The Contagion Box: Measuring Co-movements in Financial Markets by Regression Quantiles*

Lorenzo Cappiello, Bruno Gerard, and Simone Manganelli †

February 2004

Abstract

We propose a simple new semi-parametric approach to investigate whether co-dependence across markets increase in periods of extreme negative or extreme positive returns relative to quiet periods. Our empirical investigation is based on the computation of a conditional probability: given that returns on a certain market have fallen in the extreme negative or positive tail of their own distribution, we compute the probability that returns on a different market will also take on extreme values. Technically, we estimate the probability that returns on one market are lower than a given quantile, when returns on another market are also lower than the correspondent quantile. Quantiles, which are first kept constant, are next made time-varying using the CAViaR model developed by Engle and Manganelli (2004). Graphically, conditional probabilities can be represented in what we call "the contagion box", which is a square with unit side. Loosely speaking, since a 45^o line represents the case of independence between two markets, when the conditional probability lies above this line, this indicates presence of co-movements. From this insight rigorous econometric tests of contagion are derived and implemented. An application to the so-called "tequila" crisis is finally carried out.

Keywords: contagion, conditional probabilities, CAViaR

*Preliminary and incomplete. Comments welcome.

†Cappiello and Manganelli are with the European Central Bank, Kaiserstrasse 29, 60311 Frankfurt am Main, Germany. Email: lorenzo.cappiello@ecb.int, tel: +49-69 1344 8765 and simone.manganelli@ecb.int, tel: +49-69 1344 7347. Gerard is with the Norwegian School of Management BI, Elias Smiths vei 18, Box 580 N-1302 Sandvika, Norway, email: bruno.gerard@bi.no, tel: +47-67 55 71 05. The views expressed in this paper are those of the authors and do not necessarily reflect those of the European Central Bank.

1 Introduction: review of the literature and research goals

Contagion is, possibly, a recurrent phenomenon that affects financial markets. If and when a crisis in one market spills over to another country's market, the receiving country may experience detrimental instability in its financial and banking system and suffer severe dislocations in its real economy. Hence it is important for the policy maker and the economist to develop measures to assess whether contagion has or will occur and to determine the factors that affect its likelihood. In this project, we propose a novel methodology, based on the semi-parametric regression quantiles developed by Engle and Manganelli (1999) to address these issues.

According to the World Bank,[1] contagion can be defined as follows:

1) Restrictive definition:

Contagion is the transmission of shocks to other countries or [the increase in] the cross-country correlation, beyond any fundamental link among the countries and beyond common shocks.

2) Very restrictive definition:

Contagion occurs when cross-country correlations increase during "crisis times" relative to correlations during "tranquil times."

The empirical literature has been adopting several methodologies to measure contagion (see the surveys of Dungey, Fry, González-Hermosillo, and Martin, 2003, Pericoli and Sbracia, 2003, and de Bandt and Hartmann, 2000). In essence, two different approaches can be identified: estimations of first and/or second moments and measures of the probability of a crisis in one in one market, conditional on a crisis occurring in another market.[2] Within the first approach, at least two different methodologies have been used: regressions aimed at estimating cross-market correlation coefficients and estimations of volatility models such as the Generalized Autoregressive Conditional Heteroskedastic (GARCH) processes.[3] Conditional probabilities, instead, have been measured through two main techniques: Dichoto-

[1] See the web site http://www1.worldbank.org/economicpolicy/managing%20volatility/contagion/definitions.html

[2] In this paper (financial) crises are defined as extremal (positive or negative) market realisations. For instance a stock market crisis will be characterised by a sharp fall in a equity market index.

[3] Other approaches such as cointegration techniques and Markov switching models have also been employed. Fiess (2003) is an example of cointegration techniques, while Jeanne (1997) and Jeanne and Masson (2000) are examples of Markovian models.

mous and polychotomous probability models and Extreme Value Theory (EVT).

Many studies have tested whether return correlation between two markets significantly increases when moving from a tranquil to a turbulent period. Early research adopting this approach (see, for instance, King and Wadhwani, 1990, Lee and Kim, 1993, and Calvo and Reinhart, 1996) finds that cross-market correlation coefficients increase significantly during crisis periods, seemingly pointing towards contagion. Forbes and Rigobon (2002) argue that the increase in correlation coefficients during crisis times is due to an upward bias imputable to the higher variance typical of turbulent periods. When the bias is corrected, no more evidence for contagion can be detected. Importantly, an implicit assumption in Forbes and Rigobon is that returns in different markets depend on a common factor. In line with this observation, Corsetti, Pericoli, and Sbracia (2003) point out that in the correction procedure of correlation coefficients it is assumed that the variance of equity returns in the country where the crisis originates is a proxy for the variability of the common factor that affects all markets. Contrary to Forbes and Rigobon, Corsetti *et al.* find evidence for contagion by distinguishing between common and country-specific components of market returns. A similar insight is exploited by Bekaert, Harvey and Ng (2003), where, in an asset price framework, it is shown that correlation between equity returns in two different countries may increase as a consequence of exposure to a common factor. In line with this stream of the literature, Pesaran and Pick (2003) show that, in the presence of inter-dependencies, without country-specific fundamentals contagion effects cannot be consistently estimated. Ciccarelli and Rebucci (2003) estimate a VAR model of the type proposed by Forbes and Rigobon (2002), but with a Bayesian approach. The methodology allows to circumvent heteroskedasticity and omitted variable issues and does not require any knowledge of the timing of the crisis, thereby it does not need corrections in the correlation coefficients.

The methodology based on GARCH models is a natural way to capture return volatility spill overs from one market to another. Early work include, among the others, Engle, Ito, and Lin (1990), Hamao, Masulis and Ng (1990), and Chan, Karolyi and Stulz (1992). Engle *et al.* trace out the effects of news from one foreign exchange market on the volatility in other foreign exchange markets. It is found that the foreign news may play an even more important role than domestic news, which points towards volatility spill overs. Hamao *et al.* and Chan *et al.* use a Capital Asset Pricing Model (CAPM) which is estimated with a GARCH-in-Mean methodology and find evidence of equity price volatility spill overs among international markets. Edwards (1998) using data on short term nominal interest rates for some Latin America countries finds volatility contagion from Mexico to Argentina, but not from Mexico to Chile.

A number of studies seek to predict the probability that when a country has experienced a crisis also other countries will. Eichengreen, Rose and Wyplosz (1996) is perhaps the first paper that, using a probit model, estimates the probability of the spread of financial crises across countries. After controlling for the effects of political and economic fundamentals, the authors find that a speculative attack on one currency increases the probability of an attack on another currency. In particular countries which are more closely tied by international trade linkages experience currency contagion more easily than countries which share similar macroeconomics conditions. In the same vein, Bae, Karolyi and Stulz (2003) adopt a multinomial logistic model to estimate the probability of simultaneous occurrence of large returns (exceedances). Similarly to Eichengreen *et al.*, a set of control variables is employed to explain security returns and hence co-exceedances.[4] The fraction of exceedence events not attributable to the explanatory variables but to exceedancees from another region would be defined as contagion. Contagion appear to be more important for Latin American markets than for Asian markets, while the US market seems to be insulated from Asian markets. The paper of Eichengreen *et al.* is an example of dichotomous probability model, while the paper of Bae *et al.* is an example of polychotomous probability model.

