Terms of Trade, Business Cycles and Tobin's q in Developing Open Economies

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Abstract

This paper examines the relationship of business cycles, the terms of trade and Tobin's q using a three-sector dynamic stochastic general equilibrium model for a small open economy. Results show that terms of trade shocks account for half of actual volatility of GDP and stock market indices for developing countries. The model fails to replicate the actual volatility of stock market indices for developed economies.

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1 Introduction

In the Real Business Cycle (RBC) approach to developing open economics, large and recurrent fluctuations in terms of trade are widely viewed as an important driving force of business cycles. The terms of trade affect the industrial countries mainly by raising the relative price of energy. In developing countries this effect primarily affects the price of imported capital goods.

There is high volatility in the share markets of emerging economies over the long term. In the beginning of the 90's, RBC models for developing countries did not focus on the share market, given that the size of the market was small. A comparison of these series during the last decade shows that the stock market capitalization for developing countries has greatly increased in their value relative to GDP.

Tobin's q is used to account for the price of replacement of the capital. Tobin's q is the ratio of the market value of an additional unit of capital to its replacement cost. Hayashi (1982) shows that the average and the marginal values of capital are identical under neoclassical assumptions. Tobin and Brainard (1977) conjecture that the average and marginal values respond in similar ways to shocks, and the empirical evidence in Abel and Blanchard (1986) appears to support the conjecture. The stock market indices are the value for the Tobin's q used in this research.

This paper links stock market dynamics and the RBC model for developing countries. As usual in RBC literature, we focus on volatility, correlation, and cross-correlation between the series in the real data and compare these with the statistics from data generated by the model.

This paper addresses the following questions:

- Do the dynamics of share prices for RBC correspond to the dynamics and values for the stock market indices for a group of developing countries?

- How much of the variability of changes in GDP and stock market indices are implied by the terms of trade shocks?

To answer these questions this paper does an examination of the relationship between terms of trade shocks and business cycles from the perspective of a dynamic stochastic general equilibrium framework. It borrows numerical solution methods from real business cycle theory to compute stochastic processes of three sector inter-temporal model of a small open economy. It compares various features of the model's business cycles with actual business cycles.

This paper extends Mendoza (1995) with an analysis of Tobin's q. The paper does not use the endogenous discount factor; it is an exogenous variable. The data set is characterized by different frequencies, time and countries. This study uses different numerical algorithms to find the solution, and incorporates a richer sequence of the shocks.

This study follows Obstfeld and Roggff (1996) and Romer (1996) to introduce the Tobin's q theory and its implication. Both text books present the simplest situation with only one capital and one good in a partial equilibrium model in the production side. The model used here is a general equilibrium model, and the value of q is affected by the consumer preferences in each period.

To solve the model, this study follows Duffy and McNelis (2001). For assuring accuracy in the simulations, it uses the Den Haan and Marcet (1994) statistic. This study also uses two different techniques: the linear quadratic approximation method, and parameterized expectations algorithm, with a neural network approach. It supports a complex combination of the state variables. The second algorithm helps us to approximate the volatility of Tobin's q for developing economies.

The paper has the following order: the next section analyzes the real side of the economy and shows the empirical regularities and some of the stylized facts. Section three develops the model and some of its variations and the equilibrium and dynamics of the economy. The steady state and parameters are presented in section four. Section five is a presentation of both solution algorithms. Benchmark simulations are in section six. Section seven examines the robustness of the model. The last section concludes.

2 Empirical Regularities

This section studies key properties of the data, for Gross Domestic Product (GDP), Stock Market Indices (SMI), and Terms of Trade (TOT) , and their relations.

Data come from three different sources for each country. The First is from The World Bank, World Development Indicators (2001), series of Stock Market Capitalization are from this database.

The Second is The International Financial Statistics (IFS), CD-Rom, December 2001, for the series of GDP, of export prices (Px) and import prices (Pm). The series of TOT is a simple construction based on this data. The data are in constant US dollars.

The other database is from Morgan Stanley Capital International (MSCI); this database is available from 1970 but not for all countries. This paper constructs series of stock market indices, SMI, using the values of the last day of information for each quarter and this data comes in index values.

This research is limited by two restrictions: the first is the large amount of series to consider in this sample, the second is the amount of the countries.

The sample is between 1990 and 2000, for most of the countries. But a few countries have fewer observations. They are: Belgium, Singapore and Thailand, that begin in 1993; Argentina that finishes in 1998, and Poland that begins in 1995.

All off these limitations are due to missing data.

The number of the countries is limited to 24. Some of them are not developing economies; they serve as a comparison.

These countries were put in three different groups: group of seven largest industrialized countries (G7), others industrialized countries (IC's), and developing countries (DC's).

The list of the countries is:

G7: Canada, France, Germany, Italy, Japan, United Kingdom, and United States.

IC's: Australia, Belgium, Denmark, Finland, Netherlands, Norway, Sweden, and Switzerland.

DC's: Argentina, Brazil, Hong Kong, Korea, New Zealand, Poland, Singapore, Thailand, and Turkey.

Table 1:

Stock Markets Capitalization				
	$ Millons		% of GDP	
	1990	**1999**	**1990**	**1999**
Group of Seven				
Canada	241,920	800,914	42.2	126.1
France	314,384	1,475,457	25.9	103.0
Germany	355,073	1,432,190	*22.2*	67.8
Italy	148,766	728,273	13.5	62.2
Japan	2,917,679	4,546,937	98.2	104.6
United Kingdom	848,866	2,933,280	85.9	203.4
United States	3,059,434	16,635,114	53.2	181.8
mean	**1,126,589**	**4,078,881**	**48.7**	**121.3**
Industrialized Countries				
Australia	108,879	427,683	35.1	105.9
Belgium	65,449	184,942	33.1	74.5
Denmark	39,063	105,293	29.3	60.4
Finland	22,721	349,409	16.6	269.5
Netherlands	119,825	695,209	*40.5*	176.6
Norway	26,130	63,696	22.6	41.6
Sweden	97,929	373,278	41.2	156.4
Switzerland	160,044	693,127	70.1	268.1
mean	**80,005**	**361,580**	**36.1**	**144.1**
Developing Countries				
Argentina	3,268	166,068	2.3	29.6
Brazil	16,354	226,152	3.5	30.3
Hong Kong	83,397	609,090	111.5	383.2
Korea, Rep.	110,594	148,649	43.8	75.8
New Zealand	8,835	28,352	20.5	51.9
Poland	144	31,279	*0.2*	19.1
Singapore	34,308	198,407	93.6	233.6
Thailand	23,896	29,489	28.0	46.9
Turkey	19,065	69,659	12.6	60.7
mean	**33,318**	**167,461**	**35.1**	**103.5**
Other developing Countries				
Chile	13,645	60,401	45.0	101.1
Mexico	32,725	125,204	12.5	31.8
Peru	812	10,562	3.1	25.8
Israel	3,324	64,081	6.3	63.3
Saudi Arabia	48,213	67,171	40.8	43.4
Egypt, Arab Rep.	1,765	28,741	4.1	36.8
India	38,567	148,064	12.2	41.3
Indonesia	8,081	26,834	7.1	45.0
Philippines	5,927	51,554	13.4	62.8
mean	**17,007**	**64,735**	**16.0**	**50.1**
Mean Developing C.	**25,162**	**116,098**	**25.6**	**76.8**
2001 World Development Indicators, World Bank				

Table 1 contains information on market capitalization; this table is a summary of one of the tables given in The World Bank data set. This table pins down the growth of the Stock Market Capitalization, in developing countries around of the world. To clarify it, Table 1 also presents other developing countries. In the comparison, the average of the Stock Market Capitalization as percentage of GDP increased more or less in the same amount for G7 and the rest of the countries in the data set. The changes in the stock market value from 1990 to 1999 are bigger for developing countries than for the G7. This situation illustrates the importance of the stock market value for developing countries.

