Party polarization and electoral accountability

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Abstract

In this paper we model the *interaction* between *parties* and *candidates* to highlight the mechanisms by which parties selecting candidates may discipline legislators. Parties are long-lived institutions providing incentives to short-lived candidates. Citizens have preferences over a multimentional policy space comprising an ideological and a monetary dimension. Candidates are policy motivated on the ideological dimension only and have opposing interest with respect to citizens on the monetary dimension. Policy motivation implies that candidates care more about winning elections the bigger the ideological distance from the candidate of the opponent party. Therefore, parties can use strategically polarization to provide incentives to candidates. Because of this strategic use, the polarization of the *political race* may diverge from the polarization of voters' *preferences*. In general, the polarization of the political race is a compromise between policy preferences of party members and electoral goals. Finally, when parties converge to the median voter, electoral accountability is inevitably compromised.

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1. Introduction

Parties are organizations acting as political intermediaries between candidates and citizens[1]. As such parties accomplish many important functions. First, they select candidates. Second, they can provide information on candidates and monitor the performance of elected politicians[2]. In this way parties may discipline legislators and have a chance to influence policies. In this work we do not challenge the view that parties are *necessary* to select and discipline candidates. Indeed starting from the observation that in all modern democracies parties perform those functions, we ask whether they are efficient in accomplishing those tasks, i.e. whether they choose candidates serving the interests of their constituents. Therefore this paper, modelling the *interaction* between *parties* and *candidates,* develops a formal analysis of the *candidate selection* process as a tool to control elected officials.

The question of the control of elected politicians is a typical delegation problem. Party members select candidates and constituents vote for candidates to whom they delegate the selection of the policy. When candidates are policy motivated, parties and citizens know that the winner of the electoral race will implement his most preferred policy[3]. However, whenever candidates are not purely policy-motivated, policies may be contractible and policy outcomes depend on incentives faced by legislators. We can illustrate a typical delegation problem with some examples. Consider a public policy that has two dimensions, an ideological dimension and a monetary dimension. The ideological dimension can be represented by the rules to follow in the provision of goods often provided by the state such as health or education[4], and the monetary dimension is the total cost of provision, paid by the citizens through taxes. Suppose that individuals are policy motivated on the ideological dimension only. This implies that this dimension is fixed and determined by their ideology. On the other hand, the monetary dimension is not fixed and the politicians' choice will depend on the incentives they will face. For example, if private firms are involved in various ways in the provision of these goods, they could lobby the incumbent legislator to set an high cost of provision and share the benefits. Or, alternatively if a policy can be realized at a low cost, but the legislator can divert money

[1]Often parties originate as associations of citizen bounded together by ideological concerns. The political science literature on parties goals is vast. For a brief overview on the goals of political parties see J. A. Schlesinger (1975).

[2]On the informational rationale for political parties see Caillaud-Tirole [2002] and Snyder-Ting [2000] .

[3]Policy-motivated candidates cannot commit to choose policies that do not reflect their preferences (Alesina, 1988).

[4]Examples of these rules can be universal versus restricted access, different procedures to regulate the access to the service , different allocations of total speding to differents aspects of the service, etc.

from the public funding to his pocket, then the legislator may choose a high level of taxes and capture the monetary surplus from the realization of the low cost policy. Finally, it could happen that the low cost policy requires a high level of effort from the legislator , while for a high cost project less effort is required. In this case, again the legislator can gain from choosing the high cost policy. In all these examples, the politician faces incentives inducing a policy against the interests of the constituents.

In this paper we show how parties help voters to solve this accountability problem. Parties are ideological and want to win elections in order to implement their ideological line. Candidates also seek for reelection for ideological reasons, however their interests diverge from those of parties and voters on non-ideological stances (such as the monetary dimension) that generate private benefits for the politician. Hence candidates may be willing to compromise reelection to obtain private benefits. In this framework we show that purely ideological parties using *only* their *candidate selection* strategy may be able to finally bring to power *accountable legislators*. Hence our paper shows that parties are an effective institutional device for electoral accountability even though they have *limited instruments* (candidate selection) and only care about a subset of the policy space (*ideology*).

The outline of the model is as follows. Citizens have preferences over a two-dimensional policy space comprising an ideological and a monetary dimension. Preferences on the ideological dimension are single-picked and citizens to the left and right of the median voter belong respectively to the left and right party. Parties select candidates for the electoral race. In particular, each median party member puts forward one candidate. Candidates are policy motivated on the ideological dimension only. The monetary dimension of the policy is the cost that is paid by all the citizens through non-distortionary taxation. The cost can be high or low. Any policy can be implemented at a low cost, but the legislator can arbitrarily decide to set a high cost that generates a private benefit for himself. A legislator obtaining private benefits hurts the interests of the citizens and hence can loose elections. Therefore, the question arises of how citizens - as voters and party members - can find mechanisms to discipline candidates. Party members are interested in accountable candidates because they can enhance the electoral prospects of future party candidates. Hence parties pursuing their primary long term objective (control of the ideological dimension of the policy), can provide incentives to legislators to forgo private benefit and seek for the election of their future nominees. In particular, since legislators are policy motivated on the ideological dimension of the policy , they care more about the election of their party's nominee the bigger the *ideological distance* from the candidate of the opponent party. Therefore parties, shaping the political race, can control the behavior of the

incumbent legislator.

The main insight from this analysis is that parties determine the polarization of the political race and they can use strategically polarization to provide incentives to candidates. Because of this strategic use of candidate polarization we obtain that the polarization of voters' *preferences* and the polarization of the *political race* do not necessarily go in the same direction. For a medium degree of polarization, the political race tends to reflect the polarization of preferences but for more extreme distributions we can obtain different results. Hence for example, when the *polarization* of preferences is *low*, parties may choose to increase the polarization of the political race in order to provide incentives to candidates. As a consequence median party members can select candidates that are more extreme then themselves. On the other hand, if the *polarization* of preferences is *high*, median party members may choose more moderate candidates than themselves. Hence, the candidates will not necessarily represent the preferences of either of the median party members nor of the median voter. In general, the polarization of the political race turns out to be a compromise between policy preferences of party members and electoral goals as parties use polarization strategically to provide incentives to candidates. Finally, when parties converge to the median voter, accountability on the monetary dimension is inevitably compromised.

This paper contributes to several streams of literature on delegation, accountability and political parties. First, the delegation problem in political games as been widely studied by *agency models* of political competition (Banks and Sundaram (1997), Persson, Roland and Tabellini (1997), Coate and Morris (1995)). However, those models abstract from parties and consider the question of the incentives as provided by *voters* only. We also consider the incentives provided by voters, but those incentives are analyzed in a game that also includes *parties*. The introduction of *parties* with their own internal process of candidates selection is the main contributions of this paper[5], that identifies precisely in the *candidate choice* the major mechanism by which parties as *long-lived institutions* can provide incentives to *short-lived candidates*. In this sense, this paper relates to Alesina and Spear (1988) that show how long-lived parties, making transfers to short-lived candidates, can prevent them from choosing policies detrimental to parties' interests. However, while in Alesina and Spear the role of parties is to make transfers, in our model the institutional role of a party is only to guarantee the existence of a mechanism of candidate choice that *always* select a candidate with certain

[5]Agency models typically assume that voters are identical as the heterogeneity of voters will make intractable the informational problem. In our model, indeed we simplify the informational structure (actions and types are observable) to focus on the heterogeneity of voters' preferences that translates into different party locations.

ideological preferences, and we show that this is sufficient to discipline short-lived politicians. Our view of *parties* as *long-lived organizations* providing incentives to short-lived candidates is also similar the concept of *firms* as *long-term organizations* employing short-term workers. The incentive problem for parties is very similar to the problem faced by firms that try to motivate workers. In the spirit of this literature, cooperation of short-term agents can be enforced when the game is repeated[6]. In our model, incumbents that cannot re-run for office are similar to old member of organizations about to retire as in Crémer (1989). However, contrary to Crémer (1989) we do not need overlapping generations since in our model the cooperation of an incumbent in his last term[7] is possible because after he leaves office, his own interests as a citizen will be still linked to the interest of the party through the ideological dimension of the policy (*externality argument*). Hence, provided that the politician lives at least one period after retiring from office, he can act in the interest of the organization in order to enjoy future ideological reward for his current policy choice as in Harrington (1992b). However, the focus of our paper is substantially different from Harrington (1992), since beside the ideological dimension, we introduce a contractible dimension on which an agency problem arises.

This paper also contributes to the vast literature on *party convergence* initiated by the seminal work of Downs (1957). The most celebrated result of convergence to the median voter in a model of spatial competition where parties only care about winning elections is difficult to reconcile with stylized facts. Hence, several theoretical explanations for party platform divergence have been proposed (Robertson (1976), Wittman(1983),Palfrey (1984), Calvert (1985), Bernhardt and Ingberman (1985), Londregan and Romer (1993) Ingberman and Villani (1993)). In particular, Alesina (1988) shows that when parties care about policies, *non-convergence* to the median voter is possible. In our model we show that non-convergence may also arise in a multidimensional policy space with perfect information. Furthermore, we show that *non-convergence* is *necessary* to obtain electoral accountability. Importantly and contrary the case of unidimensional policies, we show that when the policy is multidimensional, party polarization is not necessarily "bad" for the median voter. Indeed in our model we find that the median voter may prefer some degree of party polarization on the ideological dimension of the policy when this leads to accountability on the contractible dimension. Hence, our model on one hand provides a further explanation for the recognized non-convergence paradox, on the other shows that some degree of *party differentiation* on ideological issues may indeed be desirable as long

[6]For a survey on the repeated game literature, see for example, R. Aunmann, 1989, Survey of repeated games, in Essays in Game Theory and Mathematical Economics in Honour of Oskar Morgenstern.

