

# VOTING LEADERS AND VOTING PARTICIPATION

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ABSTRACT. We model electoral competition between two parties in a winner take all election. Parties choose strategically first their platforms and then their campaign spending under aggregate uncertainty about voters' preferences. In the unique Nash equilibrium larger elections are characterized by a higher participation rate. Moreover, no matter what the voters' preferences are, parties spend exactly the same amounts for their campaign in equilibrium. Platforms converge to the center (median voter) and spending increases as the uncertainty over voters' preferences decreases.

Keywords: Voter's Paradox, Aggregate Uncertainty.

JEL Classification: D72

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*Date:* First Version: February 7th 2004, This Version: 16th April 2004.

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## 1. INTRODUCTION

Why do so many people turn out to vote? This is a long-standing question in social science and one that has turned out to be a major headache for the rational choice approach to political behavior used by economists and political scientist alike. In Downs's (1957) original formulation of the rational voter hypothesis, the act of voting is paradoxical since voting involves some costs (e.g. time involved in registering to vote and going to the voting booth). While those costs may be low, the probability of a single voter affecting the outcome of the election is so tiny in a large election that even very small costs should deter from going to vote any individual motivated solely by the desire to influence the election result. The game-theoretic analysis of Palfrey and Rosenthal (1985), built on earlier work by Ledyard (1984), confirms Downs's intuition: in large electorates, the voter turnout in the Nash equilibrium is extremely low, provided that voters are somewhat uncertain about the preferences or voting inclination of others. This is clearly at odds with the high turnout we observe in elections.

There must be something more than the simple desire to directly and individually affect the election that brings people to the voting booth on election day. As Palfrey and Rosenthal state "a model that addresses mobilization issues directly is needed if we are to make headway answering this normative question as well as positive questions about the nature of electoral competition." In fact political parties spend considerable money and effort to encourage people to go vote in a variety of ways. In this paper the amount of campaigning and of campaign spending of political parties and politicians determines with some randomness how many individuals go to the voting booths for each party. We add a first stage platform choice decision of the parties to the Shachar and Nalebuff type of model of political participation and we study how the uncertainty about the preferences or political inclinations of the electorate affects two important campaign variables: the platform chosen by the parties and the campaign spending of the parties. Namely, we analyze how the propensity of the electorate to vote for one or the other party (or rather the uncertainty over this propensity) affects the choices of the parties regarding their political message or agenda and the amounts of their campaign spending. Of course the parties' relative ideologies and their spending prior to the election are going to be crucial determinants of the overall expected turnout in the election. The important variable in these two-fold strategic choices of the parties which is embedded in the political inclination of potential voters is the expected closeness of the election. To

be specific, our model tells us under general conditions that if parties are free to choose how much to spend (as in the US), then parties will spend the same amounts. This equal spending result reflects the empirical evidence on campaign spending in the US in the past 40 years as shown in the picture at the end of the paper. Moreover, how much will both parties spend? The closer the election is expected to be, the more the parties will spend to attract voters because bringing to vote for one party a few more people could be crucial in a close election. The second part of the paper (first stage) deals with the positioning of the parties in the policy space, i.e., what policies the two parties choose to campaign on. The closer the election is expected to be, the more the policies of the two competing parties will tend to be similar to each other, namely they will tend to converge to the center of the policy space (median voter). In a close election between two parties in fact, parties tend to be more pragmatic and less ideological: parties may be willing to soften their stand on some controversial issues to conform more to the views of majority of the electorate and try to gain more votes that way, and the more so the closer the election is expected to be because in that case the few additional votes gained can make you win or lose the election.

## 2. THE MODEL

### List of symbols:

Two parties:  $d, r$

$p$  common propensity to vote for  $d$ , random with CDF  $F$  with support on  $\underline{d}, \bar{d}$

$d$  propensity of a member of  $d$  to vote for  $d$  and platform of party  $d$  (make them one an increasing function of the other with a lower range for the

$$\text{bias}(0, \underline{d}) \subset \left(0, \frac{1}{2}\right) \text{ platform}$$

$r$  propensity of a member of  $r$  to vote for  $r$  and platform of party  $r$

note that we will need  $d, r \leq \underline{d}, 1 - \bar{d}$

probability a  $d$  votes  $d$  is  $p + d$

probability a  $r$  votes  $r$  is  $1 - p + r$

$D$  number of  $d$  voters participating

$R$  number of  $r$  voters participating

$c$  cost of getting a voter to the polls

$V$  per capita value of winning the election

$N$  number of voters

In this model, two parties  $d$  and  $r$  choose the amount of their campaign spending  $cD$  and  $cR$  in a winner take all election.

