A model of arbitration in regulation

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Abstract

I study a regulatory process in which both the regulator and the regulated firm propose prices that in case of disagreement are settled through final-offer arbitration — a practice currently used in Chile for setting prices in the water sector. Rather than submitting a single offer, each party simultaneously submit offers for each of the cost units in which the firm is divided. This multiplicity is believed to be responsible for the great divergence between parties' offers observed in practice. I show, however, that reducing the number of offers makes little difference unless parties are required to submit a single offer.

Keywords: final-offer arbitration, price regulation, Nash equilibrium

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1 Introduction

Departing from the more traditional rate-of return and price-cap regulations, prices of public utilities in Chile are set using a particular form of yardstick regulation in which the benchmarking is based on a hypothetical efficient firm.[1] Under this price setting process—introduced first in the electricity sector in the early 1980s—both the regulator and the regulated firm have a very explicit interaction. Based on their own estimation for the long term costs of this hypothetical firm, both parties propose the price to be charged by the regulated firm for the duration of the review period (4-5 years).[2] If parties cannot agree on the price, the disagreement is settled through an arbitration process.

Since 1999 this arbitration process takes a distinct form in the water sector. In order to prevent parties' offers to significantly diverge, as has occurred in the other regulated sectors, the water sector considers a final-offer arbitration mechanism in which the arbitrator is constrained to choose one of the parties' offers as a settlement.[3] But because parties do no submit a single offer for the entire firm but rather an offer for each of the cost units in which the firm is divided,[4] the actual arbitration mechanism looks more like a hybrid between final-offer arbitration (FOA) and conventional arbitration.[5]

While the division of the regulated firm in various units was aimed at introducing greater transparency into the regulatory process and avoiding subsidization across cost units, evidence on the first round of applying this price setting process for the different water utilities in the

[1] See Vogelsang (2002) for an overview of the different regulatory approaches practiced over the last 20 years.

[2] In reality, each party constructs an efficient firm and announces the long term total cost that such firm would incur in providing the service during the review period. In this construction, parties may differ not only about unit costs but also about projections of future demand.

[3] The use final-offer arbitration is commonly seen in the settlement of labor disputes (with baseball as a classic example) but I am not aware of its explicit use elsewhere in a regulatory context.

[4] There are approximately 200 units including, for example, cost of raw water, cost of capital, cost of replacing pavement, etc. For more see Sánchez and Coria (2003).

[5] In conventional arbitration, the arbitrator is not constrained to any particular settlement. So, as the number of units goes large, FOA approaches conventional arbitration since the arbitrator is able to chose almost any settlement by using some combination of parties' offers.

country has not been uncontroversial. As shown in Table 1, we observe in most cases an important divergence between the regulator's overall offer, p^r, and the firm's overall offer, p^f (to facilitate the exposition p^r has been normalize to 100).[6] Regardless of whether privately-owned firms are more effective in reducing costs than state-owned firms (see Teeples and Glyer (1987)), the numbers of Table 1 suggest that both types of firms have incentives to "inflate" costs.[7] In addition, we observe that in five cases parties failed to negotiate the final price, p^s, and had instead resorted to FOA.

INSERT TABLE 1 HERE OR BELOW

The numbers in Table 1 also raise the issue about the factors that might characterize the contract zone of Farber and Bazerman (1989), i.e., the range of settlements that both parties prefer to disagreement. Ownership status seems to explain, at least in part, why some parties are more likely to reach agreement than others. In fact, for 3 of the 6 privately-owned companies,[8] prices were determined through arbitration while for only 2 of the 9 state-owned companies, prices were determined in such a way. Firm size, which may serve as a proxy for firm's complexity and uncertainty about the arbitrator' preferences,[9] also seems relevant (although the largest two firms also happens to be in private hands). Given the small sample size, however, there is no much else that can be said.

The great divergence in parties' offers have raised more fundamental questions. Some observers have challenged the advantages of the current regulatory mechanism over more conventional mechanisms, particularly price-caps as practiced in the UK, while others have questioned

[6] The numbers shown are based on parties' announcements of long term total costs.

[7] Even though it may no retain any profits, a state-owned firm has also incentives to inflate costs in an effort to improve its (ex-post) performance.

[8] With the exception of Aguas Cordillera, these companies have gone private only recently: 1-2 years before the price reviews.

[9] As demonstrated by Farber (1980), divergence in parties' offer increases with uncertainty about the arbitrator's preferences.

the privatization process itself arguing that the increase in information asymmetries have more than offset any productivity gains.[10] Rather than introducing radical changes in both the privatization program and the regulatory scheme, the authority is exploring ways in which the actual divergence in parties' offers could be diminished. In particular, it is proposing to substantially reduce the multiplicity of offers, i.e., the numbers of units in which the regulated firm is divided. Reducing the number of offers seems reasonable since it would make the arbitration process look less like the cheap-talk game associated to conventional arbitration.

Motivated by these concerns and proposed solutions, in this paper I develop a simple model to explore the extent to which a reduction in the number of units brings parties' offers closer to each other. In so doing, I extend the model of Farber (1980) to the case in which parties simultaneously submit offers for each of the units that are part of the item in dispute and the arbitrator is limited to choose one party's offer or the other for each unit, so in principle, he is free to fashion a compromise by awarding some offers to one party and the rest to the second party. Despite this variant of FOA was already recognized by Farber in his article (he calls it "issue by issue" FOA), its formal modelling has been postponed. Understanding the equilibrium properties of this arbitration game is not only relevant for the price setting process that motivated this paper,[11] but more generally, for any FOA in which more than one issue is in dispute (e.g., a union and a firm negotiating salaries for a group of jobs, a government and a contractor renegotiating a multi-part contract, etc.).

The model of the paper is based on a one-period game that considers two parties (i.e., the firm and the regulator) with opposing preferences that simultaneously submit offers to an arbitrator whose ideal settlement is imperfectly known by both parties (recall that parties'

[10] See Gomez-Lobo and Vargas (2002) for a further discussion on the shortcomings of the current regulatory scheme.

