A General Equilibrium Self-Employment Theory based on Human Capital Differences

Gerardo Jacobs

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1 Introduction

There have been previous works that tried to study what determines the occupational choices of agents. For example, several economists, Lucas (1978) among them, have built models where agents can choose between being either entrepreneurs or workers. These works determine endogenously the proportion of the total work force that decides to be either an entrepreneur or a worker; this way they find the average size of a firm. In this type of models, agents differ in their entrepreneurial capacities, so that agents with low capabilities will decide to be workers. On a different type of model developed by Kihlstrom and Laffont (1979), agents differ on their level of risk aversion: agents with low risk aversion will be entrepreneurs. Nevertheless, these representations of the occupational decision making agents are probably the most adequate for developed countries, where most agents are either entrepreneurs or workers. However, these models are an inadequate way of representing the labor force distribution from less developed countries, where an important proportion of the total work force are self-employed workers.

There is a strong relationship between the per capita income of a country and the ratio of wage employees and self-employed workers. For example, developed countries have on average 15 workers that earn a wage for each worker that is self-employed; on the other extreme, underdeveloped countries have on average 7 self-employed workers for each worker that earns a wage. Any research work whose main purpose is to analyze and comprehend the main economic and social problems faced by the poorest people in society, must study self-employment formation. This work builds a general equilibrium model that introduces self-employment as an occupational choice.

Banerjee and Newman (1993), have built a model where self-employment is an occupational choice. In their work, poor agents choose working for a wage over self-employment, and rich agents become entrepreneurs. Occupational decisions are based on an initial wealth distribution function. Because of the existence of a collateral, rich individuals can receive a loan in order become entrepreneurs. Middle class agents receive a smaller loan that allows them to enter to self-employment. Nevertheless, in most third world countries, most small entrepreneurs live on economic activities that provide a subsistence level of income, and most of them are poorer than wage earners. These fact contradicts Banerjee and Newman (1993), where wage earners are poorer than agents in self-employment.

Recently, more models have been introduced which attempt to study occupational choice dynamics. Antunes and Cavalcanti (2002) build a general equilibrium model with credit constraints, where agents can choose to be either workers, formal entrepreneurs or informal entrepreneurs. Agents are differentiated by their entrepreneurial ability (as in Lucas (1978) and their initial wealth (as in Banerjee and Newman (1993)). Nevertheless, since a employee can not sell its entrepreneurial skills as a worker then, as in previous models, agents with low entrepreneurial skills become workers that belong to the lower end of the income distribution. Meanwhile, in the other hand, agents with high entrepreneurial skill will choose to become entrepreneurs (formal or informal) and they will belong to the upper end of the income distribution. As we have already mentioned, in most developing countries, informal entrepreneurs are poorer than formal workers.

Some evidence from a developing country is introduced in order to support an apparent stylized fact. We choose Mexico, a developing country, since there is available data in order to study our claim. Figure 1 presents data from the National Urban Employment Survey for Mexico (ENEU 2001), describing the occupational distribution between self employees and workers for different years of schooling. Group 00-00 represents the data for individuals without schooling. In this table, 89% of the people in this group are self-employees in the informal sector, while 11% are formal self-employees. Notice that, 52% of all individuals without schooling have self-employment as its occupational choice. Workers represent 47% of the total, almost equally divided between formal and informal jobs. It should be stressed out the decreasing role of self-employment when agents are better educated and the increasing role of formal employment when schooling increases. In the other hand, it is not necessary to show that individuals with low human capital belong to the lower end of the income distribution. Therefore, this data shows that most small entrepreneurs are poorer than wage earners. Because of this, traditional models of occupational choice might not be the appropriate framework in order to study the occupation dynamics of some developing economies.

00 – 00	f	i	T	00 – 06	f	i	T	06 – 09	f	i	T
W	49	51	47	W	71	29	66	W	78	22	79
SE	11	89	52	SE	25	75	33	SE	42	58	20

09 – 12	f	i	T	12 – 16	f	i	T	16 – 22	f	i	T
W	92	08	76	W	95	05	75	W	99	01	74
SE	70	30	22	SE	70	30	21	SE	92	08	19

Figure 1: Distribution of Workers and Self Employees by Schooling Years

But, is it the lack of job opportunities what pushes agents into self-employment in the informal sector? The answer to this question is extremely important for our purposes. If the existence of self-employment is explain by lack of opportunities then, instead of using a general equilibrium setup, a disequilibrium model or one with labor market rigidities could be the most appropriate too study self-employment dynamics. We do not pretend to answer this question on a rigorous way. Nevertheless, we present some evidence also for Mexico which could suggest an answer for a developing country case.

Figure 2 shows that at low levels of education, the income of self-employees is higher than that of workers. This relationship is reverse at higher levels of education. This suggests that is probable that the choice of entering self-employment is a income maximization decision rather than the lack of employment opportunities. We do not pretend to prove extensively this hypothesis, nevertheless it justifies the choice of a general equilibrium type model in order to address occupational choice issues.

This work will build a general equilibrium model where the decision to become an entrepreneur, a worker or self-employed will be endogenous to the model. As in Lucas (1978), the variable that will play a decisive role on the occupational choice is the amount of human capital each agent has: different levels of human capital will open different market opportunities to agents. Therefore the choice of activity will depend on an initial distribution

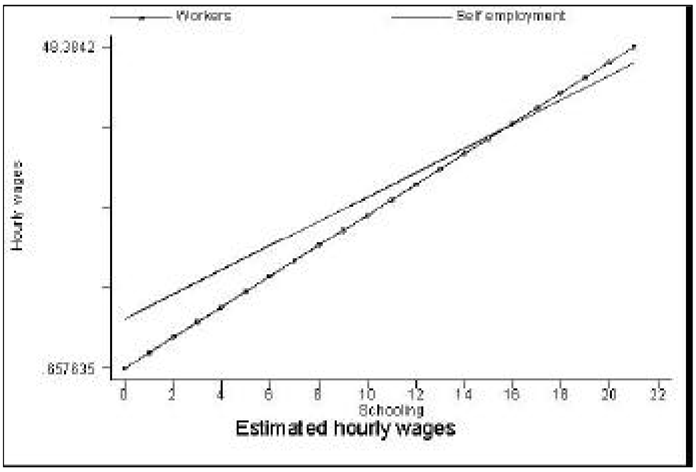


Figure 2: Income vs. Schooling

of human capital. On this model, we will prove that wage earners income is higher than those agents on self-employment, consistent with data from most LDC countries.

