Using additional information in estimating the output gap in Peru: a multivariate unobserved component approach*

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Preliminar

Abstract

One of the key inputs for inflation targeting regime is the right identification of inflationary or disinflationary pressures. These pressures are usually approximated by the output gap. In this paper we provide an estimation of the Peruvian output gap using a multivariate unobserved component (MUC) model, relying on an explicit short term relation between output gap an inflation rate (Phillips Curve) and structural restrictions over output stochastic dynamics. The estimation is carried out via Kalman Filter technique. The results show that MUC output gap estimate is less sensible to end of sample problems and exhibits more relation with the Peruvian inflation process than other estimates, calculated with the Hodrick-Prescott filter and the production function approach. Furthermore, the diagnostic statistics report that MUC estimate increases out-sample predictive power for inflation. All these features make MUC output gap more reliable than other alternatives.

JEL classification: E32, E31, C51, C52.

Keywords: Output gap, Inflation, unobserved component model.

1 Motivation

One of the inputs for a inflation-targeter central bank is the right identification of inflationary or disinflationary pressures. It is important to have a reliable indicator of these pressures because the central bank will use it for guiding its monetary policy to achieve an inflation target. The central bank will apply a

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tight (expansive) policy whenever the indicator signs inflationary (disinflationary) pressures that risk its target.

In general, the indicator used is the output gap. This variable tries to measure the short run pressures of marginal costs over inflation generated by a demand expansion and an inaccurate distribution of the productive factors of the economy. Unfortunately, the output gap is an unobservable variable and its value must be inferred from the information contained in other economic variables. To this respect, the estimation of the output gap has been the focus of considerable research effort of many central banks[1].

The most common techniques are based on univariate filters, which only use GDP information[2]. These methodologies assumed that output is an isolate process from the rest of macroeconomic series. In most of the cases, this simplicity implies a high degree of uncertainty in the output gap measure, specially at the end of the sample[3]. Moreover, in the cases that other relevant variables have affected output gap, these univariate approaches do not permit to identify them[4], thus disturbing the decisions of monetary policy[5].

Figure (1) plots the annual variation of the core Consumer Price Index of Peru and output gap, estimated with the univariate Hodrick-Prescott (HP) filter for the quarterly period 1992-2003.

Inflation process in Peru presents two outstanding phases. The first one (1992-1994) is characterized by a continuous disinflation process from high inflation rates (more than 80 percent during 1992) to moderate inflation rates (around 20 percent in 1994). In the second phase (1995-2003) the inflation rate continues decreasing, but at a lower rhythm, passing from moderate inflation rates (around 11 percent in 1995) to low inflation rates (one-digit inflation in 1997 and lower than 5 percent since 1999).

On the other hand, HP output gap during the first phase (specifically in 1994-1996) indicate high positive gaps, suggesting the presence of strong inflationary pressures. Nevertheless, this result does not seem to make sense because inflation is declining in every moment. Another similar episode of strange results is observed at the final of the sample, where inflation is relatively stable but HP output gap is positive, indicating inflationary pressures. In this context, the results obtained with the HP filter do not permit to analyze and explain correctly the inflation rate evolution, particularly during periods where output is growing significantly and inflation is falling or it is stable. This univariate

[1] See for example, Benes and N'Diaye (2002) and Butler (1996).

[2] See for example, Hodrick and Prescott (1997), Beveridge and Nelson (1981), Baxter and King (1995) and Harvey and Jaeger (1993).

[3] Several studies have addressed this problem in univariate methods. For example, Orphanides and van Norden (1999) studied uncertainty in US ouput gap estimation process and Gruen *et. al.* (2002) do the same for Australian GDP. Their results confirm that end of problem is the principal source of uncertainty affecting ouput gap estimation.

[4] For example, Haltmaier (2001) uses cyclical indicators to adjust japanese output gap estimates derived from Hodrick and Prescott filter over the most recent period.

[5] Smets (2002) and Gaudich and Hunt (2000) found that the bigger the uncertainty surrounding output gap estimates, the smaller the reaction of monetary policy to it.

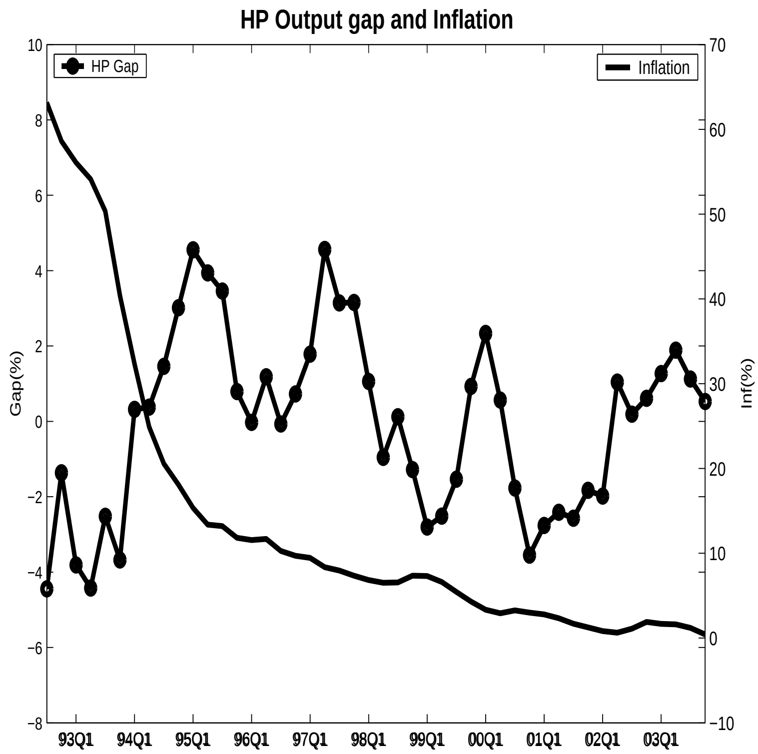

Figure 1: Ouput gap corresponds to Hodrick and Prescott estimate with smoothing parameter of 1600. Inflation is calculated on year to year base.

technique only capture the output process, without taking into account what was happening with other variables at the same time.

Inflation rate in Peru is not only determined by output gap. There are other variables like imported inflation or inflation expectations that must to be considered. Figure (2), plots core CPI inflation and imported inflation rates[6].

Core inflation seems to have a stronger relation with imported inflation, moving together except in two remarkable cases: 1994-1996 and 1998-1999. During the former period, inflation is higher than imported inflation suggesting that some inflationary pressures might have restrained the total pass-through. The opposite happens in the second case: imported inflation is higher than core inflation, and this coincide with a weak output phase. This analysis suggests that imported inflation is a key variable that have to be considered in the determination of the output gap.

From the point of view of macroeconomic analysis, the most important limitation of univariate methods is that they bring to bear no information about the

[6]Imported inflation is calculated from US inflation and nominal exchange rate depreciation (appreciation). Information about inflation expectations in Peru is not presented because there is not reliable quarterly time-series.