[11] This arbitration scheme has also been proposed in place of the current mechanisms used to settle disputes over regulated prices in the electricity and telecommunication sectors in Chile.

uncertainty regarding the arbitrator's preferences is what leads to offers divergence). As in Farber (1980) and the literature that has followed (e.g., Gibbons, 1988), I do not include a previous stage in which parties bargain over the final price before going to arbitration, so I do not intent to explain what makes parties more likely to reach an agreement rather than end in arbitration.[12] The main result of the paper is that the division of the firm in just two units introduce enough flexibility in parties' strategies so that there are multiple equilibria. The multiplicity associated to this two-offers game implies that in equilibrium the distance between the parties' overall offers is not unique but varies from that obtained for the single-offer game, which is unique, to virtually infinity.[13]

These results are interesting for both technical and practical reasons. From a technical perspective, it is interesting to observe that the introduction of just a bit of uncertainty on the arbitrator's preferences produces dramatic changes in the equilibrium of the game. If parties are fully certain about the arbitrator's ideal settlement, the equilibrium of the game shows perfect convergence regardless the numbers units that constitute the firm. Conversely, if parties are not fully certain and there are two or more units, divergence between parties' offers can be arbitrarily large in equilibrium. The practical implications of the results of the paper, on the other hand, are rather clear: the authority's proposal that call for a reduction in the number of cost units from something like 200 to 50 offers (or to two units for that matter) would make little difference, if any, in its effort to lower parties divergence.

I should emphasize that this paper is by no means an attempt to discuss the overall optimality of this regulatory approach relative to alternative approaches but rather understand the effect of regulatory design on parties behavior. With that objective in mind, the rest of the

[12] For a discussion see Farber and Bazerman (1989).

[13] Using a focal point argument, one could argue that the likely outcome of the game is one in which parties' offers locate at the limits of the interval that supports the arbitrator's ideal settlement.

paper is organized as follows. In Section 2, I introduce the model for the single-offer game (in Appendix A I explain the effect of risk aversion). In Section 3, I extend the model to multiple offers. I develop the two-offers case in the text and show in Appendix B that the results carry over to the case of three or more offers. Using the results of these two sections, in Section 4, I explore whether and to what extent a reduction in the number of offers (i.e., firm's divisions) lead to greater convergence in parties' overall offers. Concluding remarks are in Section 5.

2 The Model

Consider the following simple model of price regulation in which a regulator and regulated firm submit an offer for the price to be charged by the regulated firm. In case of price disagreement, the final price is settled through FOA. Let p^r and p^f denote the regulator's and firm's price offers. Following Farber (1980), the arbitrator is characterized by the parameter z, which describes the arbitrator's most preferred settlement.[14] If the actual settlement is p, the arbitrator's utility is $v_a(p, z) = -(p - z)^2$. In FOA, the arbitrator is constrained to choose one of the parties' offers as a settlement. Given this utility function and assuming that in equilibrium the regulator's offer will be smaller than the firm's offer, the arbitrator will choose the regulator's offer if and only if $z < \overline{p}$, where $\overline{p} = (p^r + p^f)/2$.

The parties are assumed to be risk-neutral and equally uncertain about the value of z. The parties believe that z is randomly distributed on the interval $[z_l, z_h]$ according to the cumulative distribution function $F(z)$, with density $f(z)$. Hence, given the parties' offers, the probability that the regulator's offer is accepted is $F(\overline{p})$. Contrary to Farber (1980), here parties do not have strictly opposed preferences. The firm simply seeks to maximize the expected settlement.

[14] Note that I am assuming that the arbitrator learns nothing from the parties' offers about the ideal settlement (i.e., the true cost of providing the service efficiently). I comment on this issue of learning in the concluding section.

The regulator, on the other hand, seeks to minimize the expected settlement (i.e., maximize expected consumer surplus) taking into account that the firm should obtain a fair return on their investments and not go bankrupt.

The timing of the final-offer arbitration game is as follows. First, the regulator and the firm simultaneously submit their offers to the arbitrator.[15] Second, the arbitrator chooses the offer that maximizes his utility function $v_a(p, z)$ as the settlement. The parties' Nash equilibrium offers (p^f and p^r) maximize their expected payoffs so they are found by simultaneously solving

$$\max_{p^f} p^r F(\overline{p}) + p^f[1 - F(\overline{p})] \tag{1}$$

$$\min_{p^r} (1 - \lambda_1)p^r F(\overline{p}) + [1 - \lambda_2]p^f[1 - F(\overline{p})] \tag{2}$$

where $0 < \lambda < 1$ is a known parameter intended to capture the regulator's concern about firm's profits. Since the regulator should be less concerned about firm's profits when the settlement chosen by the arbitrator is p^f, we let $\Delta\lambda \equiv \lambda_1 - \lambda_2 > 0$.

The first-order conditions for this optimization problem are[16]

$$(p^f - p^r)f(\overline{p})/2 = 1 - F(\overline{p}) \tag{3}$$

$$(p^f - p^r)f(\overline{p})/2 = F(\overline{p}) - \frac{\Delta\lambda}{2(1-\lambda_1)} f(\overline{p})p^f \tag{4}$$

that rearranged lead to

$$F(\overline{p}) = \frac{1}{2} + \frac{\Delta\lambda}{4(1-\lambda_1)} f(\overline{p})p^f \tag{5}$$

$$p^f - p^r = \frac{1}{f(\overline{p})} - \frac{\Delta\lambda}{2(1-\lambda_1)} p^f \tag{6}$$

[15] As in Farber (1980) and subsequent papers I do not explicitly model a first stage where parties can bargain before going to arbitration. We can think of p^r and p^f as the last offers during the bargaining period.

[16] Note that the convexity of the arbitrator's utility function assures the existence of equilibrium.

This Nash equilibrium reduces to the one obtained by Farber (1980) for $\Delta\lambda = 0$. In this case the parties' offers are centered around the mean of the parties' belief about the arbitrator's ideal settlement (i.e., $\overline{z}$) and the distance between the equilibrium offers decreases as this belief becomes more precise (i.e., higher $f(\cdot)$). In the limit, when there is no uncertainty about the arbitrator's preferences, both parties submit the arbitrator's ideal settlement, that is $p^r = p^f = z$. When $\Delta\lambda > 0$, however, Farber's equilibrium changes. More specifically

Proposition 1 *When the regulator puts some weight on firm's profit, the parties' offers are centered above the mean* $\overline{z}$ *and the distance between the parties' offers decreases.*

The trade-off detected by Farber (1980) still applies here. In equilibrium, each party must balance a trade-off between making a more aggressive offer and reducing the probability that the offer will be chosen by the arbitrator. When $\Delta\lambda > 0$, the regulator does not want to be as aggressive and, hence, the distance between parties' offers reduces. In addition, as the distance shrinks with λ_1 [see eq. (6)], one may wonder whether offers could eventually coincide. For example, if z distributes uniformly on $[a, b]$, it is not difficult to show that when firm's profits are less important to the regulator (i.e., $\lambda = 0$ or $\Delta\lambda = 0$), parties' offers show maximum differentiation, that is $p^f = b$ and $p^r = a$. When $\Delta\lambda > 0$, on the other hand, parties' offers are $p^f = b$ and $p^r = a + \Delta\lambda b/2(1 - \lambda_1)$. Then, for $a = b/2$ and $\lambda_2 = 0$, p^r would approach p^f as λ_1 approaches 0.5.[17]

The above example also show that the expected settlement, $E[p^s]$, increases with the weight the regulator puts on firm's profit (i.e., λ_1) and could eventually reach b.[18] This is a more general result that derives directly from Proposition 1. Provided that $E[p^s]$ is the firm's objective

[17]Note that p^r cannot be greater than p^f in equilibrium; otherwise second order conditions would not be satisfied. In such case the solution is $p^f = p^r$.