Additionally, from a dynamic point of view, there are countries that have experienced a rise in both self-employment and wage employment, while other countries have faced self-employment increase while salary employees have decreased, and viceversa. That is, self-employment for some countries is cyclical and for others counter-cyclical. Most theories typically explain the countercyclical part, especially those theories who use market imperfections arguments in order to explain the rise of self-employment. This paper will study conditions under which self-employment behaves in a cyclical or countercyclical form.

2 An Economy with Self-Employment

The economy consists of a continuum of different types of agents. All individuals are endowed with a different educational level, where a continuum between zero and one, describes the type of agent. That is,

$$i \in [0, 1]$$

where $i^* = 1$ represents the individual with the highest educational level. We choose to build a model where the level of Human Capital is an exogenous variable because of two main reasons: first of all, if agents could choose endogenously their educational level will produce dynamic decision making which will make difficult to study and characterize the equilibrium of the model, on the other hand, exogenous human capital will allow us to analyze the impact of public policy upon the distribution of human capital and the equilibrium level.

Agents can perform two types of activities: low skill and high skill. Not all agents are equally capable of performing high and low skill activities. Probably no far from reality, all agents are equally capable of performing low skill activities, that is

$$h(i) = h \text{ for all } i,$$

where the productivity of an agent on a low skill activity is independent of the educational level. On the other hand, agents with high educational levels will be better at performing high skill activities. Therefore, we have a function $H(i)$(where $H^{'}(i) > 0$) that transforms educational level i into a high skill abilities. Agents can offer, for a wage, their low or high skill abilities to the market, where the income as a low skill worker and a high skill worker are represented by

$$I_{hw}(i) = w_h h \text{ and } I_{Hw}(i) = w_H H(i),$$

where w_h represents the low skill wage and w_H the high skill wage rate. The economy has two types of production technologies: the first one requires low skill labor, where

$$Q(h, k) = \min\{k, h + l_h\}.$$

That is, an agent that decides to be a low skill technology entrepreneur requires k units of capital, contributes with h units of low skill labor, and hires (if profitable) l_h units of low skill labor. Therefore, the income of a low skill entrepreneur (LSE) is

$$I_h(i) = \min\{k, h + l_h\} - w_h l_h - rk.$$

where r represents the rental rate of capital. An agent that decides to be a LSE will choose l_h that maximizes $I_h(i)$. In order to have a well defined maximization problem, we keep the amount of capital as an exogenous variable. Notice that the interest rate does not has a subindex h. This reflects that, in order to simplify matters, we assume that there is only one market for capital. Therefore, a low skill technology requires k units of capital while, as it is explain later, a high skill technology requires K units of capital, where $K > k$. In order words, there are no low and high quality hammers, high skill (HS) technologies require more hammers than low skill (LS) technologies.

An agent i that chooses to be a HS entrepreneur: requires K units of capital, provides $H(i)$ units of administrative work, and hires l_H units of HS labor. An agent that decides to be a high skill entrepreneur (HSE) must adopt the following technology

$$Q(H(i), K) = \min\{KH(i), l_H\}.$$

The first important difference between the HS and the low skill (LS) technologies is that the HS technology needs two types of labor: HS skill labor and an administrator of capital. In order to simplify matters, there is no market for administrator activities[1], therefore a HSE must use his/her high skill abilities in order to fulfill this activity. Since $H(i)$ is increasing in i, a highly educated agent will be a better administrator. Also, note that a good administrator of scarce capital will optimally hire more labor than a less capable one.

Assuming that the HS technology uses HS labor and not LS labor, is central to the results of this work, since it introduces a labor market for HS labor where highly educated people have a higher income. As it will be explain on the following section, this assumption will allow us to have a group of middle income agents that work for a salary and that are richer than agents in self employment.

The income of a HSE will be, $I_H(i) = \min\{KH(i), l_H\} - w_H l_H - rK$, where optimally, $l_H = KH(i)$. Therefore,

$$I_H(i) = K[H(i) - w_H H(i) - r] \tag{1}$$

Now we introduce our last assumption. Recall that the income of a LSE is $I_h(i) = \min\{k, h + l_h\} - w_h l_h - rk$. In order to simplify matters, assume that $k < h$. Therefore, is redundant to hire LS workers, thus $l_h = 0$ and,

$$I_h(i) = k(1 - r). \tag{2}$$

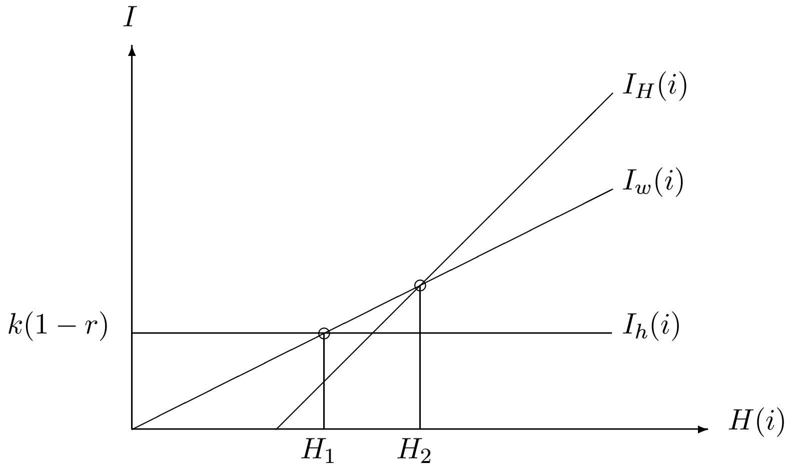
This assumption rules out the existence of a LS labor market, since there is no demand for this type of abilities. Therefore, in order to simplify notation, since there is only one market there is no need to introduce two

