A Collective Model of Household Behavior with Private and Public Goods : Theory and Some Evidence from U.S. Data

Olivier Donni*
University of Quebec at Montreal, and CIRPEE

Abstract : In the present paper, we adopt the collective approach to consumer behavior—which supposes that each household member is characterized by his/her own preferences and that the decision process results in Pareto-efficient outcomes—and assume, in addition, that agents are egoistic and consumption is either private or public. The main results are based on a conditional demand ('m-demand') framework in which household demands are directly derived from the marginal rates of substitution. We show that (i) household demands have to satisfy testable constraints and (ii) some elements of the decision process can be retrieved from observed behavior. These theoretical considerations are followed by an empirical application using the U.S. Consumer Expenditure Survey. Overall, it turns out that the data are consistent with the theoretical model.

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*Address: Département des Sciences Economiques, Université du Québec à Montréal, Case Postale 8888, Succursale Centre-Ville, Montréal (Qc), CANADA H3C 3P8. Email: donni.olivier@uqam.ca. Preliminary versions of this paper have been presented at the Econometric Society European Meeting 2002 (Venise), the Canadian Economic Association Congress 2002 (Calgary) and several seminars and workshops in Paris, Rennes, Copenhagen, Quebec, Sherbrooke, Ottawa. We specially thank Bernard Lejeune for a GMM gauss package and valuable discussions. We are also grateful to François Bourguignon, Martin Browning, Pierre-André Chiappori, Anyck Dauphin, Bernard Fortin, Alain Guay, Guy Lacroix, Nicolas Marceau, André Masson, Philip Merrigan, Bernard Salanié for useful comments and suggestions. Of course, we bear the sole responsability for any remaining errors. Financial support from the Fonds Québécois de Recherche sur la Société et la Culture is acknowledged.

1 Introduction

In microeconomics, the household as a whole is usually considered the elementary decision unit. In particular, it is characterized by a unique utility function that is maximized under a budget constraint. This is what we call the 'unitary' approach. However, it seems clear that a household comprising several adult members does not necessarily behave as a single agent.

Some years ago, Chiappori (1988, 1992) challenged the unitary approach by explicitly taking into account the bargaining between household members. He developed a 'collective' approach to household labor supply, where each agent is characterized by his/her own preferences and intra-household decisions result in Pareto efficient outcomes. The key idea underlying this model, if agents are egoistic (or altruistic in a Beckerian sense) and consumption is purely private, is very simple. Specifically, efficiency essentially means that the intra-household decision process can be decentralized by application of the Theorems of Welfare Economics. In a first step, members divide non-labor income according to some predetermined rule which depends on the household environment. In a second step, they maximize their utility subject to their own budget constraints. The main results are then twofold:

a) The collective labor supply has to satisfy testable restrictions under the form of partial differential equations;

b) The sharing rule for non-labor income can be retrieved, up to an additive constant, from the observed labor supply.

In addition, Fortin and Lacroix (1997) and Chiappori, Fortin and Lacroix (2002), have shown that these restrictions are not empirically rejected for Canada and the United States.[1]

More recently, the collective approach has been generalized to the analysis of household consumption. Bourguignon, Browning and Chiappori (1995) provide the main theoretical results for the case of constant prices. Browning et alii (1994) use Canadian expenditure data to estimate one of these models. Browning and Chiappori (1998) extend this theoretical setting to the case of variable prices. This generates further theoretical restrictions on

[1] Contributors to the theory of collective labor supply include, among others, Apps and Rees (1997), Chiappori (1997) for domestic production, and Blundell et alii (2001), Donni (2001, 2003) for corner solutions.

household behavior. In particular, they show that, under Pareto efficiency, the substitution matrix of the household demand system has to be equal to the sum of a symmetric matrix and an outer product. See also Lewbel, Chiappori and Browning (2003), who estimate equivalence scales in the collective framework, and Chiappori and Ekeland (2001a, 2001b) who develop a synthesis of collective models with price effects.

In collective models of household consumption, as well as in collective models of labor supply, the decision process can be decentralized if consumption is purely private and agents are egoistic (or altruistic in a Beckerian sense). Then, the sharing rule can be identified, under regularity conditions, from the observed behavior. This result is essential to the study of welfare at the individual level, but it crucially relies on the privateness of household consumption. This is a severe limitation because the existence of joint consumption, after all, is one of the main 'economic' justifications for the formation of a couple; see Becker (1991) for example.[2] Be that as it may, when consumption is completely or partially public, the possibility of identifying structural elements of the decision process (e.g., the personal value of public goods for each household member) has not often been tackled until now. This is the principal objective of our contribution.

In this paper, we suppose that agents are egoistic (or altruistic in a Beckerian sense) and follow the most common line of research on collective models. However, the great novelty, in contrast with previous work described above, is that public goods as well as private ones are now considered. We show that this setting allows us to recover some elements of the household decision process. More precisely, individual demands for private goods and individual prices (i.e., Lindahl prices) for public goods are partially identified. Complete identification requires additional assumptions on preferences, i.e., they must be such that public goods are separable from other goods. We also derive a set of very simple testable constraints (including a symmetry property). In principle, these constraints permit an empirical distinction between a private use and a public use of each category of goods.

The proofs of all the results in this paper are based on a second theoretical innovation, namely, a collective generalization of the marginal demands that were previously studied by Browning (1999) for the unitary approach.

[2] In empirical applications, the usual way of dealing with public consumption consists either in assuming the separability of public goods from private ones or in conditionning demands on the level of public goods. In both cases, the allocation of public goods in the household is simply ignored.

In this generalization, the quantities of private goods and the prices of public goods are modeled as functions of the prices of private goods, the quantities of public goods, and the quantities of two reference goods. We assume that, in these relationships, one reference good is exclusively consumed by the husband and the other by the wife. The idea is that the levels of these goods, if they are normal, represent a convenient indicator of the distribution of resources within the household. The advantages of this specification are twofold. First, it provides a particularly simple and intuitive way to describe the intra-household decision process. Second, as will become clearer below, the modeling of these within-period collective marginal demands is compatible with a life-cycle allocation rule and robust to some controversial assumptions often made for intertemporal allocations.[3]

The theoretical considerations are followed by empirical evidence using the U.S. Consumer Expenditure Survey. We show that the constraints of the collective model are globally satisfied but the separability of private and public goods is definitively rejected. This implies that the model cannot be completely identified. Even in that case, however, useful structural components of the model can be recovered.

As indicated above, identification results in the context of collective models with public consumption are rare.[4] Still, this question is, undoubtedly, one of the main topical ones in the research agenda on collective models. Several recent and unpublished manuscripts deal with it. Chiappori, Blundell and Meghir (2002) assume that private goods and leisure are separable from public ones and investigate the identification issue in a collective model of labor supply with private and public goods. However, our results are in some respects more general, since we also consider the case of public goods not being separable from private ones and assume that prices of goods may vary. Chiappori and Ekeland (2001a, 2001b) consider identification and testability in general collective models of consumption with public and private goods. These authors study the abstract characterization of household demands for groups of persons and do not consider the empirical implementation of these

[3]Mazzocco (2003) exploits the advantages of marginal demands to study the intertemporal allocation of consumption in a collective context. He also presents empirical evidence from the U.S. Consumer Expenditure Survey.

[4]Fong and Zhang (2001) consider a collective model of labor supply where, for each partner in a marriage, there are two distinct types of leisure: one type is each person's private (or independent) leisure and the other type is public (or spousal) leisure. This is an important, but very specific, contribution.

results. More generally, our work was conducted independently of theirs and our approach (e.g., the use of marginal demands and the underlying idea of the proofs) is simpler.

This paper is structured as follows. In Section 2, we present the main theoretical results. We specify the assumptions, define the collective marginal demands, and derive the testable properties they have to satisfy. In Section 3, these theoretical results are extended to demographic-dependent preferences. We also present further results on the identification of the model and discuss the relationships between marginal demands and more usual demands. In Section 4, the statistical model is specified. In Section 5, the data are described and the estimates are presented. In Section 6, concluding comments are given.

2 Theory—Basis

2.1 Preferences, Goods and the Decision Process

To begin, there are some goods in households that can reasonably be treated as private (e.g., alcoholic beverages) and other goods that clearly have a strong public element (e.g., heating). This distinction is a familiar one and does not require a detailed discussion. However, if there is a good that only one person in the household cares about, we prefer to categorize such a good separately as exclusive rather than public or private.[5] It will become clear below that the concept of exclusivity is essential in the theory which follows. An example of an exclusive good, which is exploited in the empirical part of this paper, is 'clothing'. If there are no externalities and if the husband consumes only men's clothing and the wife consumes only women's clothing, then we can think of men's and women's clothing as two exclusive goods. Another example in the context of labor supply is 'leisure'. Still, the absence of externalities is here more debatable.

The main objective of this paper is to analyze consumption of private and public goods in a unified framework. To do so, we consider a two-member household. There are two exclusive goods, one for each person in the household. We suppose that life-cycle utility is strongly intertemporally

[5] A pair of exclusive goods and one assignable good (i.e., a private good for which individual consumption can be observed) can be distinguished by the fact that the prices of two exclusive goods may independently vary.

separable and that the within-period preferences of member I $(I = A, B)$ can be represented by a well-behaved utility function:

$$u_I(x_I, \mathbf{q}_I, \mathbf{q}), \tag{1}$$

where x_I, $\mathbf{q}_I$ and $\mathbf{q}$ respectively denote an exclusive good, a K_1–vector of private goods consumed by member I, and a K_2–vector of public goods consumed by the household (with $K_1 + K_2 = K$). Several points must be stressed at this stage. First, the agents are 'egoistic', in the sense that their utility only depends on their own consumption, but all the results of this paper can be extended to the case of 'altruistic' agents in a Beckerian sense, where agents actually maximize some 'altruistic' index:

$$U_I\left[u_A(x_A, \mathbf{q}_A, \mathbf{q}), u_B(x_B, \mathbf{q}_B, \mathbf{q})\right], \tag{2}$$

where $U_I(\cdot)$ is a well-behaved utility function in u_A and u_B. Second, all goods, whether public or private, are assumed to be non-durable. Third, the individual demands for private goods $\mathbf{q}_I$ are treated as unobservable and the demand for these goods is only observed at the household level $(\mathbf{q}_A + \mathbf{q}_B)$. Conversely, the 'individual' demands for public and exclusive goods $\mathbf{q}$ and x_I are observable. The most important point, however, is that each good can be unambiguously designated as purely public or purely private. This excludes the possibility of 'impure' goods, like leisure in Fong and Zhang's (2001) view.[6]

We assume that there is no domestic production. The household faces a linear budget constraint and non-negativity restrictions. Thus, the within-period budget set is given by

$$\begin{gathered} y - a \geqslant \textstyle\sum_I x_I r_I + \sum_I \mathbf{q}_I' \mathbf{p} + \mathbf{q}'\mathbf{P}, \\ \mathbf{q} \geqslant 0, \quad \mathbf{q}_I \geqslant 0 \quad \text{and} \quad x_I \geqslant 0, \end{gathered} \tag{3}$$

where r_I, $\mathbf{p}$, and $\mathbf{P}$ respectively denote the price for the exclusive good, the K_1–vector of prices for private goods, and the K_2–vector of prices for public goods, y denotes household income, and a the within-period variation in household assets. The notation $\mathbf{P}$ (capitalized) for the price of public goods will become clearer in what follows.