There are  $N$  voters in the population, that have a common propensity (probability)  $p$  to vote for party d and  $1-p$  for party r. This propensity  $p$  is a random variable with CDF  $F(p)$  with support interval  $[\underline{b}, \bar{b}] \subset [0, 1]$ . Besides this unknown common propensity, it is common knowledge that voters have certain biases towards either party. That is, if party d spends an amount  $cD$  he will bring to vote  $D(p+d)$  people for his party, where  $d$  is known but  $p$  is not.

### 3. FORMULATION $a \in [0, 1]$

By spending  $cD$  party d will not only bring people to vote for him but will encourage a number

$$aD(1-p-d)$$

of people to vote for the other party. Where to keep this assumption as general as possible  $a$  is a non-negative number less than or equal to one: if  $a = 0$  one party spending does not bring votes to the opposing party.

The probability that party d wins is equal to the probability that the number of votes for d is greater than the number of votes for r, that is

$$D(p+d) + aR(p-r) > R(1-p+r) + aD(1-p-d)$$

where the spending  $D$  of the d party generates  $D(p+d)$  votes for the d party,  $aD(1-p-d)$  for the opposite r party and  $(1-a)D(1-p-d)$  abstainers. Rearranging the inequality we obtain

$$p \geq \frac{R+aD}{(R+D)(1+a)} + \frac{Rr-Dd}{(R+D)} \equiv P(D, R)$$

The value  $P(D, R)$  is a fundamental threshold that we use extensively in the paper. In the symmetric case  $d = r$  we have<sup>1</sup>

$$p \geq \frac{R+aD}{(R+D)(1+a)} + r \frac{R-D}{R+D}$$

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<sup>1</sup>The latter equation has similarities with Shachar Nalebuff's equation

$$p \geq \frac{\psi^r(R)}{\psi^r(R) + \psi^d(D)} = \frac{1}{2} + \frac{1}{2} \frac{\psi^r(R) - \psi^d(D)}{\psi^r(R) + \psi^d(D)}$$

**3.1. Parties Objectives.** The objective functions of the two parties  $d$  and  $r$  are respectively

$$\begin{aligned} W^d &= NE(v^d) - cD \\ W^r &= NE(v^r) - cR \end{aligned}$$

where  $N$  is the size of the electorate and we are taking the expected value of

$$\begin{aligned} v^d &= \begin{cases} g(d) & \text{if Win} \\ g(1-r) & \text{if Lose} \end{cases} \\ v^r &= \begin{cases} g(r) & \text{if Win} \\ g(1-d) & \text{if Lose} \end{cases} \end{aligned}$$

where  $d$  and  $r$  are the respective locations of the party on the policy unit interval and  $g$  is the benefit associated with the location of the party platform which has the symmetry and monotonicity properties

$$\begin{aligned} g(d) &= -g(1-d) \\ g(0) &> 0 \\ g'(d) &< 0 \end{aligned}$$

This means that the party  $d$  has maximum benefit from winning the election if he is located at the extreme left of the interval and decreasing gradually to zero benefit from winning if he is located at the midpoint (median voter). If party  $d$  loses the election its utility is more negative the farther to the right of the midpoint the winning party  $r$  locates himself. The importance of the election also depends of course on its size  $N$ . In formulas

$$\begin{aligned} W^d &= N (g(d) (\Pr(p \geq P(D, R))) + g(1-r) (\Pr(p < P(D, R)))) - cD \\ W^r &= N (g(r) (\Pr(p < P(D, R))) + g(1-d) (\Pr(p > P(D, R)))) - cR \end{aligned}$$

We assume first that the platform choices  $d$  and  $r$  are given and that parties choose simultaneously how much to spend  $D$  and  $R$ . We solve for this second stage in this section. Once we have solved for this second stage strategic simultaneous spending decision, we will endogenize the party platform choices in the first stage of the model.

