COINTEGRATION AND WAVELETS: AN EMPIRICAL ANALYSIS OF THE RELATIONSHIP BETWEEN MONEY AND OUTPUT IN PERU

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Abstract

This paper analyses empirically the relationship between money and output in Peru, based on an orthogonal decomposition of series by timescales obtained using wavelets, following Ramsey and Lampart (1998). Specifically, we propose the application of wavelet filtering to analyze cointegrating relationships. No evidence of cointegration between money, real output and prices is found. However, there is evidence of cointegration between non-stationary components of the series that includes different timescale details obtained using wavelets; this result could be considered as an alternative way to represent the existence of hidden co-integration. In this context, it is found that (1) the link between money and real output is not unique, and (2) the direction of causality and exogenity depends on both the timescale and the monetary aggregate considered.

Keywords: *wavelets, timescale, money, output, cointegration, hidden cointegration, Granger causality, exogenity.*

JEL Classification: C22, C49.

1. Introduction

The purpose of this paper is to provide some insights about the empirical the relationship between money and output in Peru. The analysis is based on an orthogonal decomposition of series by timescale obtained using wavelets, following Ramsey and Lampart (1998), and subsequent research by Chew (2001) and Gençay et al. (2002). The two main results of this literature, obtained in a short run context, are: (1) the link between money and real output is not unique, and (2) the direction of Granger causality depends on the timescale considered.

In this paper we go a little bit further in the empirical analysis of money-output relationship using wavelets. Specifically, we propose the application of wavelet filtering to analyze cointegrating relationships. The data for Peruvian case shows no evidence of cointegration between money, real output and prices. However, it is found evidence of cointegration between non-stationary components of the series that includes different timescale details. This result could be considered as an alternative way to represent the existence of hidden co-integration, as proposed by Granger and Yoon (2002). Furthermore, causality tests reveal that (1) the link between money and real output is not unique, and (2) the direction of Granger causality and exogenity depends on both the timescale and the monetary aggregate considered.

The paper is organized as follows. In section 2, it is provided the description of the data. In section 3, it is showed how the traditional approach –standard time series econometrics techniques- provides no clear evidence about the relationship between money and real output in the long run. In section 4, after wavelet filtering is applied to the data, it is found evidence of cointegration between non-stationary components of the series. In particular, the non-stationary series are constructed using different components of the wavelet-filtered series, which are associated to different timescales. Furthermore, the error correction model associated is not based on just first differences, but on specific time scales. In section 5, conclusions are presented.

2. Data

The analysis is based on monthly data from the Central Bank of Peru, from May 1992 to December 2002. In this way, the sample used to make the decomposition of the series using wavelets has a size that is power of 2 (in this case $n=2^7=128$)[1]. Nevertheless, the econometric analysis was made using the results from January 1993 to December 2001, a period when monetary policy followed a nominal anchor regime, where the anchor or the intermediate target was the monetary base[2].

Five nominal monetary aggregates were chosen as proxies of money: monthly average monetary base, currency, "money" (M1), broad money in domestic currency (M2) and broad money in foreign currency (denominated in domestic currency, "nuevos soles")[3]. The monetary aggregate called "money" is the sum of currency and demand deposits; broad money in domestic currency is the sum of "money" and saving deposits, time deposits and other values denominated in domestic currency; broad money in foreign currency is the sum of demand deposits, saving deposits, time deposits and other values denominated in foreign currency. The real activity was approximated through the real Gross Domestic Output (GDP) in terms of 1994 soles and the nominal Gross Domestic Output. Finally, the GDP Implicit Price deflator and the CPI (consumer price index) have been used as proxy of the price level. The series were seasonally adjusted[4] and used in logarithms.

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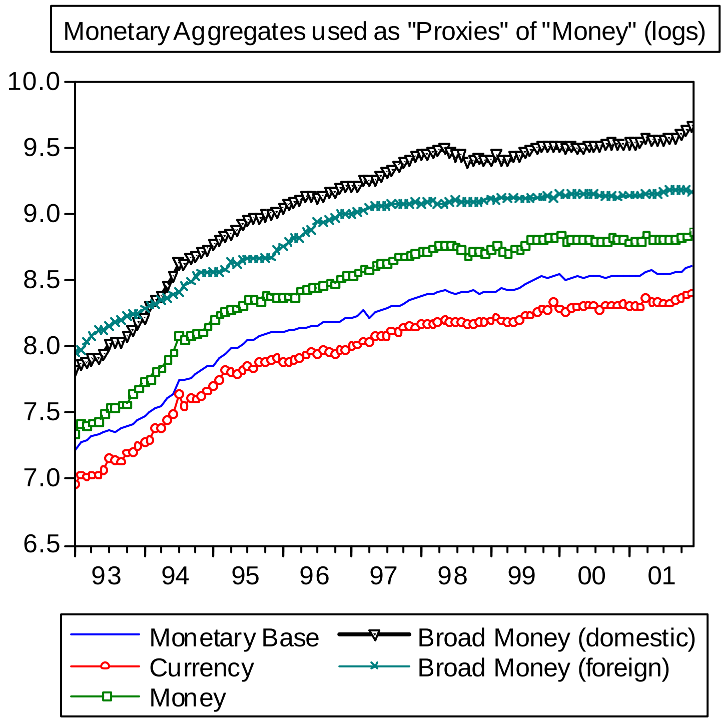
[1] *Since the filtration of the time series through wavelets has considered 20 additional periods to the analyzed ones (12 previous and eight later ones), this aids to eliminate possible problems in the ends of each one of the filtered series.*

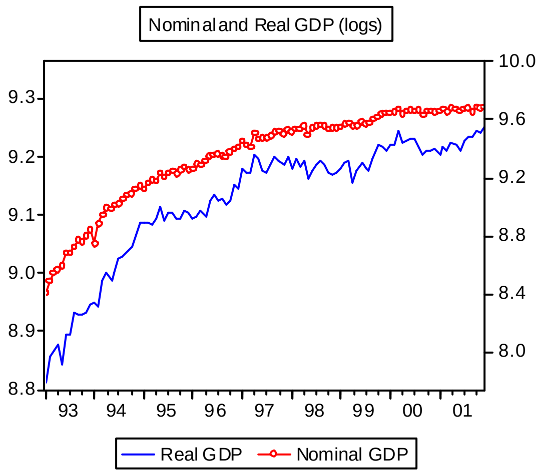
[2] *From January of 2002 the monetary policy follows an inflation objective scheme (inflation targeting), where the intermediate target is a specific inflation level.*

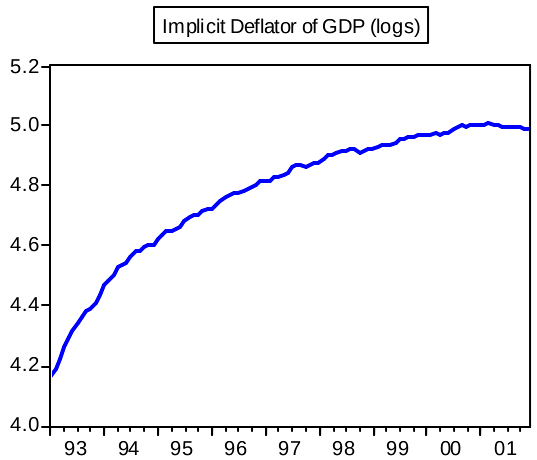
[3] *The sum of M2 and broad money in foreign currency is denominated total liquidity, and is the broaden monetary aggregate of the Peruvian economy.*

