A simple model of dynamic incentives and occupational choice with motivated agents

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Abstract

People care not only about how much they are paid, but also about what they do. The aim of this paper is to investigate the interplay between an individual's personal motivation and the structure of dynamic incentive schemes. The optimal long-term contract involves not only transfers at each date which are contingent on the whole past history of outcomes but also an initially assigned mission. A modified martingale property is shown to hold in equilibrium. Moreover, the occupational choice problem is investigated and an optimal job separation rule is derived.

Keywords: dynamic moral hazard, motivated agent, occupational choice.

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1 Introduction

Economists have traditionally modeled the employment relationship using a principal - agent framework. An implicit, but crucial, assumption of this setting is the underlying idea that the agent has no willingness to exert effort in the absence of an explicit, upward sloping, incentive contract offered by the principal.

Nevertheless, in general, people care not only about how much they are paid, but also about what they do. In other words, people value the characteristics of their task in addition to how much they are paid for their work. In particular, sociologists have identified two sources of intrinsic motivators: task involvement and goal identification. The former denotes the degree to which an agent derives utility from the actual performance of a task. The latter emphasizes that agents have objectives for accomplishments that are independent of any financial reward.

The aim of this paper is to investigate the interplay between an individual's personal motivation and the structure of dynamic incentive schemes. I use a dynamic (two-period) moral hazard model with one-sided lack of commitment to investigvate the effect of motivation on the optimal transfer scheme. It has been shown (Lambert (1983), Rogerson (1985)) that the optimal long-term contract exhibits memory. Does this result continue to hold when agents are motivated?

More generally, is it always optimal for the principal to retain the agent in the second period with probability one, once he has discovered his preferred mission (task)?

A labor contract involves both contingent (on the whole past history of outcomes) transfers and an initially assigned mission. They are signed under incomplete and symmetric information. With time (and experimentation), the agent discovers what he likes to do (his preferred task). Hence, whenever his current job is very different from his favorite one, it may be optimal to have a job separation. Clearly, the optimal contract originally offered by the principal should take into account this possibility, merely that the agent can walk away when he does not like his job.

Some of the paper's ideas can be illustrated by an example.

To aid the exposition, we shall imagine a supervisor (principal) - PhD student (agent) relationship.

At the onset of a PhD program, very few students know exactly the precise contours of the field in which they would like to conduct research in.

However, many factors, such as what the student has done previously in his undergraduate program, the trend of the job market, suggestions (or constraints) from a third party, determine the subject of the student's first paper.

Facilitated by elective coursework, seminar partecipation and research assistantship, the student learns his true type, that is to say, his favorite research area.

This area can be either closely related to the topic of his first paper or distant.

In the latter case, the student, especially when his initial research output has been successful, will face a trade-off between continuing the ongoing relationship with the current supervisor (and collecting the rewards implied by the memory property of the optimal long-term contract) or following his discovered bent (and starting from scratch a totally different dissertation under the supervision of a brand-new professor).

We will see that it may be optimal to change field when the distance between the true type and the initial one is large enough. Suppose that the student's original focus was in applied industrial organization. Then, by chance, he found out to enjoy much more informational economics. One conclusion of the analysis is that a changeover is, in this case, optimal. The supervisor, at the beginning of the relationship, fully and rationally anticipates the future events. For this reasons, he is not going to invest heavily on the student early in the relationship. Rather, rewards are smoothed intertemporally, and thus concentrated in future periods.

Outline. The paper is organized as follows. In the next section I discuss some related literature. Section three describes the basic problem. Section four studies optimal contracts. Section five both lays out some testable implications and suggests possible extensions left for future research. Section six concludes. The computation of the optimal long-term contract is contained in the appendix.

2 Related Literature

The closest paper in the literature is Besley and Ghatak (2003). They study incentives in a context of motivated agents, that is agents who pursue goals because they perceive intrinsic benefits from doing so. This paper uses the same baseline model but allows for dynamics. This is not an innocuous extension. In particular, the shape of the optimal long-term incentive scheme becomes more elaborated and an additional issue naturally arises: the occupational choice problem.

The ideas developed in the paper can be related to three main strands of the economic literature: occupational choice, intrinsic motivation and contract theory.

There is a large literature on occupational choice, started with Jovanovic (1979) and continued with Harris-Holmstrom (1982), Gibbons-Murphy (1992) and Felli-Harris (1996) among others. While these models concentrate their focus on the wage dynamics and the optimal quitting policy, I prefer to investigate in more detail the contractual aspect. From one side, the contract space I am dealing with is larger and so the related set of predictions richer. From the other side, the bandit nature of the problem faced by the agent is much simpler.

Intrinsic motivation stands for the idea that individuals undertake some actions for their own sake. Thus, individuals perform acts for the pleasure they experience in doing them: monetary incentives can be either crowded out or complemented by intrinsic motivation. Kreps (1997) draws the attention to economists' research agenda on the importance of the interaction between norms and economic incentives. Because intrinsic returns are non-contractible, Murdock (2002) shows the optimality of implicit contracts. Nevertheless, his paper focuses on the implications of goal identification rather than task involvement, as this paper does. Moreover, in his world, contracts are always, and necessarily, incomplete: the court of law is totally superfluous. Benabou and Tirole (2003) use a static informed principal framework to identify conditions under which monetary rewards are harmful, instead of beneficial. Their focus is on the motivation crowding-out effect.[1] My paper neglects this peculiarity and money has never a detrimental impact on the agent's payoff, neither directly nor indirectly.

