The US Phillips Curve and inflation expectations: A State Space Markov-Switching explanatory model.*

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Abstract

This paper proposes a new empirical representation of US inflation expectations in a Stace-Space Markov-Switching framework in order to identify the expectations regimes which are associated with short and long term Phillips curves. Results suggest that the dynamics of inflation expectation errors change across regimes.

Keywords: State-Space Markov-Switching model; Inflation expectation errors; Phillips curve; occasionally integrated process

JEL classification: E3, C32, C51

1 Introduction

Expectations, especially those for inflation, play a crucial role in many different macroeconomic models. Since they are not directly observed, strong hypotheses are made about them.

While the "traditional" approach by Hibbs (1977) about the Phillips Curve includes adaptive expectations, Sargent (1969) proposed a "rational" version of former models in which he incorporated rationality for expectations, as pioneered by Muth (1961). As these expectations hypotheses have different implications for the theory, it seems important to know which ones are the more plausible for each period of time. While there is a vast literature on this topic, no consensus has emerged among economists on how to measure these subjective magnitudes. One approach, for instance, tries to infer the expected inflation rate from prices of financial instruments (Bank of Canada, 1998, Mylonas and Schich, 1999). An alternative approach uses quantitative information

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on inflation expectations from qualitative survey data (Carlson and Parakin (1977), Bakhsi and Yates (1998)).

A different approach is adopted in this article. Following Kim (1994) methodology, the expected inflation process is estimated using a State-Space (SS) Markov-Switching (MS) model, while introducing the Phillips Curve so as to improve the identification of the regimes for the expectation errors. The measurement equations, which link observable with unobservable components, are the expectation errors and the Phillips curve equations. On the other hand, the state equations specify the unobservable equations components: the inflation expectations and the natural rate of unemployment. An assumption is made that expected inflation is the underlying component of the inflation rates, so that they share the same behaviour.

So as to test the Rational Expectations Hypothesis (REH), the following assumption is made. Whenever a shift occurs in the economy, the agents should integrate it in their expectations if they are rational. This should be introduced in the dynamics of inflation expectations, so that any structural shift should leave the expectation errors dynamics unchanged. Furthermore, the REH implies that the expectation errors follow a white noise process. The MS model developed in this paper, in which the expectation errors process is allowed to change across regimes, will enable us to check these two points.

Moreover, in order to test which expectation regime is associated with either a short or a long run Phillips curve, the slope of the curve is allowed to change along with the expectation regimes.

The rest of the article is organized as follows. In the next section, the importance of inflation expectations for the Phillips curve is emphasized in a theoretical framework. In a third part, the econometric models is set out while specifying univariate representation for the inflation rate before presenting the SS-MS model. Empirical results are presented in section 4. Section 5 sums up the results and concludes the article.

2 Inflation expectations in a Regime-Switching Phillips curve

From the modified Phillips curve, three sources of inflation are usually identified: the inflation expectations Π_t^e, the unemployment gap $U_t - \bar{U}_t$, (where $\bar{U}_t$ is the natural rate of unemployment) and the supply shocks v_t.

These three factors are encompassed in the following equation:

$$\Pi_t = \Pi_t^e - \delta(U_t - \bar{U}_t) + v_t \tag{1}$$

with δ positive. For modeling convenience, it will be preferable to rewrite the relation between unemployment and inflation in its reversed form as in the following :

$$U_t - \bar{U}_t = \lambda(\Pi_t - \Pi_t^e) + \eta_t \tag{2}$$

where $\lambda = -\frac{1}{\delta}$ is negative.

When agents have adaptive expectations, arbitrage between inflation and unemployment exists in the short run. For the agents who make believes in rational expectations, there is no arbitrage between inflation and unemployment, even in the short run, and the unemployment rate is equal to its natural level. In order to take into account long run and short run Phillips curves, we allow for a regime-dependent parameter λ. We rewrite (2) as:

$$U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \eta_t \tag{3}$$

where $S_t = (1, 0)$ can be seen as the regime of inflation expectations at time t. In the regime of adaptive expectations ($S_t = 1$), a negative λ_0 is expected, while this parameter should disappear in the case of rational expectations ($\lambda_1 = 0$).