[6] The notion of public goods in our model may, however, cover the case of externalities in consumption. Consequently, the marginal utility may be negative for some goods and one person in the household.

The originality of the efficiency approach lies in the fact that household decisions are assumed to result in Pareto-efficient outcomes and that no additional assumption is made about the decision process.[7] That means that there exists a scalar ϕ such that household behavior can be described as the solution to the following program:

$$\max_{\{x_A, x_B, \mathbf{q}_A, \mathbf{q}_B, \mathbf{q}\}} \phi \cdot u_A(x_A, \mathbf{q}_A, \mathbf{q}) + (1 - \phi) \cdot u_B(x_B, \mathbf{q}_B, \mathbf{q}), \tag{$\bar{\mathrm{P}}$}$$

subject to (3). The scalar ϕ can be interpreted as a 'distribution of power' index. It generally depends on all the exogenous variables that may affect the intra-household distribution of power:

$$\phi = \phi(r_A, r_B, y, \mathbf{p}, \mathbf{P}, \mathbf{s}), \tag{4}$$

where $\mathbf{s}$ is a vector of distribution factors.[8] By definition, these variables influence the decision process but do not affect the (within-period) budget constraint or (within-period) preferences. Specifically, the state of the market for marriage and the specific features of the marriage contract are expected to have an impact on the intra-household distribution of power, as is stressed by Becker (1991) and illustrated by Chiappori, Fortin and Lacroix (2002) with U.S. data. Moreover, the past and future values of y, r_A, r_B, $\mathbf{p}$ and $\mathbf{P}$, in a life-cycle context, can also influence the decision process. Using an Indonesian survey, Thomas, Contreras and Frankenberg (1997) have shown that the distribution of wealth by gender at the time of the marriage has a significant impact on household behavior (evaluated here by the children's health).

2.2 Collective Marginal Demands

In the unitary approach, the marginal demand ('m-demand') for a good is defined as a function of the level of another reference good rather than total expenditure or the marginal utility of money. This concept has often been exploited (either explicitly or implicitly) in life-cycle analysis of household behavior; see Altonji (1986) and Meghir and Weber (1996), for example. The theoretical properties of m-demands are studied by Browning (1999).

[7] The relevance of the efficiency hypothesis is discussed, among others, by Browning and Chiappori (1998).

[8] The function ϕ does not depend on a since, in general, a can itself be seen as a function of all the exogenous variables (including distribution factors).

We next show that the study of household behavior from a collective viewpoint is especially simple when a generalization of m-demands is used. We denote by λ the Lagrangian multiplier of $(\bar{\mathrm{P}})$ corresponding to the budget constraint, and we define $\phi_A = \phi$ and $\phi_B = 1 - \phi$. For an internal solution, the first-order conditions of the program $(\bar{\mathrm{P}})$ are then given by

$$\frac{\phi_I}{\lambda}\frac{\partial u_I}{\partial x_I} = r_I \qquad \text{and} \qquad \frac{\phi_I}{\lambda}\frac{\partial u_I}{\partial \mathbf{q}_I} = \mathbf{p} \tag{5}$$

for the allocation of exclusive and private goods, and

$$\textstyle\sum_I \frac{\phi_I}{\lambda}\frac{\partial u_I}{\partial \mathbf{q}} = \mathbf{P} \tag{6}$$

for the allocation of public goods. We can now define the system of collective marginal demands, in terms of quantity, for analyzing private consumption (cm-demands for private goods). To do that, we eliminate λ and ϕ_I in the first-order conditions (5) and obtain the allocation rule for private consumption:

$$\left.\frac{\partial u_I}{\partial \mathbf{q}_I}\right/ \frac{\partial u_I}{\partial x_I} = \frac{\mathbf{p}}{r_I}. \tag{7}$$

We suppose that this system of K_1 equations can be uniquely solved for $\mathbf{q}_I$ as a function of $r_I, x_I, \mathbf{p}$ and $\mathbf{q}$. We obtain the (individual) cm-demands for private goods:

$$\mathbf{q}_I = \mathbf{q}_I(r_I, x_I, \mathbf{p}, \mathbf{q}). \tag{8}$$

Since we generally do not observe individual consumption in surveys, we consider the (aggregate) cm-demands for private goods:

$$\mathbf{Q} = \textstyle\sum_I \mathbf{q}_I(r_I, x_I, \mathbf{p}, \mathbf{q}). \tag{9}$$

We can also define the system of collective marginal demands, in terms of prices, for analyzing public consumption (cm-demands for public goods). We eliminate λ and ϕ_I in the first-order condition (6) and obtain Samuelson's allocation rule for public consumption:

$$\mathbf{P} = \textstyle\sum_I r_I \left.\frac{\partial u_I}{\partial \mathbf{q}}\right/ \frac{\partial u_I}{\partial x_I}. \tag{10}$$

If we eliminate $\mathbf{q}_I$ in this expression, using (8), we obtain the (aggregate) cm-demands for public goods:

$$\mathbf{P} = \textstyle\sum_I \mathbf{p}_I(r_I, x_I, \mathbf{p}, \mathbf{q}), \tag{11}$$

where $\mathbf{p}_I$ corresponds to the (individual) cm-demands for public goods, or the Lindahl prices, i.e., the price at which each member values his/her public consumption.[9] Finally, using the following conventions:

$$\mathbf{d}_I' = (\mathbf{q}_I', -\mathbf{p}_I') \qquad \text{and} \qquad \mathbf{m}' = (\mathbf{p}', \mathbf{q}'),$$

we simply define the system of collective marginal demands as

$$\mathbf{D} = \textstyle\sum_I \mathbf{d}_I(r_I, x_I, \mathbf{m}),$$

where $\mathbf{D}' = (\mathbf{Q}', -\mathbf{P}')$. The variables on the left-hand side and the right-hand side are observable. These relations can be directly estimated with the usual techniques. Of course, in empirical work, we shall have to account for the probable endogeneity of x_I and $\mathbf{q}$.

A sufficient condition for the existence of the cm-demands for private goods, and consequently of the cm-demands for public goods, is the normality of the exclusive goods (conditional on the level of public goods). This is formally stated as follows.

Assumption A1 Let $x_I = \chi_I(r_I, \mathbf{m}, \rho_I)$ be the conditional Marshallian demand for the exclusive good x_I, where $\rho_I = r_I x_I + \mathbf{p}'\mathbf{q}_I$ is the level of expenditure on private and exclusive goods. Then, $\partial\chi_I/\partial\rho_I > 0$.

The underlying intuition is that the demand for exclusive goods, if (conditional) normality is assured, can be seen as a satisfactory indicator of the allocation of private goods at the individual level; see Browning (1999). For example, if a person in the household consumes a lot of his/her personal exclusive good, and if this good is normal, then we can expect that this person will consume a lot of private goods in general. In a certain sense, the pair of exclusive goods can be seen as 'sufficient statistics' for the level of private consumption and the distribution of this consumption within the household. Incidentally, the cm-approach is in line with the recent recognition that consumption may better reflect expected lifetime resources than current income. In addition, income reported in surveys may also be an insufficient indicator of material well-being because of misreporting, mismeasurement or (in-kind) transfers among extended families or friends; see Cutler and Katz (1992) and Slesnick (1993) for this argument.

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[9] Since public goods can be seen as an externality, Lindahl prices can be negative for one person (at most) in the household; see Myles (1997) for a discussion of Lindahl prices.

2.3 Testability and Identifiability

In what follows, we assume that Assumption **A1** is globally satisfied. Any cm-demand has, naturally, specific properties that can be used to check, *ex post*, the adequacy of the theory to observed behavior.

First of all, cm-demands have to be homogeneous. This is formally stated in the next Proposition.

Proposition 1 *Let us assume* **A1***. Then, under collective rationality,*

1. *The cm-demands for private goods* $\mathbf{Q}(r_A, r_B, x_A, x_B, \mathbf{p}, \mathbf{q})$ *are homogeneous of degree* 0 *in* r_A, r_B *and* $\mathbf{p}$ *;*

2. *The cm-demands for public goods* $\mathbf{P}(r_A, r_B, x_A, x_B, \mathbf{p}, \mathbf{q})$ *are homogeneous of degree* 1 *in* r_A, r_B *and* $\mathbf{p}$*.*

Proof. These properties directly result from the first order conditions (7) and (10). ■

One remarkable point here is that the homogeneity of cm-demands does not follow from the homogeneity of the distribution function ϕ. In other words, cm-demands will be homogeneous even if the bargaining power of household members is affected by money illusion.

The remaining restrictions are twofold. First, the additive structure of cm-demands can be translated into testable restrictions in the form of partial differential equations. This is what we call 'c-separability', where 'c' stands for 'consumption'. Second, household behavior is characterized by a symmetry property, as in Browning and Chiappori (1998) or Donni (2002), which results from the optimization problem. We formally introduce the following Proposition.

Proposition 2 *Let us assume* **A1***. Then, under collective rationality, the system of cm-demands* $\mathbf{D}(r_A, r_B, x_A, x_B, \mathbf{m})$ *satisfies the following:*

1. $\dfrac{\partial^2 \mathbf{D}}{\partial x_A \partial x_B} = \dfrac{\partial^2 \mathbf{D}}{\partial r_A \partial x_B} = \dfrac{\partial^2 \mathbf{D}}{\partial x_A \partial r_B} = \dfrac{\partial^2 \mathbf{D}}{\partial x_A \partial x_B} = \mathbf{0}$ *;*

2. $\left(\dfrac{\partial \mathbf{D}}{\partial \mathbf{m}'} + \sum_I \dfrac{\partial \mathbf{D}}{\partial x_I} \dfrac{\partial \mathbf{D}'}{\partial r_I} \right)$ *is a symmetric matrix.*

Proof. See Appendix A. ■

The first statement in this Proposition (c-separability) yields a particularly simple test of collective rationality under specific auxiliary assumptions, i.e., egoistic agents, absence of domestic production, and absence of impure goods. Specifically, it necessitates a verification that the four cross-terms in a second-order approximation of the cm-demands are equal to zero. This may be realized with single equation methods (or even non-parametric ones). The second statement is a translation of Slutsky symmetry into the cm-context. This condition generalizes in two directions a symmetry property previously derived by Browning (1999, Proposition 1) in the unitary framework: a) the household is characterized by two decision makers, and b) some demands are implicitly represented with their prices as dependent variables. Theoretically, cm-demands also have to satisfy a property of negativity, which is more complicated to derive.

One precision is necessary here. Consider the more natural specification of demands, where quantities are expressed as a function of prices and other variables. The previous Proposition implies, in particular, that the constraints imposed on the demands for private goods are different from those imposed on the demands for public goods.[10] This has two important consequences. First, it may be seriously misleading to analyze public consumption in a framework initially developed for private consumption. Second, discrimination between public and private uses of some categories of goods is theoretically possible.

The next important result of this section concerns the identification of structural elements of the decision process from the estimation of $\mathbf{Q}$ and/or $\mathbf{P}$. Then, we can put forward the next Proposition.

