LONG-RUN AND CYCLICAL DYNAMICS IN THE US STOCK MARKET

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Abstract

This paper examines the long-run dynamics and the cyclical structure of the US stock market using fractional integration techniques. We implement a version of the tests of Robinson (1994a), which enables one to consider unit (or fractional) roots both at the zero (long-run) and at the cyclical frequencies. We examine the following series: inflation, real risk-free rate, real stock returns, equity premium and price/dividend ratio, annually from 1871 to 1993. When focusing exclusively on the long-run or zero frequency, the estimated order of integration varies considerably, but nonstationarity is found only for the price/dividend ratio. When the cyclical component is also taken into account, the series appear to be stationary but to exhibit long memory with respect to both components in almost all cases. The exception is the price/dividend ratio, whose order of integration is higher than 0.5 but smaller than 1 for the long-run frequency, and is constrained between 0 and 0.5 for the cyclical component. Also, mean reversion occurs in all cases. Finally, we use six different criteria to compare the forecasting performance of the fractional (at zero and cyclical) models with other based on fractional and integer differentiation exclusively at the zero frequency. The results show that the fractional cyclical model outperforms the others in a number of cases.

Keywords: *Stock Market, Fractional Cycles, Long Memory, Gegenbauer Processes*

JEL classification: *C22, G12, G14*

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The empirical literature analysing stock markets typically tests whether the series of interest are I(1) (stock prices), or I(0) (stock market returns). This is because, according to the Efficient Market Hypothesis (EMH), it should not be possible to make systematic profits above transaction costs and risk premia, and therefore stock prices are characterised as an entirely unpredictable random walk process, which implies that stock returns should be I(0). Mean reversion is seen as inconsistent with equilibrium asset pricing models (see the survey by Forbes, 1996). Caporale and Gil-Alana (2002), though, stress that the unit root tests normally employed impose too restrictive assumptions on the behaviour of the series of interest, in addition to having low power. They suggest instead using tests which allow for fractional alternatives (see Robinson, 1994a, 1995a,b), and find that US real stock returns are close to being I(0) (which raises the further question whether the shocks are autocorrelated, with the implication that markets are not efficient). Fractional integration models have also been used for inflation and interest rates (see, e.g., Shea, 1991; Backus and Zhin, 1993; Hassler and Wolters, 1995; Baillie et al., 1996, etc.).

However, it has become increasingly clear that the cyclical component of economic and financial series is also very important. This has been widely documented, especially in the case of business cycles, for which non-linear (Beaudry and Koop, 1993, Pesaran and Potter, 1997) or fractionally ARIMA (ARFIMA) models (see Candelon and Gil-Alana, 2004) have been proposed. Furthermore, it has been argued that cycles should be modelled as an additional component to the trend and the seasonal structure of the series (see Harvey, 1985, Gray et al, 1989). The available evidence suggests that the periodicity of the series ranges between five and ten years, in most cases a periodicity of about six years being estimated (see, e.g., Baxter and King, 1999; Canova, 1998; King and Rebelo, 1999; Caporale and Gil-Alana, 2003).

In view of these findings, the present paper extends the earlier work by Caporale and Gil-Alana (2002) by adopting a modelling approach which, instead of considering exclusively

the component affecting the long-run or zero frequency, also takes into account the cyclical structure. Furthermore, the analysis is carried out for the US inflation rate, real risk-free rate, equity premium and price/dividend ratio, in addition to real stock returns. More precisely, we use a procedure due to Robinson (1994a), which enables one to test simultaneously for unit and fractional roots at both zero and the cyclical frequencies. This approach has several distinguishing features compared with other methods, the most noticeable one being its standard null and local limit distributions.[1] Moreeover, it does not require Gaussianity (a condition rarely satisfied in financial time series), with a moment condition only of order two required. Additionally, using a large structure that involves simultaneously the zero and the cyclical frequencies, we can solve at least to some extent the problem of misspecification that might arise with respect to these two frequencies. We are able to show that our proposed method represents an appealing alternative to the increasingly popular ARIMA (ARFIMA) specifications found in the literature. It is also consistent with the widely adopted practice of modelling many economic series as two separate components, namely a secular or growth component and a cyclical one. The former, assumed in most cases to be nonstationary, is thought to be driven by growth factors, such as capital accumulation, population growth and technology improvements, whilst the latter, assumed to be covariance stationary, is generally associated with fundamental factors which are the primary cause of movements in the series.

The structure of the paper is as follows. Section 1 briefly describes the statistical model. Section 2 introduces the version of the Robinson's (1994a) tests used for the empirical analysis. Section 3 discusses an application to annual data on several US stock market series for the time period 1871 – 1993. Section 4 is concerned with model selection for each time series, and the preferred specifications are compared with other more classical representations. Section 5 contains some concluding comments.

1. The statistical model

Let us suppose that $\{y_t, t = 1, 2, \ldots, n\}$ is the time series we observe, which is generated by the model:

$$(1 - L)^{d_1} (1 - 2\cos w\, L + L^2)^{d_2} y_t = u_t, \qquad t = 1, 2, .., \qquad (1)$$

where L is the lag operator ($Ly_t = y_{t-1}$), w is a given real number, u_t is $I(0)$[2] and d_1 and d_2 can be real numbers. Let us first consider the case of $d_2 = 0$. Then, if $d_1 > 0$, the process is said to be long memory at the long-run or zero frequency, also termed 'strong dependent', so-named because of the strong association between observations widely separated in time. Note that the first polynomial in (1) can be expressed in terms of its Binomial expansion, such that for all real d_1:

$$(1 - L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1 (d_1 - 1)}{2} L^2 - \ldots .$$

These processes were initially introduced by Granger (1980, 1981) and Hosking (1981), and were theoretically justified in terms of aggregation by Robinson (1978), Granger (1980): cross section aggregation of a large number of AR(1) processes with heterogeneous AR coefficients may create long memory. Parke (1999) uses a closely related discrete time error duration model, while Diebold and Inoue (2001) relate fractional integration with regime switching models.[3] The differencing parameter d_1 plays a crucial role from both economic and statistical viewpoints. Thus, if $d_1 \in (0, 0.5)$, the series is covariance stationary and mean-reverting, with shocks disappearing in the long run; if $d_1 \in [0.5, 1)$, the series is no longer stationary but still mean-reverting, while $d_1 \geq 1$ means nonstationarity and non-mean-reversion. It is therefore crucial to examine if d_1 is smaller than or equal to or higher than 1. Thus, for example, if $d_1 < 1$, there is less need for policy action than if $d_1 \geq 1$, since the series will return to its original level sometime in the future. On the contrary, if $d_1 \geq 1$, shocks will be permanent, and active policies are required to bring the variable back to its original long term projection. In fact, this

is one of the most hotly debated topics in empirical finance. Lo and MacKinlay (1988) and Poterba and Summers (1988) used variance-ratio tests and found evidence of mean reversion in stock returns. On the contrary, Lo (1991) used a generalized form of rescaled range (R/S) statistic and found no evidence against the random walk hypothesis for the stock indices, contradicting his earlier finding using variance–ratio tests. Other papers examining the persistence of shocks in financial time series are Lee and Robinson (1996), Fiorentini and Sentana (1998) and May (1999).

Let us now consider the case of $d_1 = 0$ and $d_2 > 0$. The process is then said to exhibit long memory at the cyclical frequency. It was examined by Gray et al. (1989, 1994), who showed that the series is stationary if $|\cos w| < 1$ and $d_2 < 0.50$ or if $|\cos w| = 1$ and $d_2 < 0.25$. They also showed that the second polynomial in (1) can be expressed in terms of the Gegenbauer polynomial C_{j,d_2}, such that, calling $\mu = \cos w$,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j, \qquad (2)$$

for all $d_2 \neq 0$, where

$$C_{j,d_2}(\mu) = \sum_{k=0}^{[j/2]} \frac{(-1)^k (d_2)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d_2)_j = \frac{\Gamma(d_2 + j)}{\Gamma(d_2)},$$

where $\Gamma(x)$ represents the Gamma function and a truncation will be required in (2) to make the polynomial operational. Of particular interest is the case of $d_2 = 1$, i.e. when the process contains unit root cycles; its performance in the context of macroeconomic time series was examined, for example, by Bierens (2001).[4] Such processes, for which the crucial issue is to have a spectral density with a peak at $(0, \pi]$, were later extended to the case of a finite number of peaks by Giraitis and Leipus (1995) and Woodward et al (1998) (see also Gray et al (1989) and Robinson (1994a)). The economic implications in (2) are similar to the previous case of long memory at the zero frequency. Thus, if $d_2 < 1$, shocks affecting the cyclical part will be

mean reverting, while $d_2 \geq 1$ implies an infinite degree of persistence of the shocks. This type of model for the cyclical component has not been previously used for financial time series, though Robinson (2001, pp. 212-213) suggests its adoption in the context of complicated autocovariance structures.