[18]For this specific example we have $E[p^s] = (a + b)/2 + \gamma^2/2(b - a)$, where $\gamma = \Delta\lambda b/2(1 - \lambda_1)$.

function, from the envelope theorem we have

$$\frac{\partial E[p^s]}{\partial \lambda_1} = [F(\overline{p}) - (p^f - p^r)f(\overline{p})/2]\frac{\partial p^r}{\partial \lambda_1} \tag{7}$$

Without explicitly solving for p^r and p^f it is possible to demonstrate that (7) is positive. The term in brackets is positive from (4). On the other hand, $\partial p^r/\partial \lambda_1$ is positive from both (5) and (6). In fact, if p^r falls with λ_1, p^f must increase by a larger amount for (5) to hold. But that would lead offers to be further apart, contradicting (4). Neither can we have, from (5), a fall in p^f accompanied of no change in p^r.

If for some reason one believes that a regulator is more likely to assign higher weight to profits of state-owned firms, then, the numbers of Table 1 would be somewhat consistent with the analysis presented here: parties' offers for state-owned companies are expected to be closer to each other and, hence, more likely to fall within the contract zone (or agreement zone). The same analysis would also indicate, however, that the final price for state-owned companies are expected to be higher than for privately-owned companies, other things equal.

Finally, it is worth asking whether risk-aversion can bring parties' offers even closer. Abstracting from profit weights to isolate the effect of risk-aversion and assuming identical utility functions, Appendix A demonstrates that risk aversion reduces the average of the parties' offers but does not necessarily decrease the distance between them.

3 Multiple offers

An important difference between Farber's model and the regulatory scheme studied in this paper is that parties do not submit a single offer but multiple offers. Consider then the case in which the regulated firm is divided in two units or production centers: 1 and 2 (e.g., water production and water distribution).[19] Since we understand the implications of not having parties with strictly opposing preferences, in what follows I omit profit weights to simplify notation.

In this multiple-offer game, the regulator and the regulated firm submit simultaneously price offers for each of the two units. The regulator's offers are denoted by p_1^r and p_2^r and the firm's offers are denoted by p_1^f and p_2^f. The arbitrator's task is to choose a price offer for each unit following a FOA procedure. The arbitrator will choose prices p_1 and p_2 that maximize its utility $v_a(p_1, p_2, z) = -(p_1 + p_2 - z)^2$. Then, there will be four possible offer combinations for the arbitrator to choose from: $\{p_1^r, p_2^r\}$, $\{p_1^f, p_2^r\}$, $\{p_1^r, p_2^f\}$ and $\{p_1^f, p_2^f\}$.

Note that since the possibility of submitting multiple offers only affect parties' strategy space but not the actual operation of the water utility (the firm will minimize costs regardless the price chosen for each unit), both parties and the arbitrator only care about the overall offer $p = p_1 + p_2$ (i.e., about the final price to be paid by consumers) and not about the price of each individual unit.

3.1 Certainty about the arbitrator's preferences

It is useful to start by studying the game in which both parties know the arbitrator's ideal settlement because it helps to illustrate equilibrium properties that may carry over to the uncertainty case. Parties' action space and arbitrator's ideal settlement z are depicted in Figure 1. More specifically, parties' offers for units 1 and 2 are in the horizontal and vertical axis, respectively. For example, point A represents a regulator's offer consisting of ${}^A p_1^r$ for the first unit and ${}^A p_2^r$ for the second unit. The line z, on the other hand, contains those combinations of p_1 and p_2 that add up to z. The arbitrator is indifferent between any two combinations that lie on this line.

[19] I shall comment later on the case with three or more offers.

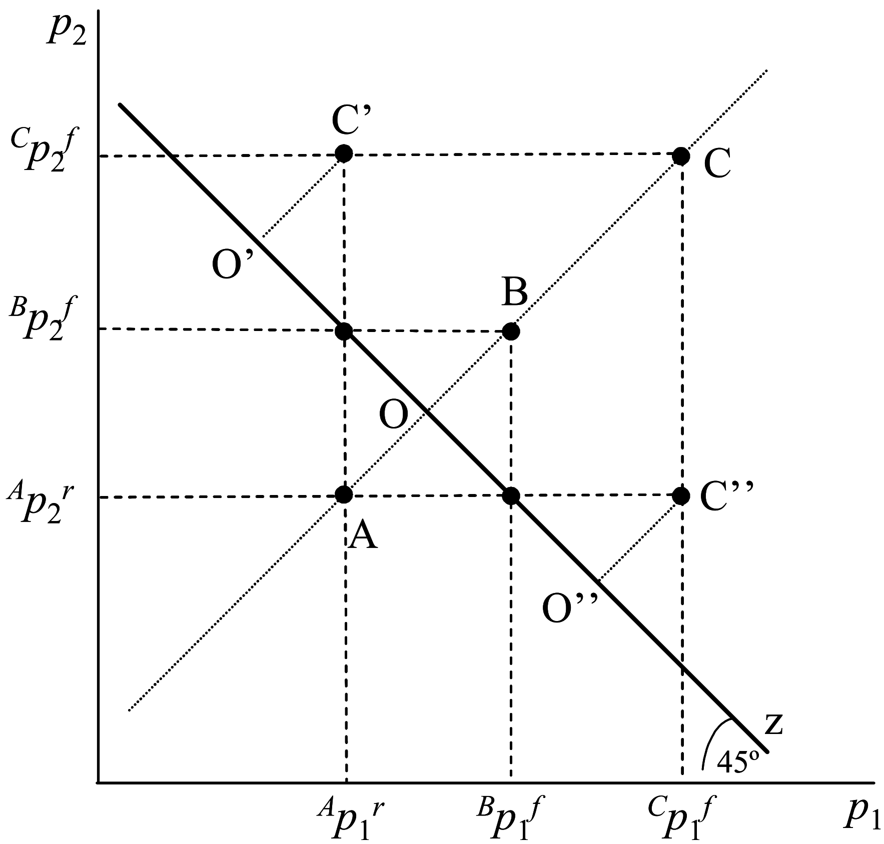

Figure 1: Two-offers game under certainty

As in the one-offer case, an obvious equilibrium of the game is for each party i to submit a pair $\{p_1^i, p_2^i\}$ where $p^i \equiv p_1^i + p_2^i = z$. We know that if party i submits an overall offer of $p^i = z$, party j's best response is not constrained to any offer because the arbitrator would pick p^i regardless his offer. But for $p^i = z$ to be a best response to party j's offer, we must necessarily have $p^j \equiv p_1^j + p_2^j = z$.

