ by μ. For given (τ, E), define a transition probability on R_+ by a function $P : R_+ \times \mathfrak{R} \times [0, 1] \times R_+ \to [0, 1]$ satisfying: for each $y \in R_+$, $\tau \in [0, 1]$, and $E \in R_+$, $P(y, .; \tau, E)$ is a probability measure on $(R_+, \mathfrak{R})$; and, for each $Z \in \mathfrak{R}$, $\tau \in [0, 1]$, and $E \in R_+$, $P(., Z; \tau, E)$ is a $\mathfrak{R}$-measurable function.

For given (τ, E) and each pair of (y, Z), the transition probability is defined by;

$$P(y, Z; \tau, E) = \begin{cases} \int_{B^{Pub}(y,Z;E)} g(\gamma)\mu(d\gamma) & \text{if } y < y^*(\tau, E), \\ \int_{B^{Pri}(y,Z;\tau)} g(\gamma)\mu(d\gamma) & \text{otherwise}, \end{cases}$$

where $B^{Pub}(y, Z; E) = \{\gamma | \gamma h(E) \in Z\}$ and $B^{Pri}(y, Z; \tau) = \{\gamma | \gamma h(e((1-\tau)y)) \in Z\}$.

Using the transition probability, a sequence of $\{\lambda_t\}$ with a given sequence $\{\tau_t, E_t\}$ is defined by;

$$\lambda_{t+1}(Z) = \int P(y, Z; \tau_t, E_t)\lambda_t(dy).$$

With the distribution of household income $\lambda_t(y)$, the cumulative distribution function of household income is given by $F_t(y) = \lambda_t(\{z|z \leq y\})$. The dynamics of income distribution can be described in terms of the cumulative distribution function as follows;

$$F_{t+1}(y) = \int P(x, \{z|z \leq y\}; \tau_t, E_t)dF_t(x). \tag{3}$$

Since there is no aggregate uncertainty, income distribution in date $t+1$ is pinned down *uniquely* from F_t for given (τ_t, E_t).

3.5 Endogenous Education Policy

Public education policy is determined by majority vote in each period. Each parent votes for her favorite policy that maximizes her (indirect) utility.

3.5.1 Household's problem

Let $(\tau(y), E(y))$ be the preferred pair of tax rate and quality of public education of a parent with pre-tax income y. A policy preferred by the parent with income y is given by;

$$(\tau(y), E(y)) = \arg\max_{(\tau,E)} V(y; \tau, E),$$

subject to the government budget constraint (1), that is, $N(\tau, E)E = \tau Y$.

Notice that voters are *foresighted* in the sense that each parent takes into account the effect of her policy choice on the *size* (N) and the *quality* (E) of public education when she chooses her favorite tax rate. One important implication of this assumption is that the voters face the tradeoff not only between current consumption and future income of child but also between *redistribution* and *the size of public education.* If the voters take the size of public education as given, their favorite tax rates are chosen so as to resolve only the first tradeoff. However, when they are foresighted, they choose their favorite tax rate to achieve the optimal size of public education as well as its quality. For example, a poor parent has an incentive to increase tax rate in order to redistribute educational resources from rich. However, she may not want to raise the tax rate because a higher tax rate leads to a larger size of public education and low quality of public education. In fact, because of this second tradeoff, households prefer a tax rate at most as high as that in the public regime where the size of public education $N = 1$ is taken as given.[8] We will see in the later part of this paper that this second tradeoff plays a crucial role in the comparison of the mixed regime with the public regime.

The optimal policy for the parent with income y is summarized as follows. Note that, on one hand, whenever the parent chooses private school, her favorite policy is always zero provision of public education, because she receives no benefit from it. Hence, the maximized utility from choosing private school is given by $V(y; 0, 0) = V^{Pri}(y; 0)$. On the other hand, if the parent chooses a public school, the maximized utility is given by $V(y; \tilde{\tau}(y), \tilde{E}(y)) = V^{Pub}(y; \tilde{\tau}(y), \tilde{E}(y))$, where $(\tilde{\tau}(y), \tilde{E}(y))$ is the favorite policy of a parent with household income of y *conditional on sending her child to public school.* That is;

$$(\tilde{\tau}(y), \tilde{E}(y)) = \arg\max_{(\tau, E)} V^{Pub}(y; \tau, E)$$

subject to $N(\tau, E)E = \tau Y$. Hence, the tax rate and the quality of public education preferred by the parent with pre-tax income y are;

$$(\tau(y), E(y)) = \begin{cases} (0,0) & \text{if } V^{Pub}(y; \tilde{\tau}(y), \tilde{E}(y)) < V^{Pri}(y; 0), \\ (0,0) \text{ or } (\tilde{\tau}(y), \tilde{E}(y)) & \text{if } V^{Pub}(y; \tilde{\tau}(y), \tilde{E}(y)) = V^{Pri}(y; 0), \\ (\tilde{\tau}(y), \tilde{E}(y)) & \text{if } V^{Pub}(y; \tilde{\tau}(y), \tilde{E}(y)) > V^{Pri}(y; 0). \end{cases}$$

If a parent is indifferent between public and private education, I assume that the parent prefers the type of school she attended.[9]

3.5.2 Majority-Voting Policy

A policy is a *majority-voting policy* if it is supported by at least 50 percent of voters in any pairwise comparison. As Stiglitz (1974) points out, there may be no majority voting policy when households choose either a public or a private school, because the political preferences are not single-peaked. However, it is possible to guarantee the existence of majority-voting policy by imposing further restrictions on the preferences of households. Epple and Romano (1996) show that there exists a majority-voting policy in which the median voter is decisive when the slope of indifference curves of $V^{Pub}(y; \tau, E)$ in the (E, τ) plane is non-decreasing in y, or equivalently, when the price elasticity of implied demand for public education exceeds the absolute value of the income elasticity of the

[8] This is proved as Lemma 2 in Appendix.

[9] More precisely, when $V^{Pri}(y; 0) = V^{Pub}(y; \tilde{\tau}(y), \tilde{E}(y))$, the parent votes for $(\tau, E) = (\tilde{\tau}(y), \tilde{E}(y))$ if she received public education, otherwise she votes for $(\tau, E) = (0, 0)$.

same. This condition holds if richer parents claim a higher marginal increase in the quality of public education than poorer parents do to compensate the utility loss from the marginal increase in the tax rate. I follow their approach and apply to the case with stochastic ability. The median voter theorem under school choice is summarized as Proposition 1.

Proposition 1 *Assume that* $-\frac{u''c}{u'} \leq 1$*. Then there exists a majority-voting policy chosen by the median voter. Moreover, the majority-voting policy is generically unique.*

Throughout this paper, I impose this assumption on the utility function $u(.)$ that guarantees the existence of majority-voting policy.