[1]For works where administrators hire other administrators see Garicano and Rossi (2002)

wages, we let $I_{Hw}(i) = I_w(i)$ and $w_H H(i) = wH(i)$.For our purposes, a LSE represents self employment decisions. Therefore,

$$I_w(i) = wH(i). \tag{3}$$

Wrapping up, there are three occupational choices: HSE, LSE, and HS worker, where equations 1, 2, and 3 represent respectively the income levels. Graphically,



In the preceding graph, all agents such that $H(i) < H_1$ will decide to be LSE, agents where $H_1 < H(i) < H_2$ will prefer to work for a wage, while HSE will be those that $H_2 < H(i)$. The graph is draw without paying attention to the exogenous and endogenous parameters of the economy. As a mater of fact, later on we will prove that at equilibrium, under specific values of the exogenous parameters, it could be the case that no agents chooses to be a LSE, and under different parameters, there is only an equilibrium with a traditional sector(only self employment activities), and no modern sector (i.e. no high skill sector).

Before proving our main hypothesis, some notation is introduced. Let,

$$\varphi_w(w) = \{i \in [0,1] : I_w(i) \geq I_h(i) \text{ and } I_w(i) \geq I_H(i)\}.$$

In other words, $\varphi_w(w)$ represents the set of all agents that at wage rate w will decide to be a LSE. Similarly, we can define the set of agents that at a given wage rate, decide to become a worker or a HSE. That is, let

$$\varphi_h(w) = \{i \in [0,1] : I_h(i) \geq I_w(i) \text{ and } I_h(i) \geq I_H(i)\}$$

and similarly,

$$\varphi_H(w) = \{i \in [0,1] : I_H(i) \geq I_w(i) \text{ and } I_H(i) \geq I_h(i)\}.$$

The main objective of this work is too study the dynamics of these three sets.

We can prove some interesting and important properties of these sets:

Lemma 1 *The sets φ_i are convex sets.*

The previous Lemma in interesting since it states that if there is a pair of agents with educational levels between a and b, and choose an specific occupation, then no agent that has an educational level between a and b will optimally choose a different occupation. Convexity is not very common to see in real economies. Nevertheless, this result will greatly simplify our analysis. It is possible to choose an alternative specification of the model that produces non convexities, but at the cost of greater complexity when characterizing the equilibrium. Also, it is easy to see from the definition of the sets φ_i that they are convex. The following Lemma states that all agents must choose at least one occupation.

Lemma 2 $\varphi_w \cup \varphi_h \cup \varphi_H = [0,1]$

The next two theorems provide the proof of one of the main hypothesis of this paper. The first theorem states that the LSE are the agents with the lowest level of Human Capital, while the HSE are agents the highest.

Theorem 1 *i) if $i \in \varphi_h(w)$ and $i^* \in \varphi_w(w)$ then $i \leq i^*$ and ii) $i \in \varphi_w(w)$ and $i^* \in \varphi_H(w)$ then $i \leq i^*$*

The second theorem will help us to fully characterized our main hypothesis,

Theorem 2 *i) if $i \in \varphi_h(w)$ and $i^* \in \varphi_w(w)$ then $I_h(i) \leq I_w(i^*)$ and ii) if $i \in \varphi_w(w)$ and $i^* \in \varphi_H(w)$ then $I_w(i) \leq I_H(i^*)$*

That is, the LSE not only are the agents with the lowest human capital but the group with the lowest income, which proves our main goal: to build a model that rationalizes the occupational choice data from some developing economies. What is left of this paper, will make some comparative statics. In order to do this, we need to introduce an equilibrium concept. The following section will do this.

3 Equilibrium

In order to characterize the demand and supply for labor, some notation needs to be introduced. In particular, we need to define who are the agents with the lowest and highest educational levels that choose an specific occupation. Let,

$$\inf(\varphi_j(w)) = \{i^* \in \varphi_j(w) : i^* < i, \forall i \in \varphi_j(w)\} \text{ for } j = w, h, H.$$

In other words, for example, $\inf(\varphi_w(w))$ represents the worker with the lowest human capital. Similarly, let

$$\sup(\varphi_j(w)) = \{i^* \in \varphi_j(w) : i^* > i, \forall i \in \varphi_j(w)\} \text{ for } j = w, h, H.$$

be the agent from set $\varphi_j(w)$ with the highest human capital. We can easily prove that:

Theorem 3 *If $\varphi_j(w)$ are not empty sets then*
i) $\inf(\varphi_h(w)) = 0$,
ii) $\sup(\varphi_H(w)) = 1$
iii) $\sup(\varphi_h(w)) = \inf(\varphi_w(w))$
iv) $\sup(\varphi_w(w)) = \inf(\varphi_H(w))$

That is, the previous theorem states that, i) the LSE with the lowest human capital is also the agent with the lowest human capital and, similarly ii) the HSE with the highest human capital is also the agent with the highest human capital of the economy.

Let $i_{wh} \equiv \sup(\varphi_h(w)) = \inf(\varphi_w(w))$. We know that, since $i_{wh} \in \varphi_h(w)$ and $i_{wh} \in \varphi_w(w)$, then $I_w(i_{wh}) = I_h(i_{wh})$(i.e. if an agent is indifferent between been a worker or a LSE is because the income from both occupations is the same). Therefore,

$$k(1 - r) = wH(i_{wh}).$$

We cannot solve for i_{wh} until we choose an specific distribution function for $H(i)$. Similarly, let $i_{wh} \equiv \sup(\varphi_w(w)) = \inf(\varphi_H(w))$. If an agent decides either to be worker or a HSE is because the income from both occupations is the same (i.e. $I_w(i_{wH}) = I_H(i_{wH})$). Therefore,

$$K[H(i_{wH})(1 - w) - r] = wH(i_{wH}),$$

which we will have to wait to solve until next section, where we introduced an specific functional form for $H(i)$.