Proposition 3 *Let us assume* **A1**. *Then, under collective rationality,*

1. *The individual cm-demands for private goods* $\mathbf{q}_I(r_I, x_I, \mathbf{m})$ *can be retrieved up to an additive function* $\mathbf{g}_I(\mathbf{m})$ *;*

2. *The individual cm-demands for public goods* $\mathbf{p}_I(r_I, x_I, \mathbf{m})$ *can be retrieved up to an additive function* $\mathbf{h}_I(\mathbf{m})$ *;*

[10] It can be shown, to be more precise, that only c-separability has implications that permit an empirical distinction between a private use and a public use of goods.

3. *The functions* $\mathbf{g}_I(\mathbf{m})$, $\mathbf{h}_I(\mathbf{m})$ *and* $\mathbf{f}_I'(\mathbf{m}) = [\mathbf{g}_I'(\mathbf{m}), \mathbf{h}_I'(\mathbf{m})]$ *have to satisfy the following properties:*

$$\sum_I \mathbf{f}_I = \mathbf{0}, \quad \frac{\partial \mathbf{g}_I}{\partial \mathbf{p}'}\mathbf{p} = \mathbf{0}, \quad \frac{\partial \mathbf{h}_I}{\partial \mathbf{p}'}\mathbf{p} = \mathbf{h}_I, \quad \frac{\partial \mathbf{f}_I}{\partial \mathbf{m}'} = \frac{\partial \mathbf{f}_I'}{\partial \mathbf{m}}.$$

Proof. See Appendix A. ■

Even if the identification of the basic components of the model is not complete—the functions $\mathbf{g}_I(\mathbf{m})$ and $\mathbf{h}_I(\mathbf{m})$ remain undefined—this result proves attractive. It indicates that differences in tastes between the husband and the wife can be revealed by the estimation of cm-demands. In addition, more powerful conclusions can be obtained with mild additional assumptions. For example, let us consider the following statement.

Assumption A2 The level of the exclusive goods is an appropriate indicator of bargaining power, i.e., if ϕ increases in program $\bar{\mathrm{P}}$, then x_A increases and x_B decreases.

This assumption says that member A (say) will consume a greater quantity of his/her personal exclusive good when member A's bargaining power increases. This seems quite natural and does not merit discussion. We observe that **A1** and **A2** are equivalent when there are no public goods.

Corollary 4 *Let us assume* **A1** *and* **A2** *and consider a variation in bargaining power that keeps the total expenditure of the household constant. Then, the sign of the effect of such a variation on the demand for private and public goods is identified.*

Proof. See Appendix A. ■

This Corollary indicates, in particular, that the effect on the demand for public goods of a shift in bargaining power can be predicted.[11]

In conclusion, we may observe that the existence of a sharing rule is not explicitly postulated in the cm-demand context, unlike in the large majority

[11] The theory here does not say how a shift in bargaining power can be achieved, though, this point is now well documented in the literature. For example, it is clear that a public transfer to one person in the household should improve the situation of this person.

of papers on collective models. However, it is possible to define the expenditure on private and exclusive goods of each household member as a function of r_I, x_I $\mathbf{p}$ and $\mathbf{q}$ as follows:

$$\rho_I(r_I, x_I, \mathbf{p}, \mathbf{q}) = x_I r_I + \mathbf{p}'\mathbf{q}_I(r_I, x_I, \mathbf{p}, \mathbf{q}).$$

This is a generalization of the sharing rule to public goods. Its derivatives with respect to r_i and x_i can obviously be identified. In particular, we have:

$$\frac{\partial \rho_I}{\partial x_I} = r_I + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_I} \quad \text{and} \quad \frac{\partial \rho_I}{\partial r_I} = x_I + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial r_I}. \tag{12}$$

This result is reminiscent of previous results on the identification of the sharing rule with exclusive goods; see Browning et alii (1994) or Donni (2002), for instance. Moreover, from (12) and the following identity:

$$x_I \equiv \chi_I \left[r_I, \mathbf{m}, \rho_I(r_I, x_I, \mathbf{m}) \right],$$

the slope of the conditional Marshallian demand for exclusive goods can be readily obtained:

$$\frac{\partial \chi_I}{\partial \rho_I} = \left(r_I + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_I} \right)^{-1}, \tag{13}$$

$$\frac{\partial \chi_I}{\partial r_I} = -\left(x_I + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial r_I} \right) \left(r_I + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_I} \right)^{-1}. \tag{14}$$

The idea that exclusive goods are normal, as demanded by Assumption **A1**, can thus be assessed with data.

3 Theory—Extensions

3.1 Preference Factors

The present model can be generalized in several ways. To begin with, the preferences of each agent generally depend on a set of socio-demographic characteristics. Therefore, we may assume:

$$u_I(x_I, \mathbf{q}_I, \mathbf{q}; \mathbf{z}_I, \mathbf{z}), \tag{15}$$

where $\mathbf{z}_I$ and $\mathbf{z}$ called 'preference factors'. We must make an important, if a little artificial, distinction between factors such as $\mathbf{z}_I = (z_I^{j_I})$ which seem to be related to a specific individual in the household (such as age, race, or level of education) and factors such as $\mathbf{z} = (z^j)$ which are common to both agents (such as the number and age of the children, the state/country of residence).

The next step is to define, as previously, the cm-demands 'extended' to preference factors. We follow the same approach as in the preceding section and obtain:

$$\mathbf{D} = \sum_I \mathbf{d}_I(r_I, x_I, \mathbf{p}, \mathbf{q}; \mathbf{z}_I, \mathbf{z}).$$

The distinction between common and specific preference factors naturally generates further testable restrictions. This is what we call 'p-separability', where 'p' stands for 'preference'. This is formally expressed in the following Proposition.

Proposition 5 *Let us assume* **A1***. Then, under collective rationality, the 'extended' cm-demands* $\mathbf{D}(r_A, r_B, x_A, x_B, \mathbf{m}; \mathbf{z}_A, \mathbf{z}_B, \mathbf{z})$ *satisfy the following:*

$$\frac{\partial^2 \mathbf{D}}{\partial z_A^{j_A} \partial r_B} = \frac{\partial^2 \mathbf{D}}{\partial z_A^{j_A} \partial x_B} = \frac{\partial^2 \mathbf{D}}{\partial z_A^{j_A} \partial z_B^{j_B}} = \frac{\partial^2 \mathbf{D}}{\partial r_A \partial z_B^{j_B}} = \frac{\partial^2 \mathbf{D}}{\partial x_A \partial z_B^{j_B}} = \mathbf{0},$$

for any j_A *and* j_B.

Proof. It suffices to use the definition of the utility functions (15) and derive the cm-demands, like before. ■

This Proposition provides a very simple test of the collective approach with egoistic agents, provided that a clear distinction between specific and common preference factors exists. This result is very useful in empirical applications because it is one of the very few results in the literature that restricts the influence of preference factors on household behavior. As such, it provides a simple way of limiting the number of parameters in the functional form. In addition, it is easy to show that the effect of the specific preference factors on the individual cm-demands can be identified as well.

3.2 Identification: Further Results

It has previously been shown that the identification of the individual cm-demands is incomplete: The function $\mathbf{f}_I(\mathbf{p}, \mathbf{q})$ remains undetermined (for convenience the preference factors are disregarded in the remainder of this section). A solution to this problem necessitates the addition of a structure on preferences. Fong and Zhang (2001) and Chiappori, Blundell and Meghir

(2002) deal with this issue in a labor supply context. Basically, they assume that preferences are such that public goods are separable from other goods. Similarly, we assume here:

$$u_I\left[\mu_I(x_I, \mathbf{q}_I), \mathbf{q}\right]. \tag{16}$$

This assumption is certainly restrictive. Nevertheless, it should be emphasized that such separability has never been tested in the literature until now. The available tests refer to household level separability, which is definitely not implied by our framework.

If the utility functions are of the form (16), the first-order conditions for the allocation of private goods become:

$$\frac{\partial \mu_I}{\partial \mathbf{q}_I} \Big/ \frac{\partial \mu_I}{\partial x_I} = \frac{\mathbf{p}}{r_I}. \tag{17}$$

If these equations are solved as previously, we obtain the cm-demands for private goods:

$$\mathbf{Q} = \textstyle\sum_I \mathbf{q}_I(r_I, x_I, \mathbf{p}).$$

The remarkable point is that, under the assumption of separability, these equations do not depend on the level of public consumption. This is sufficient to identify some important elements of the decision process. We formally have the following Proposition.

Proposition 6 *Let us assume* **A1**. *The preferences have the form represented by (16). Then, under collective rationality,*

1. *The individual cm-demands for private goods* $\mathbf{q}_I(r_I, x_I, \mathbf{p}, \mathbf{q})$ *can be retrieved up to an additive function* $\mathbf{g}_I(\mathbf{p})$ *;*

2. *The individual cm-demands for public goods* $\mathbf{p}_I(r_I, x_I, \mathbf{p}, \mathbf{q})$ *can be exactly retrieved; in particular,*

$$\mathbf{p}_I = \left(\frac{\partial \mathbf{P}}{\partial r_I}\frac{\partial \mathbf{Q}'}{\partial x_I} - \frac{\partial \mathbf{P}}{\partial x_I}\frac{\partial \mathbf{Q}'}{\partial r_I}\right)\mathbf{p} + \frac{\partial \mathbf{P}}{\partial r_I} r_I\,;$$

3. *The separability of preferences generates further testable restrictions on cm-demands.*

Proof. See Appendix A. ■

The fact that the function $\mathbf{g}_I(\mathbf{p})$ remains undetermined is not of great concern here. The important point for our purpose is that the individual demands for private goods can be derived when prices are constant (e.g., over a short period of time) and the valuation of public goods by household members is exactly identified. Moreover, some elements of the individual preferences can probably be retrieved too, but the proof of this is beyond our scope here.

One trivial consequence of this result is that the structural components of the model are completely identified if consumption is purely public ($K_1 = 0$). The underlying intuition is straightforward if we recall that individual prices are linearly homogeneous too. Then,

$$\mathbf{p}_I = r_I \cdot \frac{\partial \mathbf{P}}{\partial r_I},$$

i.e., the individual cm-demands $\mathbf{p}_I(r_I, x_I, \mathbf{q})$ are exactly identified. Chiappori and Ekeland (2001b) present another version of this result.

3.3 Relationship with 'Traditional' Collective Models

Generally, the collective models that are used in empirical applications are unconditional in the sense that each demand is represented as a function of income, prices, distribution and preference factors (they are eventually conditional on the level of public goods). However, the properties of the cm-demands that we develop above can be expressed in terms of these more traditional collective demands. To do that, we have to specify the form of the demands for the exclusive goods.

Typically, the demands for the exclusive goods depend on all the variables in the budget constraint and the bargaining function:

$$x_I = x_I^*(r_A, r_B, y, \mathbf{m}, \mathbf{s}). \tag{18}$$

These relations have theoretical properties which are studied in depth in Chiappori, Fortin and Lacroix (2002). In particular, it can be shown that there exists a sharing rule, ρ_I, such that the demands can be written as follows:

$$x_I = \chi_I \left[r_I, \mathbf{m}, \rho_I(r_A, r_B, y, \mathbf{m}, \mathbf{s}) \right],$$

with $\sum_I \rho_I = y - a$. This specification generates a set of testable restrictions and allows us to identify some elements of the decision process. One underlying difficulty in the estimation task here is that the demands for the exclusive goods may be functions of numerous variables that are not necessarily observed by the economist (e.g., past or future values of y).

