Efficient partnership dissolution under buy/sell clauses

Maria Angeles de Frutos[1]
Universidad Carlos III de Madrid

Thomas Kittsteiner[2]
Nuffield College, Oxford University and University of Bonn.

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[2]The authors have benefited from comments by Paul Klemperer, Benny Moldovanu and seminar participants in Bonn. Email: kittsteiner@uni-bonn.de

Abstract

When a partnership comes to an end partners have to determine the terms of the dissolution. A well known way to do so is by enforcing a buy/sell option. Under its rules one partner has to offer a price for the partnership and the other agent can choose whether she wants to sell her share or buy her partner's share at this price. It is well known that in a model with private valuations this dissolution rule may generate inefficient allocations. However, we here show that if partners negotiate for the advantage of being chooser, then this buy/ sell provision results in an ex-post efficient outcome. This result helps to explain why such provisions are so widely introduced in partnership contracts.

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1 Introduction

The business attorney Mary Hanson writes: "*Where two individuals or entities each own 50% of a corporation, a "forced buy/sell" provision can be very useful. Such a provision allows one shareholder to give the other a buyout offer. The recipient of the offer must either accept the offer and be bought out, or turn the offer around and purchase the interest of the first shareholder at the same price and upon the same terms as the first shareholder's offer. This arrangement assures that the price and terms are fair, since the shareholder making the offer may end up receiving the purchase price rather than paying it.*"[1]

Partners in commercial relationships have to think about the possibility that the partnership might end. Disagreement about the future strategy of the commonly owned firm, or family circumstances of one of the partners, might lead to the inevitability of splitting-up. Since it is often impossible or implausible to sell the partnership (or a share) to a third party, the partners have to discuss the terms of dissolution among each other. This might cause ressources-consuming negotiations which might even end up before court, if no method of valuing the interests of the involved parties has been fixed prior to the initiation of the dissolution. Therefore it is advisable to include an arrangement in the initial partnership agreement which predesignates a division mechanism. One of the common provisions in partnership agreements for dealing with dissolution is a buy/sell clause[2], where one party names a per share price, and the other owner has the right to buy or sell at that price. These provisions seem to be highly recommended by legal advisors. Real life examples of partnerships that have included buy/sell clauses in their partnership agreement abound[3].

[1] See "Shareholder Agreements: What you and your fellow shareholders should have in writing"by Mary Hanson, at The Businness Advisor, see http://www.bizadvisor.com/archive.htm.

[2] There exists a huge variety of different names for the described procedure in the non-economic literature. Other names given to a buy/sell clause are "Put-Call Provision", Texas shootout", "Shotgun Clause" and "Russian Roulette".

[3] Some examples are:

1.- In February 1995, Liberty and Comcast entered into a home shopping partnership that contained a forced buy-sell clause. Liberty triggered the clause this year, and Comcast agreed to sell its 57% stake in QVC for $7.9 bil.

2.- The joint venture contract between Primadonna Resorts Inc. and MGM Grand Inc. included a buy-sell clause which allowed one party to bid for the other's 50 percent interest in the $460 million New York- New York Hotel.

3.- In 1993, Levi Strauss & Co. entered into a joint venture partnership with a small company, Designs Inc., to own and operate high-concept "Original Levi's Stores" (OLS). Termination provisions of the partnership were carefully tailored, including a put-call

One of the best documented partnership arrangement is the one between Telewest Communications and Comcast UK (or NTL following the amalgamation of Comcast and NTL, which was common knowledge at the time of the agreement), respectively the second and third largest cable franchise owners in the UK. In August 1998 they devised a clause to determine ownership of their joint venture. Under their arrangement, Comcast/NTL had to make its best bid to Telewest which would then have the choice of selling its 50% stake to Comcast/ NTL or buying the Comcast/NTL 50% shareholding at the same price. In August 1999 NTL proposed a price of approx. £428 million to Telewest Communications, who decided to buy at this price. In its press release from 17 August 1998 Telewest describes the clause:"Telewest and Comcast have agreed within a certain timeframe to rationalize their interests in Cable London. Consequently, by no later than 30 September 1999, Comcast (or NTL) will notify Telewest of a price at which Telewest, at its option, will be required either to purchase Comcast's 50% interest in Cable London or sell its 50% interest in Cable London to Comcast (or NTL)"

Buy/sell clauses are a variant of the divide-and-choose method for dividing a cake, which plays an important role in the literature on fair division. This method for allocating a good among parties is generally defined as a mechanism where one party proposes a division of the good and the other party chooses the portion of the division she prefers. If the good is indivisible, the division can be accomplished with money so that one party proposes a price, and the other party chooses to, either accept the price, or, to take the good and pay the first party that price. Under complete information, this mechanism ensures an efficient dissolution and favors the proposer, since she is aware of all possible gains from trade and can completely exploit these. If, however, parties only know their own valuation and just have an estimate about the partner's valuation, it may result in inefficient allocations (see McAfee [1992]). Nevertheless, the mechanism can render efficient allocations if the proposer is the "right" partner, i.e. the partner who perceives herself as equally likely to be seller and buyer. This fact, together with the broad use of buy/sell clause brings about that there may be more to the story than what the mechanism design literature has analyzed so far. In fact, it stems from the agreement between Telewest and Comcast that the buy/sell clause is often used after a negotiation stage on the identity of the proposer. The

provision that allowed either of the partners to buy the partnership from the other partner at any point, subject to certain conditions.

4.- In December 1999 the media company News-Corporation sold its 49.9 % share in the German TV-channel VOX to CLT-UFA, which already owned 24.9% of VOX for $340 million. Before entering the partnership, both parties agreed upon a put-call provision for their shares.

particulars in this real life case has moved us to study the entire dissolution process which starts with negotiations in which partners decide on who has the right to choose.