Chiappori et al. (1994) offer an interesting survey of the literature on repeated moral hazard. Rochet and Stole (2002) provide a complete analysis of a static model with random partecipation constraints and a continuum of types, while Laffont and Tirole (1990) study a type-dependent partecipation constraint. Ma (1991) analyses a similar contractual problem. However, there are at least two differences between his paper and this one. Firstly, here agents are motivated, and thus there is an occupational choice problem. Secondly, in my model, the consequences of the provision of effort dye out at the end of the period, while in Ma's model, they have a long-run impact on the principal's production process. Hence, in his world, the information is not only incomplete but also asymmetric and renegotiation-proof contracts have to be used.

[1] Frey and Oberholzer-Gee (1997) provide an empirical investigation about the hidden costs of monetary rewards.

3 Model

3.1 Players

There are two players. One (risk-neutral) principal (P) and one (risk-averse) agent (A). P can credibly commit to a long-term contract, while A cannot commit.

I am not modeling how the P is able to commit to this contract. One possibility is a supergame equilibrium, where the deviation is severely punished by the one-shot (Nash) equilibrium (trigger strategy)[2]. Another possibility is the existence of external enforcement mechanism. The agent's inability to commit hinges on the inalianability of human capital.

3.2 Preferences and Production Technology

3.2.1 P

P has a separable (over time) von Neumann-Morgestern utility function:

$$V_P = S(q_1) - t_1 + \delta p(q_1)\left(S(q_2) - t_2\right)$$

where S (q_i) stands for the value the P assigns to q_i units of the good, with S' > 0 and S" < 0. q_i (respectively t_i) is the output (transfer) at date t= i, i $\in \{1, 2\}$. p(q_i) is the probability (A sticks to the terms of the original contract|q_1). $\delta(\geq 0)$ is the discount factor, that I allow to be greater than one

[2] Levin (2003) can be used to model it explicitely.

in cases where period 2 lasts much longer (it is more important) than period one.

One remark is noteworthy: P's utility function is standard except for p(q_1). According to the initial assumption (about commitment), the A can walk away from the current relationship at the onset of period 2, after he has learnt his true type. It may be too costly (and thus not profitable) for the P (especially for realization of θ very distant from $\theta\hat{}$) to retain the A with probability 1. Therefore, second-period profits are not only taken in expectation over q_1 and q_2, but also weighted by the probability that the A doesn't breach the original contract.

3.2.2 A

The A has intertemporal von Neumann-Morgestern utility given by:

$$U_A = u(t_1) - g(e_1) + \delta \left[p(q_1)(u(t_2) - g(e_2) - k \mid \theta - \theta\hat{} \mid) + (1 - p(q_1))U_2(q_1) \right]$$

where e_i represents A's effort exerted at date i, with $e_i \in \{0, 1\}$, for $i \in \{1, 2\}$. g(e_i) is A's disutility of effort i, with normalization as follows: g(1)$\equiv \psi$ and g (0) $\equiv$ 0. u(t_i) is the utility derived from transfer at date i, with u' > 0 and u" < 0 (risk-aversion). k is the relative importance of motivational aspects (intrinsic motivators) with respect to monetary rewards (extrinsic motivators).

$\theta\hat{}$ stands for the task assigned (once and for all) by the P to the A. Instead,

θ is the A's preferred task. Ex-ante, it is distributed according to $F(\theta)$ in $\left[\underline{\theta}, \overline{\theta}\right]$. $\delta(\geq 0)$ is the A's discount factor, for simplicity assumed equal to the P's one. Finally, $\mathrm{U}_2(\mathrm{q}_1)$ is the A's second-period outside option and, with full generality, can be contingent to the first-period output realization.

The A's preferences specification features the term $-k \mid \theta - \theta^{\wedge} \mid$. The A not only cares of money and effort, but also of the quality of the job in which he is employed. The larger the difference in taste (between the current work and his preferred one), the higher should be the monetary reward in order to compensate him for this additional source of disutility.

This additive term features only in the second period, because, by assumption (see timing of contracting), in the first period the Nature has not revealed yet this piece of information to the A. Moreover, given the irreversability of time, what I have done in the past, today can be considered as sunk. Another interpretation can be: only when I know what I enjoy, I can feel satisfied in doing it.

I assume throughout the paper that the P has the full ability to restrict the A's access to the capital market: the A is neither allow to save nor to borrow, hence all transfers are immediately consumed.

3.2.3 Production Technology

Production (q_i) is stochastic and can take only two values, $\left\{\underline{q}, \overline{q}\right\}$, with $\overline{q} > \underline{q} \geq 0$. Effort improves production in the sense of first-order stochastic dominance: $\Pr\left(q = \overline{q} \mid effort\ = 1\right) \equiv \pi_1 > \Pr\left(q = \overline{q} \mid effort\ = 0\right) \equiv \pi_0$, with

$\pi_1 - \pi_0$ denoted by $\Delta\pi$.

Stochastic returns are independently distributed over time, so that the past history of realizations does not yield any information on the current likelihood of a success or a failure of the production process.

As usually assumed in hidden action framework, effort is neither observable nor verifiable: it is a non-contractible variable.

3.3 Timing of Contracting

The timing of the game is as follows.

1) The P offers a long-term contract: $\{t_1(q_1), t_2(q_1, q_2), \hat{\theta}\}$ such that $(q_1, q_2) \in \{\underline{q}, \overline{q}\}^2$ and $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$.

2) A accepts or refuses the contract.

3) If A has accepted it (always the case in equilibrium), he decides whether exerting effort or not. Subsequently, first-period output q_1 and transfer t_1 take place.

4) At the onset of the second period. A discovers his preferred task θ (type). He can then decide to stick to the terms of the original contract or reneging it and getting the market outside option $U_2(q_1)$.