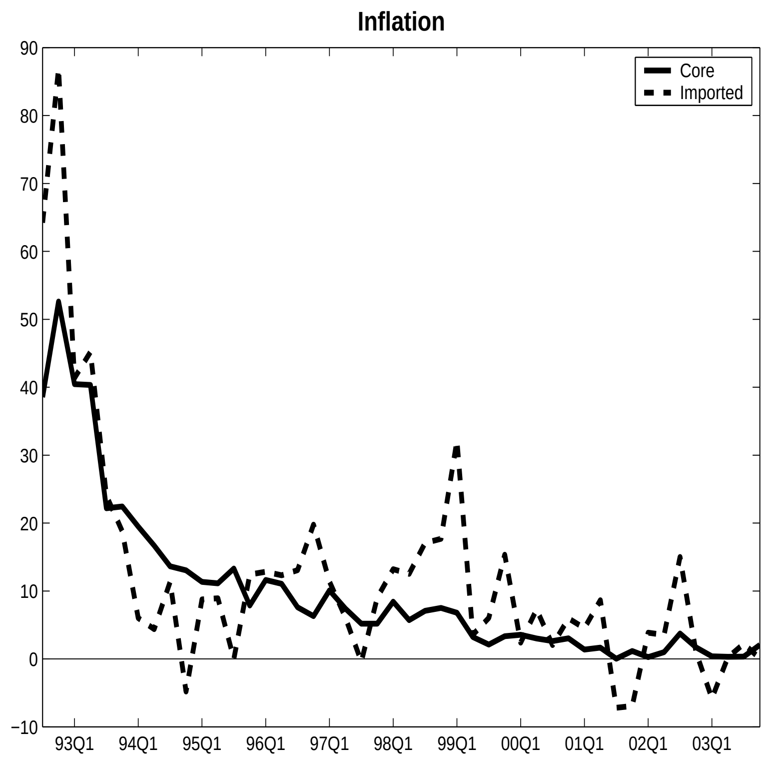

Figure 2: Quarterly core inflation and imported inflation. Imported inflation is computed by PPP formula.

structural constrains and limitations of production process as well as that they do not observe other important relations. As an alternative, multivariate methods involve different macroeconomic relationships in order to provide a better identification of the output gap.

Different multivariate methods have been developed, each one based on a particular theory and implementation technique. The first group using this approach is conformed by Production Function method, which consists on a neoclassical production function with different inputs, generally capital stock, labor force and total factor productivity (Solow residua). Often, researchers attempting to apply this technique use an univariate method to estimate the trend of productivity[7]. As a consequence, uncertainty remains on this component affecting the output gap reliability.

Another way to impose structural restrictions is using SVAR Blanchard and Quah (1989) identification. The SVAR output gap is the component not affected by permanent shocks and related to employment rate or inflation in a transitory way. This method has several limitations, it is not accurate to identify permanent and transitory shocks and its performance could be undermined

[7]See for example, Miller (2004a) and Texeira (2002).

by omitted variable problems[8].

Univariate, production function and SVAR approaches have been widely applied to Peruvian case. For example, Cabredo and Valdivia (1999), Caballero and Gallegos (2001) and Miller (2004a) compares different output gap estimates using those techniques. Their results indicate that production function output gap is the best indicator of inflation pressures in Peru.

More recently, a new group of multivariate methods combines structural relationships with statistics filter properties. Their main characteristic is that they include an explicit relation between output and inflation (Phillips Curve), and/or between the output gap and the unemployment rate (Okun´s law). Several authors have used multivariate techniques based on unobserved component models, whose estimation is based on Kalman Filter algorithm[9]. This approach benefits from correlation in the data and model structure, mixing this information according to the lower prediction error. This technique has been successfully applied, increasing the accuracy and reliability of output gap estimation[10].

The higher accuracy of multivariate estimations can determine important shifts in central bank behavior. Accessing to a better indicator enables the policy maker to avoid the shocks that affect the output gap, achieving an effective control over inflation[11]. In this sense, this paper is aimed to provide an estimation of the output gap for the Peruvian case that is directly related with the dynamic of inflation and take into account the information provided by other variables for its determination. We apply a multivariate unobserverd component (MUC) model, estimated via Kalman Filter for the period 1992-2003.

The results show that the multivariate unobserved component output gap (MUC) is less sensible to end of sample problems and presents a better relation with the Peruvian inflation process than other estimates, calculated with the Hodrick-Prescott filter and the production function approach. In particular, in periods of high output growth together with disinflationary or stable inflation environments, MUC output gap is lower than the ones obtained with the alternative methods mentioned. Besides, MUC identification is quite related to pass-through effect from imported prices to consumer prices. In particular, whenever imported inflation was higher (lower) than domestic inflation, the system found a negative (positive) output gap. Furthermore, the diagnostic statistics report that MUC estimate is more reliable than other alternatives and increases out-sample predictive power for inflation.

The document is organized in the following form. In the second section, the structure of the model used, as well as its implementation and the data, are explained and analyzed .In next section, we present the most important features

[8]See for a technical details van Norden (1995), Blanchard and Quah (1989) and Cerra and Chaman (2000).

[9]For example, see De Brouwer (1998), Scott (2000b), Camba-Mendez and Rodriguez-Palenzuela and Benes (2001) and N'Diaye (2002). Alternatively, Laxton and Tetlow (1992) and Hirose and Kamada (2001) propose a multivariate Hodrick and Prescott filter.

[10]See for example, Rünstler (2002). Although, it worths to say that the improvement depends on the structure imposed, and calibration or parameters estimation, see Butler (1996).

[11]For a discussion, see Gaudich and Hunt (2000) and Benati (2001).

of MUC estimate, and some of its properties: updating properties and inflation forecasts power. Finally, in the fourth section, we stress on the conclusive remarks of the paper.

2 The model

We use a semi-structural model for a small open economy. The system is based on three behavioral equations:

1. Uncovered interest parity.
2. Phillips Curve.
3. Aggregate demand.

The uncovered interest parity allows us to estimate the permanent and transitory components of real interest rate and real exchange rate. Combining the gaps of real interest rate and real exchange rate, we construct a real monetary condition index[12]. This index captures the general orientation of monetary policy affecting the aggregate demand with the aim to control the inflation rate[13]. Taking this index as an exogenous variable, we use the aggregate demand equation and Phillips curve to calculate the output gap related to the evolution of real activity and inflation. The model takes the following form,

$$y_t = \overline{y}_t + \widehat{y}_t \tag{1}$$

$$B(L)\,\widehat{y}_t = \kappa RMCI_t + \eta_t^y \qquad \eta_t^y \,\tilde{}\, NID(0, \sigma_{\eta y}^2) \tag{2}$$

$$\pi_t = \widetilde{\pi}_t + \varepsilon_t^\pi \qquad \varepsilon_t^\pi \,\tilde{}\, NID(0, \sigma_{\varepsilon\pi}^2) \tag{3}$$

$$\begin{aligned} \widetilde{\pi}_t \;=\;& \alpha_1 \widetilde{\pi}_{t-1} + \alpha_2 \widetilde{\pi}_t^m + (1-\alpha_1-\alpha_2)\widetilde{\pi}_{t,t+1}^e + \gamma \widehat{y}_t + \eta_t^\pi \\ & \eta_t^\pi \,\tilde{}\, NID(0, \sigma_{\eta\pi}^2) \end{aligned} \tag{4}$$

From equation (1), output y_t (in logarithms) is decomposed into potential output $\overline{y}_t$ and the output gap $\widehat{y}_t$. The second equation describes the output gap dynamics influenced by the real monetary condition index, $RMCI_t$. The lag polynomial is defined by $B(L) = 1 - \beta$, which represent an AR(1) stationary process. The equation (3) decompose the CPI core inflation, π_t, into its forecastable component[14], $\widetilde{\pi}_t$, and an stochastic shock η_t^π. The underlying inflation is modelled using a Phillips curve for a small open economy, equation (4). According to this equation, this measure is influenced by its own inertia, imported inflation $\widetilde{\pi}_t^m$, inflation expectations $\widetilde{\pi}_{t,t+1}^e$ and output gap $\widehat{y}_t$.

