Age-Differentiated Minimum Wage: An Exploratory Model*

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Abstract

The fact that minimum wages seem specially binding for young workers has led some countries to adopt age-differentiated minimum wages. We develop a dynamic two sector labor market model where workers with heterogeneous endowments of natural skills gain productivity through experience. We compare two equally binding schemes of single and age-differentiated minimum wages showing that even though with differentiated minimum wages a more equal distribution of income is achieved, such a scheme creates a more unequal distribution of wealth by forcing less skilled workers to remain in the uncovered sector for longer. We also show that relaxing minimum wage solely for young workers might be harmful for the less skilled ones. Suggestive evidence from Chile - where a differentiated minimum wage for workers under the age of 18 was introduced in 1989 - lends support to the predictions of our model.

JEL Classification: D30, D31, J31, J42.

Key Words: Age-differentiated minimum wage, income distribution, wealth distribution, segmented labor markets.

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1 Introduction

According to traditional dualistic models, the minimum wage excludes the least productive individuals from the covered sector, thus segmenting the market and increasing inequality in labor income distribution. If workers acquire productivity by way of experience, as some empirical evidence suggests, then the minimum wage will be especially restrictive for young workers. Table 1 reflects this fact for the case of Chile in the year 2000. The table shows for different population groups, with different levels of schooling, the proportion of those that are restricted by the minimum wage.

	15–17	18–21	22–24	25–29	30–39
Primary incomplete	74.1%	55.3%	51.1%	39.9%	34.5%
Primary complete	72.2%	47.7%	41.3%	31.5%	22.4%
Secondary incomplete	65.9%	43.9%	30.0%	17.7%	16.4%
Secondary complete	-	33.3%	21.8%	8.4%	4.5%

Source: Own calculations based on CASEN survey

Table 1: Workers restricted by the minimum wage for different age groups with different levels of schooling

This consideration has led some economists and policymakers to propose the replacement of the single minimum wage (henceforth SMW) with an age-differentiated minimum wage (henceforth DMW). They contend that this will avoid the exclusion of young workers from the covered sector and result in a better labor income distribution than under a SMW.

This paper aims to explore the effects of this measure on the labor market and to examine not only the current labor income distribution, but also the distribution of wealth, defining wealth as the present value of future labor income.

The model we use has two sectors: the covered sector, where the minimum wage is in force, and the uncovered sector, where it is not. Salaries are higher in the covered sector. We suppose different levels of productivity between individuals. Productivity depends on natural skills and age, that is, we allow individuals to acquire experience through age. A worker's labor income will depend on his productivity and on the sector he is employed in. Thus, if the most productive workers are employed in the covered sector (where salaries are higher) and the least productive in the uncovered sector, then the labor income distribution will be less equal than in the contrary case. In this model the minimum wage's role is to segment the two markets: individuals whose marginal labor product is valued at or above the minimum wage will be employed in the covered sector, and the remainder will be left in the uncovered sector.

In the literature we have found essentially two ways of modelling a two-sector economy. The first begins with Mincer (1976), who assumes a restrictive minimum wage in the covered sector such that an individual who seeks work in the sector will find it with a probability of less than one. In equilibrium, the expected income from seeking work in the covered sector is equal to the income obtained in the uncovered sector. A second approach (Welch [1976]) assumes that some workers are hired in the covered sector and those that remain are forced to seek work in a purely residual uncovered sector. We extend this approach by assuming heterogeneous productivity in the labor force following Pettengill (1981) and Heckman and Sedlacek (1981). In addition, we assume that productivity depends not only on natural skills but also on experience. Thus we extend the previous models by adding an intertemporal dimension to the problem. This allows us to compute in which sector an individual works at different points in his lifetime.

We conclude that although a DMW may appear preferable when viewed from a static perspective due to favorable effects on income distribution, an equally restrictive SMW achieves a more equal distribution of wealth.

Intuitively, the income distribution among all individuals improves if we move from a SMW to a DMW for the following reason. Under a SMW low productivity individuals are excluded from the covered sector: thus low productivity individuals receive low wages and high productivity individuals receive high wages. On introducing a DMW we will observe some low productivity individuals working in the covered sector and receiving a high wage as a result. These are very young, but talented workers, whose youth spares them from confronting a restrictive minimum wage. We will also observe high productivity individuals working in the uncovered sector, receiving a low wage. These workers are old, have low natural skills and face a very restrictive minimum wage because of their age. This results in a more equal distribution of income.

However, switching to a DMW worsens the wealth distribution among individuals belonging to the same generation for the following reason. With a SMW workers with low natural skills increase their productivity over time until eventually the minimum wage is no longer restrictive and they are able to work in the covered sector and receive a high wage. On the other hand, high natural skilled individuals quickly enter the covered sector. Since low skilled workers enter the covered sector when they are old and with a DMW the minimum wage for old people rises, they will take longer to enter the covered sector than with a SMW. The converse is true for the high skilled workers. This results in high skilled workers being rich from a younger age, and low skilled workers being poor for longer, leading to a worsening of the wealth distribution.

Lastly, we show that relaxing the minimum wage solely for young people is not beneficial to the less skilled individuals. This is due for two reasons. First, the reduction in the minimum wage benefits them less than the high skilled workers, since they enter the covered sector when they are old, and the minimum wage for old people barely changes. Second, as more young high natural skill workers are able to enter the covered sector, the marginal productivity of labor falls, making it more difficult for them to fulfill firm's hiring constraint.

The paper is organized as follows. In section 2 we lay out the model, and in section 3 we determine the equilibrium under SMW and DMW regimes. In section 4 the outcomes are compared assuming equally restrictive minimum wages. In section 5 the outcomes are compared assuming that the minimum wage is relaxed only for the young workers. In section 6 we show some empirical evidence that supports our predictions and in section 7 we conclude.

2 The Model

The economy is composed of two sectors: covered and uncovered. Call L_c the number of units of effective labor in the covered sector and L_u the number of units of effective labor in the uncovered sector. Total labor in the economy is thus given by $L \equiv L_c + L_u$.

We use a continuous time overlapping generations model, in a closed economy where agents are endowed with perfect foresight. Each individual lives for a period of A. At any given point in time, different generations from ages 0 to A live and coexist. The economy is assumed to be at its steady state with no population growth. These assumptions serve to reduce our treatment of dynamics to a minimum.

The Firms

There are two types of firms: those of the covered and uncovered sectors. Firms in the covered sector follow the law, thus if there is a minimum wage it will hold in this sector.

The production of firms in both sectors is a function of units of effective labor and capital, $F(L, K)$. We assume that the function exhibits constant returns to scale, decreasing returns to each factor and that labor marginal productivity is an increasing function of capital, with lower bound $F_L(L, 0) \equiv w_u$. We also assume that only the covered sector uses capital, and the amount of it is fixed and equal to $K \equiv \overline{K}$.[1]

Firms are price takers and hire factors up to the point where:[2]

$$w_c = F_L\left(L_c, \overline{K}\right) \tag{1a}$$

$$w_u = F_L\left(L_u, 0\right) \tag{1b}$$

$$r_K = F_K\left(L_c, \overline{K}\right) \tag{1c}$$

[1]Since the uncovered sector doesn't follow the law and capital is observable by the authority, this sector can't use capital.

[2]We use the final good as the *numeraire*.

Where w_c is the price of effective labor in the covered sector, w_u the price of effective labor in the uncovered sector and r_K the rental price of capital.[3]

The Government

The government owns the capital in the economy and rents it to the firms obtaining $r_K\overline{K}$ at every moment. It provides individuals with public goods g which do not affect the marginal utility of private consumption. We assume the government has an infinite horizon and it's utility comes from the discounted value of the public goods it provides:

$$\mathcal{U}_g = \int_0^\infty g e^{-\rho t} dt \tag{2}$$

As the government is indifferent between present and future resources discounted at rate ρ, it is willing to lend any amount of goods to households at the same rate.

