Hold-Up under Costly Litigation and Imperfect Courts of Law

C. Manuel Willington[¤]

July, 2003

Abstract

Two main results have been obtained on the literature on contractual solutions to the hold-up problem. First, a contract specifying a price and quantity of the …nal good to be traded will, fairly generally, induce e¢cient investments if these are 'sel…sh' in nature, i.e., each party's investment directly a¤ects only his own pro…t (Edlin and Reichelstein, 1996). Second, and in contrast, no contract however complicated is of any value in reducing the ine¢ciency if the investments are 'cooperative', i.e., each party's investment a¤ects directly only the other party's payo¤ (Che and Hausch, 1999). We show that courts of law may play a more important role in real contract disputes than has been realized. The key observation is that the presence of a court can make it valuable to specify putative investment levels in a contract - even if the court remains ignorant of the parties' actual investment levels. This is because the putative investment levels in‡uence the expected damages the court awards if it decides that breach occurred. The probability of the court deciding breach occurred is independent of the actual investment levels - they remain entirely unveri…able. It depends at most on the parties' court expenditures. These expenditures make litigation costly for the parties, and, therefore, in equilibrium they settle before trial. The presence of even such an imperfect court has a signi…cant impact on whether contracts alleviate hold-up ine¢ciencies. In the case of one-sided cooperative investment, we show that the …rst-best outcome can sometimes be achieved by the adoption of a simple non-contingent contract, contrary to the negative result of Che and Hausch (1999). Our result extends to the case of hybrid investment, provided the investment is mainly cooperative.

Keywords: hold-up, cooperative investment, incomplete contracts, costly enforcement.

JEL Numbers: D20, K40, K12, L22, C7.

Correspondent: C. Manuel Willington, ILADES, Universidad Alberto Hurtado, Erasmo Escala 1835, Santiago, Chile. Email: mwilling@uahurtado.cl

[¤] I thank Stefano Barbieri, Charles Bronowsky, Jan Eeckhout, Makoto Hanazono, George Mailath, Andrea Mattozzi, Nicola Persico, Felipe Zanna and especially Steve Matthews and Andy Postlewaite for useful comments and discussions. All remaining errors are mine.

1. Introduction

The hold-up problem arises in situations of bilateral trade when complete contracts are not available and relationship specific investments are required. The under-investment problem that may result was first analyzed by Coase in the early 1930s,[1] and, more recently, by Williamson (1985), Grout (1984), and Hart and Moore (1988). The literature later developed proposed contractual and non-contractual solutions to this hold-up problem (see Section 2 for a review of the literature). Two main results on contractual solutions to the hold-up problem have been obtained. First, a simple contract specifying the price and quantity of the final good to be traded will, fairly generally, induce efficient investments if the investments are 'selfish' in nature – i.e., if each party's investment directly affects only his own profit (Edlin and Reichelstein, 1996). Second, and in contrast, no contract however complicated is of any value in reducing the inefficiency if the investments are 'cooperative' in nature, i.e., if each party's investment directly affects only the other party's profit (Che and Hausch, 1999).

We maintain the assumption that investment is observable but not verifiable, and re-examine these previous results by taking into account the potential role of costly litigation and imperfect courts of law. The court in our model is assumed to obtain no information about the actual investment levels – they remain entirely unverifiable. To reduce the court's apparent usefulness, the probability of it deciding breach occurred is assumed to be independent of whether breach actually occurs. Since litigation is costly and there is complete information, in equilibrium the parties will settle before going to court. The presence of even such an imperfect court has a significant impact on whether contracts alleviate hold-up inefficiencies.

In the case of cooperative investment, the key observation is that the presence of a court can make it valuable to specify putative investment levels in a contract – even if the court remains ignorant of the parties' actual investment levels. This is because the putative investment levels influence the expected damages the court awards when it decides that breach occurred. In the case of one-sided cooperative investment, we show that a simple non-

[1] See Coase (2000), where he refers to his letters sent to Ronald Fowler about this issue in 1932 (p. 17).

contingent contract can be valuable, and the …rst-best outcome can sometimes be achieved, contrary to the negative result of Che and Hausch (1999). Our result extends to the case of hybrid investment (investment a¤ects both the seller's costs and the buyer's valuation), provided the main e¤ect of the seller's investment is to enhance the buyer's valuation.

In the case of sel…sh investment, the result that a simple contract can induce the …rst-best level of investment (Edlin and Reichelstein, 1996) remains valid in general.

The literature on the hold-up problem, and the contract theory literature in general, typically assumes that courts of law enforce the letter of the contract at no cost.[2] Parties are restricted to contract only on veri…able variables; but once this restriction is met, parties can rely on a court to enforce their agreement at no cost. That is, courts impose speci…c performance on any veri…able variables and they are silent about any non-veri…able variable.

On the other hand, the literature in law and economics takes a di¤erent approach to modeling litigation. Most of this literature assumes that going to court is costly and/or that the litigation outcome depends probabilistically on the parties' actions (i.e., how much they spend during trial, and, possibly, on past actions). Moreover, in reality, contracts typically include non-veri…able clauses, like the commitment to negotiate 'in good faith', or to make their 'best e¤ort'.

We take the law and economics literature approach by assuming litigation is costly and that the grounds for suing are given by a non-veri…able variable.

In our setup, only the seller invests before the transaction takes place. A simple contract will include a quantity to trade, an up-front payment, and a putative investment level.[3] The breaching of a contract does not refer, as in most of the literature on the hold-up problem, to the seller (buyer) refusing to deliver (accept) the good. Instead, we assume the buyer

[2] An exception is Anderlini, Felli and Postlewaite (2001), where the court, at the request of one of the parties, may choose to void the original contract if it considers that some 'unforeseen contingency' has occurred.

[3] Throughout the paper we use the expressions non-contingent contract and simple contract in an interchangeable way, as opposed to more complex contracts like menu contracts or contracts that include messages games.

may go to court, claim that the seller did not ful…ll the terms of the contract, and ask for damages to be awarded.[4] With some …xed probability the court …nds the seller breached the contract and with the complementary probability it …nds she did not breach it. In case of breach, the seller pays expected damages to the buyer and, we assume, parties are no longer bound by the original contract.

Court Practices - De…nitions

Expected damages are de…ned as the amount of money that would make the breach victim indi¤erent to receiving that money and to the contract being ful…lled. If the seller is judged to have fully breached the contract, the expected damages she has to pay would be the buyer's value of the goods to be traded minus what he was supposed to pay. In this case, parties would be no longer bound by the terms of the original contract.[5]

Alternatively, the court could …nd that the seller only partially breached the contract. In this case damages should only cover the di¤erence in the buyer's value from what it would have been if the contract terms were ful…lled. The concept of partial breaching is related to a duty to mitigate damages by the breach victim. If this duty applies, the victim may be forced to accept the good that does not conform to the contract and be compensated only for the di¤erence between the value of the contractually speci…ed good and what the breaching party is o¤ering.

We assume throughout that any breaching is considered total breaching. The assumption makes sense in the context of the hold-up problem where the non-veri…ability of the investment is the root of the problem. If the breaching is considered total and the court

[4] Given our contract de…nition, the buyer's claim before the court would be that the seller did not invest the contractually speci…ed amount. But there is nothing special about the breaching being related to the investment level. We could instead include an extra variable re‡ecting 'other aspects' of the contract (possibly payo¤ irrelevant), and assume the breaching refers to these other aspects.

[5] This does not mean the contract is voided. If the court ruling is to void the contract, then the buyer has no right to collect damages, only to recover any up-front payments made.

If a contract is voided, the buyer must be left in the same situation as if the contract was never signed while in the case of expected damages, the buyer must be left as if the terms of contract were ful…lled.

grants expected damages, all the court needs to know is the buyer's valuation of the goods for the contractually specified investment. On the other hand, in order for the court to award 'partial breaching damages', it would also need an estimation of what the seller really invested. Under these circumstances we may expect the court would be inclined to consider any breaching as total breaching.

Moreover, quoting Edlin (1996):

> "...This duty to mitigate [damages] is broader than often obtains. For instance, in Parker v. Twentieth Century-Fox [1970], the California Supreme Court held that Shirley MacLaine Parker did not need to accept Twentieth Century's offer to star in a western titled 'Big Country, Big Man' to mitigate damages for Twentieth Century's breach of the contract in which she was to star in a musical titled 'Bloomer Girl'. Also, in the context of the sale of goods, under the Uniform Commercial Code Section 2-601, the buyer has the right to 'reject the whole' if 'the goods or the tender of delivery fail in any respect to conform to the contract.' Moreover, under Section 2-711 a 'rightful' rejection by the buyer leaves her with the same remedies as if the seller had not performed at all..."[6]

Instead of expected damages, the court could, in principle, award liquidated damages to the victim of breach. Liquidated damages are specified by the parties in the original contract and are typically enforced "only if (a) at the time of contracting, the damage that the promisee will suffer in the event of breach (that is, the promisee's expectation) is uncertain, and (b) the amount of liquidated damages is both a reasonable estimate of (the mean of) those damages and are not disproportionate to the actual (ex post) damages. A larger amount is called a 'penalty' and is unenforceable" (Mahoney, 1999). We briefly consider the case of liquidated damages in Section 6.

A third option for the court is to award reliance damages. In case of breach, the promisee is compensated for whichever investment specific to the relationship he has made. In our

[6] His results depend on the assumption that the court considers the breaching as partial breaching and he advocates in favor of the breach victim having a broad duty to mitigate damages.

model, reliance damages make no sense since the investing party is the potential breacher.

To sum up, we assume the buyer can sue the seller for not investing the amount specified in the contract, and that the court will either uphold the contract or, if it finds breach, grant expected damages to the buyer.[7] We do not claim that this court behavior is optimal.[8] We simply take a positive standpoint here: a party can sue the other for virtually anything, and it has some chances of being successful.[9] We also assume the probability that the court finds the seller breached the contract is independent of the actual investment. This is an

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[7] Alternatively, we could assume that the buyer can go to the court and ask for the contract to be voided. Quoting Anderlini et al (2001):

> "...There are two primary categories under which this [a contract being voided] might happen. The first is impracticability of performance, that is, when unanticipated events subsequent to contracting have made the promised performance impossible. The second category is termed frustration of purpose. One view of frustration is that it will '...excuse performance where performance remains possible, but the value of performance to at least one of the parties and the basic reason recognized by both parties for entering into the contract have been destroyed by a supervening and unforeseen event'..."

Note that in this case the buyer would not need to claim the seller breached the contract at all, simply that some 'unforeseen contingencies' have arose.

Although this is not a plausible court ruling in a world with no uncertainty, we discuss in Section 6 how our results are affected if the court voids the contract rather than award expected damages (the insight derived there would also be valid in a model with uncertainty).

[8] Alternatively, an optimizing court may do better. Anderlini et al. (2001) assume the court is an optimizing player (whose interests are aligned with those of the contracting parties). Restricted to either voiding or upholding a contract, the court balances the trade-off between providing incentives to invest and providing insurance in the case of 'extreme' unforeseen contingencies.

[9] Some examples can illustrate this point: "A surfer recently sued another surfer for 'taking his wave.' The case was ultimately dismissed because they were unable to put a price on 'pain and suffering' endured by watching someone ride the wave that was 'intended for you."' (Source: CALA).

"A jury awarded $178,000 in damages to a woman who sued her former fiancée for breaking their seven-week engagement. The breakdown: $93,000 for pain & suffering; $60,000 for loss of income from her legal practice, and $25,000 for psychiatric counseling expenses." (Source: CALA) extreme assumption, but it is needed to be consistent with the standard assumption in hold-up models that investment is non veri…able. Obviously, a court with some ability to verify investment levels could enforce more complete contracts and so, in general, do better.

Results

Assuming this is the way courts work, we study whether non-contingent contracts can be used to achieve an outcome more e¢cient than the one induced with no contract.

