The Effect of Corruption on Bidding Behavior in First-Price Auctions

Leandro Arozamena* and Federico Weinschelbaum†

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Abstract

Most of the literature on auctions assumes that the auctioneer owns the object on sale. However most auctions are organized and run by an agent of the owner. This separation generates the possibility of corruption. We analyze the effect of a particular form of corruption on bidding behavior in a single-object, private-value auction with risk-neutral bidders. Bidders believe that, with a certain probability, the auctioneer has reached an agreement with one of the bidders by which, after receiving all bids, (i) she will reveal to that bidder all of her rivals' bids, and (ii) she will allow that bidder to change her original bid upwards or downwards. We study how an honest bidder would adjust her bidding behavior when facing this type of collusion between a dishonest rival and the auctioneer. In a first price auction, an honest bidder can become more or less aggressive than she would be without corruption, or her behavior can remain unchanged. We identify sufficient conditions for each of the three possibilities. We also examine the extent to which the most commonly used distributions satisfy each of the three conditions.

Keywords: Auctions; Corruption.

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*Universidad Torcuato Di Tella, Business School and Economics Dept. E-mail: larozamena@utdt.edu .

†Universidad de San Andrés, Economics Department. E-mail: fweinsch@udesa.edu.ar .

1 Introduction

Most of the literature on auctions assumes that the auctioneer owns the object on sale. However, most auctions are organized and run by an agent of the owner. This separation between the owner and the auctioneer creates the scope for corruption to appear. The auctioneer may be tempted to enter into corrupt agreements with one of the bidders to tilt the auction in her favor.

In this paper, we examine a particular form of corruption in a single-object, private-value auction with risk-neutral bidders. We focus on the case where corrupt dealings between the auctioneer and any bidder consist of revealing information on how much other participants in the auction have bid. Specifically, we assume that bidders believe that the auctioneer may have reached an agreement with one of them by which, after receiving all bids, (i) she will reveal to that bidder all of her rivals' bids, and (ii) she will allow that bidder to change her original bid upwards or downwards if she wishes to do so. We are particularly interested in the effect of possible existence of this form of corruption on how honest bidders behave in the auction. That is, we want to ascertain whether the fact that her bid will, with a certain probability, be revealed to a rival makes a bidder more or less aggressive as compared to the case where corruption is absent.

In a second-price auction, since bidding her own valuation is a weakly dominant strategy for every bidder, this form of corruption has no effect on bidding behavior or on the auction's result. In a first-price auction, however, the situation is more complex. We show below that an honest bidder can become more or less aggressive than she would be without corruption, or her behavior can remain unchanged. We provide sufficient conditions for each of the three possibilities. Namely, if F is the cumulative distribution function of a bidder's valuation for the object being auctioned and f is the corresponding density, if the ratio F/f is strictly convex (respectively, strictly concave, linear) in the valuation, the honest bidder will become more aggressive (respectively, less aggressive, equally aggressive) with corruption. Furthermore, we establish the extent to which the most commonly used distributions satisfy one of those conditions.

Other papers have dealt with similar forms of corruption. In particular, Jones and Menezes (1995), Lengwiler and Wolfstetter (2000), Burguet and Perry (2002) and Menezes and Monteiro (2003) consider cases where a corrupt arrangement, just as in this paper, implies revealing to one of the bidders what her rivals have bid. However, their analyses differ from the one we present in several respects. In Jones and Menezes (1995), bidders are not aware of the possibility of corruption when choosing their bids, so bidding behavior remains unaltered by assumption.[1] Lengwiler and Wolfstetter (2000) and Menezes and Monteiro (2003) consider situations where the auctioneer approaches the winning bidder offering the chance to lower her bid (while still winning the auction) in exchange for a bribe. In this paper, on the contrary, who the auctioneer may conspire with is independent of the auction's result. That is, their agreement is reached before the auction takes place. In addition, the favored bidder is allowed to raise or lower her bid according to her interests. Burguet and Perry (2002) is closest to our analysis. They study several variants as to the exact form corruption may take. They are particularly interested in the effect of the bargaining game between the auctioneer and the dishonest bidder on the auction's result. Only one of the variants they deal with involves allowing the favored bidder to freely revise her bid upwards or downwards, but, as in all the remaining cases, they only focus on the two-bidder case when corruption is certain to all parties. Furthermore, they do not concentrate on the effect corruption has on equilibrium bids. Here, we allow for the existence of corruption to be uncertain, extend the analysis to the general, N-bidder case and are able to provide a fuller characterization of the effect of corruption on bidding behavior. Finally, Compte et al. (forthcoming), consider the situation where the auctioneer reveals the winning bid to all participants and allows them to compete for the chance to resubmit their bids. Their focus, then, is on bribing competition and its effects.