The Individuals

Individuals are supposed heterogeneous with regard to their productivity. Workers are indexed by their productivity with the letter $p \in [0, P]$. Each individual is endowed with H hours of work at a point in time, which he supplies inelastically. For simplicity we normalize the amount of hours to one, so that $H \equiv 1$.

Individual "p" generates p units of effective labor for each hour of work at all points in time. Therefore:

$$l_p = Hp = p \tag{3}$$

This means that a worker indexed by p is twice as productive as the one indexed by $\frac{p}{2}$. Any firm will be indifferent between hiring one of the former or two of the latter.[4] The productivity component p is determined by two factors:

1. The natural skill of individual "p" which we shall call $j \in [0, J]$. This skill remains constant throughout the life of the individual. We assume skills are distributed among individuals according to the function $f(j)$.

2. Age. This is supposed to be a linear trend which we call $a \in [0, A]$. We are assuming that as the individual ages he acquires experience, independent of the sector in which he works, which causes his productivity to grow.[5] At each point in time there

[3]Since $F(L, K)$ has constant returns to scale, we can notice that w_u will be constant: $w_u = F_L = h(K_c/L_c) = h(0)$.

[4]See Lucas (1988).

[5]Strictly, all we need is p to be a function of age, not necessarily an increasing one. It could seem reasonable to suppose there is a threshold age after which productivity declines. Therefore a may be reinterpreted as a variable inversely related to the distance from this threshold.

exist $f(a)$ individuals of age a. Since the population is stable through time $f(a)$ is distributed uniformly $U[0, A]$.

As a result, we may rewrite equation (3) as:

$$l_p = p = j + a \tag{4}$$

The individual can supply his units of labor to one of the both sectors. His income depends on the product of the price per unit of effective labor in the sector in which he works and his endowment of productivity. If he is employed in the covered sector he receives an income of $w_c\,(j + a)$, while if he works in the uncovered sector he receives $w_u\,(j + a)$. The prices w_c and w_u are the equilibrium prices per unit of effective labor in each sector. Clearly, the individual will prefer to work in the covered sector that pays the highest price per unit of labor.

Each individual has a level of wealth at birth which we shall call W. Thus, if we suppose an individual of natural skill j works in the uncovered sector for the period $[0, a^*]$ and works in the covered sector for the rest of his life, his wealth at $a = 0$ will be:

$$W\,(j) = \int_0^{a^*} w_u(j + a)e^{-\rho a} da + \int_{a^*}^{A} w_c(j + a)e^{-\rho a} da \tag{5}$$

Where the interest rate ρ is determined by the government's discount factor. We assume that individuals are born without assets, they can lend or borrow from the government in order to smooth their consumption at rate ρ, and they die with no assets. As our focus is the labor market, we shall not explicitly model agents' savings and consumption decisions.

3 Equilibrium

3.1 No Minimum Wage

As mentioned above, productivity in the covered sector is greater than that in the uncovered sector, thus $w_c > w_u$. Everyone will wish to supply their labor to the covered sector and there are no restrictions to obstruct them from doing so.

Aggregate supply of labor in the covered sector in any moment is:

$$L_c = \int_0^A \int_0^J f(j, a)(j + a)dj da \tag{6}$$

The distributions $f(j)$ and $f(a)$ are independent of each other, so we may write $f(j, a) = f(j)f(a) = \frac{f(j)}{A}$. We may rewrite equation (6) as:

$$L_c = \int_0^A \int_0^J \frac{f(j)}{A}(j + a)dj da = \overline{j} + \overline{a} \tag{7}$$

Where $\overline{j} \equiv \int_0^J jf(j)dj$ and $\overline{a} \equiv \int_0^A af(a)da = \frac{A}{2}$.

Equaling the supply of labor given by equation (7) with the demand given by equation (1a) we obtain the equilibrium price of a unit of effective labor in the covered sector:

$$w_c = F_L\left(\overline{j} + \overline{a}, \overline{K}\right) \tag{8}$$

Income

We shall now obtain the distribution of labor income under this scenario. As each individual earns $w_c(j+a)$ we must calculate the density function $f(I)$, with $I \equiv w_c(j+a)$. Note that productivity, and therefore also income, grows at a rate w_c.

For ease of exposition we assume that $A > J$.[6] In Appendix A we derive the income density function. As all individuals receive an income of $w_c(j+a)$ the income distribution is simply a linear transformation of the population's distribution of productivities.

Wealth

Finally, we can calculate the wealth of an individual of natural skill j. The individual is born and works in the covered sector for his entire life, obtaining a wage of w_c per unit of effective labor. As a result his wealth will be:

$$W(j) = \int_0^A w_c(j+a)e^{-\rho a}da \tag{9}$$

3.2 Single Minimum Wage

In this section we assume the government imposes a single minimum wage, which we shall call S. As observing individual productivity is impossible for the government, this minimum wage is set per hour of work, not per unit of effective labor. It stands that every worker must be paid at least S per hour of work.

Firms in the covered sector will hire labor only if the value of the marginal product per hour is greater than S. In equilibrium the following hiring condition will hold for firms:

$$w_c(j+a) \geq S \tag{10}$$

This means that individuals whose productivity in equilibrium is less than S/w_c cannot work in the covered sector. We assume S is sufficiently low such that $S/w_c < A$ and $S/w_c < J$ hold. That is, every individual works in the covered sector at some stage in his life, and some work in that sector their whole lives. Individuals with $j = 0$ work in the covered sector from $a = S/w_c$ and individuals with $j \geq S/w_c$ work in the covered sector from $a = 0$. See Figure 1 for graphical representation.

[6]This assumption has no effect on our results.

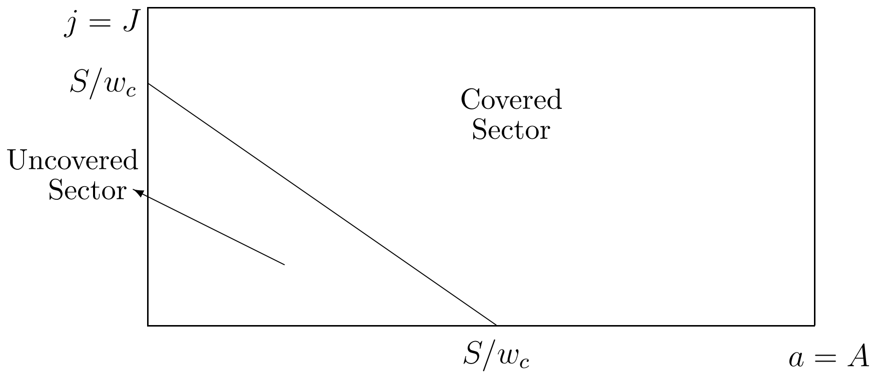

Figure 1: Participation condition under a SMW

To calculate the supply of individuals that satisfy condition (10) we subtract the labor units of those workers who do not fulfill the condition from the total units of effective labor in the economy:

$$L_c = \overline{j} + \overline{a} - \int_0^{S/w_c} \int_0^{S/w_c - j} \frac{f(j)}{A}(j+a)dadj \equiv L_c\left(w_c, S\right) \tag{11}$$

Even when total supply of effective labor is inelastic, the supply of those individuals who fulfill condition (10) will have a positive elasticity with respect to w_c.[7] This is because as w_c increases, so does the number of individuals who fulfill the hiring condition (10).

Intersecting equation (11) with equation (1a) we obtain the equilibrium price per unit of effective labor in the covered sector under a SMW:

$$w_c = F_L\left(L_c\left(w_c, S\right), \overline{K}\right) \tag{12}$$

Equation (12) has a single solution, which we may write as:[8]

$$w_c \equiv w_c\left(S\right) \tag{13}$$

It can be shown that $w_c\left(S\right)$ is increasing in S, which implies that the wage in equation (13) is greater than the wage in equation (8). The intuition is that as the minimum wage increases, fewer individuals fulfill the hiring condition (10). As labor becomes scarce in the covered sector, its price must rise.