Taking the first order condition we obtain the implicit best responses

$$\begin{aligned} U_D^d &= -g(d) f(P(D, R)) P_D(D, R) + g(1-r) f(P(D, R)) P_D(D, R) = \frac{c}{N} \\ &= -(g(d) + g(r)) f(P(D, R)) P_D(D, R) = \frac{c}{N} \end{aligned}$$

$$\begin{aligned} U_R^r &= g(r) f(P(D, R)) P_R(D, R) - g(1-d) f(P(D, R)) P_R(D, R) = \frac{c}{N} \\ &= (g(d) + g(r)) f(P(D, R)) P_R(D, R) = \frac{c}{N} \end{aligned}$$

which imply conditions on the threshold derivatives

$$-P_D(D, R) = P_R(D, R) = \frac{c}{N} \frac{1}{f(P)(g(d) + g(r))}$$

$$\begin{aligned} -P_D(R, D) &= \frac{R(1-a)}{(R+D)^2(1+a)} + \frac{(d+r)R}{(R+D)^2} = R \frac{(r+d+A)}{(R+D)^2} \\ P_R(R, D) &= \frac{D(1-a)}{(R+D)^2(1+a)} + \frac{(d+r)D}{(R+D)^2} = D \frac{(r+d+A)}{(R+D)^2} \end{aligned}$$

where

$$A \equiv \frac{1-a}{1+a} \in [0, 1]$$

Hence we obtain the following result:

**Both candidates spend the same in equilibrium**

There is a unique possible interior solution for the party spending (pending on the second order conditions)

$$\begin{aligned} \frac{d+r+A}{4R^*} &= \frac{c}{N} \frac{1}{f(P)(g(d) + g(r))} \\ D^* = R^* &= \frac{N}{4} \frac{1}{c} f\left(\frac{1}{2} + \frac{r-d}{2}\right) (g(d) + g(r)) (r+d+A) \end{aligned}$$

The fact that the condition  $-P_D = P_R$  implies that the candidates spend the same:  $D = R$  is a direct consequence of the symmetry property of the threshold value function

$$P_R(D, R) = -P_D(R, D)$$

which has a trivial interpretation. Merging the two conditions we obtain

$$P_D(D, R) = P_D(R, D)$$

and since intuitively

$$P_D(D, R) \neq P_D(R, D) \iff D \neq R$$

we obtain the equal spending result.

3.1.1. *Turnout.* The expected turnout rate is generically different for each party, total turnout is not random and is proportional to the spending

$$\begin{aligned} E(T_d) &= [(1+a)E(p) + d - ar] \frac{D^*}{N} \\ E(T_r) &= [(1+a)E(1-p) + r - ad] \frac{D^*}{N} \\ T &= [(1+a) + (1-a)(r+d)] \frac{D^*}{N} \end{aligned}$$

All the properties of the equilibrium spending described in the next section hold for the total turnout too.

**3.2. Interpretation of the Results.** Let's rewrite the equilibrium condition

$$D^* = R^* = \frac{N}{4} \frac{1}{c} f\left(\frac{1}{2} + \frac{r-d}{2}\right) (g(d) + g(r)) (A + (d+r))$$

The comparative statics is the following:

Party spending is proportional to the importance of the election measured by the size of the electorate  $N$