In this paper we consider the buy/sell mechanism with a negotiation stage to decide the identity of the chooser in the classical framework with independent private values (as introduced by Cramton et al [1987] and McAfee [1992]). We model the negotiations as an ascending auction or equivalently a sealed bid second price auction where the winner pays the loser. It can be thought of as a simplified model of a negotiation procedure in which partners make alternating offers for the right-to-choose. If one of the partners is not willing to increase her offer further, she becomes proposer and receives the last offer made as a monetary payment, i.e. her own bid in the second-price auction. Note that the information revealed in this procedure may allow the proposer to refine her beliefs about the chooser's valuation. We show that this happens in a way that leads to an efficient dissolution of the partnership.

Even though buy/sell clauses are widely used in practice, it is hardly analyzed in environments with asymmetric information[4]. The analysis of the buy/sell clause (without negotiations) in an independent private values model (like ours) is performed in McAfee [1992]. His main result shows that the clause may result in an inefficient outcome and that the proposer is disadvantaged compared to the chooser. McAfee concludes that "This result casts a shadow on the entire literature on cake-cutting type mechanisms". Given this negative result most of the literature on partnership dissolution studies a more symmetric dissolution mechanism instead: a simultaneous auction. In this auction both partners submit a sealed bid and the partner with the higher bid receives the partnership whereas the per-share price is given by a (pre-determined) convex combination of the two bids. Such an auction is shown to be efficient in Cramton et al. [1987] and McAfee [1992][5].

The main contribution of this paper is to provide an economic rationale for the broad use of buy/sell clauses.

[4]The analysis of the buy/sell clause in the literature is essentially restricted to complete information settings where it exhibits nice properties like efficiency and envy-freeness. For an overview of this literature and possible extensions see Brams and Taylor [1996].

[5]De Frutos [2000] analyzes properties of this auction in a model where partners have asymmetric prior beliefs. Kittsteiner [2003] shows the possibility of inefficiencies even in a model with symmetric priors but interdependent valuations. An overview of the literature on partnership dissolution with interdependent valuations is given in Moldovanu [2002].

2 Buy/Sell Clauses in Shareholder Agreements

We consider two risk neutral partners, 1 and 2, with valuations v_1 and v_2 respectively, for a commonly owned object (partnership). The valuation v_i is private information to partner i. Valuations are independently and identically distributed according to a commonly known distribution function F with continuous density f and compact support on $[0,1]$. In what follows, we assume that the standard hazard rate conditions are satisfied:

$$\begin{aligned} \frac{d}{dx}\left(x+\frac{F(x)}{f(x)}\right) &\geq 0, \qquad (1) \\ \frac{d}{dx}\left(x+\frac{F(x)-1}{f(x)}\right) &\geq 0. \end{aligned}$$

Each partner owns an equal share of the partnership which is due for dissolution.[6] An agent's utility is linear in payments and share, i.e. the utility of partner i who holds a share of α in the partnership and receives a payment m is given by $U_i = \alpha v_i + m$. Hence agent $i's$ utility is initially given by $\frac{1}{2}v_i$, which can also be interpreted as her outside-option.

The buy/sell clause in this environment is from an economic viewpoint a mechanism where one party specifies a price (we call this party the proposer), and the other party decides whether to buy the object at that price or sell the object at the price specified (this party is called the chooser). If p is the price specified by the proposer, and each party owns half of the good, then the chooser selects either $p/2$ or the object in which case she pays the proposer $p/2$. It can easily be verified that the chooser decides to take the money as long as the price p is larger than her valuation whereas she decides to buy her partner's share otherwise[7].

[6] Hauswald and Hege (2003) analyze four samples of joint ventures announced between January 1st, 1985 and 2000. The date shows that about 80% of all recorded transactions are two-partner joint ventures. Further, data indicate that about two thirds of two-partner joint ventures have 50-50 equity allocations. Similarly, from a sample of 668 worldwide alliances, Veugelers and Kesteloot (1996) also report that 50% of the joint ventures between two partners exhibit 50-50 share allocations.

[7] In the Guide to US Real State Investing issued by the Association of Foreign Investors in Real Estate (AFIRE), one can read: "..it is common for joint venture agreements to provide a “buy-sell” or “put-call” mechanism by which the venture can be terminated. Such a clause is usually thought of as the ultimate mechanism for resolving disputes...Procedurally, the clause typically provides for one party to offer to buy out the other...The other party then has the right to sell its interest to the offering party at a price equal to the offered total value.., or to buy the offering party's interest at a price equal to the offered total value. Alternatively (and less commonly), a buyout clause may be based on a price set by appraisal or arbitration".

The proposer's expected utility (or profit) if she proposes p is:

$$\begin{aligned} U^P(v_P, p) &= (v_P - p/2)\Pr(v_C \leq p) + p/2\Pr(v_C > p) \\ &= (v_P - p)F(p) + p/2, \end{aligned}$$

where P stands for proposer and C for chooser. Let us define the revenue maximizing price for the proposer $p^*(\cdot)$ by $p^*(v_P) = \arg\max_p U^P(v_P, p)$ and the derivative of U^P with respect to its second argument by U_2^P. It is important to note that the proposer's optimal strategy does depend on the distribution of the chooser's valuation, whereas the chooser's optimal strategy does only depend on the proposed price p and her own valuation v_C (it is therefore independent of any distributional assumptions). The next proposition characterizes the equilibrium price set by the proposer and some of its properties.

Proposition 1 *(McAfee [1992]) Denote the median of the distribution F by $v^{med} = F^{-1}\left(\frac{1}{2}\right)$. The optimal price $p^*(v)$ is the unique solution for p to $U_2^P(v, p) = 0$. It is non-decreasing and satisfies $p^*(v) = v$ if $v = v^{med}$, $p^*(v) < v$ if $v > v^{med}$, and $p^*(v) > v$ if $v < v^{med}$, where v^{med} stands for the median valuation.*

The rationale behind the properties of the equilibrium price is clear. If a partner with a valuation above the median sets a price equal to her own valuation, she will more likely end up buying the asset. She would hence improve her profits by reducing the buying price[8]. Similarly, if her valuation is below the median she is better off setting a price above her valuation, as she is more likely the selling partner.