5) If the agent have not walked away, he decides whether exerting second-period effort or not. And, finally, second-period output q_2 and transfer t_2 take place.

The timing of contracting is summarizesd in the following figure 1.

4 Analysis

I first solve for the optimal contract, then I study the implications of motivation on occupational choice.

4.1 Optimal long-run contract

In solving the model, I focus on the case where effort is extremely valuable for the P, who always wants to implement a high level of effort in both periods.

The P's program can be written as:

$$\max_{\{t_1(q_1),t_2(q_1,q_2),\hat{\theta}\}_{q_1,q_2\in\left\{\underline{q},\bar{q}\right\}^2,\hat{\theta}\in[\underline{\theta},\bar{\theta}]}} E_{q_1,q_2}\left[\begin{array}{c}(S(q_1)-t_1(q_1)+\delta p(q_1)\left(S(q_2)-t_2(q_1,q_2)\right)) \mid \\ e_1=e_2=1\end{array}\right]$$

s. to

$$E_{q_1,q_2,\theta}\left[\begin{array}{c}u(t_1(q_1))-g(e_1)+\delta\left[p(q_1)(u(t_2(q_1,q_2))-k\mid\theta-\hat{\theta}\mid-g(e_2))+(1-p(q_1))U_2(q_1)\right] \mid \\ e_1=e_2=1\end{array}\right]\geq 0 \quad (\text{IR}_1)$$

$$E_{q_1,q_2,\theta}\left[\begin{array}{c}u(t_1(q_1))-g(e_1)+\delta\left[p(q_1)(u(t_2(q_1,q_2))-k\mid\theta-\hat{\theta}\mid-g(e_2))+(1-p(q_1))U_2(q_1)\right] \mid \\ e_1=e_2=1\end{array}\right]\geq$$

$$(\text{IC}_1)$$

$$E_{q_1,q_2,\theta}\left[\begin{array}{c}u(t_1(q_1))-g(e_1)+\delta\left[p(q_1)(u(t_2(q_1,q_2))-k\mid\theta-\hat{\theta}\mid-g(e_2))+(1-p(q_1))U_2(q_1)\right] \mid \\ e_1=0,e_2=1\end{array}\right]$$

$$E_{q_2}\left[u(t_2(q_1,q_2)) - g(e_2) \mid e_2 = 1\right] \geq E_{q_2}\left[u(t_2(q_1,q_2)) - g(e_2) \mid e_2 = 0\right] \tag{$\mathrm{IC}_2(q_1)$}$$

$\bar{\theta}^*(q_1)$ and $\underline{\theta}^*(q_1)$ are implicitely defined by the following equation:

$$\pi_1 u\left(t_2(q_1,\bar{q})\right) + (1-\pi_1)\, u(t_2(q_1,\underline{q})) - \psi - k \mid \theta - \theta\hat{} \mid = U_2(q_1) \tag{$\theta^*(q_1)$}$$

where $\bar{\theta}^*(q_1) > \underline{\theta}^*(q_1)$.

Thus,

$$p(q_1) = \int_{\underline{\theta}^*(q_1)}^{\bar{\theta}^*(q_1)} f(\theta)d\theta = F\left(\bar{\theta}^*(q_1)\right) - F\left(\underline{\theta}^*(q_1)\right)$$

IR_1 is the A's intertemporal partecipation constraint: the A is at least as well off by signing the long-term contract than by rejecting it.

IC_1 is the A's first-period incentive compatibility constraint: anticipating his future stream of random payoffs (conditional on exerting effort in the second period), it is optimal for the A to exert effort in the first period.

$\mathrm{IC}_2(\mathrm{q}_1)$ is the A's second-period incentive compatibility constraint: no matter what the history of past performance has been (that is, the realization of q_1), it is optimal for the A to exert effort in the second period.

$\bar{\theta}^*(q_1)$ and $\underline{\theta}^*(q_1)$ are the cutoff values of θ, such that for $\theta \in [\underline{\theta}, \underline{\theta}^*(q_1)]$ and $[\bar{\theta}^*(q_1), \bar{\theta}]$, A's optimal strategy is to quit job. While, for $\theta \in [\underline{\theta}^*(q_1), \bar{\theta}^*(q_1)]$, A's optimal strategy is to stick to the terms of the original contract.[3]

[3] $\mid \theta - \theta\hat{} \mid$ is a measure of the distance between the A's preferred mission and his current

An implicit assumption is both that A is equally productiove for P in every task he is assigned to and P is such a large firm that he can give to A every mission in the support of the distribution of θ.

From now on, for the sake of simplicity, it is assumed that $\theta \sim U[\underline{\theta}, \bar{\theta}]$, symmetrically around 0 (that is, $\underline{\theta} = \bar{\theta}$). Hence:

$$p(q_1) = \frac{\bar{\theta}^*(q_1) - \underline{\theta}^*(q_1)}{\bar{\theta} - \underline{\theta}}$$

The main feature of the long-term optimal contract are summarized in the next proposition.

Proposition 1 *With a twice repeated moral hazard problem, the optimal long-term contract with one-sided lack of commitment exhibits memory and the "modified" martingale property*

$$h'(u_1(q_1)) = \frac{p(q_1)C'(u_2(q_1)) - \frac{\partial p(q_1)}{\partial u_2(q_1)} V(u_2(q_1))}{p(q_1) + \frac{\partial p(q_1)}{\partial u_2(q_1)} \left(u_2(q_1) - \frac{k\bar{\theta}}{2} - U_2 \right)} \qquad \forall q_1 \in \{\underline{q}, \bar{q}\}$$

is satisfied. Moreover, the first-period optimal task assignment is E(θ) = 0 and all constraints, in equilibrium, are binding.