4 Equilibrium

A sequence $\{\tau_t, E_t, F_t, N_t,\}_{t=0}^{\infty}$ (with given F_0) is an *equilibrium* if for all $t \geq 0$, (i) each household solves the utility maximization problem described in the equation (2) taking τ_t and E_t as given; (ii) the government budget constraint (1) is satisfied in every period; (iii) the amount of students in public schools is consistent with the decision of school choice made by each household, that is,

$$N_t \in [F_t^-(y^*(\tau_t, E_t)), F_t(y^*(\tau_t, E_t))], \tag{4}$$

where $F^-(y) \equiv \lim_{\epsilon \to 0+} F(y - \epsilon)$; (iv) for given F_t and (τ_t, E_t), F_{t+1} satisfies the dynamics of income distribution (3); and (v) (τ_t, E_t) is a majority-voting policy.

To see the existence and uniqueness of equilibrium, note that we can construct an equilibrium in a recursive way. At any date t, the income distribution F_t is pre-determined. Under this income distribution, each household votes on education policy (τ_t, E_t), and the majority-voting policy (which is the policy preferred by the median voter) is uniquely determined generically (Proposition 1). Under the majority-voting policy, the income distribution in the next period F_{t+1} is obtained uniquely by aggregating the individual dynamics as described in (3). Hence, we have the following proposition.

Proposition 2 *From every* F_0*, there exists generically a unique equilibrium.*

5 Steady state

A steady state is defined in the standard sense. A collection, (τ, E, F, N) is a *steady-state equilibrium* if there exists an equilibrium $\{\tau_t, E_t, F_t, N_t\}$ with $(\tau_t, E_t, F_t, N_t) = (\tau, E, F, N)$ for all $t \geq 0$.

Although there exists a unique equilibrium from every initial income distribution, the analytical study of the general model is difficult. Notice that the dynamics of household income (3) depends on the sequence of public education policies $\{\tau_t, E_t\}$, which is endogenously determined by majority vote. On top of this, policy choice by the median voter is discontinuous due to school choice. It is well-known (e.g,. Conlisk (1976), Banerjee and Newman (1993)) that without further assumptions, the characterization of dynamics is extremely hard in this class of models with interaction among households, and we cannot apply the standard argument of a stochastic Markov chain, as seen in Stokey and Lucas (1989), to this class of models.

To obtain further characterization results about steady state, I use the following specification; $u(c_t) = \ln c_t$, $v(y_{t+1}) = \beta \ln y_{t+1}$, $h(e;\gamma) = \gamma A e^{\alpha}$, $0 < \alpha < 1$, $\beta > 0$, $b > 0$, and $\ln \gamma \sim N(-\sigma^2/2, \sigma^2)$ with $\sigma > 0$. I use this specified model in the rest of this paper.

In this model, optimal investment in private education by household with income y_t under given τ_t is given by;

$$e((1-\tau_t)y_t) = \frac{\alpha\beta}{1+\alpha\beta}(1-\tau_t)y_t. \tag{5}$$

The indirect utility of the parent with income y_t from private education is;

$$V^{Pri}(y_t;\tau_t) = (1+\alpha\beta)\ln((1-\tau_t)y_t) + \ln\frac{(\alpha\beta)^{\alpha\beta}}{(1+\alpha\beta)^{1+\alpha\beta}} + \beta\ln A - \beta\frac{\sigma^2}{2}. \tag{6}$$

On the other hand, under public education with education expenditure per student E_t, the indirect utility is given by;

$$V^{Pub}(y_t;\tau_t, E_t) = \ln((1-\tau_t)y_t) + \alpha\beta\ln(E_t) + \beta\ln A - \beta\frac{\sigma^2}{2}. \tag{7}$$

Using these indirect utility functions, the threshold level of income is given by;

$$y^*(\tau_t, E_t) = \frac{E_t}{1-\tau_t}\frac{(1+\alpha\beta)^{1+\frac{1}{\alpha\beta}}}{\alpha\beta}.$$

Hence, the household income at time $t+1$ is given by;

$$y_{t+1} = \begin{cases} \gamma_{t+1} A E_t^{\alpha} & \text{if } y_t < y^*(\tau_t, E_t), \\ \gamma_{t+1} A \left[\frac{\alpha\beta}{1+\alpha\beta}(1-\tau_t)\, y_t\right]^{\alpha} & \text{otherwise.} \end{cases} \tag{8}$$

Notice that in equilibrium there always exist households who prefer private school. This is because the unbounded support of ability $(0, \infty)$ implies the existence of households whose income is larger than the finite level of threshold income. Moreover, if median income is lower than mean income, the median decisive voter prefers public school and thus we observe the mixture of public and private schools. The next proposition states that this is the case in the long run.

Proposition 3 *In the economy with the above specification, any steady state involves a mixture of public schools and private schools.*

There are two implications of this proposition. The first implication is that a purely public (private) educational regime is not sustainable when households are allowed to choose school. The second implication is that the share of students in public schools is more than 50 percent (otherwise public education is not supported by the majority). These two implications are consistent with the observations in the U.S. such that all states have a mixture of public and private schools and the share of public education is more than 50 percent in all states as we saw in Table 1.[10]

6 Comparison of Educational Regimes

In this section, I compare the long-run outcomes of the mixed-regime model with two "pure" regimes; (i) the public regime in which only public schooling is available to households; and (ii) the private regime in which only private schooling is available. This comparison is important for the following two reasons. Firstly, it provides an important implications for the evaluation of the effects of educational reforms such as centralization or privatization of educational regimes. Secondly, it reveals the costs and benefits of the mixed regime relative to these "pure" regimes.

To compare the mixed regime with the public (private) regime, I first calibrate the mixed-regime model to the U.S. economy. Then I simulate the three models (the public, the private, and the mixed regime) and compare their long-run outcomes.

6.1 Calibration

First of all, we need to choose the parameters of the model. In the model, I have four parameters to be assigned, α, β, A, and σ. The parameter α is the elasticity of future income with respect to quality of education. Some empirical studies, for instance, Card and Krueger (1992) and Wachtel (1976), suggest that α is around .2. I use this value as a benchmark model.

I calibrate the rest of the parameters using three pieces of information; the share of educational expenditure in personal income, mean income and median income. For the share of educational

[10] Looking at the U.S. Census data, we can confirm that these observations are also true for all time periods from 1960 to 2000.

spending in household income, I use the evidence reported in Fernández and Rogerson (1999). They found that in 1966 the percentage of personal income spent on K-12 education in the US was 4.6 percent and 4.69 in 1993. Its average over this period was 4.63 percent and almost constant. For the mean and median income, I take these values from Census 2000 (we have 59,339 and 45,000 dollars for mean and median income, respectively). I choose the parameters so that the long-run prediction of the model matches these empirical counterparts.