Let us explore now whether a pair of offers equally distant from the line z, such as A and B in Figure 1 ($\overline{OA} = \overline{OB}$), could also constitute an equilibrium of the game . If this were the case, we could observe offers divergence in equilibrium but with the same settlement outcome as above. In fact, the arbitrator would be indifferent between the pairs $\{{}^A p_1^r, {}^B p_2^f\}$ and $\{{}^B p_1^f, {}^A p_2^r\}$ because both yield z; her ideal settlement. However, this is not a suitable equilibrium candidate.

If the regulator is playing A, the firm's best response is not playing B but playing C, where $\overline{O'C'} = \overline{O''C''} = \overline{OA} - \epsilon$ and ϵ is a very small positive number. This play leaves the arbitrator indifferent between $C' = \{^A p_1^r, ^C p_2^f\}$ and $C'' = \{^C p_1^f, ^A p_2^r\}$ with a price settlement of $z + \overline{AO}$ $-\epsilon > z$).[20] And following the same logic, we know that A cannot be the best response to C but something further apart (more precisely, three times larger than $\overline{OC}$). As this illustration shows, there is no best-response correspondence off the z-line. To summarize

Proposition 2 *If both parties know the arbitrator's preference z, the Nash equilibria of the two-offers game are $p^i \equiv p_1^i + p_2^i = z$ for $i = r, f$.*

This proposition indicates that the introduction of multiple offers (as many as the number of units in which the firm has been divided) does not affect the perfect convergence of parties' offers when there is certainty about the arbitrator's preferences. Although it has only been formally shown for the two-offers case, it should be clear that Proposition 2 extends to the case of three or more offers.[21] This is an interesting result because one would think that as the number of offers increase the arbitration process would converge to conventional arbitration in the sense that the arbitrator can impose almost any settlement he wishes by choosing the right combination of parties' offers. But in conventional arbitration we know that in equilibrium we can observe either any offers (as in any cheap-talk game) or maximum differentiation if the arbitrator is believed to split differences.

[20]If for any reason the regulator's offer is to the north-east of the line z, the firm's best response is to play any pair equally or further distant from z in the north-east direction.

[21]A simple example should be enough here. Consider a three-offers game in which the arbitrator's ideal settlement is $z = \$10$. If the regulator submits the offer $p^r = \{1, 2, 3\}$, which is \$4 off the z-plane, the firm's best response is not to play a symmetrically distant offer such as $p_a^f = \{3, 5, 6\}$ but to play $p_b^f = \{8.99, 9.99, 10.99\}$, where 0.01 is the smallest possible number, say, a penny. By submitting the latter the firm assures itself a settlement of 13.99. Since p^r is, by the same arguments, not the regulator's best response to p_b^f, we cannot have an equilibrium with parties' offers located off the z-plane.

3.2 Uncertainty about the arbitrator's preferences

Let us now turn to the case in which the parties are uncertain about the arbitrator's preferences. To estimate the probability that the arbitrator choose a particular offer combination we need first to understand some regularities that prevail in equilibrium. From the certainty case we know that if the regulator plays something like A, the firm's best response will lie somewhere along the line ABC depending on the value of z (if by any chance the z-line falls to the south-west of A, the firm will pick A). This implies that in equilibrium we must have $p_k^f > p_k^r$ for $k = 1, 2$,[22] which, in turn, assures that $p^f > p^r$ in equilibrium.

Since p_1 and p_2 are perfect substitutes, we can adopt the convention that in equilibrium $p_2^i \geq p_1^i$ for $i = r, f$, which leads to $p_2^f - p_1^f \geq p_2^r - p_1^r$. The probabilities can then be found by dividing the z space in four different regions, each supporting the election of one particular offer combination. Depending on the parties' offers there will be values $z_1 < z_2 < z_3$ such that if z falls in the region $(-\infty, z_1)$, the arbitrator will choose $\{p_1^r, p_2^r\}$, if z falls in the region $[z_1, z_2)$ the arbitrator will choose $\{p_1^f, p_2^r\}$, if z falls in the region $[z_2, z_3)$ the arbitrator will choose $\{p_1^r, p_2^f\}$, and if z falls in the region $[z_3, +\infty)$ the arbitrator will choose $\{p_1^f, p_2^f\}$.

As before, the parties' Nash equilibrium offers maximize their expected payoffs so are found by simultaneously solving

$$\max_{p_1^f, p_2^f} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)] \\ + (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)] \quad (8)$$

[22] It is a strict inequality because in this uncertainty environment there will be at least one z-line to the north-east of A.

$$\min_{p_1^r, p_2^r} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)]$$
$$+ (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)] \quad (9)$$

where $z_1 = (p_1^r + 2p_2^r + p_1^f)/2$, $z_2 = (p_1^r + p_2^r + p_1^f + p_2^f)/2$ and $z_3 = (p_1^r + p_1^f + 2p_2^f)/2$.[23]

The first-order conditions for this optimization problem are[24]

$$[p_1^f] : 1 - F(z_3) + F(z_2) - F(z_1) + (p_1^r - p_1^f)[f(z_3) - f(z_2) + f(z_1)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (10)$$

$$[p_2^f] : 1 - F(z_2) + (p_1^r - p_1^f)[f(z_3) - f(z_2)/2] + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (11)$$

$$[p_1^r] : F(z_3) - F(z_2) + F(z_1) + (p_1^r - p_1^f)[f(z_3) - f(z_2) + f(z_1)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (12)$$

$$[p_2^r] : F(z_2) + (p_1^r - p_1^f)[f(z_1) - f(z_2)/2] + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (13)$$

Although the solution involves multiple equilibria as in the certainty case (any of the four equations is a linear combination of the other three; in particular $[p_1^f] + [p_1^r] = [p_2^f] + [p_2^r]$ where $[p_k^i]$ denotes the first-order condition for p_k^i), they all must satisfy the conditions above that rearranged leads to

Proposition 3 *When $f(\cdot)$ is a symmetric probability density function the two-offers Nash equilibria present the following characteristics: the parties' (overall) offers are centered around the mean $\overline{z}$ and the distance between them cannot be smaller than in the single-offer case.*

Proof. Let us prove first that parties' offers are centered around $\overline{z}$, i.e., $F(z_2 = \overline{p}) = 1/2$.

[23] Note that $z_3 - z_2 = z_2 - z_1 = p_2^f - p_2^r > 0$ and that $z_2 = \overline{p}$.