By looking at extremal observations, the EVT approach seeks to detect structural changes possibly caused by contagion. If, in a bivariate distribution, there were a shift in tail dependence from independence to dependence, this would indicate the presence of contagion. Tail dependence hinges on the notion of copula, which is a function that couples univariate distribution functions into a multivariate distribution function. Adopting EVT Longin and Solnik (2001) find that the conditional correlation of large negative returns, or "co-crashes" is higher than that expected under the assumption of multivariate normality, while conditional correlation does not increases in bull markets. Hartmann, Straetmans, and de Vries (2002) employ EVT to analyse equity and government bond markets as well as the linkages between

[4] "Exceedances" are extreme outcomes (for instance those falling in the bottom or top 5% tail of the distribution) taken on by financial variables (exchange rates, stock returns, interest rates, etc.). "Co-exceedances" are episodes carachterised by the fact that more than one market synchronously experiences these tail events.

the two. While single equity or bond crashes occur rarely, the conditional probability of co-crashes in equity and bond markets are higher, although it decreases for a co-crash between a stock and a bond market.

Each of these approaches suffers from several drawbacks. Correlation-based methodologies fail to describe how market linkages change when moving from a tranquil to a turbulent time. Moreover results are usually affected by heteroskedasticity, omitted variables and simultaneous equation problems. GARCH models test for contagion only indirectly, since volatility spill overs do not necessarily reflect increase in first moment correlation. While EVT and probit/logit models have the advantage of quantifying the probability of contagion, only the latter controls for economic fundamentals. Neither, though, make use of a benchmark to test whether the probability of contagion statistically differs from the probability that would prevail when correlation between two markets is low or null. We suggest an approach which provides a common framework to address these issues. Our methodology is able to measure the probability of contagion with respect to a benchmark, and, at the same time, to take into account its economic driving forces. Moreover, unlike cross-market correlation models, it does not suffer from biases due to heteroskedasticity, omitted variables and simultaneous equation. Finally, differently from GARCH-type approaches, security returns and not their volatility is the object of the analysis.

The paper proceeds as follows. In Section 2, we describe the empirical framework, provide some intuition and compare our tests to the alternatives in the literature. The formal econometrics of the tests are developed in Section 3, while Section 4 describes the data. Section 5 reports the results of the analysis. Section 6 concludes. All proofs are relegated to the appendix.

2 The contagion box

In this section, we present the intuition behind our approach and put it in perspective with other existing methodologies. Technical results are derived and discussed in the next section.

Let x_t and y_t denote two random variables. The fundamental analytical tool of our analysis are the following conditional probabilities:

$$\begin{aligned} p_t^L(\theta) &= prob\left[y_t < q_y^{\theta}\left(\beta_\theta^0, \Omega_t\right) \middle| x_t < q_x^{\theta}\left(\delta_\theta^0, \Omega_t\right), \Omega_t\right] \qquad if \quad \theta \in (0,1) \quad (1) \\ p_t^U(\theta) &= prob\left[y_t > q_y^{1-\theta}\left(\beta_\theta^0, \Omega_t\right) \middle| x_t > q_x^{1-\theta}\left(\delta_\theta^0, \Omega_t\right), \Omega_t\right] \qquad if \quad \theta \in (0,1) \end{aligned}$$

where $q_x^\theta\left(\delta_\theta^0, \Omega_t\right)$ and $q_y^\theta\left(\beta_\theta^0, \Omega_t\right)$ are the θ-quantiles of the two random variables, which possibly depend on some unknown parameters and other variables that belong to the information set at time t. Ω_t denotes the information set available at time t. If it is the empty set (i.e., $\Omega_t = \varnothing$, $\forall t$), then the θ-quantiles are constant for all t. On the other hand, if the information set contains all the available information up to time t, the θ-quantiles are not necessarily constant.

If we think of $\{x_t\}_{t=1}^T$ and $\{y_t\}_{t=1}^T$ as the time series returns of two different markets, for each probability θ, $p_t^L(\theta)$ measures the probability that on market Y the return will fall below its θ-quantile, conditional on the same event occurring in market X. By the same token, $p_t^U(\theta)$ measures the probability that on market Y the return will fall above its θ-quantile, conditional on the same event occurring in market X.

The shape of $p(\theta)$ (where $p(\theta)$ can either be equal to $p_t^L(\theta)$ or $p_t^U(\theta)$) depends on the characteristics of the joint distribution of the random variables x_t and y_t. According to the definitions 1) and 2) reported in section 1, absence of contagion is equivalent to a constant correlation between these two random variables. It is well-known that correlation is a sufficient statistic to describe the dependence between two random variables only when they have a spherical or elliptical joint distribution (see, for example, Embrechts, McNeil and Straumann 1999). There are three notable exceptions to this result: 1) perfect positive correlation, 2) independence and 3) perfect negative correlation. These and more general cases can be conveniently analysed in what we call the "contagion box" (see Figure 1). In the contagion box, $p(\theta)$ - to be read on the vertical axis - is plotted against θ - to be read on the horizontal axis. It is a square with unit side. If two markets are independent, which implies $\rho = 0$, $p(\theta)$ will be piecewise linear, with slope equal to one, if $\theta \in (0, 0.5)$, and slope equal to minus one, if $\theta \in (0.5, 1)$. In addition, $\lim_{\theta\to 0} p(\theta) = 0$ and $\lim_{\theta\to 1} p(\theta) = 0$. In other words, conditional and unconditional probabilities coincide. When there is perfect positive correlation between x_t and y_t (i.e. $\rho = 1$), $p(\theta)$ is a flat line that takes on unit value. Under this scenario, the two markets essentially reduce to one. The polar case occurs for a perfect but negative correlation, i.e. $\rho = -1$. In this case $p(\theta)$ is always equal to zero. The reason is that if y_t falls in one half of its distribution, x_t will not, because it will take on diametrically opposite values.

The shape of $p(\theta)$ for intermediate cases is unknown and can be generally derived only by numerical simulation. The above discussion suggests that the shape of $p(\theta)$ might provide key insights about the dependence between two random variables x_t and y_t. In particular, in applications to contagion we are interested in the behaviour of the tails of the distribution. Therefore, it is natural to focus the attention on the values of $p(\theta)$ for θ close to 0 or 1.

While this measure can be used to compare the dependence between different markets, by itself it is not sufficient to test for contagion. Indeed, the two definitions of contagion given in the introduction refer to an increase in correlation in crisis with respect to tranquil times. What is missing so far is a benchmark against which our measure of dependence can be compared. Denote this benchmark by $p^B(\theta)$. (We defer the discussion about the choice of the benchmark to the next section.) We can now derive a rigorous definition of contagion:

Definition 1 *(Existence of Contagion) - We say that we have* ***contagion*** *in the lower (upper) tail if and only if there exists* $\theta^* < 0.5$ $(\theta^* > 0.5)$ *such that* $p(\theta) > p^B(\theta)$, $\forall\theta \leq \theta^*$ $(\forall\theta \geq \theta^*)$.