Recall that the demand for labor from a HSE is $L_H = KH(i)$. With is mind, we now define the aggregate demand for labor,

$$\pounds_d(w) = \begin{cases} 0 & \text{if } \varphi_H(w) = \emptyset \\ K \int_{\inf(\varphi_H(w))}^{1} H(i)di & otherwise \end{cases}$$

and the supply for labor

$$\pounds_s(w) = \begin{cases} 0 & \text{if } \varphi_w(w) = \emptyset \\ \int_{\inf(\varphi_w(w))}^{\sup(\varphi_w(w))} H(i)di & otherwise \end{cases}$$

As it is done in most general equilibrium models, we introduce an arbitrary occupational distribution, in order to ask if there is a wage rate such that all agents choose voluntarily the occupational choice assign to them.

Definition 1 (Occupational Equilibrium Distribution Vector OEDV) *Let* $X = \{X_h, X_w, X_H\}$ *be an array of three subsets of* $[0,1]$ *such that* $X_w \cup X_h \cup X_H = [0,1]$. *For given values of* k, K *and* r *we say that* X *is an equilibrium occupational distribution vector (OEDV) if there is a wage rate* $\hat{w}$ *such that ,*

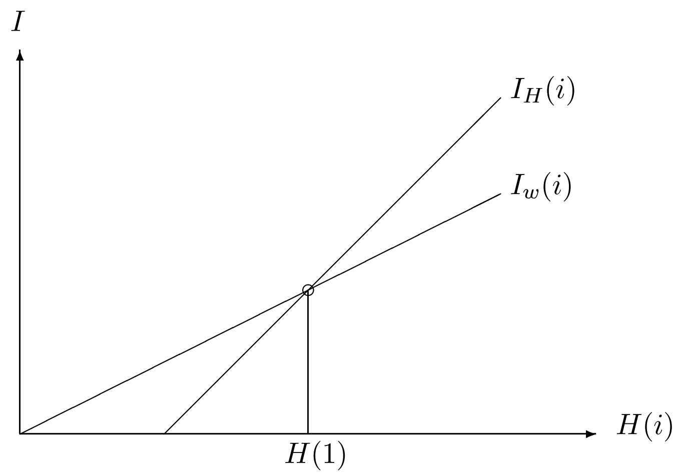
i) $X_i \subseteq \varphi_i(\hat{w})$*for i=h,w,H (income maximization)*

ii) $\pounds_s(\hat{w}) = \pounds_d(\hat{w})$*(labor market equilibrium)*

Notice that in our definition for OEDV, there is no equilibrium condition for the capital market, we could think that our economy is a small country that faces an exogenous interest rate and an inelastic supply for capital. This will allow us later on to make some comparative statics concerning changes in the exogenous interest rate and the capital requirements for both technologies. In order to find an equilibrium for this economy, we need to choose a specific representation for $H(i)$, which transforms human capital into high skill productivity. Before doing this, assuming that an equilibrium exists, we can study some important properties of an OEDV.

Suppose that at a given wage rate w^*, the income as a worker of the most educated agent is equal to its income as a LSE (i.e. $w^*H(1) = k(1-r)$). If this is the case, every agent with a human capital lower than 1 will decide to become a LSE since this occupation provides a higher return. Therefore, at a wage rate lower than $w^* = k(1-r)/H(1)$, no agent will choose to be a worker . Similarly, assume that at the wage rate w^*, the income as a worker of the most educated agent is equal to its income as a HSE (i.e.

$K[H(1)(1-w^*)-r] = wH(1)$. If this is the case, as the following graph shows, every agent we a human capital lower than one will not choose to be a HSE since becoming a worker provides a higher return.



Solving for w^* we obtain,

$$w^* = \frac{K}{1+K}\left[1 - \frac{r}{H(1)}\right]$$

Therefore, at any wage rate higher than w^* no agent in the economy chooses HSE as an occupational activity. That is,

Lemma 3 *If* $w < \frac{k(1-r)}{H(1)}$ *then* $\varphi_w = \emptyset$ *and if* $w > \frac{K}{1+K}\left[1 - \frac{r}{H(1)}\right]$ *then* $\varphi_H = \emptyset$.

In other words, lemma 3 provides necessary conditions for the existence an OEDV where there is a modern sector with high skill workers and HSE, also presents a sufficient condition for the existence of an equilibrium with only self employment. Unfortunately, the wage rate w is an endogenous variable to the model, nevertheless it is easy to combine both conditions to study an interesting result in terms of exogenous variables only:

Theorem 4 *If* $\frac{K}{1+K}\left[1-\frac{r}{H(1)}\right] < \frac{k(1-r)}{H(1)}$ *(condition a) then* $X^* = \{[0,1],\emptyset,\emptyset\}$ *is the only OEDV.*

We omit a formal proof since it is straightforward: choose $\hat{w}$ such that $\frac{K}{1+K}\left[1-\frac{r}{H(1)}\right] < \hat{w} < \frac{k(1-r)}{H(1)}$. Because of Lemma 3, $\varphi_H(\hat{w}) = \emptyset$ and $\varphi_w(\hat{w}) = \emptyset$. Therefore, $\pounds_s(\hat{w}) = \pounds_d(\hat{w}) = 0$ and, because of Lemma 1, $\varphi_h(\hat{w}) = [0,1]$. That is, $\hat{w}$ is a wage rate such that $X_i \subseteq \varphi_i(\hat{w})$ for i=h,w,H and $\pounds_s(\hat{w}) = \pounds_d(\hat{w})$,therefore $X^* = \{[0,1],\emptyset,\emptyset\}$ is an OEDV. Now, if we choose a wage rate that is lower than the LHS of the inequality, because of Lemma 3 no one will choose to be a worker and we have an excess demand for labor. Finally if w is higher than the RHS inequality, no one is a LSE and we have an excess supply for labor.

Now, imaging that at a given wage rate w^*, the income as a worker of the less educated agent is equal to its income as a LSE (i.e. $w^*H(0) = k(1-r)$). If this is the case, every agent with a level of human capital higher than 0 will choose to be worker over been a LSE since $H()$ is an increasing function. In other words, at a wage rate higher than $w^* = k(1-r)/H(0)$, no agent will choose to be a LSE since been a worker provides a higher return. Similarly, imaging that the at a given wage rate w^*, the income as a LSE of the least educated agent is equal to its income as a HSE (i.e. $k(1-r) = K[H(0)(1-w^*)-r]$). If this is the case, at any wage rate lower than w^*, no agent will choose to be a HSE since becoming a worker provides a higher return. That is, solving for w, we get

Lemma 4 *If* $w > \frac{k(1-r)}{H(0)}$ *then* $\varphi_h = \emptyset$ *and if* $w < 1 - \frac{1}{H(0)}\left[r + \frac{k(1-r)}{K}\right]$ *then* $\varphi_h = \emptyset$.

