On the Role and Effects of IMF Seniority

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Abstract

The paper presents a three period model that studies the effects of IMF loans on borrowers' and lenders' welfare highlighting the fact that the IMF has both *de jure* and *de facto* seniority rights over private creditors. It is shown that an IMF intervention affects borrowers and lenders in different ways. Ex-post, once capital is installed and a liquidity shock occurs, an IMF intervention always makes the borrower country better off. The effects on non-senior lenders depend on the size of the senior intervention and on the country's solvency situation. IMF intervention makes existing creditors worse off when the country's solvency situation is either very good or weak, but makes them better off when solvency is in an intermediate range, consistent with the nonlinearities found empirically in Mody and Saravia (2003). The possibility of future senior intervention affects the optimal level of investment ex-ante, and it may be the case that the borrower country would be better off by committing today not to borrow from the IMF in the future. Since a country has incentives to borrow from the IMF once the shock occurs, this promise is not time consistent and an institution with clear rules about when to intervene will be welfare improving.

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1 Introduction

The role that the IMF should play in the New International Financial Architecture is an important issue in the current policy and academic debate, especially after the crisis that took place in the 1990s, beginning with Mexico in December of 1994. It has been recognized that the IMF has some characteristics that make it a special player in the international lending community, capable of attracting capital flows to a country and improving in this way its economic situation. For example, it is argued that the IMF may have more information than other lenders and that its presence may be a positive signal about countries' characteristics that are not observed by other creditors (Rodrik 1996); a related argument is that the IMF can be used by less informed investors as a country's screening device (Marchesi and Thomas 2001). Another hypothesis is that the IMF could act as a delegated monitor through its conditionality and surveillance functions or could serve as a country's commitment device to behave well (for example Rodrik (1996), Tirole (2002), Mody and Saravia (2003)).[1]

This paper focuses on a different aspect of IMF lending, specifically its status as senior lender. This focus is motivated by some facts about IMF lending that have received little analytical attention. These are: (1) countries have shown a higher aversion to defaulting on IMF loans than on loans from private creditors [2], and (2) the IMF has contractual seniority on its loans. Arguably, these two characteristics imply two other characteristics of IMF lending: (1) the IMF lends at a lower interest rate than private creditors, and (2) the IMF lends in circumstances where other creditors are not willing to do so.

This paper addresses the following questions: Is IMF seniority good? For whom? Under what circumstances? Since we are interested in the seniority issue, we will study the IMF as a creditor of a country with the only difference being that it has seniority rights. The crucial distinction in the model is, therefore, between senior and non-senior lenders.

The presence of senior lending may introduce a conflict of interest between non-senior creditors and the debtor country. Consider a country that has been hit by shocks that prompt a need for

[1] Cottarelli and Gianini (2002) clasiffy the channels in which flows are "catalyzed" in five categories similar to the mentioned here as an example.

[2] For example, Argentina, Indonesia, Ecuador, Pakistan, Ukraine. "The Economist" in October of 2002 stated: "While in default to its private creditors since December, Argentina has continued to service debts to international financial institutions".

new financing. It may be the case that no new lending will be provided without seniority rights; for example a highly indebted economy would have problems attracting new non-senior funds because of credit ceiling and debt overhang considerations. A senior lender would have less problems lending since the probability of being repaid is higher than for non-senior lenders. Thus, seniority may be a necessary condition to have financing that allows the economy to cope with shocks. However, non-senior lenders may be worse off in the presence of a senior lender since in case of bankruptcy they have to wait until senior debts are repaid.

The paper presents a model with three periods: a planning period, a period when a shock hits the economy, and a final period where output is obtained, and consumption and debt repayment take place. In the planning period, the country borrows to invest in capital, which is used in the production process in order to maximize expected utility. In the middle period, the country potentially has to borrow more money to cope with a liquidity shock that hits the economy. The way that the IMF adds value in this model is by lending in circumstances where non-senior creditors are not willing to lend in equilibrium. If the shock is big enough, non-senior lenders will expect losses on new lending and, assuming initial lenders are atomistic and cannot coordinate efforts to make "emergency loans", they will not be willing to offer credit; in these cases a deep-pocket lender with seniority rights (IMF) will be necessary to cope with the shock. Once capital is installed and the initial lending and borrowing decisions have been made (i.e. ex-post), a senior intervention always makes the country better off, since senior creditors lend at a lower interest rate, allowing a higher consumption level. The effects of the IMF's lending on non-senior lenders depend on the size of the liquidity shock and on what non-senior lenders would get when the IMF does not intervene. On the one hand, having senior lending allows the economy to cope with a higher range of liquidity shocks, but on the other hand, a senior lender jeopardizes what private creditors expect to get in case of bankruptcy. As a consequence of these opposing effects, lenders may prefer to discontinue the project, and would be ex-post worse off with an IMF intervention.

Lenders take into account these effects when making their initial lending decisions (i.e. ex-ante). It may be the case that the option of a future senior intervention makes contractual conditions more onerous in the planning period and that, as a consequence, the country ends up borrowing (and investing) a lower amount than in the case where the IMF is not allowed to intervene. Moreover, it may be the case that the borrower country would be ex-ante better off by committing not to borrow from the IMF to cope with future shocks. Since the country has incentives to borrow from the IMF once the shock occurs, this promise is not time consistent and a commitment technology will be necessary to maintain it.

The paper is related to the discussion about the role of International Financial Institutions as a Lender of Last Resort (LOLR) (for example Fischer (1999), Zettelmeyer (2000) and Calomiris (1998)). This discussion is often based on models where a crisis occurs as a self-fulfilling equilibrium caused by coordination problems between creditors. An important point in this debate is the trade off between ex-post efficiency and ex-ante moral hazard. Some argue that having a LOLR institution able to fill liquidity needs reduces the probability of a crisis and ameliorates their effects once they occur. Others claim that having a LOLR would trigger debtor and other creditors' moral hazard. Our model abstracts from coordination and moral hazard issues and adds to this literature in two aspects. *First*, although the IMF contributes to ex-post efficiency by allowing projects to continue— ameliorating the effect of the crisis— we highlight the point that such an intervention affects borrowers and lenders differently. As noted above, it may be that private lenders are worse off with an IMF intervention, although the borrower country is always ex-post better off. *Second*, contrary to the *moral hazard* view that predicts that the possibility of a future bail-out will lead to excessive lending by making lenders take riskier strategies, our model predicts that the possibility of a future bail-out may lead to less lending, in equilibrium, as a consequence of the conflict of interest mentioned above.

Recent theoretical work by Corsetti et al. (2003) studies the role of the IMF in catalyzing capital flows by providing liquidity in a model with coordination problems between creditors having asymmetric information about the state of the economy.[3] In one of the extensions to their model, they consider the case where the IMF is a senior lender. They conclude that since a senior lender is more willing to intervene, the probability of a crisis would be reduced, but since the return to junior lenders is lower they would be less willing to roll over their debts. As noted above, in our paper, we are not concerned with coordination problems and roll-over of short term debt issues although we recognize they are important. Rather, our framework allows us to analyze the impact of senior

[3]Morris and Shin (2003) use a similar analysis to Corsetti et.al. to analyze the IMF's ability to catalyze capital flows. Penalver (2002) reaches similar conclusions to Morris and Shin's work with a different modelling strategy. None of these works analyzes the role of IMF seniority.

interventions on borrowers' and lenders' ex-ante and ex-post welfare, highlighting the conflict of interest between borrowers and lenders that a senior intervention may imply. This is something that previous work has abstracted from and it is what allows us to generate the result that the amount borrowed and the country's welfare may be lower when senior lending is allowed.