Table 2:

GDP and Terms of Trade								
	gdp		tot			cross correlation		
	σ_y	$\rho_{(1)}$	σ_{tot}	σ_{tot}/σ_y	$\rho_{(1)}$	$\rho_{(y,tot)}$	shift	Lags
Group of Seven								
Canada	1.132	0.781	2.082	1.840	0.864	0.253	0.488	-2
France	0.697	0.745	1.094	1.569	0.525	-0.195	0.322	9
Germany	2.705	0.726	1.871	0.692	0.679	-0.178	0.404	-5
Italy	0.747	0.687	2.643	3.537	0.591	-0.263	0.450	-6
Japan	1.112	0.498	4.013	3.609	0.833	-0.676	-0.676	0
United Kingdor	1.088	0.844	1.317	1.211	0.453	-0.253	-0.496	-4
United States	0.759	0.655	1.699	2.238	0.650	0.156	-0.361	-10
mean	**1.177**	**0.705**	**2.103**	**2.099**	**0.656**	**-0.165**		
Industrialized Countries								
Australia	1.085	0.751	3.258	3.002	0.802	0.292	0.409	-8
Belgium	0.752	0.489	1.067	1.419	0.543	-0.240	0.525	-3
Denmark	1.706	-0.334	1.151	0.675	0.508	-0.080	-0.310	-3
Finland	3.092	0.052	2.828	0.914	0.803	0.242	-0.401	-10
Netherlands	0.580	0.704	1.811	3.122	0.285	-0.410	-0.433	-1
Norway	3.724	0.359	8.334	2.238	0.602	0.764	0.764	0
Sweden	4.327	-0.290	1.702	0.393	0.695	0.146	0.386	-1
Switzerland	0.625	0.600	1.622	2.594	0.335	-0.025	-0.405	10
mean	**1.986**	**0.291**	**2.722**	**1.795**	**0.572**	**0.086**		
Developing Countries								
Argentina	4.477	0.073	23.440	5.236	0.604	0.356	0.718	-2
Brazil	66.386	0.917	14.149	0.213	0.355	-0.247	0.596	9
Hong Kong	5.011	0.175	0.872	0.174	0.799	-0.125	-0.233	-1
Korea	6.939	-0.186	4.633	0.668	0.745	-0.073	0.292	5
New Zealand	1.877	0.792	2.177	1.160	0.533	0.220	0.520	3
Poland	5.954	0.015	7.901	1.327	-0.108	0.548	0.604	-4
Singapore	6.714	-0.244	1.042	0.155	0.326	-0.022	-0.529	-2
Thailand	5.541	0.630	2.611	0.471	0.292	-0.072	0.319	-6
Turkey	16.393	0.126	4.355	0.266	0.442	-0.229	-0.324	3
mean*	**5.216**	**0.179**	**6.097**	**1.313**	**0.456**	**0.119**		

Source: IFS, for GDP and TOT. The series are Hpfilter, and GDP is log.

σ_y = Volatility of GDP

σ_{tot}/σ_y =Relative Volatility tot to y

σ_{tot} = Volatility of ToT

$\rho_{(y,tot)}$ =Cross Correlation i,j

$\rho_{(1)}$ = Autocorrelation

shift = max abs cross correlation in 10 lag and 10 lead periods

* =without Brazil and Turkey

Table 3:

GDP and Stock Market Indices	gdp		stock			cross correlation		
	σ_y	$\rho_{(1)}$	σ_{smi}	σ_{smi}/σ_y	$\rho_{(1)}$	$\rho_{(y,smi)}$	shift	Lags
Group of Seven								
Canada	1.132	0.781	9.567	8.455	0.567	0.104	0.284	-9
France	0.697	0.745	7.407	10.627	0.257	-0.006	0.304	10
Germany	2.705	0.726	9.148	3.381	0.380	-0.302	-0.343	2
Italy	0.747	0.687	12.913	17.279	0.598	-0.069	-0.404	-4
Japan	1.112	0.498	16.194	14.564	0.694	-0.173	-0.589	-6
United Kingdom	1.088	0.844	7.913	7.277	0.524	-0.023	0.379	-6
United States	0.759	0.655	7.823	10.304	0.536	-0.020	-0.291	-2
mean	**1.177**	**0.705**	**10.138**	**10.269**	**0.508**	**-0.070**		
Industrialized Countries								
Australia	1.085	0.751	7.447	6.862	0.517	0.073	0.498	-6
Belgium	0.752	0.489	9.827	13.072	0.393	-0.340	-0.450	-9
Denmark	1.706	-0.334	8.974	5.260	0.654	0.210	0.269	2
Finland	3.092	0.052	22.835	7.385	0.644	0.275	0.319	2
Netherlands	0.580	0.704	6.670	11.497	0.396	-0.096	0.476	8
Norway	3.724	0.359	14.631	3.929	0.762	0.234	-0.346	-8
Sweden	4.327	-0.290	12.775	2.952	0.496	0.249	0.249	0
Switzerland	0.625	0.600	9.065	14.493	0.456	-0.087	-0.371	-5
mean	**1.986**	**0.291**	**11.528**	**8.181**	**0.540**	**0.065**		
Developing Countries								
Argentina	4.477	0.073	28.839	6.442	0.693	0.457	0.457	0
Brazil	66.386	0.917	22.755	0.343	0.419	0.240	0.240	0
Hong Kong	5.011	0.175	17.068	3.406	0.587	0.216	0.357	2
Korea	6.939	-0.186	36.557	5.268	0.708	0.173	0.339	2
New Zealand	1.877	0.792	15.172	8.085	0.738	0.809	0.809	0
Poland	5.954	0.015	17.076	2.868	0.337	0.006	-0.370	-4
Singapore	6.714	-0.244	23.128	3.445	0.703	0.117	-0.379	-9
Thailand	5.541	0.630	41.317	7.456	0.660	0.650	0.673	1
Turkey	16.393	0.126	30.995	1.891	0.473	0.041	0.231	5
mean*	**5.216**	**0.179**	**25.594**	**5.281**	**0.632**	**0.347**		

Source: IFS, for GDP and MSCI for Stock. The seires are Hpfilter, log.

σ_y = Volatility of GDP σ_{tot}/σ_y =Relative Volatility tot to y

σ_{tot} = Volatility of ToT $\rho_{(y,tot)}$ =Cross Correlation i,j

$\rho_{(1)}$ = Autocorrelation

shift = max abs cross correlation in 10 lag and 10 lead periods

The business cycles indicators for TOT, SMI and GDP are presented in Tables 2 to 4. Moments reported there correspond to cyclical components of The Hodrick-Prescott Filter (HP) method applied to the data. The last two series are in logarithms, and all series uses a number of 1600 for the smoothing parameter corresponding to quarterly data. Reports are given for standard deviations in percentage, first order autocorrelations, contemporaneous correlations, maximum cross correlation (shift), and when that happens (Lags), of GDP, TOT and SMI.

The volatility of GDP in Brazil and Turkey are taken off for the calculation of the mean when they are compared with the simulations because the devaluation of their currency inside the sample period affects the real values of output.

The statistical evidence for the cycles in G7, IC's, and DC's presents some facts:

1. The shocks to TOT are larger than the volatility of GDP for G7 except for Germany. The shocks to TOT are smaller than the volatility of GDP for many DC's. And there is some mix for IC's.

2. The persistence of TOT is almost equal across the groups.

3. The cross correlation between TOT and GDP is small for almost all countries and its sign is different across countries.

4. The volatility of SMI, is larger in DC's than the others groups. The volatility of SMI is also larger than GDP. But the relative volatility between SMI and GDP is larger in G7 than IC's and from this group to DC's. That is a direct implication of the size of volatility of GDP for each country.

5. The cross correlation of SMI with GDP is small for almost all countries but has different signs: negative in G7 and positive in DC's.

6. The cross correlation between TOT and SMI does not have a consistent

pattern across countries though it exhibits a positive sign for more countries.

Table 4:

	SMI and TOT cross correlation			P-Values	
	$\rho_{(tot,smi)}$	shift	Lags	tot	stock
Group of Seven					
Canada	0.552	0.552	0	0.191	0.657
France	0.336	0.336	0	0.221	0.614
Germany	0.167	0.232	-1	0.284	0.035
Italy	0.275	0.323	-1	0.272	0.554
Japan	0.197	0.648	5	0.067 *	0.230
United Kingdom	0.124	-0.488	9	0.590	0.758
United States	0.467	0.467	0	0.153	0.328
mean	**0.302**				
Industrialized Countries					
Australia	0.166	0.586	-3	0.128	0.606
Belgium	0.610	0.610	0	0.125	0.285
Denmark	-0.480	-0.480	0	0.791	0.350
Finland	0.039	0.414	-4	0.829	0.287
Netherlands	0.116	-0.330	-9	0.356	0.926
Norway	0.273	-0.334	8	0.001 *	0.139
Sweden	0.174	0.412	-4	0.915	0.146
Switzerland	0.030	-0.438	1	0.312	0.670
mean	**0.116**				
Developing Countries					
Argentina	0.503	0.613	-1	0.000 *	0.097 *
Brazil	0.124	-0.279	-6	0.739	0.426
Hong Kong	-0.140	0.205	10	0.187	0.001 *
Korea	0.489	0.700	2	0.001 *	0.000 *
New Zealand	0.279	0.382	4	0.002 *	0.019 *
Poland	-0.574	-0.574	0	0.000 *	0.107
Singapore	-0.440	-0.504	2	0.246	0.874
Thailand	0.032	-0.296	-3	0.228	0.000 *
Turkey	-0.104	-0.117	-7	0.170	0.788
mean	**0.021**				

Source: IFS, for TOT and MSCI for SMI. The series are Hpfilter, SMI is in log.