[12]See appendix B.

[13]See Dennis (1997) for technical discussion.

[14]The forescastable inflation may be interpreted as a measure of underlying or trend inflation, which is filtered from high frequency fluctuations. Arguably, a central bank should be held responsible primarly for development in underlying inflation and not for high frecuency inflation which is unable to control.

Potential output $\overline{y}_t$ follows a random walk process with a stochastic slope μ_t. The slope is modelled as an stationary autorregresive process with constant, $\overline{\mu}$, reflecting the growth rate of potential output in steady state[15].

$$\overline{y}_t = \overline{y}_{t-1} + \mu_t \tag{5}$$

$$\mu_t = \phi\mu_{t-1} + (1-\phi)\overline{\mu} + \eta_t^u \qquad \eta_t^u \tilde{} NID(0, \sigma_{\eta\mu}^2) \tag{6}$$

The model is completed by the assumption that stochastic shocks $\varepsilon_t^\pi, \eta_t^\pi, \eta_t^y$,and η_t^u are normally and independently distributed and mutually uncorrelated.

2.1 Inflation expectations

One important issue is inflation expectations measurement. Typically, new keynesian Phillips curve stress on forward looking behavior in the price setting process[16]. Particularly, the forward looking component on inflation is quite important during disinflation episodes[17].

On the other hand, empirical work usually assumes totally backward looking expectations[18], recognizing that inflation is a random walk. This approach implies that inflation level is unforecastable and that output gap fluctuations have permanent effects on it[19].

Confronting this trade off between theoretical and empirical grounds, we consider a simple error correction mechanism for inflation expectations which allows us to incorporate the deceleration on Peruvian inflation without assuming random walk dynamics[20],

$$\widetilde{\pi}_{t,t+1}^e = \overline{\pi}_{t+1} + (\widetilde{\pi}_t - \overline{\pi}_t) = \widetilde{\pi}_t + \Delta\overline{\pi}_{t+1} \tag{7}$$

where $\widetilde{\pi}_t$ is underlying inflation, $\widetilde{\pi}_{t,t+1}^e$ represent inflation expectations over next quarter, and $\overline{\pi}_t$ is interpreted as inflation target rate[21]. Given (7), if underlying inflation is higher (lower) than the target, inflation expectations raises (decreases). If inflation is aligned to the target, expectations do not change. Considering this structure, replace (4) with,

$$\widetilde{\pi}_t = \frac{\alpha_1}{\alpha_1+\alpha_2}\widetilde{\pi}_{t-1} + \frac{\alpha_2}{\alpha_1+\alpha_2}\widetilde{\pi}_t^m + \frac{(1-\alpha_1-\alpha_2)}{\alpha_1+\alpha_2}\Delta\overline{\pi}_{t+1} + \frac{\gamma}{\alpha_1+\alpha_2}\widehat{y}_t + \eta_t^\pi \tag{8}$$

[15] A local linear trend model for potential ouput was proved. The results indicate that a steady state growth rate of potential output reduces end of sample revisions. For a tecnhical discussion of local level and local linear trend models see Harvey (1993).

[16] See Calvo (1983) and Clarida *et. al.* (2002)

[17] See Mankiw and Reis (2001).

[18] See for example, Rünstler (2002).

[19] This as a clear contradiction to inflation targeting policy framework, recently adopted in Peru.

[20] This approach can be related to adaptative learning expectations. See Evans and Honkapohja (2001).

[21] Formally, inflation targeting was adopted as a monetary policy framework in Peru in 2002. Nevertheless, Central Reserve Bank of Peru has been announcing inflation targets since 1994. See Rossini (2001) for details.

2.2 The state space form

For estimation, the model must be put in its state space form, which comprises two equations[22]. Measurement equation (9) relates observations y_t at time $t, t = 1, ..., T$, to the state vector of unobserved[23]. Transition equation (10) denote the stochastic dynamic behavior governing the state vector.

$$x_t = Z_t \alpha_t + \varepsilon_t \tag{9}$$

$$A_0 \alpha_t = c + A_1 \alpha_{t-1} + B\varphi_t + R_0 \eta_t \tag{10}$$

where:
$x_t = \begin{bmatrix} \pi_t & \Delta y_t \end{bmatrix}'$ is the observable vector,
$\alpha_t = \begin{bmatrix} \widetilde{\pi}_t & \widehat{y}_t & \widehat{y}_{t-1} & \mu_t \end{bmatrix}'$ is the state vector,
$\varphi_t = \begin{bmatrix} \pi^m_{t-1} & \Delta\overline{\pi}_{t+1} & RMCI_t \end{bmatrix}'$ is the exogenous vector
$\varepsilon_t = \begin{bmatrix} \varepsilon^\pi_t & 0 \end{bmatrix}'$ and $\eta_t = \begin{bmatrix} \eta^\pi_t & \eta^y_t & 0 & \eta^\mu_t \end{bmatrix}'$ are innovation vectors.

Matrices are given by:

$$Z_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 1 & \frac{-\gamma}{\alpha_1+\alpha_2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (1-\phi)\overline{\mu} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \frac{\alpha_1}{\alpha_1+\alpha_2} & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \phi \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{\alpha_2}{\alpha_1+\alpha_2} & \frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2} & 0 \\ 0 & 0 & \kappa \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Innovations η_t and ε_t are mutually uncorrelated and have diagonal covariance matrices. Both are modelled as multivariate gaussian distributions. Matrices

[22] Because output has a unit root, the model was stationarized differencing output equation. Differentiation permits direct calculation of Kalman Filter initial conditions (see appendix A) from the model structure and data, without applying diffuse initial conditions priors, which modifies severely the results at the beginning of the sample.

[23] We present an structural version of state equation, which incorporates contemporaneous effects between underlying inflation and output gap. To get the autoregressive form, invert the left matrix of the state system.

A_0, A_1, B, R_0 and vector c depend on unknown hyperparameters[24]. After fixing hyperparameters, prediction, updating and smoothing algorithms are applied[25].

To get the usual state space representation as in appendix A, take into account the following equalities,
$T_t = A_0^{-1} A_1$, is the transition matrix,
$d_t = c + A_0^{-1} B\varphi_t$, resumes exogenous variables,
$R_t = A_0^{-1} R_0$.

2.3 Calibration

The model (1)-(6) incorporates several hyperparameters, coefficients $\{\beta,\ \kappa, \alpha_1,\ \alpha_2,\ \gamma,\ \phi,\ \overline{\mu}\}$ and variances $\{\sigma^2_{\varepsilon\pi},\ \sigma^2_{\eta\pi},\ \sigma^2_{\eta y},\ \sigma^2_{\eta\mu}\}$. This hyperparameters can be estimated using maximum likelihood procedure. However there are several issues with this approximation. First, for the sample selected, the inflation rate shows a persistent dynamics which can be confused with a random walk[26]. Recognizing that inflation has a unit root implies that is unforecastable and output gap has a permanent effect over it. Those characteristics contradict inflation targeting basis. Second, the quarterly sample used is too short to permit a reliable econometric estimation. Third, we suspect that structural breaks, due to institutional changes and structural reforms in Peru, could prevent a suitable econometric identification[27].

As an alternative, we choose to calibrate the model using external information. In order to get priors, a Phillips curve and a aggregate demand function, similar to equations (2) and (8), were estimated econometrically using Hodrick and Prescott output gap[28]. The chart 1 reports the selected values.