PROOF: in appendix.

The following corollary analyses extreme cases: either when there is no second-period random IR constraint (in other words: $\frac{\partial p(q_1)}{\partial u_2(q_1)} = 0$) or when the contract is memoryless (that is, $\frac{\partial u_1}{\partial u_2(q_1)} = 0$).

task.

Corollary 2 *When $k \to 0$ and $U_2(\bar{q}) = U_2(\underline{q}) = 0$, the standard martingale property*

$$h'(u_1(q_1)) = E_{q_2}\left[h'(u_2(q_1))\right]$$

is satisfied at the optimum. While, when $k \to \infty$, renegation happens with probability one.

The intuition of the above proposition is quite straighforward. The expected marginal cost of giving up some rewards to the agent in period 1 following any output q_1 must be equal to the "corrected" expected marginal cost of giving up these rewards in the corresponding continuation of the contract, where corrected means that the principal should take into account the impact of the second-period rewards on the incentive to walk away faced by the agent.

The optimal long-term contract will be characterized by more high-powered incentives in the second-period (because they not only are useful in inducing effort, but also increase the probability of retaining the agent).

4.2 Occupational choice

The problem studied above is intrinsically dynamic, thus one question naturally arises: if the A discovers that his present task is not very agreeable, then he may be better off reneging the original contract signed at date 0 and changing occupation. In particular, the A will face a trade-off between continuing the ongoing relationship with the current employer (and collecting the rewards implied by the memory property of the optimal long-term contract) or following his preferred mission (and starting from scratch under a new employer).

Define $S \equiv \pi_1 S(\bar{q}) + (1-\pi_1)S(\underline{q})$.

I assume for the rest of the paper that

$$\pi_1 h\left(k\overline{\theta} + U_2 + \frac{1-\pi_0}{\Delta\pi}\psi\right) + (1-\pi_1)h\left(k\overline{\theta} + U_2 - \frac{\pi_0}{\Delta\pi}\psi\right) > S >$$
$$\pi_1 h\left(\frac{k\overline{\theta}}{2} + U_2 + \frac{1-\pi_0}{\Delta\pi}\psi\right) + (1-\pi_1)h\left(\frac{k\overline{\theta}}{2} + U_2 - \frac{\pi_0}{\Delta\pi}\psi\right)$$

P's expected revenue from the A's performance is such that $p(q_1) \in \left(\frac{1}{2}, 1\right), \forall q_1 \in \left\{\underline{q}, \bar{q}\right\}$. That is, the P finds optimal in the second period to retain the A with a high probability but not with certainty[4].

Given the previous assumption and proposition 1, it is possible to show that $\frac{\partial u_1}{\partial u_1(q_2)} > 0$, that is the contract exhibits covariance $(u_1, u_2(q_1)) > 0$.

Note that $U_2(q_1)$ is not a function of θ (type of the job). This is a very stark assumption. Among other things, it implies that A can switch to every occupation he wants to and the wage (in the new profession) may depend on A's track of performance but not on the quality of the task he is assigned to.

Firstly, I draw attention to the special case in which $U_2(\bar{q}) = U_2(\underline{q})$: either the market does not have record-keeping ability or the realization of q_1 is P's private information.

Proposition 3 *Whenever $U_2(\underline{q}) = U_2(\bar{q})$, there are values of θ such that:*

1) If $\mid \theta^* \mid \geq \theta_1$, then walking away is optimal for the A, independently of

[4] This assumption implies also pC' > p'V (because h' > 0): excessive transfers in the second period with respect to the first-best, where pC' = p'V.

the realization of the output in period 1;

2) If $\mid \theta^* \mid \in (\theta_2, \theta_1)$*, then it is optimal for the A to continue the ongoing relationship when* $q_1 = \bar{q}$ *and to walk away in the complementary case; and*

3) If $\mid \theta^* \mid \leq \theta_2$*, then if is optimal for the A to keep his word and do not renege the initial contract*

PROOF:

Figure 2 illustrates the idea stated by the above proposition.

Recall that, by assumption, $U_2(q_1)$ is independent on the realization of θ, hence it is an horizontal line.

Define,

$$y(q_1) \equiv \pi_1 u\left(t_2(q_1, \bar{q})\right) + (1 - \pi_1)\, u\left(t_2(q_1, \underline{q})\right) - \psi - k \mid \theta \mid$$

$\frac{\partial u_1}{\partial u_1(q_2)} > 0$ implies that, given a certain realization of θ, $y(\bar{q}) > y(\underline{q})$.

It is immediate to see that $y(q_1)$ reaches a unique global maximum at $\theta = 0$. And the claim follows.■

Because of the A's risk-aversion, the P spreads intertemporally the A's rewards and punishments to minimize the cost of implementign a high-effort in period 1[5]. This implies that $u_2(\bar{q}) > u_2(\underline{q})$. Hence, given a constant market outside option, the likelihood of retaining the A in the current job is higher under a high output realization.

Now, I consider the more general case in which $U_2(\bar{q}) \neq U_2(\underline{q})$[6].

[5] Thus, the burden of the incentive constraint is smoothed between today and tomorrow.