Definition 2 *(Intensity of Contagion) - Suppose that contagion exists, according to the previous definition. Let* θ_m *be the maximum value of* θ^* *such that* $p(\theta) > p^B(\theta)$, $\forall\theta \leq \theta^*$. *We define the lower tail* ***intensity of contagion*** *by* $\delta_L(\theta_m) \equiv \int_0^{\theta_m}[p(\theta) - p^B(\theta)]d\theta$. *Analogous definition holds for the upper tail.*

DESCRIPTION OF THE INTUITION. We can describe existing contributions to the contagion literature in terms of the contagion box just described. Our approach has close ties with extreme value theory. Indeed, $\lim_{\theta\to 0} p(\theta)$ is exactly the definition of "tail dependence" for the lower tail (similar result holds for the upper tail). However, existing contributions (e.g., Longin and Solnik 2001 and Hartmann, Straetmans and de Vries 2003) differ from ours under two important aspects. First they look at only one (extreme) point of the distribution. Second, they fail to compare this point to some benchmark which would indicate the absence of contagion. Moreover, it is not obvious how these approaches can be modified to control for economic variables. Our approach is also close to the logit/probit literature (e.g., Eichengreen, Rose and Wyplosz 1996 and Bae, Karolyi and Stulz 2000). Although these approaches can incorporate economic variables (however, see problems discussed by Rigobon), they focus on less extreme points of the distribution and suffer from the same criticisms we just discussed for extreme value theory. Finally, our approach is robust to all the problems that plague studies based on correlation (heteroskedasticity, omitted variables, simultaneous equations; see Forbes and Rigobon 2001), because we are looking directly at tail properties of the random variables of interest.

3 The econometrics of the contagion box

In this section we describe how to construct an asymptotically consistent test for contagion. A key element of our methodology is the definition of the benchmark. We propose two ways to construct the benchmarks. The first possibility is to construct the benchmark by simulation as follows. First, estimate the unconditional correlation between the two random variables of interest. Second, estimate the tail thickness of the joint distribution. Third, use these parameters to construct a joint distribution. Fourth, generate the conditional probabilities of the benchmark by Monte Carlo simulation, using the estimated joint distribution. We will use this benchmark for unconditional analysis.

The second strategy is to use the economist's knowledge to define periods where contagion is believed to have occurred. This is equivalent to splitting the sample into two subsamples, the control and the experimental samples. We will use this benchmark for conditional analysis.

The approach we describe in this section is flexible enough to accommodate both benchmarks. Define

$$I_{Yt} \equiv \begin{cases} I\left[y_t < q_y^\theta\left(\beta_\theta, \Omega_t\right)\right] & \text{if } \theta < 0.5 \\ I\left[y_t > q_y^\theta\left(\beta_\theta, \Omega_t\right)\right] & \text{if } \theta > 0.5 \end{cases}$$

where $I[\cdot]$ denotes the indicator function that takes on value one if the expression within brackets is true and zero otherwise, and we have made explicit the dependence of the quantile functions on variables that belong to the information set at time t and on some estimated parameters. These parameters can be conveniently estimated by regression quantile, as described by Engle and Manganelli (2004). Define analogously I_{Xt}. We also adopt the general convention of indicating variables evaluated at the estimated ($\hat{\beta}_\theta$ and $\hat{\delta}_\theta$) or true (β_θ^0 and δ_θ^0) parameters by ($\hat{I}_{Yt}$ and $\hat{I}_{Xt}$) or (I_{Yt}^0 and I_{Xt}^0), respectively.

The building block of our test is the conditional probability $p(\theta)$ over tranquil and crisis times. Let $p^T(\theta)$ denote the probability of co-exceedance in tranquil periods and $p^C(\theta)$ denote the probability of co-exceedance in crisis periods. Let D_t^C be a dummy that takes value 1 on days of crisis and zero otherwise. Estimation of these probabilities can be obtained by running the following regression:

$$\hat{I}_{Yt} = \alpha_\theta \hat{I}_{Xt} + \gamma_\theta D_t^C \hat{I}_{Xt} + \varepsilon_t \qquad (2)$$

Theorem 3 below will formally show that $\hat{\alpha}_\theta$ and $\hat{\alpha}_\theta + \hat{\gamma}_\theta$ are estimates of the average of $p^T(\theta)$ and $p^C(\theta)$, respectively. If instead we are interested in using the first benchmark, we simply have to put $\gamma_\theta = 0$ and compare $\hat{\alpha}_\theta$ to $p^B(\theta)$ generated by simulation. Since this second test can be obtained as a special case of the first, we will describe only this more general case.

Note that both conditional and unconditional cases are covered under this framework. For the conditional quantile, an appropriate CAViaR specification can be estimated. The unconditional quantile is just a special case of CAViaR, in which the only parameter to be estimated is the constant. In the conditional case, economic variables can be easily included. One could use the in-sample Dynamic Quantile test developed by Engle and Manganelli to test for omitted economic variables. If the test rejects the null hypothesis of correct specification, than these variables can be included in the dynamic quantile process.

It is straightforward to show that - under the assumption of correct quantile specification - the OLS estimators $\hat{\alpha}_\theta$ and $\hat{\alpha}_\theta + \hat{\gamma}_\theta$ will converge to the probability of co-exceedance in tranquil and crisis periods.

Theorem 3 (Consistency) *[ASSUMPTIONS] If the quantile models are correctly specified, then* $\hat{\alpha}_\theta \xrightarrow{p} p^T(\theta)$ *and* $\hat{\alpha}_\theta + \hat{\gamma}_\theta \xrightarrow{p} p^C(\theta)$*.*

Rewrite the regression in matrix form $\hat{I}_Y = \hat{W}\phi_\theta + \varepsilon$, where $\hat{W}' \equiv [\iota\ D]' diag(\hat{I}_X)$. The OLS estimator is $\hat{\phi}_\theta = (\hat{W}'\hat{W})^{-1}\hat{W}'\hat{I}_Y$. Approximate the indicator function by $\hat{I}^* \equiv [1 + \exp\{c_T^{-1}\hat{\varepsilon}_t\}]^{-1}$ Expanding around β^0_θ and δ^0_θ, we get: $\hat{\phi}_\theta = (\hat{W}'\hat{W})^{-1}W'I_Y + (\hat{W}'\hat{W})^{-1}[[\iota D]' diag(k_{c_T}(\varepsilon^*_t)\nabla g_t(\delta^*))I^*_Y(\hat{\delta} - \delta^0) + W^* k_{c_T}(\varepsilon^*_t)\nabla f_t(\beta^*)(\hat{\beta} - \beta^0)]$

$= \phi^0_\theta + (W'W)^{-1}[W'\varepsilon + [\iota D]' diag(h^\delta_t(0)\nabla g_t(\delta^0))I^0_Y(\hat{\delta} - \delta^0) + W^0 h^\beta_t(0)\nabla f_t(\beta^0)(\hat{\beta} - \beta^0)]$

$= \phi^0_\theta + (W'W)^{-1}[W'\varepsilon + [\iota D]' diag(h^\delta_t(0)\nabla g_t(\delta^0))I^0_Y D^{-1}_{\delta T} T^{-1}\nabla g_t(\delta^0) Hit(\delta^0) + W^0 h^\beta_t(0)\nabla f_t(\beta^0) D^{-1}_{\beta T} T^{-1}\nabla f_t(\beta^0) Hit(\beta^0)]$

This result allows us to derive an appropriate test for contagion. According to our definition, contagion exists in the lower tail if and only there exists θ_m such that $\hat{\gamma}_{\theta_i} > 0$ for all $\theta_i < \theta_m$. In addition, we can measure the intensity of contagion as follows:

$$\hat{\delta}^L(\theta_m) \equiv \sum_{i=1}^{m} \hat{\gamma}_{\theta_i}$$

where $0 < \theta_1 < \theta_2 < ... < \theta_m < 0.5$. A similar test statistic can be defined for the upper tail.

To derive the asymptotic distribution of this test statistic, we first need to derive the joint distribution of the associated regression quantile estimators. Then we need to derive the asymptotic distribution of the vector $\hat{\phi} \equiv \left[\hat{\phi}_{\theta_i}\right]$, where $\hat{\phi}_{\theta_i} \equiv \left[\hat{\alpha}_{\theta_i}, \hat{\gamma}_{\theta_i}\right]'$. Once this is done, it becomes straightforward to obtain the distribution of the above test statistic.