To calibrate the model, I take the following steps. In the first step, I construct the steady-state income distribution in the private regime for a given set of parameters in order to use it as an initial income distribution for simulations. I use it as an initial condition because it allows us to interpret the equilibrium dynamics as a departure from "laissez-faire."[11] In the second step, I simulate the equilibrium dynamics with 10,000 households for 100 generations. In the third step, I calculate the long-run values of mean and median income, and the average share of educational expenditure in income. For long-run values of these variables, I calculate the time-series average from $t = 5$ to 100, because simulations reveal that the standard deviations are small and all the values after $t = 5$ are within two standard deviations of their time-series average.[12] If these long-run values are largely different from the empirical counterparts, I repeat the above procedure for a different set of parameters. The result of the calibration is summarized in Table 2.

[*Table* 2]

The calibrated model generates 94.5 percent of the long-run share of public education. This value is within two standard deviations of the average share in the United States, but higher than the average. One reason why we observe a higher share of private education in reality may be that households have other motivations to choose private school based on, for instance, religion and race as well as quality of education, and the current model focuses only on quality-based school choice.

[11] Another interpretation of equilibrium dynamics from this initial distribution is the transition from the pure-local finance regime to the pure-state finance regime with private alternatives. Notice that we can interpret a pure-private educational regime as the pure-local finance regime, and the pure-public regime as a pure-state regime in the dynamic Tiebout model by Fernández and Rogerson (1997). Hence, current simulations also provide implications of education *finance* reforms.

[12] An alternative definition of long-run values is the values of them at $t = 100$. Using the alternative definition generates same results.

6.2 Simulation Results

Using the parameters obtained in the calibration, I simulate three models (the public, the private, and the mixed regime) and calculate the long-run mean income, the degree of income inequality, intergenerational income mobility and total welfare. In order to study the different consequences stemming purely from the difference of regimes, I simulate these models using the *same* initial income distribution and the *same* realization of stochastic shocks.

6.2.1 Mixed vs. Private Regime

Firstly, let us compare the mixed regime with the private regime. The most important difference between these regimes is that the mixed regime provides public education while the private regime does not.

[*Table* 3]

Table 3.1 summarizes the resulting long-run values in the mixed and the private regime. The long-run value of mean income in the mixed regime is higher by 6 percent than in the private regime, and the degree of income inequality measured by Gini coefficient is lower by 0.006 point in the mixed regime than in the private regime. To understand these differences, note that 94.5 percent of students are in public schools in the mixed regime. In the absence of credit markets for education, public education redistributes educational resources from rich to poor households. As a result, the degree of income inequality is lower in the mixed regime than in the private regime. Moreover, the concave production function of human capital implies that this redistribution increases the total output of the economy. Consequently, the total welfare measured by the unweighed sum of utility levels of all households is higher by 1 percent in the mixed regime than in the private regime.

The degree of intergenerational income mobility measured by the autocorrelation of household income is higher in the mixed regime than in the private regime.[13] Intuitively, in the mixed regime, most of students are in public schools in which the quality of education is "independent" of income of each household, while it crucially depends on parent's income in the private regime.

6.2.2 Mixed vs. Public Regime

Next, let us turn to the comparison of the mixed regime with the public regime. The most important difference between these regimes is that the mixed regime allows households to opt out to private

[13]Note that the higher the autocorrelation of household income, the *lower* the degree of income mobility.

school if they want.

Table 3.2 summarizes the long-run values of relevant variables in the mixed and in the public regime. The long-run mean income in the mixed regime is very close to that in the public regime, and so is the long-run total welfare. However, the degree of income inequality is higher by 0.002 point in the mixed regime than in the public regime.[14] To understand these observations, note that tax rate and expenditure per student in public schools in the mixed regime are *lower* than in the public regime. Remember that in the public regime, the decisive median household choose her preferred tax rate so as to resolve the tradeoff between current consumption and the future income of her child. However, in the mixed regime, she chooses a tax rate to resolve this tradeoff *and* the tradeoff between redistribution and size of public education. This second tradeoff leads her to choose a lower tax rate than that in the public regime.[15] Even though this lower tax rate results in a lower quality of public education, she prefers it because she obtains a higher utility from the gain in current consumption than that with a higher tax rate.[16]

Although the quality of public education is lower in the mixed regime than in the public regime, there is no significant difference in long-run mean income and total welfare between them. In the mixed regime, the income of children with public school is, on average, lower than those in the public regime, but high investment in education by rich households through private schools compensates these losses. Similarly, total welfare is similar between the mixed regime and the public regime, because the gains in the utility of rich households compensate the losses of poor households.

The degree of intergenerational income mobility is lower in the mixed regime than in the public regime. This is because in the mixed regime, rich households send their children to private schools in which the quality of education crucially depends on parent's income.

In summary, the mixed regime generates similar mean income and total welfare to those in the public regime in the long run. However, income inequality is higher in the mixed regime than in the public regime because in the mixed regime, poor households invest less in education through public schools but a significant amount of rich households opt out to private schools that provides higher quality of education. The degree of income mobility in the mixed regime is lower than in

[14]In the equilibrium, Gini coefficient in the mixed regime is higher than in the public regime *for all periods.*

[15]See Lemma 2.

[16]The lower tax rate in the mixed regime than in the public regime is observed even in the absence of stochastic ability. In the deterministic case, Tanaka (2001) shows that the long-run tax rate and the expenditure per student in public schools are lower in the mixed regime than in the public regime whenever steady state involves a mixture of public and private schools. The current simulation results imply that this finding is robust even in the presence of stochastic ability.

the public regime because of opting out to private schools by rich households.[17]

6.3 The Costs and the Benefits of the Mixed Regime

The simulations in the above revealed that the mixed regime generates higher mean income and lower income inequality than the private regime in the long run. In this comparison, provision of public education as a safety net for poor households is the most important factor to explain the different long-run performances between them.

The simulations also revealed that the difference in long-run mean income between the mixed and the public regime is very small, but the mixed regime generates higher long-run income inequality than in the public regime. In this comparison, opting out of rich households is the most important factor to explain the difference between them. Since the mixed regime allows rich households to invest more in education through private school, it possibly generates higher mean income than the public regime in which all are "forced" to go to public school. However, the mixed regime generates a lower tax rate than the public regime. In the above simulations, these two opposite effects are balanced and thus these two regimes generate similar long-run mean income.

In this subsection, I investigate the determinants of these costs and benefits of the mixed regime. To do so, I simulate the model of the mixed regime with different parameter values and compare resulting outcomes with those in the public (private) regime. In particular, I consider the variation in the elasticity of income to education (α) and the size of the variance of stochastic ability (σ).