While our interest here limits to the case where a single-dimensional object is auctioned, another related strand in the literature focuses on the possibility of corruption in multi-

[1] In addition, they consider a setting where bidders draw their valuations from uniform distributions. We prove below that, for such distributions, even if bidders were aware of the possibility of corruption their behavior would remain unaltered.

dimensional procurement auctions. Specifically, in addition to the price, the object being procured has a quality dimension that affects the procurer's welfare. The procurer delegates the assessment of quality on an agent, and the scope for corruption is thereby created. Celentani and Ganuza (2002) and Burguet and Che (2004) examine different forms of corruption in that procurement environment.

In Section 2 below, we present the auctioning context and provide sufficient conditions to characterize the effect of corruption on an honest bidder's behavior. First, we examine the case where the presence of corruption is certain to all parties and prove the sufficiency of the proposed conditions. Then, we show that sufficiency extends to the more general case with uncertain corruption. Finally, we try to ascertain the extent to which standard distributions satisfy each of the sufficient conditions. In Section 3, we provide a preliminary assessment of the effect of corruption on efficiency and revenue and conclude.

2 The Model

The owner of a single, indivisible object is selling it through an auction organized and run by an agent of hers. We assume that there are two bidders[2] in the auction whose valuations v_i ($i = 1, 2$) for the object are distributed identically and independently according to the c.d.f. F with support on the interval $[0, 1]$[3] and a density f that is positive on the whole support. The context is, then, one of independent private values. Both bidders are risk neutral. We will focus below on sealed-bid auctions with no reserve prices, and assume that the c.d.f. F is logconcave.[4] For future use, let $\alpha(v_i) = F(v_i)/f(v_i)$. Note that the logconcavity of F means that α is increasing.

Given the fact that the owner of the object being sold and the auctioneer are not the

[2] See the Appendix for the n-bidder case (to be written).

[3] This is done purely for ease of exposition and without loss of generality. We could use any interval on the real line instead.

[4] Logconcavity of the c.d.f. function holds for most well known distributions, such as the uniform, normal, logistic, extreme value, chi-squared, chi, exponential, Laplace, Pareto and any truncation of these distributions. For details see Bagnoli and Bergstrom (1989).

same, there is scope for corruption. The auctioneer may tilt the auction in favor of one of the bidders in exchange for a compensation. The exact form this collusion between the auctioneer and one of the bidders[5] may take is open to multiple possibilities. Here, we concentrate on one particular case. We assume that before the auction takes place the auctioneer may approach one of the bidders and offer to provide her with information during the auction, and that she chooses to approach each of the bidders with equal probability. Specifically, the auctioneer will reveal to the favored bidder her rival's bid, and then allow her to modify her bid upwards or downwards if she wishes to do so.

In what follows, any given bidder may be in one of two situations. If she is colluding with the auctioneer, she will be sure that her rival is not colluding at the same time —we rule out the possibility that the auctioneer colludes with both bidders. If she is not colluding with the auctioneer, she believes that her rival is with probability p. Our main object of analysis will be how an "honest" bidder who believes that her rival colludes with the auctioneer with probability $p > 0$ will bid, as compared with how she would bid in the absence of corruption (i.e. with $p = 0$).

There are at least two possible interpretations of this setup. Our analysis may be viewed as part of a more complete and specific study of how corruption affects the result of an auction. We will not model how the auctioneer and the bidder she approaches will bargain when deciding if they will collude. In addition, we will assume that both bidders will stay in the auction independently of the value of p. One of the most relevant considerations in any such bargaining game, and when a bidder decides whether or not to take part in the auction, will be what would happen if both bidders stayed and an honest bidder attached a given probability to the fact that her rival will be favored. So our results below will be crucial in any study of this form of corruption. Furthermore, examining how the auctioneer and a bidder bargain is open to many modelling alternatives. Different assumptions could be made, for instance, on what knowledge the auctioneer has of the bidder's valuation, or on how much bargaining power each party has. Our own results will be relevant to any such

[5] We are not allowing for the possibility of collusion between the bidders (see Hendricks and Porter, 1989, for a general analysis of this phenomenon), but only between one bidder and the auctioneer.

specification. We will assume, though, that the auctioneer and the colluding bidder bargain efficiently: they will reach an agreement whenever they can both gain by doing so.