[7]the equivalent of the oldest member of the organization in Crémer (1989).

as this may bring accountability on non-ideological stances. In this way our works builds a bridge between *agency models* of political competition and the literature on *party competition*, showing how *parties* competing for *elections* may help voters to obtain *accountable* legislators.

The plan of the paper is as follows. In section 2 we outline the model. In section 3 we discuss the main assumptions and we present an intuitive argument for the proof of the main results. In section 4 we solve the model and state the main propositions. In section 5 we discuss the results. In section 6 we summarize and conclude.

2. The model

2.1. Policy and preferences

The economy consists of infinitely lived citizens denoted $k = i$, that delegate to a legislator denoted $k = j$ the selection of a public policy. The legislator is a particular citizen, appointed to make policy choices for a limited number of periods, which becomes again an ordinary citizen when his mandates expires[8]. More formally, in this economy citizens live t periods where t belongs to the infinite set $T = \{0, 1, 2, 3....\}$ and for simplicity we assume that a legislator j can stay in place for one period only.

The policy is multidimensional and consists of an ideological dimension that we call *type* and a monetary dimension that we call *cost*. The *cost* of the policy is paid by the citizens. We denote $C \in \left\{C^L, C^H\right\}$, with $C^H > C^L$, the per capita cost paid by the citizens. The policy can always be implemented at a low cost, C^L. However, in each period the legislator has complete discretion on whether to set the high or the low cost. The *type* of the policy is a characteristic on which individuals have different tastes. We denote x_i the most preferred policy type of individual i and we suppose that there is a continuum of citizens distributed according to their most preferred policy *type* . We denote M the median voter of this distribution[9] that is assumed to be symmetric with respect to M. Hence, the median voter M splits the population in two groups of equal size according to their ideological preferences and these two groups constitute the two parties labelled respectively L and R. Therefore, parties in this model are associations

[8]This assumption reflects the idea that that elected officials know the date of their last mandate because of term limits or retirement age. Note however that as elections and policy choices go far beyond the last mandate of elected politicians, citizens face uncertainty about the last period where they will participate to an election or the policy will be delivered. Or put it differently, their time horizon is infinite. We will discuss later in the paper the relevance of this assumption.

[9]We are assuming that preferences of the individuals on the ideological dimension are single picked, hence citizens i can be ranked according to their most preferred policy type, x_i.

of citizens bound together by ideological concerns[10]. Given each party, we denote m_R and m_L the median party members of respectively party R and party L .

The two median party members select the two candidates for the electoral race[11]. The median voter M,voting for one of the two nominated candidates, appoints the legislator. Finally, the appointed policy maker during his mandate chooses the public policy.

We assume that policy makers are policy motivated on the ideological dimension. This means that once in office a legislator j chooses his most preferred policy type x_j . Therefore, the *policy choice* is a vector (C, x_j). We also assume that party members are ideological in the sense that they attach higher weight to the ideological dimension than to the monetary dimension of the policy. In other words, they care more about a subset of the policy space. Several observations justify this assumption. First, party charts typically specify only the party objectives on some specific issues. Second, it is widely recognized that party activists are particularly "ideological" i.e. they tend to have an exclusive interests over some dimension of the policy space. This characteristics of parties may cast doubts on the ability of those institutions to actually promote voters interests on a wider policy spectrum. Hence, it is important to understand if the ideological leaning of parties may actually compromise their ability to select candidates serving the electorate's general interests. As we will show, in our model this is not the case, since parties because of strategic considerations also influence other policy dimensions to which they attach low or even no weight.

We define $|x_k - x_j|$ the *ideological distance* between a generic individual citizen k and a legislator j . The median voter M suffers an ideological loss from the policy type equal to $-|x_M - x_j|$ and pays the percapita cost C. The legislator j obtains an ideological benefit[12] and pays the cost C, but he also receives a private benefit $B(C)$ that is increasing in his argument C. For simplicity we assume that $B\left(C^L\right) = 0$ and $B\left(C^H\right) = B > C^H$. Party members also obtain an ideological loss as citizens do, and they dislike the cost that enters in their utility function with a weight α, where $0 \leq \alpha \leq 1$. When $\alpha = 1$, party members dislike the cost exactly as any other citizen. On the other hand, when $\alpha = 0$, party members are purely ideological since they do not care about the cost of the policy.

[10]We assume the existence of parties. In other words, this is not a model of *party formation* but a more simple model of *party strategy*. Once formed parties are relatively stable institutions that can generate diverse candidate locations due to their internal democratic processes. Therefore, the focus of this paper is precisely on the effects of these internal mechanism of candidate selection on final policy outcomes.

[11]This feature of the model is meant to represent a democratic process of candidate selection inside the party. The logic of the model and the main results will go through if the candidate selection will be made by some other party member because of a different allocation of voting right inside the party in the nomination process.

[12]His ideological loss is zero since he choses his most preferred policy.

Therefore, given the ideological distance, $|x_k - x_j|$, the cost C and the private benefit $B(C)$,the per-period utility for the median voter M, the median party members, m_L and m_R, and legislator j can be written as follows:

$$v_M(C, x_j) = -|x_M - x_j| - C \tag{2.1}$$

$$v_{m_L}(C, x_j) = -|x_{m_L} - x_j| - \alpha C \tag{2.2}$$

$$v_{m_R}(C, x_j) = -|x_{m_R} - x_j| - \alpha C \tag{2.3}$$

$$v_j(C, x_j) = B(C) - C \tag{2.4}$$

Timing and strategies

The timing of the game is as follows. Initially there is a nomination process and one party's nominee is exogenously selected to become the first legislator. The nomination process works in the following way. In period $t = 0$, the two median party members, knowing which party's nominee will be the first exogenously selected legislator, announce simultaneously their candidates. The selection of candidates happens once and for ever during $t = 0$, before the first election[13]. This means that in $t = 0$ median party members decide the ideological position of all future party's candidates[14]. Hence, even if a legislator can stay in power for one period only, given the candidate selection process, he will always be replaced by a candidate with identical ideological preferences in any future electoral race . More formally, each median party member

[13]The fact that the candidate choice only happens in period $t = 0$, only means that in the future the candidates will be of the "type" intially chosen, not that they will be the same individuals. The selection of the type in $t = 0$, is a stylized representation of the the ideological party position adopted in the party constitution.

[14]As we consider a stationary environment, once party members choose optimally their candidates in the first period, they will not find optimal to change this choice later on even if they where allowed to reselect their candidates. Hence, under stationarity with no loss of generality, we carry on our analysis with candidate selection in the first stage only.

selects once and for ever at the beginning of the game a candidate within his own party [15]. We denote Γ_{m_L} and Γ_{m_R} the strategies of the two party members prescribing the selection of a candidate party member at the beginning of the game.

The appointed legislator chooses a policy in $t = 0$ and later on elections are called. In period $t = 1$ the first election takes place and the median voter decides which candidate to appoint. He can either appoint the candidate of the incumbent's party or he can appoint the opponent. The winner of the electoral competition becomes the new legislator who will choose the policy in period $t = 1$. Then again at the beginning of period $t = 2$ a new election will take place and the elected legislator choose the policy in period $t = 2$ and so on. More formally, in each period $t \geq 1$ we have an *election* followed by a *policy choice*. This is the *stage game* repeated for t periods. The players of this repeated game are the median voter M and the legislator j. At the end of each period t they observe each others actions. The action space for the median voter is the set of all possible voting decisions [16], and the action space for the legislator is the set of all possible policy choices[17]. Let h^t and e^t denote respectively the history of *past policy choices* and the history of *past electoral outcomes* up to the period t observed by all players. Then for each player, we can define a strategy that in each period maps all possible histories into actions. We denote $\sigma^t_{Mj}(h^t, e^t)$ the median voter's strategy that in every period t prescribes whether to elect or not the candidate of the incumbent legislator's party, given the history of past policy choices and past electoral outcomes[18]. Similarly, we denote $c^t_j(h^t, e^t)$ the legislator's strategy that in period t prescribes a policy (C_t, x_{j^t}), given the history of electoral outcomes and policy choices.

Payoffs

The outcome of the game induced by the strategies of the players is a *sequence* of *legislators and policies* denoted p^t , where $\{p^t = [j^t, (C^t, x_{j^t})]\}^{\infty}_{t=t+1}$. The total intertemporal payoff of each player is the sum of their per period utility, where the future is discounted according to the discount factor $\delta < 1$. Let's denote $V^t_k\left(\Gamma_{m_L}, \Gamma_{m_R}, \sigma^t_{Mj}(h^t, e^t), c^t_j(h^t, e^t)\right)$ the *intertemporal payoff* in period t of the generic player k. Let $v_k(p^t)$ be the utility the individual k associated to the outcome p^t induced by the strategies $\left(\Gamma_{m_L}, \Gamma_{m_R}, \sigma^t_{Mj}(.), c^t_j(.)\right)$. The *intertemporal payoff* of each player can be written in the following form:

[15]The set of candidates for each party coicides with his onw party members.

[16]The possible voting decisions are to vote for the candidate of the incumbent's party or to vote for the opponent.

[17]Since the ideological dimension is fixed , the possible policy choices in each period t are $C^t \in \left\{C^L, C^H\right\}$.

[18]This strategy is consistent with both retrospective and prospective voting.

$$V_k^t\left(\Gamma_{m_L},\Gamma_{m_R},\sigma_{Mj}^t\left(h^t\right),c_j^t\left(e^t\right)\right)=\sum_{\tau=t+1}^{\infty}\delta^\tau v_k\left(p^\tau\right)$$

Political equilibrium

The players of this game maximise their total intertemporal payoffs. We require the candidate choice be a Nash equilibrium of the one shot game where median party members select candidates. For the voting and policy rules in the repeated game, we require the equilibrium voting rule and the equilibrium policy to be subgame perfect Nash equilibrium. We restrict our analysis to pure strategies.