6.3.1 Elasticity of Future Income to Education

Firstly, I calibrate the mixed-regime model and simulate them for various values of α.

[*Table* 4]

Table 4 summarizes the results for $\alpha = .6$ and .8. When $\alpha = .6$ (Table 4.1), long-run mean income in the mixed regime is higher by 30 percent than in the private regime and by 4 percent than in the public regime. Although Gini coefficient in the mixed regime is lower by 0.069 point than in the private regime, it is higher by 0.017 point than in the public regime. These effects are heightened when the elasticity is higher (Table 4.2 for $\alpha = .8$).

[17] As a sensitivity analysis, I calibrated and simulated the models for $\alpha = .1$ and .3, and obtained the same conclusion.

One of the explanations for these observations is that when α is large, the production function of human capital is less concave and thereby the marginal productivity of human capital remains high even for rich households who opt out to private schools. As a result, the benefit of the mixed regime that allows rich households to invest more in education dominates its cost in the long run.

6.3.2 Variance in Ability

Next, I simulate the benchmark model with different standard deviations of the stochastic ability. These simulations uncover the role of stochastic shock for the costs and the benefits of the mixed regime.

[*Table* 5]

In Table 5.1, I report the results of the simulations with a half of the calibrated standard deviation ($\sigma = .376$). With less risk in ability, the variance of household income is smaller than in the benchmark model. As a result, the amount of households who opt out to private school is smaller than in the benchmark model (the share of students in public schools is 99.8 percent).

There are two important changes in the comparison of the mixed with the public regime. The first change is that long-run mean income and total welfare are lower in the mixed regime than in the public regime. Although the tax rate and the quality of public education are higher than in the benchmark model, they are still lower than in the public regime. Since only .2 percent of households invest more in education through private schools, the benefit of the mixed regime relative to the public one is smaller than in the benchmark economy. The second change is that there is no significant difference in the degree of income inequality. This is a consequence of a larger size of public education in this economy.

What happens if the influence of stochastic ability is heightened? In Table 5.2, I report the results of the simulations with a double of the calibrated standard deviation ($\sigma = 1.55$). With more risk in ability, the variance of household income is larger than in the benchmark model. As a result, the amount of households who opt out to private school is larger than in the benchmark model (the share of students in public schools is 92.2 percent).

Contrary to the case with a small variance of ability, long-run mean income and total welfare in the mixed regime are *higher* than in the public regime. Although the tax rate in the mixed regime is lower than in the public regime, the quality of public education is *higher* than in the public regime, because the tax revenue goes up due to the high investment in private education by 7.8

percent of rich households. Since more households opt out to private schools than in the benchmark model, the benefit of the mixed regime is larger than in the benchmark economy. However, as a consequence of a large share of students in private schools, the degree of income inequality is higher in the mixed regime than in the public regime.

In summary, simulations revealed that the mixed regime generates a higher long-run mean income and total welfare than in the public or the private regime when the variance of stochastic ability is high, and the opposite is true when it is small. The variance of stochastic ability is a crucial determinants of the size of public education and thus the benefit of the mixed regime relative to the public regime.[18]

7 Democratic Choice of Educational Regimes

In the above analysis, the educational regime itself was taken as given. However, an interesting question may be which regime is preferred by majority. The next proposition answers this question.

Proposition 4 *Suppose that the median voter prefers the public regime to the private one. Then the mixed regime is preferred by majority to the public regime.*

If the median voter prefers the public regime to the private one, the mixed regime is better for households with public education and those with private schooling. The mixed regime is better for the households with public education because some of students opt out to private and thereby the students who remain in public schools enjoy more educational resources. The mixed regime is also better for those with private education because they can invest more in education.

One implication of this proposition is that the mixed educational regime is a consequence of the democratic choice of educational regimes by majority vote in the long run. This is another reason why we observe the mixed regime in democratic countries.

8 Private Educational Vouchers

Among the recent debates on the reform of educational regime, the introduction of private educational vouchers is one of the most widely discussed. In this section, I extend the model to study the effects of private educational vouchers.[19] A private educational voucher is defined as a fixed-value

[18] I simulated the model with various values of $\sigma \in [.1, 1.55]$ and found that the benefit of the mixed regime is heightened the greater the standard deviation of stochastic ability.

[19] The effects of private educational vouchers have been studied by, for instance, Epple and Romano (1996,1998), Hoyt and Lee (1999), Bearse, Glomm and Ravikumar (2000) in a static framework. Here I look at the dynamic effect

subsidy to private tuition. More specifically, s of subsidy is provided to the households who send their children to private schools, and this subsidy can be used only for private education. With this voucher, the future income of the child with educational expenditure in private school $e_t \geq 0$ is given by;

$$y_{t+1} = \gamma_{t+1} h(e_t + s),$$

and the government budget constraint at date t is;

$$\tau_t Y_t = N_t E_t + (1 - N_t)s.$$

I simulate the benchmark model with private educational vouchers for $s \in \{0, 1000, 2000, 3000\}$.

[$Table$ 6]

Table 6 summarizes the long-run values of relevant variables for each value of vouchers. The long-run mean income and total welfare are increasing in the value of vouchers. The reasons are the following. Since education is normal goods, the households who prefer private school invest more in education the larger is the value of vouchers, and, ceteris paribus, total income in the next generation will be higher. Moreover, the introduction of vouchers raises equilibrium tax rates as long as public education is supported by the majority. Remember that the central concern in the policy determination for the median voter in the mixed regime is the tradeoff between redistribution and size of public education. Since rich households have higher incentives to opt out to private school thanks to the vouchers, the median voter can raise the tax rate more than to cover the voucher expense. If these two effects are large enough, quality of public schools improves, and therefore mean income rises in the long run. Consequently, the introduction of vouchers raises mean income and total welfare in the long run.

Long-run degree of income inequality is *decreasing* in the value of vouchers up to $s = 2000$, but it increases when $s = 3000$. To understand this observation, note that the share of students in public schools is *increasing* in the value of vouchers when it is less than 3000. *Ceteris paribus,* the introduction of vouchers decreases the share of students in public schools because more students opt out to private school due to vouchers. However, in equilibrium, the introduction of vouchers raises the equilibrium tax rate and expenditure per student in public schools, and as a result

of the introduction of vouchers on welfare.

public schools attract more students. Since more students are in public schools where all students receive equal quality of education, the degree of income inequality falls. Moreover, the private educational voucher *reduces* the variance of educational expenditures across private schools by redistributing educational resources among them through vouchers. When these two effects dominate the inequality-increasing effect of vouchers, the degree of income inequality falls. However, when the value of vouchers is $s = 3000$, the introduction of vouchers eliminates public education, and thus the degree of income inequality rises.