2. The testing procedure

Following Bhargava (1986), Schmidt and Phillips (1992) and others in the parameterisation of unit-root models, Robinson (1994a) considers the regression model:

$$y_t = \beta' z_t + x_t \qquad t = 1, 2, ..., \tag{3}$$

where y_t is a given raw time series; z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend (in the case of $z_t = (1,t)'$); β is a (kx1) vector of unknown parameters; and the regression errors x_t are such that:

$$\rho(L;\theta)\, x_t = u_t \qquad t = 1, 2, ..., \tag{4}$$

where ρ is a given function which depends on L, and the (px1) parameter vector θ, adopting the form:

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}\,(1-L^s)^{d^s+\theta^s}\prod_{j=2}^{p-1}(1-2\cos w\,L+L^2)^{d_j+\theta_j}, \tag{5}$$

for real given numbers d_1, d^s, d_2, … d_{p-1}, integer p, and where u_t is I(0). Note that the second polynomial in (5) refers to the case of seasonality (i.e. s = 4 in case of quarterly data, and s = 12 with monthly observations). Under the null hypothesis, defined by:

$$H_o: \ \theta = 0 \tag{6}$$

(5) becomes:

$$\rho(L;\theta=0) = \rho(L) = (1-L)^{d_1}\,(1-L^s)^{d^s}\prod_{j=2}^{p-1}(1-2\cos w\,L+L^2)^{d_j}\,. \qquad (7)$$

This is a very general specification that makes it possible to consider different models under the null. For example, if $d_1 = 1$ and d^s, $d_j = 0$ for $j \geq 2$, we have the classical unit-root models (Dickey and Fuller, 1979, Phillips, 1987; Phillips and Perron, 1988, Kwiatkowski et al., 1992, etc.), and, if d_1 is a real value, the fractional models examined in Diebold and Rudebusch (1989), Baillie (1996) and others. Similarly, if $d^s = 1$ and $d_j = 0$ for all j, we have the seasonal unit-root model (Dickey, Hasza and Fuller, 1984, Hyllerberg et al., 1990, etc.) and, if d^s is real, the seasonal fractional model analysed in Porter-Hudak (1990). If $d_3 = 1$ and d^s, $d_j = 0$ for $j \neq 3$, the model becomes the unit root cycles of Ahtola and Tiao (1987) and Bierens (2001), and if d_3 is real, the Gegenbauer processes examined by Gray et al. (1989, 1994), Ferrara and Guegan (2001), etc.

In this paper we are concerned with both the long run and the cyclical structure of the series, and thus we assume that $d^s = 0$ and $p = 3$. In such a case (5) can be expressed as:

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}\,(1-2\cos w\,L+L^2)^{d_2+\theta_2}\,, \qquad (8)$$

and, similarly, (7) becomes:

$$\rho(L) = (1-L)^{d_1}\,(1-2\cos w L+L^2)^{d_2}\,. \qquad (9)$$

Here, d_1 represents the degree of integration at the long run or zero frequency (i.e., the stochastic trend), while d_2 affects the cyclical component of the series.

We next describe the test statistic. We observe $\{(y_t, z_t), t = 1,2,\ldots n\}$, and suppose that the I(0) u_t in (4) have parametric spectral density given by:

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi}\,g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi\,,$$

where the scalar σ^2 is known and g is a function of known form, which depends on frequency λ and the unknown (qx1) vector τ. Based on H_o (6), the residuals in (3), (4) and (8) are:

$$\hat{u}_t \;=\; (1-L)^{d_1}(1-2\cos w\,L+L^2)^{d_2}\,y_t \;-\; \hat{\beta}'\,s_t\,, \qquad (10)$$

where

$$\hat{\beta}=\left(\sum_{t=1}^{n} s_t\,s_t'\right)^{-1}\sum_{t=1}^{n} s_t(1-L)^{d_1}(1-2\cos w\,L+L^2)^{d_2}\,y_t\,, \qquad s_t \;=\; (1-L)^{d_1}(1-2\cos w\,L+L^2)^{d_2}\,z_t.$$

Unless g is a completely known function (e.g., g ≡ 1, as when u_t is white noise), we need to estimate the nuisance parameter τ, for example by $\hat{\tau} \;=\; \arg\min_{\tau\in T}\;\sigma^2(\tau)$, where T is a suitable subset of R^q Euclidean space, and

$$\sigma^2(\tau) \;=\; \frac{2\pi}{n}\sum_{s=1}^{n-1} g(\lambda_s;\tau)^{-1}\,I_{\hat{u}}(\lambda_s), \;\text{ with }\; I_{\hat{u}}(\lambda_s) \;=\; \left|(2\pi n)^{-1/2}\sum_{t=1}^{n}\hat{u}_t\,e^{i\lambda_s t}\right|^2; \qquad \lambda_s \;=\; \frac{2\pi s}{n}.$$

The test statistic, which is derived through the Lagrange Multiplier (LM) principle, then takes the form:

$$\hat{R} \;=\; \hat{r}'\hat{r}; \quad \hat{r} \;=\; \left(\frac{\sqrt{n}}{\hat{\sigma}^2}\right)\hat{A}^{-1/2}\,\hat{a}, \qquad (11)$$

where n is the sample size, and

$$\hat{a} \;=\; \frac{-2\pi}{n}\sum_{s}^{*}\psi(\lambda_s)\,g(\lambda_s;\hat{\tau})^{-1}\,I(\lambda_s); \qquad \hat{\sigma}^2 \;=\; \sigma^2(\hat{\tau}) \;=\; \frac{2\pi}{n}\sum_{s=1}^{n-1} g(\lambda_s;\hat{\tau})^{-1}\,I(\lambda_s)\,,$$

$$\hat{A} \;=\; \frac{2}{n}\left(\sum_{s}^{*}\psi(\lambda_s)\psi(\lambda_s)' \;-\; \sum_{s}^{*}\psi(\lambda_s)\hat{\varepsilon}(\lambda_s)'\left(\sum_{s}^{*}\hat{\varepsilon}(\lambda_s)\hat{\varepsilon}(\lambda_s)'\right)^{-1}\sum_{s}^{*}\hat{\varepsilon}(\lambda_s)\psi(\lambda_s)'\right)$$

$$\psi(\lambda_s)' \;=\left[\psi_1(\lambda_s),\,\psi_2(\lambda_s)\right]; \qquad \hat{\varepsilon}(\lambda_s) \;=\; \frac{\partial}{\partial\tau}\log g(\lambda_s;\hat{\tau})\,;$$

$$\psi_1(\lambda_s) \;=\log\left|2\,\sin\frac{\lambda_s}{2}\right|; \qquad \psi_2(\lambda_s) \;=\; \log\left|\,2\,(\cos\lambda_s \;-\cos w)\right|.$$

Based on H_o (6), Robinson (1994a) established that, under certain regularity conditions:[5]

$$\hat{R} \;\to_d\; \chi^2_2, \qquad as \quad n\to\infty. \qquad (12)$$

Thus, as shown by Robinson (1994a), unlike in other procedures, we are in a classical large-sample testing situation, and furthermore the tests are efficient in the Pitman sense against local

departures from the null.[6] Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions: a test of (6) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob ($\chi^2_{2,\alpha} > \chi^2_2$) = α. A similar version of Robinson's (1994a) tests (with $d_1 = 0$) was examined in Gil-Alana (2001), where its performance in the context of unit-root cycles was compared with that of the Ahtola and Tiao's (1987) tests, the results showing that the former outperform the latter in a number of cases. Other versions of his tests have been successfully applied to raw time series in Gil-Alana and Robinson (1997, 2001) to test for I(d) processes with the roots occurring at zero and the seasonal frequencies respectively. However, this is the first empirical finance application, which tests simultaneously the roots at zero and the cyclical frequencies, a statistical approach which is shown in the present paper to represent a credible alternative to the more conventional ARIMA (ARFIMA) specifications used for the parametric modelling of many time series.

3. An empirical application to the US stock market

The dataset includes annual data on US inflation, real risk-free rate, real stock returns, equity premium and price/dividend ratio from 1871 to 1993, and is a slightly updated version of the dataset used in Cecchetti et al (1990) (see that paper for further details on sources and definitions).

(Insert Figure 1 about here)

Figure 1 contains plots of the original series with their corresponding correlograms and periodograms. All of them, with the exception of the price/dividend ratio, appear to be stationary. However, deeper inspection of the correlograms shows that there are significant values even at some lags relatively distant from zero, along with slow decay and/or cyclical oscillation in some cases, which could indicate not only fractional integration at the zero

frequency but also cyclical dependence. Similarly, the periodograms also have peaks at frequencies other than zero. For the price/dividend ratio, the slow decay in the correlogram clearly suggests that the series is not I(0) stationary.

(Insert Figure 2 about here)

Figure 2 displays similar plots for the first differenced data. The correlograms and periodograms now strongly suggest that all series are overdifferenced with respect to the 0 frequency. On the other hand, there are significant peaks in the periodograms at frequencies different from zero. In view of this, it might be of interest to examine more in depth the behaviour of these series using a fractional model at both the zero and the cyclical frequencies.

As a first step, we focus on the long run or zero frequency and implement a simple version of Robinson's (1994a) test, which is based on a model given by (3) and (4), with z_t = (1,t)', t ≥ 1, (0,0)' otherwise, and $\rho(L; \theta) = (1 - L)^{d+\theta}$. Thus, under H_o (6), we test the model:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ... \tag{13}$$

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{14}$$

for values d = 0, (0.01), 2, and different types of disturbances. In such a case, the test statistic greatly simplifies, taking the form given by (11), with $\psi(\lambda_s)$ being exclusively defined by $\psi_1(\lambda_s)$ and $\hat{u}_t = (1 - L)^d y_t - \hat{\beta}' w_t$. The null limit distribution will then be a χ^2_1 distribution. However, if $\rho(L; \theta) = (1 - L)^{d+\theta}$, then p = 1, and therefore we can consider one-sided tests based on $\hat{r} = \sqrt{\hat{R}}$, with a standard N(0,1) distribution: an approximate one-sided 100α% level test of H_o (6) against the alternative: H_a: θ > 0 (θ < 0) will be given by the rule: "Reject H_o if $\hat{r}$ > z_α ($\hat{r}$ < - z_α)", where the probability that a standard normal variate exceeds z_α is α. Note that by testing the null hypothesis with d = 1, this becomes a classical unit-root tests of the same form as those proposed by Dickey and Fuller (1979) and others. However, instead of using autoregressive (AR) alternatives of the form: $(1 - (1+\theta)L)x_t = u_t$, we use fractional alternatives.