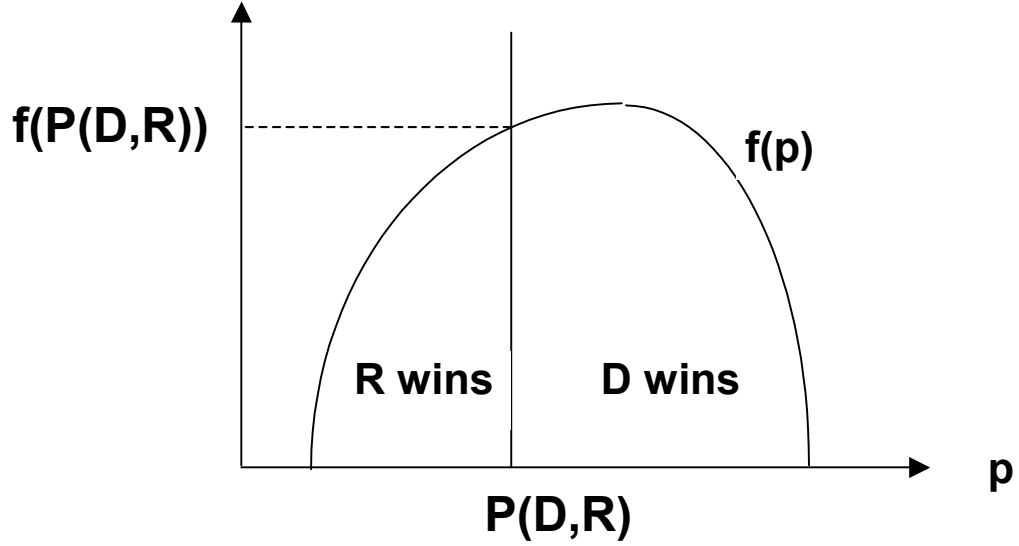
If the bias/platform increase from zero and get closer to  $\frac{1}{2}$  there are two opposite effects: you want to spend more because you are more likely to attract people (higher bias), but you also want to spend less because you care less about winning (see when we describe the first stage platform choice of the model).

Less trivially and most importantly we have the following result:

**Party spending is proportional to the value of the pdf in one single point**

$$D^* = R^* \propto f\left(\frac{1}{2} + \frac{r-d}{2}\right)$$

which is the threshold for winning



This means that parties spend more in equilibrium if this threshold value for the common propensity  $p$  (or, by continuity, something close to it) is more likely to be the true value, that is if the election is more likely to be a close election. If the density is zero around this threshold value than the parties spend zero in equilibrium. Note that, this value is not generically linked with the variance of the distribution: in the uniform case it is though and more certainty (lower variance) implies more participation.

What we are assuming is that in a larger election the race is more likely to be close. If this is true then we have higher turnout in larger elections.

**3.3. Second Order Conditions.** If the second derivatives of both objectives at the unique stationary point are negative, then the objectives are single-peaked at the stationary point and the interior solution is the unique Nash equilibrium.

Given that the second derivatives of the threshold are

$$P_{DD} = 2R \frac{r + d + A}{(R + D)^3}$$

$$P_{RR} = -2D \frac{r + d + A}{(R + D)^3}$$



then, the second order conditions become

$$\begin{aligned}
U_{DD}^d &\propto -f(P(D, R))P_{DD}(D, R) - f'(P(D, R)) [P_D(D, R)]^2 \\
&\propto -f(P(D, R)) \frac{P_{DD}(D, R)}{[P_D(D, R)]^2} - f'(P(D, R)) \\
&= -f(P(D, R)) \frac{2(R+D)}{R(d+r+A)} - f'(P(D, R)) \\
U_{DD}^d(D^*, R^*) &\propto -f\left(\frac{1}{2} + \frac{r-d}{2}\right) \frac{4}{(d+r+A)} - f'\left(\frac{1}{2} + \frac{r-d}{2}\right) < 0
\end{aligned}$$

$$\begin{aligned}
U_{RR}^r &= f(P(D, R))P_{RR}(D, R) + f'(P(D, R)) [P_R(D, R)]^2 \\
U_{DD}^r(D^*, R^*) &\propto -f\left(\frac{1}{2} + \frac{r-d}{2}\right) \frac{4}{(d+r+A)} + f'\left(\frac{1}{2} + \frac{r-d}{2}\right) < 0
\end{aligned}$$

Both conditions simultaneously imply:

$$\frac{4}{(d+r+A)} > \left| \frac{f'\left(\frac{1}{2} + \frac{r-d}{2}\right)}{f\left(\frac{1}{2} + \frac{r-d}{2}\right)} \right|$$