A dissolution mechanism ensures an ex-post *efficient* allocation if the partner with the highest valuation gets the entire partnership.[9] Using a buy/sell clause to dissolve a partnership may lead to inefficient allocations. The inefficiency might arise when both partners' valuations are either below the median valuation, or both above the median valuation. However, in either case, inefficiencies only arise when the "wrong" partner is proposing. To make this point clear, consider first that both partners' valuations are below the median. As either partner will name a price larger than her valuation,

[8] By lowering the price she would also sell with a loss to partners with valuation slightly below her own. As the price is close to her valuation this loss is of second order (whereas the gain because of the lower buying price is of first order).

[9] In the sample analyzed by Hauswald and Hege (2003), out of 193 US Joint Ventures with a 50-50 division of shares, they found that, in a time interval of fifteen years, there was no change in ownership status in 90 whereas there was a buyout, an acquisition by one partner of the entire stake, in 51.

efficiency demands that the partner with the larger valuation proposes. Similarly, if both valuations are above the median then an efficient allocation emerges whenever the partner with the smallest valuation proposes. This suggest that the partner with the lower valuation should choose if both valuations are below the median, and propose if they are above the median. A natural question to ask is then whether an endogenous determination of the proposer can overcome the problem of the "wrong" partner becoming the proposer. There are several reasons why one might be concerned with studying this issue. The main motivating factor for us derives from the fact that most lawyers recommend including buy/sell clauses in joint venture agreements. It is precisely this fact which moves us to further analyze if this division scheme can render efficient allocations.

3 Endogenous determination of the proposer

If the agreement signed by the parties does not state which party will propose it makes sense to consider that the identity of the proposer will be determined by negotiations among the parties. We examine now the outcome of a dissolution when partners must abide by the buy/sell clauses in their agreement, and they negotiate to determine the proposer[10].

We consider a dissolution procedure which consists of two stages. In the first stage, *the negotiation stage,* partners determine who becomes chooser and proposer. In the second stage, *the pricing stage,* they dissolve the partnership according to the rules of the buy/sell clause as described in the previous section. We will refer to this sequential game as the *dissolution game.* The negotiation stage, is modelled as a sealed-bid second price auction. Both partners submit a sealed bid to an auctioneer who announces the lowest bid and the identity of the partner who submitted it (we call this partner the loser and the other partner the winner of the auction). The loser proposes the price in the pricing stage and gets a monetary compensation equal to her own bid. The winner pays to her partner the lower bid and chooses in the pricing stage. Note that in the framework of this paper this sealed-bid second-price auction is strategically equivalent to an ascending-price auction (which does not require an auctioneer to announce either the winner or the payment).[11] As explained in the introduction, the sealed-bid second-price

[10] It might be that these negotiations already take place some time before the dissolution and therefore the identity of the proposer is already specified in the contract (as in the Telewest/ Comcast example in the introduction). This is also consistent with the procedure we model in this section (in the Telewest/Comcast case both firms were already quite confident that they will split up some month after they negotiated the chooser).

[11] In the ascending auction the price rises continuously and either partner can stop the

(or the ascending) auction can be interpreted as a negotiation game.

Since the mechanism is sequential, we will first study the pricing stage, i.e., the continuation game that follows the negotiation stage in which the identity of the proposer has been determined.

3.1 The Pricing Stage

An important aspect of this dissolution game is that information about partners' valuations is revealed by the strategies played in the first round. Partners can hence update their beliefs about the distribution of the other partner's valuation. This is only crucial insofar as it affects the proposer's belief about the chooser's valuation. Any information revealed to the chooser does not change her strategy, as her decision only depends on the comparison of her own valuation with the proposed price.

Since partners' willingness to pay is U-shaped in the buy/sell mechanism described in the last section (this is proven in Proposition 7 in the Appendix) we assume this to be true for bidding functions in the first stage[12]. If equilibrium bidding functions $b(v)$ of the first-round auction are U-shaped with a minimum at some valuation v^m, then as illustrated in Figure 1, the loser who bid $b(v_P)$ knows that the chooser's valuation v_C must fulfill $b(v_C) > b(v_P)$. Further, because of the U-shaped form of the bidding function, there exists another valuation $\widetilde{v}_P$ resulting in the same bid $b(v_P) = b(\widetilde{v}_P)$. Assuming that $\widetilde{v}_P > v_P$ the proposer knows that the chooser's valuation must be either above $\widetilde{v}_P$ or below v_P.

Lemma 2 *If the bidding strategies in the negotiation stage are U-shaped, and the proposer in the negotiation stage submitted a bid $\widehat{b}$ such that there is $v^* < v^{**}$ with $b(v^*) = b(v^{**}) = \widehat{b}$ then the proposer"s updated beliefs are given by*

$$\begin{aligned} F^C(x) &= \Pr\left(v_C \leq x \mid v_C \in [0, v^*] \cup [v^{**}, 1]\right) \\ &= \begin{cases} \frac{F(x)}{F(v^*)+1-F(v^{**})} & \textit{if} \quad x \in [0, v^*] \\ \frac{F(v^*)}{F(v^*)+1-F(v^{**})} & \textit{if} \quad x \in [v^*, v^{**}] \\ \frac{F(x)-F(v^{**})+F(v^*)}{F(v^*)+1-F(v^{**})} & \textit{if} \quad x \in [v^{**}, 1]. \end{cases} \end{aligned}$$

If there are no two types with $b(v^*) = b(v^{**})$ then this distribution is given by the formula above and $v^{**} = v^*$, with $b(v^*) = \widehat{b}$.

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auction (and hence the rising price) at any time. The partner who stops the auction becomes proposer and receives a payment equal to the price at which the auction was stopped.

[12] As will become clear later on, this is still valid if partners learn more about their partner's valuation after the negotiations.