We define the equilibrium of this game as follows:

Definition 1. *A political equilibrium consists of the following elements:*

(a) pair of candidate that are best response to each other, given the equilibrium voting strategy and the equilibrium policy choice

(b) a voting strategy and a policy choice that are subgame perfect

To summarize, a political equilibrium consists in a *pair of candidates* appointed by the two parties to participate to the political race, in an *equilibrium voting strategy* for the citizens that elect the legislator and in an *equilibrium policy* chosen by the legislator .

3. Policy preferences, political salience and party loyalty

The median party member selects the candidate, while the median voter determines the result of the election. The median voter unambiguously prefers a low cost policy to an high cost policy. On the other hand, in terms of ideology he prefers a candidate that is as close as possible to himself. Ideally, a median voter would prefer a median candidate *type* choosing a low cost policy since his most preferred policy is $\left(x_M,C^L\right)$. On the other hand, a median party member would like to have his own most preferred policy type implemented for ever. To do so, he needs the support of the median voter. Clearly, when the low cost policy has been chosen, the candidate of the incumbent's party is more likely to be elected by the median voter. Note that, even when median party members do not care directly for the cost of the policy (i.e. $\alpha=0$), still their intertemporal payoff depends on the cost of the policy because of electoral considerations. Hence our main goal is to verify if electoral considerations are sufficient to

induce median party members to select candidates that will be accountable on the monetary dimension.

We can provide a simple intuition for how we will proceed to characterize an equilibrium with accountable legislators. An incumbent choosing a high cost policy obtains a private benefit B. Since he can stay in office only for one mandate, he can eventually obtain this private benefit for one period only . On the other hand, he obtains the maximum "ideological" benefit choosing the policy. Furthermore, after his mandate expires, he can still obtain a maximum ideological benefit provided that his party nominee (whose ideological preferences are the same of the incumbent) will win elections.

Therefore, we can note here a striking difference between the *private benefit* from the policy and the *ideological benefit*. A candidate can enjoy private benefits only during the limited number of political mandates he can serve[19], on the other hand he receives the ideological benefit (loss) for the rest of his life after he leaves office and this benefit will be maximum if a legislator of his own party will be in office. Clearly, if opponent party wins elections, then incumbents will suffer an electoral loss that will be higher the bigger the ideological distance between the candidates of the two parties. This ideological loss can be interpreted as the cost to an incumbent of future electoral defeats of his own party.

Because of the ideological loss, an incumbent whose political mandate cannot be renewed may still have an interest to enhance the electoral prospect of his own party. Hence, he may be willing to give up the private benefit from the policy to guarantee the electoral success of his party's nominees in future elections. In our model, this outcome is possible when the median voter is willing to reward (punish) with election (no election) the candidate of the incumbent's party when the incumbent has chosen the low cost (high cost), i.e. when the cost of the policy is politically salient[20]. Hence, the possibility to discipline an incumbent on the monetary dimension of the policy relies on two factors: (a) the existence of a credible *reward/punishment mechanism* and (b) the size of the *ideological loss*. The reward/punishment mechanism is the *voting rule* adopted by the median voter M. The ideological loss depends on the distance between the candidates of the two parties, i.e. the *polarization* of the political

[19]In our model for simplicity we report the results with one mandate, however all the results hold for any other finite number of mandates.

[20]Clearly, since the policy is multidimensional, the political equilibrium depends crucially on the salience of the two issues. Formally, the cost of the policy is politically salient when the following holds: $|x_M - x_j| + C^L \leq C^H$.

In words, the inequality says that the median voter M prefers a legislator j choosing the low cost policy to a legislator M choosing the high cost policy. Note that, with a continuum of types, there is always a candidate $j \neq M$ but close enough to M, so that the previous inequality holds. However, this candidate j needs not to be the median party member.

race. We can immediately note that if the two parties would converge to the median voter M, than an incumbent will never suffer an ideological loss being replaced by an "identical" challenger. Therefore, if a political equilibrium where the incumbent is accountable on the monetary dimension exists, in this equilibrium parties must *not converge* to the median voter.

In the next sections we will characterize the political equilibrium in two steps. First, we will show that there is a credible reward/punishment voting rule that the median voter can adopt toward the incumbent. Second, given this credible punishment/reward rule, we will show there are cases where parties, choosing candidates, do not converge to the median voter. In some of these non-convergent locations, incumbents are accountable on the monetary dimension of the policy. Hence we show how parties, as long lived institutions, can use strategically their location on the ideological stance so as to provide incentives to short-lived candidate to forgo current private benefit in order to avoid future ideological losses.

4. Political equilibrium

4.1. Voting rule and policy choice

At the beginning of the game one of the two parties' nominee is exogenously selected to become the first legislator. Without loss of generality suppose that the first incumbent legislator j belongs to party L and his challenger j' belongs to party R. As all future nominees of a party share the same ideological preferences, for simplicity we will call *"incumbent type"* all future nominees of the incumbent's party. In terms of ideological location, the two candidates can be either symmetric with respect the median voter M or asymmetric. We can summarize the possible candidates locations in three cases:

• *case 1*: the challenger j' is closer to the median voter then the incumbent j. For simplicity, let's assume that $j' = M \neq j$

• *case 2*: the incumbent j is closer to the median voter then the challenger j'. For simplicity, let's assume that $j = M \neq -j$

• *case 3*: the incumbent j and the challenger j' are equally distant from the median voter.

We illustrate in detail how to solve the game in *case 1.* Then the same logic will be used for the other two cases. Remember that we are interested in an equilibrium where the incumbent legislator chooses the low cost policy, i.e. he is accountable to voters on the monetary dimension.

The incumbent legislator $j \neq M$ chooses the policy maximising his total intertemporal payoff. He chooses his most preferred policy type x_j while in power and then he resigns.

However if in the following period his party's nominee is elected, the incumbent legislator j will obtain again his most preferred policy type. Hence, the appointment of the incumbent's party nominee can be seen as a "reward" for the past incumbent's policy choice, and the replacement with the challenger as a "punishment". The median uses his voting strategy to reward or punish the incumbent for his decision on the cost dimension. However, since the policy is multidimensional, a reward/punishment scheme contingent on the cost is not necessarily optimal when the incumbent has ideological preferences different from the median voter. On one hand the median voter would like to reward the incumbent's party nominee to obtain the low cost policy, on the other he may be tempted to replace him with a challenger with closer ideological preferences. Since punishments and rewards may be costly to carry on we need to verify that they are credible, or put it differently, subgame perfect.

We can represent this game with a game three where in each period t a legislator decides whether to choose C^L or C^H and the identity of the legislator at each decision node from period $t = 1$ onward can be either $j = M$ or $j \neq M$, depending on the equilibrium voting strategy of the median voter M.

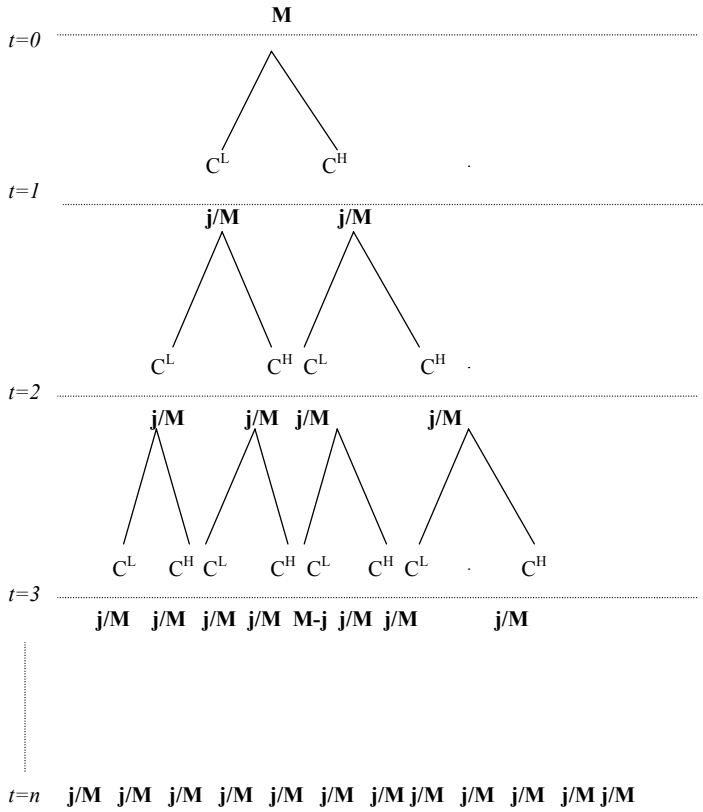

Figure 1 - game tree (a)

Suppose that the median voter M follows the strategy "*reappoint the incumbent type j if j has chosen the low cost policy, appoint the challenger j' if the incumbent j has chosen the high cost policy*". We have to prove that punishment and reward are sub-game perfect. We can easily prove the subgame perfection of the punishment:

Lemma 1. *Suppose that in $t = 1$ there are two candidates $j \neq M$ and $j' = M$ where j is the incumbent type and j' is the challenger, then the voting strategy prescribing to not elect the incumbent type j if the incumbent j has ever chosen C^H in the past, is an equilibrium voting strategy.*

Proof. See Appendix.