Moreover, the use of AR alternatives involves a dramatic change in the asymptotic behaviour of the tests. Thus, if $\theta < 0$, x_t is stationary; it contains a unit root if $\theta = 0$, and it becomes nonstationary and explosive for $\theta > 0$. On the contrary, under fractional alternatives of the form as in (14), the behaviour of x_t is smooth across d, this being the intuitive reason for its standard asymptotic behaviour.

The results presented in Table 1 correspond to the 95%-confidence intervals of those values of d where H_o (6) cannot be rejected, using white noise disturbances.[7] We examine separately the cases of $\beta_0 = \beta_1 = 0$ a priori (i.e., with no regressors in the undifferenced model (13)); β_0 unknown and $\beta_1 = 0$ (with an intercept); and β_0 and β_1 unknown (an intercept and a linear time trend). The inclusion of a linear time trend may appear unrealistic in the case of financial time series. However, it should be noted that in the context of fractional (or integer) differences, the time trend disappears in the long run. Thus, for example, suppose that u_t in (14) is white noise. Then, testing H_o (6) in (13) and (14) with $d_o = 1$, the series becomes, for $t > 1$, a pure random walk process if $\beta_1 = 0$, and a random walk with an intercept if both β_0 and β_1 are unknown. The results vary substantially from one series to another. For instance, for inflation and real risk-free rates, the values are always higher than 0 but smaller than 0.5, oscillating between 0.07 (inflation rate with a linear trend) and 0.49 (real risk-free rate with no regressors). For real stock returns and equity premium, the values of d where H_o (6) cannot be rejected widely oscillates around 0, ranging between –0.18 (equity premium with a linear trend) and 0.14 (stock returns with no regressors). Finally, for the price/dividend ratio, all the non-rejection values are higher than 0.5, implying nonstationarity with respect to the zero frequency.

(Insert Tables 1 and 2 about here)

The significant results in Table 1 may be in part due to the fact that I(0) autocorrelation in u_t has not been taken into account. Thus, we also performed the tests imposing AR(1)

disturbances (see Table 2). Higher AR orders were also tried and the results were very similar. For all series, except the price/dividend ratio, the values oscillate around 0, implying that the series may be I(0) stationary. However, for the price/dividend ratio, the values are still above 0, ranging from 0.13 (with a linear time trend) to 0.83 (in the case of no regressors). Comparing the results of this table with those of Table 1 (white noise u_t), we are left with the impression that the orders of integration are smaller by about 0.20 when autocorrelation is allowed for. This may be related to the fact that the estimates of the AR coefficients are Yule-Walker, which entails AR roots that, although automatically less than one in absolute value can be arbitrarily close to one. Hence, they might compete with the order of integration at the zero frequency when describing the behaviour at such a frequency.

It may also be of interest to examine d, independently of the way of modelling the I(0) disturbances, at the same zero frequency. For this purpose, we use a semiparametric procedure due to Robinson (1995a), which we now describe. The Quasi Maximum Likelihood Estimator (QMLE) of Robinson (1995a) is basically a 'Whittle estimator' in the frequency domain, considering a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{s=1}^{m} \log \lambda_s \right), \qquad (15)$$

$$\overline{C(d)} = \frac{1}{m}\sum_{s=1}^{m} I(\lambda_s)\,\lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{n}, \qquad \frac{m}{n} \to 0,$$

where $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi n}\left| \sum_{t=1}^{n} x_t\, e^{i\lambda_s t} \right|^2,$$

and $d \in (-0.5, 0.5)$.[8] Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \rightarrow_d N(0, 1/4) \quad as \quad n \rightarrow \infty,$$

where d_o is the true value of d, with the only additional requirement that $m \rightarrow \infty$ slower than n. Robinson (1995a) showed that m must be smaller than n/2 to avoid aliasing effects. A multivariate extension of this estimation procedure can be found in Lobato (1999). There also exist other semiparametric procedures for estimating the fractional differencing parameter, for example, the log-periodogram regression estimator (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995b), and the averaged periodogram estimator (APE) of Robinson (1994b). However, we have chosen to use here the QMLE, primarily because of its computational simplicity. Note that, when using the QMLE, one does not need to employ any additional user-chosen numbers in the estimation (as in the case of the LPE and the APE). Also, there is no need to assume Gaussianity in order to obtain an asymptotic normal distribution, the QMLE being more efficient than the LPE.

(Insert Figure 3 about here)

Figure 3 reports the results based on the QMLE of Robinson (1995a), i.e., $\hat{d}$ given by (15) for a range of values of m from 1 to n/2.[9] It also displays the confidence intervals corresponding to the I(0) hypothesis for all series and the unit root for the price/dividend ratio. We see that, for inflation and the real risk-free rate, there are some estimates that are within the I(0) interval, especially if m is small; however, for most of the values of m, the estimates are higher than those corresponding to the confidence interval. For real stock returns and equity premium, almost all values are within such intervals, while for the price/dividend ratio they are clearly not. Also, for the latter series, the values are lower than those within the unit root interval, clearly suggesting that d is greater than 0 but smaller than 1. Consequently, the findings are the same as with the parametric procedure, namely there is strong evidence in favour of I(0) stationarity for real stock returns and equity premium, some evidence of long

memory for inflation and real risk-free rates, and strong evidence of fractional integration for the price/dividend ratio.

The above approach to investigating the long-run behaviour of a time series consists of testing a parametric model for the series and estimating a semiparametric one, relying on the long run-implications of the estimated models. The advantage of the first procedure is the precision gained by providing all the information about the series through the parameter estimates. A drawback is that these estimates are sensitive to the class of models considered, and may be misleading because of misspecification. It is well known that the possibility of misspecification can never be settled conclusively in the case of parametric (or even semiparametric) models. However, the problem can be partly addressed by considering a larger class of models. This is the approach used in what follows, where we employ another version of the tests of Robinson (1994a) that enables us simultaneously to consider roots at zero and the cyclical frequencies.

For this purpose, let us consider now the model given by (3) and (4), with $\rho(L; \theta)$ as in (8) and $z_t = (1,t)'$. Thus, under H_o (6), the model becomes:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ... \tag{16}$$

$$(1 - L)^{d_1} (1 - 2\cos w L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, ..., \tag{17}$$

and, if $d_2 = 0$, the model reduces to the case previously studied of long memory exclusively at the long-run or zero frequency. We assume that $w = w_r = 2\pi j/n$, $j = n/r$, and r indicating the number of time periods per cycle.

(Insert Table 3 about here)

We computed the statistic $\hat{R}$ given by (11) for values of d_1 and $d_2 = -0.50, (0.10), 2$, and $r = 2, \ldots, n/2$,[10] assuming that u_t is white noise. For brevity, we do not report the results for all statistics. In brief, the null hypothesis (6) was rejected for all values of d_1 and d_2 if r was smaller than 4 or higher than 9, implying that, if a cyclical component is present, its periodicity

is constrained to be between these two years. This is consistent with the empirical finding in Canova (1998), Burnside (1998), King and Rebelo (1999) and others that cycles have a periodicity between five and ten years. We report in Table 3 the non-rejection cases at the 5% level, with an intercept and with r = 6. The reason for giving the results only for the case of an intercept is that those based on a linear time trend were very similar, together with the fact that the coefficient corresponding to the linear time trend was found to be insignificantly different from zero in virtually all cases. Note that the test statistic is obtained from the null differenced model, which is assumed to be I(0), and therefore standard t-tests apply. Further, we focus on r = 6 since the non-rejection values with r = 4, 5, 7, 8 and 9 formed a proper subset of those non-rejections obtained with r = 6. We see that for inflation and real risk rate the non-rejection values oscillate between 0.10 and 0.40 for d_1, and between 0 and 0.3 for d_2. They are slightly smaller for d_2 in the case of stock returns and equity premium, in some cases being even negative. Finally, for the price/dividend ratio, the values of d_1 range between 0.5 and 1, while d_2 seems to be constrained between 0 and 0.5.

(Insert Figure 4 about here)

In order to have a more precise view about the non-rejection values of d_1 and d_2, we re-computed the tests but this time for a shorter grid, with d_1, d_2 = -0.25, (0.01), 2. Figure 4 displays the regions of (d_1, d_2) values where H_o cannot be rejected at the 5% level. Essentially, the series can be grouped into three categories: inflation rate and real risk-free rate; real stock returns and equity premium; finally price/dividend ratio. Starting with the first group (inflation and real risk-free rates), we observe that the values of d_1 range between 0.1 and 0.5 while d_2 seems to be constrained between 0 and 0.3. Thus, we observe a slightly higher degree of integration at the long run or zero frequency compared to the cyclical one. For real stock returns and equity premium, the values of both orders of integration oscillate around 0. Finally, for the price/dividend ratio the values of d_1 range between 0.5 and 1, while d_2 is between 0 and

0.5, implying nonstationarity with respect to the zero frequency but stationarity with respect to the cyclical component, and mean reversion with respect to both. Consequently, shocks to the latter series will disappear in the long run, with those affecting the cyclical part tending to disappear faster than those affecting its long-run or trending behaviour.[11]

4. Forecasting and comparisons with other models

In this section, we try first to determine the best model specification for each time series. Then, we compare the selected models with other approaches based on I(0) and I(1) hypotheses.