As we did in the previous theorem, we can combine the previous lemmas in a theorem which states necessary conditions under which an OEDV exists with LSE.

Theorem 5 *If* $\frac{k(1-r)}{H(0)} < 1 - \frac{1}{H(0)}\left[r + \frac{k(1-r)}{K}\right]$ *(condition b) and an OEDV exists then* $\varphi_h(\hat{w}) = \emptyset$.

Their is an important difference between theorems 4 and . First of all, theorem 4 states sufficient conditions for the existence of an OEDV, while 5 needs to assume that an OEDV exists. Later on, we will build economies which, under certain parameters, even if *condition b* holds, an equilibrium fails to exists, while in the other hand, if *condition a* holds an OEDV with

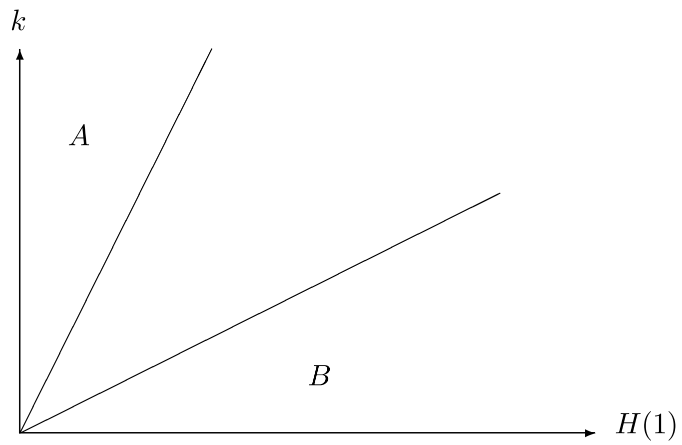
$X^* = \{[0,1], \emptyset, \emptyset\}$ will always exist. We will use theorems 4 and 5 to present a graph that illustrates our results this far. First, notice that *condition a* on theorem 4 implies

$$\frac{K}{1+K}\frac{1}{(1-r)}\left[H(1)-r\right] < k,$$

while *condition b* together with defining $\alpha \equiv H(0)/H(1)$, implies

$$\frac{K}{1+K}\frac{1}{(1-r)}\left[\alpha H(1)-r\right] < k.$$

Graphically



Where, if the exogenous variables $H(1)$ and k are in the area A of the graph, because of theorem 4, then at equilibrium $\varphi_w = \emptyset$ and $\varphi_H = \emptyset$ (no modern sector). On the other hand, if $H(1)$ and k are in the area A, and an equilibrium exists, then $\varphi_h = \emptyset$. The variables $H(1)$ and k were chosen to draw the graph since there are the ones that have a linear relationship between them; this simplifies the presentation. These results are highly intuitive, if there is a big enough increase in k, since the productivity and income of LHS increases, less agents will be willing to choose other occupations. Similarly, if $H(1)$ increases, more agents will choose to be HSE, increasing the demand for labor, thus the wage rate. Because of this, more agents will leave the low skill sector of the economy and work for a wage.

This is as far as we can go without defining an specific functional form for $H(i)$.

Nevertheless, these results have interesting implication in public policy: the road towards economic development is not only a movement into the area B of the graph (i.e. an economy with a modern sector and without self employment). Since any point that is further away from the origin represents a higher per capita income, moving into the area A (i.e. only a traditional sector) is also a feasible way of increasing income and promoting development. In other words, a contraction of the modern sector together with an expansion of the self employment sector, does not necessarily produces economic stagnation.

Assume for a moment that $H(0) = H(1)$, which means that schooling provides no value added into improving high skill abilities. If this is the case, notice that conditions a and b are the same but with the inequality sign reverse. This means that, if there is no value added from schooling, the economy will have either only a modern sector or a traditional one. The simultaneous presence at equilibrium of both sectors is the result of an educational sector that provides value added.

4 A Uniform Distribution

Assume that the function $H : [0, 1] \rightarrow \Re_{\geq 0}$ has the following uniform distribution,

$$H(i) = \alpha + \beta i,$$

where the lowest level of high skill is $H(0) = \alpha$ and the highest is $H(1) = \alpha + \beta$. Recall from the previous section that, $i_{wH} = \inf(\varphi_H(w)) = \sup(\varphi_w(w))$ then, since i_{wH} is indifferent between been a worker or a HSE, it must be the case that $K[H(i_{wH})(1 - w) - r] = wH(i_{wH})$. Now we know the functional form of $H(i)$, thus substituting $H(i)$ and solving for i_{wH} we get

$$i_{wH} = \frac{r}{\beta(1 - w - \frac{w}{K})} - \frac{\alpha}{\beta}$$

Therefore, if $\varphi_H(w) \neq \emptyset$, we can find the demand for labor where $\pounds_d(w) = K \int_{\inf(\varphi_H(w))}^{1} H(i)di$. Similarly, recall that $i_{wh} = \inf(\varphi_w(w)) = \inf(\varphi_h(w))$ where $k(1 - r) = wH(i_{wh})$ therefore we substitute $H(i)$ and we solve for i_{wh}, and we get

$$i_{wh} = \frac{k(1 - r)}{w\beta} - \frac{\alpha}{\beta}$$

At equilibrium, $\pounds_d(w) = \pounds_s(w)$,that is

$$K\int_{i_{wH}}^{1} H(i)di = \int_{i_{wh}}^{i_{wH}} H(i)di$$

which, after evaluating the integral, we get $K[H(1)]^2 - K[H(i_{wH})]^2 = [H(i_{wH})]^2 - [H(i_{wh})]^2$. It is not possible to find a close form solution for this equation, therefore we present some numerical solutions.