[24] Identical FOCs will be obtained if we adopt the alternative convention that in equilibrium $p_1^i \geq p_2^i$ for $i = r, f$.

Combine (10) with (12) and (11) with (13) to obtain, respectively

$$F(z_2) = F(z_1) + F(z_3) - 1/2 \tag{14}$$

$$F(z_2) = 1/2 + (p_1^f - p_1^r)[f(z_1) - f(z_3)]/2 \tag{15}$$

In addition, we know that

$$z_3 - z_2 = z_2 - z_1 \tag{16}$$

Given the perfect colinearity between first-order conditions (which implies that we have 3 equations for 4 unknowns), we can make an unrestricted selection for one of the 4 offers, or alternatively, for $\Delta \equiv p_1^f - p_1^r > 0$. Furthermore, any particular value of Δ leads to a unique equilibrium given the parties' objective functions (including the arbitrator's) that we are considering here.[25] And since $f(z_1) = f(z_3)$ and $F(z_2) = 1/2$ is an equilibrium candidate in that solves the system (14)–(16) for any $\Delta > 0$ and a symmetric density function, uniqueness implies that $z_2 = \overline{z}$. On the other hand, to find an expression for the distance between parties' offers add (10) and (12) and rearrange to obtain

$$p^f - p^r = \frac{1}{f(z_2)} - (p_1^f - p_1^r)\left[\frac{f(z_3) + f(z_1) - 2f(z_2)}{f(z_2)}\right] \tag{17}$$

where $p^f = p_1^f + p_2^f$ and $p^r = p_1^r + p_2^r$. Replacing $f(z_3) = f(z_1)$ and $z_2 = \overline{z}$, eq. (17) can be re-written as

$$p^f - p^r = \frac{1}{f(\overline{z})} - 2(p_1^f - p_1^r)\left[\frac{f(z_1) - f(\overline{z})}{f(\overline{z})}\right] \tag{18}$$

[25] Uniqueness can be easily proved using the results from the certainty case. If the regulator's offer is, say, the pair A of Figure 1, the firm's best response for a given value of z is unique and equal to the pair C of Figure 1 (if for some value of z the pair A falls to the north-east of the z-line, the firm's best response is A). And since the firm's best response is a non-decreasing function of z (strictly increasing if A is to the south-west of the z-line), the firm's best response to A is unique when $z \in [z_l, z_h]$ according to $F(z)$.

Since $\Delta \equiv p_1^f - p_1^r > 0$ can be arbitrarily small and $f(z_1) \leq f(\overline{z})$ for any distribution function that is not U-shaped, the distance between offers will be equal or larger than in the single-offer case. ■

Proposition 3 suggests that decreasing the number of offers (i.e., firm's divisions) may not necessarily reduce the distance between the parties' offers as intended in the authority's proposal. Without specifying $f(\cdot)$, however, it is hard to provide further insights about the equilibrium properties of this arbitration game. In the next section I add more structure to the equilibrium solution by considering a couple of density functions and I investigate, among other things, the possibility that in equilibrium both z_1 and z_3 fall outside the interval $[z_l, z_h]$.

Before moving to the next section there are three issues worth mentioning. The first is about the equilibrium properties for an asymmetric density function. We know that in the single-offer case parties' offers are centered around the mean $\overline{z}$ regardless of $f(\cdot)$. In the two-offers case, however, parties' offers are no longer centered around $\overline{z}$ for an asymmetric density function but they can be above or below $\overline{z}$. The second is about the equilibrium properties for a U-shaped density function. Although it is hard to conceive such a shape in practice, eq. (18) indicates that the distance could in principle decrease by going from a single offer to two offers if, in equilibrium, $z_1 \in [z_l, z_h]$. Since it is possible to show that in equilibrium z_1 and z_3 could fall either outside or inside $[z_l, z_h]$ (see footnote 28), an increase in convergence is a possibility.

A final issue concerns the extension of the model to three or more offers. As shown in the Appendix B, the equilibrium properties contained in Proposition 3 carry over to the three-offers game. Following the same procedure, it can be shown that they carry over to a game with a higher number of offers as well.

4 Convergence of offers

The authority's new proposal rests on the intuition that a reduction in the number of offers (i.e., number of units in which the regulated firm is divided) should bring parties' offers somehow closer. To be more precise about whether and how a reduction in the numbers of offers affects the convergence of parties' offers we need to add more structure to the model by specifying the density function $f(\cdot)$. To work with closed-form solutions, I consider first a uniform distribution and then a triangular distribution.

When $f(z)$ is a uniform distribution over the interval $[a, b]$, the equilibrium solution for the single-offer case is straightforward: parties' offers exhibit maximum differentiation among the arbitrator's possible ideal settlements, that is $p^r = a$ and $p^f = b$ (note that nothing prevents parties to submit offers that fall outside the interval $[a, b]$).

Obtaining the equilibria for the two-offers case is more involved. We need to know whether in equilibrium z_1 and z_3 fall inside or outside $[a, b]$. Let assume first (to be checked later) that z_1 and z_3 fall inside $[a, b]$, that is $z_1 \geq a$ and $z_3 \leq b$. From Proposition 3 we know that $p^f - p^r = b - a$ and $(p^f + p^r)/2 = (a + b)/2$, so in equilibrium we have that $p^r \equiv p_1^r + p_2^r = a$ and $p^f \equiv p_1^f + p_2^f = b$, as in the single-offer case. For this to be indeed an equilibrium we need to corroborate that there is a combination of individual offers (i.e., p_1^r, p_2^r, p_1^f and p_2^f) satisfying $z_1 \geq a$ and $z_3 \leq b$. To do this, let $p_1^r = a/2 - \alpha$ and $p_1^f = b/2 - \beta$, where α and β are two arbitrarily chosen parameters that define an specific equilibrium (we have one extra degree of freedom than usual because $f(z)$ is constant, or more precisely, symmetric). Using $p^r = a$ and $p^f = b$ to obtain that $p_2^r = a/2 + \alpha$ and $p_2^f = a/2 + \beta$, and replacing these values into z_1 and z_3 we obtain

$$z_1 = \frac{p_1^r + 2p_2^r + p_1^f}{2} = a + \frac{b-a}{4} - \frac{\beta - \alpha}{2} \tag{19}$$

$$z_3 = \frac{p_1^r + p_1^f + 2p_2^f}{2} = b - \frac{b-a}{4} + \frac{\beta - \alpha}{2} \tag{20}$$

As long as α and β are chosen such that $\beta - \alpha \leq (b-a)/2$, eqs. (19) and (20) indicate that there are multiple equilibria in which the distance between parties' offers remains unchanged from the single-offer case, i.e., $p^f - p^r = b - a$.