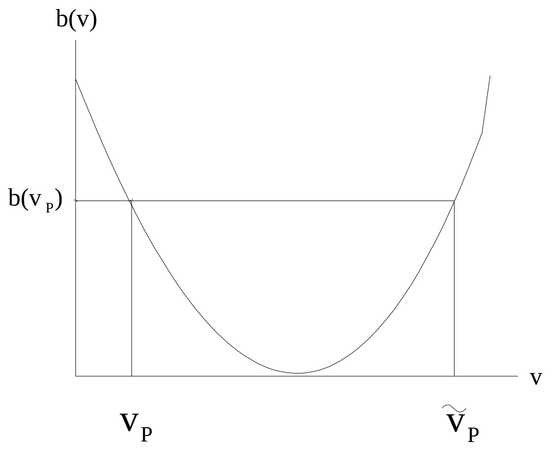

Figure 1:

Lemma 3 *If bidding functions at the negotiation stage are U-shaped, there is a unique optimal price for the proposer in the pricing stage.*

3.2 The negotiation stage

In this subsection we focus on the bidding functions that will be optimal for the partners given the continuation pricing game described before. We first note that the overall utility of a partner in the dissolution game can be decomposed in the expected payoff from being either chooser or proposer in the pricing game, plus the payments she expects to receive/ pay in the auction.

To determine their optimal bids, partners will take into account their willingness to pay for becoming chooser rather than proposer. To illustrate this point assume first that there is an equilibrium with no information revelation in the negotiation phase. If $U^C(v)$ denotes the interim utility of a partner with valuation v who is chooser, and $U^P(v)$ the interim utility as proposer, the difference in utilities, $U^C(v) - U^P(v)$, is a U-shaped positive function (with a minimum at v^{med}). This can be seen from Figure 2 for the uniform distribution, and is shown to hold for any cdf in Proposition 7 in the Appendix. The intuition for this result is that choosers are benefiting from the fact that the proposer has uncertainty about whether she will sell or buy. Because of this uncertainty prices are set close to v^{med}, which favors choosing partners with "more extreme" valuations. In addition, such a partner cannot take much advantage of her "extreme" valuation if she proposes

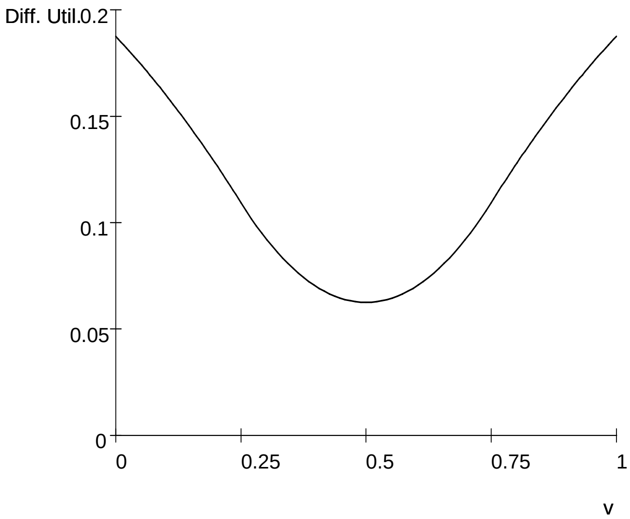

Figure 2: $U^C(v) - U^P(v)$ for uniform distribution.

since she does not want to set a price close to v^{med}, which would result in unprofitable trade with high probability. Since these extreme partners have more to gain from being choosers they should deviate at the bidding game to make more likely the event of choosing. This contradicts that optimal bidding functions are flat. Further, it stresses the relationship between the optimal bidding functions and the willingness to pay. The main result of this paper (Theorem 5 below) shows that this happens in equilibrium. It turns out that equilibrium bidding functions $b(v)$ of the first-round auction are U-shaped with a minimum at the median valuation v^{med}.

Theorem 4 *The following strategies constitute an equilibrium of the dissolution game:*

- In the first stage, both partners bid according to the following bidding function

$$b(v) = \begin{cases} \frac{\frac{1}{2}\int_0^v (s(t)-t)F(t)f(t)dt}{2F^2(v)} & \text{if } \quad v \leq v^{med} \\ \frac{\frac{1}{2}\int_v^1 (t-s(t))(1-F(t))f(t)dt}{2(1-F(v))^2} & \text{if } \quad v > v^{med}, \end{cases} \tag{2}$$

where

$$s(v) := F^{-1}(1 - F(v)).$$

- In the second stage, the proposer sets a price equal to her valuation, i.e. $p = v_P$.

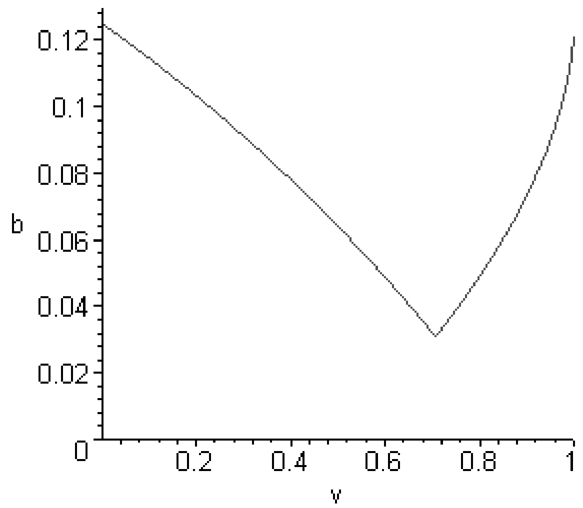

Figure 3: The bidding function b_B.

Corollary 5 *The equilibrium above is ex-post efficient.*

The equilibrium bidding functions are strictly decreasing for valuations below the median v^{med} and strictly increasing for $v > v^{med}$. In addition we have that $b(v) = b(s(v))$, i.e. for any v we have that the mass of valuations that submit a higher bid is equally distributed on valuations smaller than v and valuations that are larger than v.