Our setup could be understood in a more specific way as well. We may assume that, before the auctioneer may approach any of them, both bidders believe that the auctioneer is corrupt —and then expect her to make an approach— with probability q. Then, if a bidder has not been approached, this may mean that the auctioneer is not corrupt or that the auctioneer has approached the rival. The updated probability that an honest bidder attaches to the fact that the auctioneer be corrupt is then $p = \frac{q}{2-q}$.

All that will matter below, though, is that an honest bidder believes that her rival is colluding with the auctioneer with probability p. When deciding how much to bid, any bidder will find herself in one of two situations. If she is not colluding with the auctioneer, then with probability $(1-p)$ she will be competing against another honest bidder in her exact same situation (i.e. the rival will herself believe that the original bidder is colluding with the auctioneer with probability p). With probability p, she will face a rival that will be informed of her bid and be allowed to rebid accordingly. She will have to choose her bid weighing both possibilities. If she is colluding with the auctioneer, her bid will be irrelevant, since she will later be allowed to change it in any way she may wish after learning her rival's bid.

The effect of corruption on bidding behavior will clearly differ according to the sealed-bid auction format. In a second price auction, bidding her own valuation is a weakly dominant strategy for every bidder without corruption, and, of course, remains so once the possibility of corruption appears. Thus, bidding behavior remains unaltered. Furthermore, the result of the auction will not change. Knowing the rival's bid, no bidder will have any incentive to modify her original bid. It remains true that the bidder with the highest valuation wins in equilibrium, and she pays her rival's valuation, exactly as occurs when the possibility of corruption is absent.

In a first-price auction, however, the possibility of corruption has a significant effect. If a bidder colludes with the auctioneer and learns her rival's bid, he may have an incentive to change her original bid. If, according to her original bid, she is winning the auction, then she will revise her bid down to her rival's.[6] If she is losing the auction, there are two possibilities. If her rival's bid lies below the colluding bidder's valuation, then she will raise her bid up to her rival's and win the auction. If her rival's bid lies above her valuation, then her original bid will remain unchanged, since she would have to bid above her valuation to win the auction. Viewing the auction from the standpoint of an honest bidder that faces a colluding rival, this means that the former will have to bid above the latter's valuation to win. In other words, she will be competing against the rival's valuation instead of competing against her bid.

Let $b_i^p : [0,1] \longrightarrow \mathbb{R}$ be bidder i's bidding function in this setup. For convenience, we will use the inverse of her bidding function, $\phi_i^p(.)$. An honest bidder with valuation v_i who faces a rival j that, if honest, has a bidding function with inverse $\phi_j^p(.)$ that is strictly increasing will choose her bid by solving the following expected utility maximization problem

$$\max_b \, (v_i - b) \left[(1-p)F(\phi_j^p(b)) + pF(b)\right]$$

with first-order condition

$$v_i - b = \frac{(1-p)F(\phi_j^p(b)) + pF(b)}{(1-p)f(\phi_j^p(b))\phi_j^{p\prime}(b) + pf(b)}$$

In a symmetric equilibrium —i.e. one where both bidders exhibit the same behavior when they are honest— we will have $\phi_i^p = \phi_j^p = \phi^p$, and the first-order condition becomes

$$\phi^p(b) - b = \frac{(1-p)F(\phi^p(b)) + pF(b)}{(1-p)f(\phi^p(b))\phi^{p\prime}(b) + pf(b)} \tag{1}$$

In the absence of corruption, we would have a standard symmetric first-price auction with independent private values and risk-neutral bidders. Let ϕ_i be bidder i's inverse bidding function in this case. Thus, when bidder i has valuation v_i and faces a rival that behaves according to the inverse bidding function ϕ_j she would solve the problem

$$\max_b \, (v_i - b)F(\phi_j(b))$$

[6] We assume that, in the event of a tie, the auctioneer chooses the winner. Therefore, she will always chose the bidder she is trying to favor.