[4] *Wavelets can capture the seasonal components of the series. However, the seasonal adjusted series were chosen to be able to compare the results of the analysis using traditional econometrics with the alternative approach using wavelets.*

Figure 1: Data







3. Traditional Approach

The first step was the evaluation of the existence of unit root in the series. The ADF and Phillip-Perron tests showed that it is not possible to reject the hypothesis of unit root; Zivot and Andrews (1992) and Perron (1997) tests were then applied to evaluate the hypothesis of unit root vs. the alternative of stationary series with breaks. The results showed no evidence in favor of the stationary hypothesis. Given this result, the time series econometric analysis – the one we called "traditional approach"- involves the analysis of the series in terms of their first differences, their gaps or, if there exists any cointegrating vector, combining levels and first differences in an Error Correction Model (ECM) framework.

To evaluate the existence of any cointegrating vector, the Engle and Granger (1987) and the Johansen (1988) methodologies[5] were used. Both showed evidence in favor of cointegrating vectors at 1 and 5 percent of significance, between different monetary aggregates and real output, but only under the following assumptions: (a) no deterministic trend in the data, (b) the cointegrating vector does not present intercept neither a linear trend, and (c) no intercept in the error correction model. The existence of a cointegrating relation between output and broad money in foreign currency appeared under the same assumptions, except (b), because in this case it was necessary to assume that the cointegrating vector had an intercept but not a linear trend.

Table 1

COINTEGRATION AND GRANGER CAUSALITY:

1993:01 - 2001:12 [1/]

Null Hipothesis	VEC	Levels
PBIR does not cause BASE	0.4618	YES
BASE does not causePBIR	0.0592	YES
Lags	**12**	**12**
PBIR does not cause CIR	0.0164	YES
CIR does not cause PBIR	0.0651	NO
Lags	**14**	**14**
PBIR does not cause DIN	0.0060	YES
DIN does not cause PBIR	0.0472	NO
Lags	**14**	**14**
PBIR does not cause LIQMN	0.0000	YES
LIQMN does not cause PBIR	0.0645	NO
Lags	**24**	**24**
PBIR does not cause LIQME	0.5739	NO
LIQME does not cause PBIR	0.1010	YES
Lags	**21**	**21**

1/ In all cases, exists a cointegrating vector at 1% y 5% of significance, except, to the model with currency (only at 5%)

Source: Own elaboration.

[5] *In the case of the bivaried relations analysis between each monetary aggregate with output, the results provided by the Engle and Granger methodology were similar to those obtained by the Johansen methodology, thus the presentation of the results was based on the statistics ones obtained by this last methodology.*

Table 1 shows the results of the Granger causality analysis in a cointegrating context. As it can be read, output is weakly exogenous when "money" is considered, but strongly exogenous when currency and broad money in domestic currency are considered. Furthermore, it is found that monetary base and output are both weakly exogenous. Finally, the broad money in foreign currency is strongly exogenous.

The assumptions that underlie the cointegrating vectors are not consistent with the nature of the data. In particular, the assumption of no deterministic trend in the data is not suitable, especially for monetary aggregates. In particular, assumptions (a), (b) and (c) become relevant only when the series have zero average. Under assumption (c) it was not possible to find any cointegrating vector between each monetary aggregate, the real GDP and the GDP Implicit Price deflator.

Table 2

STATIONARY SERIES AND GRANGER CAUSALITY:
1993:01 - 2001:12

Null Hipothesis	First Differences	Gaps HP
PBIR does not cause BASE	0.0082	0.0812
BASE does not PBIR	0.3290	0.4044
Lags	**4**	**6** 1/
PBIR does not cause CIR	0.0181	0.0899
CIR does not cause PBIR	0.1046	0.0001
Lags	**14** 1/	**22**
PBIR does not cause DIN	0.0092	0.1350
DIN does not cause PBIR	0.0339	0.0017
Lags	**20**	**21**
PBIR does not cause LIQMN	0.3630	0.0467
LIQMN does not cause PBIR	0.0221	0.0945
Lags	**1**	**21**
PBIR does not cause LIQME	0.4795	0.4172
LIQME does not cause PBIR	0.0029	0.0145
Lags	**24**	**26**

1/ First order autocorrelation

Source: Own elaboration.

Considering these results, the empirical analysis of the relationship between money and output causal relationship between it was analyzed the existence of causality in Granger sense using the first differences of the logarithms of the series and their gaps. The results are contained in Table 2.

Using the first differences of the logarithms of the series (rates of growth), it was found that output Granger causes money when the latter is represented by monetary base, currency, or currency plus demand deposits (money); the causality reverses when broader monetary aggregates are considered, as much in domestic currency as in foreign currency. For the case of gaps, the money Granger causes output when currency, currency plus demand deposits (money) and broad money in foreign currency are considered; the only case where output Granger causes money is when the latter is measured as broad money in foreign currency; when the monetary base is considered, nothing can be concluded about the existence of Granger causality.

In short, the results do not show a clear Granger causality between output and the different monetary aggregates, as much in domestic as foreign currencies. These results are similar to those obtained including the real GDP implicit Price deflator.

4. Alternative Approach: Wavelets and Multiresolution Analysis

As an alternative to the traditional approach, the empirical analysis of the relationship between output and different monetary aggregates was based on the multiresolution analysis of the series using wavelets, following Ramsey and Lampart (1998). Specifically, the series were filtered using the mother wavelet function denominated Symmlet of order 12 (Sym12) characterized by ortonormality, compact support and for being almost symmetric[6].

The multiresolution analysis was made considering six details for each series: $D_1, D_2, D_3, D_4, D_5, D_6$ and a smoothed component S_6. The detail D_1 contains information of movements from the series (mainly of high frequency) that occur between $2^1 = 2$ y $2^2 = 4$ months; the detail D_2 movements from the series between $2^2 = 4$ y $2^3 = 8$ months, the detail D_3 movements from the series between $2^3 = 8$ y $2^4 = 16$ months,…, the detail D_6 movements from the series between $2^6 = 64$ y $2^4 = 128$ months[7].

The analysis was made using two measures of output: the real output and the nominal output. The following relations were considered:

(a) A short run relation between the real money and the GDP. For that reason, the causality analysis in the sense of Granger was made through a vector autoregressive or VAR model.

(b) Two long run relationships: (1) the money and the nominal GDP, and (2) money, the real GDP and price level. In these cases, the causality analysis in the sense of Granger was made through an error correction model (ECM) for the cointegrated series.