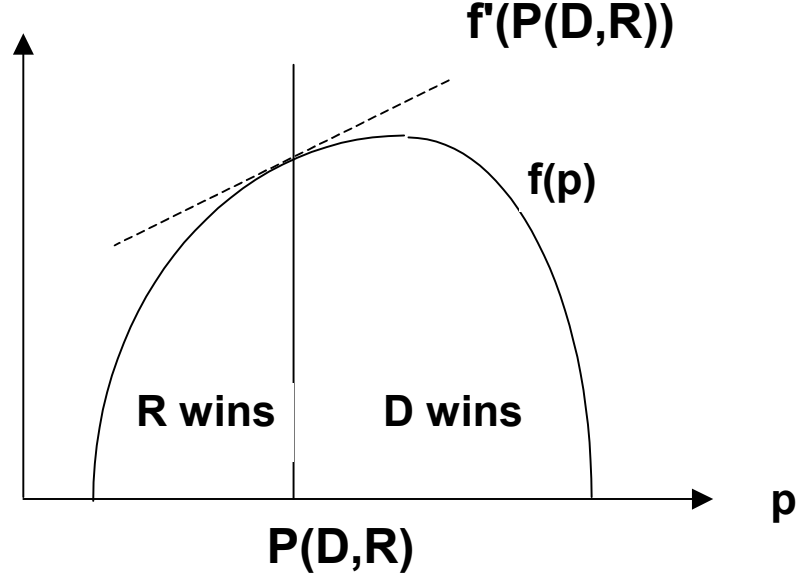
symmetric case  $d = r$  and  $f$  symmetric around  $\frac{1}{2}$  the second order condition is satisfied because  $f'(1/2) = 0$  and

$$D^* = R^* = \frac{N}{4} \frac{1}{c} f\left(\frac{1}{2}\right) (1 - 2d) (2d + A)$$

So, the second order conditions (SOCs) have the property:

**The SOCs fail to hold only if the density changes very much in the threshold point**

In this case one of the parties has the incentive to deviate because by spending a little more than the other party its probability of winning changes significantly.



Only in cases similar to the picture above the second order conditions fail and no interior solution exists. For all other cases the SOCs hold and there is a unique solution, the interior solution we found.

#### 4. FIRST STAGE

We have solved for how much parties would spend given an exogenous location in the ideological unit interval. Assume now that the two parties actually can choose their platforms simultaneously before deciding how much to spend on the campaign.

Solving backwards taking into account the second stage solution obtained, the objective becomes

$$\begin{aligned} W^d &= N \begin{bmatrix} g(d) (1 - F(\frac{1}{2} + \frac{r-d}{2})) - g(r) (F(\frac{1}{2} + \frac{r-d}{2})) \\ -\frac{1}{4} f(\frac{1}{2} + \frac{r-d}{2}) (g(d) + g(r)) (A + (d+r)) \end{bmatrix} \\ &= N \begin{bmatrix} g(d) - (g(d) + g(r)) F(\frac{1}{2} + \frac{r-d}{2}) \\ -\frac{1}{4} f(\frac{1}{2} + \frac{r-d}{2}) (g(d) + g(r)) (A + (d+r)) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} W^r &= N \begin{bmatrix} g(r) F(\frac{1}{2} + \frac{r-d}{2}) - g(d) (1 - F(\frac{1}{2} + \frac{r-d}{2})) \\ -\frac{1}{4} f(\frac{1}{2} + \frac{r-d}{2}) (g(d) + g(r)) (A + (d+r)) \end{bmatrix} \\ &= N \begin{bmatrix} -g(d) + (g(d) + g(r)) F(\frac{1}{2} + \frac{r-d}{2}) \\ -\frac{1}{4} f(\frac{1}{2} + \frac{r-d}{2}) (g(d) + g(r)) (A + (d+r)) \end{bmatrix} \end{aligned}$$

The first order conditions are

$$\begin{aligned} \frac{\partial}{\partial d} \left( g(d) - (g(d) + g(r)) F \left( \frac{1}{2} + \frac{r-d}{2} \right) \right) &= \frac{1}{4} \frac{\partial}{\partial d} \left( \begin{array}{c} (g(d) + g(r)) \\ ((A + (d+r)) f \left( \frac{1}{2} + \frac{r-d}{2} \right)) \end{array} \right) \\ g'(d) (1 - F) - (g(d) + g(r)) \left( -\frac{1}{2} \right) f &= \frac{1}{4} \left( \begin{array}{c} g'(d) (A + (d+r)) f + (g(d) + g(r)) \\ (f - \frac{1}{2} (A + (d+r)) f') \end{array} \right) \end{aligned}$$