Given the lack of efficient procedures for estimating the parameters involved in the model given by (16) and (17), we have decided to use the following strategy: first, we recompute the values of the test statistic for d_{1o}, d_{2o} = -0.50, (0.01), 2 and r = 2, …, n/2, for the three cases of no regressors, an intercept and an intercept with a linear time trend. Then, we discriminate between the three cases according to the t-values of the estimated coefficients in (16), and choose the values of d_{1o}, d_{2o} and r which produce the lowest statistic in absolute value. The selected model for each time series is reported in the second column in Table 4. We observe that for inflation rate and real risk-free rate, both orders of integration are constrained to be between 0.10 and 0.30, the order of integration at zero being slightly higher than the cyclical one; for real stock returns and equity premium, the values of the d's are close to zero, being slightly negative for the zero frequency; finally, for the price-dividend ratio we see that it is nonstationary at the long-run frequency (d_1 = 0.68), and stationary with d_2 close to zero for the cyclical component.

(Insert Table 4 about here)

The third column of the table reports the selected models taking into account exclusively the component affecting the long run or zero frequency, while the fourth refers to the case of integer differentiation with respect to such a frequency. In both cases, we model the

cyclical structure using ARMA specifications. Starting with the case of fractional integration, we observe that the highest degree of integration is obtained for the price/dividend ratio (d = 0.73), followed by inflation (d = 0.19). For the remaining three series, the values are practically zero (0.03 for real risk-free rate; 0.01 for real stock returns, and –0.04 for equity premium). Imposing integer orders of integration, for the first four variables, we use d = 0 while for the price-dividend ratio we try both d = 0 and 1. With respect to the short-run components we use ARMA(p, q) models, with p, q ≤ 3, and choose the best model specification using both LR tests and likelihood criteria (AIC, BIC). We see that, for most of the series, the short-run structure can be described by simple MA models, the only exception being the real risk-free rate where an AR(1) process is imposed.

Next, we compare the various models in terms of their forecasting performance. Standard measures of forecast accuracy are the following: Theil's U, the mean absolute percentage error (MAPE), the mean-squared error (MSE), the root-mean-squared error (RMSE), the root-mean-percentage-squared error (RMPSE) and mean absolute deviation (MAD) (Witt and Witt, 1992). Let y_t be the actual value in period t; f_t the forecast value in period t, and n the number of periods used in the calculation. Then:

a) Theil's U: $\frac{\sqrt{\sum (y_t - f_f)^2}}{\sqrt{\sum (x_t - x_{t-1})^2}}$;

b) Mean absolute percentage error (MAPE): $\frac{\sum |(x_t - f_t)/x_t|}{n}$;

c) Mean squared error (MSE): $\frac{\sum (x_t - f_t)^2}{n}$;

d) Root-mean-percentage-squared error (RMSP): $\sqrt{\frac{\sum (x_t - f_t)^2 / f_t}{n}}$;

e) Root-mean-squared error (RMSE): $\sqrt{\frac{\sum (x_t - f_t)^2}{n}}$;

f) Mean absolute deviation (MAD): $\frac{\sum |x_t - f_t|}{n}$.

The first type of evaluation criteria measures the spread or dispersion of the forecast value from its mean. The MAD belongs to this category. It measures the magnitude of the forecast errors. Its principal advantages are ease of interpretation and the fact that each error term is assigned the same weight. However, by using the absolute value of the error term, it ignores the importance of over or underestimation.

The second type of accuracy measure is based on the forecast error, which is the difference between the observation, x_t, and the forecast, f_t. This category includes MSE, RMSE and RMSPE. MSE is simply the average of squared errors for all forecasts. It is suitable when more weight is to be given to big errors, but it has the drawback of being overly sensitive to a single large error. Further, just like MAD, it is not informative about whether a model is over- or under-estimating compared to the true values. RMSE is the square root of MSE and is used to preserve units. RMSPE differs from RMSE in that it evaluates the magnitude of the error by comparing it with the average size of the variable of interest. The main limitation of all these statistics is that they are absolute measures related to a specific series, and hence do not allow comparisons across different time series and for different time intervals. By contrast, this is possible using the third type of accuracy measure, such as MAPE, which is based on the relative or percentage error. This is particularly useful when the units of measurement of x are relatively large. However, MAPE also fails to take over or under estimation into consideration.

Unlike the measures mentioned above, Theil's U is a relative measure, allowing comparisons with the naïve ($x_t = x_{t-1}$) or random walk model, where a U = 1 indicates that the naïve method is as good as the forecasting technique, whilst U < 1 means that the chosen

forecasting method outperforms the naïve model. The smaller the U-statistic, the better the performance of the forecasting technique relative to the naïve alternative. Despite some attractive properties, the U-statistic has the disadvantage of not being as easily interpretable as MAPE; further, it does not have an upper bound, and therefore is not robust to large values.

The three selected time series models (fractional and cyclical differencing, FCD; fractional differencing, FD; and integer differencing, ID) for each of the series were used to generate the 5-year-ahead out-of-sample forecasts. Each forecast value was calculated and compared with the actual value of the series. Then, the above six criteria were used to rank the three forecasting models for each series. The ranking in terms of forecasting performance is given in Table 5. We observe that for inflation and real risk-free rate the FCD model outperforms FD and ID for all the criteria. For real stock returns and equity premium, the ID specification seems to be the most adequate, while for the price/dividend ratio the results are mixed. Therefore, on the basis of the MAPE, MSE, RMSP and RMSE criteria, the fractional and cyclical (FCD) model emerges as the best specification, while the other two criteria, MAD and Theil's U, suggest that the simple fractional model (with d = 0.73) is the most adequate one.

(Insert Table 5 about here)

5. Conclusions

In this paper we have examined the time series behaviour of the US stock market for the time period 1871 - 1993 by means of new statistical techniques based on long memory processes. Specifically, we have used a procedure due to Robinson (1994a) that has enabled us to test for unit and fractional roots not only at zero but also at the cyclical frequencies. These tests have standard null and local limit distributions and can easily be applied to raw time series.[12]

We initially focused exclusively on the long run or zero frequency, performing a suitable version of Robinson's (1994a) parametric tests along with a semiparametric estimation procedure. We used these methods because of the distinguishing features that make them particularly relevant in the context of financial time series. Specifically, they do not require Gaussianity (which is an assumption that is not satisfied in most financial data), but only a moment condition of order 2. Additionally, they have standard null limit distributions, which is another advantage of these tests compared to other procedures based on AR alternatives. The order of integration estimated using these methods varies considerably, but nonstationarity is found only in the case of the price/dividend ratio.

However, the non-rejection values obtained at the zero frequency could be partly due to the fact that attention has not been paid to other possible (cyclical) frequencies of the process. Thus, we adopted a method suitable for simultaneously testing for the presence of roots at zero and the cyclical frequencies, as in Robinson (1994a). For the latter frequencies, the model is based on Gegenbauer processes. The results suggest that the periodicity of the series ranges between 5 and 10 years, which is consistent with most of the empirical literature on cycles finding a periodicity of about six years (see, e.g., Baxter and King, 1999, Canova, 1998, and King and Rebelo, 1999). Further, the series can be grouped into three different categories: inflation and real risk-free rates, with the order of integration at the zero frequency fluctuating between 0 and 0.5 and d_2 (cyclical integration) between 0 and 0.3; real stock returns and equity premium, with both orders of integration fluctuating around 0; and finally, the price/dividend ratio, with d_1 ranging between 0.5 and 1 and d_2 between 0 and 0.5. Thus, we found evidence of stationary long memory with respect to both components for inflation and real risk-free rates; I(0) stationarity for stock returns and equity premium; and nonstationary long memory at the zero frequency but stationary at the cyclical component for the price/dividend ratio. Finally, the fact that all orders of integration are smaller than 1 suggests that mean reversion takes place

with respect to both components for all series, though the rate of adjustment varies across series.

An argument that could be employed against this type of models for the cyclical component is that, unlike seasonal cycles, business cycles are typically weak and irregular and are spread evenly over a range of frequencies rather than peaking at a specific value. A strong counterargument is that, in spite of the fixed frequencies used in this specification, flexibility can be achieved through the first differenced polynomial, the ARMA components and the error term. In fact, Bierens (2001) uses a model of this kind (with $d_2 = 1$) to test for the presence of business cycles in the annual change of monthly unemployment in the UK. Our analysis also yields clear-cut results, which are consistent with earlier findings on the periodicity of cycles.

The selected models for each time series were then compared with other approaches based on fractional and integer differentiation with respect to the zero frequency. Six forecasting criteria were employed and the results showed that the fractional cyclical model outperforms the others in a number of cases.

It would also be worthwhile to obtain point estimates of the fractional differencing parameters in this context of trends and cyclical models. For the trending component the literature is vast (see, e.g., Fox and Taqqu, 1986; Dahlhaus, 1989; Sowell, 1992; Tanaka, 1999, etc.). For the cyclical part, there are fewer contributions such as Arteche and Robinson (2000) and Arteche (2002). However, the goal of this paper is to show that a fractional model with the roots simultaneously occurring at the zero and the cyclical frequencies can be a credible alternative to the conventional ARIMA (ARFIMA) specifications. In fact, our approach leads us to some unambiguous conclusions, with the periodicity ranging between 4 and 10 years and most of the orders of integration within the intervals (0, 0.5) and (0.5, 1) depending on the series and the component under study.