4.3.1. Nominal Money and Real Output

Table 3 presents the Granger causality test results between different nominal monetary aggregates and the real GDP (short run relation), using each one of the details of the series obtained from the *MRA* of the same ones. It is seen that the causality relation between money (measured by different monetary aggregates) and output (measured by the real GDP) **is not unique** and **it changes with the time scale considered;** furthermore, the results about causality in the sense of Granger differs between monetary aggregates. These results can be summarized as it follows:

[6] It was chosen a length of 12 for the wavelet filter denominated Symmlet, to get good properties in terms of regularity. See Gencay, et al. (2002) and Odgen (1997) for a discussion about desired properties of wavelets.

[7] The multiresolution graphs are presented in the annex.

(1) For all monetary aggregates, output Granger causes money at scale 1; that is, using "detail 1" of the multiresolution analysis, which contains movements from 2 to 4 months of the series.

(2) When considering greater scales, Granger causality changes: unidirectional causality of money to output and vice versa, double causality and absence of causality are observed.

(3) The most interesting case is when considering the monetary aggregate called "money", defined as the sum of currency plus demand deposits. In this case, at sacel 1 (movements from 2 to 4 months), output Granger causes money; then Granger causality reverses at scale 2 (movements from 4 to 8 months), and money Granger causes output; when movements from 8 to 16 months (scale 3) are considered, output Granger causes money again[8]; finally, at scales 4 and 5 (movements from 16 to 32 and from 32 to 64 months), double causality between output and money[9] is found.

These results shows how, in contrast to the traditional approach, the use of wavelets and multiresolution analysis –"the alternative approach"- allows to establish the existence of causality in the sense of Granger and the possibility of different directions, depending on time scales[10] considered.

Table 3

GRANGER CAUSALITY USING WAVELETS: 1993:01 - 2001:12

(Seasonal adjusted monthly series)

Null Hipothesis	D1 (2 a 4 m.)	D2 (4 a 8 m.)	D3 (8 a 16 m.)	D4 (16 a 32 m.)	D5 (32 a 64 m.)	D6 (64 a 128 m.)
PBIR does not cause BASE	0.0157	0.0138	0.2558	0.0005	0.3396	
BASE does not cause PBIR	0.7242	0.0119	0.3445	0.0000	0.0018	**UNSTABLE**
Lags	**16**	**23**	**9**	**18**	**19**	
Autocorrelation	**NO**	**NO**	**YES**	**NO**	**NO**	
PBIR does not cause CIR	0.0000	0.0000	0.0020	0.0017	0.0000	
CIR does not cause PBIR	0.2075	0.2754	0.0079	0.0000	0.0000	**UNSTABLE**
Lags	**13**	**22**	**23**	**18**	**27**	
Autocorrelation	**NO**	**NO**	**YES**	**NO**	**NO**	
PBIR does not cause DIN	0.0472	0.3146	0.0032	0.0000	0.0000	0.1856
DIN does not cause PBIR	0.9915	0.0004	0.2547	0.0000	0.0000	0.0000
Lags	**23**	**18**	**13**	**23**	**23**	**9**
Autocorrelation	**NO**	**NO**	**YES**	**NO**	**NO**	**YES**
PBIR does not cause LIQMN	0.0007	0.0289	0.2545	0.1518	0.3431	0.0000
LIQMN does not cause PBIR	0.6918	0.2427	0.0000	0.0000	0.0000	0.0000
Lags	**29**	**20**	**13**	**20**	**13**	**20**
Autocorrelation	**NO**	**NO**	**YES**	**NO**	**YES**	**NO**
PBIR does not cause LIQME	0.0206	0.5486	0.0258	0.0001	0.1929	0.0001
LIQME does not cause PBIR	0.9991	0.2839	0.0002	0.0000	0.0000	0.0000
Lags	**10**	**14**	**5**	**26**	**6**	**20**
Autocorrelation	**YES**	**YES**	**YES**	**NO**	**YES**	**NO**

Source: Own elaboration.

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[8] *Although in this case, exists an autocorrelation generated by an autoregressive process of order 4, only 2 and 4 lags are significant.*

[9] *This result is in the line of the evidence presented by Ramsey and Lampart (1998b), Chew (2001) and Gencay, et. al (2002).*

[10] *This diversity of causality relations in the sense of Granger also is obtained when the price level is included in the analysis.*

4.3.2. Money and Output: Long run Relationship

The theoretical reference for analyzing a long run relationship between money and output is the money quantitative equation $MV = PY$. This equation relates nominal amount of money, M, the velocity of circulation V, the level of prices P and the level of real activity Y. The empirical implications of this equation come from two-long run assumptions: (a) the velocity of money is stationary, and (b) output is constant (equilibrium level).

The quantitative equation can be expressed in logarithms as it follows:

(4.1) $\log M + \log V = \log PY$

or, in terms of the logarithm of the velocity:

(4.2) $\log M - \log PY = \log V$

The equation (4.2) implies that, if $\log V$ is stationary, $\log M$ and $\log PY$ are ***cointegrated*** and the cointegrating vector is also a vector with parameters equal to one (in absolute value). An alternative expression is given by:

(4.3) $\log M + \log V = \log P + \log Y$

or, in terms of the logarithm of the velocity:

(4.4) $\log M - \log P - \log Y = \log V$

The equation (4.4) implies that, under the assumption that $\log V$ is stationary (a stable velocity of money), $\log M$, $\log P$ and $\log Y$ ***are cointegrated*** and the cointegrating vector is a vector with parameters equal to one (in absolute value).

Since the series are unit root, the Engle and Granger (1987) and Johansen (1988) cointegration tests[11] were applied, but it was not possible to find any cointegrating vector for models (4.2) and (4.4). Nevertheless, and due to the existence of a possible cointegrating relation between these variables, it was evaluated the existence of cointegration between non-stationary components of the series constructed using ***the details and the smooth term of the multiresolution analysis of the series.*** This kind of cointegration is similar to the concept of ***hidden cointegration,*** proposed by Granger and Yoon (2002). According to these authors, it is possible to find components of each one series that are nonstationary, integrated of order 1 such that there is a cointegrating relationship. When this occurs, exists ***a hidden cointegrating vector for*** the original variables, or that ***they cointegrate in a hidden way*** and the ECM is called ***crouching error correction model***. Under these considerations, Granger and Yoon (2002) show evidence of the existence of hidden cointegration between the accumulated positive changes of the short and long run interest rates, although the original series of interest rates are not cointegrated.

[11] *Only for the model with two variables represented by (4.2).*

4.3.3. Cointegration between the money and the nominal GDP

To evaluate the presence of hidden cointegration between money and nominal GDP, the details 5 and 6 (D5 and D6) were eliminated of each original series, producing:

$$LDINSA\_65 = LDINSA - LDINSA\_D6 - LDINSA\_D5$$
$$LPBINSA\_65 = LPBINSA - LPBINSA\_D6 - LPBINSA\_D5$$

where *LDINSA* is the logarithm of seasonal adjusted money and *LPBINSA* is the logarithm of seasonal adjusted nominal GDP, both nonstationary and integrated of order 1. Engle and Granger (1987) and Johansen (1988) methodologies show the existence of a cointegrating vector between *LDINSA_65* and *LPBINSA_65* or ***hidden cointegrating vector*** between money and output**.** The first row of Table 4 shows there is bi-directional Granger causality between *LDINSA_65* and *LPBINSA*_65 and that the latter is weakly exogenous.