Note that, given this equilibrium voting strategy, in the first mandate an incumbent legislator j choosing the high cost policy, is permanently replaced by the challenger M. Also, from second mandate onward the legislator M will always choose the high cost policy, since he cannot be punished for this. Hence, we can conclude that from period $t = 1$ onward the policy (C^H, x_M) will be chosen. Given *lemma 1*, we can illustrate our game with the following figure. It is useful to call G_t the game starting after any deviation from the low cost policy. Since we know the solution to the game G_t, we can easily compute the payoff of each player in this game. In the game three of the figure 2, the bold line illustrates the equilibrium path whenever the incumbent j chooses the high cost policy:

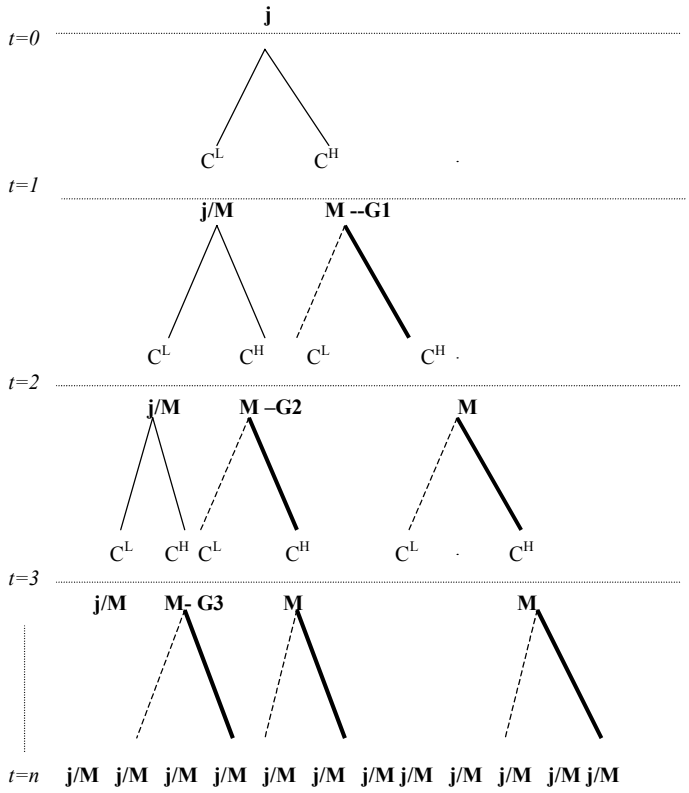

Figure 2 - game tree (b)

Let's consider now the part of the game three following the low cost policy. The argument proving sub-game perfection of the reward for the low cost policy is more subtle. The credibility problem of future rewards comes from the fact that the median voter M is closer to the challenger $j' = M$ than to the incumbent $j \neq M$ on the ideological dimension. Hence, once the incumbent has chosen the low cost policy, the median voter may be tempted to replace the incumbent type with the challenger.

To prove that re-election is credible, we first *conjecture* that rewarding the incumbent type with reelection if the incumbent has chosen C^L is an equilibrium voting strategy, and then we show that, under some parametric restrictions this conjecture is correct. In particular, we can prove that for the median voter it is optimal to vote according to this conjecture, provided that this conjectured voting strategy induces the incumbent to choose the low cost policy.

Assume that the *conjectured voting strategy,* denoted $\sigma^*_{Mj}(.)$ for the incumbent with respect to the median voter decision is "*reappoint the incumbent type j if j has always chosen C^L in the past and replace the incumbent type j with the challenger j' if j he has ever chosen C^H in the past*" .

The next lemma state the conditions under which the incumbent in $t = 0$, given the conjectured voting strategy $\sigma^*_{Mj}(.)$finds optimal to choose the low cost .

Lemma 2. *Given the candidates M and j , the conjectured voting strategy $\sigma^*_{Mj}(.)$ and any future policy sequence chosen by successive legislators, in period $t = 0$ the incumbent legislator j will choose the policy $\left(C_0^L, x_j\right)$ if and only if $|x_j - x_M| \geq \frac{B}{\delta}$*

Proof. See Appendix.

Intuitively the lemma says that if the future ideological benefit from the low cost policy $\delta |x_j - x_M|$ is higher than the current private benefit loss B, then the first incumbent chooses the low cost policy. Importantly, the future ideological benefit depends on the *polarization* of the political race, i.e. the ideological distance between the incumbent j and the challenger M. Given the *stationarity* of the game, this result holds for any incumbent legislator in place for the first time at any period t.

Finally, we can prove that under the same parametric restrictions of *lemma 2*, the conjectured voting strategy is an equilibrium voting strategy:

Lemma 3. *Suppose that in $t = 1$ there are two candidates $j \neq M$ and $j' = M$, where j is the incumbent and j' is the challenger and assume that the cost is politically salient, i.e. $|x_M - x_j| + C^L < C^H$. If $|x_j - x_M| \geq \frac{\beta}{\delta}$,then the voting strategy prescribing to elect the incumbent type j if j has always chosen C^Lin the past, is an equilibrium voting strategy.*

Proof. See Appendix.

Intuitively, when the conjectured strategy implies an equilibrium where the incumbent j is accountable in every period, for the voter it is optimal to vote accordingly since if he doesn't, in any future period accountability will be lost. Note that the assumption of political salience is crucial to obtain this result, since when C is politically salient the median voter prefers an equilibrium where $j \neq M$ is accountable to an equilibrium where $j = M$ is not accountable.

As a final but important remark, the fact that the game is repeated is important. If the game had a final period, by backward induction we can easily show that the voter could not credibly reward the incumbent for choosing the low cost policy and as a consequence the low cost policy will never be chosen. Proceeding backward, in the final period t, because no future election will be called, any j legislator would choose the high cost policy. If this is the case, an incumbent legislator in period $t - 1$, knows that the median voter will never "reward" him for choosing a low cost policy since he will certainly prefer the challenger M choosing C^H to the

successor j choosing C^H. As a consequence, the incumbent j in $t-1$ will choose C^H. Hence, proceeding backward up to period $t=0$ we can conclude that the policy C^H is the equilibrium policy in every period $t \geq 0$.

Having characterized the equilibrium of the repeated game when the two candidates are asymmetric and the challenger is closer to the median voter than the incumbent (*case 1*), we can now easily solve the repeated game in the other two cases where the incumbent is closer to the median voter than the challenger (*case 2*) and where candidates are symmetric (*case 3*).

We can represent the game of *case 2* with the following picture :

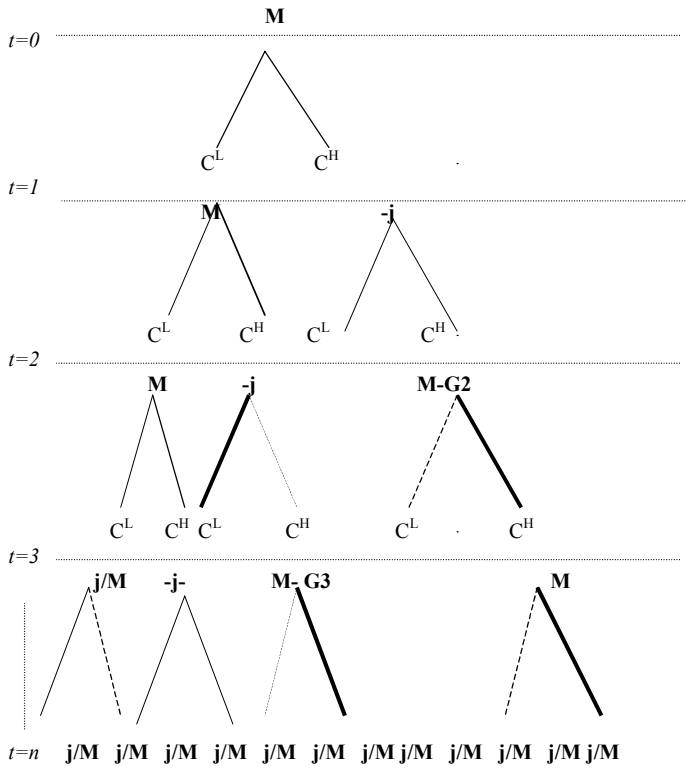

Figure 3 - game tree (c)

Note that, the game starting after each deviation of the incumbent legislator M is identical to the game we solved in *case 1.* Therefore, for the solution the same logic applies[21].

Lemma 4 states the conditions for the incumbent legislator $j = M$ to be accountable to voters:

[21]We prove in the appendix that, when *lemma 3* holds, the equilibrium voting strategy in case 2 coincide with the equilibrium voting strategy $\sigma^*_{Mj}(.)$ in case 1.

Lemma 4. *Given the candidates $j = M$ and $j' \neq M$, the conjectured voting strategy $\sigma^*_{Mj}(.)$ and any future policy sequence chosen by successive legislators, the incumbent legislator M in $t = 0$ will choose the policy $\left(C^L_0, x_M\right)$ if and only if $|x_M - x_j| \geq \frac{B+\delta(C^H-C^L)}{\delta}$.*

Proof. the lemma is straightforward since we simply need to impose that the following inequality holds: $\delta C^H + \sum_{t=2}^{\infty}\delta^t\left(|x_M - x_{-j}| + C^L\right) \leq -B + \sum_{t=1}^{\infty}\delta^t\left(|x_M - x_{-j}| + C^L\right)$.

Note that, when the incumbent is $j = M$, the threshold level $\frac{B+\delta(C^H-C^L)}{\delta}$, is bigger than the threshold level we found in *lemma 2* where the incumbent is $j \neq M$ [22].

We conclude with *case 3*, where the candidates j and j' are symmetric with respect to the median voter. In this case the equilibrium voting strategy is the same as in the asymmetric case where the incumbent is $j \neq M$ and the challenger is $j = M$ (*case 1).* Consequently, the condition insuring that the incumbent legislator will choose the low cost is the same of *lemma 1* (proof in appendix).