Chart 1: Model parameters

Parameter	*Calibrated value*
α_1	0.70
α_2	0.15
γ	0.70
β	0.70
κ	0.10
ϕ	0.80
$\overline{\mu}$	0.01

In the Phillips curve, we calibrate the parameter α_1 in 0.7. The inflation elasticity to output gap (γ) is calibrated in 0.7. This value is higher than the ones

[24]The next section focus on the criteria utilized for hyperparameters determination.
[25]See appendix A.
[26]This seudo random walk behavior can be explained by a non-stationary homogenous component in the stochastic dynamic equation of inflation. This phenomenon may invalidate any econometric estimation. For a technical discussion, see Enders (1995) chapter 1.
[27]There exist some evidence about structural breaks in Peruvian data, see León (1999). In general, structural breaks could distort inflation - output relationship, see Clark and McCracken (2003).
[28]We must take these results with caution because the presence of persistent dynamics on inflation can invalidate statistical inference and also the use of an incorrect output gap measure distort coefficient values. Nevertheless, the estimations give useful priors to model calibration.

found for other countries[29]. However, it reflects the low sacrifice ratio during the disinflation process in the last ten years[30]. Additionally, we set the pass-through effect from imported inflation over CPI core inflation captured by α_2 in 0.15, according to those found by Miller (2004b) and Winkelried (2004).

For the output gap equation we use the econometric estimation to set the inertia parameter β in 0.7, the effect of the real monetary condition index κ in 0.1 and the value of ϕ in 0.8. The steady state growth rate of potential output was fixed in 4 per cent (annualized), according to the mean growth rate of potential output calculated using production function approach[31].

All variances, except that for the growth rate of potential output, were normalized. For filtering process, we have to identify the permanent and transitory components of output. The signal extraction problem is basically related to the variance ratio between growth rate of potential output and output gap, $\sigma^2_{\eta\mu}/\sigma^2_{\eta y}$. We set this value in 1/64[32], confering smoothness to potential output and increase the relation between the cyclical component of output and inflation.

2.4 The data

We use Central Reserve Bank of Peru quarterly data base. The sample selected starts on 1992 and ends on 2003. As output we utilized the real GDP calculated using 1994 prices. Inflation is represented by core CPI inflation and nominal exchange rate by soles/US$ parity. As an international interest rate we use monthly LIBOR rate. External inflation is approximated by United States CPI inflation rate. Imported inflation is constructed as PPP formula: $\pi_t^m = \pi_t^{US} + \Delta e_t$, where π_t^{US} is US CPI inflation and Δe_t is the exchange rate depreciation (appreciation).

Real exchange rate is represented by imported prices index deflated by core consumer prices index. On the other hand, ex-post real interest rate is measured as: $r_t = i_t - \pi_t^{core}$, where i_t is the annualized interbank interest rate and π_t^{core} is year to year core inflation rate. Real monetary condition index is constructed with real interest rate and real exchange rate gaps. Risk premium is calculated as uncovered interest parity condition residua.

Finally, the inflation target rate is the Hodrick and Prescott trend of core inflation, restricted to the last announced target (2.5 percent) as a final level prior since 2002[33].

[29] For Germany 0.40 and United States 0.44, see Ball (1994), and Czech Republic 0.22, see Benes *et. al.* (2002).

[30] See Zegarra (2000).

[31] See Miller (2004a).

[32] Signal extraction problem is practically intractable without imposing some ad-hoc restrictions, see Quah (1992) for a technical discussion. For example, the direct estimation of variance ratio between transitory and permanent component of a time series tend to differ to those recomend by Hodrick and Prescott, see for example, Blith *et. al.* (2001).

[33] For a discussion of priors inclusion on Hodrick and Prescott filter, see St. Amant y van Norden (1997).

3 Results

We have divided this section in three parts. In the first part, we describe MUC output gap, comparing it to Hodrick-Prescott filter and the production function estimates. Additionally, properties of revisions in the output gap estimates and inflation forecast performance are discussed. The results indicate that MUC estimate shows more relation with Peruvian inflation process, reduces end of sample uncertainty and improves inflation forecast.

3.1 Output gap estimates

Panel (a) of figure (3) plots the MUC output gap estimate using the multivariate unobserved component approach, based on the model defined by (1)-(6) and (8).

According to the results, peruvian output gap has fluctuated inside the range of -7 to 2 per cent. Four periods of inflationary pressures can be identified: 1994Q2-1995Q4, 1997Q1-1997Q4, 1999Q4-2000Q2, and more recently 2002Q2-2002Q4. The first two periods have been the most outstanding and the longest, reaching levels near 2 per cent. With regard to the disinflationary pressures' episodes, they have been longer and have presented a higher average magnitude than inflationary ones. Four periods have been also identified: 1992Q3-1994Q2, 1996Q1-1996:Q4, 1998Q2-1999Q4, and 2000Q4-2002Q1, being the first one the most significant, -7 per cent.

Panel (b) of figure (3) plots quarterly underlying inflation and imported inflation. Both series show a high correlation during the ninety's. However, this relation breaks in two remarkable periods: 1994-1995 and 1998-1999. In the first one, underlying inflation is higher than imported inflation. At the same time, a positive output gap is identified, explaining the incomplete pass-through. The opposite happens in the second period: underlying inflation is lower than imported inflation, phenomenon accompanied with a negative output gap[34].

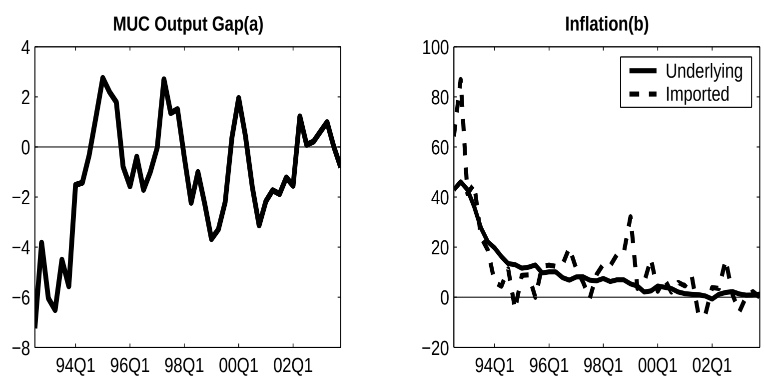

Figure 3: Ouput gap correspond to MUC smoother estimate. Underlying inflation is computed using semi-structural model and imported inflation is computed by PPP formula.

[34]This kind of non linear pass-through has been discussed recently in Winkelried (2004).

Figure (4) displays MUC output gap estimate beside to Hodrick-Prescott filter (HP) and the production function (PF) estimates[35].

MUC, HP and PF output gap estimates are very similar for all the sample. However, our estimate is lower than the alternatives in two periods: 1994-1997 and at the end of the sample (2003). The most remarkable feature about those episodes is that combine high output growth rates with a disinflationary process (1994-1997) or stable inflation environment (2003). On one hand, HP and PF methods tends to link the output gap evolution with the economic cycle, even when this cycle had not affected the inflation rate. On the other hand, MUC estimate is influenced not only by output behavior but also by domestic and imported inflation.