We focus for most of the paper on the case of cooperative investment; speci…cally, the investment is made by the seller and a¤ects only the the buyer's valuation for the good. We choose a very simple model in which the court is characterized by three parameters and a function: the buyer's probability of winning, the buyer's and seller's litigation costs, and the 'damage function', which is assumed to depend only on the contractual terms (but not on the actual investment). This is for expositional simplicity only. The results hold in a more general setup in which the buyer's probability of winning and damages are functions of the parties' endogenously chosen court expenditures. We discuss in some detail this extension in Section 6.

The simpler model, however, helps highlight the main point of the paper: in the case of cooperative investment, where contracts have no value if enforcement is costless (Che and Hausch, 1999), the parties can use the fact that litigation is costly to implement a more e¢cient outcome even though no information is revealed during the trial process.

The idea is fairly intuitive. Although no information about the actual investment is revealed during the trial, the buyer's incentives to bring a suit do depend on the actual investment: the less the seller invested, the more valuable it is for the buyer to get out of the contract. Therefore, the parties can design a contract such that if the seller invests a certain level the buyer is exactly indi¤erent to either suing or not suing. The exact value of this critical investment level can, to some extent, be determined by the players when they sign the contract.

If the investment is slightly below that critical level, the buyer prefers to sue. In such a case the buyer's payo¤ increases, since it is equal to his expected payo¤ of going to court (approximately equal to his not-suing payo¤), plus the fraction of the total trial costs he appropriates in the renegotiation process. Since the total surplus is nearly the same, the seller su¤ers a loss when she slightly decreases her investment. This gives the seller incentives to invest.

The relevance of our results relies on the fact that we consider simple contracts and a realistic court game. It is well known, after Maskin and Tirole (1999), that the parties could achieve a …rst-best outcome if courts were able to enforce contracts that specify ex-post ine¢cient outcomes. There is an open debate in the literature on which mechanisms parties can use to be committed to ex-post ine¢cient outcomes. Beyond the potential enforceability of such mechanisms, we do not observe them in reality. Our results are a potential answer to this divorce between theory and reality. When parties face litigation costs and courts award expectation damages, simple contracts that can later be renegotiated can give the parties the right incentives. In these cases, there is no need for complicated message game contracts and devices to prevent renegotiation.

Outline

Section 2 discusses previous literature on the hold-up problem and some related law and economics literature. Section 3 presents the basic model. In Section 4, a numeric example is presented to illustrate our results in a simple way. The general results are presented in Section 5 and in Section 6 we discuss possible extensions of the basic model. We conclude in Section 7 by discussing our results in the context of the mechanism design literature.

2. Related Literature

Our work belongs to the literature on contractual solutions to the hold-up problem.[10] Chung (1991), Aghion, Dewatripont and Rey (1994) and Noldeke and Schmidt (1995) assume that parties can, in di¤erent ways, manipulate the mechanism they will use to revise the original

[10] A di¤erent approach looks at the possible role of di¤erent institutional arrangements such as vertical integration (Klein, Crawford, and Alchian, 1978), shifting property rights (Grossman and Hart, 1986 and Hart and Moore, 1990) and authority relationships (Aghion and Tirole, 1997).

contract and assign all the bargaining power to one party. This party becomes the residual claimant at the renegotiation stage and, therefore, has the right incentives to invest. By choosing the appropriate quantity in the original contract they affect the threat point in the renegotiation process and give the second party incentives to invest efficiently.

Edlin and Reichelstein (1996) highlight the importance of integrating to the analysis the breach remedies courts may use. Unlike Chung (1991), Aghion et al. (1994) and Noldeke and Schmidt (1995), they assume the bargaining powers are exogenously given (and both parties have some). For the case of one sided selfish investment, they show that both specific performance and expected damages allow the parties to design a contract such that the first-best is achieved. If both parties are supposed to invest, only the specific performance remedy allows the parties to attain the first-best.[11]

In the case of cooperative investment, Che and Hausch (1999) show that when parties cannot manipulate the renegotiation process (and both parties have some bargaining power), then not only is the first-best not achievable, but also no contract out-performs the null contract.[12]

As it is standard in the literature, Che and Hausch (1999) and Edlin and Reichelstein

[11] A related branch of the literature compares the efficiency of remedies to breach. Early works by Shavell (1980) and Rogerson (1984) show the potential incentives to overinvest in a relationship if the legal remedy is either the expectation damage rule or the reliance damage rule. In contrast, appropriate stipulated damages can induce an efficient level of investment.

Edlin (1994) shows that the result of Shavell (1980) and Rogerson (1984) does not hold when we combine 'cadillac contracts' (that ensure only one party will have incentives to breach) with up-front payments. Efficiency can be achieved under the expected damage rule combined with a broad duty to mitigate damages.

Ishiguro (1999) shows the convenience of an expected damage remedy over specific performance in a setup where only the seller invests and this investment reduces its costs. Che and Chung (1999) compare the efficiency of different legal remedies when investment is cooperative. In contrast to previous results, they find that specific performance is optimal when the court can without bias estimate the investment made.

[12] Earlier, MacLeod and Malcomson (1993) studied the case of one sided cooperative investment. Assuming the court can distinguish the cases where the parties do not trade at all from those where they exercise an outside option, and that the court enforces penalty payments in the latter case, then the efficient level of investment can be induced.

(1996) assume that parties are constrained to contract only on veri…able variables and the court can always at no cost enforce speci…c performance if one of the parties requires it. Our paper assumes a completely di¤erent enforcement technology, exploring the potential role of simple contracts when litigation is costly and courts are imperfect.

The fact that we choose a more realistic court game relates our paper to the law and economics literature. Concerned with di¤erent issues (optimality of the fee shifting rule, comparing the inquisitorial vs. the adversarial regimes, etc.), several papers model the litigation process as a rent-seeking game (Froeb and Kobayashi, 2000, and Gong and McAfee, 2000).[13]

Bull and Watson (2001) model the court game as one where parties present documents and the court rules according to the evidence presented. Di¤erent sets of documents may be available in di¤erent states, and this a¤ects the set of outcomes that may be implemented. Within this basic framework, Bull (2001a) studies the relative merits of an inquisitorial system vs. an adversarial system (when there are costs of producing and suppressing evidence). Bull (2001b) departs from the basic framework of Bull and Watson (2001) by assuming documents are costly to produce; he analyzes the potential role of redundant documents (two documents are redundant if they are available in the same states) to enlarge the set of implementable outcomes.

Ishiguro (2002) is related to our paper, he also shows the value of the expected damage rule in an incomplete contracting setting. He considers a moral hazard model where the action the agent takes is observable and 'endogenously' veri…able. He models a court game where the agent can spend resources, and his action is veri…ed by the court with some probability that depends on the amount he spends. Ishiguro provides a necessary and su¢cient

[13] Bernardo et al. (2000) and Sanchirico (2000) consider models where the outcome of the court game depends not only on what the parties do in that stage (i.e., how much they spend), but also on some past actions. For example, in a car accident, the probability of being found guilty is larger for someone that has been negligent than for someone that took adequate precautions. Alternatively, it might be cheaper for an innocent defendant to generate exculpatory evidence and, depending on the way the court rules, he might be able to signal his innocence by outspending a guilty one.

condition for the existence of a contract that induces the …rst-best action.[14] A contract in his setting is a menu that speci…es a payment for each di¤erent action. In equilibrium, the agent takes the …rst-best action, the principal breaches the contract by paying a wage less than the contractually speci…ed wage for the …rst-best action, and, nonetheless, the agent has no incentives to go to court. Our result has a similar ‡avor: the contract is typically breached in equilibrium and the buyer (the principal) has no incentives to go to court when the seller invested e¢ ciently.

Our model di¤ers in several aspects from Ishiguro (2002). First and most importantly, we assume that no information about the agent's action is revealed in the court process. Second, we make use of simpler contracts that specify a single action and payment rather than a menu.[15] Third, although litigation is costly and there is complete information, in Ishiguro (2002) parties cannot avoid trial by settling out of court.

3. The Model

Timing

We consider a situation where after contracting at time 0, the seller (she) chooses an investment level x 2 $[0; X]$ at time 1; and both players observe it. This investment is purely cooperative: it increases the valuation of the good for the buyer (he), but it does not a¤ect her own production costs. He decides whether to sue the seller – claiming she did not invest the stipulated amount– at time 2 and, if he does, the court game is played at 3. At time 4 the transaction is completed and payo¤s realized.

[14] This condition relates the productivity of the resources spent in court, the costs for the agent of undertaking the …rst best e¤ort and the cost of the lowest possible e¤ort.

[15] This di¤erence is not trivial. In Ishiguro (2002), only the principal can breach the contract by paying the agent less than what is speci…ed in the contract for the action taken. In our setup, only the agent (seller) can breach the contract by taking an action di¤erent from the one speci…ed in the contract.

Willington (2002) presents an example in which the veri…cation technology does not satisfy the necessary and su¢ cient condition identi…ed in Ishiguro (2002), and shows that if this veri…cation technology is available to the principal, then the …rst-best can be attained with a non-contingent contract.

We allow parties to renegotiate before the buyer decides whether to sue, and then before the final transaction has to be made. Parties will do so before wasting resources in the court, and, off the equilibrium path, after the court's ruling.[16] We assume the outcome of the renegotiation process is ex-post efficient, and the bargaining powers of the parties are exogenously given: μ for the buyer and $1 - \mu$ for the seller.

The time of the game is summarized below:

contract k is signed	investment x is made and observed	buyer decides to sue or not	if buyer sues, court game is played	transaction and payoffs are realized
↓	↓	↓	↓	↓
0	1	2	3	4

——————————————————————————————————→

	settle out of court?		renegotiation?	
	↑		↑	

A contract is a triple $k = \left(\bar{x}, \bar{q}, \bar{t}\right)$, where $\bar{x} \in [0, X]$ is the level the seller agrees to invest, $\bar{q} \in \Re_+$ is the quantity to be transacted, and $\bar{t} \in \Re$ is an up-front payment from the buyer to the seller.[17] Although the investment level is not verifiable, a value for it is included in the contract. In case of dispute, if the court rules in favor of the buyer, the seller pays damages that depend on the contracted levels of investment $(\bar{x})$, and the quantities to be traded $(\bar{q})$. We assume that at date 4 the court enforces, at no cost, whichever transaction $\left(\bar{q}', \bar{t}'\right)$ the parties agreed to (it could be the result of a renegotiation either before date 2 or after the court ruling).

Notation and Assumptions

We denote the buyer's valuation of q units when the seller invests x as $V(x, q)$, and the

[16] Allowing the parties to renegotiate after the court ruling implies that the court can not impose ex-post inefficient outcomes on the parties.

[17] In our model, $\bar{t}$ plays only a role for dividing the surplus. We will often refer to a contract as a pair $(\bar{x}, \bar{q})$.

seller's cost as $C(q)$. Both are assumed to be continuous and twice differentiable, increasing in q; equal to zero when $q = 0$; and $V(x; q) - C(q)$ is assumed to be less than zero for q large enough. We further assume:[18]

$$V_x(x; q) > 0 \text{ and } V_{x;q}(x; q) > 0 \text{ (if } q > 0). \tag{A1}$$

We denote by $q(x)$ the efficient quantity to be traded when the seller invests x:

$$q(x) \equiv \arg\max_q V(x; q) - C(q).$$

We assume $q(x)$ is unique and strictly positive. Given (A1), this implies $q(x)$ is increasing in x. Let the total surplus when production is efficient be

$$S(x) = V(x; q(x)) - C(q(x)),$$

and assume it is strictly concave. We denote the maximizer of $S(x) - x$ by x^{FB}.

If no contract is signed, the seller will solve

$$\max_x (1 - \mu) S(x) - x.$$

We denote the solution to this problem by x^w, and assume $x^w > 0$ (note that $x^{FB} > x^w$).[19]

A court in our model is a quadruple $(F; \beta; \sigma; D(\overline{x}; \overline{q}))$, where F is the probability the buyer wins the trial, β and σ are the buyer's and seller's court costs, and $D(\overline{x}; \overline{q})$ are the damages the buyer collects from the seller in case of breach when the contract is $(\overline{x}; \overline{q})$.