[6] Many situations can arise (some of which are both (either) unrealistic and (or) uninter-

Proposition 4 *If $U_2(\bar{q})$ is much larger than $U_2(\underline{q})$, then there are values for θ such that:*

1) If $\mid \theta^* \mid \geq \theta_3$, then walking away is optimal for the A, independently of the realization of the output in period 1;

2) If $\mid \theta^* \mid \in (\theta_4, \theta_3)$, then it is optimal for the A to continue the ongoing relationship when $q_1 = \underline{q}$ and to walk away in the complementary case; and

3) If $\mid \theta^* \mid \leq \theta_4$, then it is optimal for the A to keep his word and do not renege the initial contract

PROOF:

Figure 3 is very similar to figure 2, except that now $U_2(\bar{q}) > U_2(\underline{q})$. And this implies that there are cases where the result of proposition 3 is reversed: it is more likely to retain first-period unsuccessful As.■

The above idea can be illustrated by an example. Suppose there is a professor of economic theory that has published a revolutionary article in financial economics. Subsequently, Wall Street banks try to allure him with a high salary and perks package. These incentives crowd out the university, because the professor is likely to quit his current job, unless the mission of the academy closely coincides with the one of the scholar. However, when the professor's research is unsuccessful, a large distance between the assigned mission and the preferred one can be sustained, because his outside option is low (nobody wants to hire him).

esting): a complete analysis is available upon request.

5 Applications

Firstly, I suggest possible extensions of the basic model presented in section four and, then, I point out some testable implications.

5.1 Extensions

The model can be easily extended in various directions.

Instead of a two-period model, it is possible to study a stationary problem: that is, an infinitely repeated relationship. In this latter framework, and given the same timing of the game[7], my (tentative) guess is that it is almost surely optimal for the A to change job, unless his type coincides with E(θ).

Another possible extension can allow for a competitive market of Ps: the number of Ps in the economy goes to infinity. The demand of the final product q$_i$ is the key determinant of the equilibrium quantity (because supply is infinitely elastic). Rochet and Stole (2002) have looked at this interesting case of competition among Ps.

Finally, one further extension consists in endogenizing the outside option $U_2(q_1)$. Suppose that the market is so large that the agent can choose (in period one) to do exactly what he likes. The problem becomes a one-period moral hazard, with effort still extremely important. Maybe that it is not an efficient solution to have the agent to leave his original principal with probability one. In fact, for θ close enough to $\theta\hat{}$, the original long-term contract provides a (valuable to the agent and, hence, to the principal) form of insurance.

[7] The crucial part is about the A knowing his true type early in the relationship.

5.2 Predictions

It is possible to identify four main empirical implications stemming from the previous analysis.

- There exists a lower bound to wages, determined by the characteristics of the production process $(S(q), \pi_0, \bar{q}, \underline{q})$. Thus, given the commitment technology, the wage is downward rigid (see, for instance, Harris and Holmstrom, 1982).
- Small firms have to pay on average higher wages. Assume that at date one, once the agent has discovered his type, the employer has some job openings. Larger firms have ceteris paribus more vacancies available and therefore can economize on monetary incentives: the insurance premium they have to pay to the agent is lower.
- The identity of the employer matters only through the kind of task that is offered to the agent. Hence, when we observe a change of employer (job separation) that is not accompanied by a change of mission, a variation in the wage scheme should have taken place.
- The endogenized outside option is a crucial variable in understanding and pinning down the characteristics (either successful or unsuccessful) of people that leave their current job. The investigation about the formation of the outside option can shed light on possible differences between markets.

6 Conclusions

This paper makes one primary contribution: it investigates the shape of the optimal long-term incentive scheme when agents are intrinsically motivated. In particular, a modified martingale property is shown to hold in equilibrium. Moreover, the occupational choice problem is investigated and an optimal job separation rule is derived.

At the end, in section five, some issues, both theoretical and empirical, left for future research have been laid down.

7 References

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8 APPENDIX

Define (in order to simplify notation):

$u(t_2(q_1, \bar{q})) \equiv \overline{u}_2(q_1)$

$u(t_2(q_1, \underline{q})) \equiv \underline{u}_2(q_1)$

$u(t_1(\bar{q})) \equiv \bar{u}_1$

$u(t_1(\underline{q})) \equiv \underline{u}_1$

$h(.) \equiv u^{-1}(.)$ (Grossman-Hart (1983) kind of trick)

From $\theta^*(q_1)$ equation, it is immediate to show:

$$\begin{cases} \overline{\theta}^*(q_1) = \frac{\pi_1 \overline{u}_2(q_1) + (1-\pi_1)\underline{u}_2(q_1) - \psi - U_2}{k} + \theta\hat{} \\ \underline{\theta}^*(q_1) = \frac{\psi + U_2 - \pi_1 \overline{u}_2(q_1) - (1-\pi_1)\underline{u}_2(q_1)}{k} + \theta\hat{} \\ p(q_1) \in [0,1] \Rightarrow \overline{\theta} \geqslant \overline{\theta}^* \geqslant \underline{\theta}^* \geqslant \underline{\theta} \end{cases}$$

Thus,

$$p(q_1) = \frac{\bar{\theta}^*(q_1) - \underline{\theta}^*(q_1)}{\bar{\theta} - \underline{\theta}} = 2\frac{\pi_1 \overline{u}_2\,(q_1) + (1-\pi_1)\,\underline{u}_2(q_1) - \psi - U_2}{k(\overline{\theta} - \underline{\theta})} = \frac{u_2(q_1) - U_2}{k\overline{\theta}}$$

Proposition 5 *In the second-period, the problem is the same as the static moral hazard problem.*

PROOF

$$\max_{\overline{u}_2(q_1), \underline{u}_2(q_1)} E_{q_2}\left[S(q_2) - h(u_2(q_1)) \mid e_2 = 1\right]$$

s.to

$$\pi_1 \bar{u}_2(q_1) + (1-\pi_1)\underline{u}_2(q_1) - \psi \geq u_2(q_1) \qquad \forall q_1 \in \{\underline{q}, \overline{q}\}$$

$$\overline{u}_2\,(q_1) - \underline{u}_2(q_1) \geqslant \frac{\psi}{\Delta\pi} \qquad \forall q_1 \in \{\underline{q}, \overline{q}\}$$

Both constraints are binding at the optimum. To see it formally, write down the Lagrangian and use Kuhn-Tucker conditions, that in this case are both necessary and sufficient for characterizing optimality: concave problem and linear constraints.