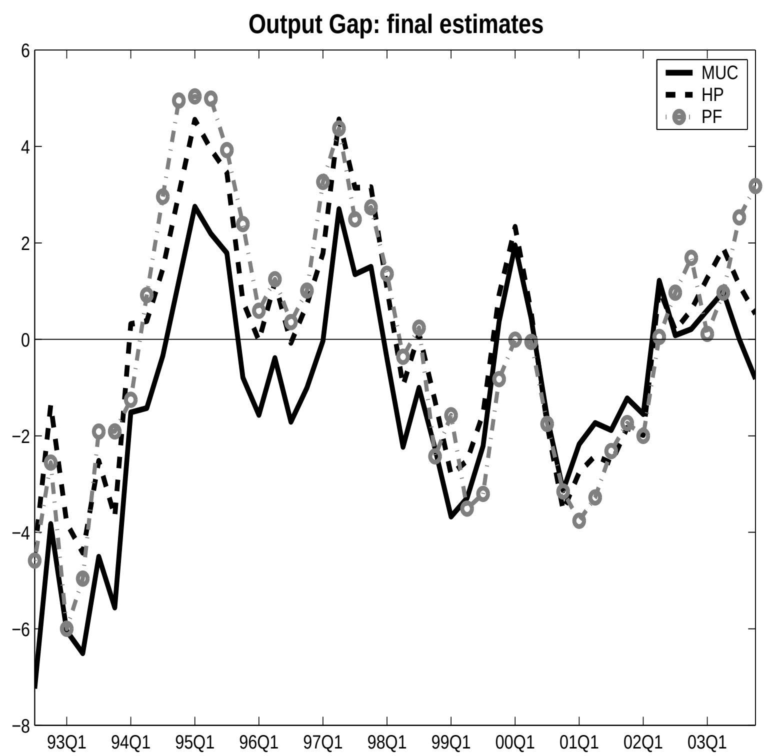

Figure 4: Smoother ouput gap estimates.

3.2 Properties of revisions: end of sample problem

In this section, we analyze the updating properties of MUC, HP and PF estimates. Measuring output gap revisions due to additional observations is one way of evaluating the uncertainty surrounding different methods[36]. In fact, the

[35]See appendix C for details concerning to HP and PF techniques.
[36]See Gruen *et.al.* (2002) and Orphanides and van Norden (1999).

higher uncertainty around output gap estimate, the lower is the sensitivity of monetary authority reactions to it[37].

In order to compare the reliability of each method we calculated the real-time estimates and the final estimates of the output gap[38]. The uncertainty that each method introduce in the output gap estimation is determined by the comparison between the final estimate and the real time estimate.

The results of this exercise are shown in the figure (5). The left panel plots the real-time and final estimates calculated with MUC, HP and PF methods. It is evident that updating of the MUC estimate of the real-time output gap in response to new data are much smaller than those of HP and PF. In the right panel, the scat graphs between real-time and final estimates are presented. Those graphs allow to observe clearer the uncertainty degree underlying each method. The scats graphs are divided in 4 areas: Areas I and III show the points where the final and real-time estimates provide contradictory signals, while areas II and IV present those occasions in which both estimations give similar signals. The results indicate that MUC estimates are grouped around the 45°line (areas II and IV), while HP and PF provide contradictory signals (areas I and II).

With the aim of quantify the uncertainty degree, we calculated the correlation coefficients and concordance indices[39]. Additionally, we test the reliability of output gap estimates using the Pesaran and Timmermann (1992) test[40]. The results are summarized in chart 2. The correlation coefficients indicate that the output gaps calculated (final and real-time) with MUC present higher co-movements (0.65) than those obtained with HP (0.26) and PF (0.16). In the same way, the concordance statistic indicates that real-time and final estimates with MUC provides similar signals (0.73), better than HP (0.63) and PF (0.52) do. Moreover, the application of the Pesaran and Timmermann test shows that the acceptance probability of similar signals in the case of MUC is 70.89 per cent, in contrast with the 0.01 per cent and 0.00 per cent of HP and FP, respectively. Those results suggest that the multivariate approach provides more reliable estimates[41].

[37]See Gaudich and Hunt (2000) and Smets (2002).

[38]The real time estimates correspond to updated state estimate in Kalman Filter recursion. In that sense, real-time estimate are conditional to past and current data. On the other hand, final estimates are equivalent to smoother estimates in Kalman Filter, reflecting all available information to forecast sequentially observable variables. See appendix A. To get updated estimates of Hodrick and Prescott filter we use its state space representation, see Scott (2000b)

[39]The concordance index is simply a non parametric statistic method that measures the time proportion in which two time series are in the same state. Thus, the degree of concordance will be 1 if both output gap measures have the same sign for a determined period. By contrast, it will take a zero value if the sign of both measures (final and real-time) are always opposite. For more details of this indicator see McDermott and Scott (1999).

[40]Quoted in Camba-Mendez and Rodriguez-Palenzuela (2001).

[41]A variety of studies evaluates the reliability of different univariate and multivariate methods, comparing the real-time estimates relative to final ones. Butler (1996), Conway and Hunt (1997), Camba-Mendez and Rodriguez-Palenzuela (2001), De Brouwer (1998) and Scott (2000a) compare the updating properties of different output gaps measures for Canada, European Union, United States, Australia and New Zealand. Their results suggest that multivariate output gaps estimates are statistically more reliable. Rünstler (2002) concentrated

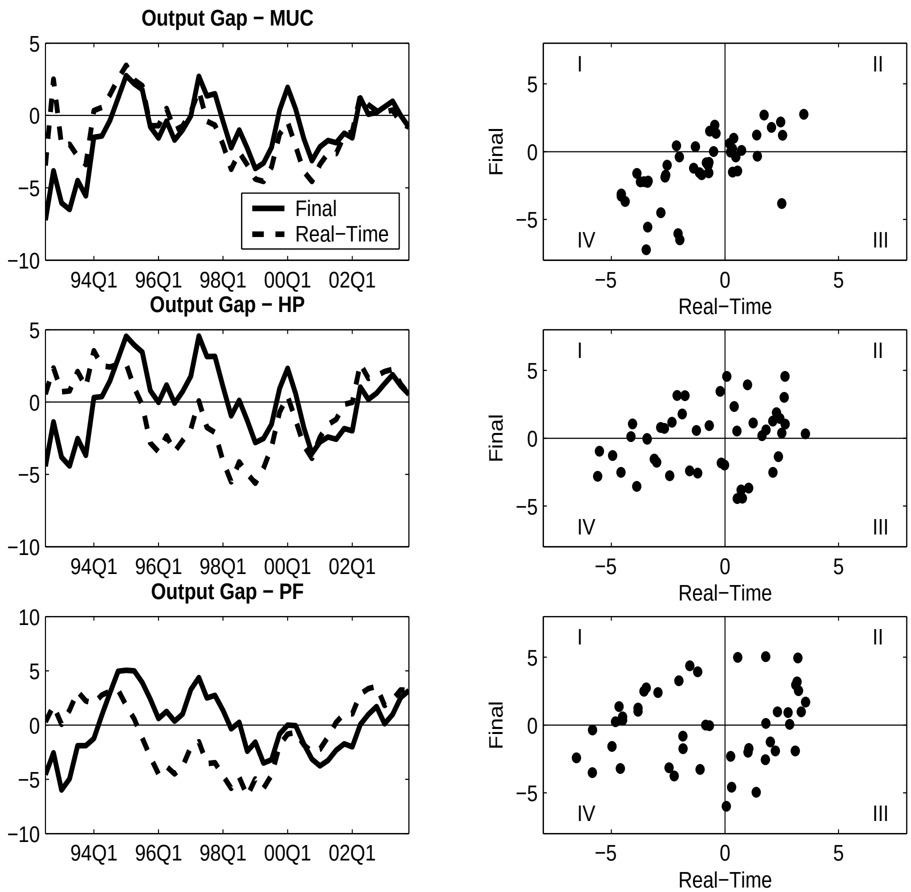

Figure 5: Final estimates correspond to smoother estimates. Real-Time estimates correspond to updated estimates.