Section 2 describes the elements of the model. Section 3 solves the model backwards. We compare a situation where senior lending is not allowed with one where it is. Analyzing period 1, when capital is installed and the shock hits the economy, we will examine ex-post effects of senior intervention on the country's and private creditors' welfare. In period 0, when borrowing and lending decisions are made, we study how the possibility of a senior intervention affects the initial level of investment and the country's welfare ex-ante. Section 4 relates this work to the empirical evidence presented in Mody and Saravia (2003). Section 5 concludes.

2 Model

Time. There are three periods, indexed by t=0,1,2. In period 0, agents make real investment and borrowing decisions. In period 1, the economy can be hit by a shock that affects the production process. In order to cope with this shock, agents have to borrow again. In period 2, output is realized, debt issued in period 0 and 1 is repaid and consumption takes place.

Agents and production. The economy is populated by a continuum of identical consumer-producers with linear preferences over consumption of a single good at date 2; i.e their utility function is $U(c_0, c_1, c_2) = c_2$. The production process has a time-to-build aspect: investment is realized in period 0 and 1 and output is realized in period 2. It is assumed that agents do not have any endowment of goods in period 0 and 1, so they have to borrow from abroad in order to import goods used as inputs in the production process. In period 0, agents borrow to install capital, k_0, which will be depreciated totally at the end of period 2.

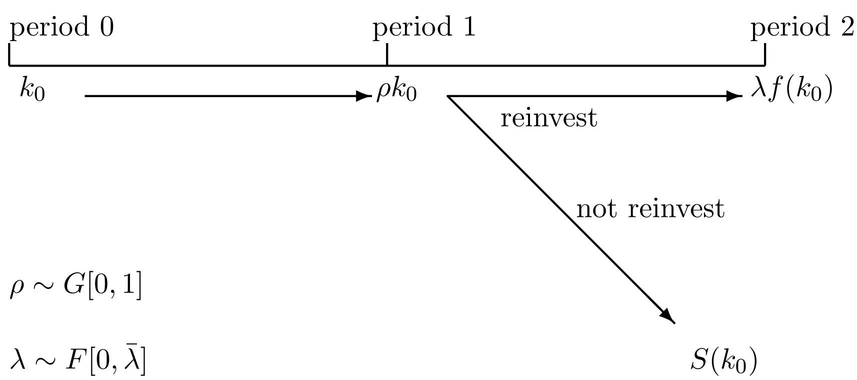
To avoid borrower's moral hazard considerations, we assume that investment is verifiable, or alternatively, that there is no storage technology available, so that the amount borrowed has to be invested in the production process.

Following Holmstrom and Tirole (1998) and Caballero and Krishnamurthy (2001) we introduce a liquidity shock in period 1 as a production shock that the economy has to cope with by borrowing additional funds.

Let ρ be the aggregate liquidity shock that hits the economy in period 1. Agents will need a reinvestment of ρk_0 to continue the project. If they do not reinvest this amount, then the project cannot continue and a scrap value, $S(k_0)$, is obtained in period 2. S is assumed to be quasiconcave, increasing in k_0 and satisfies $S \leq k_0$.

Assume ρ is a random variable distributed between $[0, 1]$ with cumulative distribution function $G(\rho)$. In order to introduce market incompleteness, we assume that ρ is observable but not verifiable, so that contracts in period 0 cannot be made contingent on realized values of the shock in period 1. We do not consider idiosyncratic shocks since we are interested in cases in which the economy as a whole needs liquidity, and we are not concerned with heterogeneity between residents.

If reinvestment is made in period 1, then the project continues and output in period 2 is $\lambda f(k_0)$, where λ is a random productivity shock distributed between $[0, \bar{\lambda}]$ with cumulative distribution $F(\lambda)$, and where $f(k_0)$ is a concave function. It is assumed that $E(\lambda)f(k_0) > k_0$; otherwise, investors will not invest in period 0.



Financial contracts. As noted above, residents have to borrow from abroad in order to produce. This is an ability-to-pay model with no deadweight losses associated with bankruptcy. That is, when realized output is lower than debt face value or when the project is discontinued, lenders can seize output or the scrap value.

It is assumed that debt issued in period 0 and debt issued in period 1 both mature in period 2. International lenders are risk neutral, act in a competitive environment and have enough wealth to provide liquidity to the country when needed. Clearly, for any amount lent they will charge a positive interest rate since the default risk is positive (remember that the minimum value that λ can take is zero).

Without loss of generality, it is assumed that the gross international interest rate is equal to 1. At date 0 domestic agents borrow an amount L_0 (equal to k_0) and agree to pay a total amount of D_0 (i.e. initial amount borrowed plus interest) in period 2. At date 1 they borrow an amount L_1 (equal to ρk_0) whose face value in period 2 is D_1.

3 Equilibrium

In what follows we will solve the model backwards beginning with period 2. In period 1, when the shock hits, we will consider what happens when a senior lender(s) is allowed in that period. Then we will consider period 0.

3.1 Period 2

In period 2, if reinvestment has been made in period 1, output is realized, debt is repaid, and consumption takes place. Consumption will be greater than zero if and only if output is greater than the total face value of debt contracted in period 0 (D_0) and in period 1 (D_1), which occurs when:

$$\lambda f(k_0) - D_0 - D_1 > 0$$

or, equivalently:

$$\lambda > \frac{D_0 + D_1}{f(k_0)} \equiv \lambda^*. \tag{1}$$

Thus, total debt will be repaid and consumption will be positive if and only if the productivity shock is higher than a threshold value λ^*.

Assumption 1. *In case of default (i.e. $\lambda < \lambda^*$) the proportion of output that goes to each creditor equals the share of his loan in total loans, i.e* $\frac{L_i}{L_i + L_{-i}}$.

That is, absent seniority, creditors have *equal footing* on output in case of bankruptcy. We have not assumed that the share of output going to each creditor is equal to the share of his debt in total debt, i.e. $\frac{D_i}{D_i + D_{-i}}$, for simplicity and because, if this were the case, second period debt could be made effectively senior by having a high enough D_1. Since $\frac{L_i}{L_i + L_{-i}}$ need not be the same as $\frac{D_i}{D_i + D_{-i}}$, it is possible that the output due to a creditor in case of default is higher than his debt face value. To rule this out, assume:

Assumption 2. *In case of default, if* $\frac{L_i}{L_i+L_{-i}}\lambda f(k_0)$ *is greater than* D_i *then lender* i *gets* D_i*.*

Thus, a creditor's repayment in period 2 will be the maximum of his contractual value of debt and his share of output under the equal footing scheme.

If reinvestment has not taken place in period 1, the scrap value of the project, $S(k_0)$, is divided between creditors, and consumption is equal to zero (remember that by assumption $S(k_0) < k_0$ and, consequently, $S(k_0) < D_0$).

3.2 Period 1

At the beginning of this period the random variable ρ is observed, there is installed capital (k_0), and the economy inherits a stock of debt contracted in period 0 (D_0). Agents need to borrow ρk_0 in order to continue the project. Since it is assumed that if reinvestment is not made the project ends and consumption is zero, the borrower country will always want to reinvest as long as the highest possible output level is higher than the total value of debt. So the demand for loans is determined by the size of the shock.

3.2.1 Supply of loans under equal footing

As noted above, international capital markets are competitive and the international gross interest rate is equal to 1. Competition between lenders will ensure that expected profits from lending to the country will be zero.