Regression quantiles were introduced by Koenker and Bassett (1978). Let $\{y_t\}_{t=1}^T$ be a random variable and $q_t\left(\beta^0_{\theta_i}\right) \equiv q\left(\beta^0_{\theta_i}, \Omega_t\right)$ the associated θ_i-quantile function, which depends on the vector of parameters $\beta^0_{\theta_i}$ and some variables that belong to the information set Ω_t. Define $H_i \equiv I\left(y_t < q_t\left(\beta_{\theta_i}\right)\right) - \theta_i$. Throughout the paper we use the convention that $\hat{H}_i$ and H_i^0 denote variables evaluated respectively at $\hat{\beta}_{\theta_1}$ and $\beta^0_{\theta_i}$. The unknown parameters can be consistently estimated by maximising the following objective function:

$$\max_{\beta_{\theta_i}} T^{-1} \sum_{t=1}^T H_i \left[y_t - q_t\left(\beta_{\theta_i}\right)\right]$$

See Engle and Manganelli (2004) for the consistency result. Being estimated on the same data, the different parameter estimates $\left\{\hat{\beta}_{\theta_i}\right\}_{i=1}^m$ will be correlated. Let

$$\hat{\beta} \equiv [\hat{\beta}'_{\theta_1}, ..., \hat{\beta}'_{\theta_m}]'$$

$$D_T^{\theta_i} \equiv E\left[T^{-1} \sum_{t=1}^T h_t^{\theta_i}(0|\Omega_t) \nabla' q_t\left(\beta^0_{\theta_i}\right) \nabla q_t\left(\beta^0_{\theta_i}\right)\right] \qquad D_T \equiv diag\left[D_T^{\theta_1}, ..., D_T^{\theta_m}\right]$$

$$A_T^{ij} \equiv E\left[T^{-1} \sum_{t=1}^T \nabla' q_t\left(\beta^0_{\theta_i}\right) \nabla q_t\left(\beta^0_{\theta_j}\right) H_i^0 H_j^0\right] \qquad A_T \equiv \left[A_T^{ij}\right] \qquad (i, j = 1, ..., m)$$

The following theorem derives the distribution of $\hat{\beta}$.

Theorem 4 *Under some assumptions,* $\sqrt{T}\left(\hat{\beta} - \beta^0\right) \xrightarrow{d} N\left(0, D_T^{-1} A_T D_T^{-1}\right)$.

Theorem 5 *Under the null of constant correlation and some technical assumptions,* $\sqrt{T}\hat{\gamma} \xrightarrow{d} N\left(0, \Sigma\right)$, *where* $\Sigma \equiv \theta^{-\prime}\Gamma\theta^{-}$.

Proof. (sketch) - Note that $T^{-1}\sum_{t=1}^{T} I_t^X\left(\hat{\delta}_\theta\right)' \odot I_t^X\left(\hat{\delta}_\theta\right) \xrightarrow{p} \theta$, by the law of large numbers. Let $\hat{\theta}^- \equiv \left[T^{-1}\sum_{t=1}^{T} I_t^X\left(\hat{\delta}_\theta\right)' \odot I_t^X\left(\hat{\delta}_\theta\right)\right]^{-1}$ and $\theta^- \equiv \left[\theta_1^{-1}, ..., \theta_p^{-1}\right]'$. Then, $\hat{\theta}^- \odot T^{-1}\sum_{t=1}^{T}\left\{I_t^X\left(\hat{\delta}_\theta\right) \odot I_t^Y\left(\hat{\beta}_\theta\right) - I_t^X\left(\delta_\theta^0\right) \odot I_t^Y\left(\beta_\theta^0\right)\right\} \xrightarrow{p} \hat{\gamma}$. The result follows from the previous lemma. ■

Theorem 6 *Under the same conditions of the previous theorem* $\sqrt{T}\hat{\psi}_L(X,Y) \xrightarrow{d} N(0, \iota'\Sigma\iota)$, *where* ι *is a vector of ones.*

Proof. Immediate. ■

4 Data

The empirical analysis is carried out on returns on equity indices for four Latin American countries, Brazil, Mexico, Chile and Argentina. Equity returns are continuously compounded and computed with Morgan Stanley Capital International (MSCI) world indices, which are market-value-weighted and do not include dividends. The data set covers a period from January 2 1989 to August 27 2003 and data points are observed at daily frequency.

Descriptive statistics for the asset data are given in table 1. Not surprisingly, all distributions exhibit skewness and leptokurtosis at 1% significance level, a clear sign of non-normality. This is confirmed by the Jarque-Bera normality test.

5 Empirical results: an application to Latin America

In this section we estimate average conditional probabilities of co-movements between equity market pairs for some selected Latin American countries. In figures 1A-1L two lines are plotted. The blue line indicates the conditional probability of co-movements under the benchmark or, equivalently, over tranquil times. This line is the graphical representation of $\hat{\alpha}_\theta$ in equation (2). The green line, instead, shows the probability of co-movements during crisis times and plots $\hat{\alpha}_\theta + \hat{\gamma}_\theta$ of equation (2). Following Forbes and Rigobon (2002), the turbulent period of our sample commences on November 1 1994 and ends on March 31 1995. If the green line lies above the benchmark, this can be interpreted as evidence for contagion. When the two lines approximately coincide, no contagion can be detected. Finally, if the green line lies below the benchmark, during crises time the co-movement between two different markets actually decreases.

When analysing the left tails of the distributions, visual inspection suggests that there is contagion between Brazil and Argentina, Brazil and Chile, and Argentina and Chile. There seems to be no contagion, instead, between Mexico and Argentina, Mexico and Chile, and probably between Brazil and Mexico. Notice that these results are in line with those of Forbes and Rigobon (2002). When they correct correlation coefficients for the upward bias due to heteroskedasticity, they do not find any evidence "of a significant change in the magnitude of the propagation mechanism from Mexico to any other country in the sample" (Forbes and Rigobon, 2002). Our results, however, are richer than those obtained by Forbes and Rigobon since we detect contagion for certain country pairs.

The analysis of the right tails of the distributions also delivers appealing outcomes. When Mexico is coupled with other countries, for small values of θ co-movements in turbulent times is higher than the benchmark correlation. For larger values of θ, instead, the green line shifts below the benchmark, indicating that correlation seems to decrease. A similar outcome is seen for the country pairs Brazil-Argentina and Brazil-Chile, with the difference that the shift between the two lines occurs for relatively higher values of θ. When returns (heavily) bounce back during crisis times (low values of θ) correlations remain quite high, while the less extreme (positive) returns are, the less the correlation is. These results are somehow consistent with previous research (see, for instance, Ang and Bekaert, 2002, and Longing and Solnik, 2001 and 1995), which finds that during bull markets correlation seems to decrease. Finally, the equity returns between Argentina and Chile show increased interdependence only for small values of θ, while the two lines approximately coincide for relatively larger values of θ.

Table 1
Descriptive statistics of daily returns on stock market indices
Panel 1A: Distributional statistics of equity returns

	Argentina	**Brazil**	**Chile**	**Mexico**
Mean	0.0023	0.0045	0.0007	0.0010
Min.	-0.7280	-0.2174	-0.0605	-0.1269
Max.	0.3904	0.2466	0.0860	0.1214
St.dev.	0.0355	0.0268	0.0112	0.0156
Skew.	-0.8495^{**}	0.4073^{**}	0.2215^{**}	0.0202^{**}
Kurt.	59.3969^{**}	9.9651^{**}	6.7044^{**}	7.5669^{**}
J-B	507104.4^{**}	7833.24^{**}	2217.18^{**}	3322.51^{**}

** denotes 1% significance level.

The significance level for skewness (skew.) and excess kurtosis (kurt.) is based on test statistics developed by D'Agostino, Belanger and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as χ^2_m with $m = 2$ degrees of freedom.