$\rho_{(tot,smi)}$ = Cross Correlation tot and smi

shift = max abs cross correlation in 10 lag and 10 lead periods

P- Values are for all lags toghether in each variable with 3 lags

$GDP_t=a_1+b_1*GDP_{t-1}+...+b_n*GDP_{t-n}+c_1*TOT_{t-1}+...+c_n*TOT_{t-n}+$
$D_1*Stock_{t-1}+...+d_n*Stock_{t-n}+error_t,$

In Table 4, there is also a test with the p-value of the Wald test to check if the lags of TOT or SMI affect current GDP, on a VAR model with these three series.

P-values are for three lags together for each variable, TOT and SMI, are significally different from zero,

$$
\begin{aligned}
GDP_t &= a_1 + b_1 * GDP_{t-1} + ... + b_n * GDP_{t-n} + c_1 * TOT_{t-1} + ... \\
&\quad + c_n * TOT_{t-n} + d_1 * SMI_{t-1} + ... + d_n * SMI_{t-n} + \varepsilon_t
\end{aligned}
$$

The result shows that for many developing countries the terms of trade and stock market indices shocks explain part of the GDP volatility, but it does not happen for G7 and industrial countries.

3 Model

In this economy, there is a representative agent who lives forever. The preferences are given by maximizing the present value of expected utility given by a composite good, C_s, leisure time, L_s, and subjective exogenous discount factor, β.

$$
\max U(C) = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(C_s \left(L_s\right)^{\omega}\right)^{1-\gamma}}{1-\gamma} \right] \tag{1}
$$

$$
L_s = 1 - \bar{l}^x - \bar{l}^f - l_s^n
$$

where E_t is the expectations operator, conditional on information available at time t. In each period of the utility function, leisure time enters in unitary - elasticity form where ω governs labor supply elasticity. Leisure time is equal to total time minus the time spend in each production sectors; constant in both tradeable sector: exportable, $\bar{l}^x$, and importable, $\bar{l}^f$, and variable in nontradeable sector, l_s^n.

The composite good has the following function:

$$C_t = \left[(C_t^{\tau})^{-\mu} + n_t^{-\mu} \right]^{\frac{1}{-\mu}} \tag{2}$$

$$C_t^{\tau} = x_t^{\alpha} f_t^{1-\alpha} \tag{3}$$

$$\gamma > 1 \ , \ \mu > -1, \ 0 > \beta, \alpha > 1, \omega > 0$$

Tradeable, C_t^{τ}, and nontradeable, n_t, goods are represented in constant-elasticity of substitution (CES) form and $\frac{1}{1+\mu}$ is its elasticity of substitution. Tradeable are expressed in Cobb-Douglas unitary-elasticity form with α is the share of exportable goods, x_t, and $(1-\alpha)$ is the share in importable goods, f_t. The intertemporal elastiticy of substitution in aggregate consumption is $\frac{1}{\gamma}$.

The output is given by the aggregate production function for each sector, exports, (Y_t^x), imports, $\left(Y_t^f\right)$, and non-traded, (Y_t^n):

$$Y_t^x = \varepsilon_t^x A^x \left(K_t^x\right)^{1-\alpha_x} \left(\bar{l}^x\right)^{\alpha_x} \tag{4}$$

$$Y_t^f = \varepsilon_t^f A^f \left(K_t^f\right)^{1-\alpha_f} \left(\bar{l}^f\right)^{\alpha_f} \tag{5}$$

$$Y_t^n = \varepsilon_t^n A^n \left(\bar{K}^n\right)^{1-\alpha_n} \left(l_t^n\right)^{\alpha_n} \tag{6}$$

$$K_t = K_t^x + K_t^f \tag{7}$$

Production for each sector follows a Cobb-Douglas technology, but with differences in the shares for capital; the tradeable sector has a constant labor supply.

Nontradeable sector has constant capital and variable labor supply. K_t^x, K_t^f and $\bar{K}^n$ are capital stocks; since trade sector capital is homogenous, the capital for each period is the sum of the capital in both sectors. A^x, A^f and A^n are total factor productivity effects. ε_t^x, ε_t^f and ε_t^n are random shocks for each function of production.

$$\varepsilon_t^p P^x x_t + f_t + I_t + \psi_t = \varepsilon_t^p P^x Y_t^x + Y_t^f \tag{8}$$

$$P_t^n n_t = P_t^n Y_t^n \tag{9}$$

$$K_{t+1} = (1-\delta) K_t + I_t \tag{10}$$

Equation (8) and (9) are the household budget constraint. All the prices are expressed in term of importables (as numeraire), so P^x is world relative price of exportables and P_t^n is the endogenous domestic relative price of nontradeables. ε_t^p is an exogenous shock to exportable goods and also the shock to the terms of trade.

The equation (10) is the change of capital stock between dates t and $t+1$ that evolves with net investment, I_t.

Capital has a depreciation rate, δ. The household-firm pays a deadweight installation cost of capital following a quadratic adjustment cost function,

$$\psi_t = \frac{\chi}{2K_t} (I_t)^2 \quad "OR" \tag{11}$$

$$\text{or} \;\; \psi_t = \frac{\chi}{2} (I_t)^2 \qquad "M" \tag{12}$$

Two different functions are considered: The first uses Obstfeld and Roggoff (1996), "OR". The second uses Mendoza (1995), "M". Using "OR" presents the advantage that the marginal and average Tobin's q are equal. However the value of adjustment costs are relatively small.

This specific cost function shows an increasing marginal cost of investment; and captures the observation that a faster pace of change requires a greater than proportional rise in installation costs. The representative agent-firm pays this cost. With that, it is possible to distinguish a net investment value, I_t, and a gross investment that is the sum of net investment and the cost function, $I_t + \psi_t$.

$$\varepsilon_t^i = \exp\left(\upsilon_t^i\right)$$

$$\upsilon_t^i = (1-\rho)\,\bar{\upsilon} + \rho\upsilon_{t-1}^i + \vartheta_t^i \qquad\qquad i = p, x, f, n$$

$$\vartheta_t \backsim iid \quad N\left(0, \Sigma\right),$$

The random shocks ε_t^p, ε_t^x, ε_t^f and ε_t^n are assumed to follow first order Markov Processes. The random variable υ_t^i follows an autoregressive process AR(1).

For simplicity the model presented here does not incorporate international assets and thus does not have capital mobility. The openness of financial markets with a new international asset which the household can borrow or lend at the international fixed interest rate requires a new endogenous variable to solve the problem of steady state. One approach uses an endogenous discount factor function, as Mendoza (1991), Smith-Grohé and Uribe (2001), or Epstein and Zin (1989). Those functions imply at least another Euler equation, and they use many more parameters for the parameterized expectation algorithms.

3.1 Equilibrium and Dynamic Programming Formulations

The competitive equilibrium is defined by stochastic processes $\{ I_t,\ K_{t+1},\ x_t,\ f_t,\ n_t,\ l_t^n,\ p_t^n,\ K_t^x,\ K_t^f \}_{t=0}^{\infty}$, where the household optimizes the expected value of utility subject to the budget constraint in the tradeable sector, the market clears in non-traded sector, and the two restrictions on capital, equations (7) and (10), are satisfied.

The Lagrangian for solving the model is:

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{1}{1-\gamma} \left(\left(x_s^{\alpha} f_s^{1-\alpha} \right)^{-\mu} + n_s^{-\mu} \right)^{\frac{(1-\gamma)}{-\mu}} \left(1 - \bar{l}^x - \bar{l}^f - l_s^n \right)^{\omega(1-\gamma)} \right. \right.$$

$$+ \phi_s \left(\varepsilon_s^n A^n \left(\bar{K}^n \right)^{1-\alpha_n} \left(l_s^n \right)^{\alpha_n} - n_s \right) +$$

$$\theta_s \left(K_s - K_s^x - K_s^f \right) - q_s \left(K_{s+1} - (1-\delta) K_s - I_s \right) +$$

$$\lambda_s \left(\ \varepsilon_s^p P_s^x \varepsilon_s^x A^x \left(K_s^x \right)^{1-\alpha_x} \left(\bar{l}^x \right)^{\alpha_x} - I_s - \varepsilon_s^p P_s^x x_s \right.$$

$$\left. \left. + \varepsilon_t^f A^f \left(K_t^f \right)^{1-\alpha_f} \left(\bar{l}^f \right)^{\alpha_f} - f_s - \frac{\chi}{2K_s} \left(I_s \right)^2 \right) \quad \right) \right]$$

The variable λ_t is the familiar Lagrangian multiplier representing the marginal utility of wealth. The term q_t, known as Tobin's q, represents the Lagrange multiplier for the evolution of capital - it is the "shadow price" for new capital.