Be that as it may, if these demands were estimated, they could be introduced into the individual cm-demands to give the more traditional collective demands. That is:

$$\mathbf{D} = \sum_I \boldsymbol{\delta}_I \left[r_I, \mathbf{m}, \rho_I(r_A, r_B, y, \mathbf{m}, \mathbf{s}) \right], \tag{19}$$

$$= \sum_I \mathbf{d}_I^*(r_A, r_B, y, \mathbf{m}, \mathbf{s}). \tag{20}$$

Similar expressions can be found in Bourguignon, Browning and Chiappori (1995), for instance. However, the originality of the theory here is that the structure of demands for public goods is also specified. Moreover, using Proposition 3 and Relation (18), it is easy to compute the effect of most exogenous variables on the individual traditional demands. For example, differentiating $\mathbf{d}_I^*$ with respect to y or $\mathbf{s}$ yields:

$$\frac{\partial \mathbf{d}_I^*}{\partial y} = \frac{\partial \mathbf{d}_I}{\partial x_I} \frac{\partial x_I^*}{\partial y} \qquad \text{and} \qquad \frac{\partial \mathbf{d}_I^*}{\partial \mathbf{s}'} = \frac{\partial \mathbf{d}_I}{\partial x_I} \frac{\partial x_I^*}{\partial \mathbf{s}'}, \tag{21}$$

where the right-hand side derivatives are known. This relation allows us to measure the impact of a change in y or $\mathbf{s}$ on $\mathbf{d}_I$.

Several points must be stressed at this stage. First, the constraints given in Proposition 3 necessarily have a transposition in the form of restrictions on the partial derivatives of the demands (20). Still, these restrictions, in particular the symmetry restriction, may be extraordinarily complicated. Second, it can be shown that the demands in (20) must satisfy additional constraints. For example, using (21) and assuming $\partial x_I / \partial y \neq 0$, we obtain:

$$\frac{\partial \mathbf{d}_I^*}{\partial y} = \theta_j \cdot \frac{\partial \mathbf{d}_I^*}{\partial s^j}, \tag{22}$$

where $\theta_j = \left(\partial x_I / \partial s^j \right) / (\partial x_I / \partial y)$ is a scalar and s^j is a typical element of $\mathbf{s}$. The underlying intuition is clear: y and $\mathbf{s}$ influence the cm-demands only through a variation in the level of the exclusive goods. This 'distribution' property, in Browning and Chiappori's (1998) terminology, will be implicitly used for the construction of the instruments in the empirical part of this paper. Third, in the literature on collective models, there exists another concept of demands. In y-demands (see Bourguignon, Browning and Chiappori (1995), Donni (2002) and Donni and Moreau (2003)), the quantity of goods is a function of various exogenous variables, total income, and one exclusive good. The properties of these demands can also be related to the cm-framework.

4 Statistical Specification

In this section, we use a parametrization of the cm-demand system to derive the implications of the collective setting. We follow Browning (1999) for the individual cm-demands and model relative expenditures, i.e., the ratio of expenditures on the good to be modeled to expenditure on the conditioning good. The structure of individual demands is as follows:

$$\frac{\mathbf{m} \odot \mathbf{d}_I}{r_I x_I} = \boldsymbol{\Delta}_I + \frac{\mathbf{m} \odot \mathbf{f}_I}{r_I x_I},$$

where $\odot$ stands for the Hadamard product, i.e., the element-by-element product, and the function $\mathbf{f}_I(\mathbf{m}, \mathbf{z})$ is not identifiable, as previously explained. Since $\mathbf{d}_I$ is not observed, the following transformation from individual to household demands is used:

$$\frac{\mathbf{m} \odot \mathbf{D}}{\sum_I r_I x_I} = \frac{\sum_I r_I x_I \boldsymbol{\Delta}_I}{\sum_I r_I x_I}. \tag{23}$$

The role of the econometrician is then to select a functional form for the component $\boldsymbol{\Delta}_I$ of the cm-demands that can be identified. After considerable experimentation, we adopted a variation of the AI Demand System. The functional form is the following:

$$\boldsymbol{\Delta}_I = \mathbf{a}_I^*(\mathbf{z}_I, \mathbf{z}_J, \mathbf{z}) + \mathbf{B} \ln \mathbf{m} + \textstyle\sum_{n=1}^N \mathbf{c}_n (\ln r_I x_I - \ln \Phi_n)^n + \mathbf{d} \ln r_J x_J, \tag{24}$$

with $J \neq I$, where $\mathbf{a}, \mathbf{B}, \mathbf{c}_n$ and $\mathbf{d}$ are conformable vectors and matrices of parameters[12] and Φ_n are functions defined as follows:

$$\ln \Phi_1 = 0 \qquad \text{and} \qquad \ln \Phi_n = \mathbf{c}_n' \ln \mathbf{m} \text{ if } n > 1.$$

[12] A positive translation must be applied to x_I and $\mathbf{q}$ for the computation of the logarithms in the functional form. The reason is that, for some households, the observed expenditure on these goods is simply equal to zero during the period of observation.

One advantage of this form is that the flexibility of the model can be arbitrarily increased by choosing a large N. However, it is advisable to be cautious and perform some simple empirical tests, because the levels and prices of the exclusive goods enter the functional form only through expenditures on them. Moreover, the individual demands are assumed, for the sake of parsimony, to have the same parameters (except for the intercepts, as is quite usual in econometrics). This assumption must also be carefully examined.

The next step is to make some allowance for observable heterogeneity and derive the theoretical constraints. To do this, we choose a functional form for the index $\mathbf{a}_I^*$ and allow for the fact that different people will have different tastes. A linear specification is adopted:

$$\mathbf{a}_I^*(\mathbf{z}_I, \mathbf{z}_J, \mathbf{z}) = \mathbf{a}_I + \mathbf{a}_1\mathbf{z}_I + \mathbf{a}_2\mathbf{z}_J + \mathbf{a}_3\mathbf{z}, \tag{25}$$

where $\mathbf{a}_I, \mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$ are conformable vectors and matrices of parameters. The parameters of the functional forms (24) and (25) then have to satisfy several constraints for the model to be consistent with the theory. First, the vectors and matrices of parameters are partitioned as follows:

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}^{11} & \mathbf{B}^{12} \\ \mathbf{B}^{21} & \mathbf{B}^{22} \end{pmatrix}, \quad \mathbf{c}_n = \begin{pmatrix} \mathbf{c}_n^1 \\ \mathbf{c}_n^2 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} \mathbf{d}^1 \\ \mathbf{d}^2 \end{pmatrix},$$

where the dimension of $\mathbf{B}^{11}$ is $K_1 \times K_1$, the dimension of $\mathbf{B}^{12}$ is $K_1 \times K_2$ and so on. The constraints are summarized in Table 1. Most of them are intuitive. The derivation of the symmetry restriction is tedious but not difficult. The separability of public goods only implies that the cm-demands for private goods do not depend on the quantity of public goods. Other implications of this separability are automatically satisfied.

To complete this specification, we also have to introduce unobservable heterogeneity. The most satisfactory treatment would be to develop a fully stochastic model, but we adopt the more conventional approach of simply adding error terms to the aggregate demands:

$$\frac{\mathbf{m} \odot \mathbf{D}}{\sum_I r_I x_I} = \frac{\sum_I r_I x_I \boldsymbol{\Delta}_I}{\sum_I r_I x_I} + \boldsymbol{\varepsilon},$$

where $\boldsymbol{\varepsilon}$ is a stochastic term. Potential sources of such heterogeneity in the cm-context comes only from preferences and measurement/optimization

Table 1: List of constraints

Name of constraints	Parameter restrictions
Homogeneity	$\mathbf{B}^{11}\boldsymbol{\iota}+\mathbf{c}_1^1+\mathbf{d}^1=\mathbf{0}$,
	$\mathbf{B}^{21}\boldsymbol{\iota}+\mathbf{c}_1^2+\mathbf{d}^2=\mathbf{0}$,
	$\mathbf{c}_n^1\boldsymbol{\iota}=\boldsymbol{\iota}, n=2,\ldots,N$,
	with $\boldsymbol{\iota}=(1,\ldots,1)'$
C-separability	$\mathbf{d}=\mathbf{0}$
Symmetry	$\mathbf{B}=\mathbf{B}'$
P-separability	$\mathbf{a}_2=\mathbf{0}$
Separability of public goods	$\mathbf{B}^{12}=\mathbf{0}$,
	$\mathbf{c}_n^2=\mathbf{0}, n=1,\ldots,N$

errors.[13] This implies the probable endogeneity of the level of exclusive and public goods, i.e.,

$$E(\boldsymbol{\varepsilon}|x_A, x_B, \mathbf{q}) \neq 0.$$

The basic reason for this endogeneity is that the level of all goods are variables of choice and are simultaneously determined by unobservable members' tastes. To make things clearer, let us consider the example of 'clothing' and 'personal care services'. If some persons take particular care of their physical appearance and others do not, then the error terms on 'clothing' and 'personal care services' are expected to be positively correlated. There is another important cause of endogeneity, though. In surveys, observations on expenditure are generally contaminated by the infrequency of purchases. These 'errors in variables' create another form of endogeneity.

However that may be, there is a natural assumption which permits the econometric identification of the model, i.e.,

$$E(\boldsymbol{\varepsilon}|r_A, r_B, y, \mathbf{p}, \mathbf{P}, \mathbf{s}, \mathbf{z}_A, \mathbf{z}_B, \mathbf{z}) = 0. \tag{26}$$

One point must be stressed here. The natural instruments for the level of exclusive goods are income and distribution factors, since the theory states that, conditional on the level of the exclusive goods, income and distribution factors should not affect the cm-demands. The exclusion of these variables

[13] Contrary to more usual collective demands, the heterogeneity related to the decision process is directly summarized here by the level of the exclusive goods and does not enter the error terms.

from the cm-demands is, therefore, a prediction that can also be tested. This restriction corresponds to (22) in the more traditional context.

5 Data and Results

5.1 The Consumer Expenditure Survey

The data are drawn from the 'Interview' component of the 'Consumer Expenditure Survey'. This survey, which has been extensively used since the early eighties (see Nelson (1988), or Meghir and Weber (1996), for example), contains global estimates of total spending on food at home and other items for the three-month period immediately preceding the interview. It is a rotating panel. Households are interviewed five times, but the first quarter of expenditure data are used only for bounding purposes.

The complete sample includes about 100,000 households from 1980 to 1999. The data used in this study come from each of the overlapping 12-month periods January 1980–December 1980, February 1980–January 1981, and so on through the 12 month period April 1998–March 1999. We then select a subsample of married couples without children, since children are expected to increase problems related to domestic production. We also restrict the sample to couples in which the husband and wife both work full-time (whose yearly labor supply falls between 1500 and 3000 hours) and who are less than 65 years old. These selection rules and the exclusion of missing data[14] leave us with a total of 2,604 cases for the empirical analysis. The construction of the sample is more precisely described in Appendix B.