At a symmetric equilibrium, $\phi_i = \phi_j = \phi$, and the first-order condition is

$$\phi(b) - b = \frac{F(\phi(b)}{f(\phi(b))\phi'(b)} = \frac{\alpha(\phi(b))}{\phi'(b)} \tag{2}$$

Our main objective is to compare ϕ^p with ϕ. That is, we want to establish whether the fact that her rival will learn her bid with probability p makes a bidder more aggressive ($\phi^p(b) < \phi(b)$ for all $b \in (0,1]$) or less aggressive ($\phi^p(b) > \phi(b)$ for all $b \in (0,1]$) as compared to how she would behave if corruption were absent. We view the case where $p < 1$ as more relevant and realistic in the analysis of corruption. With $p = 1$, every party to the auction except the owner of the object on sale is aware of the existence of corruption and of who will be favored by the auctioneer. With $p < 1$, the existence of corruption is uncertain and so is the identity of the favored bidder.

For expositional ease, we make this comparison in two steps. The next subsection deals with the case where $p = 1$. Using the results obtained for this case, subsection 2.2 derives analogous results for any $p > 0$.

2.1 First-Price Auctions with Certain Corruption

We assume here that the auctioneer has agreed to reveal to one of the bidders the value of her rival's bid and then allow her to modify her own bid if necessary. This can be viewed as a sequential game were the honest agent bids first and her rival bids after observing the honest bid. We compare bidding behavior in this environment with the standard, non-corruption case.

Let ϕ^c be the honest bidder's inverse bidding function when $p = 1$. The first-order condition (1), which characterizes the honest bidder's behavior, becomes

$$\phi^c(b) - b = \frac{F(b)}{f(b)} = \alpha(b) \tag{3}$$

The comparison between (2) and (3) yields Proposition 1.

Proposition 1 *If $\alpha(v)$ is strictly convex (respectively, strictly concave, linear), the honest bidder will become more aggressive (respectively, less aggressive, equally aggressive) with corruption. That is, for all $b \in (0,1]$, $\phi^c(b) < \phi(b)$ (respectively, $\phi^c(b) > \phi(b)$, $\phi^c(b) = \phi(b)$).*

Proof. We provide a proof only for the case where $\alpha(v)$ is strictly convex. In the remaining two cases, the proof is analogous.

We proceed in two steps:

(a) First, we show that when $\alpha(v)$ is convex, for all $b \in (0,1]$, $\phi(b) = \phi^c(b) \Rightarrow \phi\prime(b) > \phi^c\prime(b)$. That means that if there is a value $b > 0$ at which the bidding functions cross, to the right of this value we have $\phi(b) > \phi^c(b)$.

Let $\gamma(b) = \phi(b) - \phi^c(b)$. From (2) and (3),

$$\gamma(b) = \phi(b) - b - (\phi^c(b) - b) = \frac{\alpha(\phi(b))}{\phi'(b)} - \alpha(b)$$

Using (3) again,

$$\gamma'(b) = \phi'(b) - \phi^{c\prime}(b) = \phi'(b) - 1 - (\phi^c(b) - 1) = \phi'(b) - 1 - \alpha'(b)$$

When $\gamma(b) = 0$, it must be the case that $\frac{\alpha(\phi(b))}{\phi'(b)} = \alpha(b)$. Consequently,

$$\gamma'(b) = \frac{\alpha(\phi(b))}{\alpha(b)} - 1 - \alpha'(b) = \frac{\alpha(\phi(b)) - \alpha(b)}{\alpha(b)} - \alpha'(b) = \frac{\alpha(\phi) - \alpha(b)}{\phi - b} - \alpha'(b)$$

where the last equality, once more, follows from (3).

Given that $\alpha(.)$ is convex, and that $b < \phi(b)$, we have

$$\frac{\alpha(\phi(b)) - \alpha(b)}{\phi(b) - b} > \alpha'(b)\ .$$

Hence $\gamma'(b) > 0$.

(b) It can be easily checked that $\phi^c(0) = \phi(0) = 0$. Our second step is to show that, arbitrarily to zero, $\phi^c(b) < \phi(b)$. If this is the case, then, by step (a) above we will know that both bidding functions cannot cross for any $b \in (0,1]$, and the proof will be complete.