In the symmetric equilibrium we have  $d = r$ . Hence for a symmetric distribution we derive and define the following levels

$$F \left( \frac{1}{2} + \frac{r-d}{2} \right) = \frac{1}{2} \quad f \left( \frac{1}{2} + \frac{r-d}{2} \right) = h \quad f' \left( \frac{1}{2} + \frac{r-d}{2} \right) = 0$$

where  $h$  represents the likelihood of a close race. The marginal benefit equals the marginal cost in the optimum, that is

$$\begin{aligned} \frac{g'(d)}{2} + g(d) h &= \frac{1}{4} (g'(d) (A + 2d) + 2g(d)) h \\ \frac{2}{h} g'(d) + 4g(d) &= g'(d) (A + 2d) + 2g(d) \\ \left( d - \frac{g}{g'} \right) &= \left( \frac{1}{h} - \frac{1}{2} A \right) > 0 \end{aligned}$$

By the implicit function theorem we obtain

$$\begin{aligned} G(d, h) &\equiv 2g(d) - g'(d) \left( A + 2d - \frac{2}{h} \right) = 0 \\ \frac{\partial d}{\partial h} &= -\frac{G_h}{G_d} = \frac{g'(d) \frac{2}{h^2}}{g''(d) \left( A + 2d - \frac{2}{h} \right)} > 0 \quad \iff \quad g''(d) < 0 \end{aligned}$$

The condition  $g'' < 0$  satisfies the second order conditions (see below). Under the latter condition the more likely the probability of a close election (higher  $h$ ) the higher the bias that the parties choose in equilibrium.

Note that

$$\frac{\partial}{\partial d} \left( d - \frac{g}{g'} \right) = \frac{gg''}{(g')^2} < 0$$

Under the concavity condition  $g'' < 0$ ,  $\left( d - \frac{g}{g'} \right)$  is monotonic so for any given  $h$  if an interior solution exists it is unique.

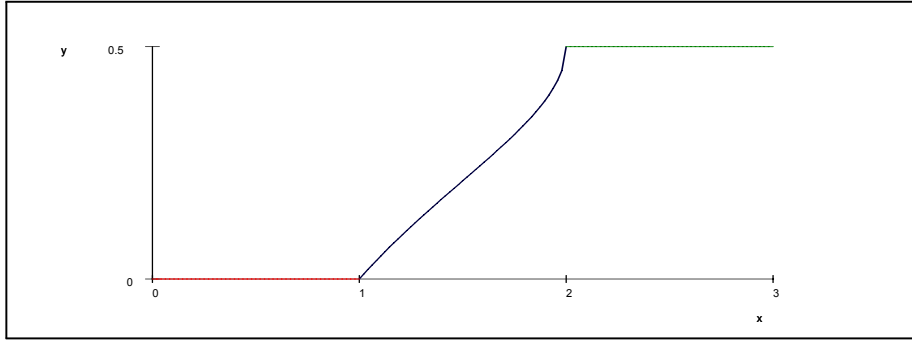
A closed form example is given by the following function

$$g(d) \equiv \left(2 + \frac{1}{d-1}\right) \quad \text{if } d < \frac{1}{2}$$

$$\left(\frac{g(d)}{g'(d)} - d\right) = \left(-2 \left(d - \frac{1}{2}\right)^2 - \frac{1}{2}\right) = -\frac{1}{h} + \frac{1}{2}A$$

$$d = r = \frac{1}{2} \left(1 - \sqrt{\frac{2}{h} - (1 + A)}\right)$$

Note that if  $A = 1$  then we get interior solutions in the interval:  $h \in (\frac{2}{3}, 1)$ , whereas if  $A = 0$  then we get interior solutions in the interval:  $h \in (1, 2)$   $d$  is always non-decreasing in  $h$ . In this case the position of the parties as a function of the closeness are depicted in the picture below



More certainty increases the spending directly and indirectly because a higher bias induces higher spending too.