We assume $D(\overline{x}; \overline{q})$ is continuous, equal to zero if $\overline{q}$ is zero, and strictly increasing in both arguments unless $\overline{q} = 0$. Moreover, we assume

$$D(\overline{x}; \overline{q}) - (1 - \mu) V(x; \overline{q}) - \mu C(\overline{q})$$

is strictly quasi-concave, bounded above, and negative for some $\overline{q}$ large enough.

[18] Subindexes denote partial derivatives.

[19] The 'w' is standard in the literature and refers to Williamson, who was among the first to address this under-investment problem.

The canonical example of a damage function is simply $D(\bar{x}, \bar{q}) = V(\bar{x}, \bar{q})$, the expectation damage function.[20]

Our approach is to take the court $(F, \beta, \sigma, D(\cdot))$ as given and characterize the optimal contract for this court and the corresponding equilibrium investment level. We then characterize the set of courts for which contracting is valuable, and the set for which the first-best is achievable.

Two generalizations of this simple court game are discussed in section 6. The first one is to assume there is a set of courts from which parties can choose. The selection of the court is then included in the original contract. The second generalization is to model the court game as a rent-seeking game, where the parties optimally choose β and σ, and their decisions affect the trial outcome.

Payoffs

Note that the renegotiation between dates 1 and 2 can take place under two qualitatively different scenarios. Given a contract k and an investment level x, it might be optimal for the buyer not to sue the seller (N) even if no agreement is reached at this renegotiation stage. In this case, the threat point is determined exclusively by the original contract, and, therefore, the payoff for the buyer after renegotiating would be

$$\Pi^{B,N}(k, x) = V(x, \bar{q}) + \mu\left[S(x) - (V(x, \bar{q}) - C(\bar{q}))\right],$$

where $V(x, \bar{q})$ is what he would obtain under the original contract (if the parties do not renegotiate), and the second term is the fraction of the renegotiation surplus the buyer appropriates.[21]

[20] Note that $D(\bar{x}, \bar{q})$ being an approximation to $V(\bar{x}, \bar{q})$ corresponds to the definition of expected damages because we assumed that $\bar{t}$ is an up-front payment. If we were to assume that $\bar{t}$ is a payment to be made at the moment of the transaction, then the expected damages the buyer should be entitled to collect in case of breach would be only $D(\bar{x}, \bar{q}) - \bar{t}$ (as an approximation to $V(\bar{x}, \bar{q}) - \bar{t}$). In both cases, the $\bar{t}$ only serves the role of dividing the surplus.

In order to implement expectation damages perfectly the court would have to know the buyer's valuation function $V(x, q)$. This might be a strong assumption.

[21] All payoffs are net of the up-front payment $\bar{t}$.

On the other hand, if, given $(k;x)$, it is optimal for the buyer to go to court (C) when no agreement is reached, then the threat point for the renegotiation is given by the payoffs the players would obtain if they go to court. Since the parties will settle out of court (S), the payoff for the buyer in this case will be what he would obtain if they go to court, $\pi^{B,C}(k;x)$, plus a fraction μ of the renegotiation surplus,[22] that is

$$\pi^{B,S}(k;x) = \mu(\beta+\sigma) + \pi^{B,C}(k;x),$$

where the buyer's payoff from going to court is

$$\pi^{B,C}(k;x) = -\beta + F\left[D(\overline{x};\overline{q}) + \mu S(x)\right] + (1-F)\,\pi^{B,N}(k;x).$$

If the buyer wins the trial – with probability F – he collects damages $D(\overline{x};\overline{q})$ and, since they are then no longer bound by the original contract, he appropriates a fraction μ of the total surplus. If he loses – with probability $1-F$ – he gets what he would have received if there had been no trial. In both cases, he has to pay his court costs (β).

Similarly, we can define seller's payoff in the three situations:

$$\begin{aligned}
\pi^{S,N}(k;x) &= -C(\overline{q}) + (1-\mu)\left[S(x) - V(x;\overline{q}) + C(\overline{q})\right] - x \\
\pi^{S,C}(k;x) &= -\sigma + F\left[-D(\overline{x};\overline{q}) + (1-\mu)S(x) - x\right] + (1-F)\,\pi^{S,N}(k;x) \\
\pi^{S,S}(k;x) &= (1-\mu)(\beta+\sigma) + \pi^{S,C}(k;x).
\end{aligned}$$

Note that we implicitly assume that after they agree on a new contract, the buyer cannot sue the seller. This is with no loss of generality. One possibility for the players is to agree on a contract that specifies only a quantity to be traded and a payment, which leaves the buyer with no grounds to sue the seller (in our model the buyer sues claiming the seller did not invest what she committed to investing). Alternatively, the parties can get the payoffs specified above by, between 1 and 2, annulling the contract and agreeing on a payment t', and later splitting the surplus $S(x)$ according to their bargaining powers. The annulled contract guarantees the buyer cannot sue.

[22] By settling out of court the parties are saving the court costs $\beta+\sigma$. This is the surplus 'generated' in this renegotiation process.

4. An Illustrative Example

We present a simple example that illustrates the main point of the paper: even though investment is not verifiable (the enforcement technology is independent of the investment), players can use a costly and imperfect court to improve on their implementation problem.

Assume that only one or two units can be sold and that the seller can only choose to invest 0, 2, or 4. Assume the bargaining power for the buyer is $\mu = 0.7$ and the buyer's valuation and seller's production cost are

$V(x, q)$

$x \backslash q$	1	2
0	3.0	4.9
2	4.5	7.5
4	5.5	8.9

$C(q)$

q	1	2
C	1.5	3.5

Notice that the efficient quantity is one if investment is zero and two if investment is either two or four. Overall, the efficient investment level is two:

$$S(2) - 2 = 7.5 - 3.5 - 2 > 3 - 1.5 - 0 = S(0);$$
$$S(2) - 2 = 7.5 - 3.5 - 2 > 8.9 - 3.5 - 4 = S(4) - 4.$$

Note also that, given the assumption that μ is 0.7, there is a hold-up problem: if no contract is signed and the seller invests 2, she would get 0.3 of the post-investment surplus and therefore his total payoff would be

$$0.3(7.5 - 3.5) - 2 = -0.8;$$

while if she chooses $x = 0$ she would get

$$0.3(3 - 1.5) = 0.45.$$

Claim 1 below is an implication of Che and Hausch (1999).

Claim 1. When the parties are unable to commit themselves to avoiding renegotiation and courts only impose specific performance on verifiable variables at no cost, $x = 2$ is not an equilibrium whatever the contract.

Che and Hausch (1999) prove this for arbitrary 'message game' contracts. To illustrate the point, we argue here the simpler case of non-contingent contracts.

Note that the seller's payoff when she invests x and the contract specifies a quantity $\bar{q}$ and a payment $\bar{t}$ is

$$\bar{t} - C(\bar{q}) - x + (1-\mu)[S(x) - V(x;\bar{q}) + C(\bar{q})].$$

The terms $\bar{t}$ and $C(\bar{q})$ are independent of the investment x. Therefore, by increasing x, the seller only affects $S(x)$ and $V(x;\bar{q})$.

Note that the seller appropriates only a fraction $(1-\mu)$ of the increase in the surplus, but bears the full cost of the investment x. If this were the only effect of increasing x, the seller would choose $x = 0$ (we have already shown that $(1-\mu)S(0) > (1-\mu)S(2) - 2$).

The second effect of increasing x is on $V(x;\bar{q})$. The larger x is the better the buyer's bargaining position will be (unless $\bar{q}$ is zero); and, therefore, this effect reinforces the incentives to underinvest.

More formally, and after some trivial manipulation, the seller's payoff differential between investing 2 and 0 can be written as

$$\Pi^{S,N}(k;2) - \Pi^{S,N}(k;0) = \{(1-\mu)[S(2) - S(0)] - 2 + 0\} - (1-\mu)[V(2;\bar{q}) - V(0;\bar{q})].$$

The first term is negative (that is why there is a hold-up problem in the first place) and the second one is strictly negative for $\bar{q} > 0$. It is therefore optimal for the seller to choose $x = 0$.

We now want to illustrate how our alternative enforcement technology can help. Assume the court is characterized by

$$F = 0.75$$
$$\beta = 3.4$$
$$\sigma = 0$$
$$D(x;q) = V(x;q).$$

That is, if the buyer decides to go to court and spends $\beta = 3.4$, the court rules, with probability 0.75, that the seller has breached the contract, and awards damages $D(\bar{x}, \bar{q})$. For simplicity we assume seller's court costs are zero.

How can this court, which does not extract any information about x and is costly to use, be helpful? The intuition is the following: the parties sign a contract such that if the seller invests 2, the buyer prefers not to sue $[\Pi^{B,N}(k;2) \geq \Pi^{B,C}(k;2)]$, but if the seller invests 0, the buyer strictly prefers to sue $[\Pi^{B,N}(k;0) < \Pi^{B,C}(k;0)]$.

This implies that the renegotiation threat point changes: if $x = 2$, the threat point is $[\Pi^{B,N}(k;2), \Pi^{S,N}(k;2)]$, while if $x = 0$, it is $[\Pi^{B,C}(k;0), \Pi^{S,C}(k;0)]$. If the damages that the seller has to pay in case the court finds breaching are large enough, then the seller might be deterred from investing $x = 0$. Note, however, that if the damages are too high, then the buyer will always prefer to sue (i.e., the inequality $\left[\Pi^{B,N}(k;2) \geq \Pi^{B,C}(k;2)\right]$ will not hold), and the seller, given that she will be sued anyway, will choose to underinvest. Claim 2 formalizes the argument.

Claim 2. The contract $k = \{\bar{x}, \bar{q}, \bar{t}\} = \{4, 2, 6.1\}$ induces an investment level $x = 2$ and gives the buyer and seller payoffs of 1.4 and 0.6, respectively.

Proof: Note that if $x = 0$ and the parties do not reach an agreement between dates 1 and 2, the buyer prefers to sue:

$$\Pi^{B,N}(k;0) = 4.9 + 0.7(1.5 - 1.4) = 4.97 <$$
$$0.25\,(4.97) + 0.75\,(8.9 + 0.7 \cdot 1.5) - 3.4 = 5.305 = \Pi^{B,C}(k;0) - \bar{t}.$$

Therefore, the threat point for the renegotiation is given by $\left(\Pi^{B,C}(k;0), \Pi^{S,C}(k;0)\right)$.

By settling out of court the players then get

$$\begin{aligned}
\Pi^{B,S}(k;0) &= 5.305 + 0.7 \cdot 3.4 = 7.685 \\
\Pi^{S,S}(k;0) &= S(0) - \Pi^{B,S}(k;0) = -6.185.
\end{aligned}$$

On the other hand, if $x = 2$, the buyer will not sue the seller:

$$\Pi^{B,N}(k;2) = 7.5 \geq 0.25\,(7.5) + 0.75\,(8.9 + 0.7 \cdot 4) - 3.4 = 7.25 = \Pi^{B,C}(k;2),$$

therefore, the threat point is given by $\left(\pi^{B,N}(k,2), \pi^{S,N}(k,2)\right)$. Since the contract specifies the efficient quantity ($\bar{q} = 2$), given the investment $x = 2$, there is no room for renegotiation. Therefore, the players' payoffs will be $\left(\pi^{B,N}(k,2) - \bar{t}, \pi^{S,N}(k,2) + \bar{t}\right) = (1.4, 0.6)$.

Since 0.6 is greater than $\pi^{S,S}(k,0) + \bar{t} = -0.085$, the seller would choose to invest $x = 2$. This proves Claim 2.