4.2. Party location and accountability

In this section we show how parties can choose accountable candidates, given the credible reward/punishment voting rule illustrated in the previous section. Since the most important ingredient in the characterization of the political equilibrium is the *polarization* of the political race, i.e. the ideological distance between the two parties' candidates, the following definition is useful to lay out the main results:

Definition 2. *Let $\sigma_{Mj}(.)$ be a credible voting strategy that prescribes re-election of the incumbent type when the incumbent j has chosen the low cost and no re-election if the incumbent has chosen the high cost policy. We define* "accountability threshold" *the minimum ideological distance between any incumbent j and any opponent j', such that , given the voting strategy $\sigma_{Mj}(.)$, the incumbent candidate j chooses the low cost policy, $\left(C^L, x_j\right)$.*

Corollary 1. *given any two candidates j and j', the* accountability threshold *is equal to $\frac{B+\delta(C^H-C^L)}{\delta}$.*

[22] This result is due to the fact that the incumbent M choosing C^H is permanently replaced by the challenger j choosing C^L. Therefore, although the incumbent M in the future (as a citizen) will suffer a loss on the ideological dimension, still on the monetary dimension he will get his most preferred policy C^L.

Proof. Proof: corollary 1 follows from definition 1 and lemmata 1-4.

Using now *corollary 1* and *definition 2* we can state the main results of the model. Remember that, when $\alpha < 1$ party members care less than citizens about the cost of the policy. Clearly, the less parties care for the cost of the policy, the more difficult it is to obtain an equilibrium where they select accountable legislators. In particular, the most difficult case for accountability arises when parties are purely ideological, i.e. $\alpha = 0$. Hence, if we can show that purely ideological parties can choose accountable candidates, then *a fortiori* the results of the model will hold even for less ideological candidates. In the next proposition we prove how purely ideological parties can select accountable legislators. The conditions we find are then necessary for accountability when $\alpha = 0$ and sufficient for $\alpha > 0$.

Proposition 1. *Suppose that the party of the first incumbent is L and the opponent is R. If the distance between the median party member m_L and the median voter M is bigger or equal than the "accountability threshold", then the party L runs the median party member m_L, wins the election against any candidate of the opponent party R, and chooses the policy $\left(C_t^L, x_{m_L}\right)$ in any period $t \geq 0$.*

Proof. See Appendix.

The previous proposition characterizes the political equilibrium highlighting the role of policy preferences. When the distance between the median party member and the median voter is sufficient to induce the incumbent legislator m_L to choose the low cost policy, then the incumbent can be held accountable even if the opponent party would converge to the median voter in his candidate choice! Hence, in this case the polarization of policy preferences has two implications. The median party member will realize his most preferred policy. The median voter will not receive his most preferred policy on ideological grounds but he will discipline the incumbent on the monetary dimension of the policy.

If we assume that, although the cost is politically salient, the ideological distance between the median party member of the incumbent party, m_L, and the median voter, M, is not sufficient to guarantee accountability, then we obtain the following results:

Proposition 2. *Suppose that the party of the first incumbent is L and the party of the opponent is R. Let L_C be the set of candidates $j \in L$ more centrist than the median party member m_L and let R_C be the set of candidates $j' \in R$ more centrist than the median party member m_R. Let j_{E1} be the first extreme candidate such that $\mid x_{j_{E1}} - x_M \mid = \mid x_{m_L} - x_{m_R} \mid$ and let's define the following threshold level for the discount rate, $\delta_j = \frac{\left|x_{m_L} - x_j\right|}{\left|x_{m_L} - x_{j'}\right|}$, where $j = \{j_{E1}, m_L, j \in L_C\}$. Suppose that the distance between the median party member m_L and the median voter M is strictly less than "accountability threshold". then the following holds:*

- *when the distance between the two median party members m_L and m_R is strictly bigger than the "accountability threshold" and $\delta < \delta_{j_{E1}}$, then the party L runs the extreme candidate j_{E1} and the opponent party R runs the median voter M. The candidate of the incumbent party, j_{E1}, chooses the low cost policy and wins the elections in every period $t \geq 1$.*

- *When the distance between the two median party members m_L and m_R is weakly smaller than the "accountability threshold" two cases arise. If $\delta > \delta_j$ for every $j \in L_C$, then party L runs the median party member m_L, party R runs the most extreme candidate in R_C. The incumbent m_L chooses the high cost policy and looses the elections in every period $t \geq 1$. On the other hand, if $\delta_j < \delta < \delta_M$ for every $j \in L_C$, then party L runs $j = M$, party R runs $j' = M$. The incumbent M chooses the high cost policy and wins the elections in every period $t \geq 1$;*

Proof. See Appendix.

Intuitively, the proposition states that if the distance between the median party member of the incumbent party, m_L, and the median voter, M, is not sufficient to guarantee accountability because it is inferior to the accountability threshold , but the distance between the median party member of the incumbent party, m_L, and the median party member of the opponent party m_R is sufficient for accountability, then the party of the first incumbent can delegate the policy choice to a more extreme policy type than himself so as to obtain accountability. In other words, when the opponent party locates his candidate on the median voter M, the incumbent party, choosing a more extreme type than himself, can locate sufficiently far away from the median voter to obtain an accountable candidate. However, this is an optimal strategy provided that the median party member is closer to the first extreme candidate that guarantees accountability than to the median voter M. In fact when m_L delegates to a more extreme candidate, he is facing an ideological loss, although this will guarantee him to win the elections. Clearly if the

median party member is closer to the median voter than to the extreme candidate, than for him it is better to loose the election and have a median opponent in place than to win the election with an extreme candidate. Geometrically, we can show that the when the distance between the median party members is sufficient for accountability, then the distance between the median party member and the first extreme candidate that will be accountable is not bigger than the distance from the median voter. Furthermore, the median party member could run a candidate m_L, obtain his most preferred policy in one period and then be replaced for ever by the opponent j'. Hence, for delegation to the more extreme j_{E1} to be optimal, it must be that the median party member prefers to give up his most preferred policy in the first period in order to obtain in all future period the policy chosen by his candidate j_{E1} instead of the policy of the opponent j'. When the distance between the median party members is either necessary or not sufficient for accountability, than the median party member is closer to the median voter than to the first extreme candidate that will be accountable. Therefore, in this case, delegation to a more extreme candidate to obtain accountability is never optimal. On the other hand, when the median party member is closer to j_{E1} than to M, delegation to a more extreme is optimal when δ sufficiently small, i.e. , $\delta < \delta_{j_{E1}}$. If this is not the case, then either the party of the first incumbent prefers to run the median party member, obtain his most preferred policy in one period and then be replaced by the opponent, or there is full convergence the median voter M. In both cases , the high cost policy will be chosen[23].

Finally, let's consider the following case. Suppose that the distance between the median party member m_L and the median voter M is such that if m_L ,choosing the low cost policy, could be reelected, then he would prefer to choose the low cost policy and be in office instead of obtaining the private benefits and loose the possibility to determine the ideological dimension of the policy. In other words, the incumbent legislator cares enough about the ideological dimension to forgo private benefits. However, suppose that, given the distance between m_L and M , the median voter M would rather prefer a median candidate choosing the high cost policy than a the more extreme candidate m_L choosing a low cost policy. In this example, the distribution of preferences is such that the polarization is enough to provide incentives to legislators but the distribution is too polarized for the median voter to reward a legislator for choosing the low cost policy. Given this distribution of preferences, we can show the following result:

[23]The characterization of the political equilibrium when $\delta > \delta_{j_{E1}}$ is reported in appendix.

Proposition 3. *Suppose that the party of the first incumbent is L and the opponent is R. Assume that* $|x_M - x_{m_L}| + C^L > C^H$ *but there exist a more centrist candidate* $j \in L_C$ *such that* $|x_M - x_j| + C^L < C^H$ *. Let* j_{C1} *be the first centrist candidate such that* $|x_M - x_{j_{C1}}| + C^L < C^H$*. Let's define the following threshold level for the discount rate,* $\delta_C = \frac{|x_{m_L} - x_{j_{C1}}|}{|x_{m_L} - x_{j'}|}$*. If the distance between the candidate* j_{C1} *and the median voter M is bigger or equal than the accountability threshold and* $\delta < \delta_C$*, then party L runs* j_{C1} *which chooses the low cost policy and beats any opponent in every period* $t \geq 1$*.*

Proof. See Appendix.

Intuitively the proposition says that if the median party member,m_L, is "too far" from the median voter, M, and there is a more centrist candidate j_{C1} that can guarantee accountability, then the median party member prefers to delegate to the less extreme candidate. It is also trivial to show that when δ does not satisfy the restriction required in *proposition 3*, we obtain the same candidate locations described in *proposition 2* (proof in appendix).

Using the main propositions, we can draw some conclusions on the link between *distribution of preferences* and *party location*. From *proposition 1* we learn that, if the distance between the median party member and the median voter is sufficiently high to provide incentives to legislators to forgo private benefits in order to avoid ideological losses, then accountability can be achieved. On the other hand, if the distance between the median party member and the median voter is not enough to induce accountability, then the median party member may solve the accountability problem choosing a more extreme policy type than himself. Hence, to gain an everlasting control on the ideological dimension of the policy the median party member may find optimal to delegate the policy choice to a more extreme candidate. In this case we may say that parties use polarization of candidates as a *discipline device*. However polarization as a discipline device may work only if the median party member gains more from having in power his candidate (although more extreme than himself) than he would from the candidate of the opponent party member. In fact, if he is closer to the median voter than to the first extreme candidate that can guarantee accountability, then he would rather prefer the median candidate of the other party than the extreme candidate of his own party (*proposition 2*). Finally, when the preferences are very polarized so that the median voter is not willing to reelect the median party member of the incumbent party, the incumbent party to win the elections is again willing to sacrifice partly his ideological preferences choosing a less extreme candidate than himself.