Maximizing the Lagrangian with respect to I_t, K_{t+1}, x_t, f_t, n_t, l_t^n, K_t^x, K_t^f, q_t,

θ_t, λ_t, ϕ_t, yields the following first order conditions (Using "OR"):

$$I_t : q_t = \lambda_t \left(\frac{\chi I_t}{K_t} + 1 \right) \tag{13}$$

$$K_{t+1} : q_t = \beta E_t \left[\theta_{t+1} + (1-\delta) q_{t+1} + \frac{\chi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \lambda_{t+1} \right] \tag{14}$$

$$x_t : \left(\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)-\mu}{\mu}} \left(1 - \bar{l}^x - \bar{l}^f - l_t^n \right)^{\omega(1-\gamma)}$$
$$\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} \frac{\alpha}{x_t} = \lambda_t \varepsilon_t^p P_t^x \tag{15}$$

$$f_t : \left(\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)-\mu}{\mu}} \left(1 - \bar{l}^x - \bar{l}^f - l_t^n \right)^{\omega(1-\gamma)}$$
$$\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} \frac{(1-\alpha)}{f_t} = \lambda_t \tag{16}$$

$$l_t^n : \left(\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)}{\mu}}$$
$$\left(1 - \bar{l}^x - \bar{l}^f - l_t^n \right)^{\omega(1-\gamma)-1} \omega = \phi_t \alpha_n \varepsilon_t^n A^n \left(\bar{K}^n \right)^{1-\alpha_n} (l_t^n)^{\alpha_n - 1} \tag{17}$$

$$n_t : \left(\left(x_t^\alpha f_t^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)-\mu}{\mu}} \left(1 - \bar{l}^x - \bar{l}^f - l_t^n \right)^{\omega(1-\gamma)} n_t^{-\mu-1} = \phi_t \tag{18}$$

$$K_t^x : \theta_t = \lambda_t (1-\alpha_x) \varepsilon_t^p \varepsilon_t^x P_t^x A^x (K_t^x)^{-\alpha_x} \left(\bar{l}^x \right)^{\alpha_x} \tag{19}$$

$$K_t^f : \theta_t = \lambda_t (1-\alpha_f) \varepsilon_t^f A^f \left(K_t^f \right)^{-\alpha_f} \left(\bar{l}^f \right)^{\alpha_f} \tag{20}$$

$$\lambda_t : \varepsilon_t^p P_t^x x_t + f_t + I_t + \frac{\chi}{2K_t} (I_t)^2 = \varepsilon_t^p P_t^x Y_t^x + Y_t^f \tag{21}$$

$$\phi_t : n_t = \varepsilon_t^n P_t^n A^n \left(\bar{K}^n \right)^{1-\alpha_n} (l_t^n)^{\alpha_n} \tag{22}$$

$$\theta_t : K_t = K_t^x + K_t^f \tag{23}$$

$$q_t : K_{t+1} = (1-\delta) K_t + I_t \tag{24}$$

The market clearing conditions are as follow: first, the Current Account is in equilibrium and equal to zero for each period. By definition it gives a restriction on the total resources available for the country. In this economy there are no international bonds, so the current account implies the value of exported, (X_t), is equal to the value of imported goods, (M_t). Second, home production is equal to home consumption for non-traded goods.

$$CA_t = 0 = \varepsilon_t^x P^x \left(Y_t^x - X_t\right) - \left(Y_t^f - F_t\right) \tag{25}$$

$$Y_t^n = n_t \tag{26}$$

The first order conditions combining with market clearing conditions and production functions can be re-expressed as:

$$x_t = \frac{\alpha}{(1-\alpha)\,\varepsilon_t^x P_t^x} f_t \tag{27}$$

$$n_t = \left(\frac{\alpha_n\left(1 - \bar{l}^x - \bar{l}^f - l_t^n\right) - l_t^n \omega}{\omega l_t^n}\right)^{\frac{1}{\mu}} x_t^{\alpha} f_t^{1-\alpha} \tag{28}$$

$$\varepsilon_t^p P_t^x Y_t^x + Y_t^f = \frac{f_t}{(1-\alpha)} + \left(1 - \frac{\lambda_t}{q_t}\right)\frac{K_t}{\chi} + \frac{K_t}{2\chi}\left(1 - \frac{\lambda_t}{q_t}\right)^2 \tag{29}$$

$$\lambda_t = \left(\left((x_t)^{\alpha}(f_t)^{1-\alpha}\right)^{-\mu} + n_t^{-\mu}\right)^{\frac{(\gamma-1)-\mu}{\mu}} \frac{(1-\alpha)\left(1 - \bar{l}^x - \bar{l}^f - l_t^n\right)^{\omega(1-\gamma)}}{f_t\left((x_t)^{\alpha}(f_t)^{1-\alpha}\right)^{\mu}} \tag{30}$$

$$Y_t^x = \varepsilon_t^x A^x \left(K_t^x\right)^{1-\alpha_x} \left(\bar{l}^x\right)^{\alpha_x} \tag{31}$$

$$Y_t^f = \varepsilon_t^f A^f \left(K_t^f\right)^{1-\alpha_f} \left(\bar{l}^f\right)^{\alpha_f} \tag{32}$$

$$n_t = \varepsilon_t^n A^n \left(\bar{K}^n\right)^{1-\alpha_n} \left(l_t^n\right)^{\alpha_n} \tag{33}$$

$$\left(K_t^f\right)^{-\alpha_f} \left(\bar{l}^f\right)^{\alpha_f} (1-\alpha_f)\,\varepsilon_t^f A^f = (1-\alpha_x)\,\varepsilon_t^p \varepsilon_t^x P_t^x A^x \left(K_t^x\right)^{-\alpha_x} \left(\bar{l}^x\right)^{\alpha_x} = \frac{\theta_t}{\lambda_t} \tag{34}$$

$$K_t = K_t^x + K_t^f \tag{35}$$

$$K_{t+1} = \left((1-\delta) + \frac{1}{\chi} \left(\frac{q_t}{\lambda_t} - 1 \right) \right) K_t \tag{36}$$

$$p_t^n = \frac{\phi_t}{\lambda_t} = \frac{n_t^{-(\mu+1)} f_t}{\left((x_t)^\alpha (f_t)^{1-\alpha} \right)^{-\mu} (1-\alpha)} \tag{37}$$

$$q_t = \left(1 + \chi \frac{I_t}{K_t} \right) \lambda_t \tag{38}$$

$$q_t = \beta E_t \left[\theta_{t+1} + (1-\delta) q_{t+1} + \frac{\chi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \lambda_{t+1} \right] \tag{39}$$

Equation (38) states that the shadow price of capital equals the marginal cost of investment, including installation costs. The condition can be rewritten as a version of the investment equation posited by Tobin (1969), with the only difference that traditional Tobin's q is that here q is a "nominal" variable, because it is multiplied by marginal value of wealth of the representative agent.

The last equation is the only Euler equation in this model; this condition is an investment Euler equation. The above equation also shows that the solution for q comes from forward-looking stochastic process. It states that at, an optimum for the household-firm, the date t shadow price of an extra unit of capital is the discounted sum of:

1. The capital's marginal product next period.
2. The shadow price of capital on the next date, $t+1$, net of depreciation.
3. The capital's marginal contribution to lower installation costs next period.

The price of non-traded goods adjusts instantaneously to clear the market for non-traded goods.

Thus the model has only one "forward-looking" stochastic Euler equation, which determine q_t. This variable, together with rest of the equations gives the solution for each period with a initial capital for each period and a particular realization of the shocks.

4 Solution Algorithm

Several algorithms are used to solve a model like this.

Mendoza (1995) uses value function iteration and transition probability iterations using discrete grids to approximate the state space. This method is memory intensive and uses a limited number of state variables and few shocks. For this reason this paper uses two other algorithms: first, Linear Quadratic approximation (LQ) and second, the Parameterized Expectation Algorithm (PEA).

The LQ method uses a second order approximation to the steady state. It is necessary to combine the FOC's to reduce the number of equations to equal the number of state variables and the restrictions are limited to linear equations. Linear quadratic methods are widely used in RBC models. The optimal linear decision rule is the same for the deterministic and stochastic versions this is the well-known certainty equivalence property of this algorithm. This gives a good approximation when the shocks are small and the model stays close to its steady state.

The PEA uses a parametric function of the state variables to approximate each expectation term in the Euler equations. This method adjusts these function to minimize the error between the expected value and the ex-post value of this expectation. The advantage of this algorithm is that it permits a richer model with more variables, shocks and no linearity. The limitations are that it is time consuming, in computational terms.

The PEA approximates the forward-looking expectation as non-linear functional forms of the information available at each period. The polynomial approach works well when the model has few parameters and there are few constraints as in Maliar and Maliar (2003). However it is difficult to find the solution when there are many constraints as in this model. For this reason, this study uses a neural network approach, where variables and parameters enter as non linear functions. For this reason the algorithm uses a global search, for the optimization problem.