This example also illustrates a typical feature of the optimal contract: it specifies an investment level larger than the one parties are trying to induce, and, therefore, it is breached in equilibrium.[23,24]

The next section shows, in the general setup, when the first-best can be induced and when, even if the first-best is not achievable, contracting is still valuable in the sense that it induces an investment level above the one chosen by the seller if no contract is signed (x^w).

5. General Results

The Hold-Up Problem with Costless and Perfect Courts

In the standard hold-up model with purely cooperative investment, any contract $\left(\bar{q}, \bar{t}\right)$ with $\bar{q} > 0$ induces lower investments than the null contract; this is the result of Che and

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[23] This is not necessarily the case. The reader can check that if $\beta = 2.15$, then the contract $k = (2, 2, 6.1)$ (which specifies both the efficient investment and the efficient quantity) induces the investment $x = 2$. In a more general setup, the contracted quantity may also differ from the quantity finally traded.

[24] This enforcement technology can be reinterpreted in the setup of Bull (2001b). The natural way to 'translate' the non-verifiability assumption to the setup in which documents are the primitives, is by assuming that the set of available documents is not affected by the investment level. We can then reinterpret our example: a set of documents M is available with a 75% probability (independently of the investment made by the seller) if the buyer spends $\$3.4$. The parties can then sign the following contract:

> 'The buyer will pay $\$6.1$ to the seller in exchange for 2 units, unless the buyer presents the documents M. In such a case, the seller will pay $\$2.8$ $(8.9 - 6.1)$ to the buyer, and the parties will not be obliged to any other transaction.'

In equilibrium, the seller will choose to invest $x = 2$ and the buyer will not try to obtain the documents M. (Note that in this case $\bar{t}$ is not an up-front payment, but still does not play any role.)

Hausch (1999). Given a contract $\left(\bar{q}, \bar{t}\right)$, the seller would solve

$$\underset{0 \le x}{\text{Max}}\ \bar{t} - C(\bar{q}) + (1-\mu)\left\{S(x) - [V(x,\bar{q}) - C(\bar{q})]\right\} - x.$$

The first order necessary condition is:

$$S'(x) - \frac{1}{1-\mu} \le V_x(x,\bar{q}), \quad (= \text{ if } x > 0).$$

Let us remember that x^w is defined as the investment level the seller will choose if no contract is signed ($x^w = \arg\max_{x \ge 0}(1-\mu)S(x) - x$; it satisfies $S'(x^w) = (1-\mu)^{-1}$). Since $S'(x)$ is positive and decreasing in x, and $V_x(x,\bar{q})$ is positive, the first order condition cannot be satisfied if $x > x^w$. Moreover, since $V_x(x,\bar{q})$ is strictly positive unless $\bar{q}$ is zero, the equilibrium investment (which must satisfy the first order condition) is maximized if $\bar{q} = 0$.

The intuition for this result is the same we discussed in the previous section. By increasing her investment, the seller does two things: first, she increases the total surplus; and second, she improves the buyer's bargaining position ($V(x,\bar{q})$ increases). Because the seller appropriates only a fraction $(1-\mu)$ of the total surplus, the first effect will induce her to invest up to the x such that $(1-\mu)S'(x) = 1$ (i.e., up to x^w). The second effect induces the seller to invest even less. Since $V_{x,q}(x,\bar{q}) > 0$ for any $\bar{q} > 0$, the higher $\bar{q}$ is the stronger the incentive to underinvest. The optimal contract must then specify $\bar{q} = 0$, and it induces x^w.

The Hold-Up Problem with Costly and Imperfect Courts

The intuition from the example in the previous section is still valid in the more general setup: if we want to induce an investment level $x^* > x^w$, we need a contract k such that the buyer has no incentives to sue when x^* is invested, but he strictly prefers to sue if the seller underinvests. That is:

$$\Pi^{B,C}(k,x^*) - \Pi^{B,N}(k,x^*) \le 0, \text{ and}$$
$$\Pi^{B,C}(k,x') - \Pi^{B,N}(k,x') > 0, \text{ for any } x' < x^*.$$

After manipulating the expressions for $\Pi^{B,C}$ and $\Pi^{B,N}$, we see that the buyer's payoff differential between suing and not suing is:

$$\Delta(k,x) \equiv \Pi^{B,C}(k,x) - \Pi^{B,N}(k,x) = F\left[D(\bar{x},\bar{q}) - (1-\mu)V(x,\bar{q}) - \mu C(\bar{q})\right] - \beta.$$

This expression is strictly decreasing in x when $\overline{q} > 0$. Therefore, the above two conditions reduce to $\Delta(k; x^*) = 0$. Lemma 1 shows that this is a necessary condition to induce the desired investment level x^*.

Lemma 1. For all $x^* > 0$, if $x^* > x^w$ and $\Delta(k; x^*) \neq 0$, then x^* is not an equilibrium investment level of the game defined by contract k.

Proof: see Appendix A.

The intuition for this result is simple: if $\Delta(k; x^*) > 0$ and the seller invests $x = x^*$, the buyer will choose to go to court. Given that she will be sued, it is easy to show that the optimal investment for the seller is some x smaller than x^w. On the other hand, if $\Delta(k; x^*) < 0$, there will exist an investment $x^{*\prime} \in [x^w; x^*)$ such that she will not be sued ($\Delta(k; x^{*\prime}) \leq 0$) and her payoff will be larger ($\Pi^{S,N}(k; x^{*\prime}) \geq \Pi^{S,N}(k; x^*)$).

Note that the necessary condition $\Delta(k; x^*) = 0$ cannot be satisfied if β is too large or if F is too small. Intuitively, in such cases it will never be optimal for the buyer to sue the seller and, therefore, the seller would have the same incentives as if there were not a costly court game.[25]

Let $X^* = \{x^* : \exists k \text{ s.t. } \Delta(k; x^*) = 0\}$. Note that if

$$\max_{\overline{x};\overline{q}} F\left[D(\overline{x}; \overline{q}) - (1-\mu) V(x^*; \overline{q}) - \mu C(\overline{q})\right] - \beta > 0,$$

there will be a continuum of contracts such that $\Delta(k; x^*) = 0$.[26]

$\Delta(k; x^*) = 0$ is not only necessary for x^* to be induced, but it also implies that the seller would not 'slightly' underinvest: if she chooses to invest $x^* - \varepsilon$, then it will be optimal for the buyer to sue and the threat point for the renegotiation will be $\left(\Pi^{B,C}, \Pi^{S,C}\right)$ rather than $\left(\Pi^{B,N}, \Pi^{S,N}\right)$. For ε small, the total surplus will only be slightly affected and $\Pi^{B,C}(k; x^* - \varepsilon)$ will be nearly the same as $\Pi^{B,N}(k; x^*)$.[27] Since the total court expenditures $(\beta + \sigma)$ are

[25] This would also be the case if $D(\overline{x}; \overline{q})$ is too small relative to $V(x; \overline{q})$ and $C(\overline{q})$.

[26] Let us remember the assumptions that all functions are continuous, that $V(x; 0) = C(q) = 0$, and that the expression $[D(\overline{x}; \overline{q}) - (1-\mu) V(x; \overline{q}) - \mu C(\overline{q})]$ is negative for $\overline{q}$ large enough.

[27] Recall that $\Delta(k; x^*) = 0 \Leftrightarrow \Pi^{B,N}(k; x^*) = \Pi^{B,C}(k; x^*)$.

positive and non-negligible and the buyer's payoff is $\Pi^{B,C}(k, x^* - \varepsilon) + \mu(\beta + \sigma)$; then it necessarily holds that the seller's payoff – equal to the total surplus minus the buyer's payoff – is discretely reduced when she invests $x^* - \varepsilon$ instead of x^*.

To make sure that the seller does not prefer to 'severely' underinvest, we need to compare her payoff when she invests x^* and will not be sued with her payoff if she underinvests and the parties settle before trial. The proof of Proposition 1 is straightforward.

Proposition 1. An investment level $x^* > x^w$ is induced with a contract k if and only if

$$\begin{aligned} \Pi^{S,N}(k, x^*) - \max_{0 \leq x'} \Pi^{S,S}(k, x') &\geq 0, \text{ and} \\ \Delta(k, x^*) &= 0. \end{aligned} \qquad (*)$$

Proof: see Appendix A.

Although correct, Proposition 1 is not very informative. It says nothing about the optimal contract or when condition (*) holds.

In the case the first-best level of investment is achievable, there would typically be a continuum of contracts that can induce it; and all we can say in that case is that the contract k chosen will satisfy (*) for x^{FB} and $\Delta\left(k, x^{FB}\right) = 0$.

However, if the first-best is not achievable, then it has to be either because (*) is binding for the equilibrium x^*, or because for any $\varepsilon > 0$ $x^* + \varepsilon \notin X^*$ (i.e., there is no contract k satisfying $\Delta(k, x^* + \varepsilon) = 0$).[28]

We need to further explore the restrictions imposed by $\Delta\left(k, x^{FB}\right) = 0$ and (*) to be able to say something about the optimal contract when x^{FB} is not achievable and to characterize under which circumstances this contract improves over the no contract case.

Lemma 1 implies that, if we want to induce $x^* \in X^*$, we can restrict attention to contracts satisfying $\Delta(k, x^*) = 0$. When this condition is satisfied, equation (*) can be written as

$$\mu(\beta + \sigma) + (1 - \mu)\left[S(x^*) - S(x^c)\right] - x^* + x^c - (1 - F)(1 - \mu)\left[V(x^*, \bar{q}) - V(x^c, \bar{q})\right] \geq 0, \qquad (*')$$

[28] As will become apparent, if an investment $x > x^w$ cannot be induced in equilibrium, then no other $x' > x$ will be inducible. Moreover, if x can be induced in equilibrium, then any $x'' \in [x^w, x]$ will be inducible.

where x^c solves seller's 'cheating' problem: $\max_{0 \leq x} \Pi^{S,S}(k; x)$. The above expression is decreasing in $\bar{q}$ since $x^* > x^w \geq x^c$ (in the proof of Lemma 1 we show that $x^w \geq x^c$).[29] Therefore, we can restrict attention to the contract with the lowest possible $\bar{q}$ that also satisfies $\Delta(k; x^*) = 0$. If such a contract does not satisfy (*'), then x^* cannot be induced.

For all $x^* \in X^*$, define

$$\hat{\bar{q}}(\bar{x}; x^*) \equiv \min\{\bar{q} : F[D(\bar{x}; \bar{q}) - (1-\mu)V(x^*; \bar{q}) - \mu C(\bar{q})] - \beta = 0\}.$$

Recall that we assumed that $D(\bar{x}; \bar{q})$ is increasing in $\bar{x}$; then it is straightforward to show that $\hat{\bar{q}}(\bar{x}; x^*)$ is decreasing in $\bar{x}$ and increasing in x^* and, therefore, we can restrict attention to contracts where $\bar{x} = X$. Let $\hat{q}(x^*) \equiv \hat{\bar{q}}(X; x^*)$.

Proposition 2. Let x^M be the maximum investment level that can be induced by any contract. If $x^M > x^w$ and $F < 1$, the unique contract that induces x^M is $(\bar{x}; \bar{q}) = \left(X; \hat{q}\left(x^M\right)\right)$.

Proof: see Appendix A.

Corollary 1. If an investment level $x^* > x^w$ can be induced with a contract $(\bar{x}; \bar{q})$, then it can also be induced with the contract $(X; \hat{q}(x^*))$.

Proof: see Appendix A.

Knowing what the optimal contract looks like (Proposition 2) and what conditions are necessary and sufficient to induce a particular investment level (Proposition 1), it is immediate to characterize for which courts contracting will be valuable (an $x^* > x^w$ can be induced), and when the first-best will be achievable. This is done in Propositions 3 and 4 and illustrated with a numerical example below.

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[29] The problem $\max_{0 \leq x'} \Pi^{S,S}(k; x')$ can be rewritten as

$$\max_{0 \leq x'} K + (1-\mu)S(x') - x' - (1-F)(1-\mu)V(x'; \bar{q});$$

where K is a constant. The terms including x' in the above expression are the same (with opposite sign) that we have in (*'); therefore, by the envelope theorem, we can restrict attention to the direct effect of $\bar{q}$ on (*').