Chart 2: Evaluation statistics

	MUC	HP	PF
Correlation coefficient	0.65	0.26	0.16
Concordance index	0.73	0.63	0.52
Pesaran and Timmermann test*	70.89%	0.01%	0.00%

* Acceptance probability of null hypothesis.

What explain these results? Because future data always contains relevant information to the current decomposition of transitory and permanent shocks, the most recent estimates of output gap will invariably change as the persistence characteristics of past shocks become more apparent. With structural restrictions, the MUC approach exploits the correlation in the data, guiding the output gap estimation at the end of the sample.

only on unobservable components methods, univariate and multivariate. As well as in the preceding cases, his study indicates that bigger information levels increase the confidence of the ouput gap estimate in European Union.

3.3 Inflation forecast

The predictive power of the output gap for inflation through short term supply curve (Phillips curve) is an essential precondition for the economic validity of any output gap estimates. This section test the information content of different real-time output gap estimates as a leading indicator for future inflation change. For this purpose, we analyze the following regression,

$$\Delta\widetilde{\pi}_t = \theta\widehat{y}_{t|t} + \sum_{i=1}^{k}\Delta\widetilde{\pi}_{t-i} + \varepsilon_t \tag{11}$$

where $\widehat{y}_{t|t}$ is the real time output gap estimate and $\widetilde{\pi}_t$ is the underlying inflation measure calculated from Kalman Filter recursion[42].

We apply this equation on real-time output gap estimates computed with different methods: MUC, HP and PF. Additionally an ARIMA regression is estimated, which is taken as a benchmark. In all cases, order lags k is found from Akaike criterion minimization.

We evaluate out-sample performance using the following steps. First, the equation (11) and ARIMA equation are estimated for the sample selected. Second, the out of sample forecasts of inflation changes over the next four quarters are computed. Third, another observation to the sample is added and the first two steps are applied. We start this procedure with a sample from 1992Q1 to 1997Q4, expanding it until 2002Q2.

Chart 3 reports the mean square error of forecast for underlying inflation using different output gap estimates in relation to the benchmark equation.

Chart 3: Out-sample forecasts performance

MSE-ratio (in percentages)	*Forecast Horizon*			
	+1	+2	+3	+4
MUC	57	55	65	67
HP	66	73	77	84
PF	75	77	78	85

MSE-ratio denotes the mean square error of the inflation forecast relative to MSE of the random walk forecast. MSE's of out-of-sample random walk forecasts are given by 1.56, 1.61, 1.76 and 1.29 for 1, 2, 3, 4 quarters ahead, respectively. Initial sample: 1992:Q1-1997Q4.

Out-sample forecasts using output gap estimates improve substantially on the random walk forecast at all the four quarters ahead. However, the improvement varies across different methodologies considered. MUC output gap increases inflation predictability more than HP and PF gaps estimates do. Additionally, the forecast performance of MUC estimate is nearly stable as the forecast horizon increases. In that sense, HP and PF perform worse, showing MSE-ratios increments as forecast horizon is expanded. At best, the four quarter forecast of HP and PF improve only slightly relative to the random walk model.

[42] The use of inflation changes eliminates the excessive persistence on inflation. Econometrically, this approach is optimal since improves regression identification and short term forecast.

4 Final remarks

With the objective of improving output gap measurement in Peru, we develop a semi-structural model for a small open economy, The model was estimated as a multivariate unobserved component model using Kalman Filter technique. The system incorporates explicitly a short term relation between output gap and inflation process through a Phillips Curve and also adds some other structural restrictions over potential output stochastic dynamics. Instead of econometrically estimated it, the model was calibrated using external information sources. Our results indicate that MUC output gap estimate outperforms other alternatives, computed using HP filter and PF.

The results indicate that MUC output gap is quite similar to alternatives measures. However, in periods of high output growth rate together with a disinflation or stable inflation context, our estimate indicate lower demand pressures than other estimates do. Besides, MUC output gap identification is quite related to pass-through effect from imported prices to consumer prices. In particular, whenever imported inflation was higher (lower) than domestic inflation, the system found a negative (positive) output gap.

Furthermore, we studied updating properties comparing the smoother estimates and the updated estimates of the three competitive approaches. The diagnostic statistics report that MUC estimate is the most reliable of the group. Finally we explore the out-sample predictive power for inflation of different output gap estimates. The results indicate that MUC estimates forecasts better inflation changes, confirming the essential precondition for the economic validity of any output gap estimate.

The advantages above-mentioned prove the importance of adding structural information on output gap calculation. For monetary policy purposes, this outcome could imply a significant uncertainty reduction and could improve future inflation control. Given that, the agenda could be oriented to explore additional cyclical indicators to improve output decomposition, in that sense, we recommend Rünstler (2002). Further, as the model presented here was calibrate, uncertainty involved in this process must be quantified. Regarding to this, Bayesian analysis of posterior densities of hyperparameters as in Harvey *et. al.* (2002) could be implemented.

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Technical appendix

Appendix A: Kalman Filter

The Kalman Filter is a recursive procedure based on two stages: prediction and updating[43]. Additionally, the smoother algorithm provides two sides estimates which are based on full-sample information.

Before Kalman Filter application, the model should be put in its state space form. The state space form comprises two equations. Measurement equation (A1) relates observations x_t at time $t, t = 1, ...T$, to the state vector of unobserved components, α_t. Transition equation (A2) denote the stochastic dynamic behavior governing the state vector.

$$x_t = Z_t \alpha_t + \varepsilon_t \tag{A1}$$

$$\alpha_t = d_t + T_t \alpha_{t-1} + R_t \eta_t \tag{A2}$$

where innovations η_t and ε_t are mutually uncorrelated. Both are modelled as multivariate gaussian distribution, with covariances matrix given by Q_t and H_t, respectively. System matrices Z_t, T_t, d_t, H_t and Q_t are deterministic and time-invariant. T_t and R_t matrices depend on unknown hyperparameters, which can be computed using maximum likelihood algorithm or calibrated. After fixing hyperparameters, Kalman Filter recursion with prediction and updating steps can be applied.

Prediction and updating

Based on all available information (including x_t), a_t is the estimate of state vector α_t and P_t is its mean square error,

$$P_t = E\left[(\alpha_t - a_t)(\alpha_t - a_t)'\right]$$

Assuming that a_{t-1} and P_{t-1} are given, the conditional estimate based on information set of period $t-1$ by one-step forecast from equation (A2) and its mean square error matrix can be computed as,

$$a_{t|t-1} = d_t + T_t a_{t-1} \tag{A3}$$

$$P_{t|t-1} = T_t P_{t-1} T_t^{'} + R_t Q_t R_t^{'} \tag{A4}$$

Taking into account the measurement equation (A1), the conditional estimate of x_t can be computed directly,

$$\widetilde{x}_{t|t-1} = Z_t a_{t|t-1} \tag{A5}$$

Starting from the above equation, the forecast error and its mean square error matrix can be calculated as,

$$\begin{aligned} v_t &= x_t - \widetilde{x}_{t|t-1} = Z_t(\alpha_t - a_{t|t-1}) + \varepsilon_t \\ F_t &= Z_t P_{t|t-1} Z_t^{'} + H_t \end{aligned}$$

[43] Further discussion can be found in Harvey (1994) and Hamilton (1994)

Once the forecast error is known, is possible to update the conditional prediction state vector and its mean square error matrix. The updating equations are,

$$a_{t|t} = a_{t|t-1} + P_{t|t-1}Z_t^{'}F_t^{-1}(x_t - Z_t a_{t|t-1}) \tag{A6}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}Z_t^{'}F_t^{-1}Z_t P_{t|t-1} \tag{A7}$$