Following a similar procedure it is possible to show that there are also multiple equilibria in which z_1 and z_3 fall outside $[a, b]$ and the distance between parties' offers is greater than in the single-offer case, i.e., $p^f - p^r > b - a$. Since $f(z_1) = f(z_3) = 0$, from Proposition 3 we know that $p^f - p^r = b - a + 2(p_1^f - p_1^r)$ and $(p^f + p^r)/2 = (a+b)/2$. To corroborate that there is a combination of individual offers (i.e., p_1^r, p_2^r, p_1^f and p_2^f) satisfying $z_1 < a$ and $z_3 > b$, let again $p_1^r = a/2 - \alpha$ and $p_1^f = b/2 - \beta$. Given these definitions and the equilibrium conditions of Proposition 3 we have that $p_2^r = a - b/2 + \beta$ and $p_2^f = b - a/2 + \alpha$. Replacing these values into z_1 and z_3 we obtain

$$z_1 = a - \frac{b-a}{4} + \frac{\beta - \alpha}{2} \tag{21}$$

$$z_3 = b + \frac{b-a}{4} - \frac{\beta - \alpha}{2} \tag{22}$$

As before, as long as α and β are chosen such that $\beta - \alpha < (b-a)/2$, eqs. (21) and (22) indicate that there are multiple equilibria in which the distance between parties' offers is greater than in the single-offer case, i.e., $p^f - p^r > b - a$.[26] Given the multiplicity of equilibria this exercise illustrates that increasing the numbers of offers does not necessarily lead to greater divergence in parties' offers. One can even argue that because the pair $\{p^r = a, p^f = b\}$ is the only focal outcome of this game (especially if offers way off the arbitrator's range of preferences may be perceived as unreasonable and disregarded as in Farber and Bazerman (1986)), an increase in

[26] Provided that $p^f - p^r = 2(b-a) + 2(\alpha - \beta)$ and that $\beta - \alpha$ can take an arbitrarily large negative value, there is no limit on the distance between partie's offers that can be observed in equilibrium. Note also that as $\beta - \alpha$ approaches $(b-a)/2$, $p^f - p^r$ approaches $b - a$.

the number of offers is unlikely to affect convergence when both parties assume that arbitrator's preferences are uniformly distributed.

It remains to be seen whether these results also hold for distribution functions in which parties assign more weight to intermediate values, which can be regarded as a more reasonable assumption for parties' priors. For simplicity, let us assume that $f(z)$ is a symmetric triangular distribution over the interval $[0, 2a]$, which means that $f(z) = z/a^2$ if $0 \leq z \leq a$, $2/a - z/a^2$ if $a \leq z \leq 2a$ and 0 otherwise (offers can take negative values). Following the analysis of Section 2 for $\lambda = 0$, it can be shown that the unique equilibrium for the single-offer case is $p^r = a/2$ and $p^f = 3a/2$, i.e., the distance between parties' offers is a.

If we now extend the game to two-offers, Proposition 3 indicates that in equilibrium it holds

$$p^f - p^r = a - 2(p_1^f - p_1^r)[z_1/a - 1] \tag{23}$$

Using $z_2 - z_1 = (p_2^f - p_2^r)/2$ and $z_2 = \overline{z} = a$, we have that $p^f - p^r = 2(a - z_1) + p_1^f - p_1^r$. Replacing the latter into (23) and making $\Delta = p_1^f - p_1^r > 0$ our arbitrary choice (alternatively, one can pick one of the four offers), we obtain that $z_1 = a/2$ and, hence, $p^f - p^r = a + \Delta$. This result and $(p^f + p^r)/2 = \overline{z} = a$ allows us then to establish that in equilibrium $p^r \equiv p_1^r + p_2^r = (a - \Delta)/2$ and $p^f \equiv p_1^f + p_2^f = (3a + \Delta)/2$.[27]

Since there are no restrictions on the (arbitrary) selection of Δ other than it has to be positive, the distance between parties' offers associated to each of these multiple of equilibria can be anything from a (when $\Delta \approx 0$) to infinity. Although we cannot rule out that moving from one to two offers (or vice versa) may have no effect on offers convergence, the focal argument

[27] The particular shape of the density function ($f(z_1) = f(z_2)/2$ in equilibrium) adds an additional degree of freedom, so the determination of a particular equilibrium (i.e., p_1^r, p_2^r, p_1^f and p_2^f) requires, in addition to Δ, an arbitrary selection of one of the four offers (since $\Delta > 0$ and $p_1^i < p_2^i$ by convention, the selection's only restriction is $p_1^r < a/2$ or $p_2^f > 3a/2$). For example if we take $\Delta = 3a$ and $p_1^r = -3a$, the rest of the equilibrium is given by $p_2^r = 2a$, $p_1^f = 0$ and $p_2^f = 3a$.

deployed above would suggest the pair $\{p^r = 0, p^f = 2a\}$ as the likely outcome of the game resulting in a doubling of the distance between parties' offers from the single-offer case.[28]

What is remarkable from both of these exercises (and more generally)[29] is that the division of the regulated firm into two units already provide parties with enough flexibility for their offers to exhibit, in equilibrium, a degree of convergence that can be anywhere from that in the single-offer case to virtually infinity. This is a most important result because proposals to foster offers convergence consider reductions in the number of offers to no less than 50 units,[30] which, according to the results of this paper, would prove innocuous.

To finish, it may be worth indicating that allowing parties not to have strictly opposing preferences (as in Section 2) does not introduce any substantial changes to the results. Parties offer's will be centered above the mean $\overline{z}$ (see Proposition 1) but the multiplicity of equilibria will be maintained.

5 Concluding remarks

Prices of public utilities in Chile are set using a particular form of yardstick regulation in which the benchmarking is a hypothetical efficient firm. Based on their own estimation for the long term costs of this hypothetical firm, both the regulator and the regulated firm propose the price to be charged by the regulated firm. If parties cannot agree on the final price, the disagreement is settled through and arbitration process that in the water sector takes the form of final-offer arbitration (FOA) applied to each of the cost units that constitute the firm. Motivated by the large divergence in parties' offers that we observe in practice, I have extended Farber's

[28] This analysis can be easily extended to a U-shaped density function. Consider, for example, a symmetric inverted triangular function over the interval $[0, 2a]$ for which $f(0) = f(2a) = 2/3a$ and $f(a) = 1/3a$. It follows from Proposition 3 that in equilibrium holds that $z_2 = \overline{z} = a$ and $z_1 = a(3\Delta - a)/[2(\Delta + a)]$. Therefore, as long as $a/3 < \Delta < 3a$, the two-offers equilibrium will exhibit more convergence than the single-offer equilibrium.

[29] Numerical results for other distributions (including the unimodal symmetric beta distribution and other no-symmetric distributions) are qualitatively the same.