Define λ^1 as the threshold productivity level above which period 1 lenders' output share, computed under equal footing, is greater than their contractual debt value,

$$\lambda^1 \equiv \left[\frac{L_0 + L_1}{L_1}\right]\frac{D_1}{f(k_0)},$$

or equivalently, since L_1 equals ρk_0 and L_0 equals k_0:

$$\lambda^1 \equiv \left[\frac{1+\rho}{\rho}\right]\frac{D_1}{f(k_0)}. \tag{2}$$

Similarly, define λ^0 as the threshold value above which period 0 lenders' output share is greater than D_0:

$$\lambda^0 \equiv [1+\rho]\,\frac{D_0}{f(k_0)}. \tag{3}$$

This last expression follows from the fact that $[1+\rho]$ is equivalent to $\left[\frac{L_0+L_1}{L_0}\right]$.

Note that $\left[\frac{\rho}{1+\rho}\right]\lambda^1 + \left[\frac{1}{1+\rho}\right]\lambda^0 = \lambda^*$, so that the threshold productivity shock above which all debts are repaid (λ^*) is a weighted average of λ^1 and λ^0. When λ^1 is lower than λ^*, it means that D_1 is totally repaid when the productivity shock is at least λ^1; for productivity shocks between λ^1 and λ^*, D_0 holders get output in excess of D_1; and when the productivity shock is higher than λ^*, output is enough to repay both D_0 and D_1. A comparable analysis holds when λ^0 is lower than λ^*. Also, note that λ^0 will be higher than λ^1 if and only if the interest rate charged on period 0 loans is higher than the interest rate charged in period 1; both interest rates are determined in equilibrium below.

Thus, period 1 lenders' zero profit condition under equal footing satisfies:

$$\rho k_0 = \left[\frac{\rho}{1+\rho}\right]\int_0^{\min\left(\lambda^1,\lambda^0\right)} \lambda f(k_0)dF(\lambda) + \int_{\min(\lambda^0,\lambda^*)}^{\lambda^*} \left[\lambda f(k_0) - D_0\right] dF(\lambda) + \int_{\min[\lambda^1,\lambda^*]}^{\bar{\lambda}} D_1 dF(\lambda). \tag{4}$$

The right hand side is period 1 lenders' expected repayment from investing in the country and the left hand side is the amount lent. Alternatively, we can express the same condition in terms of each unit lent:

$$1 = \left[\frac{1}{1+\rho}\right]\int_0^{\min\left(\lambda^1,\lambda^0\right)} \frac{\lambda f(k_0)}{k_0}dF(\lambda) + \frac{1}{\rho}\int_{\min(\lambda^0,\lambda^*)}^{\lambda^*} \left[\frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0}\right] dF(\lambda) + \int_{\min[\lambda^1,\lambda^*]}^{\bar{\lambda}} r_1 dF(\lambda),$$

where $r_1 = \frac{D_1}{\rho k_0}$ is the gross interest rate charged to the country by international lenders.

Lemma 1. *The interest rate r_1 is increasing in the amount lent.*

Proof in the appendix.

So, the higher period 1 shock is, i.e. the higher the amount needed to continue the project, the more expensive, per dollar, it will be for the borrower to continue.

Proposition 1. *If*

$$\int_0^{\bar{\lambda}} \lambda \frac{f(k_0)}{2k_0} dF(\lambda) + \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \left[\frac{1}{2} \frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0} \right] dF(\lambda) < 1, \tag{5}$$

there is a set of liquidity shocks sufficiently close to 1 for which no credit is supplied in period 1 under equal footing.

Proof. A necessary and sufficient condition to have lending in period 1 that satisfies the zero profit condition under equal footing is:

$$\rho k_0 \leq \frac{\rho}{1+\rho} \int_0^{\bar{\lambda}} \lambda f(k_0) dF(\lambda) + \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \left[\frac{1}{1+\rho} \lambda f(k_0) - D_0 \right] dF(\lambda). \tag{6}$$

This is because, given the loan size (ρk_0) and the value of debt issued in period 0 (D_0), period 1 lenders' expected repayment is increasing in D_1; and the right hand side of (6) is lenders' expected repayment when the value of D_1 is high enough that total debt ($D_1 + D_0$) is greater than or equal to the highest possible repayment ($\bar{\lambda} f(k_0)$).[4] If condition (6) is not satisfied then period 1 creditors will expect losses on any loan of size ρk_0. The set of values for ρ satisfying (6) is not empty. The right hand side is unambiguously greater than the left hand side for values of ρ near zero since $\int_0^{\bar{\lambda}} \lambda \frac{f(k_0)}{k_0} dF(\lambda)$ is greater than one.

Since the first term of the right hand side of (6) is a continuous, increasing and concave function of ρ and the second term is continuous and decreasing in ρ, a necessary and sufficient condition to have a range of liquidity shocks where expected profits are negative is that (6) is not satisfied when ρ is equal to one. So, if condition (5) holds, there will be a threshold value of ρ strictly less than one above which expected profits to lenders are negative. Since the expected repayment function is increasing and continuous in D_1, there will be a value of D_1 such that expected repayment equals the loan size.

In what follows we assume that condition (5) holds, in which case there is a $\hat{\rho}$ less than 1 that satisfies:

$$\hat{\rho} k_0 = \frac{\hat{\rho}}{1+\hat{\rho}} \int_0^{\bar{\lambda}} \lambda f(k_0) dF(\lambda) + \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \left[\frac{1}{1+\hat{\rho}} \lambda f(k_0) - D_0 \right] dF(\lambda) \tag{7}$$

[4] If $D_1 + D_0 > \bar{\lambda} f(k_0)$, then $\lambda^* > \bar{\lambda}$ and $\lambda^1 > \bar{\lambda}$. Thus, the left hand side of (6) follows from replacing λ^* by $\bar{\lambda}$ in the left hand side of (4), taking into account that the third term vanishes.

such that for $\rho > \hat{\rho}$ there will be no lending under equal footing. A sufficient condition to have $\hat{\rho} < 1$ is that (5) is true even in the case where D_0 is equal to k_0, which is the lowest possible interest rate on period 0 debt and thus the case most likely to favor lending in period 1. Therefore, a sufficient condition is:

$$E(\lambda)\frac{f(k_0)}{2k_0} + \int_{\min[\frac{2k_0}{f(k_0)},\bar{\lambda}]}^{\bar{\lambda}} \left[\frac{\lambda f(k_0)}{2k_0} - 1\right] dF(\lambda) < 1.$$

Note that it may be in the interest of period 0 lenders, as a group, to lend in period 1 at an expected loss in order to protect their initial claims. However, any individual lender will be better off if the other lenders provide liquidity allowing the project to continue. That is, there is a conflict between private and collective interests; each period 0 lender has incentive to 'free-ride'.[5] This free rider problem has been discussed in the sovereign debt literature; see for example Krugman (1988) and Eichengreen (2002).

Clearly, creditors that have not lent in period 0 do not have any incentive to lend at an expected loss in period 1. In this paper we assume that lenders are atomistic, act in a purely competitive market and can not coordinate actions to pursue their collective interests (i.e. the free-rider issue is severe).[6]

3.2.2 Senior Lender allowed in period 1

Consider the case where a senior lender(s) is allowed to intervene in credit markets in period 1. The concept of seniority is relevant when contractual obligations cannot be totally satisfied; i.e. in the case of bankruptcy. If this is not the case, there is no conflict of interest between creditors and the concept of seniority is not important.

Since senior creditors have priority on output in case of default, they do not consider the stock of existing debt when making their own lending decisions.