In summary, the introduction of private educational vouchers increases mean income and total welfare in the long run, and reduces the degree of income inequality as long as public education is supported by the majority.[20]

9 Concluding Remarks

In this paper, I studied a dynamic political economy model of the mixed regime and analyzed the costs and benefits of the mixed regime in comparison with the public or the private regime. Using the model, I showed why we observe the mixed regime and the public and the private regimes are not sustainable. This result is consistent with the observations in the U.S. where all states have a mixture of public and private schools and the majority of students are in public schools.

Next, I compared the mixed regime with the public or the private regime using a numerical method. Simulations of the model revealed that in the mixed regime long-run mean income and total welfare are higher, and income inequality is lower than in the private regime. These results indicate that the economy may suffer from a loss in total welfare and a rise in income inequality from privatization of educational regimes.

In the comparison between the mixed and the public regime, simulations revealed that there is no significant difference in long-run mean income and total welfare between them, but the tax rate and the quality of public education in the mixed regime are significantly lower, and thus long-run income inequality is higher in the mixed regime. These results imply that centralization of educational regimes may reduce the degree of income inequality without reducing mean income and total welfare.

[20] Here is one caveat about the evaluation of the effects of vouchers on welfare. The introduction of vouchers increases the total welfare *in the long run*, but it may reduce the total welfare when they are introduced. In these simulations, it is true that the introduction of educational vouchers *reduces* the total welfare at $t = 0$, but the total welfare with vouchers after $t = 1$ is always higher than that without vouchers. The evaluation of the full effects of the vouchers depends on how to aggregate their welfare across time.

To investigate the determinants of the costs and benefits of the mixed regime relative to the public or private regime, I simulated the model with various values of the elasticity of future income to education and the variance of stochastic ability. I found that the mixed regime generates higher long-run mean income and total welfare than the public regime if the elasticity of future income to education is high. I also found that long-run mean income and total welfare are higher (lower) in the mixed regime than in the public regime if the variance of stochastic ability is large (small). These results indicate that the elasticity of future income to education and the size of the risk in individual ability are very important in the evaluation of educational reforms.

Finally, I analyzed the effects of private educational vouchers on long-run welfare. Simulations of the calibrated model revealed that private educational vouchers can increase the long-run mean income and total welfare, and reduce income inequality as long as public education is maintained. However, if the value of the voucher is so high that the decisive household prefers private school, then public education is abandoned, resulting in higher income inequality. This result indicates that in the evaluation of the effects of vouchers, we need to be careful about its effect on political choice of public education.

A Appendix: Proofs

A.1 Lemma 1

Lemma 1 *Suppose that a household with income level* y^* *is indifferent between the public school and a private school. Then all households whose income is higher than* y^* *strictly prefers a private school and all household whose income is lower than* y^* *strictly prefers the public school.*

Proof. Firstly, note that

$$\frac{\partial V^{Pub}(y;\tau,E)}{\partial y} = u'((1-\tau)y)(1-\tau),$$

$$\frac{\partial V^{Pri}(y;\tau)}{\partial y} = u'((1-\tau)y - e((1-\tau)y))(1-\tau).$$

Since $h'(0)=\infty$, $e((1-\tau)y)>0$. Thus, $(1-\tau)y > (1-\tau)y - e((1-\tau)y)$. Moreover, since $u(.)$ is strictly concave,

$$\frac{\partial V^{Pub}(y;\tau,E)}{\partial y} < \frac{\partial V^{Pri}(y;\tau)}{\partial y},$$

for all y. This implies that if $V^{Pub}(y^*;\tau,E) = V^{Pri}(y^*;\tau)$, $V^{Pub}(y;\tau,E) < V^{Pri}(y;\tau)$ for $y > y^*$ and vice versa. Q.E.D.

A.2 Lemma 2

Lemma 2 *Assume that* $F(y)$ *has a density function* $f(y)>0$ *for all* $y>0$*. For each* $y>0$*, a preferred tax rate of the household with income* y *is at most as high as that in the public regime in which* $N=1$ *is taken as given.*

Proof. Fix $y>0$. Let τ^{Mix} and E^{Mix} be the tax rate and the quality of public school the household with income y prefers, respectively. Similarly, let τ^{Pub} and E^{Pub} be the policy the household with income y prefers when she takes $N=1$ as given. I show that $\tau^{Mix} \leq \tau^{Pub}$.

Firstly, note that if the household prefers private school,$\tau^{Mix}=0$. Since $h'(0)=\infty$ implies $\tau^{Pub}>0$,$\tau^{Mix}\leq\tau^{Pub}$.

Next, consider the case in which the household prefers public school. Suppose $\tau^{Mix}>\tau^{Pub}$. Define $\tilde{v}(.) \equiv \int v(\gamma h(.))dG(\gamma)$. Note that $\tilde{v}(.)$ is strictly increasing and concave. Since $u((1-\tau^{Mix})y) + \tilde{v}(E^{Mix}) \geq u((1-\tau^{Pub})y) + \tilde{v}(E^{Pub})$, $E^{Mix} > E^{Pub}$.

Let $E^*(\tau)$ be educational expenditure per student in public school when all households make utility-maximizing choices. Using the government budget constraint (1), this is defined implicitly in $E^*(\tau)N(\tau, E^*(\tau)) = \tau Y$. Due to Epple and Romano (1996), this is continuous and differentiable almost everywhere. Hence, the first order condition;

$$u'((1-\tau^{Mix})y)y = \tilde{v}'(E^{Mix})\frac{dE^*}{d\tau}$$

holds. Similarly, when she takes $N = 1$ as given, the first order condition;

$$u'((1-\tau^{Pub})y)y = \tilde{v}'(E^{Pub})Y$$

holds. Since $\tau^{Mix} > \tau^{Pub}$,

$$\tilde{v}'(E^{Pub})Y = u'((1-\tau^{Pub})y)y < u'((1-\tau^{Mix})y)y = \tilde{v}'(E^{Mix})\frac{dE^*}{d\tau}.$$

Note that

$$\frac{dE^*}{d\tau} = \frac{E^*}{\tau}\frac{1-\epsilon_{N,\tau}}{1+\epsilon_{N,E}} = \frac{Y}{N}\frac{1-\epsilon_{N,\tau}}{1+\epsilon_{N,E}} < Y,$$

where $\epsilon_{N,\tau} = \frac{\tau y^* f(y^*)}{(1-\tau)N} > 0$ and $\epsilon_{N,E} = \frac{\tilde{v}'(E)Ef(y^*)}{(1-\tau)N\Delta} > 0$ with $\Delta \equiv u'((1-\tau)y - e((1-\tau)y)) - u'((1-\tau)y) > 0$. This implies that $\tilde{v}'(E^{Pub}) < \tilde{v}'(E^{Mix})$, which is a contradiction to $E^{Mix} > E^{Pub}$. Therefore, $\tau^{Mix} \leq \tau^{Pub}$. Q.E.D.