The rest of this section works with both methods LQ and PEA to solve the model.

4.1 Linear Quadratic Approximation

This subsection summarizes the features for solving the model with linear quadratic approximation, following to McGrattan (1994), Urrutia (1998) and Pacharoni (2000). The optimization problem is simplified by expressing it as a dynamic programming problem in the state space comprised by K_t and the shocks. All variables with hat, $(\hat{\ })$,are the equilibrium values:

Using equations: (34), (35), (31), and (32), production in tradeable sector is:

$$\left(\hat{K}_t^f\right)^{-\alpha_f}\left(\bar{l}^f\right)^{\alpha_f}\varepsilon_t^f A^f = \frac{(1-\alpha_x)}{(1-\alpha_f)}\varepsilon_t^p \varepsilon_t^x P_t^x A^x \left(\hat{K}_t - \hat{K}_t^f\right)^{-\alpha_x}\left(\bar{l}^x\right)^{\alpha_x} \tag{40}$$

$$\hat{K}_t^x = K_t - \hat{K}_t^f \tag{41}$$

$$\hat{Y}_t^x = \varepsilon_t^x A^x \left(\hat{K}_t^x\right)^{1-\alpha_x}\left(\bar{l}^x\right)^{\alpha_x} \tag{42}$$

$$\hat{Y}_t^f = \varepsilon_t^f A^f \left(\hat{K}_t^f\right)^{1-\alpha_f}\left(\bar{l}^f\right)^{\alpha_f} \tag{43}$$

Equilibrium for consumption of exportable goods follows from equation (27):

$$\hat{x}_t = \frac{\alpha}{(1-\alpha)\,\varepsilon_t^x P_t^x} f_t \tag{44}$$

Labor in the non-traded sector solves the next nonlinear equation. This result helps to solve for production and consumption in non tradeable sector, using equations (28) and (33)

$$\varepsilon_t^n A^n \left(\bar{K}^n\right)^{1-\alpha_n}\left(\hat{l}_t^n\right)^{\alpha_n} = \left(\frac{\alpha_n\left(1-\bar{l}^x-\bar{l}^f-\hat{l}_t^n\right)-\hat{l}_t^n\omega}{\omega \hat{l}_t^n}\right)^{\frac{1}{\mu}} \hat{x}_t^{\alpha} f_t^{1-\alpha} \tag{45}$$

$$\hat{n}_t = \varepsilon_t^n A^n \left(\bar{K}^n\right)^{1-\alpha_n}\left(\hat{l}_t^n\right)^{\alpha_n} \tag{46}$$

Investment is a function of total capital in the next period and the current account is equal to zero for each period and gives the optimal allocation of importable goods, as a function of investment. To find it, use equations (29), (30) and (38)

$$f_t = (1-\alpha)\left(\varepsilon_t^p P_t^x Y_t^x + Y_t^f - \left(I_t \frac{K_t}{\chi} + \frac{K_t}{2\chi} I_t^2\right)\right) \tag{47}$$

$$\hat{\lambda}_t = \left(\left(\hat{x}_t^\alpha f_t^{1-\alpha}\right)^{-\mu} + \hat{n}_t^{-\mu}\right)^{\frac{(\gamma-1)-\mu}{\mu}} \frac{(1-\alpha)\left(1-\bar{l}^x-\bar{l}^f-\hat{l}_t^n\right)^{\omega(1-\gamma)}}{f_t\left(\hat{x}_t^\alpha f_t^{1-\alpha}\right)^{\mu}} \tag{48}$$

$$\hat{q}_t = \hat{\lambda}_t \left(\frac{\chi I_t}{K_t} + 1\right) \tag{49}$$

$$\varepsilon_t^i = \exp\left(\upsilon_t^i\right) \qquad\qquad i = p, x, f, n \tag{50}$$

The price for non-traded goods is found in equilibrium for each period endogenously using equation (37),

$$\hat{p}_t^n = \frac{\hat{n}_t^{-(\mu+1)} f_t}{\left(\hat{x}_t^\alpha f_t^{1-\alpha}\right)^{-\mu}(1-\alpha)} \tag{51}$$

The problem of the maximization can be written as a Bellman equation:

$$V\left(K_t, \vartheta_t^p, \vartheta_t^x, \vartheta_t^f, \vartheta_t^n\right) = \max_{\{K_{t+1}\}}\left[(1-\gamma)^{-1}\left(\left[\left(\left(\hat{x}_t\right)^\alpha \left(f_t\right)^{1-\alpha}\right)^{-\mu} + \hat{n}_t^{-\mu}\right]^{\frac{1}{-\mu}}\right.\right.$$
$$\left.\left.\left(1-\bar{l}^x-\bar{l}^f-\hat{l}_t^n\right)^{\omega}\right)^{1-\gamma} + \beta E_t\left[V\left(K_{t+1}, \vartheta_{t+1}^p, \vartheta_{t+1}^x, \vartheta_{t+1}^f, \vartheta_{t+1}^n\right)\right]\right] \tag{52}$$

The laws of motion of capital and the law of shocks are given by:

$$K_{t+1} = (1-\delta) K_t + I_t$$

$$\upsilon_t^i = (1-\rho)\bar{\upsilon} + \rho\upsilon_{t-1}^i + \vartheta_t^i$$

$$i = p, x, f, n$$

$$\vartheta_t \backsim iid \quad N(0, \Sigma),$$

where the shocks are iid normal with a variance and covariance matrix Σ.

With a second order Taylor expansion around the steady state the problem is reduced to

$$V(x) = \max_{\{y\}} \left[x^T Q x + 2y^T W x + y^T R y + \beta V(x')\right] \tag{53}$$

subject to :

$$x' = Ax + By + \varepsilon' \tag{54}$$

Following Urrutia (1998), to solve this Bellman equation it is useful to consider a guess of this expectation and to check if this guess is the solution:

$$V(x) = x^T P x + d$$

$$\text{where} \qquad d = \frac{\beta}{(1-\beta)}\text{trace}(P\Sigma)$$

After, it is necessary to compute the optimal decision rule through the partial derivative. With this, the decision rule is $y = Gx$, where

$$G = -\left(Q + \beta B^T P B\right)^{-1}\left(W + \beta B^T P A\right)$$

This optimal decision rule has two properties: First, it is a linear function of the state variables. Second, G is independent of the stochastic structure of the problem, and in particular of the variance – covariance of the shocks, Σ. Both properties are specific to linear quadratic models.

To check if the guess is a valid solution, one verifies that it solves the following equation.

$$P = R + \beta A^T PA - \left(W^T + \beta A^T PB\right)\left(Q + \beta B^T PB\right)^{-1}\left(W + \beta B^T PA\right)$$

The last equation is known as Ricatti's equation. The matrix P is solved by iteration.

The next step in this methodology is to find the steady state of the model.

Define the next auxiliary equations:

$$\begin{aligned}
D &= \left(\frac{\alpha}{(1-\alpha)\,P^x}\right)^{\alpha} \\
E &= (1+\chi\delta)\left(\frac{1}{\beta} - 1 + \frac{\delta}{2}\right) + \frac{\delta}{2} \\
H &= \left(\frac{\alpha_n\left(1 - \bar{l}^x - \bar{l}^f - l_*^n\right) - l^n\omega}{\omega l^n}\right)^{\frac{1}{\mu}} \\
Z &= D^{1-\gamma}\left(\left(1 + H^{-\mu}\right)\right)^{\frac{(\gamma-1)-\mu}{\mu}}\left(1 - \bar{l}^x - \bar{l}^f - l^n\right)^{\omega(1-\gamma)}(1-\alpha)
\end{aligned}$$

With them and equations (25) to (38) and assuming the variables are deterministic and stable for all periods, the steady state values are given by the following equations

$$
\begin{aligned}
K^f &= \left(\frac{A^f\left(1-\alpha_f\right)}{E}\right)^{\frac{1}{\alpha_f}} \\
K^x &= \left(\frac{A^x P^x\left(1-\alpha_x\right)}{E}\right)^{\frac{1}{\alpha_x}} \\
K &= K^x + K^f \\
I &= \delta K \\
Y^f &= A^f\left(K^f\right)^{1-\alpha_f}\left(\bar{l}^f\right)^{\alpha_f} \\
Y^x &= A^x\left(K^x\right)^{1-\alpha_x}\left(\bar{l}^x\right)^{\alpha_x} \\
f &= \left(1-\alpha_f\right)\left(P^x Y^x + Y^f - I - \frac{\chi}{2K} I\right) \\
\lambda &= f^{-\gamma} Z \\
q &= \lambda\left(1+\chi\delta\right) \\
\theta &= E\lambda \\
x &= \frac{\alpha}{\left(1-\alpha\right) P^x} f \\
n &= Y^n = HDf \\
\bar{K}^n &= A^n\left(\bar{K}^n\right)^{1-\alpha_n}\left(l^n\right)^{\alpha_n}
\end{aligned}
$$

To find labor in the non-tradeable sector in steady state, l^n, I use the value of the calibration and find the value of constant capital in this sector.