Thus, $\begin{cases} \bar{u}_2(q_1) = u_2(q_1) + \frac{(1-\pi_0)\psi}{\Delta\pi} \\ \\ \underline{u}_2(q_1) = u_2(q_1) - \frac{\pi_0\psi}{\Delta\pi} \end{cases}$

Note: this problem looks exactly the same as a (standard) static moral hazard problem.■

4 equations in 4 unknowns: given $u_2(q_1)$, $\left\{\bar{u}_2(\bar{q}), \bar{u}_2(\underline{q}), \underline{u}_2(\bar{q}), \underline{u}_2(\underline{q})\right\}$ are uniquely determined.

Lemma 6 *The first-period optimal task assignment will be E(θ).*

E(θ) maximizes the probability of retaining the employee given a certain monetary reward.

PROOF:

$$\max_{\theta^{\hat{}}} -E_\theta(\mid \theta - \theta^{\hat{}} \mid)^8$$

[8]Recall, argmax_x constant·f(x) = argmax_x f(x) whenever constant > 0. In this case, in

is the same as

$$\min_{\theta^{\hat{}}} \int_{\underline{\theta}}^{\theta^{\hat{}}} (\theta^{\hat{}} - x)\, dx + \int_{\theta^{\hat{}}}^{\overline{\theta}} (x - \theta^{\hat{}}) dx$$

Using Leibniz' rule: $\theta^{\hat{}} = \frac{\overline{\theta}+\underline{\theta}}{2}$

Note, the FOC is necessary and sufficient.■

Proposition 7 *With a twice repeated moral hazard problem, the optimal long-term contract with one-sided lack of commitment exhibits memory and the "modified" martingale property*

$$h'(u_1(q_1)) = \frac{p(q_1)C'(u_2(q_1)) - \frac{\partial p(q_1)}{\partial u_2(q_1)} V(u_2(q_1))}{p(q_1) + \frac{\partial p(q_1)}{\partial u_2(q_1)} \left(u_2(q_1) - \frac{k\overline{\theta}}{2} - U_2 \right)} \qquad \forall q_1 \in \{\underline{q}, \bar{q}\}$$

is satisfied.

PROOF:

Define:

$V_2(u_2(q_1)) \equiv S - C(u_2(q_1))$ (P's continuation value of the contract)

$C(u_2(q_1)) \equiv \pi_1 h(\bar{u}_2(q_1)) + (1-\pi_1)h(\underline{u}_2(q_1))$ (cost of implementing a high effort in period 2 following the promise of a second-period utility $u_2(q_1)$)

Note that $-kE_\theta \mid \theta - \theta^{\hat{}} \mid$ subject to $\theta^{\hat{}} = 0$, is equal to $-\frac{k}{\overline{\theta}-\underline{\theta}}\overline{\theta}^2$, and because of symmetric support around 0, this is the same as $-\frac{k\overline{\theta}}{2}$, a sort of insurance premium[9].

IR_1, the constant is equal to $p(\bar{q})\pi_1 + p(\underline{q})(1-\pi_1)$ and, in IC_1, $\Delta\pi(p(\bar{q}) - p(\underline{q}))$ (both strictly positive).

[9] The premium represents a fixed cost for the P and this creates a discontinuity in the

The first-period P program can be written as follows:

$$\max_{\bar{u}_1,\underline{u}_1,u_2(\bar{q}),u_2(\underline{q})} \pi_1(\bar{S}-h(\bar{u}_1))+(1-\pi_1)\left(\underline{S}-h(\underline{u}_1)\right)+\delta p(\bar{q})\pi_1[S-C(u_2(\bar{q}))]+\delta p(\underline{q})(1-\pi_1)[S-C(u_2(\bar{q}))]$$

$$\text{s. to}\begin{cases} \pi_1\left[\overline{u}_1+\delta p(\bar{q})(u_2\left(\overline{q}\right)-\frac{k\overline{\theta}}{2}-U_2)\right]+(1-\pi_1)\left[\underline{u}_1+\delta p(\underline{q})(u_2(\underline{q})-\frac{k\overline{\theta}}{2}-U_2)\right]\geq\psi-\delta U_2 & (\mu)\\ \bar{u}_1+\delta p(\bar{q})(u_2\left(\overline{q}\right)-\frac{k\overline{\theta}}{2}-U_2)-\left(\underline{u}_1+\delta p(\underline{q})(u_2(\underline{q})-\frac{k\overline{\theta}}{2}-U_2)\right)\geq\frac{\psi}{\Delta\pi} & (\lambda)\end{cases}$$

$$\begin{aligned}\mathcal{L} = {} & \pi_1(\bar{S}-h(\bar{u}_1))+(1-\pi_1)\left(\underline{S}-h(\underline{u}_1)\right)+\delta p(\bar{q})\pi_1[S-C(u_2(\bar{q}))]+\delta p(\underline{q})(1-\pi_1)[S-C(u_2(\bar{q}))]+\\ & \mu\left[\pi_1\left[\overline{u}_1+\delta p(\bar{q})(u_2\left(\overline{q}\right)-\frac{k\overline{\theta}}{2}-U_2)\right]+(1-\pi_1)\left[\underline{u}_1+\delta p(\underline{q})(u_2(\underline{q})-\frac{k\overline{\theta}}{2}-U_2)\right]-\psi+\delta U_2\right]+\\ & \lambda\left[\bar{u}_1+\delta p(\bar{q})(u_2\left(\overline{q}\right)-\frac{k\overline{\theta}}{2}-U_2)-\left(\underline{u}_1+\delta p(\underline{q})(u_2(\underline{q})-\frac{k\overline{\theta}}{2}-U_2)\right)-\frac{\psi}{\Delta\pi}\right]\end{aligned}$$