We derive our results in terms of the court game parameters $(F;\beta;\sigma)$ and the damage function $D(\overline{x};\overline{q})$. Abusing notation, we denote by $x^c(\overline{q};F)$ the optimal 'cheating' investment:

$$x^c(\overline{q};F) \equiv \arg\max_{0\leq x} \Pi^{S;S}(k;x);$$

and by $\phi(x;\beta/F)$ the smallest $\overline{q}$ that satisfies

$$F\left[D(X;\overline{q}) - (1-\mu)V(x;\overline{q}) - \mu C(\overline{q})\right] - \beta = 0.$$

Proposition 3. Contracting has value, i.e. an investment $x^* > x^w$ can be induced, if and only if

$$\max_{\overline{q}} F\left[D(X;\overline{q}) - (1-\mu)V(x^w;\overline{q}) - \mu C(\overline{q})\right] - \beta > 0; \text{ and}$$
$$\mu(\beta+\sigma) + G(x^w;F;\beta) > 0;$$

where

$$\begin{aligned} G(x;F;\beta) \equiv{} & (1-\mu)\left[S(x) - S(x^c(\phi(x;\beta/F);F))\right] - x + x^c(\phi(x;\beta/F);F) - \\ & (1-\mu)(1-F)\left[V(x;\phi(x;\beta/F)) - V(x^c(\phi(x;\beta/F);F);\phi(x;\beta/F))\right]. \end{aligned}$$

Proof: see Appendix A.

The first inequality is required for the existence of a contract k satisfying $\Delta(k;x^*) = 0$ for some $x^* > x^w$; and the second inequality implies that the contract $(X;\phi(x^w;\beta/F))$ satisfies (*') for x^w as a strict inequality. Then, by continuity, there must be a contract that induces an investment level above $x^* > x^w$.

Proposition 4. The first-best level of investment can be induced if and only if

$$\max_{\overline{q}} F\left[D(X;\overline{q}) - (1-\mu)V\left(x^{FB};\overline{q}\right) - \mu C(\overline{q})\right] - \beta \geq 0; \text{ and}$$
$$\mu(\beta+\sigma) + G\left(x^{FB};F;\beta\right) \geq 0.$$

Proof: see Appendix A.

The proof is similar to the one of Proposition 3. It relies on the fact that an optimal contract is $\left(X;\phi\left(x^{FB};\beta/F\right)\right)$.

Example to Illustrate Propositions 3 and 4

Assume:

$$\begin{aligned} V(x;q) &= D(x;q) = 4q\left(1 + x^{1/4}\right) \\ C(q) &= 4q + 2q^2 \\ \mu &= 0.5 \\ \sigma &= 0.25 \end{aligned}$$

It is easy to show that for this example $x^{FB} = 1$ and $x^w = 0.25$.

Given a value of β and an investment level $x^* \in [0;X]$ that we want to induce, we define $F_m(\beta;x^*)$ as the smallest $F \in [0;1]$ that satisfies

$$\begin{aligned} \max_{\overline{q}} F\left[D(X;\overline{q}) - (1-\mu)V(x^*;\overline{q}) - \mu C(\overline{q})\right] - \beta &\geq 0; \quad \text{and} \\ \mu(\beta + \sigma) + G(x^*;F;\beta) &\geq 0. \end{aligned}$$

Note that the above expressions are both increasing in F. Then, if the buyer's probability of winning in court (F) is larger than $F_m(\beta;x^*)$, there will exist a contract k that induces an investment x^* (i.e., there will exist k such that $\Delta(k;x^*)$ is zero and (*') is satisfied). If for a particular pair $(\beta;x^*)$ there is no $F \in [0;1]$ that satisfies both conditions, then x^* cannot be induced.

Figure B.1 illustrates this discussion for the numerical example proposed above. Figure B.2 shows how F_m is affected when σ increases to 0.35.

The case of $F = 1$

Two definitions are needed to characterize what can be implemented for the particular case of $F = 1$. Let $x(\alpha)$ be the $x \geq x^w$ that satisfies

$$(1-\mu)\left[S(x^w) - S(x)\right] - x^w + x = \alpha \quad \text{(see Figure B.3)};$$

and let $\hat{x}$ be the largest x for which there is a contract that satisfies $\Delta(k;\hat{x}) = 0$; that is:

$$\hat{x} \equiv \begin{cases} X & \text{if} \quad \max_{\overline{q}} D(X;\overline{q}) - (1-\mu)V(X;\overline{q}) - \mu C(\overline{q}) - \beta \geq 0; \\ x : \max_{\overline{q}} D(X;\overline{q}) - (1-\mu)V(x;\overline{q}) - \mu C(\overline{q}) - \beta = 0 & \text{otherwise.} \end{cases}$$

Corollary 2. Assume $\hat{x} \geq x^w$. If $F = 1$, any $x \in [x^w, \min\{\hat{x}, x(\mu(\beta, \sigma))\}]$ can be induced.

Proof: see Appendix A.

The proof of Corollary 2 uses the fact that when $F = 1$ the optimal 'cheating' investment level is $x^c = x^w$ and condition (*') simplifies to $\mu(\beta + \sigma) + (1 - \mu)[S(x^*) - S(x^w)] - x^* + x^w + \geq 0$. The definition of $x(\mu(\beta + \sigma))$ implies that the previous expression is non-negative for any $x^* \in [x^w, x(\mu(\beta + \sigma))]$.

Comparative Statics

The following corollaries are derived from Propositions 1 and 2.

Corollary 3. Assume the maximum inducible investment level x^M is smaller than X. If σ increases, then x^M strictly increases unless

$$\max_{\bar{q}} F\left[D(X, \bar{q}) - (1 - \mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta = 0.$$

Proof: see Appendix A.

Intuitively, if $\max_{\bar{q}} F\left[D(X, \bar{q}) - (1 - \mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta > 0$ and x^M is the maximum investment level that can be induced, then it has to be that (*') is binding. An increase in σ eases this constraint.

Corollary 4. Assume the maximum inducible investment level x^M is smaller than X. If F increases, then x^M strictly increases.

Proof: see Appendix A.

A larger F eases (*') and increases $\max_{\bar{q}} F\left[D(X, \bar{q}) - (1 - \mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta$, which allows us to find a contract k' such that $\Delta(k', x') = 0$ for some $x' > x^M$ and (*') is still satisfied.

Corollary 5. If $\max_{\bar{q}} F\left[D(X, \bar{q}) - (1 - \mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta = 0$ and β is increased, then x^M strictly decreases.

The proof is straightforward: $\bar{q}$ being the maximizer of the left hand side of the above expression implies that if β increases then the only way the equality can be satisfied is by decreasing x^M.

The effect of β on x^M is not always negative. If $\max_{\bar{q}} F\left[D(X;\bar{q}) - (1-\mu)V\left(x^M;\bar{q}\right) - \mu C(\bar{q})\right] - \beta > 0$ we cannot tell which is the effect of β on x^M. In that case (and since x^M is the largest x we can induce), it necessarily holds that the inequality (*') is binding:

$$\mu(\beta+\sigma) + G(x;F;\beta) = 0.$$

Then, an increase in β has two effects that go in opposite directions: the first term above obviously increases, but the increase in β also affects $G(x;F;\beta)$ negatively.

Recall the definition, $G\left(x^M;F;\beta\right)$ is

$$\begin{aligned}(1-\mu)&\left[S\left(x^M\right) - S\left(x^c\left(\phi\left(x^M;\beta/F\right);F\right)\right) - x^M + x^c\left(\phi\left(x^M;\beta/F\right);F\right)\right] - \\ &(1-\mu)(1-F)\left[V\left(x^M;\phi\left(x^M;\beta/F\right)\right) - V\left(x^c\left(\phi\left(x^M;\beta/F\right);F\right);\phi\left(x^M;\beta/F\right)\right)\right].\end{aligned}$$

When β is increased, the contracted quantity ($\bar{q} = \phi(x;\beta/F)$) has to increase in order to satisfy $\Delta\left(k;x^M\right) = 0$. That increase in $\bar{q}$ makes the last term above larger in absolute value (since $V_q\left(x^M;\bar{q}\right) - V_q(x^c;\bar{q}) > 0$), so that $G(x;F;\beta)$ decreases as β increases (assuming $F < 1$).[30]

6. Extensions of the Basic Model - Discussion

Endogenous Court Expenditure

We have characterized above for which 'courts of law' – parameters $(F;\beta;\sigma)$ and function $D(\bar{x};\bar{q})$ – contracting has value and when the first-best can be achieved. We have chosen such a simple model only for expositional ease and to highlight the main result: a costly court, unable to obtain any information about the facts of the case, can help the parties.

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[30] Consider Figure B.1 and assume $x^M = 1$ and $F = 0.5$. Then if β is small (approximately 0.4) an increase in β would increase x^M; while if β is large (approximately 0.9) then its increase would decrease x^M.

There are at least two ways in which the model can be enriched and the results not be affected (or strengthened). First, we could endogenize β and σ by assuming the parties' expenditures affect their probability of winning the trial and the amount of damages to be paid. To remain consistent with the assumption that investment is not verifiable, neither the probability of winning nor the damages should depend directly on the investment made. Indirectly, the investment made by the seller will affect the buyer's incentives to go to court (as in the simplest model) and to spend money on the litigation process.

In this more general model, we would have $F(\cdot)$ and $D(\cdot)$ as functions of β and σ, and the parties would choose them to maximize their expected payoffs. The court game would then look like a rent-seeking game with an endogenous prize. Under suitable assumptions, we can show that the equilibrium of this court game, $(\beta^*;\sigma^*)$, is unique and strictly positive. The outcome of this game, i.e. $\beta^*;\sigma^*;F(\beta^*;\sigma^*);$ and $D(k;\beta^*;\sigma^*)$, is what we considered here as our primitive 'court of law.'

This way of extending the court game is appealing. The quality of lawyers (and their salaries), the time spent to prepare for trial, the expert witnesses hired, etc. all affect the probability of winning or losing, and the amount of damages.[31]

Optimal Choice of Courts

Another extension (that could be combined with the previous one) is to assume that parties have some discretion over which court to choose. In the simpler setup, we could assume there is a set of available courts from which the parties can choose and they will pick the one that allows them to induce the most efficient outcome possible. A way of considering

[31] An example that comes immediately to mind is the criminal case against O.J. Simpson. Despite the enormous physical evidence, the defendant, after hiring a 'dream team' of lawyers and spending something between 4 and 7 million dollars (Source: CNN) was found not guilty.

This is true not only in criminal cases, quoting S. Macaulay (1985), "if lawyers of equal ability represent clients with equal resources and willingness to invest them in a case falling under Article II [of the Uniform Commercial Code], the case will end in a tie. Those who can afford to play invest in the skills of large law firms. They play the litigation game by expanding procedural complexity to draw out the process. Others who cannot afford to invest as much must drop out."

this alternative together with the previous one is by assuming that F and $D(\overline{x};\overline{q})$ depend not only on what the parties spend in court, but also on the contract the parties sign.

This extension is consistent with what we observe in reality. Parties can choose, to some extent, the particular court they want to solve their potential disputes, and they do so strategically. According to the Uniform Commercial Code (§1-105(1)), "...when a transaction bears a reasonable relation to this state and also to another state or nation the parties may agree that the law either of this state or of such other state or nation shall govern the rights and duties."[32]

The particular court chosen will affect the litigation costs of the parties. If two corporations are located in two different states and they choose to be governed by the state law of one of the parties' state, we should expect the other one to face higher litigation costs derived from having to hire lawyers licensed in the other state. If parties choose the law of a third state,[33] then both parties would probably face higher litigation costs.

The election of the particular state court may also affect the probability of one or the other party winning and the potential damages. Some state courts are more experienced than others in particular matters and therefore their ruling may be easier to predict. Juries in some states may be biased against large corporations and be prone to award large damages.