A.3 Proposition 1

Proof. Using $\tilde{v}(.)$, the expected utility from sending child to a public school is given by;

$$V^{Pub}(y;\tau,E) = u((1-\tau)y) + \tilde{v}(E),$$

and we can apply the results of the deterministic model by Epple and Romano (1996) to the model with stochastic ability.

Due to Epple and Romano (1996), the sufficient condition for the median voter theorem is that the slope of indifference curve on (E,τ) plane is non-increasing in income. Under the assumption that $-u''c/u' \leq 1$, the slope of indifference curve is non-increasing in y. Therefore, there exists a majority-voting policy in which the median voter is decisive.

For the uniqueness, it is sufficient to show that the median voter prefers (τ_t, E_t) uniquely in the generic case.

Suppose that the median voter with income y^m is indifferent between (τ^1, E^1) and (τ^2, E^2), both of which maximize her indirect utility. Then there are two cases.

In the first case, the median voter is indifferent between a policy with positive provision of public education and purely private schooling. Without loss of generality, set $(\tau^2, E^2) = (0, 0)$ and $\tau^1 > 0, E^1 > 0$. If the median voter (parent) attended a public school in her childhood, she votes for (τ^1, E^1), otherwise she votes for $(0, 0)$ (if the median voter is indifferent between them at $t = 0$, assume that she votes for (τ^1, E^1)). Even if there are more than one median voter, we can obtain a unique majority-voting policy by the majority voting (tie is possible, but it is not generic).

In the second case, the median voter is indifferent between two policies with positive provision of public education. Without loss of generality, set $\tau^1 > \tau^2 > 0$ and $E^1 > E^2 > 0$. Since the median voter is indifferent between (τ^1, E^1) and (τ^2, E^2), we have;

$$V^{Pub}(y^m; \tau^1, E^1) = V^{Pub}(y^m; \tau^2, E^2).$$

Under the assumption that the slope of indifference curve on (E, τ) plain is decreasing in y (that is, $-u''c/u' \leq 1$), $V^{Pub}(y; \tau^1, E^1) > (<) V^{Pub}(y; \tau^2, E^2)$ for $y < (>) y^m$. This implies that for y arbitrarily close to y^m, either (τ^1, E^1) or (τ^2, E^2) are strictly better than the other. Hence, if both (τ^1, E^1) and (τ^2, E^2) are the median voter's favorite public education policy, these are majority-voting policies, but these majority-voting policies are knife-edge case and thus a unique majority-voting policy is the generic outcome. Q.E.D.

A.4 Proposition 2

Proof. Fix F_t. Since (τ_t, E_t) is uniquely determined by majority voting (Proposition 1), N_t is also uniquely determined. Under the (τ_t, E_t), F_{t+1} is uniquely determined by (3). Since the above argument is true for any $t \geq 0$, we can construct a unique equilibrium $\{\tau_t, E_t, F_t, N_t,\}_{t=0}^{\infty}$ from every F_0. Therefore, there exists a generically-unique equilibrium from every F_0. Q.E.D.

A.5 Proposition 3

To prove Proposition 3, I first describe briefly steady state of the private regime and the public regime.

A.5.1 Private Regime

In the private regime, the tax rate is zero because there is no public education. Hence, optimal investment in education by the parent with income y_t is given by the equation (5) with $\tau_t = 0$, and the utility of this parent is given by the equation (6) with $\tau_t = 0$.

The income at time $t+1$ of this household with ability γ_{t+1} is given by;

$$y_{t+1} = \gamma_{t+1} A \left(\frac{\alpha\beta}{1+\alpha\beta} y_t \right)^{\alpha}. \tag{9}$$

Define μ_t^{Pri} and $\sigma_t^{Pri,2}$ as the mean and the variance of $\ln y_t$ in a purely private educational regime, respectively. Taking the logarithm of the equation (9), we have;

$$\mu_{t+1}^{Pri} = \alpha \mu_t^{Pri} + \ln A + \alpha \ln \left(\frac{\alpha\beta}{1+\alpha\beta} \right) - \frac{\sigma^2}{2}.$$

Note that in steady state, $\mu_{t+1}^{Pri} = \mu_t^{Pri} = \mu_\infty^{Pri}$. Hence, in steady state;

$$\mu_\infty^{Pri} = \frac{1}{1-\alpha} \ln A + \frac{\alpha}{1-\alpha} \ln \frac{\alpha\beta}{1+\alpha\beta} - \frac{\sigma^2}{2(1-\alpha)},$$

$$\sigma_\infty^{Pri,2} = \frac{1}{1-\alpha^2} \sigma^2.$$

Suppose that the initial income distribution is also the log-normal distribution with mean μ_0 and variance σ_0^2, that is; $\ln(y_0) \sim N(\mu_0, \sigma_0^2)$. Then the income distribution in the pure-private steady state is $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$. Indeed, this is a unique invariant distribution of the economy which is consistent with the private regime. I show the uniqueness of steady state in the private regime as Lemma 3.

Lemma 3 *$LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ is a unique long-run income distribution of the economy in the private regime.*

Proof. It is obvious that $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ is a long-run income distribution. For the uniqueness, it is sufficient to show that the income distribution of the economy converges to $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ from *any* initial income distribution.

Define $z_t \equiv \ln y_t - \mu_t^{Pri}$ and $\epsilon_{t+1} \equiv \ln \gamma_{t+1} + \sigma^2/2 \sim N(0, \sigma^2)$. The dynamics of a household income can be characterized by the following $AR(1)$ process;

$$z_{t+1} = \alpha z_t + \epsilon_{t+1}.$$

Iterating this equation, the individual income at date t starting from z_0 is

$$z_t = \alpha^t z_0 + \sum_{i=1}^{t} \alpha^{t-i} \epsilon_i.$$

Since $\epsilon \sim N(0, \sigma^2)$ i.i.d., given z_0, $z_t \sim N(\alpha^t z_0, \frac{1-\alpha^{2t}}{1-\alpha^2}\sigma^2)$. Taking the limit, $z_\infty \sim N(0, \frac{\sigma^2}{1-\alpha^2})$. Since $\lim_{t\to\infty} \mu_t^{Pri} = \mu_\infty^{Pri}$, $\ln y_\infty \sim N(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$. In words, the probability distribution of the log of a long-run household income is normally distributed, regardless of the initial income level. Since there exist infinitely many individuals in the economy, the probability distribution of individual income corresponds to the actual income distribution of the economy in the long run. Therefore the income distribution of the economy converges to $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ from any initial income distribution. Q.E.D.