The first term on the right side of (A6) is the conditional estimate of state vector using information available until $t-1$ period, which is updated according to the unanticipated component, $x_t - Z_t a_{t|t-1}$ (or forecast error over observable variables, v_t). A higher forecast error implies a higher adjustment over $a_{t|t-1}$. The factor $P_{t|t-1}Z_t^{'}F_t^{-1}$, is called Kalman gain. Kalman gain weights optimally the uncertainty forces affecting the linear prediction recursion. On one hand, the more conditional variance (uncertainty) around the conditional estimate, measure by $P_{t|t-1}$, the more weight on v_t information. The opposite, the more uncertainty on measurement equation, captured by F_t^{-1}, the less weight will be conferred to forecast error.
The prediction equations (A3) - (A5), and updating equations (A6) - (A7), defines the Kalman Filter formulae. Once all observations have been processed, the a_T estimate contains all available information to forecast. This final update estimate is employed as initial condition in smoothing backward recursion.
Kalman filter initial conditions
Kalman Filter recursion starts with initials state vector and its mean square error matrix,

$$\begin{aligned} E(\alpha_0) &= a_0 \\ Var(\alpha_0) &= P_0 \end{aligned}$$

where the innovations ε_t and η_t are neither correlated between them nor with the initial state vector, α_0.

$$E(\varepsilon_t \eta_s^{'}) = 0; \qquad E(\varepsilon_t \alpha_0^{'}) = 0; \qquad E(\eta_t \alpha_0^{'}) = 0$$

If transition matrix T_t eigenvalues are inside unite circle, (A2) is an stationary process. In this case, initial conditions are given by non-conditional mean and variance, derived from time-invariant form of equation (A2)[44],

$$\begin{aligned} a_0 &= (I-T)^{-1}d_0 \\ vec(P_0) &= [I - T \otimes T]^{-1} vec(RQR') \end{aligned}$$

However, if the model is non-stationary, $[I-T\otimes T]^{-1}$ does not exist. In this case, (a_0, P_0) can not be calculated from data and model structure. One solution is to stationarized the model. Alternatively, the state form keeps unaltered, but ad-hoc values for (a_0, P_0) must be assumed. There are two ways of implement this procedure. The first assumes that a_0 is fixed and known with certainty, which

[44]Taking expectations on (A2), $E(\alpha_t) = d_t + T_t E(\alpha_{t-1})$. Given mean stationary process, it can be concluded that, $E(\alpha) = (I-T)^{-1}d$. To compute P_0, vec operator is applicated and its following property, $vec(ABC) = (C' \otimes A)\ vecB$

implies $P_0 = 0$. In this case, a_0 should be estimated like any other hyperparameter. The second approximation is based on assumption that a_0 is stochastic with diffuse distribution, whose initial mean error square matrix is given by $P_0 = \kappa I$. Supposing a higher but finite value for κ is a good approximation of the uncertainty surrounding that initial condition.

Smoother

Once the prediction and updating processes are finished, a_T estimate contains all available information to forecast sequentially observable variables, y_t. At this point, is possible to apply smoother algorithms to adjust the state vector to all the information set. The smoother estimate, denote by $a_{t|T}$, is based on more information than predicted and updated estimates. Given that, the corresponding matrix $P_{t|T}$, is generally smaller than $P_{t|t}$ and $P_{t|t-1}$.

To compute $a_{t|T}$ a backward recursion is applied taking as initial values, a_T and P_T,

$$a_{t|T} = a_t + P_t^*(a_{t+1|T} - T_{t+1}a_t) \tag{A8}$$

$$P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*'} \tag{A9}$$

where,

$$P_t^* = P_t T_{t+1}^{'} P_{t+1|t}^{-1}$$

Appendix B: Real monetary condition index

A real monetary condition index summarizes the principal transmission channels of monetary policy, directly through real interest rate, and indirectly through real exchange rate[45]. The index is calculated as a linear combination of real interest rate and real exchange rate gaps,

$$rmci_t = -\theta \widehat{r}_t + (1-\theta)\widehat{q}_t \tag{B1}$$

where $\widehat{r}_t$ and $\widehat{q}_t$ are the gaps of real interest rate and real exchange rate, respectively. The coefficient θ measures the relative importance of real interest and real exchange rate channels in monetary transmission phenomenon[46]. In that sense, a positive (negative) real monetary condition index implies an expansive (contractive) monetary policy stance.

Real interest rate and real exchange rate gaps were computed using Kalman filtering technique. The model applied is based on uncovered interest parity condition.

$$r_t - r_t^* = 4\Delta q_t + p_t \tag{B2}$$

where r_t is the domestic real interest rate; r_t^* is the external real interest rate; q_t is the real exchange rate (in logarithms) and p_t represents risk premium level. We can decompose every variable in UIP equation into transitory (gap) and trend components.

[45] See Dennis (1997).

[46] A higher value of θ indicates that real interest rate channel is more important than real exchange rate channel. Therefore, a higher real depreciation (appreciation) is requiered to off set the effects of real interest rate increment (reduction).

$$
\begin{aligned}
r_t - 4\Delta q_t - p_t &= \overline{r}_t + \widehat{r}_t - 4\left(\Delta\overline{q}_t\right) - 4\left(\widehat{q}_t - \widehat{q}_t\right) - \overline{p}_t - \widehat{p}_t \\
r_t - 4\Delta q_t - p_t &= \left[\overline{r}_t - 4\left(\Delta\overline{q}_t\right) - \overline{p}_t\right] + \left[\widehat{r}_t - 4\left(\widehat{q}_t - \widehat{q}_t\right) - \widehat{p}_t\right]
\end{aligned}
$$

Taking UIP as a cointegration relation implies that real interest rate, change in real exchange rate and risk premium move together around a long term equilibria[47].

$$\overline{r}_t - \overline{r}_t^* = 4\Delta\overline{q}_t + \overline{p}_t$$

Considering the above-mentioned, we rearrange the UIP equation as,

$$x_t = \overline{r}_t^* + \left[\widehat{r}_t - 4\left(\widehat{q}_t - \widehat{q}_t\right) - \widehat{p}_t\right]$$

where $x_t = r_t - 4\Delta q_t - p_t$.

We need to specify the stochastic laws of motion of real interest rate, real exchange rate and risk premium. We assume that real interest rate, change in real exchange rate and risk premium follow a local level model[48] as,

$$
\begin{aligned}
z_t &= \overline{z}_t + \widehat{z}_t \\
\overline{z}_{t-1} &= \overline{z}_{t-1} + \eta_t^{\overline{z}} \qquad \eta_t^{\overline{z}}\tilde{\ }NID(0, \sigma^2_{\eta\overline{z}}) \\
\widehat{z}_t &= \eta_t^{\widehat{z}} \qquad \eta_t^{\widehat{z}}\tilde{\ }NID(0, \sigma^2_{\eta\widehat{z}})
\end{aligned}
$$

where $\overline{z}_t$ and $\widehat{z}_t$ represent the transitory (gap) and permanent (trend) component, $z_t = r_t, \Delta q_t, p_t$. To compute the gaps we calibrate the signal extraction ratio between transitory and permanent shocks.

The figure (6) plots the real monetary condition index calculated..We set $\theta = 0.88$. The results indicates two stages of monetary policy stance. Expansive during 1992Q3-1995Q4 and 2001Q1-2003Q4 and contractive during 1996Q1-2001Q4.

Appendix C: Hodrick Prescott filter and production function approach

Hodrick and Prescott filter

The Hodrick-Prescott (HP) filter[49] is probably the most widely used detrending method. This is mainly due to the fact that it can be easily calculated and implemented in virtually any econometric package.