Further research could be carried out in this context. For instance, the tests of Robinson (1994a) can be extended to allow for more than one cyclical component underlying the process. The existence of multiple cycles in financial series has not yet been examined empirically, and might be of interest in the context of various latent variates. Further, daily data could also be used to examine intraday periodicity, e.g. in the volatility of asset returns. As an alternative to the cyclical fractional approach, Andersen and Bollerslev (1997) modelled periodicity in returns by means of deterministic weights. The inclusion of deterministic components is possible in Robinson's (1994a) set-up, and its significance can be tested by means of a joint test of the deterministic regressors and of the order of integration. The univariate nature of the present study is also a limitation in terms of theorising, policy-making or forecasting. Theoretical models and policy-making involve relationships between many variables, and forecast performance can be improved through the use of many variables (e.g., factor based forecasts based on data involving hundreds of time series beat univariate forecasts, as shown, e.g., in Stock and Watson, 2002). However, the univariate approach taken in the present paper is useful, as it enables one to decompose the series into a long run and a cyclical component. Moreover, theoretical econometric models for both long run and cyclical fractional structures in a multivariate framework are not yet available. In this respect, the present study can be seen as a preliminary step in the analysis of financial data from a different time series perspective. Of particular interest in future work would be a more extensive study of the out-of-sample forecasting performance of our preferred model. In order to increase the number of out-of-sample observations and gain power, a rolling design could be used. Alternatively, larger sample could be obtained using higher frequency data, such as quarterly series. Data mining is an additional relevant issue worth exploring.

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Footnotes

1. Note that, for example, most of the "classical" unit root tests (i.e., Dickey and Fuller, 1979; Kwiatkowski et al., 1992; etc.) are non-standard, in the sense that the critical values have to be calculated numerically on a case by case simulation study.

2. For the purposes of the present paper, we define an I(0) process as a covariance stationary process with spectral density function that is positive and finite at any frequency on the spectrum.

3. Crozcek-Georges and Mandelbrot (1995), Taqqu et al. (1997), Chambers (1998) and Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes.

4. Unit root cycles were also examined by Ahtola and Tiao (1987), Chan and Wei (1988) and Gregoir (1999a, b).

5. These conditions are very mild and concern technical assumptions to be satisfied by $\psi_1(\lambda)$ and $\psi_2(\lambda)$.

6. In other words, if the tests are implemented against local departures of the form: H_a: $\theta = \delta n^{-1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi_2^2(v)$ with a non-centrality parameter v, which is optimal under Gaussianity of u_t.

7. The confidence intervals were built up according to the following strategy. First, choose a value of d from a grid. Then, form the test statistic testing the null for this value. If the null is rejected at the 95% level, discard this value of d. Otherwise, keep it. An interval is then obtained after considering all the values of d in the grid.

8. Velasco (1999a, b) has recently showed that the fractionally differencing parameter can also be consistently semiparametrically estimated in nonstationary contexts by means of tapering.

9. In the case of the price/dividend ratio, and in order to ensure stationarity, the estimates were based on the first differenced data, adding then one to the estimated values of d to get the proper orders of integration.

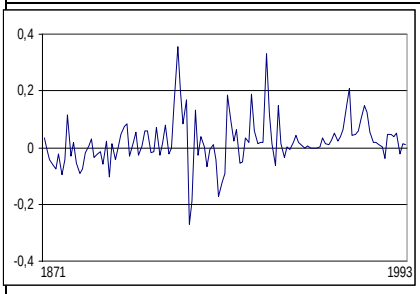
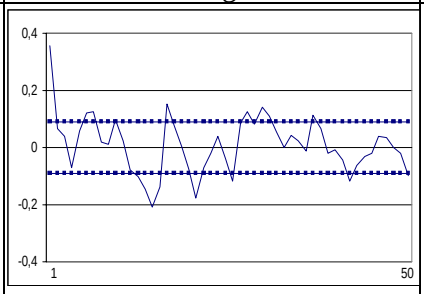
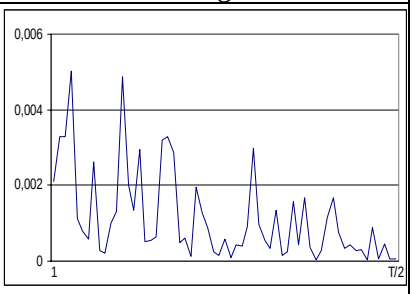
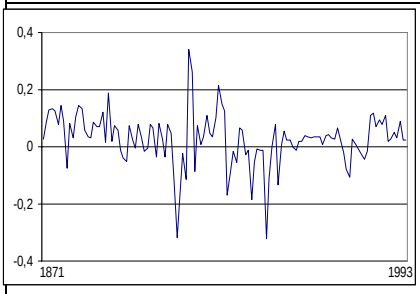
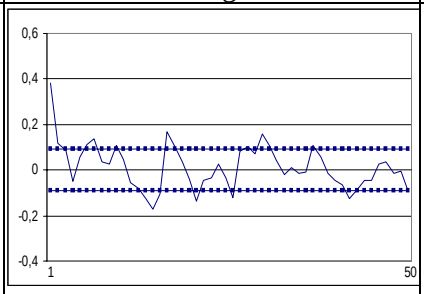
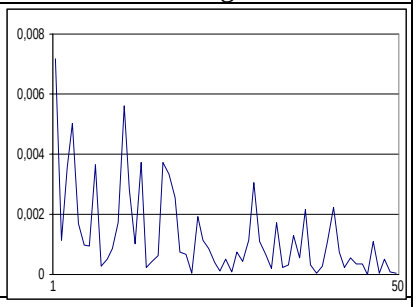
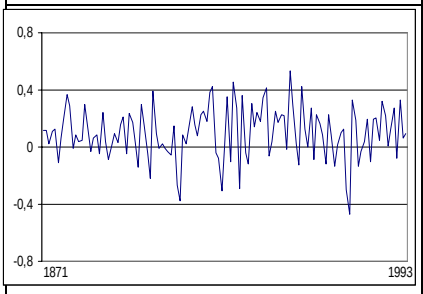
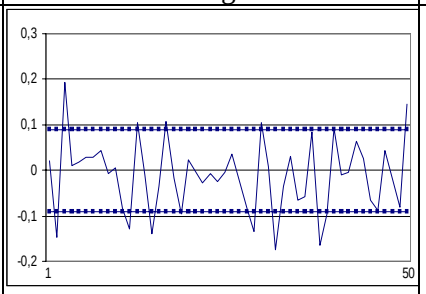
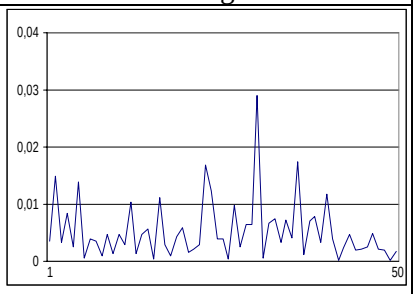
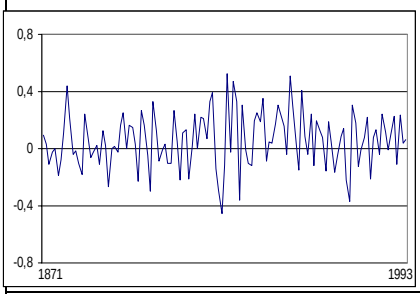
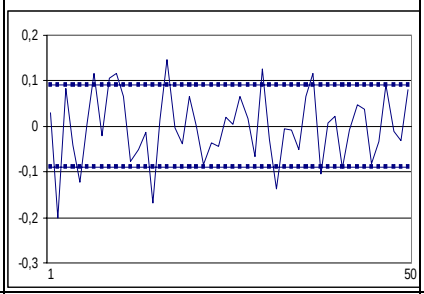
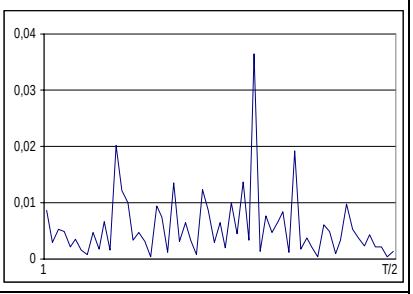
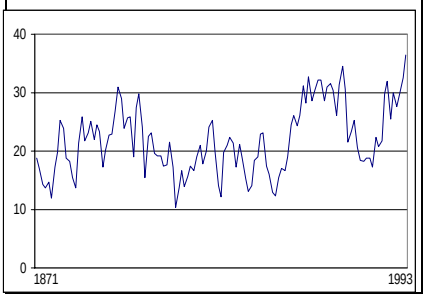
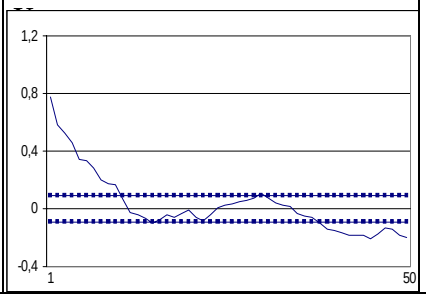
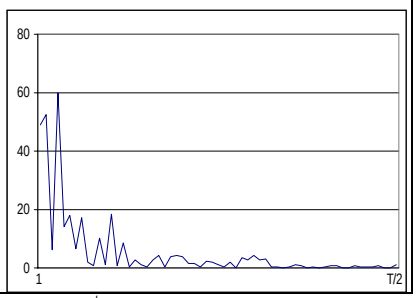
10. Note that in the case of r = 1, the model reduces to the case previously studied of long memory exclusively at the long run or zero frequency.