5. Party competition and electoral discipline

Our model allows us to analyze the effects of *party competition* on the two dimensions of the policy. The main objective of parties is to obtain an ideological policy as close as possible to the ideological preferences of the majority of the party members. The median party member has the possibility to choose a candidate with his same preferences. However, as the control of the ideological dimension of the policy over time depends on the elections, when parties choose candidate they have also to consider the preferences of the median voter. The median voter decides whether to reappoint an incumbent type nominee evaluating the incumbent's policy choice on the ideological dimension as well as on the monetary dimension. The salience of these two dimensions is crucial for the equilibrium policy outcome. In particular, if the ideological dimension is politically salient, the median voter will always obtain his most preferred type of policy but he will always pay a high cost for it. In this case the party competition cannot help to solve the *accountability* problem. On the other hand, if the monetary dimension of the policy is politically salient, then the *party competition* can help to *discipline* legislators. However, the ability of the electoral mechanism to discipline legislators depends on the distribution of preferences on the ideological dimension, as the distance between median party members and median voter plays a crucial role in the determination of the political equilibrium.

Our theoretical framework also provides useful insight on the function of parties. In our model, the role of a party as a long-lived institutions is to insure the selection of a precise *type* of candidate, i.e. the implementation of a given ideology. Hence, even if candidates are short-lived and cannot stay in place for ever, still because they are policy motivated on the ideological dimension of the policy, they can be loyal to the party so as to obtain a candidate with their same preferences in place (at least for one period) in the future. Therefore, we conclude that party loyalty due to policy preferences is a key factor for legislators' accountability. Note that this mechanism of party discipline is very different from the mechanism proposed by Alesina and Spear (1988) where the role of parties, as long-lived institutions, is to make monetary transfers to candidates after they leave office. Those transfers can be either pension payments or, most often, career paths inside the administration associated with high monetary reward. In other words, candidates loyal to the party are rewarded with positions ensuring high monetary returns when they leave office. Therefore, *de facto* the role of parties is to make future monetary transfers. In our model instead, the role of parties is to select *ideologies* that will be implemented on the long term. Therefore, provided that candidates are policy motivated on those dimensions, there is no need to make future monetary transfers to induce them to be loyal to the party,

since their interests will be aligned with the party's interests through the ideological dimension of the policy.

Note also that although there is a similarity between parties and firms as long-lived organization that try to discipline short-lived agents, still parties are long-lived institutions with a peculiar characteristic. As they determine the ideological dimension of the policy, which is a non-monetary benefit that former candidates receive even after they leave office, then parties may be able to obtain cooperation from policy motivated incumbents even when their mandate cannot be renewed, provided that the future ideological benefit from cooperation is bigger than the current private benefit sacrifice. This mechanism of discipline is similar to Harrington (1992b) where incumbents care about policy after they live office. However the focus of our model is different since the ideological location affects electoral outcomes jointly with the *non-ideological* dimension, on which parties have limited control. Hence, our paper on one hand generalizes the party location problem to a multidimensional context. On the other shows that, under certain distributions of policy preferences, even an agency problem arising on non-ideological issues can be solved by party competition. Therefore, we conclude that some degree of party differentiation on ideological issues is not only a characteristics often observed in real world elections, but may indeed be a desirable property insofar as policy motivated incumbents are more likely to be accountable to the electorate when they face the threat of political rivals with substantially different ideological agendas.

6. Conclusions

Parties are associations of citizens that play a fundamental role in representative democracies. They are the primary actor of the electoral competition since they choose the candidates of the political race. Furthermore, choosing candidates, parties may influence policies. Clearly, how parties can actually affect policies through the selection of candidates crucially depends on the candidates preferences and on the relationship between parties and candidates.

In this paper we propose a model where legislators choose a multidimensional policy. We assume that candidates are policy motivated on the ideological dimension of the policy but they are not on the monetary dimension. In particular, a legislator can obtain private benefits from a policy that goes against the interests of the citizens on the monetary dimension. In this set up, we show how parties as long lived institutions can provide incentives to short-lived candidates to reject private benefits and choose policies in the interests of the majority of their constituents.

The role of a party as a long-lived institutions is to insure the selection of a precise type of candidate, i.e. the implementation of a given ideology. Hence, even if candidates are short-lived and cannot stay in place for ever, still because they are policy motivated on the ideological dimension of the policy, they can be loyal to the party so as to obtain a candidate with their same preferences in place in the future. However, loyalty to the party is costly for incumbent legislators, since in order to enhance the reelection of future nominees of their own party they have to forgo private benefits. Hence, an incumbent legislator is willing to forgo current private benefits for future ideology provided that the ideological gain is bigger than the benefit loss. The ideological gain depends on the ideological distance between the incumbent legislator and the challenger of the opponent party, since the bigger the distance, the higher the loss from being replaced by the challenger. As a consequence, parties can use strategically the polarization of the political race to provide incentives to candidates.

The strategic use of party polarization implies that the polarization of *preferences* and the polarization of the *political race* do not necessarily go in the same direction. In general, the polarization of the political race turns out to be a compromise between policy preferences of party members and electoral goals. Furthermore, party converge to the median voters is always bad for accountability as ideologically identical candidates do not have incentives to reject private benefits.

This model is a further step toward the understanding of the role of political parties in representative democracies. We propose a relatively simple model of party competition where median party members select candidates and party polarization is the only instrument that parties can use to provide incentives and win elections. However, parties can typically use other instruments such as campaign spending to influence electoral outcomes. Also parties are complex institutions that require consistent funding for their ordinary functioning. Transfers from candidates to parties can be an important source of party financing. Therefore, as money seems to play an increasingly important role in politics, we may expect that parties may choose their candidates looking not only to their ideological preferences but also at their ability to obtain transfers from the private sector. Hence, the effect of fund raising ability on the nomination process is an interesting question that goes beyond the scope of this paper but certainly deserves further investigation.

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Appendix

a) Equilibrium voting strategy

Lemma 1

Proof:

To prove lemma 1, let's start with the part of the game three following C^H in the $t = 0$. First, we show that the voting rule "*replace the incumbent type with the challenger if he has chosen the high cost policy*" is an equilibrium voting strategy in $t = 1$. The argument proving this result goes as follows. In period $t = 0$ the incumbent legislator j has chosen C^H. When elections are called, the median voter M will face a legislator $j \neq M$ who has chosen the high cost policy and a challenger $-j = M$ who can choose any policy $C \in \left\{C^L, C^H\right\}$. Clearly, since $C < (x_M - x_j) + C^H$ for any $C \in \left\{C^L, C^H\right\}$, the median voter can credibly carry on his threat and punish the incumbent.

Let's continue now along the game three and let's check if the same strategy is an equilibrium voting strategy for any $t \geq 1$. Given $\sigma^1_{Mj}\left(h^1\right)$, we know that in period $t = 1$ the incumbent will be replaced by the challenger $-j = M$. Assume again, now that in period $t = 2$, the voter follows the strategy $\sigma^2_{Mj}\left(h^2\right) = 0$ if $h^2 = \left\{\left(C^H, x_j\right), \left(C^H, x_M\right)\right\}$. In other words, he punishes the incumbent M if he chooses C^H . We can immediately show that in period $t = 2$, the voter cannot credibly punish the incumbent M. The reason for this is that the two types of candidates, j and M, have both chosen C^H in the past, but M is closer to the median voter on ideological ground. By the same argument, this is also an equilibrium voting strategy for any subsequent period $t > 1$. Also, in our game three, the strategy that prescribes to permanently replace the incumbent $j \neq M$ with the challenger $-j = M$ whenever j chooses C^H is an equilibrium voting strategy. Therefore we conclude that if in $t = 1$ there are two candidates $j \neq M$ and $-j = M$, where j is the incumbent and $-j$ is the challenger, then the voting $\sigma^t_{Mj}\left(h^t\right) = 0$ if j has ever chosen C^H in the past, is an equilibrium voting strategy.

Lemma 4

To prove lemma 4, we consider all the possible candidates' location (cases 1-3).

Claim: the conjectured voting strategy:

$\sigma^t_{jj'}\left(h^t\right) = 1$ if the legislator j has always chosen C^L in the past;

$\sigma^t_{jj'}\left(h^t\right) = 0$ if the legislator j has ever chosen C^H in the past;

is an equilibrium voting strategy for any j and j'.

• case 1

Let $j \neq M$ be the incumbent legislator in $t = 0$ and let $-j = M$ be the challenger. We show that if $(x_j - x_M) \geq \frac{\beta}{\delta}$ claim 4 is true. We have already shown that the punishment, $\sigma^t_{Mj}\left(h^t\right) = 0$ if the legislator j has ever chosen C^H in the past, is an equilibrium voting strategy.

Now we show that the strategy: $\sigma^t_{Mj}\left(h^t\right) = 1$ if the legislator j has always chosen C^L in the past

is an equilibrium voting strategy.

Suppose that this is not the case and suppose that after the legislator j has chosen C^L in $t=0$, the voter does not carry on his reward. If this happens, the equilibrium will permanently switch to a legislator M choosing C^H for ever. Hence, the payoff for the median voter who does not reward a legislator j will be:

$$V_M^1\left(\sigma_{M_j}^1\left(C^L, x_j\right)=0, c_j^t\left(e^t\right)\right)=-\left|x_M-x_j\right|-C^L-\sum_{t=1}^{\infty} C^H$$

On the other hand, if the median voter carries on his reward, given that $(x_j - x_M) \geq \frac{\beta}{\delta}$, the equilibrium will be an incumbent j choosing C^L. The payoff from carrying on the reward is:

$$V_M^1\left(\sigma_{M_j}^1\left(C^L, x_j\right)=1, c_j^t\left(e^t\right)\right)=-\sum_{t=0}^{\infty}(\left|x_M-x_j\right|-C^L)$$

therefore given the assumption of political salience the voter will prefer to carry on the reward in the second mandate. If we move to $t=1$, under the same parameter restrictions, we have exactly the same payoffs from carrying on the reward or deviating. Hence we conclude that the conjectured voting strategy is an equilibrium voting strategy.