We follow Browning et alii (1994) and suppose that 'men's clothing' (MCLO) and 'women's clothing' (WCLO) are exclusive goods. One notable (and especially important in the cm-context) feature of the Consumer Expenditure Survey is that, since expenditures are recorded over the year, there are far fewer zeros for goods such as clothing than one finds in surveys based on short diaries. We then model the demand for four private goods: 'food and beverages at home' (FDAH), 'food and beverages away from home' (REST), 'alcoholic beverages and tobacco' (VICE), 'personal care services' (CARE), and the demand for two public goods: 'oil fuel and utility natural gas ser-

[14] The most important source of incomplete observations is attrition in the panel and the fact that the region of residence is not recorded for households living in rural areas. These observations are simply removed from our sample.

vice' (FUEL) and 'electricity, water and sewer and trash collection services' (UTIL).[15] We assume that the preferences for these goods are separable from those for all other goods (allowing the exception of labor force status since we select full-time working members).

Prices ('Consumer Price Index for all urban consumers') are taken from the Bureau of Labor Statistics. The price of composite goods is created as the weighted geometric mean of the component prices with budget shares averaged across the sample for weights. Moreover, since the series are recorded over twenty years at the region level (North, South, West and Midwest), the variation in prices in our data is appreciable; see Appendix B.[16]

5.2 Parameter Estimates and Test Statistics

Before presenting estimates of the parameters, we have to address some econometric issues. The equations are estimated using conventional iterated GMM techniques to allow for the probable endogeneity of the conditioning goods. This estimation method is also consistent with heteroscedasticity of unknown form in the errors. The econometric tests are performed with LR-type tests. The corresponding statistics are computed as the difference between the J-statistics computed for the constrained and the unconstrained model; see Newey and West (1987).[17]

For the most general specification, and after numerous tests, we finally retained fifteen factors of preference in the $\mathbf{a}_I^*$ index. We take the age, the square of the age, the years of education for each spouse, and dummies for black households, Hispanic households[18] and home owner households. We

[15] Such a classification—even if it is always debatable in the end—seeks a broad consensus. Therefore, the classification of goods such as 'entertainment' or 'transportation' as public or private seems definitively too conjectural. A more ambitious line of attack consists in letting data determine the best classification of goods. This is, however, beyond the scope of this paper.

[16] We use prices recorded at the country level for CARE (and for MCLO and WCLO and components of VICE, FUEL, UTIL after January, 1998) since the regional information is not available.

[17] Following the common practice (see Hayashi, 1998), we computed the J-statistics with the weighting matrix of the unconstrained model. We ascertained, however, that the tests are robust to the choice of other consistent weighting matrices.

[18] In principle, in cm-demands these variables can be considered specific factors of preferences. However, it is made difficult because there is a strong collinearity between the characteristics of the partners. That is, mixed marriages are rare.

also include the logarithm of time and its square to allow for possible shifts in taste, and four dummies for the region of residence.[19] As a preliminary, we decided that the optimal number of terms in expenditure, i.e., the value of N, must be equal to 2. We will return to that later. Thus, we have twenty-five (free) parameters per equation for the most general model.

The usual technique for choosing instruments in GMM consists in selecting the order of a polynomial for the exogenous variables. The set of instruments in our application includes the logarithms of the prices of the public and private goods, a second-order polynomial in the logarithms of the prices of the exclusive goods, the socio-demographic variables used in the estimation process, a third-order polynomial in the logarithm of total income (composed of spouses' earnings and non-labor incomes), a second-order polynomial in the logarithms of the wife's and husband's earnings, and dummies for negative wife's and husband's earnings.[20] Spouses' earnings can be seen here as distribution factors. In all, we have thirty-six instruments per equation. This gives eleven overidentifying restrictions per equation and a total of sixty-six degrees of freedom for the six-good system. The explanatory power and the possible endogeneity of these instruments have been extensively investigated, but we have not found any evidence of non-orthogonality or weakness.

To begin, we present the description of the different models we estimated and the corresponding set of J-statistics in Table 2. These models are nested and the different constraints are successively tested. The test of overidentifying restrictions for the unconstrained model indicates that the set of instruments, as a whole, is not rejected by the data at the usual levels.[21] This means, in particular, that household demands, when conditioned on the level of two exclusive goods, are not influenced by the various incomes of the household. Moreover, the different sets of collective restrictions (homogeneity, c-separability, symmetry, p-separability) are not rejected at any

[19] To avoid collinearity between these variables and the individual constants, we assume that the sum of the parameters for the region dummies is equal to zero.

[20] The fact is that incomes may include loss from business and, therefore, may be negative. Thus, the logarithm of incomes is not computable for all observations, and the inverse hyperbolic sine of earnings and total income is actually used. This transformation is approximately logarithmic for high values of incomes and linear for values close to zero.

[21] The power of the overidentification test can be limited if the number of overidentifying restrictions is high. However, the computation of J-statistics with subsets of instruments confirmed the hypothesis of exogeneity.

Table 2: J-stats and tests of constraints

The models and their characteristics	J-stats	D.F.	p-value
Model I (unconstrained)	74.30	66	0.23
Model II (Model I + homogeneity)	82.16	73	0.35
Model III (Model II + c-separability)	87.50	79	0.50
Model IV (Model III + symmetry)	104.54	94	0.32
Model V (Model IV + p-separability)	122.72	112	0.44
Model VI (Model V + separability of public consumption)	369.69	122	0.00

Notes: The first row of the last column indicates the p-value for the test of overidentification. The other rows indicate the p-value for the test of one model against the preceding one (e.g., the second row gives the p-value for the test of homogeneity and so on).

conventional level. Thus, the data are consistent with the collective setting.[22] However, the idea that preferences could be separable in public and private goods is much more unlikely. This assumption is definitively rejected and, of course, this casts doubts on the possibility of identifying individual cm-demands. Incidentally, the numerous empirical studies that are implicitly based on this assumption of separability could be seriously misleading.[23]

We now turn to the parameter estimates of the various models. To save on space, we focus more particularly on two specifications. We first present the estimates of the parameters of Model II in Table 3.[24] This table provides the estimates of the parameters of the functional form of individual demands when homogeneity is imposed. Except for the constant, these parameters are the same for both household members. The number of coefficients which are significant at the 5% (10%) level is equal to 31 (35) out of 150. Specifically, we must make two remarks here. First, the estimates of $\mathbf{d}$, in contrast with those of $\mathbf{c}_1$ and $\mathbf{c}_2$, are not significant at the 10% level. That confirms the formal test presented in Table 2. Second, the estimates of $\mathbf{B}$ are not very

[22] We also tested c-separability in a more general model with a squared term in the expenditure of the partner. The conclusions are not altered.

[23] To the best of our knowledge, we here present the first empirical test—and the first rejection at the same time—of this type of separability at the household level.

[24] For the sake of efficiency, the estimation procedure for this model is based on the weighting matrix computed with the parameter estimates of Model V. As indicated in Table 2, this is the most constrained model not rejected by the data.

significant. Still, a careful examination of these estimates and their standard deviations reveals that the price-and-quantity effects are, as required by the theory, approximately symmetrical.

We next consider the estimates for Model V. For this model, a greater proportion of parameters are precisely estimated: 39 (49) coefficients out of 132 are significant at the 5% (10%) level. In particular, the majority of the estimates of $\mathbf{c}_1$ and $\mathbf{c}_2$ are very significant (with t-tests higher than 2 in general). Still, the estimates of $\mathbf{B}$ remain imprecise: only 9 coefficients out of 36 are significant at the 10% level. This lack of precision can, of course, be explained by the strong collinearity in prices. Furthermore, the dependent variable here is the ratio of expenditures on one private/public good to expenditures on both exclusive goods. Also, the estimates of the parameters will not be very significant if the expenditures in the numerator and the denominator are strongly correlated. This may explain, in particular, why the estimates are generally not significant for the CARE equation.

As to the socio-demographic variables, we observe that racial and ethnic minorities have similar patterns of consumption for the REST and VICE equations. These patterns are more different for the CARE and UTIL equations. We also point out that more educated people spend relatively more on CARE and REST than on MCLO and WCLO and, quite surprisingly, home owners spend relatively less on FUEL. Finally, age is an important variable to explain the demands for FDAH, REST or VICE and the region dummies are, in general, not very significant.

Before turning to a more detailed investigation of these estimates, several empirical tests were conducted to check the adequacy of the present specification. Specifically, a third-order term in $(\ln r_I x_I - \Phi_n)$ was introduced into the functional form. However, the resulting decrease in the J-statistics turned out to be quite small—going from 122.72 to 119.21—and the present specification is not rejected (the χ^2- statistic for this test is equal to 3.51 with a p-value = 0.74). At the same time, it confirms our preliminary intuition that N must be equal to 2.

Other tests were performed. We checked whether the effect of the level of the exclusive goods and the effect of prices are distinct. We also examined the assumption that, except for constants, individual demands are the same for both members. All in all, the functional form that we adopted seems to conveniently fit the data.