4.1 Numerical Simulations

Since we can not find a solution for the equilibrium wage rate, we do some numerical simulations. These results will allow us to do some comparative statics. To this end, choose the following values for the exogenous parameters of the model,

$$\alpha = 1; \beta = 2; k = 2; K = 5; r = .2$$

These values of the parameters were chosen in such a way that is feasible to find an inner solution, that is one where at equilibrium he have a modern as well as a self employment sector (i.e. $\varphi_j(w) > 0$ for $j = w, h, H$). Therefore, this parameters violate *conditions a and b*,otherwise we would have a corner solution. The equilibrium values for some endogenous parameters of the model are

$$w = 0.77, i_{wH} = 0.93, i_{wh} = 0.53$$

Recall that i_{wH} represents the agent which is indifferent between been either a worker or a HSE, therefore $i_{wH} = 0.93$ means that 7% of the agents will choose to be HSE, while i_{wh} represents the agent that is indifferent to choose between been a worker or a LSE, thus $i_{wh} = 53$ means that 53% are LSE, while $i_{wH} - i_{wH} = 40\%$ will be workers.

4.1.1 *Impact of Changes on Human Capital*

Since all agents are indexed in the interval $[0, 1]$,that represents schooling, we cannot change Human Capital. Nevertheless, we can change parameters of the $H(i)$ function, which represents how Human Capital transfers into productive skills. On our model, low skill is constant to all agents and independent of i, and since low skill productivity is higher than the available capital k, we cannot study the impact of an increase in low skill productivity without changing, as mentioned in the section 2, the structure of the model. Nevertheless, we can study the impact in changes in the parameters of the function the transfers schooling into high skill productivity. Recall that $H(i) = \alpha + \beta i$, therefore an increase in β symbolizes an overall increase high skill productivity. We can expect an increase in the number of HSE, therefore an increase in wages, generating an incentive to leave LSE sector. To verify our intuition, we increase one parameter to $\beta = 5$ while keeping constant the rest of the parameters. As before, the new parameters satisfy the necessary conditions in order to avoid a corner solution. The equilibrium values are,

$$w = 0.80, i_{wh} = 0.20, \text{ and } i_{wH} = 0.91$$

where, as expected, there is an increase in wage, a sharp drop in the number of LSE, 69% been workers, and 9% entrepreneurs. These results are also consistent with figure 4 from the previous section in the sense that the increase in β is similar to an increase in $H(1)$, therefore the economy moves closer to region B from figure 4, which means that it moves closer to an economy where there is only a modern sector, consistent with the increase in HSE from this exercise.

We omit the exercise of an increasing α since the impact is similar. Notice that the LSE sector moved on a countercyclical form. That is, while the total product in the economy increased, the LSE sector contracted.

4.1.2 *Impact of Changes in Low Skill Capital*

We now turn to study the impact of an increase in k. A change on this variable can be read in two different ways: it could represent a relaxation of a borrowing constraint, or a technological change in the LS sector, where more capital is required. In either case, a more productive LS sector, will reduce agents willingness to work for a wage, thus reducing the supply for labor, increasing the wage rate, thus reducing the amount HSE. To illustrate this movement we return the original parameters selected for model, but instead we choose $k = 2.5$. In equilibrium,

$$w = 0.78, i_{wh} = 0.79 \text{ and } i_{wH} = 0.97$$

which confirms the increase in wages, LSE, and the decrease in workers and HSE.

4.1.3 *Impact of Changes in the Interest Rate*

A decrease in r could produce ambiguous results on the number of LSE. First of all, a decrease in the interest rate, increases profits of the HSE, thus increasing the demand for labor and the wage rate. The LSE confront two opposing forces: the decrease in r improves the incentives to become a LSE, nevertheless the increase in wages reduces these incentives. The same for HSE, the cost of capital decreases, but in the other hand, they face a higher wage rate. To show this, we return to the original parameters of the model and we set $r = .1$, and we get,

$$w = 0.80, i_{wH} = 0.94, i_{wh} = 0.62$$

where the wage rate has increased, together with a contraction of the HSE sector and an expansion of the LSE. Nevertheless, with a different set of parameters these results do not hold. Let $\alpha = 0; \beta = 1; k = 1/2; K = 2$ and $r = .2$.The equilibrium values are: $w = 0.52$, $i_{wH} = 0.928$, and $i_{wh} = 0.764$. Now decrease the cost of capital to $r = .1$.We obtain,

$$w = 0.59, i_h = 0.756 \text{ and } i_H = 0.926$$

where the results have reversed: the wage rate still increases (as predicted) but the LSE reduces and the HSE increases. Is it possible to find a set of parameters for which these results always hold? It is not possible, but the necessary conditions to avoid a corner solution could give us a hint. Recall that a necessary condition for the existence of an OEDV with HSE and workers is (i.e. Theorem 4),

$$\frac{K}{1+K}\left[1 - \frac{r}{H(1)}\right] > \frac{k(1-r)}{H(1)}$$

Rearranging terms and substituting $H(i)$ yields,

$$\frac{K}{1+K}(\alpha + \beta) + rZ - k > 0$$

where

$$Z = (k - \frac{K}{1+K})$$

It is clear what happens to the LHS of the previous equation if $\alpha + \beta$ increases, but if the interest changes, the impact will depend on the sign of Z. If the LHS decreases, we are moving closer to the area A from figure 4, where there is an equilibrium with LSE only. Now, if the interest rate decreases and $Z > 0$ then the LHS decreases. Therefore, if this is the case, we could expect an expansion of the LSE .

In the first numerical solution, there is an expansion of the LSE when the interest rate decreased. Notice that $Z = 2 - 5/6 > 0$. In the second example, the HSE increased and Z$= 1/2 - 2/3 < 0$. The economic intuition of these results is straightforward: if k is large enough, a reduction of the interest rate will provide strong incentives to move into LS entrepreneurial activities. If not, the increase in wages will upset the small decrease in costs, therefore some LSE will change occupation, deciding to become workers.

Notice that the income of all agents of the economy has increased in both cases, as a result of the lower interest rate. Nevertheless, depending on the parameters of the model, the LSE sector could expand or contract. That is, this exercise provides conditions under which the LSE sector behaves on a cyclical or countercyclical form.