Let $\delta > 0$ be an arbitrarily small number. We want to show that for all $b \in (0, \delta)$, $\gamma(b) > 0$.

Assume this is not true. Then, there is a $b^0 \in (0, \delta)$ such that, $\gamma(b^0) \leq 0$. By step (a) above, if $\gamma(b^0) = 0$, $\gamma(b) < 0$ for b lower than, but close to, b^0. Then, without loss of generality we can concentrate on the case where $\gamma(b^0) < 0$.

Since $\gamma(0) = 0$, there has to exist a $b^1 < b^0$ such that $\gamma(b^1) < 0$ and $\gamma'(b^1) < 0$ ($b^1 > 0$, because -as can be easily shown- $\gamma'(0) = 0$).

For any $b > 0$, if $\gamma'(b) < 0$ it follows that $\phi'(b) - 1 < \phi^{c\prime}(b) - 1$, or

$$\phi'(b) - 1 < \alpha'(b) < \frac{\alpha(\phi(b)) - \alpha(b)}{\phi(b) - b}$$

where the second inequality derives form the strict convexity of $\alpha(v)$. Therefore,

$$\phi'(b) - 1 < \frac{\alpha(\phi(b)) - \alpha(b)}{\phi(b) - b}$$

or

$$\phi'(b)(\phi(b) - b) - (\phi(b) - b) < \alpha(\phi(b)) - \alpha(b)$$

Using (2),

$$\alpha(\phi(b)) - (\phi(b) - b) < \alpha(\phi(b)) - \alpha(b)$$

which yields

$$\alpha(b) < \phi(b) - b$$

From (3), then,

$$\phi^c(b) - b < \phi(b) - b$$

Hence, $\gamma'(b) < 0$ implies that $\phi^c(b) < \phi(b)$, or $\gamma(b) > 0$..We conclude that it is not possible that there exist a b^1 fulfilling the conditions mentioned above. ■

2.2 The General Case: p<1

We return now to the most interesting and realistic case. We model corruption as an uncertain phenomenon. We will show that the conditions mentioned in the previous subsection are also sufficient to characterize the effect of corruption on bidding behavior in the general case. Furthermore, as Proposition 2 asserts, the bidding function of an honest bidder who is facing a bidder who is corrupt with probability p lies between the two bidding functions considered in the previous subsection. That is, between the one that results from the absence of corruption and the one used by bidder who is sure that her rival is corrupt.

Proposition 2 *The bidding function of a bidder that faces a rival who is corrupt and will learn her bid with probability $p \in (0, 1)$ lies between her bidding function in the absence of corruption and her bidding function when corruption is certain. Consequently,* *(i) if $\alpha(v)$ is strictly convex, $\phi^c(b) < \phi^p(b) < \phi(b)$ for all $b \in (0,1]$,*

(ii) if $\alpha(v)$ is strictly concave, $\phi(b) < \phi^p(b) < \phi^c(b)$ for all $b \in (0,1]$,

(iii) if $\alpha(v)$ linear, $\phi^c(b) = \phi^p(b) = \phi(b)$ for all $b \in [0,1]$.

Proof. Once again, we concentrate on (i) in our proof. The proofs of (ii) and (iii) are analogous.

The reasoning is very similar to the one used in the proof of Proposition 1. However, now we proceed in four steps, two for the comparison between $\phi^p(b)$ and $\phi^c(b)$ and the remaining two for the comparison between $\phi^p(b)$ and $\phi(b)$.

1. We first prove that $\phi^c(b) < \phi^p(b)$ for all $b \in (0,1]$.

   (a) We show that for all $b \in (0,1]$, $\phi^p(b) = \phi^c(b) \Rightarrow \phi^{p\prime}(b) > \phi^{c\prime}(b)$. As above, this means that if there is a point at which the bidding functions cross, to the right of this point $\phi^p(b) > \phi^c(b)$. We define $\gamma^p(b) = \phi^p(b) - \phi^c(b)$ Then, from (1) and (3),

   $$\gamma^p(b) = \phi^p(b) - b - (\phi^c(b) - b) = \frac{pF(b) + (1-p)\,F(\phi^p(b))}{pf(b) + (1-p)\,f(\phi^p(b))\phi^{p\prime}(b)} - \frac{F(b)}{f(b)},$$