11. This procedure was also conducted in the context of autocorrelated (AR(1) and AR(2)) disturbances and the results did not substantially differ from those reported here based on white noise u_t.

12. A diskette containing the FORTRAN codes for the programs is available from the authors upon request.

FIGURE 1

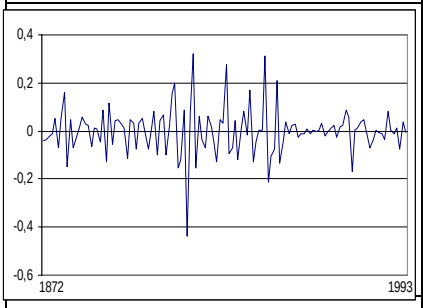
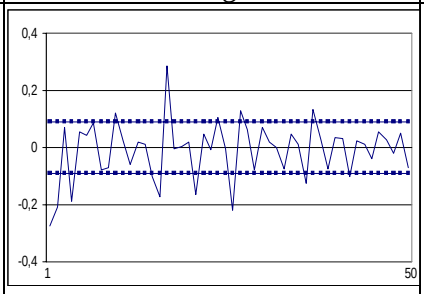
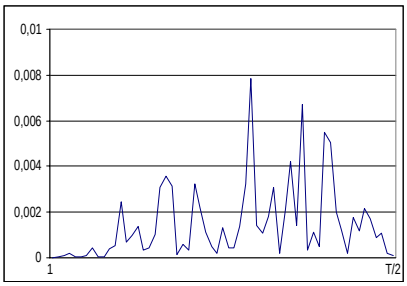
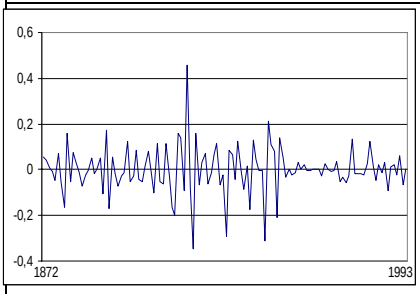
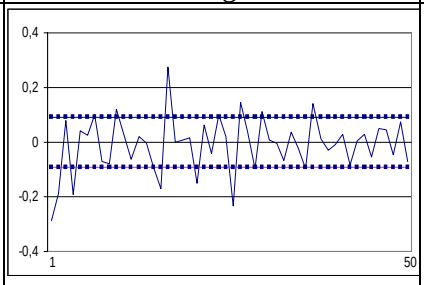
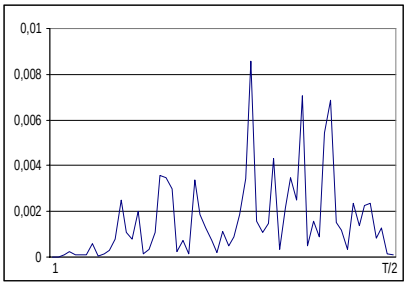
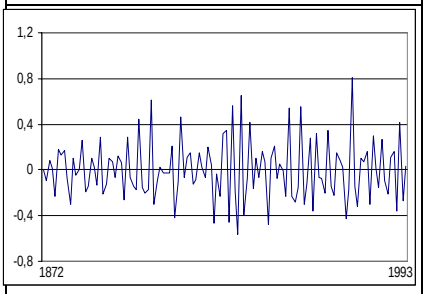
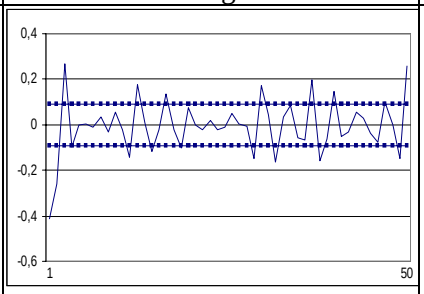
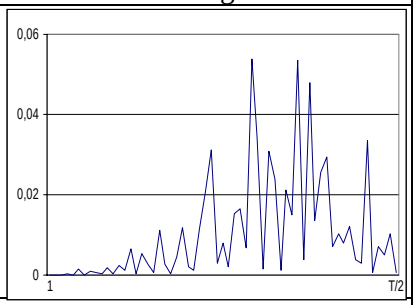
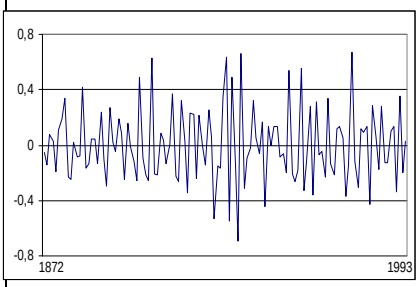
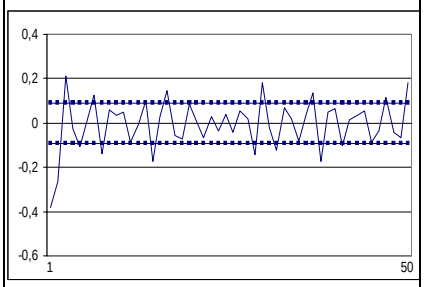
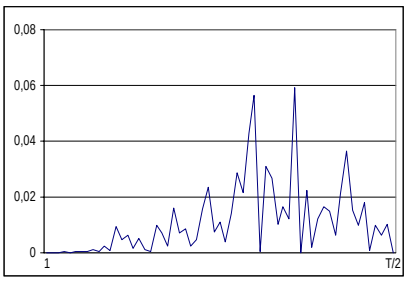
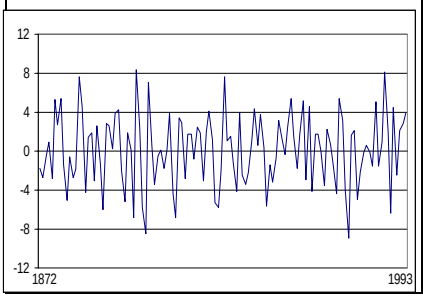
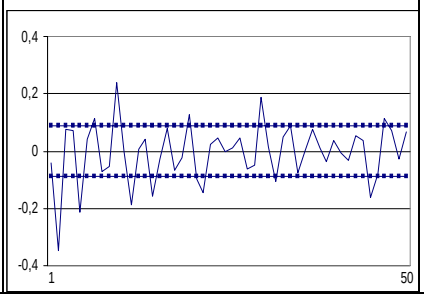
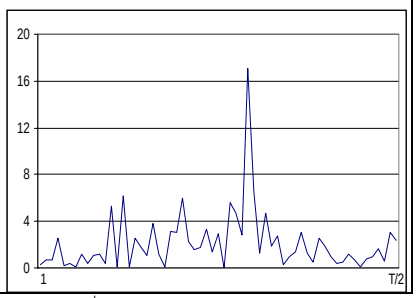
Raw time series, with their corresponding correlograms and periodograms

Inflation rate	Correlogram*	Periodogram
		
Real risk free rate	**Correlogram***	**Periodogram**
		
Real stock return	**Correlogram***	**Periodogram**
		
Equity premium	**Correlogram***	**Periodogram**
		
Price / Dividend ratio	**Correlogram***	**Periodogram**
		

* The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{n}$ or roughly 0.09 for series of length considered here.

FIGURE 2

First differenced time series, with their corresponding correlograms and periodograms

Inflation rate	Correlogram*	Periodogram
		
Real risk free rate	**Correlogram***	**Periodogram**
		
Real stock return	**Correlogram***	**Periodogram**
		
Equity premium	**Correlogram***	**Periodogram**
		
Price / Dividend ratio	**Correlogram***	**Periodogram**
		

* The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{n}$ or roughly 0.09 for series of length considered here.

TABLE 1			
Confidence intervals of the non-rejection values of d using $\hat{R}$ in (11) with $\rho(L; \theta) = (1 - L)^{d+\theta}$ and white noise u_t			
Time Series	No regressors	An intercept	A linear trend
INFLATION RATE	[0.12 - 0.45]	[0.13 - 0.46]	[0.07 - 0.44]
REAL RISK FREE RATE	[0.19 - 0.49]	[0.17 - 0.47]	[0.15 - 0.47]
REAL STOCK RETURN	[-0.09 - 0.14]	[-0.10 - 0.13]	[-0.10 - 0.13]
EQUITY PREMIUM	[-0.12 - 0.10]	[-0.14 - 0.10]	[-0.18 - 0.08]
PRICE / DIVIDEND RATIO	[0.72 - 1.02]	[0.58 - 0.92]	[0.59 - 0.92]

We test the null hypothesis: $d = d_o$ in a model given by $(1-L)^d x_t = \varepsilon_t$.

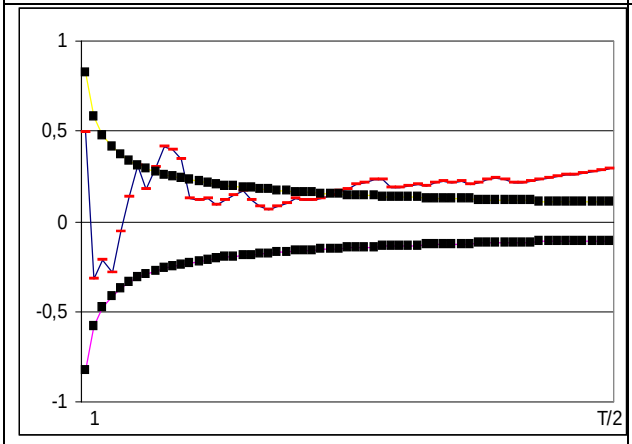
TABLE 2			
Confidence intervals of the non-rejection values of d using $\hat{R}$ in (11) with $\rho(L; \theta) = (1 - L)^{d+\theta}$ and AR(1) u_t			
Time Series	No regressors	An intercept	A linear trend
INFLATION RATE	[-0.13 - 0.19]	[-0.18 - 0.20]	[-0.44 - 0.11]
REAL RISK FREE RATE	[-0.11 - 0.33]	[-0.08 - 0.28]	[-0.14 - 0.27]
REAL STOCK RETURN	[-0.17 - 0.20]	[-0.25 - 0.18]	[-0.26 - 0.18]
EQUITY PREMIUM	[-0.22 - 0.00]	[-0.30 - 0.00]	[-0.41 - -0.04]
PRICE / DIVIDEND RATIO	[0.24 - 0.83]	[0.15 - 0.58]	[0.13 - 0.60]

We test the null hypothesis: $d = d_o$ in a model given by $(1-L)^d x_t = u_t$; $u_t = \tau u_{t-1} + \varepsilon_t$.