Example 6 *Assume that valuations are distributed according to* $F(v) = v^2$, $v \in [0,1]$. *The following bidding function* $b_B(v)$ *(see Figure 3) is part of an ex-post efficient equilibrium:*

$$b_B(v) = \begin{cases} \frac{\frac{1}{4}\sqrt{1-v^2}\left(\frac{2}{5}v^4-\frac{2}{15}v^2-\frac{4}{15}\right)-\frac{1}{10}v^5+\frac{1}{15}}{v^4} & \text{if } 0 \leq v \leq \frac{1}{\sqrt{2}} \\ \frac{\frac{1}{10}v^5-\frac{1}{6}v^3-\frac{1}{4}\sqrt{1-v^2}\left(\frac{2}{5}v^4-\frac{4}{5}v^2+\frac{2}{5}\right)+\frac{1}{15}}{\left(v^2-1\right)^2} & \text{if } \frac{1}{\sqrt{2}} \leq v \leq 1. \end{cases}$$

Theorem 5 provides a rationale for the broad use of buy/sell clauses. To further understand why the dissolution game is ex-post efficient, we look at the particular shape of the bidding functions. Its U-shaped form is a requirement for efficiency since we need that for any losing bidders valuation v_P some partners with valuations smaller than v_P as well as some partners with valuations larger than v_P bid above $b(v_P)$. The U-shaped form is also a natural consequence of the fact that bidding functions reflect the willingness to pay for becoming chooser rather than proposer. The proposer will never profit from the offer she makes in the second-round (since she offers a price that equals her valuation) but solely from the money she receives in the auction, whereas the proposer's payoff in the second-round is higher if she has a more extreme valuation (for the reasons given above). To understand why partners with valuations v and $s(v)$ bid the same amount[13] we investigate the effects of a small (marginal) change in their bids. If this has the same (marginal) effect on their expected utilities in the pricing game, partners with valuations v and $s(v)$ have the same (local) incentives to bid the same amount $b(v)$. To see that these incentives are indeed equal consider a partner with valuation $v < v^{med}$ who increases her bid marginally to $b(v - dv)$. As a result of this increase she becomes chooser rather than proposer with respect to the marginal types v and $s(v)$. To calculate the difference in expected utility from being chooser rather than proposer we just have to look at the change in utility that results from trade with the marginal types v and $s(v)$, as the proposed prices do not change if the bid is increased[14]. As proposer she will set a price of v. She will hence buy from another partner with valuation v getting $\frac{v}{2}$ with "probability" $f(v)$; in addition she will sell to a valuation $s(v)$ partner, giving her $\frac{v}{2}$ which also happens[15] with "probability" $f(v)$. The overall decrease in her utility from not being proposer (with respect to the marginal types) is $vf(v)$. On the other hand, as chooser from a partner with valuation v (who proposes a price of v) she gets $v - \frac{v}{2} = \frac{v}{2}$, and from a partner with valuation $s(v)$ (who offers a price of $s(v)$ and to whom she sells) she gets $\frac{s(v)}{2}$. Since both events happen with probability $f(v)$, the overall increase in her utility from being chooser equals $\frac{v+s(v)}{2}f(v)$. Deducting $\frac{v+s(v)}{2}f(v)$ from $vf(v)$ gives $\frac{s(v)-v}{2}f(v)$. This amount must equal the difference in expected utility from being chooser rather than proposer for a partner with valuation $s(v)$ (who bids as if she were a type $v - dv$ or equivalently type $s(v - dv)$). A similar argument to the one we used above gives that as proposer she expects to get $s(v) f(v)$. If she were chooser instead, she would buy from a valuation v partner and pay $\frac{v}{2}$ yielding $s(v) - \frac{v}{2}$, and sell to a valuation $s(v)$ partner giving her a utility of $\frac{s(v)}{2}$. Hence the overall expected utility

[13] This is determined by the willingness-to-pay which is now given by the transfers they will make/receive in the negotiations and by the expected utility they will accrue from the pricing stage.

[14] This is different if the bid is decreased. In that case the considered partner will not propose a price that equals her valuation. The calculations in that case are a bit more involved and require an Envelope-Theorem argument. See the proof of Theorem 5 (in the Appendix) for details.

[15] Note that a decrease in v does increase $s(v)$ hence the "probability" that the partner faces a valuation $s(v)$ partner is given by -$\frac{d}{dv}F(s(v))$ =-$\frac{d}{dv}(1 - F(v)) = f(v)$.

she would receive as chooser from the marginal partners is $\left(\frac{3}{2}s(v) - \frac{v}{2}\right) f(v)$. This shows that the difference in expected utility from being chooser rather than proposer (with respect to the marginal valuations v and $s(v)$) of a partner with valuation $s(v)$ is $\frac{s(v)-v}{2} f(v)$, i.e. the same as for a valuation v partner. Putting all this together, we know that in an efficient equilibrium this change in expected utility has to equal the change in transfers in the first-round auction. The latter includes the direct effect of an increase in $b(v)$ which increases payments in the case of loosing the auction, resulting in a change of utility given by $2F(v) \frac{d}{dv} b(v)$. It also includes the effect of a change in winning probabilities. The difference in payments between proposer and chooser is $2b(v)$ (since one receives b instead of paying it), and this change happens with probability $2f(v)$. Therefore the total (marginal) loss in the auction from increasing the bid adds up to $2F(v) \frac{d}{dv} b(v) + 4b(v) f(v)$. In equilibrium, the cost and revenues from deviating must cancel out which implies that the following differential equation has to be fulfilled:

$$2F(v) \frac{d}{dv} b(v) + 4b(v) f(v) = \frac{s(v) - v}{2} f(v).$$

The bidding function $b(v)$ in Theorem 5 solves this differential equation.

4 Conclusion

We analyzed the predominant dissolution mechanism for partnerships, the so called buy/sell or cake-cutting mechanism. Due to its asymmetric rules, the theoretical literature considers this procedure inferior (i.e. less efficient) compared to other more symmetric mechanisms, like e.g. a simultaneous auction. We provide a rational for the use of the buy/sell clauses if partners negotiate the different roles (i.e. who becomes proposer and chooser). Such negotiations take place in practice and might lead to an efficient dissolution of the partnership. An important role is played by the information that is revealed during the negotiations since this is incorporated in the proposed price which, in contrast to the buy/sell clauses with exogenously fixed proposer, always enforces efficient trade.