The vector and matrices implied by this method for this study are (where the capital $U_{i,j}$ are the partial derivative of utility function at steady state value with respect variable i and j :

$$x = \begin{bmatrix} K \\ \vartheta^p \\ \vartheta^x \\ \vartheta^f \\ \vartheta^n \\ 1 \end{bmatrix}; \qquad Q = \frac{1}{2}\begin{bmatrix} U_{KK} & U_{K\vartheta^p} & U_{K\vartheta^x} & U_{K\vartheta^f} & U_{K\vartheta^n} & U_K \\ U_{\vartheta^p K} & U_{\vartheta^p\vartheta^p} & U_{\vartheta^p\vartheta^x} & U_{\vartheta^p\vartheta^f} & U_{\vartheta^p\vartheta^n} & U_{\vartheta^p} \\ U_{\vartheta^x K} & U_{\vartheta^x\vartheta^p} & U_{\vartheta^x\vartheta^x} & U_{\vartheta^x\vartheta^f} & U_{\vartheta^x\vartheta^n} & U_{\vartheta^x} \\ U_{\vartheta^f K} & U_{\vartheta^f\vartheta^p} & U_{\vartheta^f\vartheta^x} & U_{\vartheta^f\vartheta^f} & U_{\vartheta^f\vartheta^n} & U_{\vartheta^f} \\ U_{\vartheta^n K} & U_{\vartheta^n\vartheta^p} & U_{\vartheta^n\vartheta^x} & U_{\vartheta^n\vartheta^f} & U_{\vartheta^n\vartheta^n} & U_{\vartheta^n} \\ U_K & U_{\vartheta^p} & U_{\vartheta^x} & U_{\vartheta^f} & U_{\vartheta^n} & 2U \end{bmatrix}$$

$$y = \begin{bmatrix} I \\ L^n \end{bmatrix}; \qquad W = \frac{1}{2}\begin{bmatrix} U_{IK} & U_{I\vartheta^p} & U_{I\vartheta^x} & U_{I\vartheta^f} & U_{I\vartheta^n} & 2U_I \\ U_{L^n K} & U_{L^n\vartheta^p} & U_{L^n\vartheta^x} & U_{L^n\vartheta^f} & U_{L^n\vartheta^n} & 2U_{L^n} \end{bmatrix}$$

$$R = \frac{1}{2}\begin{bmatrix} U_{II} & U_{IL^n} \\ U_{L^n I} & U_{L^n L^n} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \qquad A = \begin{bmatrix} 1-\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_p & 0 & 0 & 0 & 1-\rho_p \\ 0 & 0 & \rho_x & 0 & 0 & 1-\rho_x \\ 0 & 0 & 0 & \rho_f & 0 & 1-\rho_f \\ 0 & 0 & 0 & 0 & \rho_n & 1-\rho_n \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vartheta = \begin{bmatrix} 0 \\ \vartheta_p \\ \vartheta_x \\ \vartheta_f \\ \vartheta_n \\ 0 \end{bmatrix}; \qquad \Sigma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\vartheta^p\vartheta^p} & \sigma_{\vartheta^p\vartheta^x} & \sigma_{\vartheta^p\vartheta^f} & \sigma_{\vartheta^p\vartheta^n} & 0 \\ 0 & \sigma_{\vartheta^x\vartheta^p} & \sigma_{\vartheta^x\vartheta^x} & \sigma_{\vartheta^x\vartheta^f} & \sigma_{\vartheta^x\vartheta^n} & 0 \\ 0 & \sigma_{\vartheta^f\vartheta^p} & \sigma_{\vartheta^f\vartheta^x} & \sigma_{\vartheta^f\vartheta^f} & \sigma_{\vartheta^f\vartheta^n} & 0 \\ 0 & \sigma_{\vartheta^n\vartheta^p} & \sigma_{\vartheta^n\vartheta^x} & \sigma_{\vartheta^n\vartheta^f} & \sigma_{\vartheta^n\vartheta^n} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This method does not impose any discretization or grid for the space state variables. This is a good approximation only when the model is around the steady state. This method is inappropriate when the initial conditions are far away from the steady state or for economies where the shocks have a large variance. Some researches find this method sufficient for almost any RBC Model. However in this study this method fails to generate volatility, as it is observed in the data.

4.2 Parameterized Expectations Algorithm

This subsection studies the parameterized expectations approach to this model. Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to "parameterize" the forward-looking expectation in this model, with non-linear functional forms on the Euler equation

(39),

$$q_t = \beta E_t \left[\theta_{t+1} + (1-\delta)\, q_{t+1} + \frac{\chi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \lambda_{t+1} \right] \tag{55}$$

Combining this equation with (38) and (30) finds the time used in labor in the non-traded sector production as a function of some expectation function and that is the variable that is parameterized.

$$\lambda_t = \left(\left((x_t)^{\alpha} (f_t)^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)-\mu}{\mu}} \frac{(1-\alpha) \left(1 - \bar{l}^x - \bar{l}^f - l_t^n \right)^{\omega(1-\gamma)}}{f_t \left((x_t)^{\alpha} (f_t)^{1-\alpha} \right)^{\mu}}$$

$$I_t = \left(\frac{q_t}{\lambda_t} - 1 \right) \frac{K_t}{\chi}$$

$$l_t^n = E_t \left[1 - \bar{l}^x - \bar{l}^f - \left(\frac{\beta f_t \left(x_t^{\alpha} f_t^{1-\alpha} \right)^{\mu} \left[\theta_{t+1} + (1-\delta)\, q_{t+1} + \frac{\chi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \lambda_{t+1} \right]}{\left(\frac{\chi I_t}{K_t} + 1 \right) (1-\alpha) \left(\left(x_t^{\alpha} f_t^{1-\alpha} \right)^{-\mu} + n_t^{-\mu} \right)^{\frac{(\gamma-1)-\mu}{\mu}}} \right)^{\frac{1}{\omega(1-\gamma)}} \right] \tag{56}$$

$$l_t^n \simeq \psi_t^l(\mathbf{z}_{t-1}; \Omega)$$

$$\mathbf{z}_{t-1} = \{ K_t/\bar{K}, \varepsilon_t^p, \varepsilon_t^x, \varepsilon_t^f, \varepsilon_t^n \}$$

The term ψ_t^l is the expectation approximation function. The symbol $\mathbf{z}_{t-1}$ represents a vector of observable "instrument" variables known at time t: in fact, I use the state variables: the initial capital at t which is predetermined at that moment, and the realization of the different shocks at t. The term $\bar{K}$ is the value of capital in steady state. The symbol Ω represents the parameters for the approximation function ψ_t^l.

Judd (1996) classifies this approach as a "projection" or a "weighted residual" method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). These authors pointed out that the conditional expectation of the future grain price is a "smooth function" of the current state of the market, and that this conditional expectation can be used to characterize equilibrium.

Parameterizing equation (56) rather than equation (55) has at least three advantages. First, it prevents small errors in the approximation of q_t from being amplified in the variation of I_t. Second, Parameterizing equation (56) the remaining equations have closed form solutions. Third, from the FOC's there is a condition between parameters and labor in non-traded sector that must be satisfied in each period:

$$l_t^n < \frac{\alpha_n \left(1 - \bar{l}^x - \bar{l}^f\right)}{\alpha_n + \omega}$$

and putting the right side of the last equation as the maximun value of the parameterized expectation on labor in non tradeable goods, gives no violation of the condition for each period. The combination of these three advantages gives more accurate and faster solutions.

The functional form for ψ_t^l is usually a second-order polynomial: see, for example, Den Haan and Marcet (1994), Schmitt-Grohé and Uribe (2002). However, Duffy and McNelis (2001) have shown that neural networks have produced results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

The model was simulated until convergence was obtained for the expectational errors. In the algorithm, the following non-negativity constraints for consumption and the stocks of capital for next period were imposed:

$$C_t^x > 0 \tag{57}$$

$$K_{t+1} \geq 0 \tag{58}$$

The latter was achieved by restricting capital for next period to be bigger than zero, which implies that is there is some degree of reversibility on the investment.

5 Parameters

The section discusses the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model.

The selections of the parameters are from other studies. The there three set of the parameters: First, Argentina (ARG), follows Pacharoni (2000). Second and third, developing countries (DC's), and industrialized countries (G7), follow Mendoza (1995). However there are some changes. These changes are due to data frequency, quarterly data. This paper uses the volatility and autocorrelation of the series presented in section 2.