Lemma 2. *Senior lenders are willing to lend for any shock ρ.*

[5] The best way to coordinate creditors' actions in the case of a debt crisis, in order to overcome the free-rider problem, is an important issue in current policy and academic debate about the way to construct the New International Financial Architecture.

[6] In a recent speech Anne Krueger states: "...These far-reaching developments in capital markets over the last three decades have not been matched by the development of an orderly and predictable framework for creditor coordination. Because the creditor community is increasingly diverse and diffuse, coordination and collective action problems result when scheduled debt service exceeds a country's ability to pay" (see IMF survey April 2000).

Proof: Senior lenders are willing to lend any amount up to $E(\lambda)f(k_0)$, which is greater than ρk_0, for all ρ, by previous assumption.

Thus, senior lenders are willing to lend in more states of nature than non-senior creditors; seniority allows the economy to overcome more severe liquidity shocks.

Let D_1^s be the value of debt owed to a senior creditor; the threshold productivity shock above which senior lenders are totally repaid is:

$$\lambda^s \equiv \frac{D_1^s}{f(k_0)}. \tag{8}$$

If the productivity shock is lower than this threshold value, senior creditors will not be totally repaid and non-senior creditors will get nothing. The interest rate charged by a senior lender satisfies:

$$\frac{1}{L_1^s}\int_0^{\lambda^s} \lambda f(k_0)dF(\lambda) + \int_{\lambda^s}^{\bar{\lambda}} r_1^s dF(\lambda) = 1, \tag{9}$$

where L_1^s and r_1^s are the amount lent by a senior creditor and the interest rate charged, respectively. The interest rate charged by a senior lender will not be the same as that charged by a non-senior one. In particular:

Lemma 3. *For a given sized loan, the interest rate charged by a senior lender is lower than that charged by a lender without seniority rights.*

Proof in the appendix.

This result implies that total expected consumption in period 2 is higher when a senior lender intervenes and, consequently, the country is ex-post (i.e. conditional on k_0) better off under seniority. Obviously, borrowers prefer to pay less for a given amount lent.

At the beginning of period 1 there is a stock of debt issued in period 0 (D_0) that matures in period 2. The period 1 value of this stock of debt will be affected by the size of the liquidity shock and by the nature (senior or non-senior) of period 1 lenders.

To see the impact of a senior intervention on the period 0 lenders' position, we have to consider whether the liquidity shock is greater or less than $\hat{\rho}$, the threshold value above which non-senior creditors are unwilling to lend.

Consider first the case when $\rho < \hat{\rho}$. In this situation non-senior lenders are willing to lend to the borrower country and a senior intervention will make period 0 lenders worse off. To see why this is the case note that output is divided in period 2 between the country, period 0 and period 1 creditors. At the beginning of period 1, the expected value of output is given, since with $\rho < \hat{\rho}$ the project will continue whether period 1 lenders are senior or not. Meanwhile period 1 lenders, independent of their seniority rights, set the price of the new debt (r_1 or r_1^s) so that expected repayments in period 2 are equal to the size of the loan (ρk_0), by the zero profit condition.

Since expected output and expected repayment to period 1 lenders are the same with and without senior lending, but expected consumption is higher in the first case, it must be the case that period 0 lenders' expected repayment (or, equivalently, the period 1 value of their claims) is lower under a senior intervention. A senior lender does not add value when the country is able to finance the liquidity shock using non-senior sources, but instead merely transfers resources from period 0 debt holders to the country. So, a senior intervention when $\rho < \hat{\rho}$ reduces the period 1 price of the debt issued in period 0.

Consider now the case where $\rho > \hat{\rho}$. In this case, the only way to finance the liquidity shock is by issuing senior debt.

To see how senior lending affects existing creditors in this situation, we compare the period 1 value of existing debt with and without seniority. When senior lending is not allowed, the project is cancelled and the scrap value is obtained. Since this is an ability-to-pay model, period 0 lenders get the entire scrap value (remember that we have assumed that the scrap value is less than k_0). Let V^n be the period 1 value of D_0 when there is no refinancing, that is:

$$V^n(k_0) = S(k_0)$$

and let V^s be the period 1 value of D_0 when a senior intervention is allowed,

$$V^s = \int_{\lambda^s}^{\lambda^B} \left[\lambda f(k_0) - D_1^s(\rho)\right] dF(\lambda) + \int_{\lambda^B}^{\bar{\lambda}} D_0 dF(\lambda)$$

where

$$\lambda^B \equiv \frac{D_0 + D_1^s}{f(k_0)} \tag{10}$$

and

$$\lambda^s \equiv \frac{D_1^s(\rho)}{f(k_0)}.$$

The period 1 value of debt issued in period 0 is equal to the face value (D_0) times the probability of being fully repaid, which occurs when the productivity shock is higher than the threshold value λ^B, plus what existing creditors expect to get when output is not enough to cover total contractual obligations. When the productivity shock is between λ^s and λ^B output is enough to cover senior debt in full but covers only part of non-senior debt. When the shock is less than λ^s, output is not enough to cover senior debt, and non-senior creditors get nothing.

Define the function $\psi(S,\rho)$ as the difference between the period 1 value of debt when a senior intervention is allowed and when it is not:

$$\psi(S,\rho) \equiv V^s - V^n.$$

That is, positive values of ψ imply that period 0 lenders are better off with a senior intervention.

ψ is a function of the liquidity shock and of the scrap value, since both parameters affect the present value of debt with and without senior lending. We have:

$$\frac{\partial \psi}{\partial \rho} = -\int_{\lambda^s}^{\lambda^B} \frac{\partial D_1^s}{\partial \rho} dF(\lambda) < 0^7$$

and

$$\frac{\partial \psi}{\partial S} = -1 < 0.$$

Thus, $\psi(S,\rho)$ is a decreasing function in both arguments.

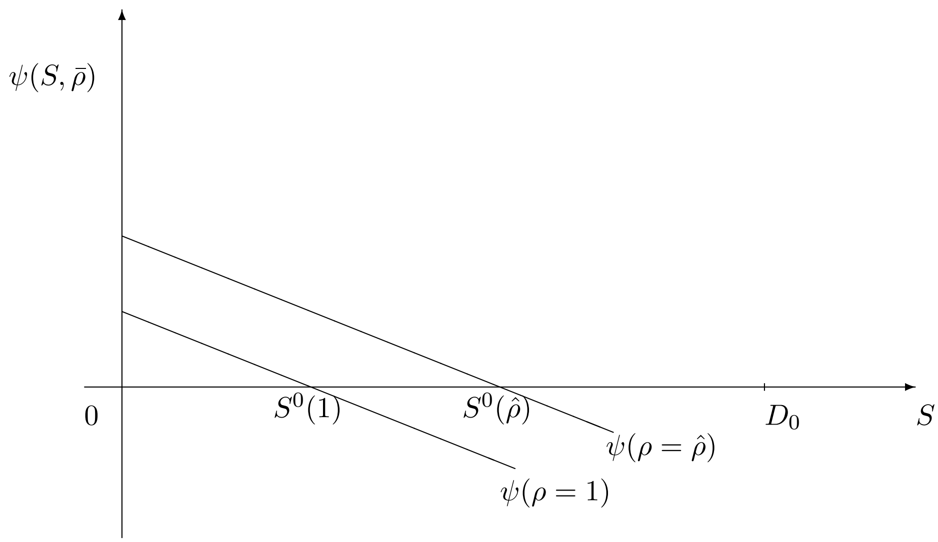
Note that when there is no scrap value (i.e. $S=0$), $\psi(0,\rho)$ is greater than zero for all values of ρ. This is because cancellation leaves existing creditors with zero, while continuation leaves existing creditors with strictly positive expected returns.[8] Also note that if the scrap value is equal to D_0, $\psi(D_0,\rho)$ is strictly negative for all values of ρ since cancellation gives period 0 debt holders the full value of debt with certainty, while a senior intervention reduces the probability of repayment

[7] The terms derived from the differentiation of the integration limits cancel each other out.