It is important to note that this result relies (like most efficiency results in the literature on auctions) on the fact that partners are ex-ante symmetric[16]. If valuations are distributed according to different distribution functions we cannot achieve efficiency with the analyzed procedure. In this case a comparison with the simultaneous auction would be interesting, which is not

[16] Even though we need that both partners valuations are identically distributed we do no need this distribution to be symmetric.

efficient either. It is not clear to us which mechanism performs better. The asymmetric case is much harder to analyze, since no closed form solutions for bidding functions exist, and the proposer will not set prices that equal her valuation. Another interesting extension is the introduction of common value components. In such a model efficiency might not be possible in general (see Fieseler et al. [2000] or Kittsteiner [2003]) and it is not clear a priory how buy/sell clauses with negotiations will perform.

5 Appendix

Proposition 7 *The chooser has an interim utility strictly greater than that of the proposer. Further, the difference in expected utility between being chooser or proposer is U-shaped and it has a minimum at* v^{med}.

Proof.
The expected utility of partner i if she is the proposer equals

$$\begin{aligned} U^P(v_i) &= (v_i - p^*(v_i))F(p^*(v_i)) + p^*(v_i)/2 \\ &= U^P(0) + \int_0^{v_i} F(p^*(z))dz. \end{aligned}$$

The expected utility of partner i if she is the chooser equals

$$U^C(v_i) = E_{v_j}\left[\max(v_i - \frac{p^*(v_j)}{2}, \frac{p^*(v_j)}{2})\right]$$

which can be written as:

$$U^C(v_i) = \begin{cases} \frac{1}{2}E_{v_j}(p^*(v_j)) & if \quad v_i \leq p^*(0) \\ v_i F((p^*)^{-1}(v_i)) + \frac{1}{2}E_{v_j}(p^*(v_j)) \\ \quad - \int_0^{(p^*)^{-1}(v_i)} p^*(z)f(z)dz & if \quad p^*(0) < v_i \leq p^*(1) \\ v_i - \frac{1}{2}E_{v_j}(p^*(v_j)) & if \quad p^*(1) \leq v_i. \end{cases}$$

The interim difference between the expected utilities $(U^C(v_i) - U^P(v_i))$ becomes
If $v_i \leq p^*(0)$, then

$$\frac{1}{2}E_{v_j}(p^*(v_j)) - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

If $p^*(0) < v_i \leq p^*(1)$,

$$v_i F((p^*)^{-1}(v_i)) + \frac{1}{2}E_{v_j}(p^*(v_j)) - \int_0^{(p^*)^{-1}(v_i)} p^*(z)f(z)dz - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

And, if $p^*(1) \leq v_i$,

$$v_i - \frac{1}{2}E_{v_j}(p^*(v_j)) - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

We must hence distinguish three cases:

1. If $v_i \leq p^*(0)$ then it is straightforward to see that $U^C(v_i) - U^P(v_i)$ is strictly decreasing in v_i.

2. If $p^*(0) < v_i \leq p^*(1)$ then $\frac{\partial\left(U^C(v_i) - U^P(v_i)\right)}{\partial v_i} = F((p^*)^{-1}(v_i)) - F(p^*(v_i))$ which is negative for $v_i \leq v^{med}$ and positive otherwise.

3. If $p(1) \leq v \leq 1$ then $\frac{\partial\left(U^C(v) - U^P(v)\right)}{\partial v} = 1 - F(p(z)) \geq 0$.

Hence the difference in expected utility is U-shaped and it has a minimum at $v = v^{med}$. ∎

Proof of Lemma 3:
Assume that the winner of the negotiation bidded according to a U-shaped function $b(v)$. Assume that a bidder who submitted a bid of $\widehat{b}$ (in the range of $b(v)$) loses the auction. Assume further that we have $b(v^*) = b(v^{**}) = \widehat{b}$ with $v^* < v^{**}$ (if such a v^{**} does not exist the following proof will also hold for $v^{**} = v^*$). The distribution of the winner's type v_C conditional on having submitted a bid higher than $\widehat{b}$ is given by

$$\begin{aligned} F^C(x) &= \Pr\left(v_C \leq x \mid v_C \in [0, v^*] \cup [v^{**}, 1]\right) \\ &= \begin{cases} \frac{F(x)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [0, v^*] \\ \frac{F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [v^*, v^{**}] \\ \frac{F(x)-F(v^{**})+F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [v^{**}, 1]. \end{cases} \end{aligned}$$

Note that the hazard rate conditions (1) imply that for any $K \in [0, 1]$ we have that

$$\frac{d}{dx}\left(x + \frac{F(x) - K}{f(x)}\right) \geq 0.$$

It is then easy to verify that $F^C(x)$ also satisfies the standard hazard rate conditions (1) for $x \in [0, v^*] \cup [v^{**}, 1]$. Assume that $v_P \leq v^*$. The proposer's (i.e. loser's) utility, if she sets a price of p is given by:

$$U(p) = \left(v_P - \frac{p}{2}\right) F^C(p) + \frac{p}{2}\left(1 - F^C(p)\right).$$

Using the abbreviation $U_p(p) = \frac{d}{dp} U(p)$ we obtain for $p \notin [v^*, v^{**}]$

$$\begin{aligned} U_p(p) &= (v_P - p) f^C(p) - F^C(p) + \frac{1}{2} \\ &= f^C(p)\left[v_p - p - \frac{F^C(p) - \frac{1}{2}}{f^C(p)}\right]. \end{aligned}$$

In addition we have that if $U_p(p) = 0$ we get:

$$\frac{d^2}{dp^2} U_{v_P}(p) = f^C(p) \left[-\frac{d}{dp} \left(p + \frac{F^C(p) - \frac{1}{2}}{f^C(p)} \right) \right] < 0.$$

This shows that there is at most one optimum of U_P in $[0, v^*]$ and also at most one optimum in $[v^{**}, 1]$.
To show uniqueness and existence of the global maximum it is necessary to compare the candidates (if they exist) of optima in the intervals $(0, v^*)$ and $(v^{**}, 1)$ (given by the first order condition above) and at the boundaries v^* and v^{**}. This tedious mechanical exercise makes use of concavity properties of the utility and is omitted.