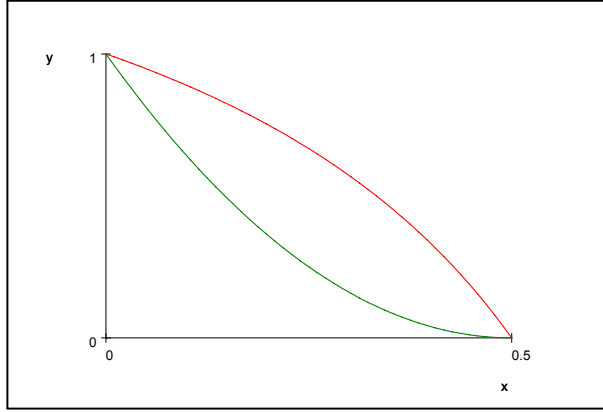
In the case of a convex gain  $g'' \geq 0$  I conjecture that the platforms can either be located at the extremes or in the middle, that is

$$d = r = \begin{cases} 0 & \text{if } h \leq \bar{h} \\ \frac{1}{2} & \text{if } h > \bar{h} \end{cases}$$

This is clear in the linear case, but in the strictly convex case it remains to be shown. The intuition is clear if you draw a convex and a concave gain function

$$g = 4 \left(d - \frac{1}{2}\right)^2 \quad (\text{convex})$$

$$g = \left(2 + \frac{1}{d-1}\right) \quad (\text{concave})$$



In the concave case you may want to move from zero because the loss is not very large and you may want to move from  $1/2$  because you would have a significant gain from doing so. In the convex case you incur a large loss if you move from zero and you don not gain much if you move from  $1/2$ . So in the convex case you are stuck at the extremes.

How does certainty impact the ex-post closeness of the election ?

4.0.1. *Second Order Conditions.* We need to check the second order conditions to see if the interior solution we found is in fact a maximum, in which case the second order condition should be negative.

The second derivative of the objective is

$$\begin{aligned}
& g''(1-F) + g' \left( \frac{1}{2} \right) f - (g(d) + g(r)) \left( \frac{1}{4} \right) f' - (g' + g') \left( -\frac{1}{2} \right) f + \\
& - \frac{1}{4} \left( g''(A + (d+r)) f + g'(f + (A + (d+r)) f') + (g' + g') \left( f - \frac{1}{2}(A + (d+r)) \left( -\frac{1}{2} \right) f' \right) \right) \\
& + (g + g) \left( f' - \frac{1}{2}(A + (d+r)) \left( -\frac{1}{2} \right) f'' - \frac{1}{2} f' \right)
\end{aligned}$$

The optimum obtained by the first order conditions implies that

$$\left( d + \frac{A}{2} \right) < \frac{1}{h}$$

Hence if  $g'' \leq 0$  and  $f'' \geq 0$  then we have

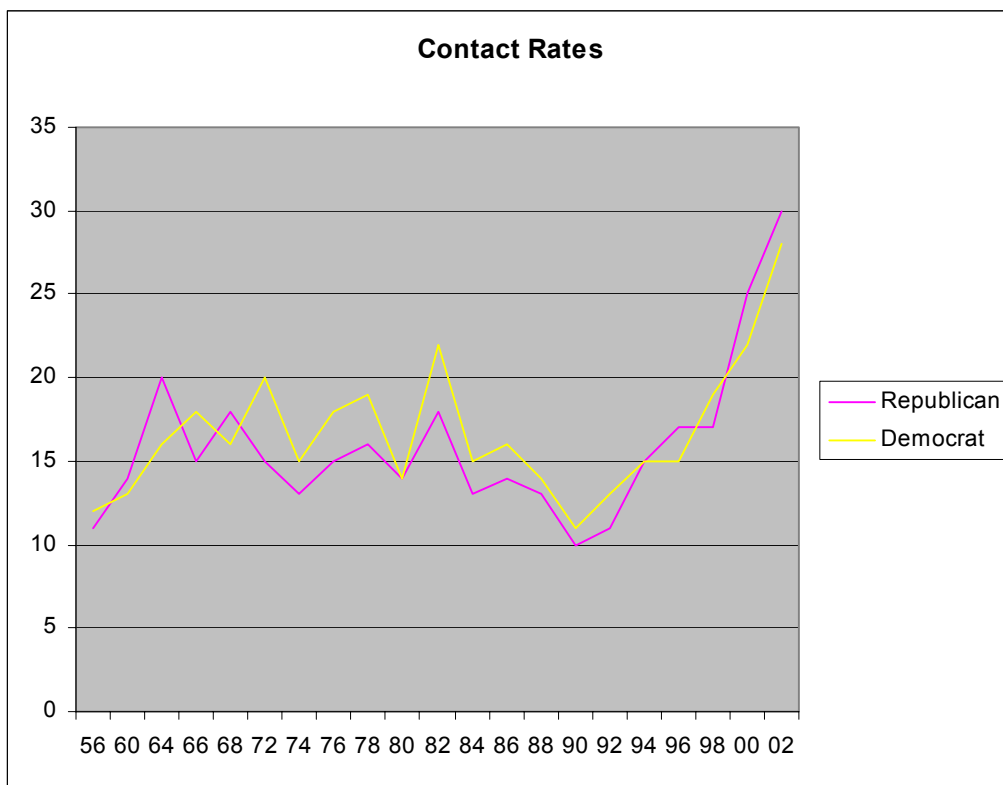
$$\begin{aligned}
& g'' \frac{1}{2} + g' \left( \frac{1}{2} \right) h + (2g') \left( \frac{1}{2} \right) h - \frac{1}{4} \left( \frac{g''(A + 2d)h + g'(h + A) +}{(2g')h + (2g) \left( \frac{1}{2}(A + 2d) \left( \frac{1}{2} \right) f'' \right)} \right) = \\
& g'' \left( \frac{1}{2} - \left( \frac{A}{2} + d \right) \frac{h}{2} \right) + g' \left( \frac{3}{4}h + A \right) - \frac{1}{4} \left( (A + 2d) \left( \frac{1}{2} \right) f'' \right) < 0
\end{aligned}$$