[8] The only case when period 0 debt holders expect to get nothing in case of continuation is when D_1^s is equal to $\bar{\lambda} f(k_0)$; but in this case senior lenders' expected profits will be strictly positive (since k_0 is lower than $E(\lambda)f(k_0)$) contradicting the zero profit condition.

below one.

Since $\psi(S,\rho)$ is a continuous and decreasing function in both arguments, and since $\psi\left(0,\rho\right) > 0\ \forall\rho$ and $\psi\left(D_0,\rho\right) < 0\ \forall\rho$, there is for each ρ a unique value of S, denoted by $S^0(\rho)$, where $\psi(S,\rho) = 0$. The higher the liquidity shock, the lower the value of S^0. We can express this in the following figure:



Thus, existing creditors' view of senior intervention depends on the size of the liquidity shock and the project's scrap value. We can distinguish three situations. First, when the scrap value is lower than $S^0(1)$, a senior intervention will raise the value of existing debt for all $\rho > \hat{\rho}$. In this case, the value of liquidation is so low that even in the worst possible scenario (highest senior debt) period 0 lenders prefer to continue the projects.

Second, when the scrap value is between $S^0(1)$ and $S^0(\hat{\rho})$ there is a set of liquidity shocks in the vicinity of 1 where a senior intervention makes period 0 debt holders worse off. Moreover, there is a set of liquidity shocks close enough (from the right) to $\hat{\rho}$ where a senior intervention makes period 0 debt holders better off. So, in this zone seniority has ambiguous effects on existing creditors depending on the size of the liquidity shock. In particular, there is a nonlinear effect of senior intervention on the price of the debt issued in period 0 that is consistent with the empirical evidence, as will be seen in section 4 below. When the shock is small ($\rho < \hat{\rho}$) a senior intervention reduces this price (i.e. increases spreads over the international interest rate); when the shock is not too far above $\hat{\rho}$, a senior intervention increases this price; and when the shock is close to 1 the price is reduced by senior intervention again.

Finally, when the scrap value is higher than $S^0(\hat{\rho})$, a senior intervention always makes period 0 debt holders worse off. Because the scrap value is so high, initial lenders prefer to get that value for sure rather than continuing the project and taking the risk of not being repaid.

We can summarize the findings of this section in the following proposition:

Proposition 2. *Conditional on k_0 a senior intervention will improve debtors' situation in all cases since it allows a higher level of consumption. The effect on period 0 debt holders depends on ρ and S:*

- *If $\rho < \hat{\rho}$ a senior intervention will always make existing creditors worse off.*
- *If $\rho > \hat{\rho}$ we have three possible scenarios:*
  1. *If $S < S^0(1)$ senior lending makes existing creditors better off for all values of ρ.*
  2. *If $S^0(1) < S < S^0(\hat{\rho})$ existing creditors' situation will improve if ρ is close enough to $\hat{\rho}$ and will be worsened if ρ is close enough to 1.*
  3. *If $S^0(\hat{\rho}) < S$ senior lending always makes existing creditors worse off.*

That is, senior lending may affect borrowers and lenders differently; in some cases, it will allow for the continuation of projects when existing creditors would prefer to liquidate them. In these cases, there is a conflict of interest between the borrower and the lenders since the former is always willing to finish the project.

3.3 Period 0

Period 0 is the planning period. Borrowers decide how much to invest and borrow in order to maximize their expected utility (expected consumption in period 2), and lenders set the price of their loans in order to attain zero expected profits.

In period 0 individuals have uncertainty about two shocks: the liquidity shock (ρ) and the productivity shock (λ). That is, expectations have to be taken over two random variables. We consider the case where all agents have perfect foresight about the nature of future interventions. That is, borrowers and lenders take their decisions knowing whether interventions in period 1 will be senior or equal footing.

3.3.1 Equal footing in period 1

Agents make their decisions taking into account that if the liquidity shock in period 1 is high enough the project will have to be discontinued and there will be no consumption and only partial debt repayment.

In equilibrium, borrowers in period 0 decide the amount they want to borrow in order to maximize their expected utility, taking into account how their decisions affect the credit conditions they face. Borrowers maximize:

$$V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left\{ \int_{\lambda^*}^{\bar{\lambda}} \left[\lambda f(k_0) - D_0 - D_1(\rho k_0)\right] dF(\lambda) \right\} dG(\rho) \tag{11}$$

subject to

$$\begin{aligned} k_0 = & \int_0^{\hat{\rho}(k_0)} \left\{ \left[\frac{1}{1+\rho}\right] \int_0^{\min\left(\lambda^1,\lambda^0\right)} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^1,\lambda^*)}^{\lambda^*} \left[\lambda f(k_0) - D_1\right] dF(\lambda) + \right. \\ & \left. + \int_{\min[\lambda^0,\lambda^*]}^{\bar{\lambda}} D_0 dF(\lambda) \right\} dG(\rho) + \int_{\hat{\rho}(k_0)}^1 S(k_0) dG(\rho) \end{aligned} \tag{12}$$

and

$$\rho k_0 = \left[\frac{\rho}{1+\rho}\right] \int_0^{\min\left(\lambda^1,\lambda^0\right)} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^0,\lambda^*)}^{\lambda^*} \left[\lambda f(k_0) - D_0\right] dF(\lambda) + \int_{\min[\lambda^1,\lambda^*]}^{\bar{\lambda}} D_1 dF(\lambda). \tag{13}$$

V_0 is borrowers' expected utility, and λ^* , λ^1 and λ^0 are as defined above in (1),(2) and (3) respectively. The outer integral of (11) corresponds to expectations taken over the liquidity shock, recognizing that if $\rho > \hat{\rho}(k_0)$ consumption is zero under equal footing. The inner integral corresponds to expectations taken over the productivity shock, knowing that consumption will be positive if output is enough to cover the total value of debt contracted in period 0 and in period 1. That is, consumption will be positive if and only if $\rho > \hat{\rho}(k_0)$ and $\lambda > \lambda^*$.

Equation (12) is the zero expected profit condition for period 0 lenders who face uncertainty about both the liquidity shock and the productivity shock. They know that if $\rho > \hat{\rho}(k_0)$, the project will not continue and they will get the scrap value. If $\rho < \hat{\rho}(k_0)$ (i.e. there is no liquidation in period 1) what they expect to get in period 2 depends on the productivity shock. Analogously with the period 1 lenders' zero profit condition in equation (4), if output is not enough to cover either D_0 or D_1, period 0 lenders receive a share $\frac{1}{1+\rho}$ $\left(\text{i.e. } \frac{L_0}{L_0+L_1}\right)$ of output. If the proportion of output that corresponds to period 1 lenders allows D_1 to be repaid for output levels lower than that required to cover total debts $\left(\text{i.e. } D_0 + D_1\right)$, then period 0 debt holders get output minus D_1 until output is enough to pay also D_0. When output is higher than this amount, they are repaid in full.

Equation (13) is lenders' zero profit condition in period 1 for a given ρ, as analyzed above in equation (4).