Panel 1B: Unconditional correlations of equity returns

	Argentina	**Brazil**	**Chile**	**Mexico**
Argentina	1.000	**	**	**
Brazil		1.000	**	**
Chile			1.000	**
Mexico				1.000

* and ** denote 5% and 1% significance levels, respectively.

Figure 1 - The contagion box: an application to Latin America

Figure 1A: Brazil-Mexico (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

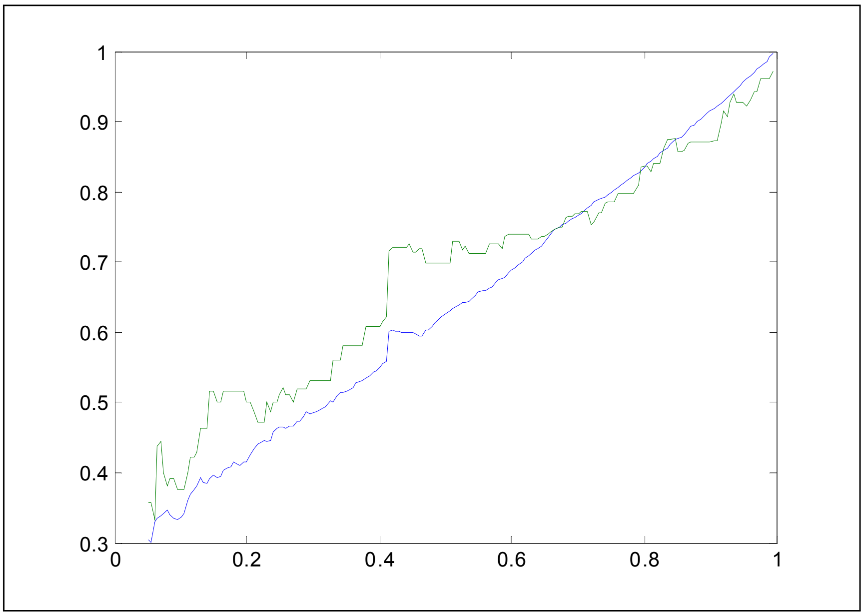


Figure 1B: Brazil-Argentina (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

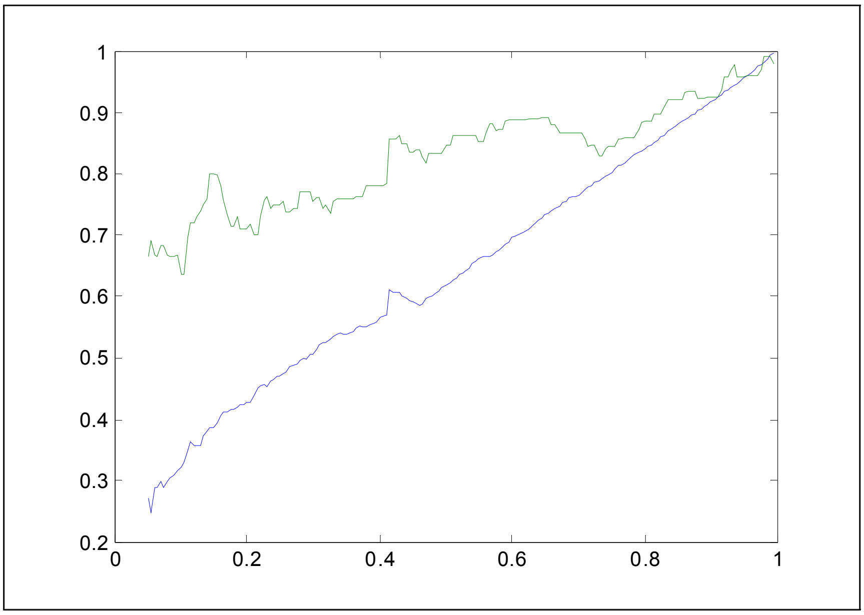


Figure 1 - Cont'd

Figure 1C: Brazil-Chile (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

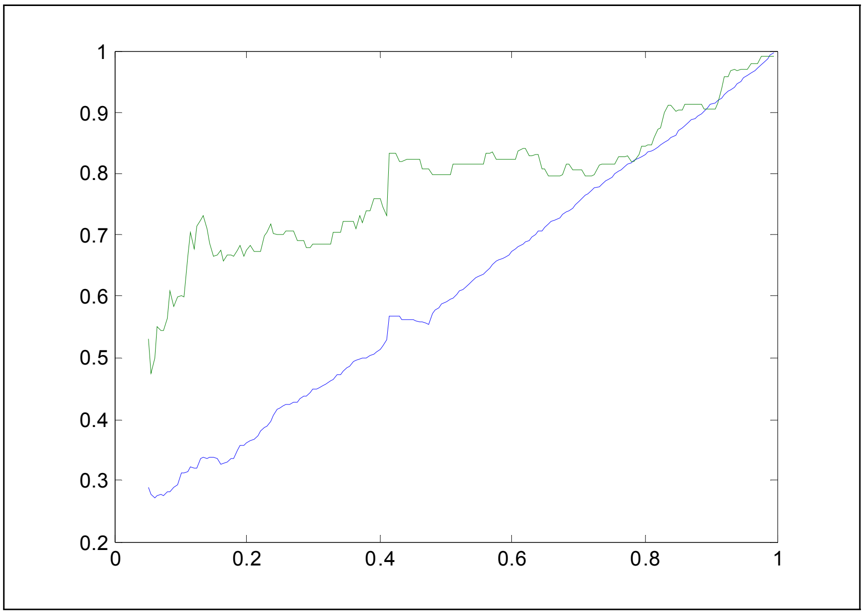


Figure 1D: Mexico-Argentina (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