   Using (3) again, it follows that

   $$\gamma^{p\prime}(b) = \phi^{p\prime}(b) - \phi^{c\prime}(b) = \phi^{p\prime}(b) - 1 - (\phi^{c\prime}(b) - 1) = \phi^{p\prime}(b) - 1 - \alpha'(b)$$

   Examining the first-order conditions, it is immediate that in the case where $\gamma(b) = 0$ we must have $\frac{\alpha(\phi^p)}{\phi^{p\prime}(b)} = \alpha(b)$. Replacing in the last expression,

   $$\gamma^{p\prime}(b) = \frac{\alpha(\phi^p(b))}{\alpha(b)} - 1 - \alpha'(b) = \frac{\alpha(\phi^p(b)) - \alpha(b)}{\alpha(b)} - \alpha'(b) = \frac{\alpha(\phi^p(b)) - \alpha(b)}{\phi^p(b) - b} - \alpha'(b),$$

   using (3) once more. Given that $\alpha(v)$ is convex , we have

   $$\frac{\alpha(\phi^p(b)) - \alpha(b)}{\phi^p(b) - b} > \alpha'(b),$$

   so $\gamma^{p\prime}(b) > 0$.

   (b) Again, it can be easily checked that $\phi^p(0) = \phi^c(0) = 0$. Our second step is, again, to show that close enough to zero it has to be true that $\phi^c(b) < \phi^p(b)$.

Let $\delta > 0$ be an arbitrarily small number. We want to show that, $\forall b \in (0, \delta)$, $\gamma^p(b) > 0$. Assume this is not true. Then there exists a $b^0 \in (0, \delta)$ such that, $\gamma^p(b^0) \leq 0$. Again, without loss of generality we can focus on the case where $\gamma^p(b^0) < 0$, since if $\gamma^p(b^0) = 0$ step (a) above implies that $\gamma^p(b) < 0$ for b lower but arbitrarily close to b^0. So there has to exist a bid $b^1 < b^0$ such that $\gamma^p(b^1) < 0$ and $\gamma^{p\prime}(b^1) < 0$ ($b^1 > 0$, since it can be easily shown that $\gamma^{p\prime}(0) = 0$). But, for any b, $\gamma^{p\prime}(b) < 0$, implies

$$\phi^{p\prime}(b) - 1 < \alpha'(b) < \frac{\alpha(\phi^p(b)) - \alpha(b)}{\phi^p(b) - b},$$

where the second inequality comes form the convexity of $\alpha(v)$. So we know that:

$$(\phi^{p\prime}(b) - 1) < \frac{\alpha(\phi^p(b)) - \alpha(b)}{\phi^p(b) - b}$$

or

$$\phi^{p\prime}(b)\,(\phi^p(b) - b) - (\phi^p(b) - b) < \alpha(\phi^p(b)) - \alpha(b)$$

If $\gamma^p(b) < 0$, then $\phi^p(b) - b < \phi^c(b) - b$. Using (3) the expression above turns into

$$\phi^p(b) - b < \frac{\alpha(\phi^p(b))}{\phi^{p\prime}(b)}$$

By (1), then,

$$\frac{(1-p)F(\phi^p(b)) + pF(b)}{(1-p)f(\phi^p(b))\phi^{p\prime}(b) + pf(b)} < \frac{\alpha(\phi^p(b))}{\phi^p(b)}$$

Which means that

$$\alpha(b) < \frac{\alpha(\phi^p(b))}{\phi^p(b)}.$$

But this, in turn, implies that

$$\phi^c(b) < \phi^p(b)$$

Hence, $\gamma^{p\prime}(b) < 0$ is incompatible with $\gamma(b) > 0$. We conclude that a b^1 satisfying the above-stated conditions cannot exist. Therefore, arbitrarily close to zero we must have $\phi^c(b) < \phi^p(b)$.

2. We prove now that $\phi^p(b) < \phi(b)$ for all $b \in (0, 1]$.

(a) In our first step, again, we show that $\phi^p(b) = \phi(b) \Rightarrow \phi^p\prime(b) < \phi\prime(b)$.