Integrating equation (13) from zero to $\hat{\rho}(k_0)$ and adding this expression to equation (12) we get:

$$k_0 + \int_0^{\hat{\rho}(k_0)} \rho k_0 dG(\rho) = \int_0^{\hat{\rho}(k_0)} \left\{ \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda^*}^{\bar{\lambda}} \left[D_0 + D_1\right] dF(\lambda) \right\} dG(\rho) + \int_{\hat{\rho}(k_0)}^{1} S(k_0) dG(\rho). \tag{14}$$

Adding and subtracting $\int_0^{\lambda^*} \lambda f(k_0) dF(\lambda)$ in equation (11) we get:

$$V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left\{ \int_0^{\bar{\lambda}} \lambda f(k_0) dF(\lambda) - \left[\int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda^*}^{\bar{\lambda}} \left(D_0 + D_1\right) dF(\lambda) \right] \right\} dG(\rho). \tag{15}$$

Inserting equation (14) into (15) we can express the borrower value function as:

$$V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left[\int_0^{\bar{\lambda}} \lambda f(k_0) dF(\lambda) \right] dG(\rho) - k_0 \left(1 + \int_0^{\hat{\rho}(k_0)} \rho dG(\rho) \right) + \int_{\hat{\rho}(k_0)}^{1} S(k_0) dG(\rho). \tag{16}$$

For simplicity, assume that the scrap function is linear in the investment level; i.e. $S(k_0) = sk_0$. Then, the optimal investment (and borrowing) level under equal footing satisfies the following first-order condition:

$$\int_0^{\hat{\rho}(k_0)} \left[E\left(\lambda\right) \frac{\partial f(k_0)}{\partial k_0} \right] dG(\rho) + \int_{\hat{\rho}(k_0)}^{1} s \, dG(\rho) = 1 + \int_0^{\hat{\rho}(k_0)} \rho \, dG(\rho) - \left\{ E\left(\lambda\right) f(k_0) - \hat{\rho} k_0 - s k_0 \right\} G'(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial k_0}, \tag{17}$$

where $\frac{\partial \hat{\rho}}{\partial k_0} < 0$; that is, the higher the level of investment, the lower the range of liquidity shocks for which continuation in period 1 will be possible without senior lending. See the appendix for the proof.

To set the optimal investment level borrowers balance the marginal benefit, given by the marginal productivity of capital and by the effect that one more unit invested has on the scrap value; and the marginal costs, given by the cost of investing in period 0, the expected cost of reinvesting in period 1 and the negative effect that one more unit of investment has on the threshold value $\hat{\rho}(k_0)$. Since higher scrap values allow period 0 lenders to offer better terms (see equation (12)), the optimal level of investment increases in s.[9]

3.3.2 Senior lending in period 1

Assuming that senior lending is allowed in period 1, the objective function is:

$$V_0^s = \max_{k_0^s} \int_0^1 \left\{ \int_{\lambda^B}^{\bar{\lambda}} \left[\lambda f(k_0) - D_0^s - D_1^s(\rho k_0) \right] dF(\lambda) \right\} dG(\rho) \tag{18}$$

subject to:

$$k_0^s = \int_0^1 \left\{ \int_{\lambda^s}^{\lambda^B} \left[\lambda f(k_0^s) - D_1(\rho k_0^s) \right] dF(\lambda) + \int_{\lambda^B}^{\bar{\lambda}} D_0 dF(\lambda) \right\} dG(\rho) \tag{19}$$

and

$$\rho k_0^s = \int_0^{\lambda^s} \lambda f(k_0^s) dF(\lambda) + \int_{\lambda^s}^{\bar{\lambda}} D_1^s dF(\lambda), \tag{20}$$

where the superscript "s" implies that senior lending is allowed; and λ^B and λ^s are as defined in (10) and (8) above. Now individuals choose investment knowing that the projects will continue in period 1 for all possible values of the liquidity shock, so the expectation in (18) is taken over the whole range of ρ.

Equation (19) and equation (20) are the zero profit conditions for period 0 and 1 respectively. Period 0 lenders know that there will not be liquidation in period 1 and, consequently, they do not consider scrap value in their zero profit condition. They know that senior lenders will have priority on output and they will begin receiving repayment if and only if senior debts are totally repaid. Equation (20) is the same as equation (9) above.

As before, integrating equation (20) over all possible values of ρ and adding this expression to

[9] Analytically, this follows from applying the implicit function theorem to (17), taking into account that the second order condition is satisfied.

equation (19) we obtain:

$$k_0^s + \int_0^1 \rho k_0^s dG(\rho) = \int_0^1 \left\{ \int_0^{\lambda^B} \lambda f(k_0^s) dF(\lambda) + \int_{\lambda^B}^{\bar{\lambda}} \left(D_1^s + D_0^s\right) dF(\lambda) \right\} dG(\rho). \tag{21}$$

Adding and subtracting $\int_0^{\lambda^B} \lambda f(k_0^s) dF(\lambda)$ in equation (18) and plugging equation (21) in the resulting expression, the borrowers' value function is:

$$V_0^s = \max_{k_0^s} \int_0^1 \left[\int_0^{\bar{\lambda}} \lambda f(k_0^s) dF(\lambda) \right] dG(\rho) - k_0^s \left[1 + \int_0^1 \rho dG(\rho) \right]. \tag{22}$$

Optimal investment satisfies the following first order condition:

$$\int_0^1 \left[E\left(\lambda\right) \frac{\partial f(k_0^s)}{\partial k_0^s} \right] dG(\rho) = 1 + \int_0^1 \rho \, dG(\rho). \tag{23}$$

Thus, borrowers balance the expected marginal product of capital with the expected marginal cost of investing one more unit, given by the marginal cost at date 0 plus the expected marginal cost of continuation in period 1.

3.3.3 Comparison

In this section we compare how the optimal level of investment and borrowers' welfare is affected by allowing senior lending in period 1.[10] As noted above, having senior lending allows the project to continue in circumstances where it otherwise would have had to be liquidated. Although borrowers always prefer to continue ex-post, non-senior lenders would prefer to liquidate the project if the scrap value is high enough. In this case, the anticipation of senior lending makes period 0 lenders offer more onerous terms in their lending, leading to a lower level of investment. When the scrap value is low enough, so that period 0 lenders prefer a senior intervention in period 1, the expectation of the intervention leads to a higher level of investment.

To see how optimal investment is affected, compare equation (17) and equation (23). First, assume that there is no scrap value in case of liquidation (i.e. $s = 0$ in (17)). In this case, the term in brackets that multiplies $\frac{\partial \hat{\rho}}{\partial k_0}$ in (17) is positive (otherwise there will be no investment in period

[10]Since lenders always set the price of their period 0 loans in such a way that expected profits are zero, allowing a senior lender in period 1 does not affect period 0 lenders' welfare ex-ante as long as lenders are fully informed about the nature of future interventions.

0), implying that $\int_0^1 [E(\lambda)f'(k_0^s) - \rho]\, dG(\rho) < \int_0^{\hat{\rho}} [E(\lambda)f'(k_0) - \rho]\, dG(\rho)$. This inequality can be expressed as:

$$E(\lambda)f'(k_0^s)\left[1 - \frac{f'(k_0)}{f'(k_0^s)}\Pr(\rho \leq \hat{\rho})\right] < E(\rho/\rho > \hat{\rho})\left[1 - \Pr(\rho < \hat{\rho})\right].$$

Since the first term on the left hand side is greater than one (by (17)), while the first term on the right hand side is less than one by definition, it must be the case that $f'(k_0) > f'(k_0^s)$ implying that $k_0 < k_0^s$.