Proof of Theorem 5:
Consider a bidder with valuation v who bids $b(\widehat{v})$ and assume that the other bidder bids according to (2). Let us assume $v \leq v^{med}$. The case $v > v^{med}$ can be shown similarly and it is hence omitted. We denote the interim utility (of the dissolution game) of the considered bidder by $U(v, \widehat{v})$. By imitating a bidder of type $\widehat{v}$ she will be a chooser in the pricing stage if the other agent's valuation is within the interval $[\widehat{v}, s(\widehat{v})]$ and a proposer otherwise. $U(v, \widehat{v})$ can be decomposed in the expected payoff from being chooser (denoted by $U^C(v, \widehat{v})$) and proposer (denoted by $U^P(v, \widehat{v})$) in the pricing stage, and the payments she expects to receive/ pay in the auction. Her expected utility is then

$$U(v, \widehat{v}) = U^P(v, \widehat{v}) + U^C(v, \widehat{v}) - \int_{\widehat{v}}^{s(\widehat{v})} b(x) f(x) \, dx + 2F(\widehat{v}) \, b(\widehat{v}). \quad (3)$$

Differentiating the overall expected utility of the deviating bidder with respect to its second argument we have

$$\begin{aligned} U_2(v, \widehat{v}) &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) - b(s(\widehat{v})) f(s(\widehat{v})) \frac{ds(\widehat{v})}{d\hat{v}} \\ &\quad + 3b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}} \\ &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) + 4b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}}, \end{aligned}$$

where the second equality follows from the symmetry of the bidding function with $b(\hat{v}) = b(s(\hat{v}))$ and from the definition of $s(\cdot)$ which gives $f(s(\widehat{v}))\frac{ds(\widehat{v})}{d\hat{v}} = -f(\widehat{v})$.

For $b(\cdot)$ to be an equilibrium strategy, it must be optimal for type v to bid $b(v)$, which implies the following necessary condition

$$U_2(v, \hat{v}) \mid_{\hat{v}=v} = 0 \text{ for all } v \in [0, v^{med}]. \quad (4)$$

We must then show that $\max_{\widehat{v}} U(v,\widehat{v}) = U(v,v)$. Note that we only need to show this for $\widehat{v} \leq v^{med}$ by the symmetry of $b(\cdot)$. Since the probability of winning, the payments and the information revealed is exactly the same when bidding $b(\hat{v})$ and $b(s(\hat{v}))$, a deviation to a bid $b(\widehat{v})$ with $\widehat{v} > v^{med}$ is equivalent to deviate to a bid $b(s(\tilde{v}))$ for some $\tilde{v} \leq v^{med}$.
In order to derive $U_2(v,\hat{v})$ we first compute $U_2^P(v,\hat{v})$. A losing bidder who bids $b(\widehat{v})$ correctly infers that the other partner valuation is either smaller than $\tilde{v}$ or larger than $s(\tilde{v})$. She uses this information to update her beliefs, and to set as proposer a price p which maximizes

$$U_{\widehat{v}}(v,p) = \left(v - \frac{p}{2}\right) F_{\widehat{v}}^C(p) + \frac{p}{2}\left(1 - F_{\widehat{v}}^C(p)\right)$$

where

$$F_{\widehat{v}}^C(x) = \begin{cases} \frac{F(x)}{2F(\widehat{v})} & \text{if} \quad x \in [0,\widehat{v}] \\ \frac{F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [\widehat{v}, s(\widehat{v})] \\ \frac{F(x)-F(s(\widehat{v}))+F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [s(\widehat{v}),1]. \end{cases}$$

The differentiation of the proposer's utility gives

$$\frac{d}{dp} U_{\widehat{v}}(v,p) = \begin{cases} (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \leq \widehat{v}, \\ 0 & \text{if} \quad p \in (\widehat{v}, s(\widehat{v})) \\ (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)-1+2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \geq s(\widehat{v}), \end{cases}$$

It is easy to see from the expression above that the optimal price she will set in the pricing stage depends upon $\tilde{v}$. Two cases have to be distinguished depending on the relationship between v and $\tilde{v}$. If $\widehat{v} \leq v$ then the following two inequalities hold:

$$\begin{array}{ll} (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} > 0 & \text{for} \quad p \leq \widehat{v}, \text{ and} \\ (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)-1+2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 & \text{for} \quad p \geq s(\widehat{v}). \end{array}$$

Hence, setting a price in the interval $[\widehat{v}, s(\widehat{v})]$ is optimal, resulting in a utility as proposer of

$$U^P(v,\widehat{v}) = (v - \frac{1}{2}p^{opt})F(\widehat{v}) + \frac{1}{2}p^{opt}(1 - F(s(\widehat{v}))) = vF(\widehat{v}).$$

Since $\widehat{v} \leq v$, we further obtain

$$\lim_{\widehat{v} \nearrow v} U_2^P(v,\widehat{v}) = vf(v).$$

Consider now that $v^{med} \geq \widehat{v} \geq v$. In this case we have

$$(v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)-1+2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 \text{ and } (v-\widehat{v})\frac{f(\widehat{v})}{2F(\widehat{v})} - \frac{F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} \leq 0.$$