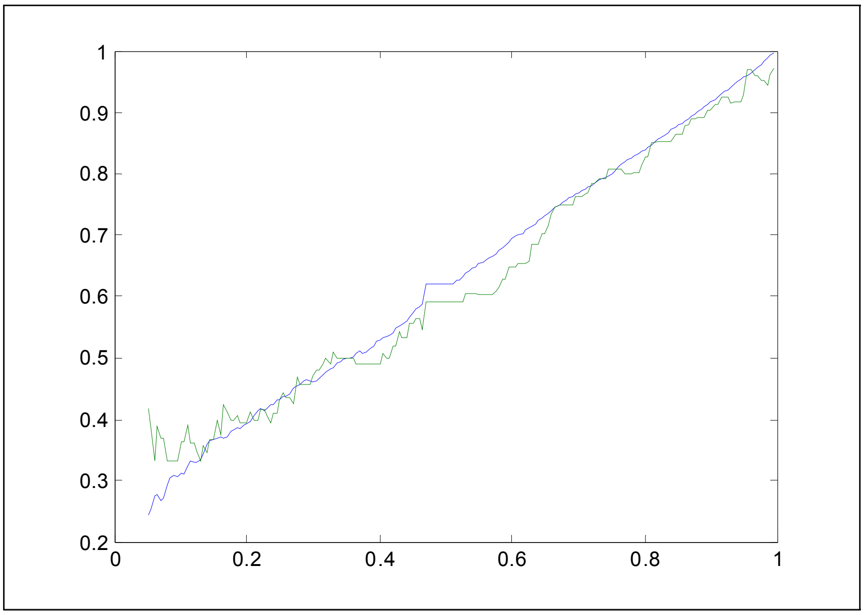


Figure 1 - Cont'd

Figure 1E: Mexico-Chile (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

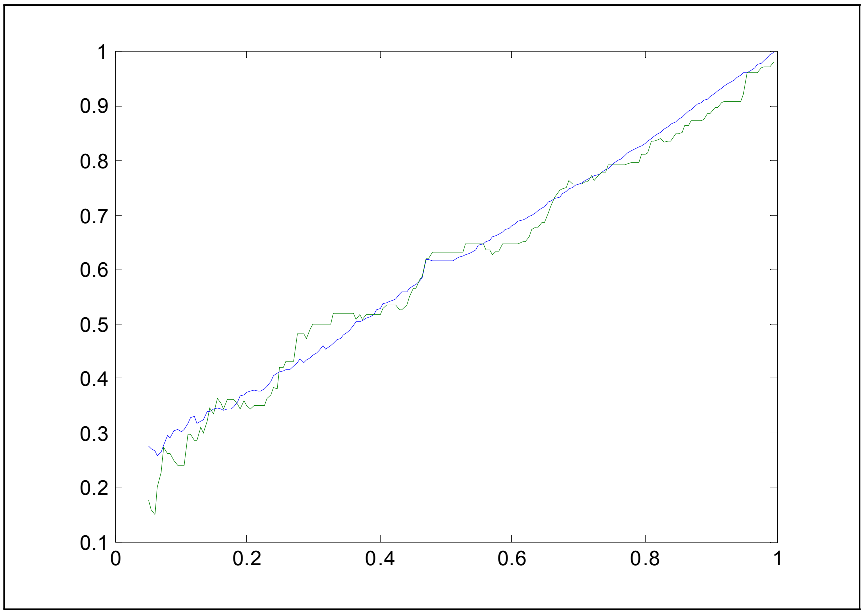


Figure 1F: Argentina-Chile (left tail) - Contagion period: 1 Nov '94 - 31 Mar '95

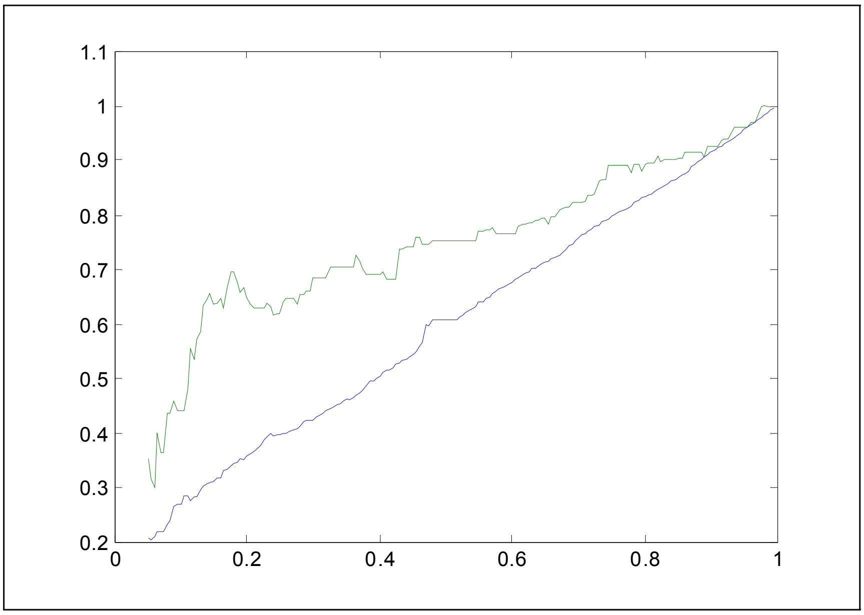


Figure 1 - Cont'd

Figure 1G: Brazil-Mexico (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95

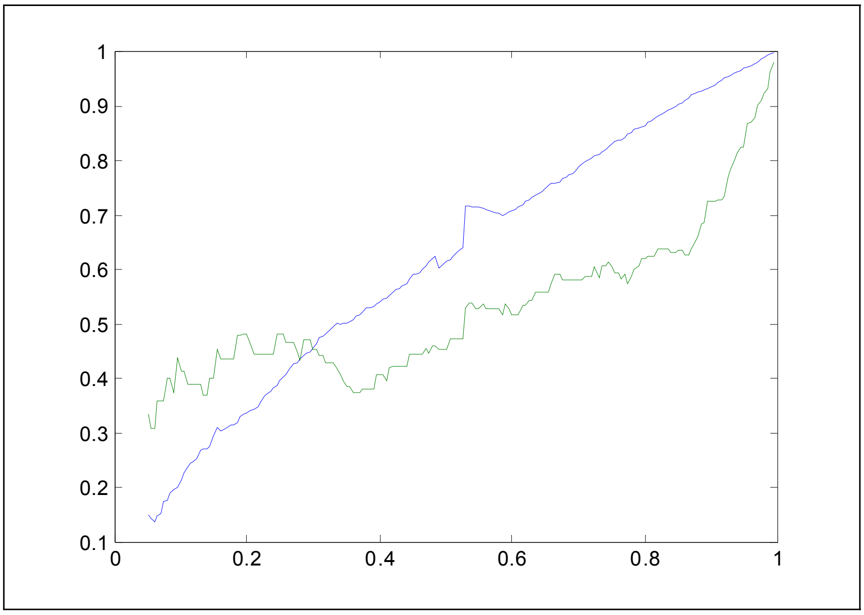


Figure 1H: Brazil-Argentina (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95