The next step was the analysis ***various hidden cointegrating vectors considering different time scales in the ECM.*** This strategy makes possible the ***evaluation of the different causality directions between money and nominal output*** considering the existence of a long run relationship.

Table 4

GRANGER CAUSALITY AND HIDDEN COINTEGRATION BETWEEN MONEY AND NOMINAL GDP USING WAVELETS : 1993:01 - 2001:12

(Seasonal adjusted monthly series)

Null Hipothesis	VEC	Levels	eliminated components
PBIN does not cause DIN	**0.0455**	YES	D6, D5
DIN does not cause PBIN	**0.0057**	NO	
Lags	10		
PBIN does not cause DIN	**0.0000**	YES	D6, D5, D3, D2
DIN does not cause PBIN	0.0692	YES	
Lags	2		
PBIN does not cause DIN	0.0735	YES	D6, D5, D1
DIN does not cause PBIN	**0.0000**	NO	
Lags	8		

Source: Own elaboration.

Two additional hidden cointegrating relations were obtained. The hidden cointegration 1 was defined in terms of the original series after removing details 2 and 3 (D2 and D3), which contain movements from 4 to 8 months and 8 to 16 months, respectively:

$$LDINSA\_6532 = LDINSA - LDINSA\_D6 - LDINSA\_D5 - LDINSA\_D3 - LDINSA\_D2$$
$$LPBINSA\_6532 = LPBINSA - LPBINSA\_D6 - LPBINSA\_D5 - LPBINSA\_D3 - LPBINSA\_D2$$

Thus, the series involved in the hidden cointegrating 1 contain –in addition to the component *D4*– the first detail or *D1*. Engle and Granger (1987) and Johansen (1988) methodologies show the existence of a cointegrating vector between *LDINSA_6532* and *LPBINSA_6532* or ***hidden cointegrating vector*** between money and output**.** The second row of Table 5 establishes that both series are weakly exogenous.

The hidden cointegration 2 was defined in terms of the original series after removing only detail 1 of the series, which contains movements from 4 to 8 months and 8 to 16 months:

$$LDINSA\_651 = LDINSA - LDINSA\_D6 - LDINSA\_D5 - LDINSA\_D1$$
$$LPBINSA\_651 = LPBINSA - LPBINSA\_D6 - LPBINSA\_D5 - LPBINSA\_D1$$

Again, it was possible to obtain a cointegrating vector between the filtered series *LDINSA_651* and *LPBINSA_651*, and so a hidden cointegrating vector between money and nominal output. The third row of Table 5 shows that money Granger causes output, but that nominal output is weakly exogenous.

4.3.4. Cointegration between money, real GDP and prices

The first step in the analysis of hidden cointegration between money, prices and real GDP, was the elimination of details 5 and 6 (D5 and D6) of each original series, producing:

$$LDINSA\_65 = LDINSA - LDINSA\_D6 - LDINSA\_D5$$
$$LPBIRSA\_65 = LPBIRSA - LPBIRSA\_D6 - LPBIRSA\_D5$$
$$LDEFLACTOR\_65 = LDEFLACTOR - LDEFLACTOR\_D6 - LDEFLACTOR\_D5$$

where *LDINSA* is the logarithm of the seasonal adjusted money, *LPBIRSA* is the logarithm of the seasonal adjusted real GDP and *LDEFLACTOR* is the logarithm of the real GDP Implicit Price deflator. Johansen (1988) test shows the existence of a cointegrating vector between the filtered series and thus the existence of hidden cointegration between money, prices and the real GDP. The first row of Table 5 shows that real output Granger causes money and that they both are weakly exogenous.

The next step was the evaluation of the existence of ***various hidden cointegrating vectors considering different time scales in the ECM.*** The hidden cointegration 1 was defined in terms of the original series after removing detail 2 (D2), which contains movements from 4 to 8 months:

$$LDINSA\_652 = LDINSA - LDINSA\_D6 - LDINSA\_D5 - LDINSA\_D2$$
$$LPBIRSA\_652 = LPBIRSA - LPBIRSA\_D6 - LPBIRSA\_D5 - LPBIRSA\_D2$$
$$LDEFLACTOR\_652 = LDEFLACTOR - LDEFLACTOR\_D6 - LDEFLACTOR\_D5 - LDEFLACTOR\_D2$$

Johansen (1995) test shows the existence of a cointegrating vector between the filtered series *LDINSA_652*, *LPBIRSA_652* and *LDEFLACTOR_652*. Thus, there is evidence of hidden cointegration between money, prices and real GDP. The second row of Table 5 shows that considering scales 1, 3 and 4, output Granger causes money, but they both are weakly exogenous.

Table 5:

GRANGER CAUSALITY AND HIDDEN COINTEGRATION BETWEEN MONEY, REAL GDP AND PRICES USING WAVELETS: 1993:01 - 2001:12

(Seasonal adjusted monthly series)

Model with money, real GDP and prices

Null Hipothesis	VEC	Levels	Eliminated components
PBIR does not cause DIN	**0.0228**	YES	D6, D5
DIN does not cause PBIR	0.2349	YES	
Lags	6		
PBIR does not cause DIN	**0.0495**	YES	D6, D5, D2
DIN does not cause PBIR	0.1961	YES	
Lags	6		
PBIR does not cause DIN	0.6896	NO	D6, D5, D3, D1
DIN does not cause PBIR	**0.0062**	YES	
Lags	2		

Source: Own elaboration.

The hidden cointegration 2 was defined in terms of the original series after removing details 1 and 3, which contains movements from 4 to 8 months and from 16 to 32 months:

$$LDINSA\_6531 = LDINSA - LDINSA\_D6 - LDINSA\_D5 - LDINSA\_D3 - LDINSA\_D1$$

$$LPBIRSA\_6531 = LPBIRSA - LPBIRSA\_D6 - LPBIRSA\_D5 - LPBIRSA\_D3 - LPBIRSA\_D1$$

$$LDEFLACTOR\_6531 = LDEFLACTOR - LDEFLACTOR\_D6 - LDEFLACTOR\_D5 - LDEFLACTOR\_D3 - LDEFLACTOR\_D1$$

Again, it was possible to obtain a cointegrating vector between the filtered series *LDINSA_6531* and *LPBINSA_6531*, i.e. a hidden cointegrating vector between money, real GDP and prices. The third row of Table 5 shows that money Granger causes real output and that money is strongly exogenous. Then, money is useful for forecasting real output considering movements at scale 2.