The HP filter is a linear filter that decomposes a series into its two components: a trend and a cycle by minimizing the size of the actual series fluctuations around its trend, subject to a constrain on the maximum allowable change in the growth of the trend series between two periods. If y denotes real GDP, potential output ($\overline{y}$) in the HP filter is the series of values that minimize the following expression:

[47] In this long term equilibrium, external real interest rate is taking as an exogenous variable, which do not adjust to any domestic disequilibrium.

[48] For technical discussion of local linear trend models and local level models see Harvey (1993).

[49] See Hodrick and Prescott (1997).

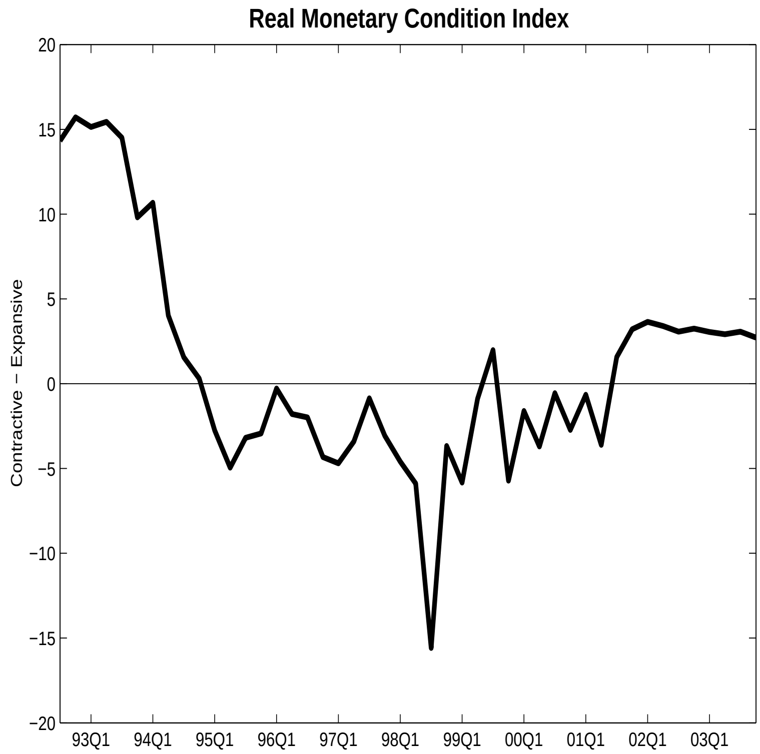

Figure 6: Real Monetary Index is computed as a linear combination of real interest rate and real exchange rate gaps.

$$Min \sum_{t=1}^{T} (y_t - \overline{y}_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\overline{y}_{t+1} - \overline{y}_t) - (\overline{y}_t - \overline{y}_{t-1}) \right]^2 \tag{C1}$$

where T is the sample size and λ is a weighting factor that determines the smoothness of the trend, that is, it represents the variances ratio of the trend relative to the cycle. Hodrick and Prescott set λ equal to 100, 1600 and 14400 for annual, quarterly and monthly data, respectively.

Production Function approach

This method consists in estimate potential output from a production function, usually derived from a Cobb-Douglas specification, in which output (y_t) depends on the stock of physical capital (k_t), the labor force (l_t) and the level of total factor productivity (A_t):

$$y_t = A_t l_t^{\alpha} k_t^{\beta} \tag{C2}$$

The parameters α and β refer to the share of labor force and capital in production, respectively. It is assumed constant returns to scale which implies that $\alpha + \beta = 1$. Since the level of productivity can not be measured directly, it has to be obtained as the residual of the equation once the parameters α and β are known. Thus, productivity factor is calculated as:

$$A_t = \frac{y_t}{l_t^{\alpha} k_t^{\beta}}$$

that is the part of product that cannot be explained by capital and labor (Solow residua).
There are two alternatives ways to calculate the parameters α and β. The first one consists in pick them from national accounts. The second way consists in find a long run relation among product, capital and labor force, what imply to assume that the economy reach an stationary level and productivity factor is constant. For the case of Peru, these parameters were calculated using the first way in Miller (2004) $\alpha = 0.486$ and $\beta = 0.514$.
To calculate potential output, it is further assumed that the growth in the level of productivity is comprised of two parts: an upward trending component representing deterministic productivity growth, and a stochastic component that corresponds to the shortfall or surplus of output around potential. Usually, the trend of the productivity factor is calculated using a statistic filter such as the Hodrick-Prescott or Baxter and King. Once it is estimated, the productivity trend is replaced in the original equation (C2) instead of productivity and the level of potential output is obtained.

Appendix D: Concordance index

For concordance evolution between estimates we use a concordance index, originally propose by Pagan and Harding (1999)[50].
This concordance index measures the proportion of periods two time series, x_i and x_j are in the same state. $S_{i,t}$ denote a time series that takes the value of 1 when x_i is positive and 0 when less or equal to zero. The same procedure is applied to calculate $S_{j,t}$. The degree of concordance of the time series x_i and x_j is then,

$$C_{ij} = T^{-1} \left\{ \sum_{t=2}^{T} (S_{i,t} S_{j,t}) + (1 - S_{i,t})(1 - S_{j,t}) \right\} \tag{D1}$$

where T is the sample size.
The concordance index the value of 1 if both time series coincide at the same state (positive or negative) every period over the sample. Otherwise, the index is 0.

Appendix E: Pesaran and Timmermann test

Pesaran and Timmermann (1992)[51] propose a directional change of forecast test. This test can be employed to evaluate the reliability of real time output gap versus final estimate. Given the real time output gap sequence, r_t, and the final estimate , f_t, define the following variables,
$R_t = 1$ if $r_t > 0$, otherwise 0; $F_t = 1$ if $f_t > 0$, otherwise 0; and $Z_t = 1$ if $r_t f_t > 0$, otherwise 0.
Under the null hypothesis that the sign of both series is the same the following holds:

[50]Cited in MacDermott and Scott (2000).
[51]Quoted in Camba-Mendez and Rodriguez-Palenzuela (2001).

$$S_n = \frac{\widehat{P} - \widehat{P}_*}{\left[V(\widehat{P}) - V(\widehat{P}_*)\right]^{\frac{1}{2}}} \rightarrow N(0,1) \tag{E1}$$

where $\widehat{P} = n^{-1}\sum_{t=1}^{n} Z_t$ and $\widehat{P}_* = \widehat{P}_r\widehat{P}_f + (1-\widehat{P}_r)(1-\widehat{P}_f)$ is the estimated probability of Z_t being 1 under the assumption of r_t and f_t being independently distributed. $\widehat{P}_r$ and $\widehat{P}_f$ are the estimated probabilities of r_t and f_t being 1, respectively. Finally, $V(\widehat{P})$ and $V(\widehat{P}_*)$ are defined as $\widehat{P}$ and $\widehat{P}_*$ variances, computed by:

$$\begin{aligned}
\widehat{V}(\widehat{P}) &= n^{-1}\widehat{P}_*(1-\widehat{P}_*) \\
\widehat{V}(\widehat{P}_*) &= n^{-1}(2\widehat{P}_f - 1)^2\widehat{P}_r(1-\widehat{P}_r) \\
&\quad + n^{-1}(2\widehat{P}_r - 1)^2\widehat{P}_f(1-\widehat{P}_f) \\
&\quad + 4n^{-2}\widehat{P}_f\widehat{P}_r(1-\widehat{P}_r)
\end{aligned}$$