A.5.2 Public Regime

Next, let us turn to the public regime. In the public regime, education is provided only through public schools. The tax rate is determined by majority vote. For given τ, t and E_t, the utility of the parent with income y_t is given by the equation (7). The preferred tax rate and expenditure per student of this parent are given by;

$$(\tau(y_t), E(y_t)) = \left(\frac{\alpha\beta}{1+\alpha\beta}, \frac{\alpha\beta}{1+\alpha\beta}Y_t\right).$$

Note that the preferred tax rate is independent of household income. In our model, all households prefer the same tax rate in a purely public regime. Hence, $\tau = (\alpha\beta)/(1+\alpha\beta)$ is chosen as a majority-voting policy in all periods.

The household income at time $t+1$ is given by;

$$y_{t+1} = \gamma_{t+1} A E_t^\alpha, \tag{10}$$

where $E_t = \tau_t Y_t$. The household income evolves *stochastically* because of the uncertainty in ability.

The dynamics of mean income Y_t is;

$$Y_{t+1} = A\left(\frac{\alpha\beta}{1+\alpha\beta}\right)^\alpha Y_t^\alpha.$$

Hence, mean income evolves *deterministically.*

Taking the logarithm of the equation (10), we obtain;

$$\ln y_{t+1} = \ln \gamma_{t+1} + \ln A + \alpha \ln E_t.$$

Since all children receive the same quality of education in public schools, the only source of the variation in income level across households is ability shocks. Consequently, the distribution of y_{t+1} is also given by the log-normal distribution, $\ln y_{t+1} \sim N(\ln A + \alpha \ln E_t - \sigma^2/2, \sigma^2)$.

Define μ_t^{Pub} and $\sigma_t^{Pub,2}$ as the mean and variance of $\ln y_t$ in the public regime at date t, respectively. Since $Y_t = \exp[\mu_t^{Pub} + \sigma_t^{Pub,2}/2]$;

$$\ln y_{t+1} = \ln \gamma_{t+1} + \ln A + \alpha\mu_t^{Pub} + \alpha\frac{\sigma_t^{Pub,2}}{2} + \alpha \ln \frac{\alpha\beta}{1+\alpha\beta}.$$

Hence, in steady state,

$$\mu_\infty^{Pub} = \frac{1}{1-\alpha}\ln A + \frac{\alpha}{1-\alpha}\ln\frac{\alpha\beta}{1+\alpha\beta} - \frac{\sigma^2}{2},$$

$$\sigma_\infty^{Pub,2} = \sigma^2.$$

That is, the steady-state income distribution in the public regime is given by $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$. This is the unique invariant distribution of the economy that is consistent with the public regime. I show the uniqueness of the steady state in the public regime as Lemma 4.

Lemma 4 *$LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ is a unique long-run income distribution of the economy in the public regime.*

Proof. Note that, since $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ is a long-run income distribution, there exists at least one long-run income distribution. For the uniqueness, it is sufficient to show that the income distribution of the economy converges to $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ from *any* initial income distribution.

Given an (any) initial income distribution (with finite mean Y_0), the distribution of $\ln y_1$ is given by $\ln y_1 \sim N(\ln b + \alpha \ln(\tau Y_0), \sigma^2)$. Moreover, $\ln y_t \sim N(\ln b + \alpha\ln(\tau Y_{t-1}), \sigma^2)$ for all $t \geq 1$. Thus the log of individual income $\ln y_t$ is always normally distributed except for $t = 0$.

Since $\tau = \frac{\alpha\beta}{1+\alpha\beta}$ and $\lim_{t\to\infty} Y_t = \exp(\frac{\sigma^2}{2(1-\alpha)})\tau^{\frac{\alpha}{1-\alpha}}$, the log-income distribution of the economy converges to $N(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$. Therefore the income distribution of the economy converges to $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ from *any* initial income distribution. Q.E.D.

A.5.3 Proposition 3

Proof. First I will show that $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ cannot be a steady state in the mixed regime. In order for $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ to be a steady state, the median voter must prefer private school. Since the median income under this income distribution is $y^m = b^{\frac{1}{1-\alpha}}\left(\frac{\alpha\beta}{1+\alpha\beta}\right)^{\frac{\alpha}{1-\alpha}}$, it is necessary to have

$$V^{Pri}(y^m; 0) \geq V^{Pub}(y^m; \tau, E),$$

where $E = \frac{\tau Y}{N}$, $\tau = \frac{\alpha\beta}{1+\alpha\beta}$ and $Y = \exp[\mu_\infty^{Pri} + \sigma_\infty^{Pri,2}/2]$. However,

$$V^{Pri}(y^m; 0) - V^{Pub}(y^m; \tau, E) = -\left(1 + \frac{1}{\alpha\beta}\right)\ln(1+\alpha\beta) - \frac{\sigma_\infty^{Pri,2}}{2} + \ln N < 0.$$

(Note that $N \in [0, 1]$ implies $\ln N \leq 0$ and that $(1 + \alpha\beta) > 1$ implies $\ln(1 + \alpha\beta) > 0$.) Therefore the median voter prefers public school, and thus $LN(\mu_\infty^{Pri}, \sigma_\infty^{Pri,2})$ cannot be a steady state in the mixed regime.

Next, I will show that $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ cannot be a steady state in the mixed regime. Suppose $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ is a steady state. Since $E = \frac{\alpha\beta}{1+\alpha\beta}Y < \infty$ ($Y = \exp[\mu_\infty^{Pub} + \sigma_\infty^{Pub,2}/2]$), $y^*(\tau, E) < \infty$. Since $\ln\alpha \sim N(0, \sigma^2)$, there always exists a household whose income level is higher than $y^*(\tau, E)$. This implies that there always exists a household that prefers the private school. Therefore $LN(\mu_\infty^{Pub}, \sigma_\infty^{Pub,2})$ cannot be a steady state in the mixed regime. Q.E.D.

A.6 Proposition 4

Proof. Suppose that the median voter prefers the public regime to the private regime. Then in the mixed regime, the median voter prefers public school to private one, and thus public education is maintained. Suppose that there is no opting out to private schools in the mixed regime. Then all households are indifferent between the mixed regime and the public regime. If some of households opt out to private schools, the majority strictly prefer the mixed regime to the public regime, because for given tax rate, the quality of public education rises and the households with private education are better off by opting out to private school. Therefore, the majority prefer the mixed regime to the public regime. Q.E.D.