Tables 6 and Table 7 show the results of analogous analysis using the remaining monetary aggregates. In particular, the results in Table 6 involve series for which details 1 and 3 were removed, while the results in Table 7 involves series where only detail 2 was not considered:

Table 6

GRANGER CAUSALITY AND HIDDEN COINTEGRATION BETWEEN DIFFERENT MONETARY AGGREGATES, REAL GDP AND PRICES, USING WAVELETS: 1993:01 - 2001:12

(Seasonal adjusted monthly series)

Model with money, real GDP and prices

Null Hipothesis	VEC	Levels	Eliminated components
PBIR do not cause BASE	0.2851	NO	
BASE do not cause PBIR	**0.0180**	**YES**	D6, D5, D3, D1
Lags	4		
PBIR do not cause CIR	0.2813	YES	
CIR do not cause PBIR	0.0603	YES	D6, D5, D3, D1
Lags	3		
PBIR do not cause DIN	0.0665	NO	
DIN do not cause PBIR	**0.0292**	**YES**	D6, D5, D3, D1
Lags	4		
PBIR do not cause LIQMN	**0.0250**	YES	
LIQMN do not cause PBIR	0.8975	YES	D6, D5, D3, D1
Lags	1		
PBIR do not cause LIQME	0.5719	YES	
LIQME do not cause PBIR	0.0750	NO	D6, D5, D3, D1
Lags	5		

Source: Own elaboration.

Table 7

GRANGER CAUSALITY AND HIDDEN COINTEGRATION BETWEEN DIFFERENT MONETARY AGGREGATES, REAL GDP AND PRICES, USING WAVELETS: 1993:01 - 2001:12

(Seasonal adjusted monthly series)

Model with money, real GDP and prices

Null Hipothesis	VEC	Levels	Eliminated components
PBIR does not cause BASE	**0.0261**	YES	
BASE does not cause PBIR	0,1845	YES	D6, D5, D2
Lags	3		
PBIR does not cause CIR	**0.0396**	YES	
CIR does not cause PBIR	**0.0505**	YES	D6, D5, D2
Lags	4		
PBIR does not cause DIN	**0.0211**	YES	
DIN does not cause PBIR	**0.0001**	YES	D6, D5, D2
Lags	3		
PBIR does not cause LIQMN	0.2338	YES	
LIQMN does not cause PBIR	0.8906	YES	D6, D5, D2
Lags	6		
PBIR does not cause LIQME	0.7771	YES	
LIQME does not cause PBIR	0.4805	NO	D6, D5, D2
Lags	5		

Source: Own elaboration.

Therefore, the use of wavelets and multiresolution allow the evaluation of different causality relations between money and real output in a cointegration context.

5. Conclusions

The purpose of this paper was to provide some insights about the empirical the relationship between money and output in Peru. The analysis was based on an orthogonal decomposition of series by timescale obtained using wavelets, following Ramsey and Lampart (1998), and subsequent research by Chew (2001) and Gençay et al. (2002).

The two main results of this literature, obtained in a short run context (Ramsey and Lampart 1998, Chew 2001, Gençay 2002), are: (1) the link between money and real output is not unique, and (2) the direction of Granger causality depends on the timescale considered.

In this paper we went a little bit further in the empirical analysis of money-output relationship using wavelets. In particular, it was proposed the application of wavelet filtering to analyze cointegrating relationships. The data for Peruvian case show no evidence of cointegration between money, real output and prices. However, it is found evidence of cointegration between non-stationary components of the series that includes different timescale details. This result could be considered as an alternative way to represent the existence of hidden co-integration, as proposed by Granger and Yoon (2002).

Given the possibility of cointegration between non-stationary series constructed using wavelet filtering, it is found that (1) the link between money and real output is not unique, and (2) the direction of Granger causality and exogenity depends on both the timescale and the monetary aggregate considered.

The methodology proposed by this paper could be applied to analyze theoretical long run relationships which have not yet found empirical support; for instance, the PPP theory. At the same time, it could be useful for analyzing empirical causality between non stationary series which are related in the long run: real output and financial development, real output and trade, real output and fiscal spending, among others.

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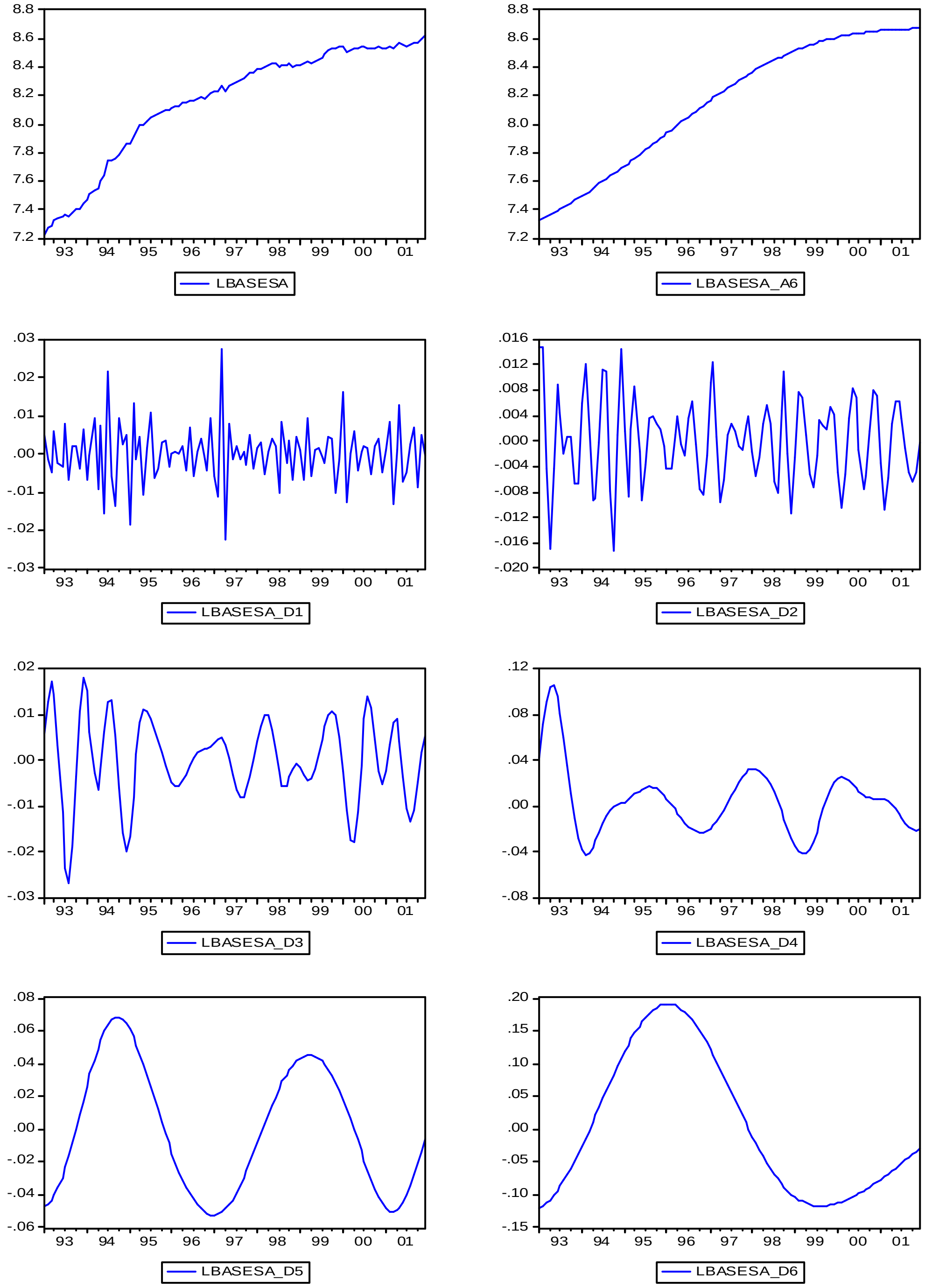
Annex: Multiresolution analysis.