Table 3: Estimates of the parameters for Model II

Parameters and labels	Dependent variables ($\mathbf{m} \odot \mathbf{D} / \Sigma_I r_I x_I$)					
	FDAH	REST	VICE	CARE	FUEL	UTIL
a_A: intercept (men)	8.77	0.10	1.14	1.19	-0.43	-1.51
	(2.96)	(3.05)	(1.53)	(0.75)	(0.55)	(1.06)
a_B: intercept (women)	8.64	-5.12	2.83	-0.66	0.11	-0.50
	(1.85)	(1.70)	(0.99)	(0.48)	(0.36)	(0.74)
a_1: North	-0.30	-0.05	-0.13	-0.04	0.01	-0.01
	(0.28)	(0.32)	(0.17)	(0.08)	(0.07)	(0.09)
a_1: Midwest	-0.25	-0.01	-0.05	0.01	0.10	0.05
	(0.16)	(0.17)	(0.10)	(0.04)	(0.03)	(0.05)
a_1: West	0.41	0.00	0.16	0.01	-0.12	0.04
	(0.31)	(0.33)	(0.19)	(0.09)	(0.07)	(0.10)
a_1: South	0.13	0.06	0.02	0.01	0.012	-0.08
	(0.17)	(0.18)	(0.09)	(0.04)	(0.03)	(0.06)
a_1: log. of a trend	-0.79	-0.43	-0.43	0.02	-0.25	-0.23
	0.39	(0.40)	(0.24)	(0.10)	(0.09)	(0.11)
a_1: log. of a trend exp2	0.32	0.16	0.15	0.01	0.10	0.09
	0.13	(0.14)	(0.08)	(0.04)	(0.03)	(0.04)
a_1: black	-0.28	-0.76	-0.32	0.17	0.03	-0.13
	0.20	(0.19)	(0.10)	(0.06)	(0.04)	(0.08)
a_1: Spanish-speaking	-0.03	-0.39	-0.35	-0.03	-0.01	0.05
	0.06	(0.22)	(0.11)	(0.06)	(0.06)	(0.07)
a_1: home owner	0.03	-0.26	0.06	0.09	0.18	0.06
	0.06	(0.63)	(0.31)	(0.16)	(0.11)	(0.20)
a_2: education in decades	0.29	-0.47	-0.78	-0.11	0.07	0.09
	(0.59)	(0.58)	(0.51)	(0.13)	(0.10)	(0.19)
a_2: age	-0.07	-0.10	-0.09	-0.06	0.13	-0.09
	(0.25)	(0.25)	(0.14)	(0.07)	(0.05)	(0.07)
a_2: age in decades exp2	0.15	0.11	0.09	0.07	-0.15	0.11
	(0.30)	(0.30)	(0.17)	(0.08)	(0.06)	(0.09)
a_3: education in decades of the partner	-0.35	0.53	0.77	0.14	-0.08	-0.12
	(0.54)	(0.57)	(0.50)	(0.13)	(0.10)	(0.18)
a_3: age of the partner	0.23	0.18	0.19	0.06	-0.13	0.09
	(0.26)	(0.26)	(0.15)	(0.07)	(0.05)	(0.08)
a_3: age in decades of the partner exp2	-0.29	-0.19	-0.20	-0.06	0.16	-0.10
	(0.30)	(0.03)	(0.17)	(0.08)	(0.06)	(0.09)
B: price of FDAH	10.70	-3.57	2.05	0.03	0.25	-0.72
	(3.78)	(3.21)	(1.64)	(0.77)	(0.64)	(1.06)
B: price of REST	-2.38	0.15	-0.94	0.50	-0.12	-0.83
	(3.26)	(3.15)	(1.85)	(0.76)	(0.74)	(0.86)
B: price of VICE	0.47	-1.56	-1.13	-0.28	0.02	-0.63
	(1.63)	(1.56)	(0.91)	(0.37)	(0.28)	(0.49)
B: price of CARE	-1.79	2.03	2.27	-0.39	-0.61	0.10
	(2.97)	(3.35)	(2.03)	(0.82)	(0.91)	(0.80)
B: quantity of FUEL	0.32	0.06	0.17	-0.00	-0.72	0.02
	(0.65)	(0.64)	(0.35)	(0.15)	(0.13)	(0.19)
B: quantity of UTIL	-2.49	0.62	-0.96	-0.11	0.01	-1.03
	(0.99)	(0.90)	(0.49)	(0.25)	(0.20)	(0.37)
c_1: expenditure on clothing	-5.30	3.24	-1.41	0.15	0.14	2.42
	(1.68)	(1.56)	(1.05)	(0.42)	(0.35)	(0.61)
c_2: expenditure on clothing exp2	1.23	-0.65	0.44	-0.03	0.02	-0.24
	(0.28)	(0.25)	(0.18)	(0.07)	(0.07)	(0.09)
d: expenditure on clothing of the partner	-1.71	-0.29	-0.84	-0.00	0.33	-0.34
	(1.44)	(1.44)	(0.83)	(0.36)	(0.29)	(0.41)

Notes: Standard deviations are in parentheses

Table 4: Estimates of the parameters for Model V

Parameters and labels	Dependent variables ($\mathbf{m} \odot \mathbf{D} / \Sigma_I r_I x_I$)					
	FDAH	REST	VICE	CARE	FUEL	UTIL
a_A: intercept (men)	6.43	2.25	1.18	1.71	-0.36	-1.96
	(2.17)	(1.84)	(1.03)	(0.54)	(0.40)	(0.88)
a_B: intercept (women)	9.91	-5.01	2.40	-0.58	0.02	-0.66
	(1.65)	(1.54)	(0.86)	(0.47)	(0.35)	(0.70)
a_1: North	-0.02	0.02	0.06	-0.01	-0.01	0.03
	(0.13)	(0.12)	(0.08)	(0.04)	(0.04)	(0.04)
a_1: Midwest	-0.22	0.08	0.07	0.03	0.075	0.07
	(0.10)	(0.09)	(0.06)	(0.03)	(0.03)	(0.03)
a_1: West	0.10	-0.04	-0.12	0.00	-0.08	-0.05
	(0.16)	(0.14)	(0.10)	(0.05)	(0.05)	(0.06)
a_1: South	0.14	-0.06	-0.01	-0.01	0.02	-0.05
	(0.10)	(0.10)	(0.05)	(0.03)	(0.03)	(0.04)
a_1: log. of a trend	-0.62	-0.36	-0.22	0.03	-0.28	-0.29
	(0.32)	(0.32)	(0.20)	(0.09)	(0.08)	(0.10)
a_1: log. of a trend exp2	0.18	0.11	0.03	0.00	0.11	0.09
	(0.10)	(0.10)	(0.06)	(0.03)	(0.02)	(0.03)
a_1: black	-0.02	-0.70	-0.27	0.17	0.03	-0.07
	(0.19)	(0.15)	(0.08)	(0.06)	(0.04)	(0.07)
a_1: Spanish-speaking	-0.10	-0.44	-0.36	-0.04	-0.03	0.07
	(0.17)	(0.19)	(0.10)	(0.05)	(0.05)	(0.06)
a_1: home owner	0.09	0.23	0.05	0.17	0.15	-0.01
	(0.33)	(0.29)	(0.18)	(0.11)	(0.08)	(0.16)
a_2: education in decades	-0.04	0.09	0.02	0.03	-0.02	-0.03
	(0.04)	(0.05)	(0.03)	(0.01)	(0.01)	(0.01)
a_2: age	0.14	0.08	0.08	0.01	-0.01	0.00
	(0.03)	(0.03)	(0.02)	(0.01)	(0.01)	(0.01)
a_2: age in decades exp2	-0.12	-0.07	-0.09	0.01	0.01	0.00
	(0.04)	(0.04)	(0.02)	(0.01)	(0.01)	(0.01)
a_3: education in decades of the partner	0.00	0.00	0.00	0.00	0.00	0.00
	—	—	—	—	—	—
a_3: age of the partner	0.00	0.00	0.00	0.00	0.00	0.00
	—	—	—	—	—	—
a_3: age in decades of the partner exp2	0.00	0.00	0.00	0.00	0.00	0.00
	—	—	—	—	—	—
B: price of FDAH	8.38	-3.38	1.82	0.06	0.05	-1.63
	(2.95)	(1.87)	(0.94)	(0.57)	(0.38)	(0.75)
B: price of REST	-3.38	0.19	0.13	0.28	-0.24	0.09
	(1.87)	(1.46)	(0.73)	(0.49)	(0.26)	(0.48)
B: price of VICE	1.82	0.13	-0.07	0.06	-0.21	-0.50
	(0.94)	(0.73)	(0.57)	(0.21)	(0.16)	(0.32)
B: price of CARE	0.06	0.28	0.06	-0.41	-0.08	-0.23
	(0.57)	(0.49)	(0.21)	(0.51)	(0.09)	(0.19)
B: quantity of FUEL	0.05	-0.24	-0.21	-0.08	-0.65	-0.02
	(0.38)	(0.26)	(0.16)	(0.09)	(0.10)	(0.13)
B: quantity of UTIL	-1.63	0.09	-0.50	-0.23	-0.02	-0.74
	(0.75)	(0.48)	(0.32)	(0.19)	(0.13)	(0.35)
c_1: expenditure on clothing	-6.89	2.78	-1.94	0.01	0.48	2.27
	(1.07)	(0.87)	(0.56)	(0.28)	(0.25)	(0.44)
c_2: expenditure on clothing exp2	1.12	-0.50	0.36	0.02	0.01	-0.28
	(0.24)	(0.22)	(0.15)	(0.07)	(0.06)	(0.08)
d: expenditure on clothing of the partner	0.00	0.00	0.00	0.00	0.00	0.00
	—	—	—	—	—	—

Notes: Standard deviations are in parentheses

Table 5: Median of the distribution of cm-demand elasticities computed from the estimates for Model V

	Dependent variables (**D**)					
	FDAH	REST	VICE	CARE	FUEL	UTIL
Price of FDAH	0.01	-1.50	0.96	0.38	-0.18	-1.95
	(0.49)	(0.95)	(1.12)	(0.94)	(0.64)	(0.96)
Price of REST	-0.69	-4.40	1.90	2.05	-0.96	-1.24
	(0.44)	(1.64)	(1.32)	(1.72)	(0.65)	(1.11)
Price of VICE	0.17	0.64	0.75	-0.40	-0.75	-0.85
	(0.19)	(0.45)	(1.17)	(0.64)	(0.33)	(0.62)
Price of CARE	0.03	0.39	-0.23	-0.01	0.48	0.31
	(0.08)	(0.33)	(0.36)	(1.22)	(0.18)	(0.21)
Quantity of FUEL	-0.02	-0.25	-0.57	0.64	1.10	-0.12
	(0.07)	(0.17)	(0.25)	(0.24)	(0.51)	(0.23)
Quantity of UTIL	-0.26	-0.30	-0.63	0.53	-0.12	-1.09
	(0.12)	(0.29)	(0.47)	(0.37)	(0.23)	(0.85)
Expenditure	0.17	2.35	0.15	0.28	-0.22	-0.11
on men's clothing	(0.03)	(0.48)	(0.14)	(0.09)	(0.05)	(0.03)
Expenditure	0.35	-0.54	0.32	0.44	-0.05	-0.06
on women's clothing	(0.16)	(0.41)	(0.22)	(0.14)	(0.12)	(0.11)

Notes: Standard deviations (computed by bootstrap) are in parentheses.

5.3 Elasticities and Identification

As previously shown, the separability of the public goods from the other goods is clearly rejected by our data. Consequently the complete identification of the individual cm-demands is not possible. Still, some interesting information on the decision process can be obtained. As a first step, we may show, using (13), that the income elasticites for the exclusive goods are positive for the large majority of observations (all the income-elasticities for men computed from the sample are positive, and less than 5% of these elasticities for women are negative) and conclude that these goods are (conditionally) superior, as required by **A1**.

We next turned to the computation of the elasticities of the cm-demands but, as a preliminary, we checked the equality of the individual intercepts, $a_A = a_B$, for each equation. It turns out that only the intercepts for REST are significantly different from each other. We therefore imposed the conditions $a_A = a_B$ for all goods except REST—this is restrictive but greatly increases the precision of the estimates—and we computed, for each observation, the price-and-quantity and expenditure elasticities from the parameter estimates of the constrained model. The median of the distribution of these elasticities is given in Table 5. We first remark, to make things clearer, that a rise in men's and women's clothing expenditure, if **A1** is satisfied, should increase (decrease) the demand for any superior private (public) good.[25]

In this context, the first conclusion we can draw is that the majority of goods are superior. The only exception is the demand for REST by women, but it is not very significant. To make up for that, the elasticity of the demand for REST by men is positive and very large. More precisely, if **A2** obtains, an increase of the bargaining power of the husband can be shown to imply an increase in the demand for REST by the household. Similarly, an increase in the bargaining power of the husband can be shown to imply an increase in the demand for FUEL and UTIL. Such a result, linking the intra-household distribution of power to the demand for public goods, has never been seen in the literature.

The second notable point is that the price-and-quantity elasticities are now fairly well estimated: 19 parameters out of 36 are significant at the 20% level. The own price-and-quantity elasticities for REST and FUEL are significant and have the expected sign. The other own price-and-quantity elasticities are not significant.

6 Conclusion

One of the main topical themes of research in collective models concerns the structure of demands for public goods. Our objective in this paper was to develop and estimate a simple collective model of household behavior with public and private goods. In a few words, our main results can be summarized as follows.