Choosing a particular court is not limited to the election of the state law that shall govern the relationship. Even for the same state law, parties can still affect their expected litigation costs when designing the contract. For example, parties can agree on the contract that any disputes will be solved by arbitration, which is typically cheaper than going to trial.[34] Moreover, once parties agree in the contract they will solve any disputes through arbitration, they can still include different clauses that will affect the cost of the arbitration

[32] If the two parties do not stipulate which state's law they want to govern their contract, then, should a dispute develop, each state will have a set of default rules called 'choice of law rules.' The default rules typically say that the state that has the 'greatest interest in the case' should rule on the dispute.

[33] They can do so only if the transaction is somehow related to this third state.

[34] Arbitration is one of the several Alternative Dispute Resolution (A.D.R.) methods parties can choose. Others are mediation, early neutral evaluation, and conciliation.

mechanism.[35]

Hybrid Investment

Although we developed the case of purely cooperative investment, the main results extend to the case of hybrid investment (investment affects both the buyer's value and the seller's costs), provided it is not 'too selfish'.

In the case of cooperative investment our mechanism works because, given a contract that satisfies $\Delta(k; x^*) = 0$; if the seller underinvests, the buyer has an incentive to go to court. The intuition behind this is simple: the lower the investment, the lower the value of the $\bar{q}$ units the buyer is committed to buy. Below a critical value of x (that depends on $\bar{q}$) the buyer simply prefers to go to court and, with probability F, avoid the commitment to buy those $\bar{q}$ units.

In the case of hybrid investment, this intuition is incomplete. If the seller decreases her investment, it will also affect her costs. If her costs increase as she decreases x,[36] her threat point for the renegotiation worsens and this favors the buyer, who appropriates a fraction μ of the renegotiation surplus.

Analytically, the buyer's payoff differential between going to court or not is given by

$$\Delta(k; x) = F\left[D(\bar{x}; \bar{q}) - (1 - \mu) V(x; \bar{q}) - \mu C(x; \bar{q})\right] - \beta.$$

In the case of cooperative investment, $\Delta(k; x) = 0$ guarantees $\Delta(k; x') > 0$ for any $x' < x$. But in the case of hybrid investment, $\Delta(k; x)$ will be increasing or decreasing in x depend-

[35] Quoting DiCarlo (2002), "they [arbitration clauses] can limit or eliminate the depositions, interrogatories, document requests, and pretrial motions that are responsible for much of the sometimes crushing expense of litigation. In recent years, courts have allowed the contracting parties broad discretion to make up any rules they wish concerning who will hear the dispute and what rules will govern the outcome."

[36] So far we assumed x is a quality enhancing investment and, therefore, it increases the buyer's valuation for the good.

In the case the investment affects both the quality of the good and the production costs, there is no 'natural' assumption about the effect of x on the production costs. For example, a higher x could be a better quality and more expensive painting for cars that requires a different painting process. This high quality painting process could, in principle, be cheaper or more expensive than the low quality one.

ing on the sign of $-(1-\mu)V_x(x;\bar{q})-\mu C_x(x;\bar{q})$. Then, our results would extend, mutatis mutandis, to the case of hybrid investment as long as $(1-\mu)V_x(x;\bar{q})>-\mu C_x(x;\bar{q})$.

Purely Selfish Investment

Consider now the case when seller's investment affects only her own costs and assume $C_x(x;q)<0$ and $C_{xq}(x;q)<0$. In this case, Edlin and Reichelstein (1996) show that the first-best can be achieved with a non-contingent contract. This result is derived for two different court remedies, expected damages and specific performance. As is standard in the literature, the breaching refers to one of the parties refusing to complete the transaction and it is assumed that the court, at no cost, either enforces the contract (imposes specific performance) or awards damages at no cost for the parties.

We reconsider the result in our setup assuming, as we have so far, that the buyer can claim that the seller is not fulfilling other terms of the contract, different from the delivery of the goods (as in most of the literature, we are assuming the delivery/acceptance of the goods can be enforced at no cost).

The incentives for the buyer to sue are driven, as in the cooperative investment case, by the payoff differential between suing and not suing. In the case of selfish investment, and assuming $D(q)=V(q)$; this differential can be written as

$$\Delta(\bar{q};x)=F\mu\left[V(\bar{q})-C(x;\bar{q})\right]-\beta,$$

the buyer would sue iff $\Delta(\bar{q};x)>0$.[37]

Logically, if $\Delta\left(\bar{q};x^{FB}\right)\leq 0$ for any $\bar{q}$, then the buyer will never sue the seller and the efficiency result of Edlin and Reichelstein (1996) will hold in our setup. Assume, for the contrary, that $\Delta\left(\bar{q};x^{FB}\right)>0$ for some $\bar{q}$, and define

$$\begin{aligned}
q_L\left(x^{FB}\right) &\equiv \min\left\{\bar{q}:\Delta\left(\bar{q};x^{FB}\right)=0\right\}\\
q_H\left(x^{FB}\right) &\equiv \max\left\{\bar{q}:\Delta\left(\bar{q};x^{FB}\right)=0\right\}
\end{aligned}$$

[37] Note that since the investment is selfish it does not affect the buyer's valuation. Then, the only relevant variable for the contract is $\bar{q}$.

Proposition 5 below identifies under which conditions the first-best is not achievable. For simplicity we maintain the assumption that $D(q) = V(q)$.[38]

Proposition 5. The first-best cannot be achieved if

$$q_L\left(x^{FB}\right) < q\left(x^{FB}\right) < q_H\left(x^{FB}\right); \text{ and} \qquad (+)$$

$$C_x\left(x^{FB}, q_H\left(x^{FB}\right)\right) > -\frac{1}{1-F}. \qquad (++)$$

Proof: see Appendix A.

The intuition for this result is as follows. An investment level x^* can, in principle, be induced with two different types of contracts: one in which the buyer has no incentives to sue ($\Delta(k; x^*) \leq 0$); and one in which the buyer would optimally sue ($\Delta(k; x^*) > 0$).

If we want to induce x^{FB} with a contract of the first type, then $\bar{q} = q\left(x^{FB}\right)$ is required (see Appendix A). However, if $q\left(x^{FB}\right) \in (q_L(x), q_H(x))$, it will be optimal for the buyer to sue if $\bar{q} = q\left(x^{FB}\right)$ and, therefore, there is no contract of the first type that induces x^{FB}.

We show in Appendix A that if we want to induce x^{FB} with the second type of contracts, then $\bar{q}$ has to satisfy $C_x\left(x^{FB}, \bar{q}\right) = -\frac{1}{1-F}$ (note that for any $F > 0$ such a $\bar{q}$ will be greater than $q\left(x^{FB}\right)$). However, if $C_x\left(x^{FB}, q_H\left(x^{FB}\right)\right) > -\frac{1}{1-F}$, then the required $\bar{q}$ will be too large and the buyer will have no incentives to sue. Therefore x^{FB} cannot be induced with the second type of contract.

Note that the converse of Proposition 5 is not true. If $(+)$ does not hold, then a contract specifying $\bar{q} = q\left(x^{FB}\right)$ will induce $x = x^{FB}$. However, if $(+)$ still holds but $(++)$ does not, then $x = x^{FB}$ is not guaranteed. It could be the case that the seller, facing a contract that specifies $\bar{q}$ such that $C_x\left(x^{FB}, \bar{q}\right) = -\frac{1}{1-F}$, finds it optimal to choose an investment $x' < x^{FB}$ such that the buyer will not sue her. $\bar{q}$ satisfying $C_x\left(x^{FB}, \bar{q}\right) = -\frac{1}{1-F}$ only guarantees that the seller will not deviate to an x' such that the buyer still finds it optimal to sue.

It is important to highlight that the negative result of Proposition 5 does not hold if there is uncertainty either about the buyer's valuation or the seller's costs. Suppose the

[38] We maintain our assumptions about $D(\bar{x}, \bar{q}) - (1 - \beta) V(x, q) - \beta C(q)$ made in Section 3. For the case of selfish investment and assuming $D(q) = V(q)$, the assumptions are simply that $V(q) - C(x, q)$ is strictly quasi-concave and negative for q large enough.

uncertainty is captured by the variable λ that is continuously distributed on its support $\left[\underline{\lambda}, \bar{\lambda}\right]$. The payoff differential between going and not going to court for the buyer will now depend also on λ, so we can define

$$q''_H\left(x^{FB}\right) \equiv \min\left\{\bar{q} : q' > \bar{q} \Rightarrow \Delta\left(q', x^{FB}, \lambda\right) \le 0, \forall (q, \lambda)\right\}.$$

Now, if $q\left(x^{FB}\right)$ is larger than $q''_H\left(x^{FB}\right)$, then a contract specifying $\bar{q} = q\left(x^{FB}\right)$ will give the buyer no incentives to sue (for any λ) if $x = x^{FB}$, and $x = x^{FB}$ will be the solution to the seller's problem given that she will not be sued.

Suppose then that $q\left(x^{FB}\right)$ is smaller than $q''_H\left(x^{FB}\right)$ and let $Q > q''_H\left(x^{FB}\right)$ be such that $V(Q, \lambda) - C(X, Q, \lambda) \le 0 \;\forall \lambda \in \left[\underline{\lambda}, \bar{\lambda}\right]$. Then, if a contract specifies $\bar{q} = Q$, the buyer will not sue the seller (for any x). It is trivial to show that the seller will choose an investment level $x > x^{FB}$.

On the other hand, if the contract specifies a very small $\bar{q}$ (i.e., $\bar{q} = 0$), the seller will choose some $x' < x^{FB}$. Since the seller's payoff varies continuously with $\bar{q}$, by the intermediate value theorem there has to be a $\bar{q} \in (0, Q)$ such that the seller invests x^{FB}.

Liquidated Damages - Voided Contracts

Similar results to those obtained in Section 5 can be derived if we assume courts would enforce any level of liquidated damages or if we assume that the court, rather than awarding damages to compensate the buyer, would simply void the contract with probability F.

The case of liquidated damages is straightforward. Let L be the level of damages stipulated in a contract k',[39] the payoff differential for the buyer between suing and not suing is now:

$$\Delta(k', x) = F\left[L - (1 - \mu) V(x, \bar{q}) - \mu C(\bar{q})\right] - \beta.$$

To induce some $x^{*} > x^{w}$ we still have to restrict attention to contracts satisfying $\Delta(k', x) = 0$. The only difference with the case of expected damages is that now a large

[39] The parties will include an investment level in the contract only if it is necessary to give the buyer grounds for suing. For the analysis made, the relevant contract will just be $(L, \bar{q})$. Again, $\bar{t}$ only determines the division of the surplus.

β is not necessarily a problem. In the case of expected damages and for β 'too large', it might be that $\max_k \Delta(k; x^*) < 0$ and, therefore, x^* would not be inducible. With liquidated damages, by simply stipulating a larger L; this problem is solved. The only relevant constraint is then (*').

The case in which the court voids the contract when it finds breach is analytically identical to the case of liquidated damages. If the contract is voided, then the parties will not be committed to any transaction and the seller will have to return any money paid by the buyer in advance.

The payoff differential for the buyer is

$$\Delta(k''; x) = F\left[\bar{t} - (1 - \mu) V(x; \bar{q}) - \mu C(\bar{q})\right] - \beta.$$

Note that in this case $\bar{t}$ plays a non-trivial role; it has to be chosen to satisfy $\Delta(k''; x) = 0$.[40] Therefore, the parties cannot use $\bar{t}$ to achieve the division of the surplus they desire. They must rely on a side payment to do this. This payment cannot be part of the contract nor can it be legally associated to the transaction, or else it would be reversed when the contract is voided.

7. Conclusions

We have considered a hold-up model with the following characteristics: only one party invests, her investment is purely cooperative, and the parties can renegotiate the original contract before going to court and/or after the court rules. The renegotiation process is exogenously given. The main difference with previous literature is the 'enforcing technology'.