In this case borrowers are ex-ante better off with a senior intervention. The intuition is that, when the scrap value is low, the effect of a senior intervention on project continuation dominates its effect on the cost of period 0 loans. Note that in this model the expectation of senior lending does not make individuals take riskier actions, so the increase in borrowing and lending in period 0 is not the consequence of *moral hazard* but of avoiding inefficient liquidation.

Now consider the case where the scrap value is different than zero. As noted above, the scrap value makes period 0 credit conditions under equal footing less onerous, because it represents a positive payoff in case of liquidation. From equation (17) we can see that the higher is s, the higher the level of investment under equal footing. When s is equal to one, the term in brackets on the right hand side of (17) is less than or equal to zero (see equation (7)), and a comparison of (17) and (23) yields

$$E(\lambda)f'(k_0^s)\left[1 - \frac{f'(k_0)}{f'(k_0^s)}\Pr(\rho \leq \hat{\rho})\right] > E(\rho/\rho > \hat{\rho})\left[1 - \Pr(\rho < \hat{\rho})\right].$$

In this case we can not rule out the possibility of k_0^s being lower than k_0. If this is the case, the effect of seniority on the cost of loans is stronger than its effect on project continuation.

Note that a higher scrap value increases the ex-ante utility level when senior intervention is not allowed in period 1. A comparison of (16) and (22) yields that borrowers may be ex-ante better off when senior lending is not allowed in period 1, depending on the size of s.

Numerical exercise. We present a numerical example to show that for scrap values sufficiently high it is possible to have a lower level of investment and welfare when a senior lender is allowed. Consider the case where $f(k_0) = k^{0.8}$, λ is uniformly distributed in [0,3], ρ is uniformly distributed in [0,1], and $s = 1$. In this case we obtain that $V_0^s = 0.12 \ < \ V_0 = 0.15$ and $k_0^s = 0.32 \ < \ k_0 = 0.59$.

As noted above, there may be circumstances where senior lending creates a conflict of interest between lenders and borrowers in period 1. Ex-post, lenders may want liquidation although it is always in borrowers' interest to continue; if senior lending is made by institutions, interventions will be determined by which interests they are identified with (that of borrowers or that of lenders).

Assume borrowers are able to set institutions in period 0 that govern the availability of senior lending in period 1. If $V_0 < V_0^s$, borrowers will allow for senior lending in period 1, and lenders will set the price of debt, knowing that there will be senior lending, in such a way that expected profits are zero.

If $V_0 > V_0^s$, borrowers will maximize ex-ante expected utility by committing not to allow senior lending in period 1. Note that this promise is not time consistent, since ex-post, borrowers would always prefer senior lending to equal footing lending in period one. If no commitment technology is available, then period 0 lenders will set the price of debt anticipating senior intervention in period 1 and the borrower country will be worse off.

4 Empirical Evidence

There are several empirical papers that study the effects that IMF interventions have on countries' access to capital markets, with varying conclusions among them.[11] The study most related to this paper is Mody and Saravia (2003). They study the effects of IMF loans on spreads and on the probability of issuing bonds by emerging markets economies. The empirical findings that are related to this work are:

- The impact of IMF lending on spreads depends on the level of countries' indebtedness. In particular, there is a 'U' shaped effect on spreads; IMF intervention raises spreads when the country's solvency situation is at the extremes, either solid or weak, and reduces spreads for intermediate levels.

- 'Precautionary programs', in which the country does not disburse the money made available by the IMF, reduce spreads and increase the probability of issuing bonds.

The first finding implies that when the countries' solvency situation is either good or weak, an IMF intervention raises spreads and reduce them when solvency is in an intermediate range.

[11] See Cotarelli and Giannini (2002) for a survey.

In our model, the higher the period 1 (middle period) liquidity shock, the worse is the country's solvency situation. The model is able to show that for small liquidity shocks (when non-senior credit is available) an IMF loan raises spreads; but when shocks are higher than a threshold value above which non-senior lending is not available, the effect on spreads depends on what lenders' expect to get in the case that reinvestment does not take place (the project's scrap value in the model). When the scrap value is in an intermediate range, an IMF intervention will reduce spreads when the liquidity shock is not to far above the threshold value, and will increase spreads when the shock is in the upper tail of the distribution. Thus, there is a nonlinear effect consistent with the empirical evidence.

The second empirical finding is related to our model's planning period. A precautionary program is a proxy for the possibility of future interventions, since it is money that has already been lent to the country but is not being used (insurance). We have seen that in equilibrium the initial borrowing level and its cost are affected by the possibility of a future senior intervention, and that the model replicates the empirical finding when the project's scrap value is not too high.

5 Conclusions

This paper presents a model that emphasizes the effects of a senior creditor (such as the IMF) on the borrower country's and on creditors' welfare. When the shock that hits the economy is big and markets are incomplete, seniority allows continuation of projects that otherwise would have to be abandoned; in this sense the IMF completes markets by financing liquidity needs when other creditors are not willing (or can not coordinate efforts) to do so. Ex-post, once the shock has occurred, an IMF loan would increase borrower welfare by providing cheaper funds than non-senior lenders, allowing for a higher consumption level. The effects on non-senior creditors depend on the size of the shock and on what they expect to get when projects are discontinued. When non-senior financing can be attracted to the country a senior intervention makes existing creditors worse off, since it does not improve the country's repayment capacity but worsens their relative position. Even when senior lending is necessary to cope with the shock, other creditors may be worse off with an IMF intervention, depending on the size of the shock and the project's scrap value.

In the absence of clear rules set ex-ante governing the types of permissable intervention, an institution providing senior lending would have to weigh the potentially conflicting wishes of borrowers and lenders, and decide when to intervene according to whose interests it more closely represents.

The anticipation of a senior lender affects the optimal level of investment and the borrower country's welfare in the planning period. On the one hand, having an institution that completes markets induces a higher level of investment by increasing the states of nature where reinvestment takes place; but on the other hand, the fact that senior lenders have priority in case of default discourages investment. If the second effect is stronger than the first one, optimal investment will be lower when senior intervention is anticipated. It may be the case that the country would maximize expected utility by committing itself not to borrow from a senior lender to cope with shocks that hit the economy. Since the country has incentives to borrow from a senior institution once the shock occurs, this promise is not time consistent. An institution with clear rules about when to intervene will be necessary to achieve credibility and will be Pareto improving.

A Proof of Lemma 1

From zero expected profit condition we write the implicit function

$$Q(\rho, r_1) \equiv 1 - \left[\frac{1}{1+\rho}\right] \int_0^{Min\left(\lambda^1, \lambda^0\right)} \frac{\lambda f(k_0)}{k_0} dF(\lambda) - \frac{1}{\rho} \int_{Min(\lambda^0, \lambda^*)}^{\lambda^*} \left[\frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0}\right] dF(\lambda) - \int_{Min[\lambda^1, \lambda^*]}^{\bar{\lambda}} r_1 dF(\lambda) = 0$$

First consider the case where $\lambda^0 < \lambda^*$; applying the implicit function theorem we have that $\frac{\partial r_1}{\partial \rho} = -\frac{\frac{\partial Q(.)}{\partial \rho}}{\frac{\partial Q(.)}{\partial r_1}}$

$$\frac{\partial Q(.)}{\partial \rho} = \frac{1}{(1+\rho)^2} \int_0^{\lambda^0} \lambda \frac{f(k_0)}{k_0} dF(\lambda) + \frac{1}{\rho^2} \int_{\lambda^0}^{\lambda^*} \left[\lambda \frac{f(k_0)}{k_0} - \frac{D_0}{k_0}\right] dF(\lambda) +$$

$$+ \left[\frac{\lambda^0 f(k_0)}{\rho k_0} - \frac{D_0}{\rho k_0} - \frac{1}{1+\rho} \frac{\lambda^0 f(k_0)}{k_0}\right] F'(\lambda^0) \frac{\partial \lambda^0}{\partial \rho} + \left[r_1 - \frac{\lambda^* f(k_0)}{\rho k_0} + \frac{D_0}{\rho k_0}\right] F'(\lambda^*) \frac{\partial \lambda^*}{\partial \rho}$$

Taking into account that $\lambda^0 = \frac{(1+\rho)D_0}{f(k_0)}$ and that $\lambda^* = \frac{D_0 + D_1}{f(k_0)}$ we have that the last two terms are both equal to zero. Thus, $\frac{\partial Q(.)}{\partial \rho} > 0$. Moreover,

$$\frac{\partial Q(.)}{\partial r_1} = -\int_{\lambda^*}^{\bar{\lambda}} r_1 dF(\lambda) < 0.$$

Thus, $\frac{\partial r_1}{\partial \rho} > 0$.