Now we proceed to calculate the critical instant, which we call a_{smw}, when an individual of natural skill j moves from the uncovered to the covered sector. This occurs when

[7]In fact, $\frac{\partial L_c}{\partial w_c} = \frac{1}{4T}\frac{S^2}{w_c^3} f\left(\frac{S}{w_c}\right) > 0$.

[8]Since $\frac{\partial L_c}{\partial w_c} > 0$ and $\frac{\partial F_L}{\partial L_c} < 0$ we know that $\frac{\partial F_L}{\partial w_c} < 0$. Thus a fixed point exists such that $w_c = F_L\left(L_c\left(w_c\right), \overline{K}\right)$.

$a \geq \frac{S}{w_c} - j$. As a cannot be negative, the lowest plausible a_{smw} is zero. Thus a_{smw} is given by:

$$a_{smw}(j, S) = \max\left\{\frac{S}{w_c(S)} - j; 0\right\} \tag{14}$$

Returning to Figure 1 we observe that for any j, a_{smw} corresponds to the coordinate on the horizontal axis where j intersects the straight line.

Income

The income distribution in this case will depend on the sector in which the individual is employed. If he works in the uncovered sector his wage income is $I_u \equiv w_u(j + a)$, while if he works in the covered sector it is $I_c \equiv w_c(j + a)$. In Appendix A we derive the density function of income under a SMW. We now observe market segmentation: the income distribution is still a linear transformation of the productivity distribution, but it is now segmented at the minimum wage level. The least productive individuals are those that work in the uncovered sector, where the price per unit of effective labor is lower than in the covered sector, thus they receive a low income. High productivity individuals work in the covered sector where they receive a higher price per unit of labor, and thus a higher income.

Wealth

An individual of natural skill j will work in the uncovered sector for the period $[0, a_{smw}]$, and in the covered sector for the rest of his working life. His wealth given a SMW is:

$$W_{smw}(j) = \int_0^{a_{smw}} w_u(j + a)e^{-\rho a}da + \int_{a_{smw}}^{A} w_c\left(S\right)(j + a)e^{-\rho a}da \tag{15}$$

3.3 Age-Differentiated Minimum Wage

In this section we suppose the relevant authorities impose an age-differentiated minimum wage according to the following formula:

$$S_a = \theta a + \beta \tag{16}$$

Thus the hiring condition becomes:

$$w_c(j + a) \geq \theta a + \beta \tag{17}$$

Individuals whose productivity in equilibrium is less than $\left(\theta a + \beta\right)/w_c$ are excluded from the covered sector. We suppose $\theta < w_c$ so that the rate of growth of productivity through age is greater than the rate of increase in the minimum wage and also that $w_c - w_u < \theta$.[9] In addition we assume that $\beta/(w_c - \theta) < A$ and $\beta/(w_c - \theta) < J$, which means that every individual will work in the covered sector at some point and some will do so for their whole working lives. Individuals with $j = 0$ work in the covered sector from $a = \beta/(w_c - \theta)$ and individuals with $j \geq \beta/w_c$ work in the covered sector from $a = 0$. See Figure 2 for graphical representation.

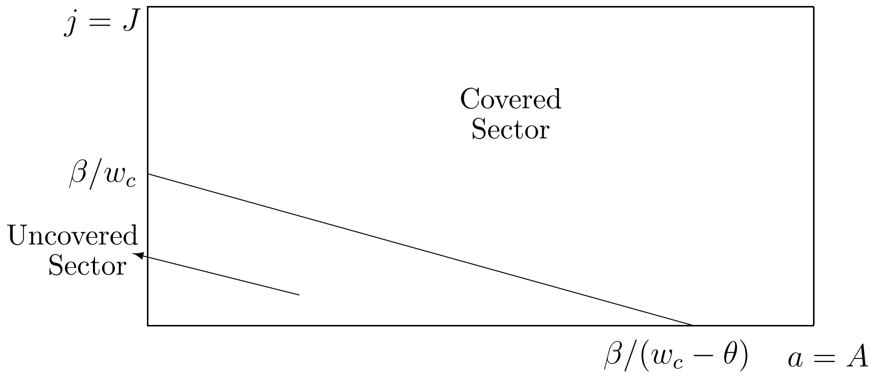

Figure 2: Participation condition under a DMW

We then calculate the labor supply of the individuals that fulfill condition (17):

$$L_c = \overline{j} + \overline{a} - \int_0^{\beta/w_c} \int_0^{(\beta - w_c j)/(w_c - \theta)} \frac{f(j)}{A}(j + a)da dj \equiv L_c(w_c, \beta, \theta) \tag{18}$$

Again we observe a positive supply elasticity with respect to w_c among individuals that satisfy condition (17).

Intersecting equation (18) with equation (1a) we obtain the equilibrium price for units of effective labor in the covered sector under a DMW:

$$w_c = F_L\left(L_c(w_c, \beta, \theta), \overline{K}\right) \tag{19}$$

Equation (19) has a single solution which we write as:

$$w_c \equiv w_c(\beta, \theta) \tag{20}$$

We call a_{dmw} the critical instant at which the individual moves from the uncovered to the covered sector with the DMW scheme. The individual will move from the uncovered to the covered sector when $a \geq \frac{\beta - w_c j}{w_c - \theta}$. This implies that:

$$a_{dmw}(j, \beta, \theta) = \max\left\{\frac{\beta - w_c(\beta, \theta) j}{w_c(\beta, \theta) - \theta}; 0\right\} \tag{21}$$

[9]The first assumption is not necessary to obtain our results but it considerably simplifies the analysis.

Income

In Appendix A we derive the income density function under a DMW. Once again we observe market segmentation; however in this case the workers in the uncovered sector are not necessarily the least productive. Under a DMW some low productivity individuals will work in the covered sector: young, high skilled workers who confront a low minimum wage because of their youth. On the other hand we will observe some high productivity individuals working in the uncovered sector: older, low skilled workers who face high minimum wages because of their age.

Wealth

The wealth of an individual of natural skill j under a DMW scheme is given by:

$$W_{dmw}(j) = \int_0^{a_{dmw}} w_u(j+a)e^{-\rho a}da + \int_{a_{dmw}}^{A} w_c\left(\beta,\theta\right)(j+a)e^{-\rho a}da \tag{22}$$

4 Comparing Both Schemes: Case I

In this section we compare the two schemes under the assumption of equally binding minimum wages. To this end we choose a vector (β, θ) such that the units of effective labor supplied in the covered sector under a SMW ($L_c(w_c, S)$ of equation [11]) are equal to the units of effective labor supplied under a DMW ($L_c(w_c, \beta, \theta)$ of equation [18]). As the units of labor supplied to the covered sector are the same under both schemes and the demand for labor is constant, the equilibrium in both cases is identical, implying that $w_c(S) = w_c(\beta, \theta)$.

This way of comparing both schemes can be thought of as follows. In this model minimum wage has effects on two distributions. On the one hand, it affects the functional distribution (between capital and labor), and on the other hand it affects the distribution between workers. We will introduce an age-differentiated minimum wage that keeps unchanged the distribution of income between labor and capital and analyze it's effects on income and wealth distribution within workers. This comparison also has the advantage that production in both sectors will be the same under both schemes, thus we can compare income and wealth distribution keeping efficiency fixed.