4.1.4 An Exponential Distribution

This section introduces a different distribution for the transfer of schooling into High skill abilities, this will allow us to study some public policy choices.

Assume that $H(i)$ has the following exponential uniform distribution.

$$H(i) = \alpha e^{\beta i}$$

where the most educated individual has αe^{β} units of high skill, while the least educated, could supply α units of high skill labor to the market. With this new distribution we look for the new values for i_{wh} and i_{wh}, where

$$i_{wh} = \frac{\ln[k(1-r)] - \ln(w\alpha)}{\beta}$$

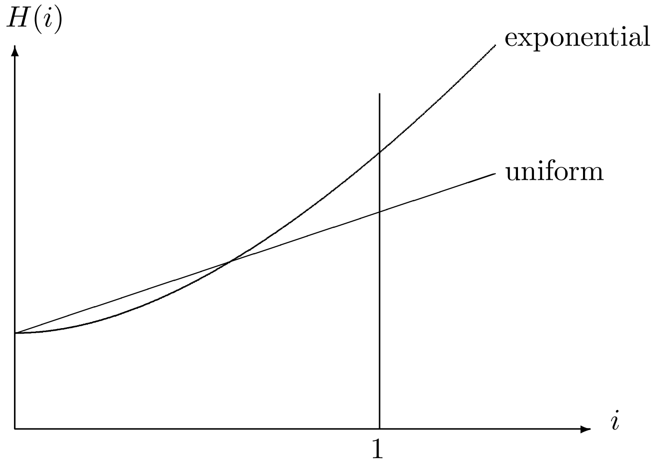
and

$$i_H = \frac{\ln(r) - \ln[\alpha(1 - w - \frac{w}{K})]}{\beta}$$

We can prove that if $\pounds_d(w) = \pounds_s(w)$ and evaluating the integral we get $KH(1) - KH(i_{wH}) = H(i_{wH}) - H(i_{wh})$. Again, is not possible to find an analytical solution for w therefore a numerical solution is presented.

We compare results between the uniform and the exponential distributions, by introducing a public policy option. Assume that the policy maker has the possibility of shifting resources in other to increase to effectivity of schooling to the latest years of education. That is imaging that we reduce resources in elementary and secondary and we increase them in higher education.

The following picture captures this policy alternative. The goal is to choose the parameters for both distributions in such a way that the area under the curve is the same for both distributions, reflecting that the total amount of value added of education is the same one, the only difference is that the exponential distribution generates higher returns on the later years of education, while punishing the returns from early schooling (better teachers at the college level).



We choose parameters in such a way that H(0) is the same for both distributions since the amount of high skill level which agents are born with it is not a policy variable, the way value added is distributed by education is the only policy variable, therefore in both cases $H(0) = \alpha$. Our task is to choose β in such way that the area under the geometric distributions is the same one than the area under the uniform distribution. We can proof that

$$\beta = \ln(k\beta_u + 1)$$

where β_u represents the parameter for the uniform distribution and k the area under its curve. Choose the parameters from the first numerical example in the previous section, $\alpha^* = \alpha = 1$and $\beta^* = 2$, therefore we can prove that $\beta = 3.513$. Now choosing as before $k = 2, K = 5$, and $r = .2$, we get,

$$w = 0.827\,44, i_H = 0.951\,36, i_h = 0.187\,71$$

where the number of LSE decreased from $i_{wh} = 0.53$ to $i_{wh} = .18$, while the HSE have decreased from 7% to 4.9% of the total population. The change in the distribution $H(i)$ while keeping constant the total value added into HS productivity, has produced higher returns from HSE, therefore generating an increase n the demand for worker, which rises the wage rate, driving this way agents out of self employment. It is not difficult to proof that new eqilibrium is Pareto improving.

5 Conclusions (in progress)

Empirical data seems to show that traditional models are not the best way to describe some facts from some developing economies. Nevertheless, is possible to build a model that rationalizes these observations. This model could allow us to study developing aspects from some developing economies. In particular studies alternative paths towards development. The most important aspect of this work is to build a model that analyses the parameters that determine the size of the self-employment sector under a general equilibrium framework. Even an analytical solution was not found, it was possible to study the properties of equilibria, assuming that one exist. The proof for existence was complete for the self-employment case, nevertheless we built economies for which an equilibrium exists with both the modern and traditional sector. Also, we studied the conditions under which the self-employment sector behaves on a cyclical or counter cyclical form. Some policy issued were studied. Nevertheless, is open the study of most of the exogeneous parameters. Changes in the amount of physical capital could be of special interest.

6 Appendix

1. **Proof (Lemma 1) .** a) Convexity of φ_h. We want to prove that if $i \in \varphi_h(w)$ and $i^{'} \in \varphi_h(w)$, then $i^{''} \in \varphi_h(w)$ where $i^{''} = \alpha i + (1-\alpha)$ $i^{'}$ and $\alpha \in [0,1]$. Assume that where $i < i'$. Since $i^{'} \in \varphi_h(w)$ we know that $k(1-r) > wH(i^{'})$. Since $i^{'} \geq i^{''}$ and $H()$ is increasing in i, then $k(1-r) > wH(i^{''})$. It is left to prove that $k(1-r) > K[(1-w)H(i^{''})-r)]$. Again, since $i^{'} \in \varphi_h(w)$ we know that $k(1-r) > K[(1-w)H(i^{''})-r)]$ and since $i^{'} \geq i^{''}$ and $H()$ is increasing in i, then $k(1-r) > K[(1-w)H(i^{''})-r)]$.

   b) Convexity of φ_w.Since $i \in \varphi_w(w)$ we know that $wH(i) > k(1-r)$,therefore since $i < i^{''}$ and $H()$ is increasing in i, then $wi^{''} > k(1-$

r).It is left to prove that $wH(i^{''}) > K[(1-w)H(i^{''})-r)]$. Since $i^{'} \in \varphi_w(w)$ then $wH(i^{'}) > K[(1-w)H(i^{'})-r)]$. Solving for $H(i)$ we get $H(i)[(1-w)-\frac{w}{K}] < r$. If $[(1-w)-\frac{w}{K}] < 0$, then the inequality holds for all i. In the other hand if $[(1-w)-\frac{w}{K}] > 0$, since $H()$ is an increasing function, the inequality holds for all $i \leq i'$,therefore it holds for i''.