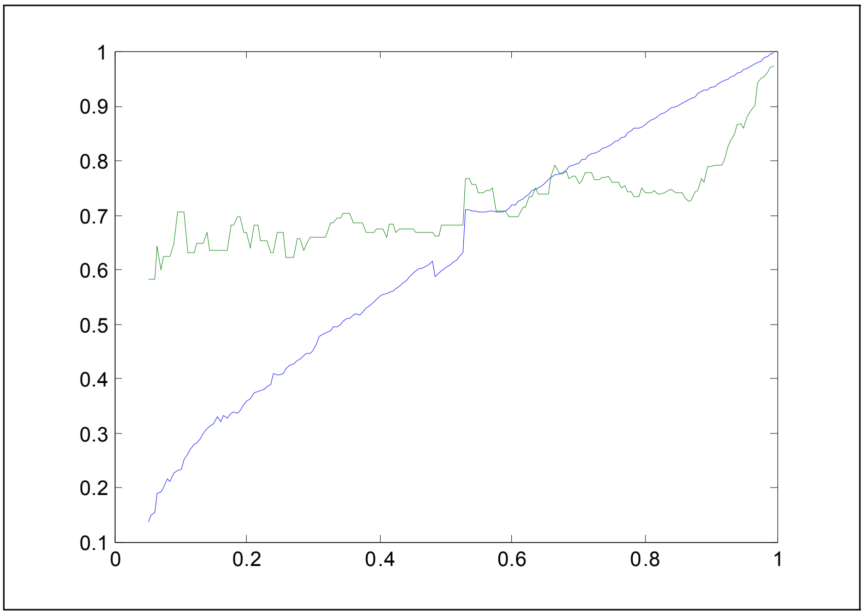


Figure 1 - Cont'd

Figure 1I: Brazil-Chile (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95

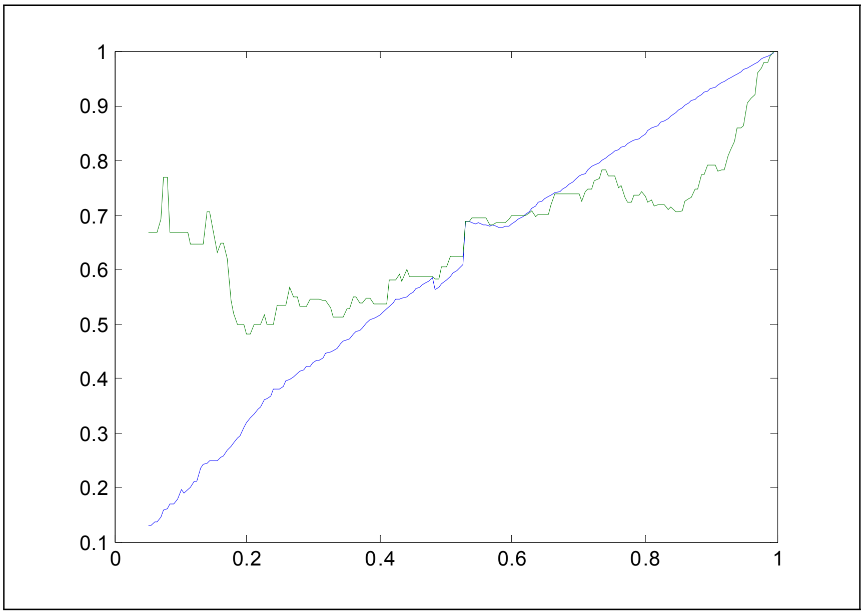


Figure 1J: Mexico-Argentina (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95

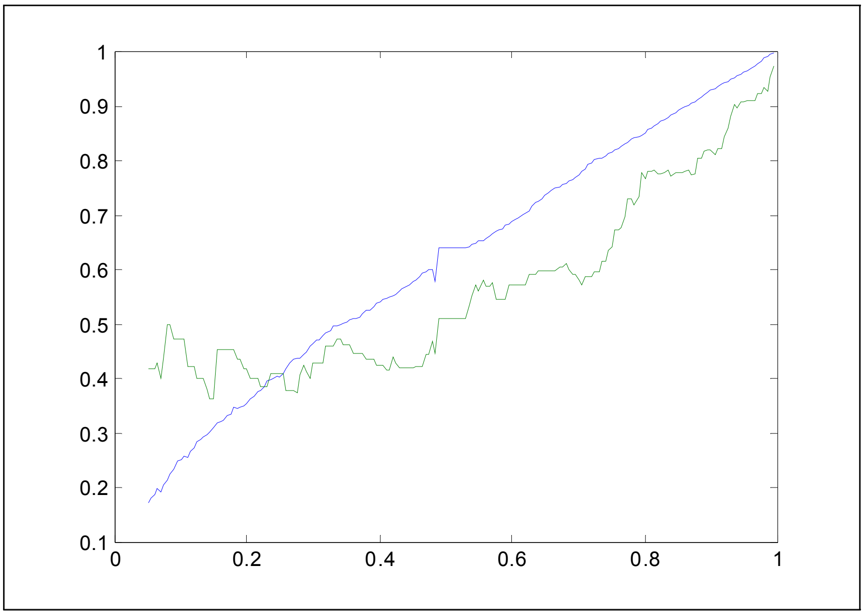


Figure 1 - Cont'd

Figure 1K: Mexico-Chile (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95

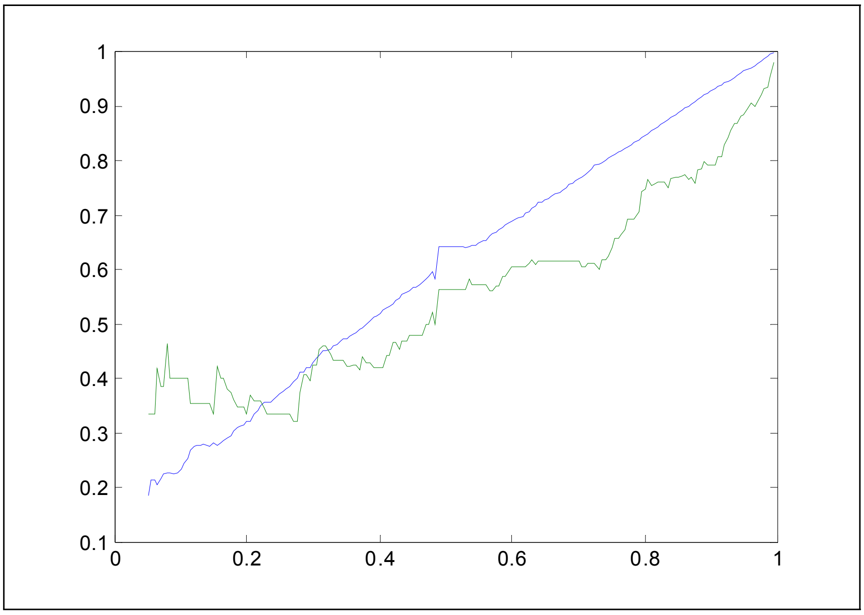
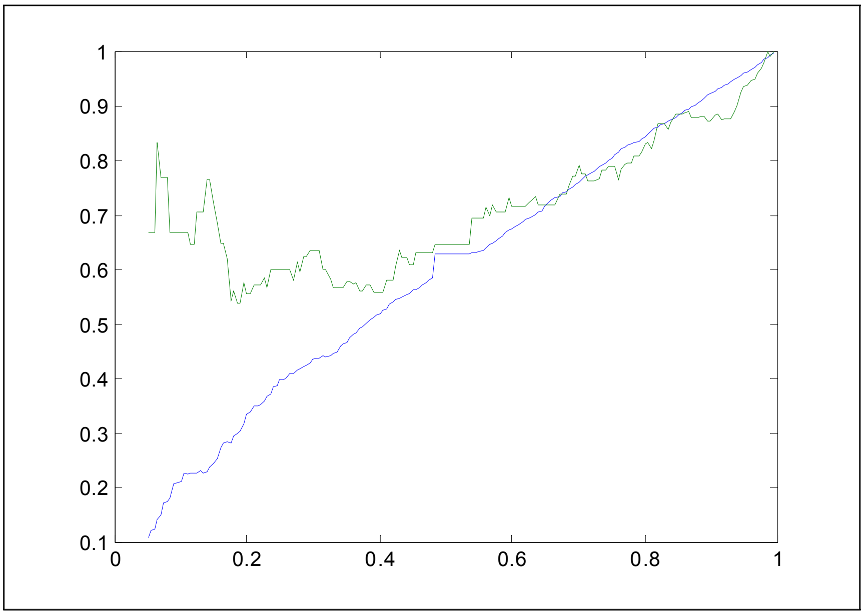


Figure 1L: Argentina-Chile (right tail) - Contagion period: 1 Nov '94 - 31 Mar '95



References

[1] Ang, Andrew and Geert Bekaert, 2002, International Asset Allocation with Regime Shifts, *Review of Financial Studies* 15, 1137-1187.

[2] Bae, Kee-Hong, G. Andrew Karolyi, and René M. Stulz, 2003, A New Approach to Measuring Financial Contagion, *Review of Financial Studies* 16, 717-763.

[3] Bekaert, Geert, Campbell R. Harvey and Angela Ng, 2003, Market Integration and Contagion, *Journal of Business*, forthcoming.

[4] Ciccarelli, Matteo and Alessandro Rebucci, 2003, Measuring Contagion with a Bayesian, Time-Varying Coefficient Model, ECB Working Paper No. 263.