3 Econometric analysis

3.1 Univariate representation of inflation

Two unobservable components (inflation expectations and unemployment natural rate) are present in equation (2).

Concerning the inflation expectations, they are considered as an underlying component of the observed inflation, so that their dynamics are both described by an occasionally integrated specification:

$$\Delta\Pi_t = \mu_{S_t} + (\rho_{S_t} - 1)\Pi_{t-1} + \sum_{i=1}^{k} \phi_{iS_t}\Delta\Pi_{t-i} + \theta_{S_t} w_t \tag{4}$$

with $\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t$, $\rho_{S_t} = \rho_0(1 - S_t) + \rho_1 S_t$, $\phi_{iS_t} = \phi_{i0}(1 - S_t) + \phi_{i1} S_t$, $\theta_{S_t} = \theta_0(1 - S_t) + \theta_1 S_t$, $w_t \rightsquigarrow nid(0,1)$, $S_t = \{0, 1\}$, $p = \Pr(S_t = 1/S_{t-1} = 1)$, $q = \Pr(S_t = 0/S_{t-1} = 0)$.

The constant μ, the coefficient for persistence ρ, the autoregressive parameters ϕ_i and the volatility θ may change with the regime S_t, which follows a first order Markov process. This model will be used to test if an unit root is present within regimes ($\rho_{S_t} = 1$)[1].

3.2 State-Space Markov-Switching model of inflation expectations

In this section,inflation expectations are identified while allowing for shifting regimes in the inflation expectations process.

The measurement equations are:

$$\Pi_t - \Pi_t^e = \alpha_{S_t} + \gamma_{S_t}(\Pi_{t-1} - \Pi_{t-1}^e) + \sigma^{\Pi} u_t \tag{5}$$

$$U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \sigma^{U} v_t \tag{6}$$

where u_t and v_t are white noise process with standard errors equal to one.

Here, the constant α and the autoregressive coefficient γ may be different across regimes, allowing for different type of inflation expectations. For the regime of adaptive expectations ($S_t = 0$), we expect γ_0 to be significantly different from zero. For $S_t = 1$ (regime of rational expectations) we expect $\alpha_1 = \gamma_1 = 0$.

The inverted MS Phillips curve developed in section 1 is added in order to improve the identifications of expectation errors and to check respectively that adaptive expectations are associated with a short run curve ($\lambda_0 < 0$) and rational expectations are associated with a long run one ($\lambda_1 = 0$).

Π_t^e (inflation expectations) and $\bar{U}_t$ (natural rate of unemployment) are unobservable components, specified in the following State equations:

$$\Pi_t^e = \mu_{S_t}^{\Pi^e} + \rho_{S_t}^{\Pi^e}\Pi_{t-1}^e + \sigma_{S_t}^{\Pi^e}\varepsilon_t^{\Pi^e}$$

[1] The same type of model was first proposed by Ang and Beckaert (1998) to describe the behaviour of interest rates.

$$\bar{U}_t = \bar{U}_{t-1} + \sigma^{\bar{U}} \varepsilon_t^{\bar{U}} \tag{7}$$

$\varepsilon_t^{\Pi^e}$ and $\varepsilon_t^{\bar{U}}$ are white noise process with standard errors equal to one.

As explained in the introduction, the same specification is considered for the inflation expectations and the observed inflation rate. The specification of the unemployment rate in (7) as well as the calibration for $\sigma^{\bar{U}}$ are chosen according to Gordon (1997).

The State Space representation is the following:

$$\begin{bmatrix} \Pi_t \\ U_t \end{bmatrix} = \begin{bmatrix} \alpha_{S_t} \\ \lambda_{S_t}\alpha_{S_t} \end{bmatrix} + \begin{bmatrix} \gamma_{S_t} \\ \lambda_{S_t}\gamma_{S_t} \end{bmatrix} \Pi_{t-1} + \begin{bmatrix} 1 & -\gamma_{S_t} & 0 \\ 0 & -\lambda_{S_t}\gamma_{S_t} & 1 \end{bmatrix} \begin{bmatrix} \Pi_t^e \\ \Pi_{t-1}^e \\ \bar{U}_t \end{bmatrix} + \begin{bmatrix} \sigma^{\Pi} & 0 \\ \lambda_{S_t}\sigma^{\Pi} & \sigma^{U} \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \Pi_t^e \\ \Pi_{t-1}^e \\ \bar{U}_t \end{bmatrix} = \begin{bmatrix} \mu_{S_t}^{\Pi^e} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_{S_t}^{\Pi^e} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Pi_{t-1}^e \\ \Pi_{t-2}^e \\ \bar{U}_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{S_t}^{\Pi^e}\varepsilon_t^{\Pi^e} \\ 0 \\ \sigma^{\bar{U}}\varepsilon_t^{\bar{U}} \end{bmatrix} \tag{9}$$

4 Empirical results

Tests are applied on quarterly US data for the estimation period: 1973:02-2003:03 and the inflation is computed from the Consumer Price Index.

4.1 Univariate representation of inflation rate

Results of the univariate model (4) estimation are presented in Table 1. Hereafter, we pose : $\beta_{S_t} = \rho_{S_t} - 1$.

From the result ($\hat{\beta}_0 = -0.70$ with a $t-stat$ equal to -2.95), the stationarity hypothesis for the inflation process in regime 0 is checked. As the probability distribution is not known in the presence of switching regimes , the critical values of the distribution of $(\frac{\hat{\beta}_0}{\hat{\sigma}(\hat{\beta}_0)})$ under the null hypothesis $H_0 : \{\beta_0 = 0\}$ are computed by Monte Carlo simulations.

The Data Generating Process G is $\Delta y_t = (\hat{\mu}_0 + \sum_{i=1}^{k} \hat{\phi}_{i0} \Delta y_{t-i} + \hat{\theta}_0 w_t)(1 - S_t) + (\hat{\mu}_1 + \sum_{i=1}^{k} \hat{\phi}_{i1} \Delta y_{t-i} + \hat{\theta}_1 w_t) S_t$

where $w_t \rightsquigarrow nid(0,1)$, $\hat{p} = \Pr(S_t = 1/S_{t-1} = 1)$, $\hat{q} = \Pr(S_t = 0/S_{t-1} = 0)$ and $\hat{\mu}_{S_t}$, $\hat{\phi}_{iS_t}$, $\hat{\theta}_{S_t}$, $\hat{p}$, $\hat{q}$ are the estimation of the coefficients of (4).

And we estimate the model E: $\Delta y_t = (\mu_0 + \beta_0 y_{t-1} + \sum_{i=1}^{k} \phi_{i0} \Delta y_{t-i} + \theta_0 w_t)(1 - S_t) + (\mu_1 + \sum_{i=1}^{k} \phi_{i1} \Delta y_{t-i} + \theta_1 w_t) S_t$

The $t - stat$ of $\tilde{\beta}_0$ of the i-th replication under the null $(\beta_0 = 0)$ is $(\frac{\tilde{\beta}_0}{\tilde{\sigma}(\tilde{\beta}_0)})$ where $\tilde{\sigma}(\tilde{\beta}_0)$ (the standard deviation of $\tilde{\beta}_0$) is calculated with a numerical procedure.

So as to test $\{H_0 :$ the process is integrated in both regimes $(\beta_0 = \beta_1 = 0)\}$ against $\{H_1 :$ the process is occasionally integrated $(\beta_0 < 0, \beta_1 = 0)\}$, we generate G and estimate E. The simulation results are reported in table 2 (cf appendix). In state 0, the null hypothesis H_0 is rejected for a risk level of 2.5% $(-2.95 < -2.41)$ so that we conclude the process is occasionally integrated.

4.2 State-Space Markov-Switching representation

As an occasionally integrated process for inflation is found from the previous section, one of the states for the inflation expectations process is assumed to be integrated $\left(\rho_1^{\Pi^e} = 1\right)$. On the other hand, the persistence parameter is left free in the other regime. For the natural rate of unemployment, $\hat{\sigma}^{\bar{U}}$ is fixed to be equal to 0.2 as in Gordon (1997). Table 3 in appendix shows the result of the estimation of the SS-MS model.