Hence, the optimal price is $p^{opt} \leq \widehat{v}$. Consequently, the utility that the proposer accrues equals

$$\begin{aligned} U^P(v,\widehat{v}) &= (v-\frac{1}{2}p^{opt})F\left(p^{opt}\right)+\frac{1}{2}p^{opt}\left[F\left(\widehat{v}\right)-F\left(p^{opt}\right)+1-F\left(s(\tilde{v})\right)\right] \\ &= p^{opt}\left(F\left(\widehat{v}\right)-F\left(p^{opt}\right)\right)+vF\left(p^{opt}\right), \end{aligned}$$

where p^{opt} satisfies the following FOC

$$\left(v-p^{opt}\right)f\left(p^{opt}\right)-F\left(p^{opt}\right)+F\left(\widehat{v}\right)=0. \tag{5}$$

Since $\lim_{\widehat{v}\searrow v} p^{opt}=v$, we obtain[17]

$$\begin{aligned} \lim_{\widehat{v}\searrow v} U_2^P(v,\widehat{v}) &= \lim_{\widehat{v}\searrow v}\left[F\left(\widehat{v}\right)-F\left(p^{opt}\right)-p^{opt}f\left(p^{opt}\right)+vf\left(p^{opt}\right)\right]\frac{d}{d\widehat{v}}p^{opt} \\ &\quad +p^{opt}f\left(\widehat{v}\right) \\ &= \lim_{\widehat{v}\searrow v} p^{opt}f\left(\widehat{v}\right) \\ &= vf\left(v\right). \end{aligned}$$

The analysis above ensures that for all $\hat{v} \leq v^{med}$ we obtain $U_2^P(v,\hat{v})\mid_{\hat{v}=v}= vf(v)$.

We now compute $U_2^C(v,\hat{v})$. As the proposer always sets a price that equals her valuation, the expected utility as chooser will be

$$U^C(v,\widehat{v})=\begin{cases} \int_{\widehat{v}}^{v}\left(v-\frac{x}{2}\right)f(x)\,dx+\int_{v}^{s(\widehat{v})}\frac{x}{2}f(x)\,dx & \text{if } \ \widehat{v}\leq v \\ \int_{\widehat{v}}^{s(\widehat{v})}\frac{x}{2}f(x)\,dx & \text{if } \ \widehat{v}\geq v. \end{cases}$$

Differentiating the chooser's utility with respect to $\tilde{v}$ yields

$$U_2^C(v,\widehat{v})=\begin{cases} \left(\frac{\widehat{v}-s(\widehat{v})-2v}{2}\right)f(\widehat{v}) & \text{if } \ \widehat{v}\leq v \\ -\left(\frac{\widehat{v}+s(\widehat{v})}{2}\right)f(\widehat{v}) & \text{if } \ \widehat{v}\geq v, \end{cases}$$

Evaluating $U_2^C(v,\widehat{v})$ at $v=\widehat{v}$ gives

$$U_2^C(v,v)=-\left(\frac{s(v)+v}{2}\right)f(v).$$

Putting these results together the first order condition (4) becomes

$$-\left(\frac{s(v)-v}{2}\right)f(v)+2F(v)\frac{d}{dv}b(v)+4b(v)f(v)=0.$$

[17] To make sure that p^{opt} is uniquely defined (given v and $\widehat{v}$) we need the hazard rate condition to hold. For the complete argument see McAfee [1992].

For (4) to hold at $v = 0$ we need that $b(0) = \frac{1}{8}$. Thus a differentiable equilibrium has to be a solution of the boundary value problem:

$$\begin{aligned} b(0) &= \frac{1}{8}, \\ -\frac{s(v)-v}{2} f(v) + 2F(v) \frac{d}{dv} b(v) + 4b(v) f(v) &= 0 \text{ for } v \leq v^{med}. \end{aligned}$$

The differential equation can be written as

$$\frac{s(v)-v}{2} F(v) f(v) = \frac{d}{dv} \left(2b(v) F^2(v)\right),$$

and then integrated to obtain (2).

We next show that bidding $b(v)$ indeed does not result in a lower payoff (for a bidder with valuation v) than bidding $b(\widehat{v})$ with $\widehat{v} \leq v^{med}$. Observe first that for $\widehat{v} \leq v$ we have that

$$U_2(v, \widehat{v}) = -\frac{s(\widehat{v}) - \widehat{v}}{2} f(\widehat{v}) + 2F(\widehat{v}) \frac{d}{dv} b(\widehat{v}) + 4f(\widehat{v}) b(\widehat{v})$$

with $U_{2,1}(v, \widehat{v}) = 0$, and therefore a bid of $b(\widehat{v})$ does not give larger payoffs than a bid of $b(v)$. Assume next that $\widehat{v} \geq v$. To show the optimality of $b(v)$ in this case, it is sufficient to show that $U_{2,1}(v, \widehat{v}) \geq 0$ for all $v \leq \widehat{v} \leq v^{med}$ (see McAfee [1992]). Using the abbreviation $p_v := \frac{\partial}{\partial v} p^{opt}$ we have that

$$\begin{aligned} U_{2,1}(v, \widehat{v}) &= U^P_{2,1}(v, \widehat{v}) \\ &= p_v f(\widehat{v}). \end{aligned}$$

From (5) we obtain that

$$p_v = -\frac{f(p^{opt})}{(v - p^{opt}) f'(p^{opt}) - 2f(p^{opt})},$$

which shows that

$$\begin{aligned} U_{2,1}(v, \widehat{v}) &= \frac{f(p^{opt}) f(\widehat{v})}{2f(p^{opt}) - (v - p^{opt}) f'(p^{opt})} \\ &= \frac{f(p^{opt}) f(\widehat{v})}{2f(p^{opt}) - \frac{F(p^{opt}) - F(\widehat{v})}{f(p^{opt})} f'(p^{opt})} \\ &= \frac{f(\widehat{v})}{\left.\frac{d}{dp}\right|_{p=p^{opt}} \left(p + \frac{F(p)-F(\widehat{v})}{f(p)}\right)} \geq 0. \end{aligned}$$

The second equality follows form the optimality of p^{opt} (recall (5)) and the last inequality from the hazard rate conditions (1).

We can hence conclude that the candidate equilibrium bid indeed maximizes the expected utility in (3). ■

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