[30] I understand that a reduction to three units was at some point considered but ultimately discarded.

single-offer model to the case in which parties submit two or more offers. I found that moving from a single offer to just two offers provide parties with such flexibility that there are multiple equilibria. More importantly, the distance between the parties' overall offers in this two-offers game is not unique but varies from that obtained for the single-offer game, which is unique, to virtually infinity.

The above result is interesting for technical and practical reasons. From a technical perspective, it is interesting to observe that the introduction of just a bit of uncertainty on the arbitrator's preferences produces dramatic changes in the equilibrium of the game. If parties are fully certain about the arbitrator's ideal settlement, the equilibrium of the game shows perfect convergence regardless the numbers units that constitute the firm. Conversely, if parties are not fully certain and there are two or more units, there are equilibria of the game that exhibit unlimited divergence between parties' offers. Using a focal point argument, one could certainly argue that the likely outcome of the game is one in which parties' offers locate at the limits of the interval that supports the arbitrator's ideal settlement.

The practical reasons, on the other hand, are rather clear. According to the results of the paper, the authority's proposal that call for a reduction in the number of offers (i.e., units) from something like 200 to 50 offers (or to two offers for that matter) would make little difference, if any, in its effort to lower parties divergence.

There are several issues related to this regulatory approach, and more generally, to any FOA procedure with multiple offers that are not covered in the paper. First, I assumed that the arbitrator's preferences are not affected by parties' offers. Empirical studies of arbitrator behavior shows (e.g., Farber and Bazerman, 1986; Ashenfelter and Bloom, 1984), however, that arbitrators do use parties' offers to compute their ideal settlement and then choose the offer closer to this ideal. Gibbons (1989) studies the equilibrium properties of a single-offer FOA game in which both parties share a common perception about the true value of the item in question (here, the cost of the efficient firm). The arbitrator is less informed about the true value of the item than both of the parties but he upgrades his beliefs after observing the parties' offers (signals). Despite parties consider the gain from misleading the arbitrator when choosing their offers, Gibbons (1989) shows that in perfect Bayesian separating equilibrium parties find it optimal not to so.[31] Rather they submit truthful offers that are closer to each other because when arbitrator learns about the value of the item parties' uncertainty regarding the arbitrator's ideal settlement is necessarily reduced.

It remains to be seen whether in a two-offers game parties still find it optimal to not mislead the arbitrator in equilibrium. If so, parties' offers could not exhibit the unlimited divergence that we found in the no-learning case; otherwise there seems to be no learning (it is hard to believe that the arbitrator would learn the same regardless whether parties' offers are close to each other or very far apart). But since the equilibrium structure of the single-offer learning game is the same as the structure of the no-learning game,[32] it may very well be that learning becomes irrelevant in a game with two or more offers. Along these lines, it would be also important, particularly in regulation, to study the case in which one of the parties (here, the firm) is much better informed about the true value of the item than the other party. This asymmetric information game has not yet been studied, so it is an open area for future research.

The paper is also silent about the question of why parties came to be in arbitration. Empirical and experimental work comparing conventional and final-offer arbitration shows that it is not clear whether dispute rates (i.e., number of negotiations that end in arbitration) and distance between parties' offers are greater in conventional arbitration than in FOA (Farber and Bazerman, 1986 and 1989; and Ashenfelter et al., 1992). If we believe that a multi-offer FOA

[31] There is also continuum of pooling equilibria in which the arbitrator learns nothing from the parties' offers.
[32] Compare eqs. (6) and (7) with (17) and (18) of Gibbons (1989).

is not very different form conventional arbitration in that any distance between the parties' offers can be observed in equilibrium (as in cheap-talk game), one could argue that reducing the number of offers to a single offer could, on the one hand, increase convergence among parties that are going to arbitration but, on the other hand, increase the number of parties that end up in arbitration. Therefore, if negotiated settlements are valuable from a policy standpoint because it allows parties more discretion in negotiating their own settlement (Farber, 1980), a drastic reduction to a single-offer may lead to undesirable outcomes, i.e., too much arbitration.

Finally, there is the question about the overall optimality of this regulatory approach relative to alternative approaches such as cost-of-return and price-cap schemes. Perhaps more realistic within the existing regulatory scheme, it is to ask for ways in which the construction of the hypothetical efficient firm could be improved. Following the yardstick regulatory scheme practiced in the water sector in the UK, one possibility it is to require, at least partially, the use of actual costs from previous review periods and from other water utilities.

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Appendix A: The effect of risk-aversion

Let the utility function of both the regulator and the firm be the same and denoted by $u(p)$, with $u' > 0$ and $u'' < 0$. The first order conditions for p^f and p^r are, respectively

$$f(\overline{p})[u(p^r) - u(p^f)]/2 + u'(p^f)[1 - F(\overline{p})] = 0 \tag{24}$$

$$f(\overline{p})[u(p^r) - u(p^f)]/2 + u'(p^r)F(\overline{p}) = 0 \tag{25}$$

where $\overline{p} = (p^r + p^f)/2$. Substracting (25) from (24) and rearranging gives $F(\overline{p}) = u'(p^f)/(u'(p^r) + u'(p^f))$. And since $u''(\cdot) < 0$, we have that $F(\overline{p}) < 1/2$, i.e., $\overline{p} < \overline{z}$. On the other hand, we know from the mean value theorem that there exist a $\xi \in (p^r, p^f)$ such that $u(p^f) - u(p^r) = u'(\xi)[p^f - p^r]$. Replacing the latter in (25) yields

$$p^f - p^r = \frac{1}{f(\overline{p})} \frac{F(\overline{p})}{1/2} \frac{u'(p^r)}{u'(\xi)}$$

While $F(\overline{p}) < 1/2$, $u'(p^r) > u'(\xi)$ and $f(\overline{p}) \leq f(\overline{z})$;[33] so it remains ambiguous whether the distance between parties' offers increases or decreases relative to the case in which parties are risk neutral.