As the units of effective labor excluded from the covered sector under DMW increase in both θ and β, if we set $\theta > 0$ then $\beta < S$ must be true for $L_c(w_c, S) = L_c(w_c, \beta, \theta)$ to hold. On the other hand if $\beta < S$, then $\beta + \theta A > S$ or else SMW would be more restrictive than DMW for individuals of all ages. In a similar way, as the units of effective labor excluded from the covered sector under DMW increases in both θ and β, the greater the

β we set the lower the θ must be for $L_c(w_c, S) = L_c(w_c, \beta, \theta)$ to hold. As a result, there exists a negative relationship between θ and β. These conditions imply that the minimum wage under DMW is lower at $a = 0$, and higher at $a = A$, than the minimum wage under a SMW regime. As under a DMW the minimum wage is increasing in a there is a single instant at which the minimum wages are the same under both schemes.

Given the assumptions of the model, it is straightforward to show that $S/w_c > \beta/w_c$ and $S/w_c < \beta/(w_c - \theta)$. If we intersect the hiring condition given by equation (10) with the hiring condition given by equation (17) we obtain a critical level of natural skill, which we shall call $\widehat{j}$. An individual with this level of natural skill will face the same minimum wage under both schemes since he enters to the covered sector in the instant at which minimum wages are the same under both schemes:

$$\widehat{j} = \frac{\beta w_c - (w_c - \theta)S}{w_c \theta} \tag{23}$$

Figure 3 illustrates the situation.

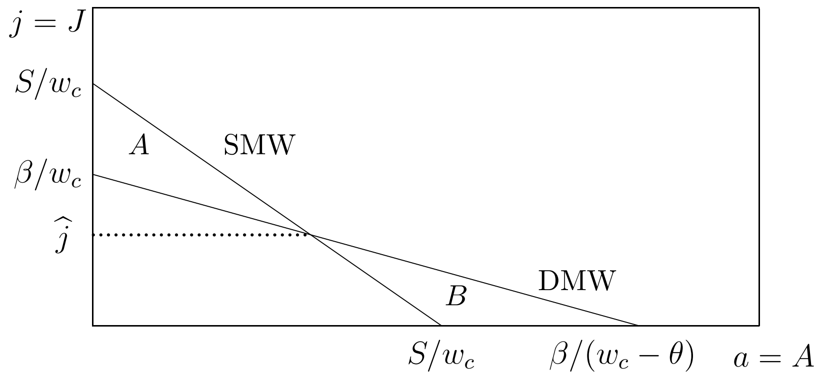

Figure 3: Participation condition comparison: SMW and DMW

As can be observed from the figure, the following holds:

$$\begin{cases} a_{dmw} > a_{smw} & \forall j \in \left[0, \widehat{j}\right) \\ a_{dmw} < a_{smw} & \forall j \in \left[\widehat{j}, \frac{S}{w_c}\right) \\ a_{dmw} = a_{smw} & \forall j \in \left[\frac{S}{w_c}, J\right] \end{cases} \tag{24}$$

Individuals with low natural skills take longer to enter the covered sector under a DMW than under a SMW. This is because although their productivity grows over age, the minimum wage they face also grows. In short, they confront a more restrictive hiring condition under a DMW. Individuals with relatively high natural skills (j between $\widehat{j}$ and $\frac{S}{w_c}$) enter the covered sector more quickly under a DMW. This is due to their high natural skill and because of their youth, as it means they confront a low minimum wage, allowing them a quick transition to the covered sector. Finally, individuals with $j \geq \frac{S}{w_c}$ enter the covered sector at $a = 0$ under both schemes.

From equations (15), (22), and (24), we know the following:

$$\begin{cases} W_{smw}(j) > W_{dmw}(j) & \forall j \in \left[0, \widehat{j}\right) \\ W_{smw}(j) < W_{dmw}(j) & \forall j \in \left[\widehat{j}, \frac{S}{w_c}\right) \\ W_{smw}(j) = W_{dmw}(j) & \forall j \in \left[\frac{S}{w_c}, J\right] \end{cases} \tag{25}$$

Clearly passing from a SMW to a DMW is not pareto efficient. The wealth of individuals with low natural skills is lower under a DMW than under a SMW, as they take longer to enter the covered sector. The wealth of individuals whose level of natural skills lies between $\widehat{j}$ and $\frac{S}{w_c}$ is greater under a DMW as they take less time to enter the covered sector. And there is no change in the wealth of individuals of natural skills $j \geq \frac{S}{w_c}$ as they enter the covered sector at $a = 0$ under both schemes.

Another important point to notice is that when a DMW is introduced, there is an outflow of workers that move from the covered to the uncovered sector (represented by area B of Figure 3) and a inflow of workers that move from the uncovered to the covered sector (represented by area A of Figure 3). Since by construction the quantity of effective labor is the same under both schemes, the inflows and outflows of units of effective labor must be exactly the same.

Individuals that move from the covered to the uncovered sector are those with natural skills given by $j < \widehat{j}$ and with age given by $\frac{S-w_c j}{w_c} \leq a \leq \frac{\beta - w_c j}{w_c - \theta}$. Individuals that move from the uncovered to the covered sector are those with $j > \widehat{j}$ and $\frac{\beta - w_c j}{w_c - \theta} \leq a \leq \frac{S - w_c j}{w_c}$. Notice that although total labor in the covered sector remains unchanged, there is a substitution between old and young workers. The productivity of workers who are entering the covered sector is lower than the productivity of workers who are leaving it. In fact, the productivity of the most productive individual that enters the covered sector is $p = \widehat{j} + \frac{S - w_c j}{w_c}$, which is exactly the same as the productivity of the least productive individual that leaves the covered sector. As a result, the average productivity of the formal sector falls when a DMW is introduced. In order to obtain the same level of labor in the covered sector under both schemes, with a DMW more people will have to work in the covered sector.

Finally, we formally compare the income distribution among all individuals and the wealth distribution at birth among individuals belonging to the same generation, under both minimum wage schemes. We use the Lorenz function as the metrics to compare the different distributions. Distribution X will be held to be more unequal than distribution Y if for all proportion of population z, the Lorenz function of X, $\mathcal{L}_X(z)$, is less than or equal to the Lorenz function of Y, $\mathcal{L}_Y(z)$. That is, if $\mathcal{L}_X(z) \leq \mathcal{L}_Y(z)\ \forall z \in [0,1]$. The following two propositions summarize our results.

Proposition 1 *Under a DMW scheme the income distribution is more equal than under a SMW scheme.*

Proof. See Appendix B ■

The intuition behind this result is as follows. As we have already said, an individual's income depends on the product of the price per unit of effective labor in the sector in which he works and his productivity. If we place the least productive individuals in the uncovered sector and the most productive in the covered sector we maximize income differences. This is precisely the effect achieved by a SMW. Any other assignment of individuals - such as that produced by a DMW - will imply a more equal distribution of income. In particular, under DMW we encounter low productivity individuals receiving a high price per unit of effective labor (these are young, high natural skilled individuals working in the covered sector) and also high productivity workers that are paid a low price for their effective labor (older, low natural skilled individuals employed in the uncovered sector). This naturally implies a more equal distribution of income as compared to that obtained under a SMW.

Proposition 2 *Under a DMW scheme the distribution of wealth is more unequal than under a SMW scheme.*

Proof. See Appendix B ■

Intuitively, under a single minimum wage low natural skilled individuals begin by working in the uncovered sector, but as their productivity grows over time eventually the minimum wage ceases to be a binding restriction and they switch to the covered sector. Under DMW these same individuals take longer to transfer to the covered sector since they enter the covered sector when they are old, and the minimum wage rises for them. That is, although their productivity grows over time, the minimum wage they confront grows also. On the other hand, with a SMW high natural skilled individuals quickly enter the covered sector, and this transition is even faster under a DMW, because their youth ensures they confront a low minimum wage. Thus, lifetime incomes are reduced for low natural skilled workers and increased for high skilled workers under a DMW in comparison to under a SMW. As wealth is defined as discounted lifetime income, the wealth of low natural skilled workers falls and that of high natural skilled workers rises under a DMW, resulting in a worsening of the wealth distribution.