Monetary Base



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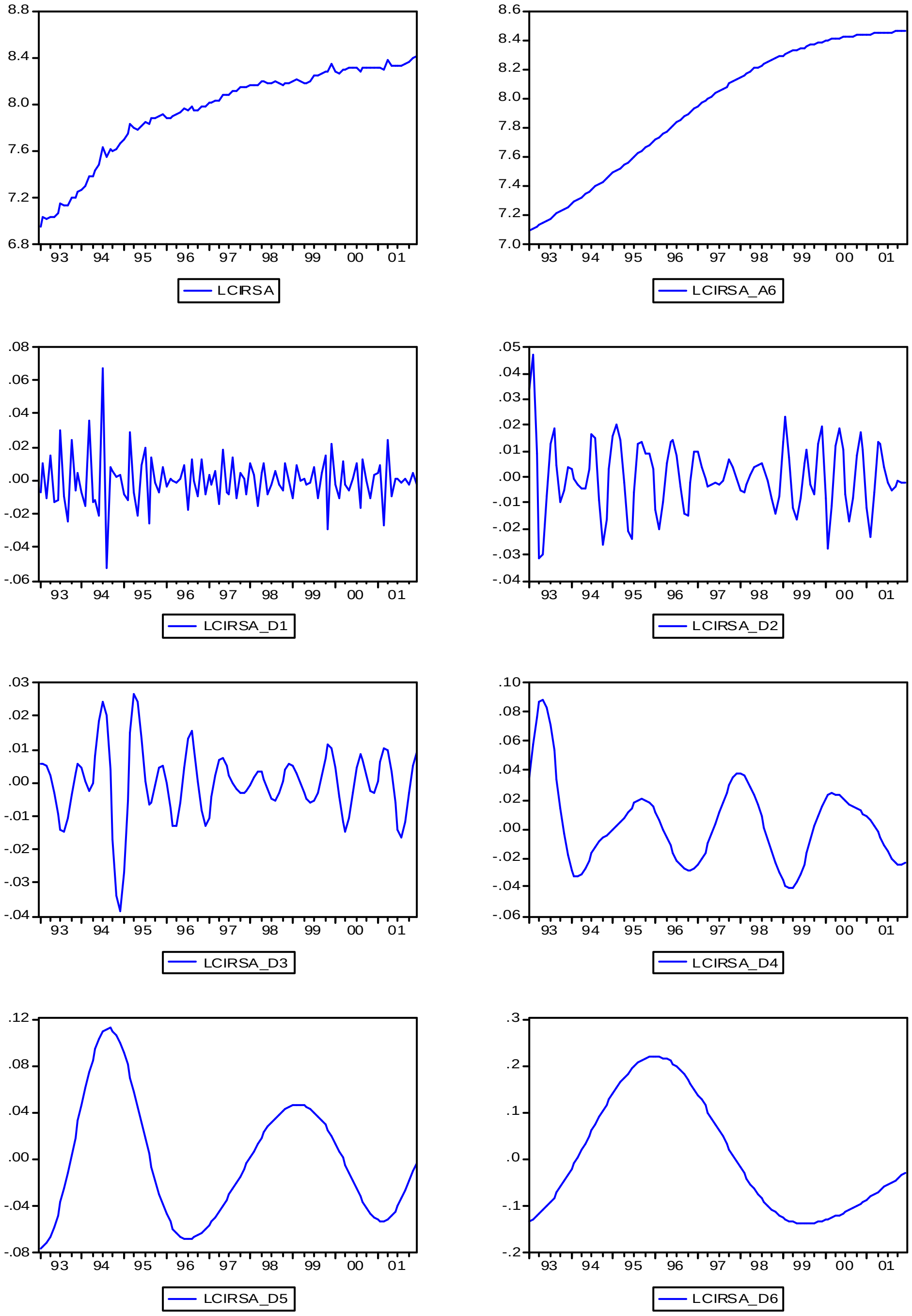
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LCIRSA_D1
LCIRSA_D2
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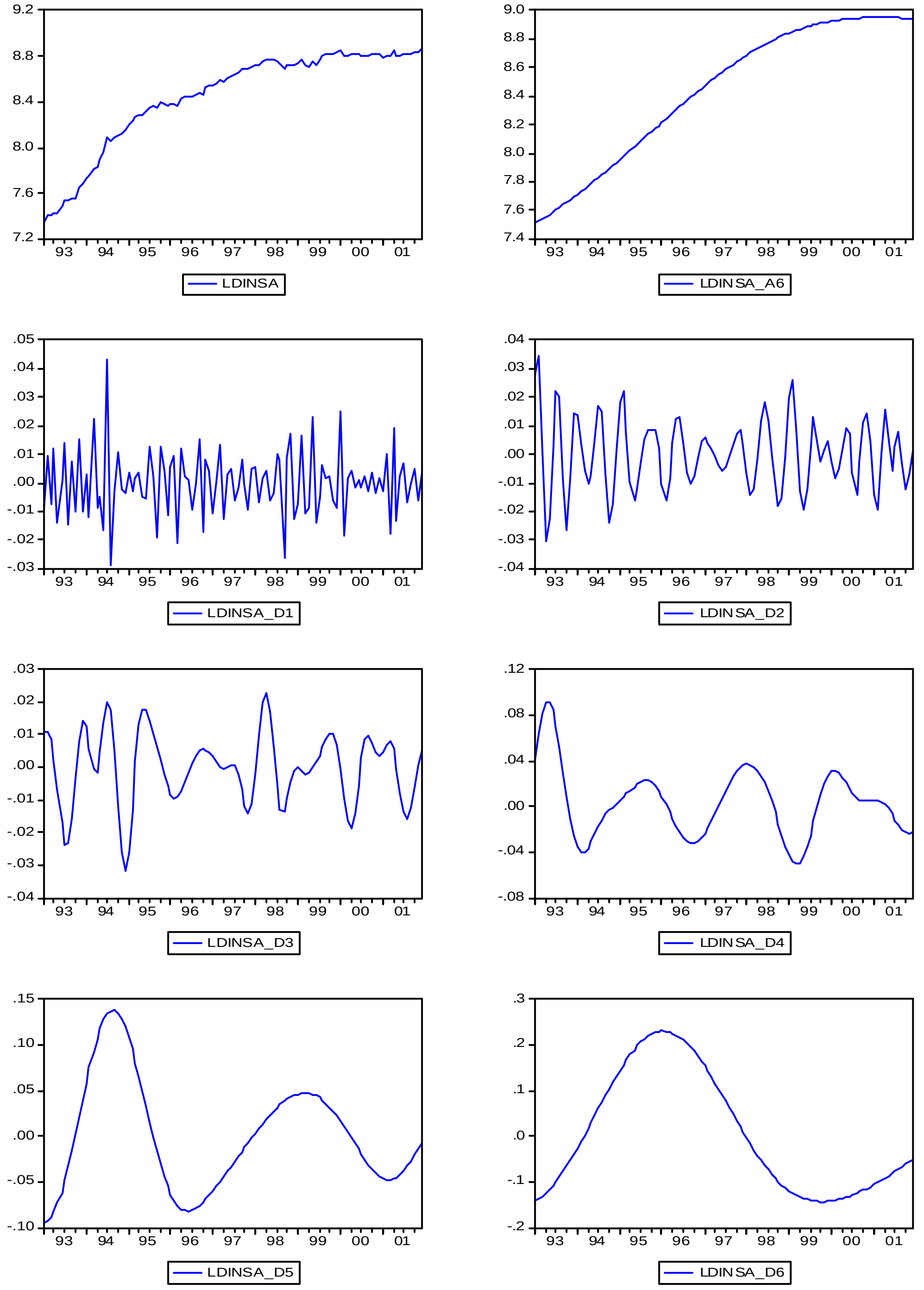
LCIRSA_D4
LCIRSA_D5
LCIRSA_D6