The parameter settings for the models appear in table 5:

Table 5

	Parameters								
	γ	α	β	μ	ω	l^x	l^f	l^n	A
Arg	2.730	0.295	0.982	-0.425	0.809	0.073	0.033	0.184	1.000
DC's	2.610	0.150	0.990	0.130	0.786	0.060	0.050	0.150	1.000
G7	1.500	0.300	0.990	0.350	2.080	0.050	0.050	0.100	1.000
	δ	χ	α_x	α_f	α_n	ρ_p	ρ_x	ρ_f	ρ_n
Arg	0.023	0.028	0.480	0.438	0.352	0.597	0.497	0.486	0.878
DC's	0.023	0.028	0.429	0.302	0.340	0.590	0.154	0.154	0.631
G7	0.023	0.028	0.510	0.730	0.560	0.670	0.154	0.154	0.631

The shocks follow an autoregresive processes and the parameter ρ_p mimics actual TOT data filtered with the HP filter. The volatility of different sectors of production are from Pacharoni (2000) and Mendoza (1995)

Table 6

	Arg	DC's	G7		Arg	DC's	G7
σ_{pp}	0.0264	0.0244	0.0208	σ_{pf}	0.0021	0.0070	0.0006
σ_{xx}	0.0145	0.0283	0.0095	σ_{pn}	0.0054	0.0081	0.0049
σ_{ff}	0.0157	0.0283	0.0095	σ_{xf}	0.0075	0.0283	0.0095
σ_{nn}	0.0252	0.0380	0.0140	σ_{xn}	0.0025	0.0082	0.0029
σ_{px}	0.0023	0.0070	0.0006	σ_{fn}	0.0031	0.0082	0.0029

6 Benchmark Simulations

This section presents the result of different exercises. In general, the model explains several qualitative features but it cannot mimic all of the stylized facts found in the data.

Table 7

Volatility - **G7**					
	Data	**LQ**		**PEA**	
		OR	**M**	**OR**	**M**
y	1.177	2.330	1.990	2.116	2.108
		(0.192)	(0.163)	(0.119)	(0.118)
qt	10.138	0.698	0.351	2.841	2.833
		(0.097)	(0.069)	(0.169)	(0.168)
c		0.386	0.997	2.078	2.072
		(0.057)	(0.1)	(0.126)	(0.126)
I		7.390	7.316	5.748	5.814
		(0.627)	(0.62)	(0.295)	(0.284)
n		0.300	0.299	0.065	0.062
		(0.03)	(0.03)	(0.004)	(0.004)
k		0.349	0.346	0.282	0.284
		(0.073)	(0.072)	(0.03)	(0.029)
		relative to y			
qt	8.613	0.300	0.176	1.344	1.345
c		0.166	0.501	0.983	0.984
i		3.172	3.676	2.722	2.765
n		0.129	0.150	0.031	0.030
k		0.150	0.174	0.134	0.135

Table 8

Volatility - **DC's**					
	data	**LQ**		**PEA**	
		OR	**M**	**OR**	**M**
y	5.216	4.050	3.552	2.757	2.853
		(0.337)	(0.298)	(0.127)	(0.133)
qt	25.878	0.559	0.501	12.052	12.133
		(0.098)	(0.079)	(0.708)	(0.745)
c		1.333	3.009	4.986	4.994
		(0.141)	(0.296)	(0.293)	(0.312)
i		8.936	7.887	5.831	5.989
		(0.764)	(0.663)	(0.274)	(0.286)
n		0.318	0.306	0.087	0.086
		(0.032)	(0.032)	(0.003)	(0.004)
k		0.323	0.288	0.259	0.265
		(0.053)	(0.047)	(0.025)	(0.025)
		relative to y			
qt	4.961	0.138	0.141	4.373	4.255
c		0.329	0.847	1.809	1.751
I		2.206	2.220	2.119	2.103
n		0.079	0.086	0.032	0.030
k		0.080	0.081	0.094	0.093

Table 9

Volatility - **Argentina**					
	data	**LQ**		**PEA**	
		OR	**M**	**OR**	**M**
y	4.476	2.115	2.228	2.0882	2.120
		(0.203)	(0.242)	(0.133)	(0.137)
qt	28.839	1.192	0.886	8.9625	9.136
		(0.145)	(0.149)	(0.655)	(0.654)
c		2.411	2.020	3.571	3.569
		(0.346)	(0.278)	(0.253)	(0.252)
i		7.158	6.924	4.7615	5.034
		(0.735)	(0.708)	(0.3)	(0.315)
n		0.825	0.809	0.1785	0.156
		(0.113)	(0.111)	(0.008)	(0.008)
k		0.404	0.392	0.3034	0.324
		(0.095)	(0.093)	(0.039)	(0.038)
		relative to y			
qt	6.443	0.564	0.398	4.293	4.311
c		1.140	0.907	1.710	1.684
i		3.384	3.108	2.286	2.381
n		0.390	0.363	0.086	0.074
k		0.191	0.176	0.145	0.153

Table 10

Correlations - **G7**					
	data	**LQ**		**PEA**	
		OR	**M**	**OR**	**M**
		First Order Autocorrelation			
y	0.705	0.510	0.500	0.554	0.552
qt	0.508	0.930	0.773	0.470	0.567
c		0.621	0.818	0.846	0.849
i		0.470	0.471	0.686	0.691
n		0.954	0.954	0.300	0.317
k		0.596	0.596	0.666	0.670
tot	0.656	0.610	0.610	0.558	0.558
		Crosscorrelation with Y			
qt	-0.070	-0.465	0.369	0.544	0.562
c		0.717	-0.218	0.023	0.016
i		0.989	0.986	0.827	0.825
n		0.165	0.185	0.549	0.536
k		-0.648	-0.638	-0.784	-0.775
tot	-0.165	0.868	0.844	0.878	0.879
		Crosscorrelation with TOT			
qt	0.302	-0.334	0.522	0.510	0.552
c		0.607	-0.316	-0.071	-0.074
i		0.856	0.856	0.683	0.682
n		0.044	0.043	0.548	0.539
k		-0.635	-0.639	-0.612	-0.603

Table 11

Correlation - **DC's**					
	data	LQ		PEA	
		OR	M	OR	M
First Order Autocorrelation					
y	0.255	0.160	0.157	0.238	0.193
qt	0.591	0.841	0.718	0.529	0.279
c		0.573	0.613	0.522	0.817
i		0.199	0.193	0.193	0.578
n		0.927	0.927	0.184	0.393
k		0.606	0.593	0.829	0.593
tot	0.443	0.543	0.543	0.432	0.491
Crosscorrelation with Y					
qt	0.301	-0.342	-0.390	-0.699	0.088
c		0.517	-0.306	0.702	0.146
i		0.982	0.985	0.249	0.698
n		-0.015	-0.034	0.333	0.157
k		-0.315	-0.409	0.084	-0.609
tot	0.040	0.532	0.544	0.446	0.470
Crosscorrelation with TOT					
qt	0.019	-0.254	-0.611	-0.275	-0.129
c		0.588	-0.312	0.306	0.144
i		0.570	0.562	0.127	0.422
n		0.013	0.002	0.052	-0.044
k		-0.532	-0.554	0.033	-0.357

Table 12

Correlations - **Argentina**					
	data	LQ		PEA	
		OR	M	OR	M
First Order Autocorrelation					
y	0.073	0.531	0.525	0.611	0.620
qt	0.693	0.898	0.784	0.229	0.268
c		0.809	0.811	0.857	0.867
i		0.645	0.644	0.681	0.694
n		0.961	0.961	0.543	0.571
k		0.798	0.796	0.684	0.695
tot	0.604	0.544	0.544	0.495	0.495
Crosscorrelation with Y					
qt	0.457	-0.476	-0.567	-0.461	-0.472
c		0.412	-0.190	0.407	0.392
i		0.802	0.806	0.915	0.918
n		0.255	0.199	-0.391	-0.413
k		0.347	0.381	-0.881	-0.879
tot	0.356	0.511	0.619	0.648	0.647
Crosscorrelation with TOT					
qt	0.503	-0.177	-0.548	-0.330	-0.350
c		0.349	-0.106	0.288	0.276
i		0.496	0.496	0.569	0.567
n		-0.075	-0.076	-0.189	-0.198
k		0.394	0.399	-0.453	-0.447

Tables 7 to 12 compare business cycles in the models with those observed in the seven largest industrialized countries, G7, developing economies, DC's, and Argentina, ARG.

Tables 7 to 9 presents the volatility of key variables: product, y, Tobin's q, qt, consumption, c, investment, i, labor, n, and capital, k. They also show the relative volatility for each variable to the GDP, y. Tables 10 to 12 present the first order autocorrelation of the same variables plus the Terms of Trade, tot, and their cross-correlation with the GDP and Tobin's q.