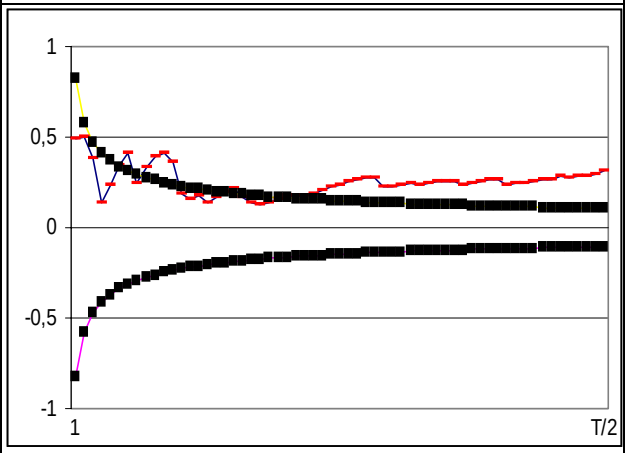
FIGURE 3

Semiparametric estimates of d based on the QMLE (Robinson, 1995a)

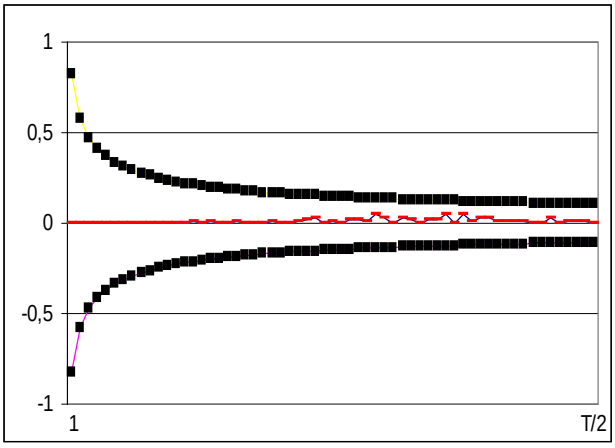
INFLATION RATE



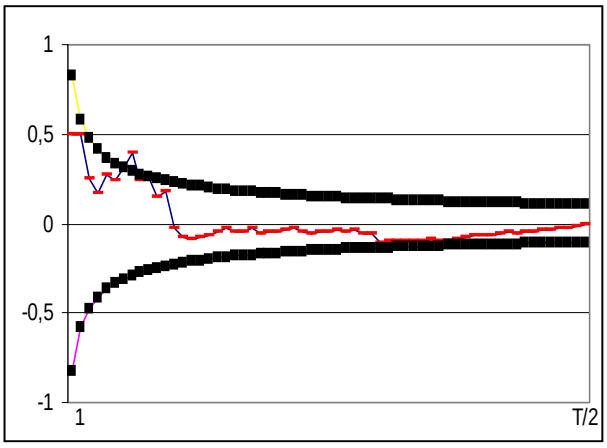
REAL RISK FREE RATE



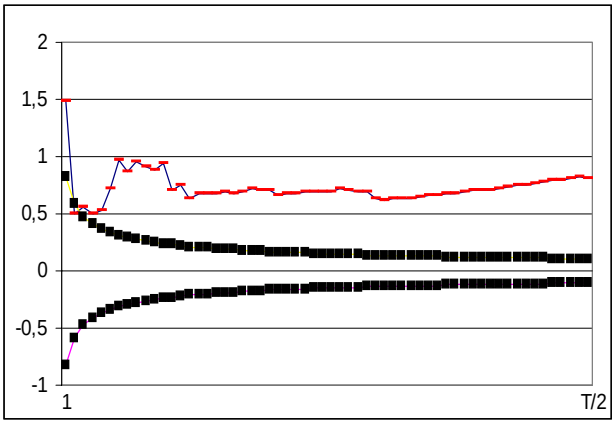
REAL STOCK RETURN



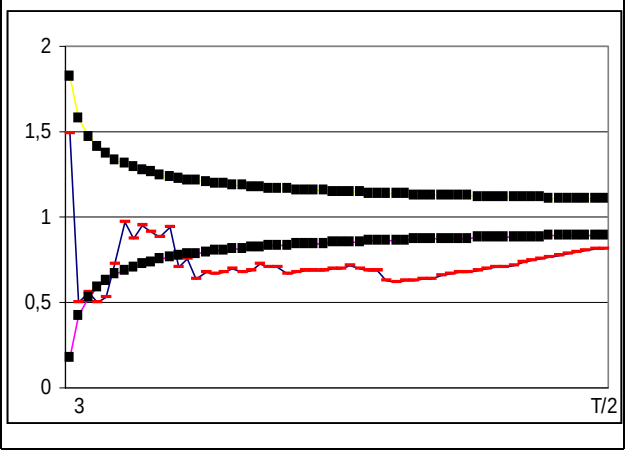
EQUITY PREMIUM



PRICE / DIVIDEND RATIO



PRICE / DIVIDEND RATIO (FIRST DIFF)



The horizontal axes corresponds to the bandwidth parameter number m, while the vertical one refers to the order of integration.

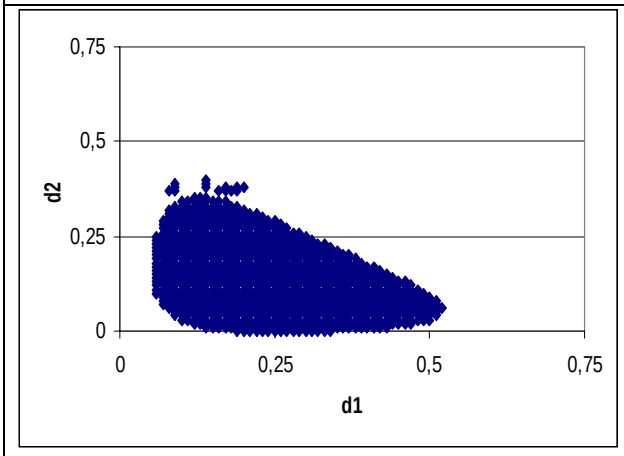
TABLE 3						
Testing H_o (6) in (16), (4) and (8) with $z_t \equiv 1$, $w = w_r$, $r = 6$ and white noise u_t						
D_1	d_2	INFLATION	RISK RATE	STOCK RT	PREMIUM	PRICE / DIV
-0.10	-0.10	39.49	51.69	**4.03**	**4.84**	236.63
-0.10	0.00	36.06	55.05	**3.38***	**0.69***	254.45
-0.10	0.10	36.86	58.98	**4.64***	**0.90***	265.93
-0.10	0.20	37.25	60.01	6.61	**3.03***	272.99
0.00	-0.10	28.25	30.70	**0.14***	**4.35***	170.35
0.00	0.00	16.73	24.09	**0.43***	**0.54***	186.83
0.00	0.10	13.29	23.03	**2.96***	**1.81***	197.19
0.00	0.20	12.66	22.78	6.60	**5.29***	202.97
0.10	-0.10	25.25	22.51	**1.32***	**5.49***	112.99
0.10	0.00	8.42	9.72	**1.95***	**2.74***	125.92
0.10	0.10	**3.04***	6.19	**5.64***	**5.11***	133.05
0.10	0.20	**2.72***	6.26	10.39	9.57	137.62
0.10	0.30	**4.70***	7.72	15.49	14.84	141.08
0.20	-0.10	24.90	20.29	**3.41***	6.91	68.85
0.20	0.00	**5.70***	**4.50***	**5.18***	**5.48***	76.73
0.20	0.10	**0.20***	**0.50***	9.78	8.87	81.48
0.20	0.20	**1.09***	**1.78***	15.19	13.99	83.13
0.20	0.30	**4.84***	**5.32***	20.69	19.63	82.15
0.30	-0.10	25.10	20.09	**5.89***	8.23	38.97
0.30	0.00	**5.65***	**3.62***	8.78	8.19	41.56
0.30	0.10	**0.98***	**0.40***	14.06	12.43	43.27
0.30	0.20	**3.32***	**3.19***	19.81	18.00	43.31
0.30	1.00	26.02	25.29	32.69	34.37	**4.71***
0.40	0.00	6.45	**4.45***	12.23	10.73	19.63
0.40	0.10	**3.30***	**2.70***	17.98	15.68	19.12
0.40	0.70	34.13	23.32	31.40	31.13	**5.73***
0.40	0.80	26.00	25.28	31.98	32.02	**5.08***
0.40	0.90	27.50	26.85	32.64	32.80	**5.11***
0.50	0.00	7.49	**5.84***	15.38	13.12	7.89
0.50	0.10	6.14	**5.83***	21.44	18.62	6.30
0.50	0.20	11.24	11,36	27.34	24.62	**5.81***
0.50	0.30	18.20	18.46	32.72	30.33	**5.86***
0.60	0.00	8.59	7.38	18.23	15..40	**2.70***
0.60	0.10	9.02	9.15	24.48	21.31	**1.00***
0.60	0.20	15.22	15.70	30.31	27.36	**1.25***
0.60	0.30	22.70	23.22	35.51	32.95	**2.59***
0.60	0.40	29.91	30.36	40.08	37.95	**4.70***
0.70	0.00	9.77	9.00	20.82	17.60	**1.22***
0.70	0.10	12.04	12.49	27.15	23.80	**0.04***
0.70	0.20	19.01	19,76	32.86	29.84	**1.26***
0.70	0.30	26.72	27.46	37.85	35.28	**3.77***
0.80	0.00	11.09	10.73	23.20	19.75	**1.72***
0.80	0.10	14.97	15.66	29.54	26.13	**1.39***
0.80	0.20	22.57	23.51	35.10	32.10	**3.57***
0.90	0.00	12.57	12.58	25.41	21.85	**3.19***
0.90	0.10	17.86	18.77	31.70	28.33	**3.82***
1.00	0.00	14.22	14.56	27.49	23.90	**5.05***

In bold and with "*", the non-rejection values of the null hypothesis at the 5% significance level.