FOC (w.r.t. $\bar{u}_1$and $\underline{u}_1$):

$$-\pi_1 h'(\bar{u}_1)+\mu\pi_1+\lambda=0 \qquad (1)$$

$$-(1-\pi_1)h'(\underline{u}_1)+\mu(1-\pi_1)-\lambda=0 \qquad (2)$$

$$\mu=\pi_1 h'(\bar{u}_1)+(1-\pi_1)h'(\underline{u}_1)$$

$$\lambda=\pi_1(1-\pi_1)\left(h'(\overline{u}_1)-h'(\underline{u}_1)\right)$$

program (when $p(q_1)$ goes from 0 to ε, P has to provide a large upfront payment). Hence, even if h(u) takes simple functional form (say, quadratic or exponential), the optimal transfer scheme can not be computed in a closed form expression (unless k takes extreme values).

FOC(w.r.t. $u_2(\bar{q})$ and $u_2(\underline{q})$):

$$\pi_1 \frac{\partial p(\overline{q})}{\partial u_2(\overline{q})} V(u_2(\bar{q})) - p(\bar{q})\pi_1 C'(u_2(\bar{q})) + (\pi_1\mu+\lambda)\left(\frac{\partial p(\overline{q})}{\partial u_2(\overline{q})}(u_2(\bar{q}) - \frac{k\overline{\theta}}{2} - U_2) + p(\bar{q})\right) = 0 \tag{3}$$

$$(1-\pi_1)\frac{\partial p(\underline{q})}{\partial u_2(\underline{q})} V(u_2(\underline{q})) - p(\underline{q})(1-\pi_1)C'(u_2(\underline{q})) + ((1-\pi_1)\mu-\lambda)\left(\frac{\partial p(\underline{q})}{\partial u_2(\underline{q})}(u_2(\underline{q}) - \frac{k\overline{\theta}}{2} - U_2) + p(\underline{q})\right) = 0 \tag{4}$$

Combining (1) and (3)

$$h'(\bar{u}_1) = \frac{p(\bar{q})C'(u_2(\bar{q})) - \frac{\partial p(\bar{q})}{\partial u_2(\bar{q})} V(u_2(\bar{q}))}{p(\bar{q}) + \frac{\partial p(\bar{q})}{\partial u_2(\bar{q})}\left(u_2(\bar{q}) - \frac{k\overline{\theta}}{2} - U_2\right)}$$

Combining (2) and (4)

$$h'(\underline{u}_1) = \frac{p(\underline{q})C'(u_2(\underline{q})) - \frac{\partial p(\underline{q})}{\partial u_2(\underline{q})} V(u_2(\underline{q}))}{p(\underline{q}) + \frac{\partial p(\underline{q})}{\partial u_2(\underline{q})}\left(u_2(\underline{q}) - \frac{k\overline{\theta}}{2} - U_2\right)}$$

■

Corollary 8 *The agent's intertemporal partecipation constraint is binding.*

PROOF (By contradiction) Suppose it is not. Then, the principal can reduce $\bar{u}_1$ and $\underline{u}_1$ by the same positive amount ε. Pick ε small enough, such that IR_1 is still not binding. IC_1 is not affected by this change in first-period transfers. But the principal is now strictly better off.

Also note, $\mu = \pi_1 h'(\bar{u}_1) + (1-\pi_1)h'(\underline{u}_1) > 0$.■

Corollary 9 *The agent's first-period incentive constraint is binding.*

PROOF Suppose it is not. Consider a new contract C'$= \left\{ \bar{u}_1', \underline{u}_1', u_2(\bar{q}), u_2(\underline{q}) \right\}$, where $\underline{u}_1' = \underline{u}_1 + \varepsilon$, $\bar{u}_1' = \bar{u}_1 - \varepsilon \left(\frac{1}{\pi_1} - 1 \right)$ (thus IR_1 holds with equality) and continuation utilities same as before. For ε sufficiently small, IC_1 is still slack. Recall h">0 (by concavity of u). The new transfer scheme has the same $\mathrm{E}_{q_1}[u(q_1)]$ but a smaller variance: $Var_{q_1}(u_1'(q_1)) < Var_{q_1}(u_1(q_1))$.

Applying Jensen's inequality to the P's objective function, it is easy to show that P is now better off using C' (contradiction).■

Thus, $\bar{u}_1 > \underline{u}_1$.