• case 2

Let $j=M$ be the incumbent legislator in $t=0$ and let $-j \neq M$ be the challenger. We show that claim 4 is true. The proof of the first part of the conjecture is trivial since the median voter always prefer a legislator $j=M$ who chooses C^L to any other legislator $-j \neq M$ choosing any policy C. For the second part, i.e. $\sigma_{MM}^t\left(h^t\right)=0$, if the legislator j has ever chosen C^H in the past, note that if the median voter does not carry on the punishment, then from $t \geq 1$ onwards the equilibrium will permanently be the incumbent $j=M$ choosing C^H. On the other hand, if *lemma 5??* holds, i.e. if $(x_M - x_{-j}) \geq \frac{\beta+\delta C^H}{\delta}$, then carrying on the punishment, the equilibrium will be the incumbent $j=M$ choosing C^L, hence clearly the median voter will prefer to carry on the punishment in $t=1$. Hence we conclude that the conjectured voting strategy is an equilibrium voting strategy.

•case 3

Let j be the incumbent legislator in $t=0$ and let $-j$be the challenger, where j and $-j$ are symmetric with respect to M. The following shows that claim 4 is true. The proof for the first part of the conjecture is trivial. Since the median voter is indifferent between the two candidates

on the ideological dimension, he cannot loose from carrying on his reward. Let's consider now the second part of the conjecture, i.e. $\sigma_{Mj}^{t}\left(h^{t}\right)=0$ if the legislator j has ever chosen C^{H} in the past. This strategy prescribes to permanently replace the incumbent j with the opponent $-j$ if ever the incumbent deviates from the low cost policy. Note that the median voter is indifferent between the two candidates on the ideological dimension. Hence, the best strategy must be the voting strategy that impliest the policy choice C^{L}. The policy C^{L} can be obtained punishing the incumbent with no re-election.

b) Policy choice

The following proves *lemma 2*

Proof:

Assume that the conjectured voting strategy $\sigma_{Mj}^{*}(.)$ is as follows:

$\sigma_{Mj}^{t}\left(h^{t}\right)=1$ if the legislator j has always chosen C^{L} in the past;

$\sigma_{Mj}^{t}\left(h^{t}\right)=0$ if the legislator j has ever chosen C^{H} in the past;

Remember the after any deviation from the low cost policy, the equilibrium path will represented by the bold line in *figure 2,* where the high cost policy is chosen forever by the legislator M. Given this conjectured voting strategy, let's compute the intertemporal payoff of legislator j when he chooses C^{L} in $t=0$. Since a deviation from the low cost policy may occur at any period t in the future, the main difficulty in computing the intertemporal payoff of the legislator choosing the low cost policy in period $t=0$ is the infinite number of possible path associated to possible deviations in $t\geq 1$. However, among all the possibilities, we can identify the path that keeps the legislator j at his lowest possible payoff. Therefore, if we can find a condition supporting the policy C^{L} when the legislator obtains his lowest payoff, then this condition will guarantee that the legislator will choose C^{L} also in any other case where he can obtain a higher payoff. We know that if he chooses C^{H} his intertemporal payoff from period $t=0$ onwards will be:

$$V_{j}^{0}\left(C^{H},x_{j}\right)=B-\sum_{t=0}^{\infty}\delta^{t}(|x_{j}-x_{M}|+C^{H})$$

If he chooses C^{L}, his intertemporal payoff will depend on the policy choice of future legislators. Let's denote $\underline{V}_{j}^{0}$ the lowest payoff for the incumbent legislator j in $t=0$, which is realized when the incumbent chooses C^{L} and his first successor chooses C^{H} . The lowest payoff for the incumbent in $t=0$ is:

$$\underline{V}_j^1 = -\delta C^H - \sum_{t=2}^{\infty} \delta^t (|x_j - x_M| + C^H)$$

Comparing the payoffs $V_j^0\left(C^H, x_j\right)$ and $\underline{V}_j^0$ we obtain thatin period $t = 0$ the incumbent legislator j will choose the policy $\left(C^L, x_j\right)$ if and only if $(x_j - x_M) \geq \frac{B}{\delta}$,which proves *lemma 2.* Finally note that, since *lemma 2* gives the condition for the legislator to be accountable in $t = 0$, given that the first successor will deviate in $t = 1$, then this condition will be sufficient for the legislator to be accountable in $t = 0$ if deviation will occur at any other date $t > 1$.

Proposition 1

Let j be the first incumbent and j' the challenger. For party L, let's define L_C and L_E the sets of the centrist candidates and extreme candidates with respect to the median party member m_L. Similarly, . for party R, let's define R_C and R_E the sets of the centrist candidates and extreme candidates with respect to the median party member m_R. Suppose that $j \in L$ and $j' \in R$. Let's define $n = \frac{1}{1-\delta}$. Given the assumption that $(x_{m_L} - x_M) \geq \frac{B+\delta(C^H - C^L)}{\delta}$, from *lemmata 1-4*, we know that the candidate $j = m_L$ chooses the low cost policy and beats any opponent j' in all future electoral rounds. Therefore, the median party m_L choosing a candidate $j = m_L$ obtains the maximum ideological payoff (zero) for ever. Suppose now that he chooses a candidate $j \neq m_L$. Given the opponent j', if j wins the elections, the median party member m_L will obtain at most the payoff $V_{m_l}^0 = -n\left|x_{m_L} - x_j\right|$ which is strictly less than the maximum payoff he obtains running the winning candidate m_L. Hence, by iterated strict dominance we can eliminate all the winning candidates $j \neq m_L$. On the other hand if j looses the election against j', the median party member m_L will obtains the payoff $V_{m_l}^0 = -n\left|x_{m_L} - x_{j'}\right|$ which again is less then the maximum payoff. Therefore, by iterated strict dominance we can eliminate all the loosing candidates $j \neq m_L$. Hence, $j = m_L$ is the only candidate location that survives iterated strict dominance. On the other hand, the opponent median party member m_R is indifferent amongst all candidate locations since for any candidate $j^{'}$ he will always obtain the payoff $V_{m_R}^0 = -n\left|x_{m_R} - x_{m_L}\right|$. Hence, the median party member m_L runs the candidate m_L that chooses the policy $\left(x_{m_L}, C^L\right)$ and wins against any candidate j'.

Proposition 2

Suppose that $\mid x_{m_L} - x_{m_R} \mid$ *is necessary for accountability, i.e.* $\mid x_{m_L} - x_{m_R} \mid = \frac{B+\delta(C^H - C^L)}{\delta}$.

First we prove that m_L is closer to the median voter, M, than to the first extreme party member that guarantees accountability, j_{E1}. In fact, if $\mid x_{m_L} - x_{m_R} \mid$ is necessary for accountability, than for j_{E1} to guarantee accountability it must be that:

$$| x_{j_{E1}} - x_M | \geq | x_{m_L} - x_{m_R} | \tag{6.1}$$

we can show that inequality 6.1 implies that m_L is closer to the median voter, M, than to j_{E1}.

claim 1: when $| x_{j_{E1}} - x_M | = | x_{m_L} - x_{m_R} |$, then $| x_{m_L} - x_{j_{E1}} | = | x_{m_L} - x_M |$

proof of claim 1: given a symmetric distribution, we have that

$$| x_{m_L} - x_{m_R} | = | x_{m_L} - x_M | + | x_{m_R} - x_M |$$

and

$| x_{j_{E1}} - x_M | = | x_{m_L} - x_M | + | x_{m_L} - x_{j_{E1}} |$

if $| x_{j_{E1}} - x_M | = | x_{m_L} - x_{m_R} |$, then $| x_{m_L} - x_{j_{E1}} | = | x_{m_R} - x_M |$ and by symmetry of the distribution :

$| x_{m_L} - x_{j_{E1}} | = | x_{m_R} - x_M | = | x_{m_L} - x_M |$

hence , $| x_{m_L} - x_{j_{E1}} | = | x_{m_L} - x_M |$

claim 2: if $| x_{j_{E1}} - x_M | > | x_{m_L} - x_{m_R} |$, then $| x_{m_L} - x_{j_{E1}} | > | x_{m_L} - x_M |$

proof of claim 2: in fact, using the proof of claim 1 it is immediate to verify that:

$| x_{m_L} - x_{j_{E1}} | > | x_{m_R} - x_M | = | x_{m_L} - x_M |$

hence we conclude that if $| x_{j_{E1}} - x_M | \geq | x_{m_L} - x_{m_R} |$ then $| x_{m_L} - x_{j_{E1}} | \geq | x_{m_L} - x_M |$ (i.e. the median voter is closer to M than to j_{E1}.)

The same logic applies when $| x_{m_L} - x_{m_R} |$ *is not sufficient for accountability,i.e.* $| x_{m_L} - x_{m_R} | < \frac{B+\delta(C^H - C^L)}{\delta}$. In this case for j_{E1} to guarantee accountability it must be that:

$| x_{j_{E1}} - x_M | > | x_{m_L} - x_{m_R} |$

hence with *strict inequality* the previous proof holds and we can conclude that again the median party member is closer to the median voter than to the first extreme accountable candidate.

Similarly , if $\mid x_{m_L} - x_{m_R} \mid$ *is sufficient for accountability, i.e.* $\mid x_{m_L} - x_{m_R} \mid > \frac{B+\delta(C^H-C^L)}{\delta}$ then for j_{E1} to be accountable the following must be true:

$\mid x_{j_{E1}} - x_M \mid \geq \frac{B+\delta(C^H-C^L)}{\delta} < \mid x_{m_L} - x_{m_R} \mid$

therefore we can prove the following:

claim 3: if $\mid x_{j_{E1}} - x_M \mid \leq \mid x_{m_L} - x_{m_R} \mid$, then $\mid x_{m_L} - x_{j_{E1}} \mid \leq \mid x_{m_L} - x_M \mid$

proof of claim 3: given a symmetric distribution, we have that

$\mid x_{m_L} - x_{m_R} \mid = \mid x_{m_L} - x_M \mid + \mid x_{m_R} - x_M \mid$

and $\mid x_{j_{E1}} - x_M \mid = \mid x_{m_L} - x_M \mid + \mid x_{m_L} - x_{j_{E1}} \mid$

hence, $\mid x_{m_L} - x_{j_{E1}} \mid \leq \mid x_{m_R} - x_M \mid = \mid x_{m_L} - x_M \mid$

therefore, the median party member is closer to the first extreme accountable candidate than to the median voter.