[25] Let us first recall that the cm-demands for public goods are defined as a negative function $-\mathbf{P}$ of $\mathbf{q}$ and other variables. Then, this property directly stems from the definition (19). Simple computation indeed yields:

$$\frac{\partial \boldsymbol{\delta}_I}{\partial \rho_I} = \frac{\partial \mathbf{d}_I}{\partial x_I} \cdot \frac{\partial x_I}{\partial \rho_I}$$

with $\partial x_I / \partial \rho_I > 0$ because of **A1**. However, our definition of 'superiority' for public goods is slightly different from the most common one since it is expressed in terms of prices or marginal rates of substitution. The definitions are equivalent when there is only one public good.

First, we propose what we call a 'cm-demand' framework, in which household demands are directly derived from the marginal rates of substitution. This framework, which turns out to be especially profitable for investigating the properties of collective demands, is extensively exploited in this paper.

Second, we show, using this framework, that household demands for public and private goods have to satisfy testable constraints and that some elements of the decision process can be retrieved from observed behavior. Moreover, the identification is complete if preferences are assumed to be such that public and private goods are separable. We also define the new concepts of 'specific' and 'common' factors of preferences.

Third, we implement this theoretical model and present empirical results with U.S. data. It turns out that, overall, the data are consistent with the theoretical model. This contribution is appreciable because, in fact, collective models of demand accounting for variable prices are rarely estimated.

We also present the first test, to the best of our knowledge, of the separability of public and private consumption in preferences. The evidence strongly suggests that such a separability is rejected. It makes one wonder, then, how individual demands could be retrieved in a collective model with public and private goods. The cm-framework can again furnish a solution. To demonstrate, let us recall that cm-demands result from the sole characteristics of the utility functions. That is to say, the decision process is completely summarized by the levels of exclusive goods. Let us now assume that the preferences of single and married persons are indistinguishable. Then, under this assumption, the m-demands obtained from a sample of singles and the individual cm-demands obtained from a sample of couples should be identical in every respect. Identification of the structural model then follows in a simple way. However, our approach in the present paper is more general and does not postulate such a similarity between single and married persons. This assumption is indeed strong. Still, it is not unusual (see Lewbel, Chiappori and Browning (2003) or Couprie (2003) for example) and certainly deserves greater attention in future.

Be that as it may, one of the main limitations of our model is that goods are assumed to be pure, either private or public. More realistically, however, most goods in households should certainly be regarded as impure. For example, expenditures on 'telephone services' include the rental as a public element and the actual use of telephone as a private element. Nevertheless, identification of the structural model undoubtedly raises further difficulties in this case. Yet, these difficulties are not insurmountable, as was shown by Fong and Zhang (2001) in a labor supply context, but more structure is probably necessary here. This is one of the most promising directions for future research.

Future research should also try to simultaneously estimate the system of cm-demands and the demands for exclusive goods, yielding a deeper understanding of the variables that influence the distribution of resources within the household. One drawback of such an investigation, however, is that the theory does not specify what distribution factors enter the bargaining. In addition, the most important distribution factors are possibly not observed by the econometrician. In fact, the sharing rule, which is not explained by the theory, cannot be easily interpreted; see Donni (2003) for more detail.

One final comment is that most public goods in households are to some extent durable. This is obvious when you think to lodging or appliances. Thus, the theory should be expanded to cover the case of durable goods.

Appendix

A List of Proofs

A.1 Proof of Proposition 2

The first statement in Proposition 2 is trivial and the proof is straightforward. We thus turn to the second statement.

Let us consider the (conditional) compensated demands and prices for each household member. They are defined in the usual way as follows:

$$x_I = x_I^c(r_I, \mathbf{m}, u_I), \qquad \mathbf{q}_I = \mathbf{q}_I^c(r_I, \mathbf{m}, u_I), \qquad \mathbf{p}_I = \mathbf{p}_I^c(r_I, \mathbf{m}, u_I).$$

The latter term is a virtual price as in Neary and Roberts (1980). Then, a result by Madden (1991, Lemma 1) indicates that the matrix

$$\begin{bmatrix} \dfrac{\partial x_I^c}{\partial r_I} & \dfrac{\partial x_I^c}{\partial \mathbf{p}'} & \dfrac{\partial x_I^c}{\partial \mathbf{q}'} \\[2ex] \dfrac{\partial \mathbf{q}_I^c}{\partial r_I} & \dfrac{\partial \mathbf{q}_I^c}{\partial \mathbf{p}'} & \dfrac{\partial \mathbf{q}_I^c}{\partial \mathbf{q}'} \\[2ex] -\dfrac{\partial \mathbf{p}_I^c}{\partial r_I} & -\dfrac{\partial \mathbf{p}_I^c}{\partial \mathbf{p}'} & -\dfrac{\partial \mathbf{p}_I^c}{\partial \mathbf{q}'} \end{bmatrix}$$

is symmetrical. The next step is to show that this result directly implies the symmetry of individual cm-demands.

Lemma 7 *Let us assume* **A1**. *Then the individual cm-demands have to satisfy the following:*

$$\left(\frac{\partial \mathbf{d}_I}{\partial \mathbf{m}'} + \frac{\partial \mathbf{d}_I}{\partial x_I}\frac{\partial \mathbf{d}_I'}{\partial r_I}\right) \text{ is symmetrical.}$$

Proof. We simply generalize here the argument of Browning (1999) and show that the individual cm-demands have to satisfy a symmetry restriction. The inversion of $x_I^c(r_I, \mathbf{m}, u_I)$ yields:

$$x_I = x_I^c(r_I, \mathbf{m}, u_I) \Leftrightarrow u_I = \psi_I(r_I, \mathbf{m}, x_I).$$

The inversion is possible if **A1** is assumed. The function $\psi_I(r_I, \mathbf{m}, x_I)$ has some properties of a preference representation but it is not a valid representation in the sense that there is a one-to-one mapping from preferences to these functions. We have, naturally, the following identity:

$$x_I \equiv x_I^c\left[r_I, \mathbf{m}, \psi_I(r_I, \mathbf{m}, x_I)\right].$$

Since this holds identically, we can take derivatives with respect to x_I, $\mathbf{m}$ and r_I:

$$\frac{\partial x_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial x_I} = 1, \tag{27}$$

$$\frac{\partial x_I^c}{\partial \mathbf{m}} + \frac{\partial x_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial \mathbf{m}} = 0, \tag{28}$$

$$\frac{\partial x_I^c}{\partial r_I} + \frac{\partial x_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial r_I} = 0. \tag{29}$$

On the other hand, the individual cm-demands are given by:

$$\mathbf{d}_I = \mathbf{d}_I^c\left[r_I, \mathbf{m}, \psi_I(r_I, \mathbf{m}, x_I)\right],$$

where $(\mathbf{d}_I^c)' = [(\mathbf{q}_I^c)', -(\mathbf{p}_I^c)']$. This procedure is very close to Cook's method for deriving Marshallian demands from the cost function. Taking the derivatives and substituting from (27)-(29), we have:

$$\frac{\partial \mathbf{d}_I}{\partial x_I} = \frac{\partial \mathbf{d}_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial x_I} = \frac{\partial \mathbf{d}_I^c}{\partial u_I}\left(\frac{\partial x_I^c}{\partial u_I}\right)^{-1},$$

$$\begin{aligned}\frac{\partial \mathbf{d}_I}{\partial r_I} &= \frac{\partial \mathbf{d}_I^c}{\partial r_I} + \frac{\partial \mathbf{d}_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial r_I} = \frac{\partial \mathbf{d}_I^c}{\partial r_I} - \frac{\partial \mathbf{d}_I^c}{\partial u_I}\frac{\partial x_I^c}{\partial r_I}\left(\frac{\partial x_I^c}{\partial u_I}\right)^{-1},\\ \frac{\partial \mathbf{d}_I}{\partial \mathbf{m}'} &= \frac{\partial \mathbf{d}_I^c}{\partial \mathbf{m}'} + \frac{\partial \mathbf{d}_I^c}{\partial u_I}\frac{\partial \psi_I}{\partial \mathbf{m}'} = \frac{\partial \mathbf{d}_I^c}{\partial \mathbf{m}'} - \frac{\partial \mathbf{d}_I^c}{\partial u_I}\frac{\partial x_I^c}{\partial \mathbf{m}'}\left(\frac{\partial x_I^c}{\partial u_I}\right)^{-1}.\end{aligned}$$

Using $\partial x_I^c/\partial \mathbf{m} = \partial \mathbf{d}_I^c/\partial r_I$ and rearranging, we obtain:

$$\frac{\partial \mathbf{d}_I}{\partial \mathbf{m}'} + \frac{\partial \mathbf{d}_I}{\partial x_I}\frac{\partial \mathbf{d}_I'}{\partial r_I} = \frac{\partial \mathbf{d}_I^c}{\partial \mathbf{m}'} + \frac{\partial \mathbf{d}_I}{\partial x_I}\frac{\partial \mathbf{d}_I'}{\partial x_I}\frac{\partial x_I^c}{\partial r_I}.$$

Since right-hand side is symmetric, so is the left-hand side.||

Since $\mathbf{D} = \sum_I \mathbf{d}_I$, $\partial \mathbf{d}_I/\partial x_I = \partial \mathbf{D}/\partial x_I$ and $\partial \mathbf{d}_I/\partial r_I = \partial \mathbf{D}_I/\partial r_I$, the proof of the second statement in Proposition 2 follows from Lemma 8. ■

A.2 Proof of Proposition 3

The proof of the first and the second statement is straightforward. The derivatives of the individual demands for private goods can be retrieved and we have:

$$\frac{\partial \mathbf{q}_I}{\partial r_I} = \frac{\partial \mathbf{Q}}{\partial r_I} \qquad \text{and} \qquad \frac{\partial \mathbf{q}_I}{\partial x_I} = \frac{\partial \mathbf{Q}}{\partial x_I}, \tag{30}$$

where the right-hand side of these expressions is known. Similarly, the derivatives of the individual prices for public goods can be retrieved as well. We have:

$$\frac{\partial \mathbf{p}_i}{\partial r_i} = \frac{\partial \mathbf{P}}{\partial r_i} \qquad \text{and} \qquad \frac{\partial \mathbf{p}_i}{\partial x_i} = \frac{\partial \mathbf{P}}{\partial x_i}. \tag{31}$$

Since the derivatives of $\mathbf{D}$ with respect to $\mathbf{p}$ and $\mathbf{q}$ are not identified, the individual cm-demands are identified up to a function $\mathbf{f}_I(\mathbf{p},\mathbf{q})$. That is, $\mathbf{d}_I = \hat{\mathbf{d}}_I + \mathbf{f}_I$ where $\hat{\mathbf{d}}_I$ is a particular solution of (30) and (31).