TIMING OF CONTRACTING

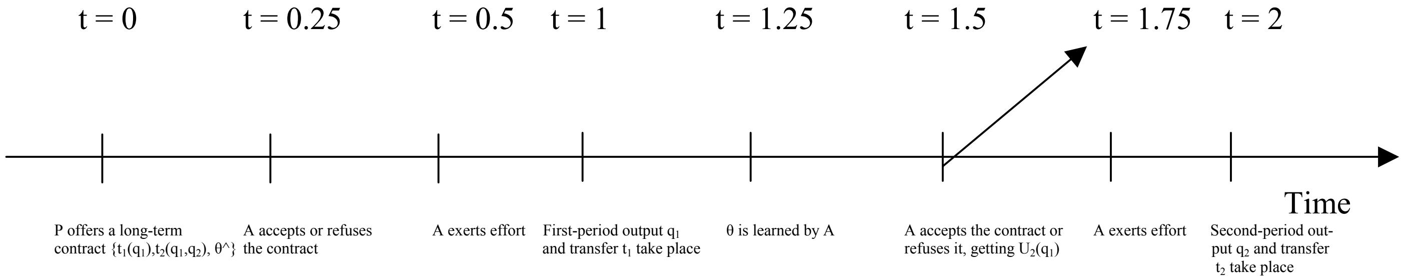

Figure 1

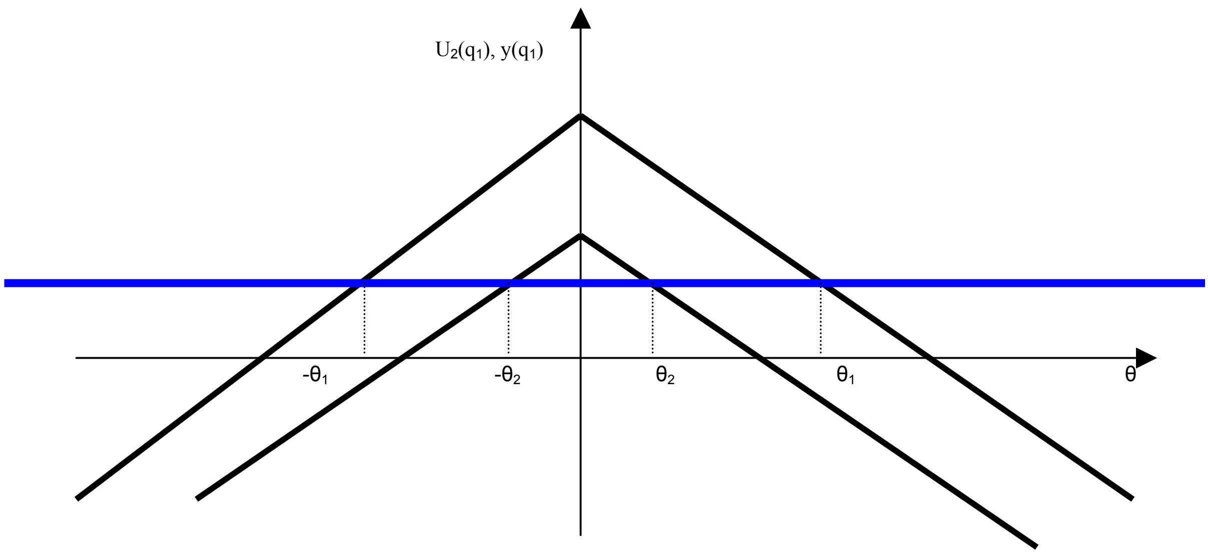

Figure 2

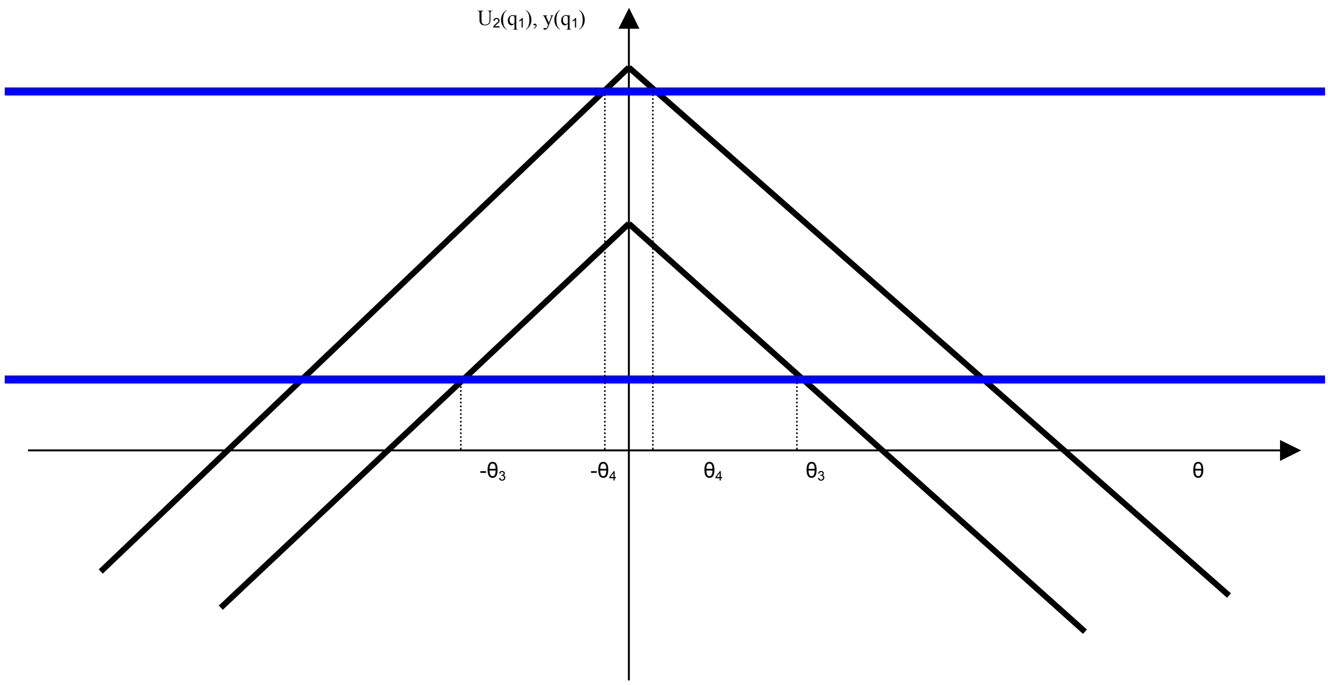

Figure 3