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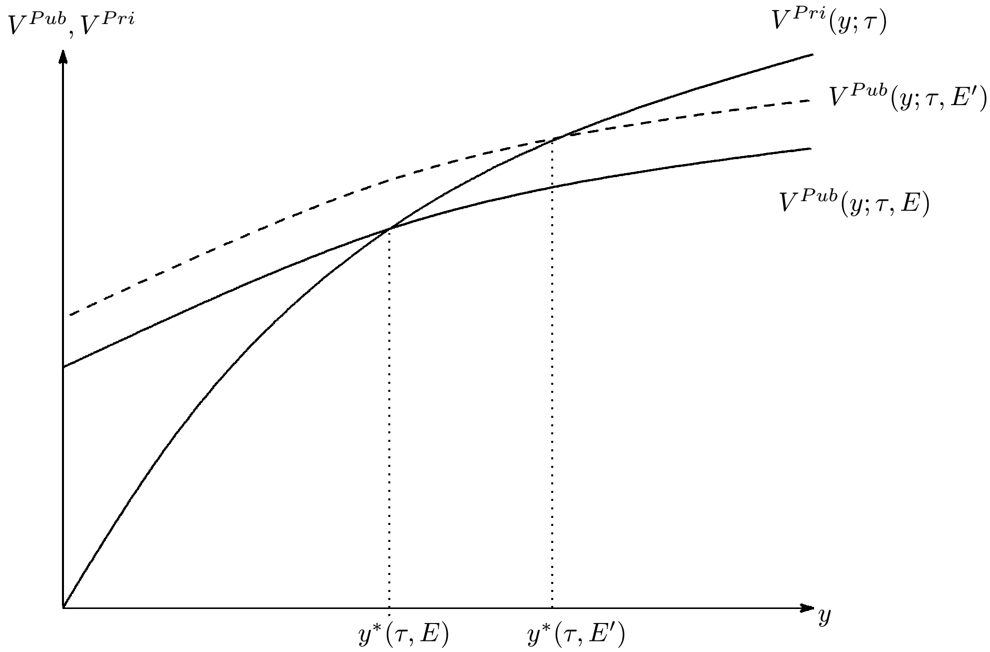

Figure 1: Indirect utility functions

Table 1: The Share of Students in Public Schools (primary and secondary, U.S., 1990)

State	Share of Public Schools
Alabama	90.14
Alaska	92.23
Arizona	92.08
Arkansas	92.77
California	87.22
Colorado	89.94
Connecticut	84.41
District of Columbia	82.51
Delaware	79.33
Florida	85.74
Georgia	90.25
Hawaii	79.99
Idaho	93.55
Illinois	83.44
Indiana	88.88
Iowa	88.24
Kansas	89.22
Kentucky	89.39
Louisiana	83.41
Maine	91.37
Maryland	82.84
Massachusetts	83.99
Michigan	87.36
Minnesota	87.82
Mississippi	89.56
Missouri	84.09
Montana	93.54
Nebraska	86.07
Nevada	92.17
New Hampshire	84.99
New Jersey	81.25
New Mexico	92.73
New York	82.14
North Carolina	92.70
North Dakota	93.24
Ohio	85.76
Oklahoma	92.43
Oregon	89.98
Pennsylvania	81.30
Rhode Island	83.03
South Carolina	90.05
South Dakota	91.42
Tennessee	90.65
Texas	91.24
Utah	94.23
Vermont	91.84
Virginia	88.62
Washington	88.12
West Virginia	93.89
Wisconsin	83.85
Wyoming	95.92
Average	88.25
Standard Deviation	4.28

Data source: U.S. Census (1990). All data are about primary and secondary education.
The share of Public Schools = (Enrollments in Public Schools) / (Enrollments in All Schools)

Table 2: Calibration Results

Parameters				Long-run Values		
α	β	A	σ	Mean Income	Median Income	The Share of Edu. Exp. in Income
0.2	0.2289	12325	0.7525	59354	44603	0.0463
				232	*297*	*0.0006*
Data				59339	45000	0.0463

Mean and Median Income are calculated using Census 2000.
The share of education expenditure is taken from Fernandez and Rogerson (1999).
Long-run values: the average of time series from t=5 to 100 (standard deviation is in *italic*).

Table 3.1: Mixed vs. Private Regime

Regime	Long-run Values						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	59354	0.4067	131170	0.0643	0.038	2383	94.5
Private	55986	0.4128	129830	0.1569	0	0	0
Difference	-3368	0.0061	-1340	0.0926	-0.038	-2383	-94.5
(percent)	-5.674	1.500	-1.022	144.012	-100.000	-100.000	-100.000

Difference: Private - Mixed

Table 3.2: Mixed vs. Public Regime

Regime	Long-run Values						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	59354	0.4067	131170	0.0643	0.038	2383	94.5
Public	59392	0.4052	131170	0	0.0438	2601	100
Difference	38	-0.0015	0	-0.0643	0.0058	218.37	5.5
(percent)	0.064	-0.369	0.000	-100.000	15.263	9.164	5.820

Difference: Public - Mixed

Table 4.1: Comparison of Educational Regimes (alpha = .6)

Edu. Regimes	**Long-run Values**						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	59699	0.4041	114900	0.2649	0.0379	2388	94.7
Public	57284	0.3868	114680	0	0.0438	2509	100
Private	45860	0.4726	110270	0.5032	0	0	0

Table 4.2: Comparison of Educational Regimes (alpha = .8)

Edu. Regimes	**Long-run Values**						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	59224	0.3921	112950	0.4195	0.0385	2390	95.4
Public	48874	0.3582	111350	0	0.0438	2141	100
Private	29652	0.562	101550	0.6936	0	0	0

Table 5.1: Comparison of Educational Regimes (sigma = .376)

Regime	Long-run Values						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	59230	0.2097	133750	0.0067	0.0431	2557	99.8
Public	59400	0.2097	133780	0	0.0438	2602	100
Private	58531	0.2139	133090	0.1884	0	0	0

Table 5.2: Comparison of Educational Regimes (sigma = 1.55)

Regime	Long-run Values						
	Mean Income	Gini Coefficient	Total Welfare	Mobility	Tax Rate	Public Edu. Exp. Per Student	% Public
Mixed	61565	0.7122	121110	0.0387	0.0397	2650	92.2
Public	59335	0.7107	120820	0	0.0438	2599	100
Private	46988	0.7208	116890	0.0722	0	0	0

Table 6: Private Educational Voucher

Value of Voucher	Long-run Values					
	Mean Income	Gini Coefficient	Total Welfare	Tax Rate	Public Edu. Exp. Per Student	% Public
0	59301	0.4064	131160	0.038	2382	9460
1,000	59912	0.4062	131263	0.041	2537	9538
2,000	60464	0.4061	131354	0.044	2680	9599
3,000	63052	0.4064	131792	0.048	0	0