The same conditions hold for the party r as well.

Under these conditions the symmetric solution to the first order conditions is the unique symmetric equilibrium.

5. EMPIRICAL EVIDENCE

Here is some first empirical evidence on the amounts of spending by democrats and republicans in the same elections since 1956 in terms of percentages within study year. Parties seem to spend exactly the same amounts for any given election year



Source: National Election Studies

## REFERENCES

- [1] Aldrich, John “Rational Choice and Turnout,” *American Journal of Political Science* 37-1 (1993), 246-278.
- [2] Becker, Gary S. and Kevin M. Murphy, *Social Economics* (2000), Cambridge, Mass. and London: Harvard University Press.
- [3] Brennan, Geoffry and James Buchanan, “Voter Choice: Evaluating Political Alternatives,” *American Behavioral Scientist* 28 (1984): 185-201.
- [4] Downs, Anthony, *An Economic Theory of Democracy* (1957), New York: Harper and Row.
- [5] Durlauf, Steven N. and H. Peyton Young (editors), *Social Dynamics* (2001), Washington, D.C.: Brookings Institution Press, and Cambridge, Mass. and London: MIT Press.
- [6] Ledyard, John, “The Pure Theory of Large Two-Candidate Elections,” *Public Choice* 44 (1984), 7-41.
- [7] Morton, Rebecca, “A Group Majority Model of Voting,” *Social Choice and Welfare* 4:1 (1987), 17-31.
- [8] Morton, Rebecca, “Groups in Rational Turnout Models,” *American Journal of Political Science* 35 (1991), 758-76.
- [9] Noelle-Neumann, Elisabeth, “Turbulences in the Climate of Opinion: Methodological Applications of the Spiral of Silence Theory,” *Public Opinion Quarterly* 41 (1977), 143-158.
- [10] Noelle-Neumann, Elisabeth, “Public Opinion and the Classical Tradition: A Re-evaluation,” *Public Opinion Quarterly* 43 (1979), 143-156.
- [11] Palfrey, Thomas R. and Howard Rosenthal, “A Strategic Calculus of Voting,” *Public Choice* 41 (1983), 7-53.
- [12] Palfrey, Thomas R. and Howard Rosenthal, “Voter Participation and Strategic Uncertainty,” *American Political Science Review* 79 (1985), 62-78.
- [13] Schuessler, Alexander A., *A Logic of Expressive Choice* (2000), Princeton and London: Princeton University Press.
- [14] Shachar, Ron and Barry Nalebuff, “Follow the Leader: Theory and Evidence on Political Participation,” *American Economic Review* 89 (1999), 525-547.
- [15] Uhlaner, Carol, “Rational Turnout: The Neglected Role of Groups,” *American Journal of Political Science* 33 (1989), 390-422.