Money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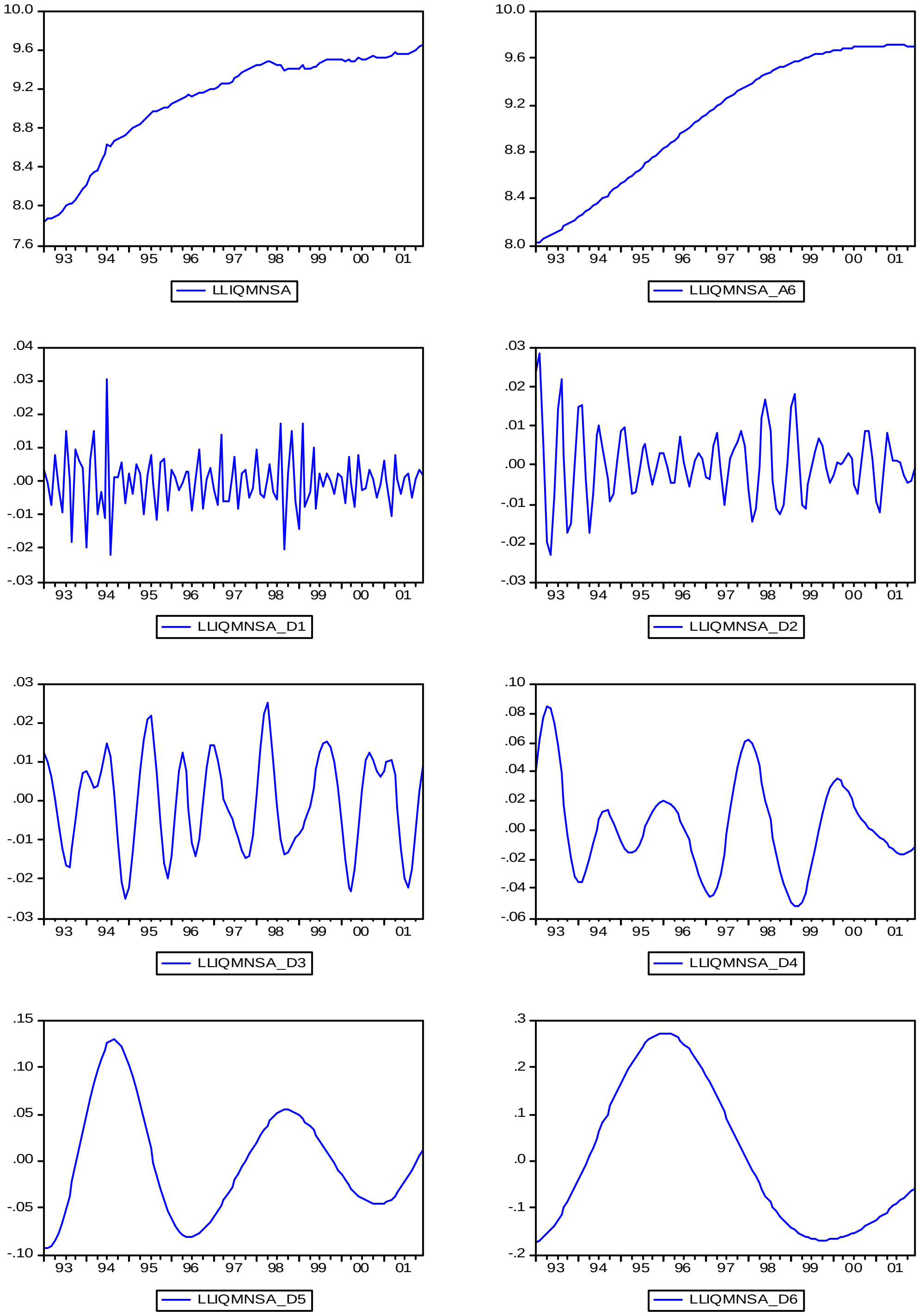
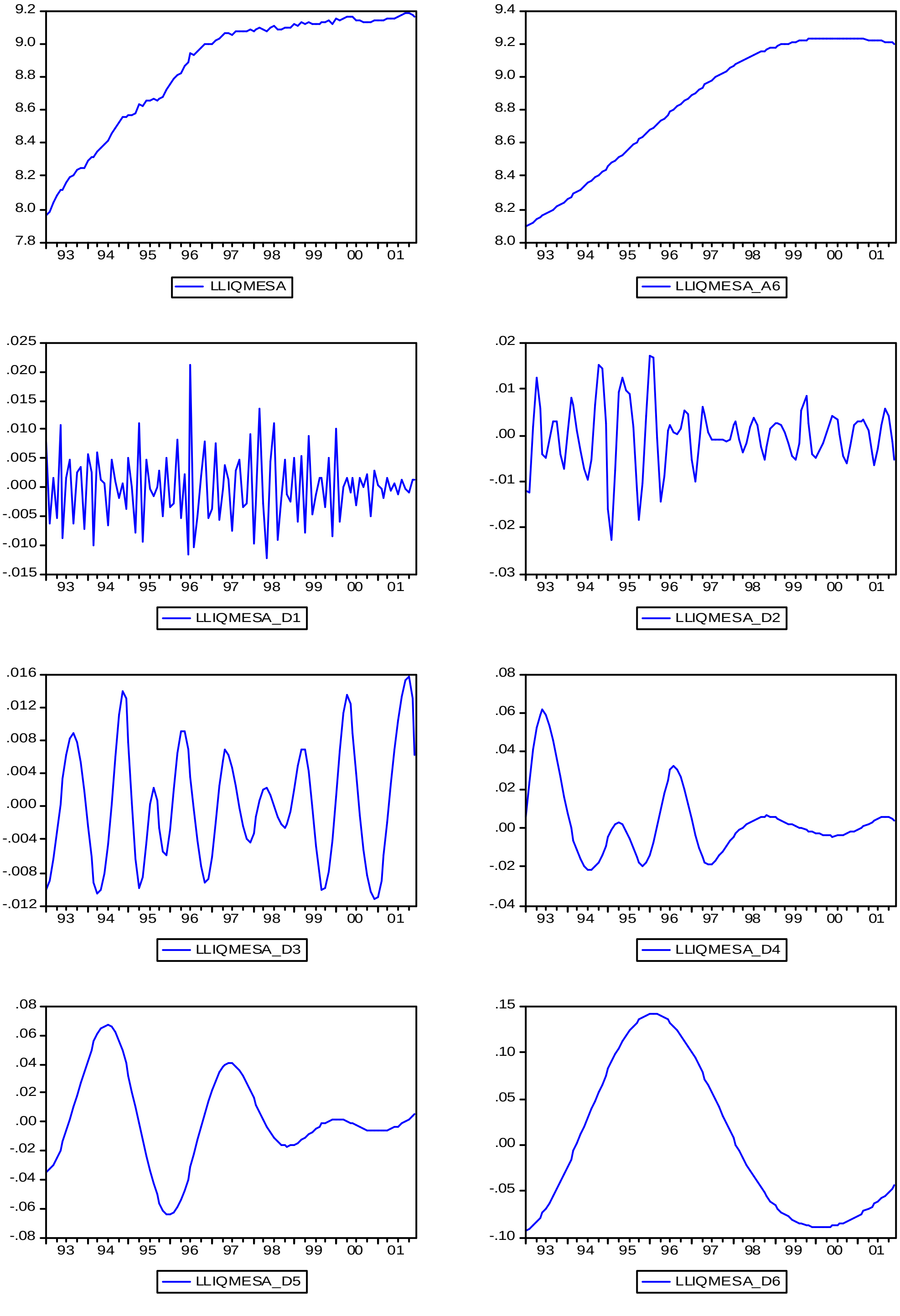
93 94 95 96 97 98 99 00 01