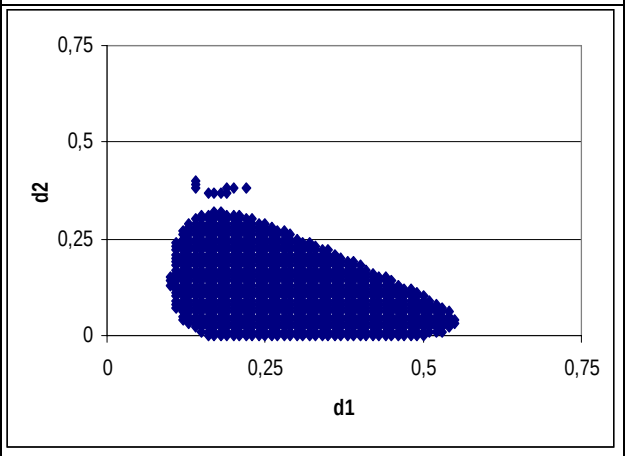
FIGURE 4

Non-rejection values of d_1 and d_2 in (16), (4) and (8) with r = 6 and white noise u_t

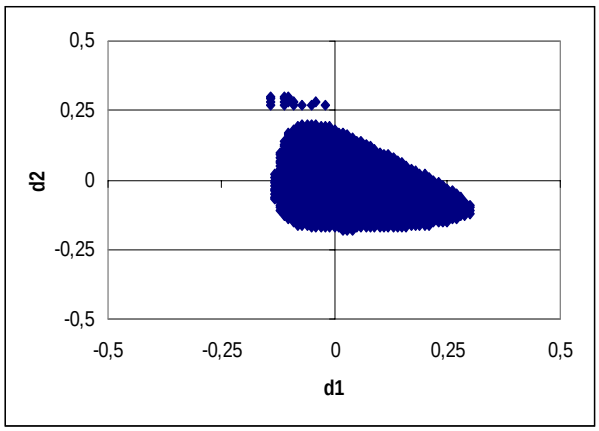
INFLATION RATE



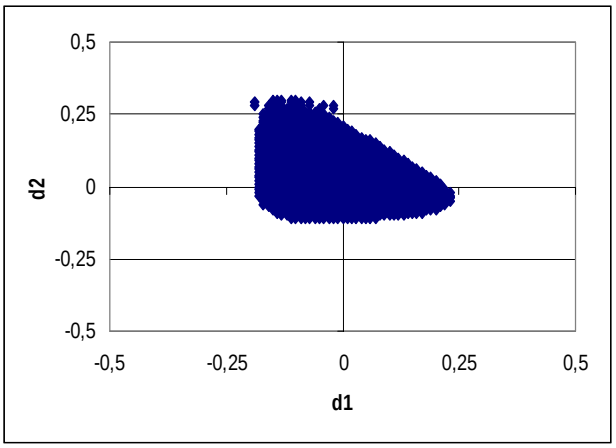
REAL RISK FREE RATE



REAL STOCK RETURN



EQUITY PREMIUM



PRICE / DIVIDEND RATIO

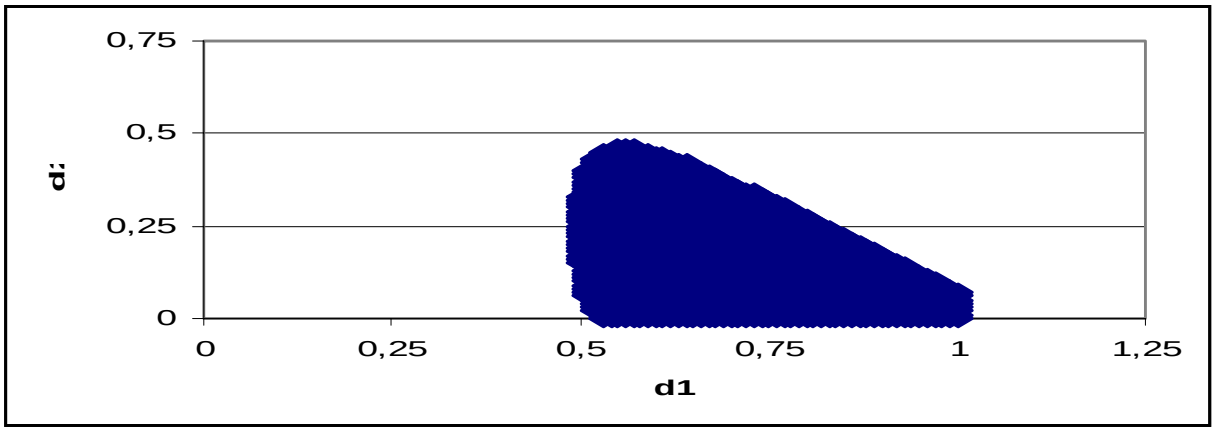


TABLE 4

Selected models for each time series

Models / Series	Fractional and cyclical differencing (FCD)	Fractional differencing (FD)	Integer differencing (ID)
Inflation rate	$y_t = 0.018 + x_t;$ (0.006) $(1-L)^{0.17}(1-2\cos w_7 L + L^2)^{0.14} x_t = \varepsilon_t$	$y_t = 0.017 + x_t;$ (0.009) $(1-L)^{0.19} x_t = \varepsilon_t$	$y_t = 0.020 + x_t;$ (0.009) $x_t = \varepsilon_t + 0.396\varepsilon_{t-1}$
Real risk free rate	$y_t = 0.0348 + x_t;$ (0.015) $(1-L)^{0.25}(1-2\cos w_6 L + L^2)^{0.10} x_t = \varepsilon_t$	$y_t = 0.021 + x_t;$ (0.011) $(1-L)^{0.03} x_t = u_t$ $u_r = 0.35 u_{t-1} + \varepsilon_t$	$y_t = 0.016 + x_t;$ (0.007) $x_t = 0.381 u_{t-1} + \varepsilon_t$
Real stock returns	$y_t = 0.0970 + x_t;$ (0.056) $(1-L)^{-0.05}(1-2\cos w_5 L + L^2)^{0.05} x_t = \varepsilon_t$	$y_t = 0.0971 + x_t;$ (0.019) $(1-L)^{0.01} x_t = u_t$ $u_r = 0.012 u_{t-1} + \varepsilon_t$	$y_t = 0.0970 + \varepsilon_t;$ (0.016)
Equity premium	$y_t = 0.0580 + x_t;$ (0.004) $(1-L)^{-0.06}(1-2\cos w_6 L + L^2)^{0.03} x_t = \varepsilon_t$	$y_t = 0.0546 + x_t;$ (0.003) $(1-L)^{-0.04} x_t = \varepsilon_t$	$y_t = 0.0574 + x_t;$ (0.011) $x_t = \varepsilon_t + 0.176\varepsilon_{t-1} - 0.239\varepsilon_{t-2}$
Price–Dividend ratio	$y_t = 18.811 + x_t;$ (6.679) $(1-L)^{0.68}(1-2\cos w_6 L + L^2)^{0.09} x_t = \varepsilon_t$	$y_t = 18.762 + x_t;$ (6.123) $(1-L)^{0.73} x_t = \varepsilon_t$	$(1-L) y_t = 0.163 + x_t;$ (0.018) $x_t = \varepsilon_t + -0.078\varepsilon_{t-1} - 0.340\varepsilon_{t-2}$

Standard errors are in parenthesis.

TABLE 5

Overall ranking of forecasting performance using different criteria

Series	Model	Theil's U	MAPE	MSE	RMSD	RMSE	MAD
Inflation rate	FCD	2	1	1	1	1	1
	FD	1	2	2	2	2	3
	ID	3	3	3	3	3	2
Real risk free rate	FCD	1	1	1	1	1	1
	FD	3	3	3	3	3	2
	ID	2	2	2	2	2	3
Real stock return	FCD	3	3	3	3	2	3
	FD	2	2	2	2	3	2
	ID	1	1	1	1	1	1
Equity premium	FCD	3	3	3	3	3	3
	FD	1	2	2	2	2	1
	ID	2	1	1	1	1	2
Price – Dividend ratio	FCD	2	1	1	1	1	2
	FD	1	2	2	2	2	1
	ID	3	3	3	3	3	3

FCD means Fractional and Cyclical Differentiation; FD is Fractional Differentiation and ID Integer Differentiation.