We can observe that the level of wealth depends on how long it takes an individual to enter the covered sector, and thus it depends solely on the rate of productivity growth in the uncovered sector. Therefore the assumption that the rate of productivity growth is the same in both sectors can be relaxed and both propositions will still hold.

The main conclusion of this section is that if the authority is interested in keeping a certain level of restriction of the minimum wage, and therefore a certain distribution between capital and labor, it faces a trade-off between income distribution and wealth distribution among workers by choosing a SMW or a DMW.

5 Comparing Both Schemes: Case II

In this section we compare the two schemes under the assumption that minimum wages for young workers are reduced while they are kept fixed for older ones. In order to do so, it's useful to develop first a simpler version of the model where the price of effective labor is exogenous.

Exogenous wages

Suppose, for a moment, that the economy is small and open, and the government buys or sells capital in the international market. In this case, capital adjusts endogenously so as to fulfill $\rho = F_K + F_{KK}K$.[10] Since $F(L_c, K_c)$ has constant returns to scale, we can write ρ and w_c as $\rho = g(K_c/L_c)$ and $w_c = h(K_c/L_c)$. Since $K_c/L_c = g^{-1}(\rho)$, we can see that $w_c = h(g^{-1}(\rho)) \equiv w_c(\rho)$. Thus the price of units of effective labor depends only on the government's discount rate.

With an exogenously determined price of units of effective labor in the covered sector, the only determinant of individual's wealth will be the time they take to enter in the covered sector. From equation (21) we know that under a DMW, the individual endowed with natural skill j will move from the uncovered to the covered sector at the critical moment a_{dmw}, where $a_{dmw} = \frac{\beta - w_c j}{w_c - \theta}$. If we total differentiate this critical moment, we obtain:

$$da_{dmw} = \frac{1}{w_c - \theta} d\beta + \frac{\beta - w_c j}{(w_c - \theta)^2} d\theta \tag{26}$$

[10]This corresponds to the first order condition of the governments' problem:

$$\begin{aligned} &\max \mathcal{U}_g = \textstyle\int_0^\infty g e^{-\rho t} dt \\ &s/t \quad f'(k)k = g + \tfrac{\partial k}{\partial t} \end{aligned}$$

From equation (26) can see that a parallel shift in the minimum wage is pareto efficient, since $da_{dmw}/d\beta$ of equation (26) is positive; it also affects all individuals symmetrically. On the other hand, an increase in the slope θ affects specially those with low natural skills, for whom the term $\beta - w_c j$ is larger. The intuition behind this result is straightforward: since less skilled workers take a long time to enter the covered sector, a minimum wage that grows faster with age is specially binding for them.

Proposition 3 *In an economy that faces an exogenously determined price of effective labor, relaxing the minimum wage for the young improves everybody's wealth, but it benefits less the least skilled workers.*

Proof. Relaxing the minimum wage for young workers while keeping it fixed for the older ones corresponds to reduce β and increase θ in order to keep $\beta + \theta A$ constant. This implies that $d\theta = \frac{-d\beta}{A}$. This allows us to express the change in the critical moment a_{dmw} as a function of the change in β :

$$da_{dmw} = \left[\frac{1}{w_c - \theta} - \frac{\beta - w_c j}{(w_c - \theta)^2 A}\right] d\beta \tag{27}$$

From equation (27) we can see that a strategy of relaxing the minimum wage for young people while keeping it fixed for the elder is pareto efficient, since $da_{dmw}/d\beta$ of equation (27) is positive; but it is biased against the less skilled workers, since they take longer to enter the covered sector.[11] The term $\frac{1}{w_c-\theta} - \frac{\beta - w_c j}{(w_c-\theta)^2 A}$ is necessarily positive for all j, since $\frac{\beta}{w_c-\theta} < A$ by construction. ■

Intuitively, since less skilled workers enter the covered sector when they are old, and the minimum wage for old people barely changes, the reduction in the minimum wage benefits them relatively little.

Endogenous wages

We now turn to the original case of a closed economy with fixed capital. In this case the price of labor is endogenous, given by $w_c = w_c(\beta, \theta)$, with $\frac{\partial w_c}{\partial \beta}$ and $\frac{\partial w_c}{\partial \theta}$ strictly positive, since a positive change in β or θ necessarily implies a more restrictive minimum wage.

Proposition 4 *In an economy with an endogenously determined price of effective labor, the strategy of relaxing the minimum wage solely for young workers delays the moment less skilled workers take to enter in the covered sector.*

[11] The effect of the change in minimum wage depends positively in the level of natural skills j. In fact $\frac{\partial \frac{da}{d\beta}}{\partial j} = \frac{w_c}{(w_c-\theta)^2 A} > 0$

Proof. If we total differentiate the critical moment a_{dmw}, we obtain:

$$da_{dmw} = \left[\frac{1}{w_c - \theta} - \frac{\beta - \theta j}{(w_c - \theta)^2}\frac{\partial w_c}{\partial \beta}\right] d\beta + \left[\frac{\beta - w_c j}{(w_c - \theta)^2} - \frac{\beta - \theta j}{(w_c - \theta)^2}\frac{\partial w_c}{\partial \theta}\right] d\theta \qquad (28)$$

A strategy of relaxing the minimum wage only for young workers in $d\beta$ while keeping it fixed for the older ones has the following effect:

$$da_{dmw} = \left[\frac{1}{w_c - \theta} - \frac{\beta - w_c j}{(w_c - \theta)^2} - \frac{(\beta - \theta j)}{(w_c - \theta)^2}\left(\frac{\partial w_c}{\partial \beta} + \frac{1}{A}\frac{\partial w_c}{\partial \theta}\right)\right] d\beta \qquad (29)$$

The expression in (29) is negative for individuals with natural skills lower than the critical skill j^*, where:

$$j^* \equiv \frac{\beta\left[1 + \left(\frac{\partial w_c}{\partial \beta} + \frac{1}{A}\frac{\partial w_c}{\partial \theta}\right)\right]}{(w_c - \theta)\left[w_c + \theta\left(\frac{\partial w_c}{\partial \beta} + \frac{1}{A}\frac{\partial w_c}{\partial \theta}\right)\right]}$$

■

Now the reduction in minimum wage is biased against the less skilled not only because they take longer to enter the covered sector (the second term in equation [29]) but also because the reduction in the price of units of effective labor in the covered sector affects them more (third term in equation [29]). This asymmetric effect due to w_c can be understood if we notice that the individual endowed with natural skill j that faces a minimum wage S_a, needs to wait until moment $a_{dmw} = \frac{S_a - j}{w_c}$ to enter in the covered sector. The lower the j is, the greater the response of a_{dmw} to w_c because most of the necessary productivity required to enter the formal sector is achieved through experience. Intuitively, since w_c can be interpreted as the rate of growth of productivity through time, a reduction of this rate affects more workers who take a long time to enter the covered sector, whom are precisely the less skilled workers.

In this case it is possible that for a level of natural skills low enough, the second and third effect in equation (29) more than outweigh the first one making $\frac{da}{d\beta}$ negative. Thus a reduction in minimum wage for the young, is harmful for them not only because when they enter the formal sector wages are lower, but also because they have to wait more to enter in the covered sector.

However notice that the distribution of wealth doesn't necessarily worsen. Wealth could fall for the least productive workers but also for the most productive ones, since individuals who work all their lives in the covered sector would confront now a lower wage.

The main conclusion of this section is that relaxing the minimum wage exclusively for young workers may be harmful for less skilled workers who will remain longer time in the uncovered sector.

6 Empirical Evidence

In 1989, Chile introduced an age-differentiated minimum wage. In May of that year, the real single minimum wage was \$15.761. In June, the real minimum wage was set at \$15.480 for workers younger than 18 years, and at \$18.000 for workers over 18 years old. In this section we compute the distribution of income and wealth before and after the introduction of the DMW, for those individuals who are likely to be affected by this change. The data lends support to the propositions of our model: the distribution of income becomes more equal but the distribution of wealth becomes more unequal.