We now turn to the proof of the third statement. We know from the preceding that $\sum_I(\hat{\mathbf{d}}_I + \mathbf{f}_I) = \mathbf{D}$ where $\mathbf{D}$ is known. Since any particular solution must also satisfy $\sum_I \hat{\mathbf{d}}_I = \mathbf{D}$, we have $\sum_I \mathbf{f}_I = \mathbf{0}$. Similarly, as indicated by the first-order conditions (7) and (10), any particular solution has to satisfy the homogeneity restriction:

$$\frac{\partial \hat{\mathbf{q}}_I}{\partial \mathbf{p}'}\mathbf{p} + \frac{\partial \hat{\mathbf{q}}_I}{\partial r_I} r_I = 0 \qquad \text{and} \qquad \frac{\partial \hat{\mathbf{p}}_I}{\partial \mathbf{p}'}\mathbf{p} + \frac{\partial \hat{\mathbf{p}}_I}{\partial r_I} r_I = -\hat{\mathbf{p}}_I$$

and, consequently, $\mathbf{g}_I$ is homogeneous of degree zero in $\mathbf{p}$ and $\mathbf{h}_I$ is homogeneous of degree one in $\mathbf{p}$. Finally, from Lemma 8, any particular solution has to satisfy the symmetry restriction:

$$\frac{\partial \hat{\mathbf{d}}_I}{\partial \mathbf{m}'} + \frac{\partial \hat{\mathbf{d}}_I}{\partial x_I}\frac{\partial \hat{\mathbf{d}}'_I}{\partial r_I} = \frac{\partial \hat{\mathbf{d}}'_I}{\partial \mathbf{m}} + \frac{\partial \hat{\mathbf{d}}_I}{\partial r_I}\frac{\partial \hat{\mathbf{d}}'_I}{\partial x_I}$$

and, consequently, $\mathbf{f}_I$ is symmetrical. ■

A.3 Proof of Corollary 4

We differentiate the cm-demand with respect to x_A, x_B, $\mathbf{q}$ and $\mathbf{Q}$ obtain:

$$\begin{aligned}
\mathrm{d}\mathbf{Q} &= \frac{\partial \mathbf{Q}}{\partial \mathbf{q}'} \cdot \mathrm{d}\mathbf{q} + \frac{\partial \mathbf{Q}}{\partial x_A} \cdot \mathrm{d}x_A + \frac{\partial \mathbf{Q}}{\partial x_B} \cdot \mathrm{d}x_B, \\
\mathrm{d}\mathbf{q} &= -\left(\frac{\partial \mathbf{P}}{\partial \mathbf{q}'}\right)^{-1} \left(\frac{\partial \mathbf{P}}{\partial x_A} \cdot \mathrm{d}x_A + \frac{\partial \mathbf{P}}{\partial x_B} \cdot \mathrm{d}x_B\right).
\end{aligned}$$

We consider a variation in x_A (say) such that the total expenditure remains the same, i.e.

$$\left(\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_A} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_A} + r_A\right) \cdot \mathrm{d}x_A + \left(\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_B} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_B} + r_A\right) \cdot \mathrm{d}x_B = 0.$$

Rearranging yields:

$$\begin{aligned}
\left.\frac{\partial \mathbf{Q}}{\partial x_A}\right|_{\mathrm{d}y=\mathrm{d}a} &= \frac{\partial \mathbf{Q}}{\partial x_A} - \frac{\partial \mathbf{Q}}{\partial \mathbf{q}'}\left(\frac{\partial \mathbf{P}}{\partial \mathbf{q}'}\right)^{-1}\frac{\partial \mathbf{P}}{\partial x_A} \\
&\quad - \left(\frac{\partial \mathbf{Q}}{\partial x_B} - \frac{\partial \mathbf{Q}}{\partial \mathbf{q}'}\left(\frac{\partial \mathbf{P}}{\partial \mathbf{q}'}\right)^{-1}\frac{\partial \mathbf{P}}{\partial x_B}\right) \cdot \frac{\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_A} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_A} + r_A}{\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_B} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_B} + r_B}, \\
\left.\frac{\partial \mathbf{q}}{\partial x_A}\right|_{\mathrm{d}y=\mathrm{d}a} &= -\left(\frac{\partial \mathbf{P}}{\partial \mathbf{q}'}\right)^{-1}\left(\frac{\partial \mathbf{P}}{\partial x_A} - \frac{\partial \mathbf{P}}{\partial x_B}\frac{\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_A} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_A} + r_A}{\mathbf{P}'\frac{\partial \mathbf{q}}{\partial x_B} + \mathbf{p}'\frac{\partial \mathbf{Q}}{\partial x_B} + r_B}\right).
\end{aligned}$$

That means that, according to **A2**, the sign of the effect of a variation in the bargaining power on the level of private and public goods is defined. ■

A.4 Proof of Proposition 6

The first statement of Proposition 6 is trivial. Since separability implies that:

$$\frac{\partial \mathbf{q}_I}{\partial \mathbf{q}'} = 0, \tag{32}$$

the individual cm-demands for private goods are identified up to a function $\mathbf{g}_I(\mathbf{p})$, which is independent of $\mathbf{q}$.

We now turn to the second statement. Using (32) and Lemma 8, symmetry (at the individual level) implies that:

$$\frac{\partial \mathbf{p}_I}{\partial \mathbf{p}'} = \frac{\partial \mathbf{p}_I}{\partial r_I}\frac{\partial \mathbf{q}_I'}{\partial x_I} - \frac{\partial \mathbf{p}_I}{\partial x_I}\frac{\partial \mathbf{q}_I'}{\partial r_I}. \tag{33}$$

On the other hand, homogeneity (at the individual level) implies:

$$\mathbf{p}_I = \frac{\partial \mathbf{p}_I}{\partial \mathbf{p}'}\mathbf{p} + \frac{\partial \mathbf{p}_I}{\partial r_I} r_I.$$

Substituting (33) and using Proposition 3 yield:

$$\mathbf{p}_I = \left(\frac{\partial \mathbf{P}}{\partial r_I}\frac{\partial \mathbf{Q}'}{\partial x_I} - \frac{\partial \mathbf{P}}{\partial x_I}\frac{\partial \mathbf{Q}'}{\partial r_I}\right)\mathbf{p} + \frac{\partial \mathbf{P}}{\partial r_I} r_I.$$

The individual cm-demands are then exactly identified.

We consider the constraints of the separability of public goods. First of all, we have seen that the demands for private goods have to satisfy (32). However, there are other constraints resulting from separability. The first-order condition can be written as follows:

$$\mathbf{p}_I = r_I \times \left(\frac{\partial u_I}{\partial \mathbf{q}} \bigg/ \frac{\partial u_I}{\partial \mu_I}\right) \times \left(\frac{\partial \mu_I}{\partial x_I}\right)^{-1}.$$

If we take the logarithm of this expression, we obtain the following functional equation:

$$\ln \mathbf{p}_I = \ln r_I + \mathbf{F}\left[f(r_I, x_I, \mathbf{p}), \mathbf{q}\right] + g(r_I, x_I, \mathbf{p}),$$

where $f = \mu_I$, $g = \ln\left(\partial \mu_I / \partial x_I\right)^{-1}$ and $\mathbf{F} = \ln\left[\left(\partial u_I / \partial \mathbf{q}\right) / \left(\partial u_I / \partial \mu_I\right)\right]$. Differentiating this expression with respect to $\mathbf{q}$ yields:

$$\frac{\partial \ln \mathbf{p}_I}{\partial \mathbf{q}'} = \frac{\partial \mathbf{F}\left[f(r_I, x_I, \mathbf{p}), \mathbf{q}\right]}{\partial \mathbf{q}'}.$$

Let p^j be a typical element of $\mathbf{p}$. Differentiating again with respect to x_I, r_I and p^j yields:

$$
\begin{aligned}
\frac{\partial \ln \mathbf{p}_I}{\partial \mathbf{q} \partial x_I} &= \frac{\partial \mathbf{F}}{\partial \mathbf{q}' \partial f} \frac{\partial f}{\partial x_I}, \\
\frac{\partial \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial r_I} &= \frac{\partial \mathbf{F}}{\partial \mathbf{q}' \partial f} \frac{\partial f}{\partial r_I}, \\
\frac{\partial \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial p^j} &= \frac{\partial \mathbf{F}}{\partial \mathbf{q}' \partial f} \frac{\partial f}{\partial p^j}.
\end{aligned}
$$

We assume that $\det (\partial^2 \ln \mathbf{p}_I / \partial \mathbf{q}' \partial x_I) \neq 0$. Since $\partial f / \partial x_I \neq 0$, we thus have:

$$
\begin{aligned}
\frac{\partial^2 \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial r_I} \left(\frac{\partial^2 \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial x_I} \right)^{-1} &= I \cdot h^1(r_I, x_I, \mathbf{p}), \\
\frac{\partial^2 \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial p^j} \left(\frac{\partial^2 \ln \mathbf{p}_I}{\partial \mathbf{q}' \partial x_I} \right)^{-1} &= I \cdot h^2(r_I, x_I, \mathbf{p}),
\end{aligned}
$$

where I is the identity matrix, $h^1(r_I, x_I, \mathbf{p}) = (\partial f / \partial r_I)/(\partial f / \partial x_I)$ and $h^2(r_I, x_I, \mathbf{p}) = (\partial f / \partial p^j)/(\partial f / \partial x_I)$. That is, the right-hand side of these expressions is equal to a symmetric matrix with identical diagonal elements and is independent of $\mathbf{q}$. There is a last constraint: the individual prices must satisfy cm-symmetry at the individual level ■

B Construction of the Data

The order in which the selection criteria were applied, and their effects in terms of the number of observations deleted using each criteria, is given in Table B1. The number of candidate observations (which could theoretically be used in the estimation process) is equal to 32,346. The most important selection is due to the deletion of households with children. The number of incomplete observations is small, since the missing values in instruments were imputed by their sample means. These incomplete observations include mainly rural households because, for confidentiality reasons, the region of residence of these households is not known. The descriptive statistics of the sample are given in Table B2.

Table B1: Selection Criteria of the Sample

Total number of observations		96,949
Attrition in the family survey	–	35,258
Single headed households	–	29,345
Total number of candidate observations		32,346
Households with members > 65 years	–	6,396
Household with children	–	18,361
Households with part-time (or non) working members	–	4,703
Incomplete observations (rural households & topcoding)	–	282
Remaining sample		2,604

Notes: Topcoding refers to the replacement of data, for confidentiality reasons, when the absolute value of the original data exceeds the allowable limits.

Table B2: Descriptive Statistics of the Sample

	Mean	St.D.
Quantities (expenditure/prices)		
Men's clothing	3.137	3.792
Women's clothing	5.183	5.895
Food and beverages at home	23.794	9.680
Food and beverages away from home	13.238	12.273
Alcoholic beverages and tobacco	5.052	5.340
Personal care services	2.411	1.840
Oil fuel and utility natural gas service	4.366	4.190
Electricity, water and sewer and trash collection services	9.048	4.790
Prices (base 1980-1984 = 100)		
Men's clothing	117.099	13.352
Women's clothing	118.036	15.209
Food and beverages at home	127.353	21.542
Food and beverages away from home	129.042	21.782
Alcoholic beverages and tobacco	147.401	37.491
Personal care services	129.950	23.088
Oil fuel and utility natural gas service	99.437	9.561
Electricity, water and sewer and trash collection services	119.576	16.742
Socio-demographic variables		
Husband's education in years	17.656	11.316
Wife's education in years	17.858	11.106
Husband's age in years	43.267	12.077
Wife's age in years	40.925	11.751
Proportion of black households	0.048	0.213
Proportion of Hispanic households	0.033	0.180
Proportion of North residents	0.217	0.413
Proportion of South residents	0.303	0.460
Proportion of Midwest residents	0.264	0.441
Proportion of West residents	0.215	0.411

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