[5] De Bandt, Olivier and Philipp Hartmann, 2000, Systemic risk: A survey, ECB Working Paper No. 35.

[6] De Santis, Giorgio and Bruno Gerard, 1997, International Asset Pricing and Portfolio Diversification with Time-Varying Risk, *Journal of Finance* 52, 1881-1912.

[7] Dungey, Mari, Renée Fry, Brenda González-Hermosillo, and Vance L. Martin, 2003, Empiricl Modelling of Contagion: A Review of Methodologies, IMF Working Paper WP/03/84.

[8] Elton, Edwin J. and Martin J. Gruber, 1992, International Diversification, in Summer N. Levine, ed.: *Global Investing* (Harper Business, New York, NY).

[9] Solnik, B., 1974c, Why Not Diversify Internationally Rather than Domestically?, *Financial Analysts Journal* 30, 48-54, reprinted in *Financial Analysts Journal* 51, 1995, 89-94.

[10] Beteror, Elisabetta and Colin Mayer, 1990, Structure and Performance: Global interdependence of stock markets around the crash of October 1987, *European Economic Review*, 34:1155-1180.

[11] Boyer, Brian H., Michael S. Gibson and Mico Loretan, 1999, Pitfalls in tests for changes in correlations, Working Paper 597R, Federal Reserve Board International Finance Division.

[12] Bodart, V. and P. Reding, 1999, Exchange rate regime, volatility and international correlations on bond and stock markets, *Journal of International Money and Finance* 18, 133–151.

[13] Calvo, Sarah, and Carmen M. Reinhart, 1996, Capital Flows to Latin America: Is there Evidence of Contagion Effects?, in Guillermo A. Calvo, Morris Goldstein, and Eduard Hochreiter, eds.: Private Capital Flows to Emerging Markets After the Mexican Crisis (Institute for International Economics, Washington, D.C.).

[14] Calvo, Guillermo A. and E. G. Mendoza, 2000, Rational Contagion and the globalization of equity markets, *Journal of International Economics* 51, 79–113.

[15] Campbell, John Y. and Yasushi Hamao, 1992, Predictable stock returns in the US and Japan, *Journal of Finance* 47(1), 43-69.

[16] Cashin, Paul A., Manmohan Kumar and C. John McDermott, 1995, International integration of equity markets and contagion effects, Working Paper 95/110, International Monetary Fund.

[17] Chan, K.C., G. Andrew Karolyi, and René M. Stulz, 1992, Global Financial Market and the Risk Premium on US Equity, *Journal of Financial Economics*, 32, 137-167.

[18] Claessens, Stijn, Rudiger Dornbusch and Yung Chul Park, Contagion: Why Crises Spread and How This Can Be Stopped, in Stijn Claessens and Kristin J. Forbes, eds., International Financial Contagion (Kluwer Academic Publishers, Boston) 19-42.

[19] Corsetti, Giancarlo, Marcello Pericoli, and Massimo Sbracia, 2003, 'Some Contagion, Some Interdependence' More Pitfalls in Tests of Financial Contagion, Yale Manuscrip, European University Institute.

[20] D'Agostino, Ralph B., Albert Belanger and Ralph B. D'Agostino Jr., 1990, A Suggestion for Using Powerful and Informative Tests of Normality, *The American Statistician* 44, 316-321.

[21] Eichengreen, Barry J., Andrew K. Rose, and Charles A. Wyplosz, 1996, Contagious Currency Crises, *Scandinavian Journal of Economics* 98, 463-84.

[22] Engle, Robert F., Takatoshi Ito, and Wen-Ling Lin, 1990, Meteor Showers or Heat Wawes? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market, *Econometrica* 58, 525-542.

[23] Engle, Robert F. and Manganelli, Simone, 1999, CAViaR: Conditional Value at Risk by Quantile Regression, NBER Working paper 7341.

[24] Erb, Claude B., Campbell R. Harvey, and Tadas E. Viskanta, 1998, Contagion and Risk, Working Paper. First Chicago NBD Investment Management Company, Chicago.

[25] Forbes, Kristin J. and Roberto Rigobon, 2001, Measuring Contagion: Conceptual and Empirical Issues, in Stijn Claessens, and Kristin J. Forbes, eds.L International Financial Contagion (Kluwer Academic Publishers, Norwell, MA).

[26] Forbes, Kristin J. and Roberto Rigobon, 2002, No Contagion, Only Interdependence: Measuring Stock Market Comovements, *Journal of Finance* 57:2223-2261.

[27] Gerard, Bruno, Kessara Thanyalakpark and Darren Filson, 2002, Testing for Contagion During the Asian Crisis", *Special Issue on Financial Risk Management, Research in International Business and Finance Series*, Vol 16, edited by T. Fetherstone and J. Batten, Elsevier May 2002.

[28] Hamao, Yasushi, Ronal W. Masulis and Victor Ng, 1990, Correlations in price changes and volatility across international stock markets, *Review of Financial Studies* 3, 281-308.

[29] Hartmann, P., S. Straetmans and C.G. De Vries, 2001, Asset Market Linkages in Crisis Periods, ECB Working paper 71.

[30] Jeanne, Olivier, 1997, Are Currency Crises Self-Fullfilling? A Test, *Journal of International Economics* 43, 263-286.

[31] Jeanne, Olivier, and Paul Masson, 2000, Currency Crises, Sunspots and Markov-Switching Regimes, *Journal of International Economics* 50, 327-350.

[32] Karolyi, G. Andrew and René. M. Stulz, 1996, Why do markets move together? An investigation of U.S.–Japan stock return comovements, *Journal of Finance* 51, 951-986.

[33] King, Mervyn and Sushil Wadhwani, 1994, Transmission of volatility between stock markets, *Review of Financial Studies* 3, 5-33.

[34] King, Mervyn, Enrique Sentana and Sushil Wadhwani, 1994, Volatility and links between national stock markets, *Econometrica* 62, 901-933.

[35] Lee, Sang B. and Kwang Jung Kim, 1993, Does the October 1987 crash strengthen the comovements among national stock markets? *Review of Financial Economics* 3, 89-102.

[36] Lin, Weng-Ling, Robert F. Engle and Takatoshi Ito, 1994, Do bulls and bears move across borders? International transmission of stock returns and volatility, *Review of Financial Studies* 7, 507-538.

[37] Longin, Francois and Bruno Solnik, 1995, Is the correlation in international equity returns constant: 1970-1990?, *Journal of International Money and Finance* 14, 3-26.

[38] Longin, Francois and Bruno Solnik, 2001, Extreme Correlation of international equity markets, *Journal of Finance* 56, 649-76.

[39] Loretan, Mico and William B. English, 2000, Evaluating "correlation breakdowns" during periods of market volatility, in Bank for International Settlements: International Financial Markets and the Implications for Monetary and Financial Stability (BIS, Switzerland).

[40] Manganelli, Simone and Robert F. Engle, 2001, Value at Risk Models in Finance, ECB Working paper 75.

[41] Pericoli, Marcello and Massimo Sbracia, 2003, A Primer on Financial Contagion, *Journal of Economic Surveys* 17, 571-608.

[42] Pesaran, M. Hashem and Andreas Pick, 2003, Econometric Issues in the Analysis of Contagion, mimeo, University of Cambridge.

[43] Roll 2002

[44] Rachmand, L. and R. Susmel, 1998, Volatility and cross correlation across major stock markets, *Journal of Empirical Finance* 5, 397-416.

[45] Susmel, R. and R.F. Engle 1994, Hourly volatility spillovers between international equity markets, *Journal of International Money and Finance* 13, 3-25.