We use data from the survey *Encuesta de Ocupación y Desocupación en el Gran Santiago*, which is a cross sectional survey with detailed information on employment and income. The survey is realized quarterly and has approximately 12.500 observations. We create a window of ± four years around the date the DMW was introduced, i.e. June 1985 - June 1994.

In our model workers are indexed by their productivity, which in turn depends on both natural skills and age. We start by indexing the individuals from the survey by these two characteristics:

1. In order to index the individuals by age, we create four age categories: 15 to 17 years, 18 to 20 years, 21 to 23 years and 24 to 26 years old. We use categories around 18 years old since the DMW was created for that age and thus they are the individuals who are likely to be affected by the change.

2. In order to index the individuals by skills, we create a proxy that depends on gender and years of schooling.[12] We use two categories of gender: masculine and feminine; and four categories of schooling: 1$^{\text{st}}$-5$^{\text{th}}$ grade of elementary school, 6$^{\text{th}}$-8$^{\text{th}}$ grade of elementary school, 1$^{\text{st}}$-2$^{\text{nd}}$ year of high school and 3$^{\text{rd}}$-4$^{\text{th}}$ year of high school. As a result, we have 8 categories that proxy natural skill.

We summarize the categories by which we index the individuals in table 2.

Once individuals have been indexed and ordered according to the characteristics mentioned above, we fill each entry of the table with the average labor income of all individuals who belong to that class of productivity. Then we proceed to compute the distributions of income and wealth as follows:

[12]The optimum would have been creating a proxy depending on several more characteristics, but the data available didn't allow us to do it.

Category	15-17	18-20	21-23	24-26
Male / 1^{st}-5^{th} elem. school				
Male / 6^{th}-8^{th} elem. school				
Male / 1^{st}-2^{nd} high school				
Male / 3^{rd}-4^{th} high school				
Female / 1^{st}-5^{th} elem. school				
Female / 6^{th}-8^{th} elem. school				
Female / 1^{st}-2^{nd} high school				
Female / 3^{rd}-4^{th} high school				

Table 2: Categories used to index individuals

1. To compute the distribution of income, we calculate the Gini index of the labor income of all individuals, which we call G_I.[13] This procedure would correspond to calculate the Gini index on the income of each entry of table 2.

2. To create a proxy for wealth, we add labor income (properly discounted) of all individuals with natural skill j from ages 0 to A.[14] This represents the lifetime income that would receive an individual with natural skill j at birth. We calculate the Gini index of wealth for every category used, which we call G_W. This procedure would correspond to create a fifth column in Table 2 that would be the sum (properly discounted) of the four first columns, and then calculate the Gini index of each entry on this new column.

	85-86	86-87	87-88	88-89	89-90	90-91	91-92	92-93	93-94
ΔG_I	-0.0062	0.0370	-0.0374	-0.0206	-0.0193	0.0349	-0.0666	0.0657	-0.0034
ΔG_W	-0.0297	0.0297	-0.0113	-0.0305	0.0296	0.0549	-0.0369	0.0411	-0.0223

Source: Own calculations based on *Encuesta de Ocupación y Desocupación* survey

Table 3: Summary of statistics computed

Table 3 presents a summary of the statistics computed. ΔG_I represents the change of the Gini coefficient of income from one year to another, and ΔG_W represents the change of the Gini coefficient of wealth. From the table we can notice that for all periods, except

[13]The Gini index is equal to one minus twice the area under the lorenz curve and was computed in the following way: $G = \frac{1}{n(n-1)2\mu} \sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|$, where x_i is the income of the i^{th} individual of a certain category and n the total number of individuals belonging to such category.

[14]We use a discount rate equivalent to a 10% annualized rate.

for the period 89-90, both statistics have the same sign, meaning that distribution of income and wealth moved in the same direction. On the other hand for the period 89-90 a negative realization of ΔG_I with a simultaneous positive realization of ΔG_W takes place, meaning that the distribution of income in that year improved and the distribution of wealth worsened respect to the previous year. This is precisely what our model predicts. This is suggestive evidence that doesn't allow us to reject the proposition that when the DMW was introduced, the distribution of income became more equal and the distribution of wealth became more unequal.

7 Concluding Remarks

This article has shown that – at the same level of efficiency – an age-differentiated minimum wage results in a more equal labor income distribution than a single minimum wage. However, low natural skilled individuals spend longer time in the uncovered sector under a DMW, leading to a more unequal distribution of wealth than that occurring under a SMW. Secondly we have show that relaxing the minimum wage solely for young workers may be harmful for the less skilled workers since they will take longer to fulfill the hiring condition of the covered sector. We think this conclusion might be extended to understand the possible outcomes of setting a minimum wage that varies with a proxy of productivity that is not perfectly correlated with it.

In this paper we are assuming individuals that are heterogeneous in natural skills and gain productivity exogenously through experience. However the distribution of natural skills might be endogenous to the minimum wage. It seems reasonable to suppose that if young workers face a high level of minimum wage they will prefer to dedicate more time to schooling. In a similar way a process of on the job training, where gains in productivity depends on the individual effort, should be explored. These are important caveats, since it seems possible that they might invalidate some of our results.

Lastly, a discussion on whether income or wealth distribution should concern the authorities should be made. From a dynamic perspective and assuming access to capital markets, wealth seems like a better proxy of welfare than current income. From this point of view a SMW seems more desirable than a DMW. The problem becomes less straightforward if there is no such access. In such a case, the concept *family* may discharge the function of capital markets. If we suppose each family is comformed of individuals of similar natural skills but differing in age, it is clear that our model predicts a better income distribution for families under a SMW than under DMW. This is because under a SMW younger family members are likely to be found in the uncovered sector and older members in the covered sector; by contrast under a DMW we may encounter entire families working in the uncovered sector. In the same way relaxing minimum wage exclusively for the young can exclude completely low skilled families from the covered sector.

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A Distribution of Income

No Minimum Wage

In the model the distribution of j is $f(j)$ with $j \in [0, J]$ and the distribution of a is $U[0, A]$. We wish to obtain the distribution of $f(I)$, with $I \equiv w_c(j + a)$. To this end we define the auxiliary variable $x \equiv a$. Probability theory tells us that $f(I, x) = f(j, a) \mid J(I, x) \mid$. In this case $f(I, x) = f(j)f(a)\frac{1}{w_c} = \frac{f(j)}{Aw_c}$. To obtain the density function $f(I)$, we compute $f(I) = \int_x f(I, x)dx$. The integration limits must fulfill conditions $0 \leq x \leq J$ and $0 \leq \frac{I}{w_c} - x \leq A$.