We know that

$$\phi^p(b) - b - (\phi(b) - b) = \frac{pF(b) + (1-p)\,F(\phi^p(b))}{pf(b) + (1-p)\,f(\phi^p(b))\phi^{p\prime}(b)} - \frac{F(\phi(b))}{f(\phi(b))\phi'(b)}$$

When $\phi^p(b) = \phi(b)$, then,

$$p\left(F\left(b\right)f(\phi(b))\phi'(b) - f(b)F(\phi(b))\right) = (1-p)\,F(\phi(b))f(\phi(b))\left(\phi^{p\prime}(b) - \phi'(b)\right) \tag{4}$$

We know that $\alpha(b) < \alpha(\phi(b))/\phi'(b)$ when $\alpha(v)$ is convex from Proposition (1) .Thus, the left-hand side of (4) is negative. So the left-hand side has to be negative as well, and hence $\phi^{p\prime}(b) < \phi'(b)$.

(b) In the second step, once more, we show that arbitrarily close to zero it must be true that $\phi^p(b) > \phi(b)$. Suppose, towards a contradiction, that $\phi(b) \geq \phi^p(b)$ in an interval $\left(0, \widehat{b}\right)$. Furthermore, step 1(a) above implies that if $\phi(b^0) = \phi^p(b^0)$ for some b^0 in that interval, then $\phi(b) > \phi^p(b)$ immediately to the left of b^0. Then, without loss of generality, we can suppose that $\phi(b) > \phi^p(b)$ for all $b \in (0, \widehat{b})$. Since we know, from Proposition 1, that the convexity of $\alpha(v)$ implies that $\frac{f(b)}{F(b)} < \frac{f(\phi)\phi'}{F(\phi)}$, the first-order conditions (1) and (2) imply that

$$\frac{f(\phi^p(b))\phi^{p\prime}(b)}{F(\phi^p(b))} < \frac{f(\phi(b))\phi'(b)}{F(\phi(b))}$$

or

$$f(\phi^p(b))\phi^{p\prime}(b)F(\phi(b)) < f(\phi(b))\phi'(b)F(\phi^p(b)).$$

Hence, for all $b \in (0, \widehat{b})$

$$\frac{d}{db}\left(\frac{F(\phi^p)}{F(\phi)}\right) < 0.$$

Integrating between 0 and $\widehat{b}$ we get,

$$\int_0^{\widehat{b}} \frac{d}{db}\left(\frac{F(\phi^p(s))}{F(\phi(s))}\right) ds < 0$$

Which implies,

$$\frac{F(\phi^p(\widehat{b}))}{F(\phi(\widehat{b}))} - \lim_{b \to 0} \frac{F(\phi^p(b))}{F(\phi(b))} < 0.$$

But

$$\lim_{b\to 0}\frac{F(\phi^p(b))}{F(\phi(b))}=\lim_{b\to 0}\frac{f(\phi^p(b))\phi^{p\prime}(b)}{f(\phi(b))\phi'(b)}=\frac{f(0)\phi^{p\prime}(0)}{f(0)\phi'(0)}=\frac{\phi^{p\prime}(0)}{\phi'(0)}=1$$

since it can be easily checked that $\phi^{p\prime}(0)=\phi'(0)=2$. Then,

$$\frac{F(\phi^p(\widehat{b}))}{F(\phi(\widehat{b}))}<1$$

this is only possible if $\phi^p(\widehat{b})<\phi(\widehat{b})$ which is a contradiction. Hence it is not possible to have $\phi^p(b)>\phi\ (b)$ for b arbitrarily close to zero. ■

We have provided sufficient conditions to characterize how bidding behavior is affected by the possibility of a corrupt arrangement between the auctioneer and one of the bidders. As noted, how an honest bidder's behavior will be influenced by the possible existence of corruption will be determined by the concavity or convexity of $\alpha(v)$. The next natural question is, of course, if most of the commonly used distribution functions imply that $\alpha(v)$ is concave or convex. The next subsection briefly deals with this issue.