Unlike most of the literature where courts enforce contracts at no cost, we assume costly litigation. The non-investing party can sue the other one claiming breach, and has a positive probability of prevailing in the trial.[41] To be consistent with the assumption of non-verifiability of the investment, we assume that no information about the investment made is

[40] The 'relevant contract' is, in this case, the pair $\left(\bar{t}; \bar{q}\right)$.

[41] The breaching may refer to the seller underinvesting or not fulfilling some other conditions of the contract.

revealed in the litigation process.

In this set up, we find that parties can improve over the non-contracting case by signing a simple contract that can later be renegotiated. The contract is such that if the seller invests the level the parties are trying to induce, the buyer's payoff is the same whether he chooses to sue or not. Therefore, for any investment level below the one parties intend to induce the buyer will find optimal to sue the seller.

Although the trial stage is never reached (parties can always renegotiate and avoid paying the court costs), when the seller underinvests the threat point for the renegotiation changes and this can be enough to discourage the underinvesting.

To make our point clear we chose a very simple enforcement technology. A more sophisticated court model could be developed where the trial outcome would depend on the players' expenditure on the court game. Moreover, in reality we can expect the parties being able to affect, by changing the contract clauses, not only the probability of success in a trial in favor of one or the other party but also the equilibrium court costs. We discussed these possibilities in Section 6.

Our results give a plausible explanation of why we do not see in reality complicated mechanisms as the ones suggested in the literature (i.e., message games or mechanisms to avoid renegotiation).[42] It may be that parties really do not need these mechanisms because with simple contracts (and courts like the ones we assumed) they may achieve (or at least approximate) the first-best outcome.

It is well known in the mechanism design literature that two parties can improve on their implementation problem if they are allowed to include a third party in the contract or if they can commit themselves to burning money. Our game structure is equivalent to a contract where the buyer, after observing the investment made by the seller, sends a message that determines, as specified in the contract, a transfer from the buyer to the seller, transfers from the buyer and seller to the third party (or simply how much money each party has to burn),

[42] For an interesting discussion about the possibility of parties committing themselves not to renegotiate see Hart & Moore (1999) and Tirole (1999).

and, with some probability, a change in the status quo point for the renegotiation. Before sending that message, the buyer and seller are allowed to renegotiate the original contract. By choosing the appropriate message-transfers functions, parties would be able to induce the …rst-best level of investment.[43]

It is also well established that this kind of contracts does not help when the third party can collude with one of the players, or if the parties have no way of committing themselves to burning money. In our model, we make use of a real institution (the court) that allows the parties to commit themselves to burning money: for the buyer to be compensated for the alleged breach, he and the seller have to waste resources in the court game (lawyer fees, etc.). Courts are particularly appealing as third parties because, ideally, they will not collude with any of the parties. The burning money process is appealing as long as we consider realistic the assumptions made about the way courts work.

[43] In our setup, the …rst-best may or may not be achievable, depending on the particular money burning and seller-buyer transfer technology. The parties can manipulate this technology to some extent with the initial contract, but they are restricted by the way the court works.

A. Appendix: Proofs Missing from the Text

Proof of Lemma 1.

- Suppose the seller chooses $x^* > x^w$ and $\Delta(k; x^*) > 0$: Then the buyer will strictly prefer to sue the seller and, since litigation is costly, they will settle out of court. The seller would solve

$$\max_{x'} \Pi^{S;S}(k; x') :$$

Let x^c be the solution to this problem. Then, the first order necessary condition requires:

$$(1-\mu) S_x(x^c) - 1 - (1-\mu)(1-F) V_x(x^c; \bar{q}) \leq 0; \quad (= \text{if } x^c > 0).$$

Recall $x^* > x^w \Leftrightarrow (1-\mu) S'(x^*) < 1$: Therefore the first order condition is not satisfied if $x^c = x^*$:

- Suppose the seller chooses $x^* > x^w$ and $\Delta(k; x^*) < 0$: By continuity, there exist ε such that $x^* - \varepsilon \geq x^w$ and $\Delta(k; x^* - \varepsilon) \leq 0$: Then

$$\Pi^{S;N}(k; x^* - \varepsilon) - \Pi^{S;N}(k; x^*) = (1-\mu)\left[S(x^* - \varepsilon) - S(x^*)\right] - x^* + \varepsilon + x^* +$$
$$(1-\mu)\left[V(x^*; \bar{q}) - V(x^* - \varepsilon; \bar{q})\right] > 0$$

since both lines on the right hand side are greater than zero. Therefore, it is not optimal for the seller to invest $x^* > x^w$: ■

Proof of Proposition 1.

$\Rightarrow$ By assumption, there is k such that

$$\Delta(k; x^*) = 0 \text{ and } \Pi^{S;N}(k; x^*) - \max_{0 \leq x'} \Pi^{S;S}(k; x') \geq 0:$$

Facing such a contract, the seller has to choose her investment level: if she chooses $x' < x^*$; then the buyer will prefer to sue

$$\Delta(k; x^*) = 0 \Rightarrow \Delta(k; x') > 0 \text{ if } x' < x^*:$$

The parties would settle out of court and the seller would end up with $\Pi^{S;S}(k; x')$; which, by assumption, is smaller than what she would get by investing x^*.

An argument similar to the one made for the proof of Lemma 1 implies she would never optimally choose $x' > x^* \geq x^w$ when $\Delta(k, x^*) = 0$.

$\Leftarrow$ (by contradiction)

-Suppose x^* is induced by contract k but $\Delta(k, x^*) \neq 0$. Lemma 1 shows this is not possible.

-Suppose that for any contract k such that $\Delta(k, x^*) = 0$, $\Pi^{S,N}(k, x^*) - \max_{0 \leq x'} \Pi^{S,S}(k, x') < 0$ holds. Then obviously the seller prefers to invest the x' that solves $\max_{0 \leq x'} \Pi^{S,S}(k, x')$, which is always smaller than or equal to x^w and therefore $x^* > x^w$ is not induced in equilibrium. ■

Proof of Proposition 2. (by contradiction).

* Suppose $(\bar{x}, \bar{q})$ induces x^M and $\bar{x} < X$.

By Proposition 1 it must be that

$$\Delta\left((\bar{x}, \bar{q}), x^M\right) = 0, \text{ and}$$

$$\Pi^{S,N}\left((\bar{x}, \bar{q}), x^M\right) - \Pi^{S,S}\left((\bar{x}, \bar{q}), x^c(\bar{q}, F)\right) \geq 0.$$

Consider the contract $\left(\bar{x}, \phi\left(\bar{x}, x^M\right)\right)$ and notice that

$$\Delta\left(\left(\bar{x}, \phi\left(\bar{x}, x^M\right)\right), x^M\right) = 0,$$

and since $\phi\left(\bar{x}, x^M\right) \leq \bar{q}$, $F < 1$, and $V_{x,q}(x, q) > 0$, then

$$\Pi^{S,N}\left(\left(\bar{x}, \phi\left(\bar{x}, x^M\right)\right), x^M\right) - \Pi^{S,S}\left(\left(\bar{x}, \phi\left(\bar{x}, x^M\right)\right), x^c\left(\phi\left(\bar{x}, x^M\right), F\right)\right) \geq 0.$$

- In the neighborhood of $\left(\bar{x}, \phi\left(\bar{x}, x^M\right), x^M\right)$, the function $\Delta(\cdot)$ is strictly increasing in $\bar{x}$, increasing in $\bar{q}$, and strictly decreasing in x. Therefore, there exist $\varepsilon > 0$, $\bar{q}'$ such that $\bar{q}' < \phi\left(\bar{x}, x^M\right)$ and $\Delta\left((X, \bar{q}'), x^M + \varepsilon\right) = 0$.

- The function $\Pi^{S,N}\left((\bar{x}, \bar{q}), x^M\right) - \Pi^{S,S}\left((\bar{x}, \bar{q}), x^c(\bar{q}, F)\right)$ is strictly decreasing in $\bar{q}$ (for $F < 1$). Using the envelope theorem, it is then immediate to conclude that

$$\Pi^{S,N}\left((X, \bar{q}'), x^M\right) - \Pi^{S,S}\left((X, \bar{q}'), x^c(\bar{q}', F)\right) > 0.$$

Then, by continuity, there exist $\delta \in (0, \varepsilon)$ such that

$$\Pi^{S,N}\left((X, \bar{q}'), x^M + \delta\right) - \Pi^{S,S}\left((X, \bar{q}'), x^c(\bar{q}', F)\right) \geq 0.$$

- If $\delta > \varepsilon$, then $x^M + \varepsilon$ can be induced with the contract $(X, \bar{q}')$.
- If $\varepsilon > \delta$, then $x^M + \delta$ can be induced with the contract $\left(X, \hat{q}\left(x^M + \delta\right)\right)$.[44]

* Suppose $(\bar{x}, \bar{q})$ induces x^M, $\bar{x} = X$ and $\bar{q} \neq \hat{q}\left(x^M\right)$.

- Since $\Delta\left(k, x^M\right) = 0$ is necessary for x^M to be implemented (Lemma 1), the definition of $\hat{q}\left(x^M\right)$ (smallest $\bar{q}$ such that $\Delta\left((X, \bar{q}), x^M\right) = 0$) implies that $\bar{q} > \hat{q}(x^M)$.
- The assumption of strict quasi-concavity of $D(\bar{x}, \bar{q}) - (1 - \mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})$ implies $\Delta\left((X, \bar{q}'), x^M\right) > 0$ for any $\bar{q}' \in \left(\hat{q}\left(x^M\right), \bar{q}\right)$.

Therefore, there exists $\varepsilon > 0$ and $\bar{q}' \in \left(\hat{q}\left(x^M\right), \bar{q}\right)$ such that $\Delta\left((X, \bar{q}'), x^M + \varepsilon\right) = 0$. Moreover, for any $\varepsilon' < \varepsilon$ there is $\bar{q}'_{\varepsilon'} \in \left(\hat{q}\left(x^M\right), \bar{q}'\right)$ such that $\Delta\left((X, \bar{q}'_{\varepsilon'}), x^M + \varepsilon'\right) = 0$.

- Since $(\bar{x}, \bar{q})$ induces x^M, $V_{qx}(x, q) > 0$, $F < 1$, and $\bar{q} > \hat{q}(x^M)$, then

$$\Pi^{S,N}\left(\left(X, \hat{q}(x^M)\right), x^M\right) - \Pi^{S,S}\left(\left(X, \hat{q}(x^M)\right), x^c\left(\hat{q}(x^M), F\right)\right) > 0.$$

- By continuity there is some $\delta \in (0, \varepsilon)$ such that

$$\Pi^{S,N}\left((X, \bar{q}'_\delta), x^M + \delta\right) - \Pi^{S,S}\left((X, \bar{q}'_\delta), x^c(\bar{q}'_\delta, F)\right) \geq 0,$$

therefore the contract $(X, \bar{q}'_\delta)$ induces the investment level $x^M + \delta > x^M$.

■

Proof of Corollary 1. Immediate from the proof of proposition 2. ■

Proof of Proposition 3.

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- Suppose $\max_{\bar{q}} F\left[D(X, \bar{q}) - (1 - \mu) V(x^w, \bar{q}) - \mu C(\bar{q})\right] - \beta \leq 0$. Then there is no $(\bar{x}, \bar{q})$ satisfying $\Delta(k, x) = 0$ for any $x > x^w$. Lemma 1 implies then that no $x > x^w$ can be induced.
- Suppose $\mu(\beta + \sigma) + G(x^w, F, \beta) \leq 0$. Note that by Corollary 1 we can restrict attention to contracts of the form $(X, \hat{q}(\cdot))$. Note also that $G(x, F, \beta)$ is decreasing in x: as x increases

[44] Note that $\hat{q}\left(x^M + \delta\right)$ is well defined: $\Delta\left((X, \bar{q}'), x^M + \varepsilon\right) = 0$ and $\varepsilon > \delta$ imply $\Delta\left((X, \bar{q}'), x^M + \delta\right) > 0$. Recall also that $\Delta\left((X, 0), x^M + \delta\right) < 0$. Therefore, by continuity, there is some $\bar{q}'' < \bar{q}'$ such that $\Delta\left((X, \bar{q}''), x^M + \delta\right) = 0$.