Using now **claims 1-3**, we can prove *proposition 2.*

Assumption 1: $|x_{m_L} - x_{m_R}| > \frac{B+\delta(C^H-C^L)}{\delta}$ and $|x_{m_L} - x_M| \leq \frac{B+\delta(C^H-C^L)}{\delta}$ and $\delta < \frac{\left|x_{m_L} - x_{j_{E1}}\right|}{\left|x_{m_L} - x_{j'}\right|}$

Let j_{E1} be the first extreme candidate such that $|x_{j_{E1}} - x_M| > \frac{B+\delta(C^H-C^L)}{\delta}$ and *lemmata 1-4* hold. From assumption 1 and *lemmata 1-4*, the candidate location $j = j_{E1}$ strictly dominates any other extreme candidate location. Hence by iterated strict dominance we can eliminate any $j \neq j_{E1} \in L_E$.

Let's consider now the candidate $j = m_L$ and the set of the centrists candidates L_C. The opponent $j^{'}$ can be the median party member m_R or an extreme R_E or a centrist R_C. First note that $j^{'} \in R_C$ beats the incumbent $j = m_L$. Therefore, the median party member of the incumbent party in this case obtains payoff $V^0_{m_L}\left(m_L, j^{'} \in R_C\right) = -n\left|x_{m_L} - x_{j'}\right|$. On the other hand, $j = m_L$ beats $j' = m_R$, therefore $V^0_{m_L}(m_L, m_R) = 0$. Finally, $j = m_L$ also beats any $j' \in R_E$, therefore $V^0_{m_L}\left(m_L, j^{'} \in R_E\right) = 0$.

Let's consider now the candidate $j \in L_C$. If $j' \in R_C$, then the median party member can get at most $V^0_{m_L}\left(j \in L_C, j^{'} \in R_C\right) = -n\left|x_{m_L} - x_j\right|$ (when j is closer to M than j') and at least $V^0_{m_L}\left(j \in L_C, j^{'} \in R_C\right) = -(n-1)\left|x_{m_L} - x_{j'}\right|$(when j' is closer to M than j) . Finally, $j \in L_C$ beats any other candidate $j' \notin R_C$, hence $V^0_{m_L}\left(j \in L_C, j^{'} \notin R_C\right) = -\frac{1}{1-\delta}\left|x_{m_L} - x_j\right|$.

Note that every $j \notin L_E$ beats every $j' \notin R_C$. This implies that $j^{'} \notin R_C$ is dominated by $j' \in R_C$. Hence by iterated dominance we can eliminate $j' = m_L$ and $j' \in R_E$.

Therefore, we are left with the following possible equilibrium locations:

$(j = j_{E1}, j = m_L, j \in L_C, j' \in R_C)$

Note that m_L is strictly dominated by j_{E1} if and only if the following holds

$\frac{1}{1-\delta}\left(|x_{m_L} - x_{j_{E1}}| - \delta\left|x_{m_L} - x_{j'}\right|\right) > 0$ for every j'.

From **claim 3** and $\delta < \frac{\left|x_{m_L} - x_{j_{E1}}\right|}{\left|x_{m_L} - x_{j'}\right|}$, the previous inequality is satisfied, hence by iterated deletion of dominated strategies m_L can be deleted.

Consider now set of candidates that have survived iterated deletion of dominated strategies. i.e. $j = j_{E1}$, L_C and R_C. First note that the most extreme candidate in R_C looses against any incumbent j. In fact if $j = j_{E1}$, we have established that j beats j'. If j is the most extreme in L_C, by symmetry he wins. If j is any more centrist in L_C, being closer the M again he wins. Hence, the most extreme candidate in R_C can be eliminated in the first round of iterated elimination. Consider now the most extreme candidate in L_C. This candidate looses against the second most extreme in R_C (i.e. the most extreme in the set of candidates R_C after the first round of iterated elimination). On the other hand, j_{E1} wins against any opponent and hence strictly dominates the most extreme candidate in the set L_C. Therefore, the most extreme candidate in L_C can be eliminated in the second round of iterated elimination. Applying the same logic in successive rounds, all the extreme candidates in the sets R_C and L_C are eliminated. Finally, $j' = M$ is the only candidate of the opponent party that survives iterated elimination. Given $j' = M$, from **claim 3** it follows that $j = j_{E1}$ is the only candidate of the opponent party that survives iterated elimination.

Assumption 2: $|x_{m_L} - x_{m_R}| > \frac{B+\delta(C^H - C^L)}{\delta}$ and $|x_{m_L} - x_M| \leq \frac{B+\delta(C^H - C^L)}{\delta}$ and $\delta > \frac{\left|x_{m_L} - x_{j_{E1}}\right|}{\left|x_{m_L} - x_{j'}\right|}$

Under assumption 2, from the previous argument if follows that j_{E1} is strictly dominated by m_L. Therefore the candidates surviving iterated eliminations are m_L, $j \in L_C$ and $j' \in R_C$. When $\delta > \frac{\left|x_{m_L} - x_j\right|}{\left|x_{m_L} - x_{j'}\right|}$ $\forall j \in L_C$, then m_L strictly dominates every $j \in L_C$. Hence, party L runs m_L that chooses the high cost policy and is permanently replaced by the most extreme $j' \in R_C$. When $\frac{\left|x_{m_L} - x_j\right|}{\left|x_{m_L} - x_{j'}\right|} < \delta < \frac{\left|x_{m_L} - x_M\right|}{\left|x_{m_L} - x_{j'}\right|}$, m_L is strictly dominated by any $j \in L_C$. Once m_L is deleted, also the most extreme j' is strictly dominated by any other $j' \in R_C$. Hence, by elimination of strictly dominated strategies in successive rounds, $j = M$ and $j' = M$ are the only candidates surviving iterated elimination.

Assumption 3: $|x_{m_L} - x_{m_R}| \leq \frac{B+\delta(C^H - C^L)}{\delta}$

From claim 1-2 we know that m_L is closer to M than to j_{E1}.

Therefore, $\frac{1}{1-\delta}\left(|x_{m_L} - x_{j_{E1}}| - \delta\left|x_{m_L} - x_{j'}\right|\right) < 0$ for every j', which means that $j = j_{E1}$ and also all the j more extreme than j_{E1} are strictly dominated by $j = m_L$. Note also that any $j' \in R_E$ is defeated by $j = m_L$ and also by any $j \in L_C$. Therefore, all $j' \in R_E$ can be eliminated by iterated deletion of strictly dominated locations. Hence, after the elimination of $j \in L_E$ and $j' \in R_E$, we are left with the following possible candidates:

$(j = m_L, j \in L_C,\ j' = m_R, j' \in R_C)$

Note that, $j' = m_R$ is defeated by any remaining j. Hence, $j = m_R$ can be deleted. Once

m_R is deleted, $j = m_L$ is defeated by any remaining j'. However, since m_L is the most preferred location of the median party member m_L, we obtain that $j = m_L$ strictly dominates any $j \in L_C$ when $\delta > \frac{|x_{m_L} - x_j|}{|x_{m_L} - x_{j'}|}$ $\forall j \in L_C$. Hence, party L runs m_L that chooses the high cost policy and is permanently replaced by the most extreme $j' \in R_C$. When $\frac{|x_{m_L} - x_j|}{|x_{m_L} - x_{j'}|} < \delta < \frac{|x_{m_L} - x_M|}{|x_{m_L} - x_{j'}|}$, m_L is strictly dominated by any $j \in L_C$. Once m_L is deleted, also the most extreme j' is strictly dominated by any other $j' \in R_C$. Hence, by elimination of strictly dominated strategies in successive rounds, $j = M$ and $j' = M$ are the only candidates surviving iterated elimination.

Proposition 3

Let $j_{C1} \in L_C$ be the first centrist candidate such that the following assumption is satisfied:

Assumption 4: $|x_M - x_{j_{C1}}| + C^L < C^H$ and $|x_{j_{C1}} - x_M| \geq \frac{B + \delta(C^H - C^L)}{\delta}$ and $\delta < \frac{|x_{m_L} - x_{j_{C1}}|}{|x_{m_L} - x_{j'}|}$

From proposition 2, candidates $j \in L_E$ and $j' \in R_E$ can be eliminated. Given assumption 4, the candidate m_L is strictly dominated by j_{C1} and can therefore be eliminated. Also, j_{C1}strictly dominates any other $j \in L_C$. Hence, j_{C1} is the only candidate of party L surviving iterated elimination. Note that j_{C1} beats any remaining possible opponent. Hence, the median party member m_R becomes indifferent amongst all candidates locations $j' = m_R$ and $j' \in R_C$.

On the other hand, using proposition 2 we obtain that when $\delta > \frac{|x_{m_L} - x_j|}{|x_{m_L} - x_{j'}|}$ $\forall j \in L_C$, then m_L strictly dominates every $j \in L_C$. Hence, party L runs m_L that chooses the high cost policy and is permanently replaced by the most extreme $j' \in R_C$. When $\frac{|x_{m_L} - x_j|}{|x_{m_L} - x_{j'}|} < \delta < \frac{|x_{m_L} - x_M|}{|x_{m_L} - x_{j'}|}$, m_L is strictly dominated by any $j \in L_C$ and also the most extreme j' is strictly dominated by any other $j' \in R_C$. Hence, by elimination of strictly dominated strategies in successive rounds, $j = M$ and $j' = M$ are the only candidates surviving iterated elimination.