Our result is the following distribution:

$$f(I) = \begin{cases} \int_0^{I/w_c} \frac{f(x)}{Aw_c}dx & = \frac{F(I/w_c)}{Aw_c} & \forall I \in & [0, w_cJ] \\ \int_0^{J} \frac{f(x)}{Aw_c}dx & = \frac{1}{Aw_c} & \forall I \in & [w_cJ, w_cA] \\ \int_{I/w_c - A}^{J} \frac{f(x)}{Aw_c}dx & = \frac{1-F(I/w_c-A)}{Aw_c} & \forall I \in & [w_cA, w_c(J + A)] \end{cases} \tag{A.1}$$

Single Minimum Wage

(i) Covered Sector

Individuals work in the covered sector iff $w_c(j + a) \geq S$, which we may rewrite as $I_c \geq S$, where $I_c \equiv w_c(j+a)$. We compute $f_{smw}(I_c) = \int_x f(I_c, x)dx$ with the following integration limits: $0 \leq x \leq J$, $0 \leq \frac{I_c}{w_c} - x \leq A$ and $I_c \geq S$. We obtain the following distribution:

$$f_{smw}(I_c) = \begin{cases} \int_0^{I_c/w_c} \frac{f(x)}{Aw_c}dx & = \frac{F(I_c/w_c)}{Aw_c} & \forall I_c \in & [S, w_cJ] \\ \int_0^{J} \frac{f(x)}{Aw_c}dx & = \frac{1}{Aw_c} & \forall I_c \in & [w_cJ, w_cA] \\ \int_{I_c/w_c - A}^{J} \frac{f(x)}{Aw_c}dx & = \frac{1-F(I_c/w_c-A)}{Aw_c} & \forall I_c \in & [w_cA, w_c(J + A)] \end{cases} \tag{A.2}$$

(ii) Uncovered Sector

Individuals work in the uncovered sector iff $w_c(j + a) < S$, which we may rewrite as $I_u < S\frac{w_u}{w_c}$, where $I_u \equiv w_u(j + a)$. We compute $f_{smw}(I_u) = \int_x f(I_u, x)dx$ with the following integration limits: $0 \leq x \leq J$, $0 \leq \frac{I_u}{w_u} - x \leq A$ and $I_u < \frac{w_uS}{w_c}$. We obtain the following distribution:

$$f_{smw}(I_c) = \begin{cases} \int_0^{I_c/w_c} \frac{f(x)}{Aw_c}dx & = & \frac{F(I_c/w_c)}{Aw_c} & \forall I_c \in & [S, w_c J] \\ \int_0^{J} \frac{f(x)}{Aw_c}dx & = & \frac{1}{Aw_c} & \forall I_c \in & [w_c J, w_c A] \\ \int_{I_c/w_c - A}^{J} \frac{f(x)}{Aw_c}dx & = & \frac{1-F(I_c/w_c - A)}{Aw_c} & \forall I_c \in & [w_c A, w_c (J+A)] \end{cases} \tag{A.3}$$

(iii) The Whole Economy

The income distribution for the whole economy, $f_{smw}(I) = f_{smw}(I_u) + f_{smw}(I_c)$, is given by:

$$f_{smw}(I) = \begin{cases} \frac{F(I/w_u)}{Aw_u} & \forall I \in & \left[0, \frac{w_u S}{wc}\right] \\ 0 & \forall I \in & \left[\frac{w_u S}{w_c}, S\right] \\ \frac{F(I/w_c)}{Aw_c} & \forall I \in & [S, w_c J] \\ \frac{1}{Aw_c} & \forall I \in & [w_c J, w_c A] \\ \frac{1-F(I/w_c - A)}{Aw_c} & \forall I \in & [w_c A, w_c (J+A)] \end{cases} \tag{A.4}$$

Age-Differentiated Minimum Wage

(i) Covered Sector

Individuals work in the covered sector iff $w_c(j+a) \geq \theta a + \beta$, which we may rewrite as $I_c \geq \beta \frac{w_c}{w_c - \theta} - \frac{\theta w_c}{w_c - \theta} x$. We compute $f_{dmw}(I_c)$ with the integration limits: $0 \leq x \leq J$, $0 \leq \frac{I_c}{w_c} - x \leq A$ and $I_c \geq \beta \frac{w_c}{w_c - \theta} - \frac{\theta w_c}{w_c - \theta} x$. We obtain the following distribution:

$$f_{dmw}(I_c) = \begin{cases} \int_{\beta/\theta - I_c\left(\frac{w_c-\theta}{w_c\theta}\right)}^{I_c/w_c} \frac{f(x)}{Aw_c}dx & = & \frac{F(I_c/w_c) - F\left(\beta/\theta - I_c\left(\frac{w_c-\theta}{w_c\theta}\right)\right)}{Aw_c} & \forall I_c \in & \left[\beta, \frac{\beta w_c}{w_c - \theta}\right] \\ \int_0^{I_c/w_c} \frac{f(x)}{Aw_c}dx & = & \frac{F(I_c/w_c)}{Aw_c} & \forall I_c \in & \left[\frac{\beta w_c}{w_c-\theta}, w_c J\right] \\ \int_0^{J} \frac{f(x)}{Aw_c}dx & = & \frac{1}{Aw_c} & \forall I_c \in & [w_c J, w_c A] \\ \int_{I_c/w_c - A}^{J} \frac{f(x)}{Aw_c}dx & = & \frac{1-F(I_c/w_c - A)}{Aw_c} & \forall I_c \in & [w_c A, w_c (J+A)] \end{cases} \tag{A.5}$$

(ii) Uncovered Sector

Individuals work in the uncovered sector iff $w_c(j+a) < \theta a + \beta$, which we may rewrite as $I_u < \beta \left(\frac{w_u}{w_u - \theta}\right) - \left(\frac{\theta w_u}{w_u - \theta}\right) x$. We compute $f_{dmw}(I_u) = \int_x f(I_u, x)dx$ with integration limits: $0 \leq x \leq J$, $0 \leq \frac{I_u}{w_u} - x \leq A$ y $I_u < \beta\left(\frac{w_u}{w_c-\theta}\right) - \left(\frac{\theta w_u}{w_c-\theta}\right)x$. We obtain the following distribution:

$$f_{dmw}(I_u) = \begin{cases} \int_0^{I_u/w_u} \frac{f(x)}{Aw_u}dx = \frac{F(I_u/w_u)}{Aw_u} & \forall I_u \in \left[0, \frac{\beta w_u}{w_c}\right] \\ \int_0^{\beta/\theta-\left(\frac{w_c-\theta}{\theta w_u}\right)I_u} \frac{f(x)}{Aw_u}dx = \frac{F\left(\beta/\theta-\left(\frac{w_c-\theta}{\theta w_u}\right)I_u\right)}{Aw_u} & \forall I_u \in \left[\frac{\beta w_u}{w_c}, \frac{\beta w_u}{w_c-\theta}\right] \end{cases} \tag{A.6}$$

(iii) The Whole Economy

The income distribution for the whole economy, $f_{dmw}(I) = f_{dmw}(I_u) + f_{dmw}(I_c)$, is given by:

$$f_{dmw}(I) = \begin{cases} \frac{F(I/w_u)}{Aw_u} & \forall I \in \left[0, \frac{\beta w_u}{w_c}\right] \\ \frac{F\left(\beta/\theta-\left(\frac{w_c-\theta}{\theta w_u}\right)I\right)}{Aw_u} & \forall I \in \left[\frac{\beta w_u}{w_c}, \beta\right] \\ \frac{F\left(\beta/\theta-\left(\frac{w_c-\theta}{\theta w_u}\right)I\right)}{Aw_u} + \frac{F(I_c/w_c)-F\left(\beta/\theta-I\left(\frac{w_c-\theta}{w_c\theta}\right)\right)}{Aw_c} & \forall I \in \left[\beta, \frac{\beta w_u}{w_c-\theta}\right] \\ \frac{F(I/w_c)}{Aw_c} & \forall I \in \left[\frac{\beta w_u}{w_c-\theta}, \frac{\beta w_c}{w_c-\theta}\right] \\ \frac{I}{Aw_c} & \forall I \in \left[\frac{\beta w_c}{w_c-\theta}, w_cJ\right] \\ \frac{1}{Aw_c} & \forall I \in [w_cJ, w_cA] \\ \frac{1-F(I/w_c-A)}{Aw_c} & \forall I \in [w_cA, w_c(J+A)] \end{cases} \tag{A.7}$$

B Proof of Propositions

Proof of Proposition 1

From Appendix A we deduce that:

$$\begin{array}{lll} (a) & f_{smw}(I) \geq f_{dmw}(I) & \forall I \in \left[0, \frac{w_u S}{w_c}\right) \\ (b) & f_{smw}(I) < f_{dmw}(I) & \forall I \in \left[\frac{w_u S}{w_c}, S\right) \\ (c) & f_{smw}(I) = f_{dmw}(I) & \forall I \in [S, w_c(J+A)] \end{array}$$

Besides, we know that:

$$\int_I f_{smw}(I)dI = \int_I f_{dmw}(I)dI = 1 \tag{B.1}$$

We shall now demonstrate that:

$$\begin{array}{lll} (a') & F_{smw}(I) \geq F_{dmw}(I) & \forall I \in \left[0, \frac{w_u S}{w_c}\right) \\ (b') & F_{smw}(I) > F_{dmw}(I) & \forall I \in \left[\frac{w_u S}{w_c}, S\right) \\ (c') & F_{smw}(I) \geq F_{dmw}(I) & \forall I \in [S, w_c(J+A)] \end{array}$$

Where $F_i(I)$ is the cumulative distribution $F_i(I) = \int_0^I f_i(x)dx$.