Proceeding in the same way we can show that this is also the case when $\lambda^1 < \lambda^*$.

B Proof of Lemma 3

To simplify the exposition of this proof consider the special case when $\lambda^0 = \lambda^1 = \lambda^*$. Without seniority, the interest rate is pinned down by:

$$\int_{\lambda^*}^{\bar{\lambda}} r_1 dF(\lambda) + \left[\frac{1}{L_0 + L_1}\right] \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) = 1$$

and with seniority by

$$\int_{\hat{\lambda}}^{\bar{\lambda}} r_1^s dF(\lambda) + \left[\frac{1}{L_1^s}\right] \int_0^{\hat{\lambda}} \lambda f(k_0) dF(\lambda) = 1$$

The proof proceeds by contradiction. Assume that $r_1 = r_1^s$. This implies that $R_1^s = R_1$ since $L_1^s = L_1$, and this implies that $\hat{\lambda} < \lambda^*$ for sure. Splitting the integral limits and equating both expressions:

$$\int_{\lambda^*}^{\bar{\lambda}} r_1 dF(\lambda) + \left[\frac{1}{L_0 + L_1}\right] \left[\int_0^{\hat{\lambda}} \lambda f(k_0) dF(\lambda) + \int_{\hat{\lambda}}^{\lambda^*} \lambda f(k_0) dF(\lambda)\right] =$$

$$= \int_{\hat{\lambda}}^{\lambda^*} r_1^s dF(\lambda) + \int_{\lambda^*}^{\bar{\lambda}} r_1^s dF(\lambda) + \left[\frac{1}{L_1^s}\right] \int_0^{\hat{\lambda}} \lambda f(k_0) dF(\lambda)$$

Rearranging we get:

$$\int_{\lambda^*}^{\bar{\lambda}} (r_1 - r_1^s)\, dF(\lambda) = \int_{\hat{\lambda}}^{\lambda^*} r_1^s dF(\lambda) + \int_0^{\hat{\lambda}} \lambda f(k_0) \left[\frac{1}{L_1^s} - \frac{1}{L_0 + L_1}\right] dF(\lambda) - \left[\frac{1}{L_0 + L_1}\right] \int_{\hat{\lambda}}^{\lambda^*} \lambda f(k_0) dF(\lambda)$$

The second term of the right hand side is positive and the first term is greater than the third one under the assumption that $r_1^s = r_1$. So the right hand side is unambiguously positive. So, the left hand side should be positive and not zero as it is under our original assumption.

There is a contradiction.

Now we have to show that r_1^s cannot be greater than r_1. Again we proceed by contradiction. Assume $r_1^s > r_1$, which implies that $R_1^s > R_1$. There are two possible cases: $\hat{\lambda} < \lambda^*$ and $\hat{\lambda} > \lambda^*$. In the first case the proof is the same as before. In the second case, split the integral limits as above, but now with $\hat{\lambda} > \lambda^*$.We get

$$\int_{\hat{\lambda}}^{\bar{\lambda}} (r_1 - r_1^s)\, dF(\lambda) = \left[\frac{1}{L_1} - \frac{1}{L_0 + L_1}\right] \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda^*}^{\hat{\lambda}} \left[\lambda f(k_0) - r_1\right] dF(\lambda)$$

The second term of the right hand side is positive under our assumption that $\hat{\lambda} > \lambda^*$. Conditional on λ being greater than λ^* and lower than $\hat{\lambda}$ output is greater than r_1. This is because output is higher than the necessary to totally repay the contractual interest rate r_1 (i.e. $\lambda > \lambda^*$). So, the left hand side is unambiguously positive and so should be the left hand side. But this contradicts our initial assumption.We conclude that r_1^s must be lower than r_1.

C Proof that $\frac{\partial\hat{\rho}}{\partial k_0} < 0$

From equation (7), define the function $F(k_0, \hat{\rho})$:

$$F(k_0,\hat{\rho}) \equiv \hat{\rho} - \frac{\hat{\rho}}{1+\hat{\rho}}\int_0^{\bar{\lambda}} \lambda \underbrace{\frac{f(k_0)}{k_0}}_{A} dF(\lambda) - \int_{Min[\lambda^0,\bar{\lambda}]}^{\bar{\lambda}} \left[\left(\frac{1}{1+\hat{\rho}}\right)\lambda \underbrace{\frac{f(k_0)}{k_0}}_{A} - \underbrace{\frac{D_0}{k_0}}_{B}\right] dF(\lambda) = 0$$

Applying the implicit function theorem to this expression:

$$\frac{\partial\hat{\rho}}{\partial k_0} = -\frac{\frac{\partial F(.)}{\partial k_0}}{\frac{\partial F(.)}{\partial \hat{\rho}}}$$

$$\frac{\partial F(.)}{\partial k_0} = -\frac{\hat{\rho}}{1+\hat{\rho}}E(\lambda)\frac{\partial A}{\partial k_0} - \int_{Min[\lambda^0,\bar{\lambda}]}^{\bar{\lambda}} \left[\left(\frac{1}{1+\hat{\rho}}\right)\lambda\frac{\partial A}{\partial k_0} - \frac{\partial B}{\partial k_0}\right] dF(\lambda) > 0$$

Since A is a concave function and B is a convex function (analogous to Lemma 1), this expression is greater than zero.

$$\frac{\partial F(.)}{\partial \hat{\rho}} = 1 - \frac{1}{(1+\hat{\rho})^2}E(\lambda)\frac{f(k_0)}{k_0} + \int_{Min[\lambda^0,\bar{\lambda}]}^{\bar{\lambda}} \frac{1}{(1+\hat{\rho})^2}\lambda\frac{f(k_0)}{k_0}dF(\lambda)$$

This expression will have the same sign as:

$$(1+\hat{\rho}) - \frac{1}{(1+\hat{\rho})}E(\lambda)\frac{f(k_0)}{k_0} + \int_{Min[\lambda^0,\bar{\lambda}]}^{\bar{\lambda}} \frac{1}{(1+\hat{\rho})}\lambda\frac{f(k_0)}{k_0}dF(\lambda),$$

from the definition of $\hat{\rho}$ (equation (7)) we have that:

$$\frac{1}{1+\hat{\rho}}E(\lambda)\frac{f(k_0)}{k_0} < 1$$

so that,

$$\frac{\partial F(.)}{\partial \hat{\rho}} > 0$$

These imply that $\frac{\partial\hat{\rho}}{\partial k_0} < 0$.

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