Vertically the tables are divided in three sections: data, Linear Quadratic, LQ, and Parametrized Expectation Algorithm, PEA. The column called "data" presents the values of actual data from tables 2 to 4. LQ has two models, with different adjustment cost functions for capital, one following Obstfeld and Roggoff (1996), "OR", and the other following Mendoza (1995), "M"; the exercise was realized on 100 simulations and 76 observations. PEA also has two models with the same adjustment cost functions. Simulations for PEA models has 500 simulations and 300 observations.

The goal of this study is to understand the implication of the volatility in the stock markets through Tobin's q, for developing countries. The benchmark simulations for developing countries underestimate these values but their relative volatility is close to the value found in the data using PEA, but do not match well with LQ.

For the G7 countries the simulation does not replicate the volatility for GDP. It is almost twice as in the actual data. For the volatility of qt, the simulations predict values greater than the GDP using PEA though far away from the value in the data.

In the DC's, the volatility of production is underestimated by half the value of PEA estimate, and something similar with LQ. But the Tobin's q has a good fit with the data for a relative volatility for PEA, given that the model underestimates also a half of the data business cycles. The consumption has the same problem as the G7. The volatility is larger than y by 1.7 times. The volatility predicted by the models for investment is twice than their productions, but this value is close to Mendoza

(1995). The volatility of the labor in the non-traded sector is underestimated in the model, all versions of the model give volatility less than 10 percent.

For Argentina, the volatility of GDP the model predicts is a little less than a half of the data, and almost one third of the stock market indices. The volatility in Tobin's q relative to GDP is more than two thirds that of the data.

The autocorrelations for TOT are close to the data by construction. They are generated to mimic the data, when the calibration was made.

In the G7 versions, the autocorrelations of GDP are around 0.7 of the data. The autocorrelations of Tobin's q are close to the data for PEA and LQ overestimates them. For DC's the values of autocorrelations of GDP and Tobin's q are close to the data for the version of the model of OR using PEA algorithm, the other versions underestimate GDP. For Argentina, the data have no autocorrelation for GDP and the model gives a number around 0.5; the Tobin's q is overestimated with LQ and underestimated with the PEA.

The cross correlation between Tobin's q and the production for the three sectors does not fit with the data; for DC's and for Argentina it has the different sign. Cross correlation of the terms of trade and product are over estimated for all the models, but they are so far away of the data for G7 (and, with different sign) and DC's. For Argentina the models overestimate them twice.

The cross correlation between Tobin's q and terms of trade is over estimated by a factor of 1.6 in G7 models. DC economies do not present cross correlations in the stock markets indices while the models predict a negative one.

7 Sensitivity Analysis

This section examines the role that some parameters play in explaining the properties of business cycles in the model.

Table 13

Volatility - **G7**			
Parameters in Expectations			
PEA-OR	**19**	**19**	**22**
	χ=0	χ=0.028	χ=0.028
y	1.957	1.961	1.949
qt	2.854	2.838	2.849
c	2.110	2.099	2.104
i	5.221	5.136	5.451
n	0.072	0.074	0.065
k	0.232	0.227	0.234
relative to y			
y	1.000	1.000	1.000
qt	1.459	1.447	1.462
c	1.078	1.071	1.080
i	2.669	2.620	2.796
n	0.037	0.038	0.033
k	0.119	0.116	0.120

Table 14

Volatility - **DC's**		
PEA	**A=0.3**	**A=1.0**
	OR	**OR**
y	2.831	2.757
	(0.13)	(0.127)
qt	11.981	12.052
	(0.692)	(0.708)
c	4.945	4.986
	(0.291)	(0.293)
i	5.902	5.831
	(0.263)	(0.274)
n	0.083	0.087
	(0.003)	(0.003)
k	0.260	0.259
	(0.025)	(0.025)
relative to y		
qt	4.234	4.373
c	1.747	1.809
i	2.088	2.119
n	0.030	0.032
k	0.092	0.094

Table 13 examines changes in the number of parameters of the function ψ_t^l, for PEA for the industrial countries. There is no theory to use one or another specific function as the parameterized function. This paper does not find any change when it uses a constant term in the last hidden layer of the neural network, the change between 19 and 22 parameters. There are no changes in the volatility of the time series when the numbers of neurons are the same, and estimates are not significantly affected by the functional form used in the nerual net.

This study pins down the two versions of the adjustment cost used in this paper. In each table there are the results for both functions. The changes in the volatility are small using one or the other. Table 13 is also a comparison of the parameter that governs the adjustment cost function, χ, when it is equal to zero or as usual in the literature 0.028.

Another analysis is study the scale of production, A. It is usually fixed at 1, but as Mendoza (1995) presents a value of 0.3 for developing economies, it is due the comparison with developed countries. Developing countries are richer than developed ones if the parameter A is the same. Table 14 examines the volatility of key variables in developing economies where their values are the same. The model does not find difference in the volatility.

Table 15

Volatility - only shock TOT						
	G7		**DC's**		**ARG**	
	(1)	**TOT**	**(2)**	**TOT**	**(3)**	**TOT**
y	2.330	1.397	2.803	0.3167	2.089	0.731
qt	0.698	0.641	11.966	0.3976	8.978	0.837
c	0.386	0.636	4.876	0.391	3.517	0.806
I	7.390	4.567	5.895	0.174	4.942	0.457
n	0.300	0.031	0.064	0.0024	0.107	0.013
k	0.349	0.183	0.270	0.007	0.334	0.019
relative to y						
y	1.000	1.000	1.000	1.000	1.000	1.000
qt	0.300	0.459	4.272	1.256	4.360	1.145
c	0.166	0.455	1.740	1.235	1.798	1.102
i	3.172	3.270	2.108	0.550	2.286	0.625
n	0.129	0.022	0.023	0.008	0.113	0.017
k	0.150	0.131	0.097	0.022	0.117	0.026
(1)-(3) from Tables 7-9						

Table 16

Volatility - **DC's**			
	data	**LQ - OR**	
		γ= 2.61	**γ= 60**
y	5.216	4.050	4.036
		(0.337)	(0.337)
qt	25.878	0.559	11.830
		(0.098)	(2.41)
c		1.333	3.331
		(0.141)	(0.347)
i		8.936	10.300
		(0.764)	(0.914)
n		0.318	0.426
		(0.032)	(0.043)
k		0.323	0.400
		(0.053)	(0.069)
relative to y			
y	1.000	1.000	1.000
qt	4.961	0.138	2.932
c		0.329	0.825
i		2.206	2.552
n		0.079	0.106
k		0.080	0.099

Table 15 examines the volatility of the model produced only by a shock in the terms of trade. The result shows small volatility in Tobin's q. It predicts more volatility in Tobin's q than in GDP for developing economies and less for industrial countries.

Table 16 presents a change in the parameter of risk aversion from 2.61 to 60, for LQ approximation, the model explains almost half of the volatility of the Tobin's q.

8 Conclusion

This paper conducts a quantitative examination of the link between terms of trade shocks and business cycles by comparing numerical solutions of the competitive equilibrium of a dynamic stochastic model of a small open economy with actual business cycles, especially to understand the implication of the volatility in the stock markets through the Tobin's q, for developing countries. In the model, a combination of consumption in exportable, importable and non-traded goods and leisure time give the welfare of the household. The firms, owned by the household, produce three different goods, using capital and labor: exportable, importable and non tradeable. World markets of good are competitive. The rest of the world has inelastic demand and supply at international prices for traded goods.

The result, that the model solved by parameterized expectations approach with global search for developing countries, shows that relative volatility of the Tobin's q to the output replicates that found in the data between Stock Market Indices and GDP during the last decade.

The comparison between the control case as the G7 and developing economies gives some implications for the different behavior of the volatility of the Tobin's q; and the case of Argentina gives one "country" example of that result.

The paper has made some comparison and sensitivity analysis: one is between different adjustment cost function. Using a marginal capital cost as Obstfeld and Roggoff (1996) the function produces more volatility and higher steady state values for capital, product and consumption than a adjustment cost function of total new capital as Mendoza (1995).

Another comparison is between linear quadratic and parameterized expectation algorithms. The LQ method fails to mimic the volatility of the Tobin's q in developing countries, but PEA gives a good approximation.

Future research could include more developing countries and some variation in this model. First, the model could incorporate capital mobility with an international asset. That will allow deviation from the current account balance for each period. That would requiere incorporating some technique to close the model, of Smith-Grohé and Uribe (2001). Second, the model could use different utility function; such as one with habit persistence or a Weil function to give more curvature to the utility function without a high inter-temporal risk parameter. Third, the model could incorporate heterogeneous agents to find more realistic market for asset prices.

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