c) Convexity of φ_H. Since $i \in \varphi_H(w)$ then $K[(1-w)H(i)-r)] > k(1-r)$. Therefore, since $H()$ is increasing in i and $i'' > i$ then $K[(1-w)H(i^{''})-r)] > k(1-r)$. It is left to prove $K[(1-w)H(i^{''})-r)] > wH(i^{''})$. Since $i \in \varphi_H(w)$ we know that $K[(1-w)H(i)-r)] > wH(i)$. Solving the previous inequality for $H(i)$ we get $H(i)[\frac{K}{w}(1-w)-1] > \frac{K}{w}r$. We know that $[\frac{K}{w}(1-w)-1] > 0$ therefore, since $i'' > i$ and $H()$ is increasing, the inequality holds for i''. ■

Proof (Theorem 1). Since $i \in \varphi_h(w)$ then $k(1-r) > wH(i)$, also since $i^* \in \varphi_w(w)$ then $wH(i^*) > k(1-r)i$ therefore $H(i^*) > H(i)$. Now, since $H(i)$ is an increasing function then $i^* > i$. b) Since $i \in \varphi_w(w)$ then $wH(i) > K[(1-w)H(i)-r)]$. Rearranging terms we get $r > H(i)[(1-w)-\frac{w}{K}]$. Also, since $i^* \in \varphi_H(w)$, it must be the case that $K[(1-w)H(i^*)-r)] < wH(i^*)$.Rearranging terms we get $H(i^*)[(1-w)-\frac{w}{K}] > r$ therefore $H(i^*) > H(i)$. Now, since $H(i)$ is an increasing function then $i^* > i$. ■

Proof (Theorem 2). i) By definition we know that $I_h(i) = k(1-r)$ and that $I_w(i^*) = wH(i)$, since $i^* \in \varphi_w(w)$ it must be the case that $wH(i^*) > k(1-r)$ therefore $I_w(i^*) > I_h(i)$.

ii) By definition we know that $I_w(i) = wH(i)$ and that $I_H(i^*) = K[(1-w)H(i^*)-r)]$, since $i^* \in \varphi_H(w)$ it must be the case that $K[(1-w)H(i^*)-r)] > wH(i^*)$.From theorem 1 we know that if $i \in \varphi_w(w)$ and $i^* \in \varphi_H(w)$ then $i > i^*$. Since $H(i)$ is an increasing function and $i^* > i$ then $H(i^*) > H(i)$, therefore $K[(1-w)H(i^*)-r)] > wH(i^*) > wH(i)$ which proves that $I_H(i^*) > I_w(i)$.

Proof (Theorem 3). i) Let $i' = \inf(\varphi_h(w))$. Assume that $i' \neq 0$,then $0 \notin \varphi_h(w)$ and either $0 \in \varphi_w(w)$ or $0 \in \varphi_H(w)$. If $0 \in \varphi_w(w)$ then it exists an $i'' = 0$ such that $i'' \in \varphi_w(w)$ where $i'' < i'$.This contradicts theorem 2 where if $i'' \in \varphi_w(w)$ then $i'' > i$ for all $i \in \varphi_h(w)$.We build the same argument for the case $0 \in \varphi_H(w)$.

ii) Let $i' = \sup(\varphi_H(w))$. Assume that $i' \neq 1$,then $1 \notin \varphi_H(w)$ and either $1 \in \varphi_w(w)$ or $1 \in \varphi_h(w)$. If $1 \in \varphi_w(w)$ then it exists an $i'' = 1$ such that $i'' \in \varphi_w(w)$ and $i'' > i'$.This contradicts theorem 2 where if

$i'' \in \varphi_w(w)$ then $i'' < i$ for all $i \in \varphi_H(w)$.We build the same argument for the case where $1 \in \varphi_h(w)$.

iii) Assume that $\sup(\varphi_h(w)) \neq \inf(\varphi_w(w))$. Let $i' = \sup(\varphi_h(w))$ and $i'' = \inf(\varphi_w(w))$.If $\sup(\varphi_h(w)) > \inf(\varphi_w(w))$ then there exist $i' \in \varphi_h(w)$ and $i'' \in \varphi_w(w)$ such that $i' > i''$.This contradicts theorem 2 where if $i'' \in \varphi_w(w)$ then $i'' > i$ for all $i \in \varphi_h(w)$. Now if $\sup(\varphi_h(w)) < \inf(\varphi_w(w))$ then it must exist an $i''' \in \varphi_H(w)$ such that $\sup(\varphi_h(w)) < i''' < \inf(\varphi_w(w))$. Recall that $1 \in \varphi_H(w)$ therefore $i''' < \inf(\varphi_w(w)) < 1$, but since $i''' \in \varphi_H(w)$,this violates the convexity of $\varphi_H(w)$ from theorem 1.

iv) Assume that $\sup(\varphi_w(w)) \neq \inf(\varphi_H(w))$. Let $i' = \sup(\varphi_w(w))$ and $i'' = \inf(\varphi_H(w))$.If $\sup(\varphi_w(w)) > \inf(\varphi_H(w))$ then there exist $i' \in \varphi_w(w)$ and $i'' \in \varphi_H(w)$ such that $i' > i''$.This contradicts theorem 2 where if $i'' \in \varphi_H(w)$ then $i'' > i$ for all $i \in \varphi_w(w)$. Now if $\sup(\varphi_w(w)) < \inf(\varphi_H(w))$ then it must exist an $i''' \in \varphi_h(w)$ such that $\sup(\varphi_w(w)) < i''' < \inf(\varphi_H(w))$. Recall that $0 \in \varphi_h(w)$ therefore $0 < \sup(\varphi_w(w)) < i'''$, but since $i''' \in \varphi_h(w)$,this violates the convexity of $\varphi_h(w)$ from theorem 1. ■ ■

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