State 0 In this state, agents seem to have rational expectations $(\hat{\alpha} \simeq \hat{\gamma} \simeq 0)$. The Phillips curve is vertical ($\hat{\lambda} \simeq 0$). Figure 1 in appendix shows that this state coincides with period of economic stability for the last twenty years (except for the early 1990s and 2000s). According to Figure 2, the unemployment rate is very close to the natural rate for this period.

State 1 In this state, expectation errors are quite persistent $(\hat{\gamma} = 0.91)$. There is a dilemma between inflation and unemployment $(\hat{\lambda} = -0.33)$. This regime can be interpreted as a regime with adaptive expectations associated with a short run negative Phillips curve. According to Figure 1, this state coincides with oil crisis periods and Volcker monetary shocks.

4.3 Test of restrictions

The Rational Expectations Hypothesis (REH) implies that $\hat{\alpha} = \hat{\gamma} = 0$. This seems to be the case in state 0. Moreover, as we want to check if rational expectations are associated with a vertical Phillips curve, we test for $\left\{\hat{\alpha}_0 = \hat{\gamma}_0 = \hat{\lambda}_0 = 0\right\}$. In table 3 the Likelihood Ratio statistic is 4.76.

The 5% critical value of a $\chi^2(3)$ is 7.8, so that we cannot reject the restrictions $\left\{\hat{\alpha}_0 = \hat{\gamma}_0 = \hat{\lambda}_0 = 0\right\}$ at the level of 5 %.

5 CONCLUSION

Throughout this article a SS-MS model has been developed with the goal of identifying expectation regimes associated with short run and long run Phillips curves. In this model, an inflation expectations augmented MS Phillips curve is used as a measurement equation and inflation expectations are specified as the underlying component of observed inflation, which is modeled as an occasionally integrated.

The persistence of expectation errors is found to be different according to the regimes. In one regime, the expectation errors process is autoregressive, whereas, in the other one, it is a white noise process. The regime with persistent expectation errors can be associated with the presence of an arbitrage between inflation and unemployment. On the other hand, this arbitrage disappears in the rational expectations regime, so that the Phillips curve is vertical.

To summarize, a Keynesian regime is identified for the period (1973-1983), whereas the periods of relative economic stability during the last twenty years are consistent with a classical regime in which agents have rational expectations and the unemployment rate is close to the natural rate.

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APPENDIX:

Table 1[2]: ML estimates of the univariate representation (1973:01-2003:03):
Inflation rate:

$$\Delta\Pi_t = (\mu_0 + \beta_0\Pi_{t-1} + \sum_{i=1}^{3}\phi_{i0}\Delta\Pi_{t-i} + \theta_0\eta_t)(1 - S_t) + (\mu_1 + \beta_1\Pi_{t-1} + \sum_{i=1}^{3}\phi_{i1}\Delta\Pi_{t-i} + \theta_1\eta_t)S_t{}^3$$

	$S_t = 0$	$S_t = 1$
$\hat{\mu}$	1.78 (2.68)	0.90 (1.69)
$\hat{\beta} = \hat{\rho} - 1$	-0.70 (-2.95)	-0.14 (-1.70)
$\hat{\phi}_1$	-0.18 (-0.88)	-0.41 (-3.61)
$\hat{\phi}_2$	-0.30 (-2.41)	-0.37 (-3.51)
$\hat{\theta}$	0.82 (5.48)	2.48 (11.47)
$\bar{\Pi}$[4]	2.66	5.9
$\hat{q}$	0.96 (2.82)	
$\hat{p}$	0.98 (3.22)	
average duration	6.5 years	12.5 years
ln(L)	-253.9	

Table 2: Tables of critical values:
DGP=G. Estimated model=E
$H_0 : \beta_0 = \beta_1 = 0$
number of replications:5000

p-value	10%	5%	2.5%
critical values of $\frac{\hat{\beta}_0}{\hat{\sigma}(\beta_0)}$	-1.64	-2.06	-2.41

[2] T-stat are into parentheses

[3] The number of lags has been tested with the k-max method. We have found $k = 2$.