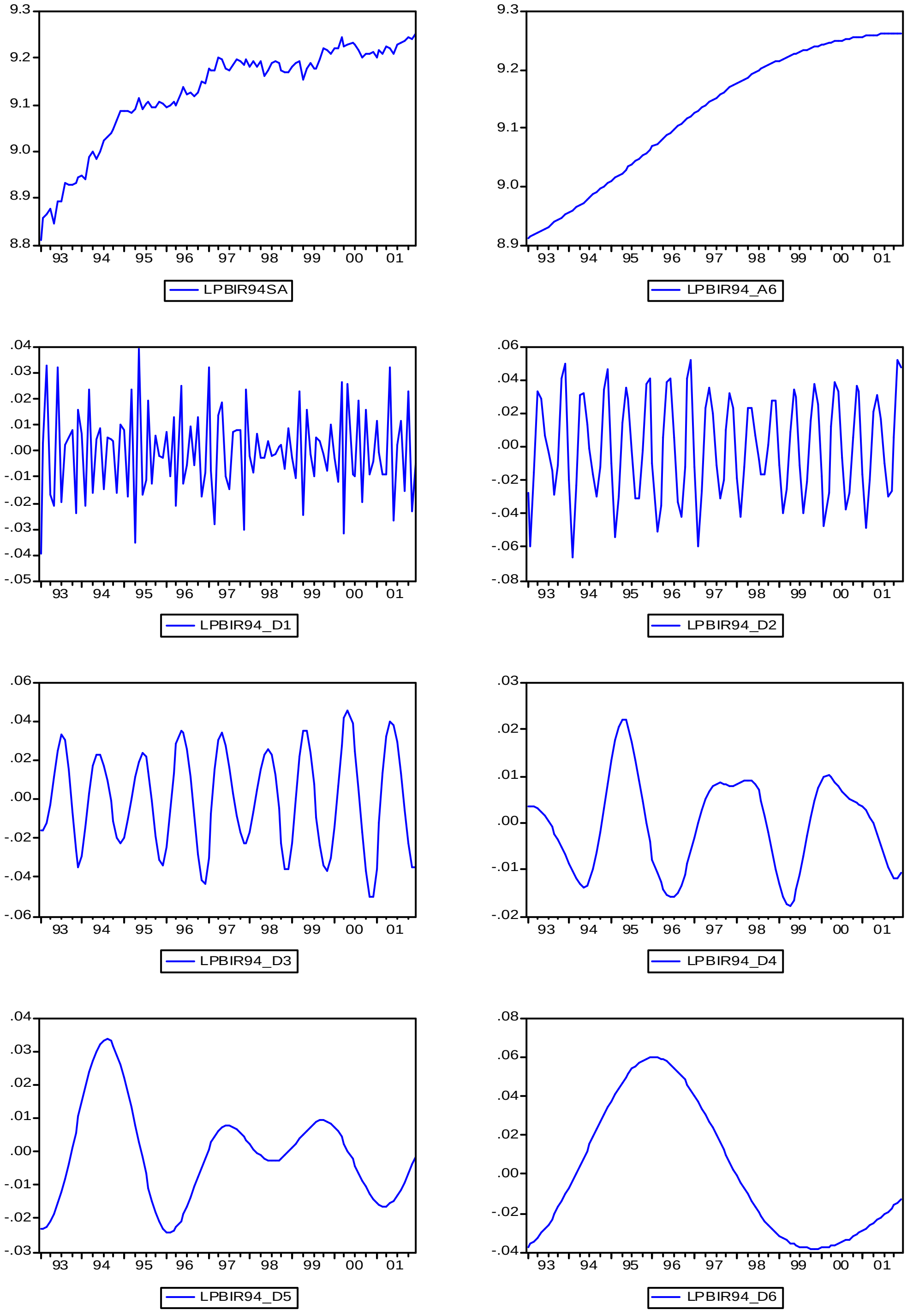
Broad money in domestic currency



Broad Money in foreign currency



Real GDP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LPBIR94_D2
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LPBIR94_D4
LPBIR94_D5
LPBIR94_D6
93 94 95 96 97 98 99 00 01
9.3 9.2 9.1 9.0 8.9 8.8
.04 .03 .02 .01 .00 -.01 -.02 -.03 -.04 -.05
.06 .04 .02 .00 -.02 -.04 -.06 -.08
.06 .04 .02 .00 -.02 -.04 -.06
.03 .02 .01 .00 -.01 -.02
.04 .03 .02 .01 .00 -.01 -.02 -.03
.08 .06 .04 .02 .00 -.02 -.04

GDP Deflator