Appendix B: Three-offers game

Consider, as in Proposition 3, that $f(\cdot)$ is a symmetric density function. Let $p^r \equiv \{p_1^r, p_2^r, p_3^r\}$ and $p^f \equiv \{p_1^f, p_2^f, p_3^f\}$ be the triplets chosen by the regulator and the firm, respectively. As in the two-offers case, we know that in equilibrium $p_k^f > p_k^r$ for $k = 1, 2, 3$. Adopting the convention that in equilibrium $p_3^i \geq p_2^i \geq p_1^i$ for $i = r, f$, the expected settlement is given by

$$\begin{aligned} E[p^s] &= (p_1^r - p_1^f)F(z_1) + (p_1^f + p_2^r - p_1^r - p_2^f)F(z_2) + (p_2^f + p_3^r - p_2^r - p_3^f)F(z_3) \\ &\quad + (p_1^r + p_2^r + p_3^f - p_1^f - p_2^f - p_3^r)F(z_4) + (p_2^f + p_3^r - p_2^r - p_3^f)F(z_5) \\ &\quad + (p_1^f + p_2^r - p_1^r - p_2^f)F(z_6) + (p_1^r - p_1^f)F(z_7) + (p_1^f + p_2^f + p_3^f) \end{aligned}$$

where $z_1 = (p_1^r + 2p_2^r + 2p_3^r + p_1^f)/2$, $z_2 = (p_1^r + p_2^r + 2p_3^r + p_1^f + p_2^f)/2$, $z_3 = (2p_1^r + p_2^r + p_3^r + p_2^f + p_3^f)/2$, $z_4 = (p_1^r + p_2^r + p_3^r + p_1^f + p_2^f + p_3^f)/2$, $z_5 = (p_2^r + p_3^r + 2p_1^f + p_2^f + p_3^f)/2$, $z_6 = (p_1^r + p_2^r + p_1^f + p_2^f + 2p_3^f)/2$ and $z_7 = (p_1^r + p_1^f + 2p_2^f + 2p_3^f)/2$. In Nash equilibrium the triplet $\{p_1^r, p_2^r, p_3^r\}$ minimize $E[p^s]$ taking $\{p_1^f, p_2^f, p_3^f\}$ as given while the triplet $\{p_1^f, p_2^f, p_3^f\}$ maximize $E[p^s]$ taking $\{p_1^r, p_2^r, p_3^r\}$ as given, so the equilibrium is found by simultaneously solving 6 first-order conditions (FOCs). Denote by $[p_k^i]$ the FOC corresponding to the offer k by party i.

[33] Unless $f(z)$ is U-shaped or significantly skewed.

Proceeding as in Proposition 3, let us combine $[p_1^r]$ and $[p_1^f]$, $[p_2^r]$ and $[p_2^f]$, and $[p_3^r]$ and $[p_3^f]$ to obtain, respectively

$$\begin{aligned} F(z_4) &= 1/2 - F(z_1) - F(z_7) + F(z_2) + F(z_6) - (p_2^f - p_2^r)[f(z_3) - f(z_5)]/2 \\ &\quad -(p_3^f - p_3^r)[f(z_5) - f(z_3)]/2 \end{aligned}$$

$$F(z_4) = 1/2 - F(z_2) - F(z_6) + F(z_3) + F(z_5) - (p_1^f - p_1^r)[f(z_7) - f(z_1)]/2$$

$$\begin{aligned} F(z_4) &= F(z_3) + F(z_5) - 1/2 - (p_1^f - p_1^r)[f(z_1) - f(z_2) + f(z_6) - f(z_7)]/2 \\ &\quad -(p_2^f - p_2^r)[f(z_2) - f(z_6)]/2 \end{aligned}$$

In addition, we know that $z_4 - z_1 = z_7 - z_4 = (p_2^f + p_3^f - p_2^r - p_3^r)/2$, $z_4 - z_2 = z_6 - z_4 = (p_3^f - p_3^r)/2$ and $z_4 - z_3 = z_5 - z_4 = (p_1^f - p_1^r)/2$.

Given that each of the 6 FOCs is a linear combination of three other FOCs (in particular, we have that $[p_1^f] + [p_1^r] = [p_2^f] + [p_2^r] = [p_3^f] + [p_3^r]$), we have only 4 equations for 6 unknowns. Letting $\Delta_2 = p_2^f - p_2^r > 0$ and $\Delta_3 = p_3^f - p_3^r > 0$ be our arbitrary choices, the same arguments employed in Proposition 3 imply that there is a unique equilibrium for any given Δ_2 and Δ_3. And since $f(z_1) = f(z_7)$, $f(z_2) = f(z_6)$, $f(z_3) = f(z_5)$ and $F(z_4) = 1/2$ solves the system above for any $\Delta_2 > 0$, $\Delta_3 > 0$ and symmetric density function, uniqueness implies that $z_4 = \overline{z}$, that is, parties' offers are centered around the mean $\overline{z}$.

On the other hand, to find an expression for the distance between parties' offers add $[p_1^r]$

and $[p_1^f]$ to obtain (after some rearrangement)

$$p^f - p^r = 1/f(z_4) - (p_1^f - p_1^r)[f(z_1) - f(z_2) - f(z_6) + f(z_7)]/f(z_4)$$
$$- (p_2^f - p_2^r)[f(z_2) - f(z_3) - f(z_5) + f(z_6)]/f(z_4) - (p_3^f - p_3^r)[f(z_3) - 2f(z_4) + f(z_5)]/f(z_4)$$

and then replace $f(z_1) = f(z_7)$, $f(z_2) = f(z_6)$, $f(z_3) = f(z_5)$ and $z_4 = \overline{z}$ to finally obtain

$$\begin{aligned} p^f - p^r &= 1/f(\overline{z}) - 2(p_1^f - p_1^r)[f(z_1) - f(z_2)]/f(\overline{z}) \\ &\quad -2(p_2^f - p_2^r)[f(z_2) - f(z_3)]/f(\overline{z}) - 2(p_3^f - p_3^r)[f(z_3) - f(\overline{z})]/f(\overline{z}) \end{aligned}$$

Since $f(z_1) \leq f(z_2) \leq f(z_3) \leq f(\overline{z})$ (unless $f(\cdot)$ is U-shaped) and $p_k^f > p_k^r$ for $k = 1, 2$, and 3, the distance between the parties' overall offers (i.e., $p^f - p^r$) cannot be smaller than the distance in the single-offer game.

Table 1. Firms' characteristics, parties' offers and settlements

Firm	Location	Size	Ownership	p^r	p^f	p^s	FOA
ESSAT	I	3.3	state	100	148	118	yes
ESSAN	II	3.3	state	100	110	106	no
EMSSAT	III	1.9	state	100	112	102	no
ESSCO	IV	4.1	state	100	128	108	no
ESVAL	V	12.9	private	100	184	141	yes
SMAPA	MR	4.7	state	100	125	107	no
Aguas Cordillera	MR	2.7	private	100	156	113	no
Aguas Andinas	MR	37.2	private	100	256	139	yes
ESSEL	VI	4.3	private	100	137	109	no
ESSAM	VII	4.7	state	100	131	113	yes
ESSBIO	VIII	10.8	private	100	115	104	no
ESSAR	IX	4.4	state	100	127	112	no
ESSAL	X	3.9	private	100	146	117	yes
EMSSA	XI	0.6	state	100	137	108	no
ESMAG	XII	1.2	state	100	119	109	no

source: Superintendencia de Servicios Sanitarios (Agency of Water Services).

Size is the fraction of consumers served from the total number of consumers