ȹ(x; ¯=F) increases and G (x; F; ¯) decreases by

$$(1 ¡ ¹) (1 ¡ F) [V_q (x; ȹ(x; ¯=F)) ¡ V_q (x^c (ȹ(x; ¯=F) ; F) ; ȹ(x; ¯=F))] :$$

Then ¹ (¯ + ¾) + G (x; F; ¯) < 0 8x > x^w; and, by Proposition 1, no x > x^w can therefore be implemented.

)

- By Corollary 1 we can restrict attention to contracts of the form (X; ȹ(¢)) : By continuity, if ¹ (¯ + ¾) + G (x^w; F; ¯) > 0 and $\max_{q̄}$F [D (X; q̄) ¡ (1 ¡ ¹) V (x^w; q̄) ¡ ¹C (q̄)] ¡ ¯ > 0; there is x > x^w such that ¹ (¯ + ¾) + G (x; F; ¯) ¸ 0 and ¢ ((X; ȹ(x; ¯=F)) ; x) = 0; and, therefore, it can be implemented. ■

Proof of Proposition 4. Immediate from Propositions 1, and 2, and Corollary 1. ■

Proof of Corollary 2.

* Note that F = 1 implies x^c = x^w for any contract k: Condition (*') then reduces to

$$¹ (¯ + ¾) + (1 ¡ ¹) [S (x) ¡ S (x^w)] ¡ x + x^w ¸ 0:$$

By de…nition of x (¹ (¯ + ¾)) ;

$$¹ (¯ + ¾) + (1 ¡ ¹) [S (x (¹ (¯ + ¾))) ¡ S (x^w)] ¡ x (¹ (¯ + ¾)) + x^w = 0:$$

S (x) being strictly concave and S^0 (x^w) = (1 ¡ ¹) then imply

$$¹ (¯ + ¾) + (1 ¡ ¹) [S (x) ¡ S (x^w)] ¡ x + x^w ¸ 0; \quad 8x 2 [x^w; x (¹ (¯ + ¾))]:$$

* The de…nition of ȿ implies that ȹ(x) is well de…ned for any x 2 [x^w; ȿ].

* Then, by Proposition 1, any x · min © x (¹ (¯ + ¾)) ; x^{FB} ª can be implemented. ■

Proof of Corollary 3. Since x^M is the maximum investment level we can induce and, we assume, $\max_{q̄}$F £ D (X; q̄) ¡ (1 ¡ ¹) V ¡ x^M; q̄ ¢ ¡ ¹C (q̄) ¤ ¡ ¯ > 0; then it has to be that equation (*') is binding. As ¾ is increased we can …nd x > x^M such that (*') is satis…ed and ¢ ((X; ȹ(x)) ; x) = 0: ■

Proof of Corollary 4.

If x^M can be induced, by Proposition 1, there must exist $(\bar{x}, \bar{q})$ such that

$$
\begin{aligned}
F\left[D(\bar{x}, \bar{q}) - (1-\mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta &= 0, \text{ and} \\
\mu(\beta+\sigma) + G\left(x^M, F, \beta\right) &\geq 0.
\end{aligned}
$$

Note that the two expressions are strictly increasing in F. Therefore, for $F' > F$ we must have

$$
\begin{aligned}
F'\left[D(\bar{x}, \bar{q}) - (1-\mu) V\left(x^M, \bar{q}\right) - \mu C(\bar{q})\right] - \beta &> 0, \text{ and} \\
\mu(\beta+\sigma) + G\left(x^M, F', \beta\right) &> 0.
\end{aligned}
$$

A continuity argument, similar to the one of Proposition 3, allows us to conclude that there is an $x > x^M$ such that $\mu(\beta+\sigma) + G(x, F, \beta) \geq 0$ and $\Delta((X, \phi(x, \beta/F)), x) = 0$ and, therefore, x can be implemented. ■

Proof of Proposition 5.

* Suppose the contract $(\bar{q})$ that induces the first-best is such that the buyer does not sue the seller. The assumptions that $V(\bar{q}) - C(x, \bar{q})$ is strictly quasi-concave and negative for $\bar{q}$ large enough imply that, if the buyer will not sue the seller, then $\bar{q} \notin \left(q_L\left(x^{FB}\right), q_H\left(x^{FB}\right)\right)$. Therefore, the seller will solve

$$
\max_x -C(x, \bar{q}) + (1-\mu)[V(q(x)) - C(x, q(x)) - V(\bar{q}) + C(x, \bar{q})] - x.
$$

For x^{FB} to be the solution to this problem it has to be true that $\bar{q}$ satisfies

$$
-\mu C_x\left(x^{FB}, \bar{q}\right) - (1-\mu) C_x\left(x^{FB}, q\left(x^{FB}\right)\right) - 1 = 0.
$$

But this equation holds only if $\bar{q} = q\left(x^{FB}\right)$, and we have assumed $q^{FB} \in \left(q_L\left(x^{FB}\right), q_H\left(x^{FB}\right)\right)$.

* Assume then that the contract $(\bar{q})$ that induces the first-best is such that the buyer would optimally sue the seller; that is $\bar{q} \in \left(q_L\left(x^{FB}\right), q_H\left(x^{FB}\right)\right)$. In this case, the seller will solve

$$
\begin{aligned}
\max_x (1-\mu)[V(q(x)) - C(x, q(x))] - F\mu[V(\bar{q}) - C(x, \bar{q})] - {} \\
(1-\mu) V(\bar{q}) - \mu C(x, \bar{q}) - x - \sigma + (1-\mu)(\beta+\sigma).
\end{aligned}
$$

For x^{FB} to be the solution to this problem it has to be true that $\overline{q}$ satisfies

$$-(1-\mu)C_x\left(x^{FB};q\left(x^{FB}\right)\right)-(1-F)\mu C_x\left(x^{FB};\overline{q}\right)-1=0;$$

which is satisfied only if $C_x\left(x^{FB};\overline{q}\right)=-\frac{1}{1-F}$: But if $\overline{q}\in\left(q_L\left(x^{FB}\right);q_H\left(x^{FB}\right)\right)$; then $C_x\left(x^{FB};\overline{q}\right)>C_x\left(x^{FB};q_H\left(x^{FB}\right)\right)>-\frac{1}{1-F}$. ■

B. Figures

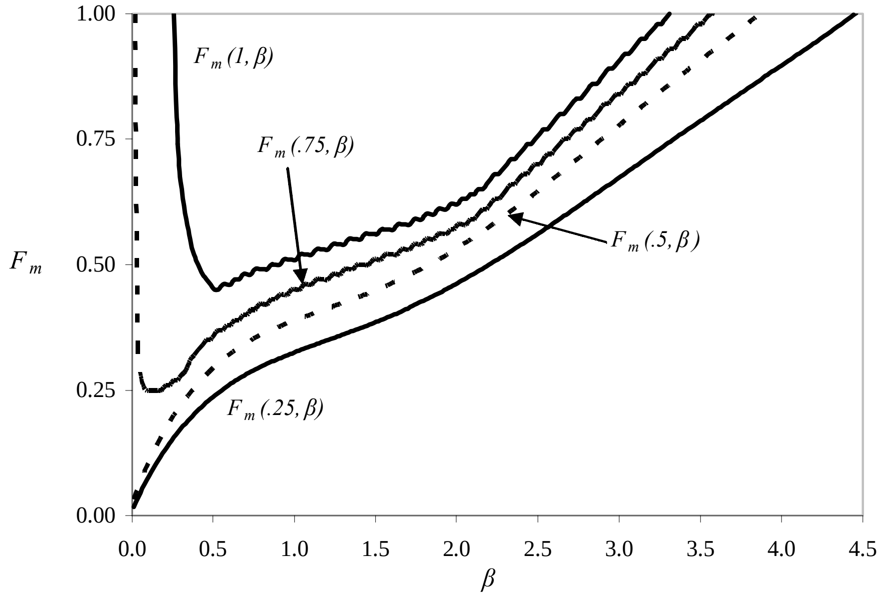

Figure B.1: Inducible Investment Levels

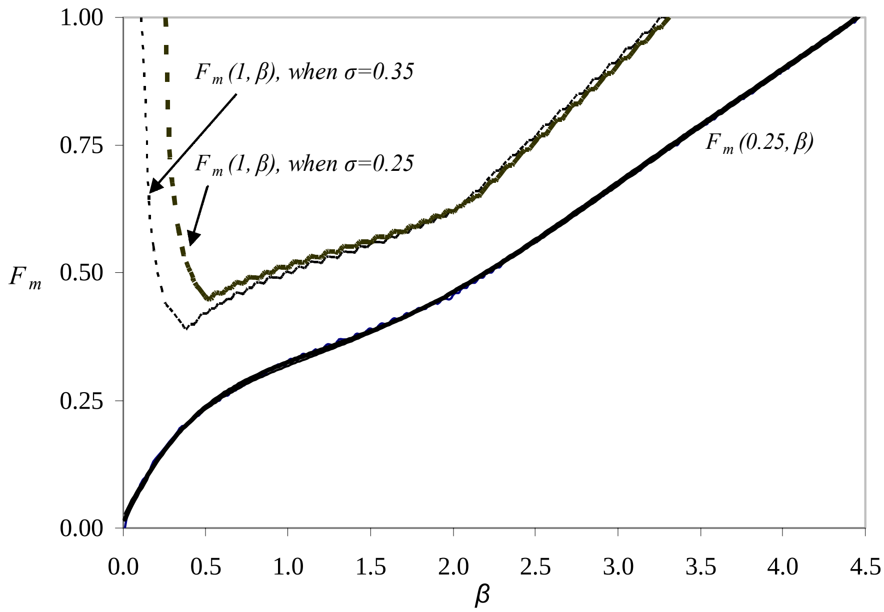

Figure B.2: The E¤ect of ¾ on the Inducible Investment Levels

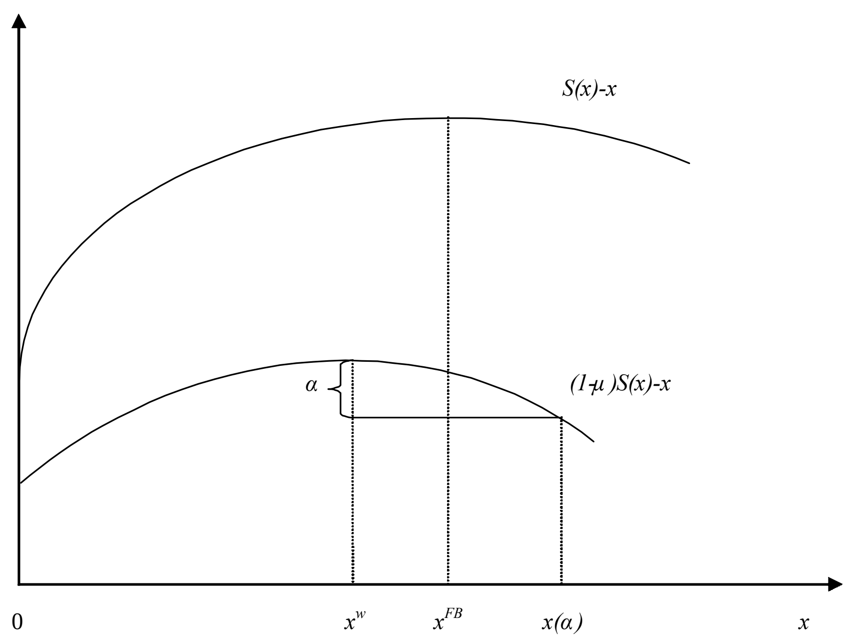

Figure B.3: De…nition of x (®)

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