[4] $\bar{\Pi}$ is the empirical mean for either regimes. $\bar{\Pi}_0 = \frac{\sum_{t=1}^{T}\Pi_t P(S_t=0)}{\sum_{t=1}^{T}P(S_t=0)}$, $\bar{\Pi}_1 = \frac{\sum_{t=1}^{T}\Pi_t P(S_t=1)}{\sum_{t=1}^{T}P(S_t=1)}$

Table 3: Maximum Likelihood estimates of model (8) and (9)

$$\left\{\begin{array}{c} \Pi_t - \Pi_t^e = \alpha_{S_t} + \gamma_{S_t}(\Pi_{t-1} - \Pi_{t-1}^e) + \sigma^{\Pi}\varepsilon_t \\ \\ U_t - \bar{U}_t = \lambda_{S_t}(\Pi_t - \Pi_t^e) + \sigma^U \eta_t \end{array}\right\}$$

$$\Pi_t^e = \mu_{S_t}^{\Pi^e} + \rho_{S_t}^{\Pi^e}\Pi_{t-1}^e + \sigma_{S_t}^{\Pi^e}\varepsilon_t^{\Pi^e}$$

		$S_t = 0$		$S_t = 1$	
		estim.	t-stat	estim.	t-stat
	$\hat{\alpha}$	-0.44	-0.46	-0.35	-1.30
$\Pi_t - \Pi_t^e$	$\hat{\gamma}$	0.20	1.53	0.91	16.45
	$\hat{\sigma}^{\Pi}$	1.28	10.06	1.28	10.06
$U_t - \bar{U}_t$	$\hat{\lambda}$	-0.008	-0.49	-0.33	-7.10
	$\hat{\sigma}^U$	1^e-6	3^e-5	1^e-6	3^e-5
	$\hat{\mu}_{\Pi}$	2.73	2.17	0.13	0.32
Π_t^e	$\hat{\rho}_{\Pi}$	0.23	0.87	1	-
	$\hat{\sigma}_{\Pi}$	9^e-5	3^e-5	2.95	10.4
	q	0.97			
	p	0.96			
LR test[5]	4.76				

[5] We test $H_0 : \alpha_0 = \lambda_0 = \gamma_0 = 0$

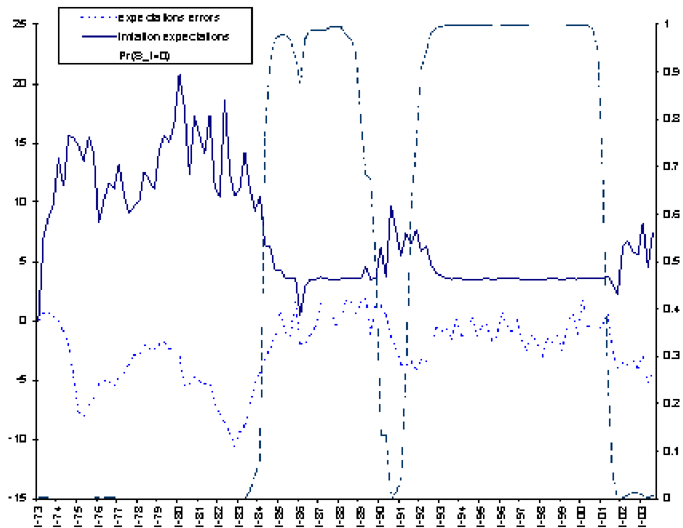

Figure 1: Expected inflation and expectation errors

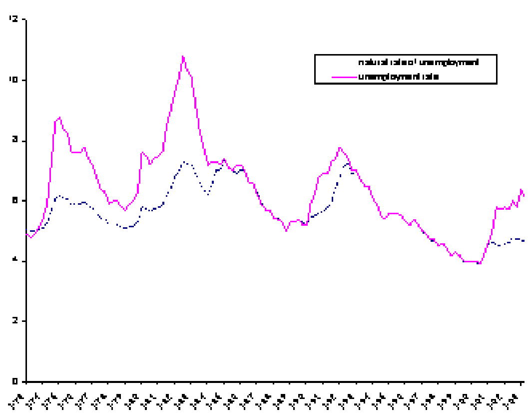

Figure 2: Unemployment rate and natural rate