- Result (a') follows directly from (a).
- Result (b') holds because of the following. Given equations (B.1), (a), (b) y (c), we know that the following is true:

$$\int_0^{w_u S/w_c} \left[f_{smw}(x) - f_{dmw}(x)\right] dx = \int_{w_u S/w_c}^{S} \left[f_{dmw}(x) - f_{smw}(x)\right] dx \tag{B.2}$$

Given that $f_{smw}(I) = 0 \ \forall I \in \left[\frac{w_u S}{w_c}, S\right]$, we may rewrite equation (B.2) as:

$$\int_0^{w_u S/w_c} f_{smw}(x)dx = \int_0^{w_u S/w_c} f_{dmw}(x)dx + \int_{w_u S/w_c}^{S} f_{dmw}(x)dx = \int_0^{S} f_{dmw}(x)dx \tag{B.3}$$

Given $f_{smw}(I) = 0 \ \forall I \in \left[\frac{w_u S}{w_c}, S\right]$, it is also true that:

$$\int_0^I f_{smw}(x)dx = \int_0^{w_u S/w_c} f_{smw}(x)dx \qquad \forall I \in \left[\frac{w_u S}{w_c}, S\right] \tag{B.4}$$

Intersecting equations (B.3) and (B.4) we obtain:

$$\int_0^I f_{smw}(x)dx = \int_0^S f_{dmw}(x)dx \tag{B.5}$$

We also now that:

$$\int_0^S f_{dmw}(x)dx = \int_0^I f_{dmw}(x)dx + \int_I^S f_{dmw}(x)dx \tag{B.6}$$

Which means that:

$$\int_0^S f_{dmw}(x)dx > \int_0^I f_{dmw}(x)dx \tag{B.7}$$

Finally, from equations (B.5) and (B.7) we can see that:

$$\int_0^I f_{smw}(x)dx > \int_0^I f_{dmw}(x)dx \qquad \forall I \in \left[\frac{w_u S}{w_c}, S\right] \tag{B.8}$$

Thus (b') holds.

- Given (b') and (c), (c') holds directly

We see that $F(I)$ under SMW is greater than or equal to $F(I)$ under DMW at all points. Moreover, mean income is the same in both cases. Thus $F_{dmw}(I)$ Lorenz-dominates $F_{smw}(I)$, which means that $\mathcal{L}_{smw}(z) \leq \mathcal{L}_{dmw}(z)\ \forall z \in [0, I]$.■

Proof of Proposition 2

From equation (25) we know that:

$$\begin{array}{lll} (d) & W_{smw}(j) > W_{dmw}(j) & \forall j \in \left[0, \widehat{j}\right) \\ (e) & W_{smw}(j) < W_{dmw}(j) & \forall j \in \left[\widehat{j}, \frac{S}{w_c}\right) \\ (f) & W_{smw}(j) = W_{dmw}(j) & \forall j \in \left[\frac{S}{w_c}, J\right] \end{array}$$

If the interest rate were zero, the wealth of an individual of skill j would be the integral of the individual's earnings between times 0 and A. Therefore the wealth of a generation at birth would be the sum of the current income of all the individuals of all ages at a single point in time. Then, given that in both schemes the adjusted supply of work and the demand are equal, the total payment to labor is the same, resulting in:

$$\int_0^J W_{smw}(j)dj = \int_0^J W_{dmw}(j)dj \tag{B.9}$$

If we suppose a positive interest rate, $\widetilde{W}_{smw} \equiv \int_0^J W_{smw}(j)dj$ will fall more than $\widetilde{W}_{dmw} \equiv \int_0^J W_{dmw}(j)dj$. As increasing ρ results in incomes received closer to $a = 0$ having greater value, the minimum wage scheme that concentrates more payments in the vicinity of $a = 0$ will be that which generates the greater wealth. From equations (15), (22) and (24), we see that a DMW concentrates more payments early in the lifetime of the worker than a SMW. Thus for any non-negative ρ it is true that:

$$\widetilde{W}_{smw} \leq \widetilde{W}_{dmw} \tag{B.10}$$

We can compute the Lorenz curve of wealth as the proportion of accumulated wealth that belongs to the proportion of the population with natural skill less than or equal to j. Given equations (d), (e) and (f), and (B.10) the results are:

$$\begin{array}{lll} (d') & \frac{1}{\widetilde{W}_{smw}} \int_0^j W_{smw}(x)dx > \frac{1}{\widetilde{W}_{dmw}} \int_0^j W_{dmw}(x)dx & \forall j \in \left[0, \widehat{j}\right) \\ (e') & \frac{1}{\widetilde{W}_{smw}} \int_0^j W_{smw}(x)dx > \frac{1}{\widetilde{W}_{dmw}} \int_0^j W_{dmw}(x)dx & \forall j \in \left[\widehat{j}, \frac{S}{w_c}\right) \\ (f') & \frac{1}{\widetilde{W}_{smw}} \int_0^j W_{smw}(x)dx \geq \frac{1}{\widetilde{W}_{dmw}} \int_0^j W_{dmw}(x)dx & \forall j \in \left[\frac{S}{w_c}, J\right] \end{array}$$

- Result (d') follows directly from (d) and from equation (B.10).

- Result (e') holds because of the following. Given equations (B.10), (d), (e) and (f), we know that the following is true:

$$\int_0^{\hat{j}} \left[W_{smw}(x) - W_{dmw}(x) \right] dx \geq \int_{\hat{j}}^{S/w_c} \left[W_{dmw}(x) - W_{smw}(x) \right] dx \tag{B.11}$$

We may rewrite equation (B.11) as:

$$\int_0^{S/w_c} W_{smw}(x)dx \geq \int_0^{S/w_c} W_{dmw}(x)dx \tag{B.12}$$

We may also write:

$$\int_0^{j} W_{smw}(x)dx = \int_0^{S/w_c} W_{smw}(x)dx - \int_j^{S/w_c} W_{smw}(x)dx \tag{B.13}$$

Thus equation (B.12) becomes:

$$\int_0^{j} W_{smw}(x)dx + \int_j^{S/w_c} W_{smw}(x)dx \geq \int_0^{S/w_c} W_{dmw}(x)dx \tag{B.14}$$

Which may be factorized to obtain:

$$\int_0^{j} W_{smw}(x)dx \geq \int_0^{j} W_{dmw}(x)dx + \int_j^{S/w_c} \left[W_{dmw}(x) - W_{smw}(x) \right] dx \tag{B.15}$$

From (e) we know that $\int_j^{S/w_c} \left[W_{dmw}(x) - W_{smw}(x) \right] > 0$. Thus it holds that:

$$\int_0^{j} W_{smw}(x)dx > \int_0^{j} W_{dmw}(x)dx \tag{B.16}$$

From equations (B.10) and (B.16), we know that (e') is true.

- Given (e') y (f), (f') follows directly.

Considering (d'), (e') y (f'), we see that the Lorenz curve of wealth under a DMW never exceeds the Lorenz curve under a SMW, which means that $\mathcal{L}_{dmw}(z) \leq \mathcal{L}_{smw}(z)$ $\forall z \in [0, I]$.■