2.3 The effect of corruption for commonly used distributions

We have shown above that how a bidder who does not take part in any arrangement with the auctioneer will alter her behavior in the presence of corruption will depend crucially on the convexity or concavity of $\alpha(v)=\frac{F(v)}{f(v)}$. Note that

$$\alpha'(v)=1-\frac{F(v)}{f(v)}\frac{f'(v)}{f(v)}$$

If F is strictly logconcave (logconvex), then $F(v)/f(v)$ will be strictly increasing (decreasing). By the same token, if f is strictly logconcave (logconvex), then $f'(v)/f(v)$ will be strictly decreasing (increasing). Recall that we have assumed that F is logconcave. We can combine these possibilities to provide simple sufficient conditions for the concavity or convexity of $\alpha(v)$.

Remark 1 *(a) If $f(v)$ is logconcave and decreasing ($f'(v)<0$), then $\alpha(v)$ is convex.*

(b) If $f(v)$ is logconvex and increasing ($f'(v)<0$), then $\alpha(v)$ is concave.

The exponential distribution is an example of part (a) in the Remark 1, while $F(v) = \frac{e^{-\lambda}}{1-e^{-\lambda}} \left(e^{\lambda v} - 1\right)$ is an example of (b). In addition, it can be easily verified as well if $\alpha(v)$ is concave or convex for any of the standard distributions. Straightforward calculations show that in the cases of the logistic, Laplace and Pareto distributions, for instance, $\alpha(v)$ is strictly convex. Hence, for all of them the existence of corruption makes an honest bidder more aggressive. Another interesting example is provided by power function distributions, which include the uniform distribution as a particular case: for all of them, $\alpha(v)$ is linear. Therefore, as proved above, the possible presence of corruption has no influence on the behavior of an honest bidder.

3 Conclusion and some further implications

We have analyzed above how a specific form of corruption affects the behavior of honest bidders in frist-price auctions. Assuming that the auctioneer may reach an agreement with one of the bidders by which the latter will be shown all of her rival's bids and will be allowed to resubmit her bid accordingly, we have provided sufficient conditions to assess how honest bidders adjust their bids when facing a rival that is possibly dishonest. Those conditions determine whether an honest bidder will behave more, equally or less aggressively than in the absence of corruption. Furthermore, we have evaluated the extent to which most commonly used distributions satisfy one of these conditions.

Let us emphasize again that, even though we have been very precise in terms of the advantages that corruption confers to a dishonest bidder in our analysis, we have been vague when referring to the negotiations between such a bidder and the auctioneer that lead to a corrupt arrangement. Our results, then, can be regarded as relevant to any specific model for such negotiation in the context of sealed-bid auctions.

Although we do not carry out a full analysis of the effect of this form of corruption on efficiency, seller's revenue, and bidders' welfare, some implications can be easily derived. In the absence of corruption, and keeping all our other assumptions, it is well known that both the first-price and the second-price auction are efficient, since the object ends up in the hands of the bidder that values it most. We also know that the presence of corruption does not change the result in the second-price auction, so efficiency remains. A natural consequence of our previous analysis, however, is that the first-price auction will be inefficient when corruption is possible. An honest bidder with the highest valuation will always shade her bid (i.e. her equilibrium bid will be lower than her actual valuation). One of her rivals may have a valuation that is higher than the honest bidder's bid but lower than the honest bidder's valuation. If such a rival is allowed to examine bids and resubmit her own by the auctioneer, then the honest bidder will lose. This means as well that the first-price auction is worse in efficiency terms than the second-price auction in the presence of corruption.

In terms of bidders' welfare, some conclusions may be reached as well. In the cases where honest bidders do not become less aggressive in the presence of corruption (that is, if $\alpha(v)$ is linear or strictly convex), it is certain that this presence is detrimental to their expected utility in the auction. With corruption, any honest bidder wins with a lower probability and, when she wins, she has to pay a (weakly) higher price. Analogously, if honest bidders do not become more aggressive with corruption (that is, if $\alpha(v)$ is linear or strictly concave) the conspiracy between a dishonest bidder and the auctioneer is better off with corruption, since the dishonest bidder wins with higher probability for any valuation and does not pay a higher price. How they will distribute those gains is undetermined in our analysis.

As regards the owner of the object, we certainly know that corruption will lower her revenue when honest bidders do not become more aggressive (i.e. when $\alpha(v)$ is linear or strictly concave). When $\alpha(v)$ is strictly convex, however, the effect of corruption on revenue is uncertain, since honest bidders shade their bids to a lesser extent.

In spite of these preliminary implications, more work is needed to determine the signs of the effects of corruption on efficiency and welfare in the general case.

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