A joint econometric model of macroeconomic and term structure dynamics*

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Abstract

We construct and estimate a joint model of macroeconomic and yield curve dynamics. A small-scale rational expectations model describes the macroeconomy. Bond yields are affine functions of the state variables of the macromodel, and are derived assuming absence of arbitrage opportunities and a flexible price of risk specification. While maintaining the tractability of the affine set-up, our approach provides a way to interpret yield dynamics in terms of macroeconomic fundamentals; time-varying risk premia, in particular, are associated with the fundamental sources of risk in the economy. In an application to German data, the model is able to capture the salient features of the term structure of interest rates and its forecasting performance is often superior to that of the best available models based on latent factors. The model has also considerable success in accounting for features of the data that represent a puzzle for the expectations hypothesis.

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1 Introduction

Understanding the term structure of interest rates has long been a topic on the agenda of both financial and macro economists, albeit for different reasons. On the

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one hand, financial economists have mainly focused on forecasting and pricing interest rate related securities. They have therefore developed powerful models based on the assumption of absence of arbitrage opportunities, but typically left unspecified the relationship of the term structure with other economic variables. Macro economists, on the other hand, have focused on understanding the relationship between interest rates, monetary policy and macroeconomic fundamentals. In so doing, however, they have typically relied on the "expectations hypothesis," in spite of its poor empirical record. Combining these two lines of research seems fruitful, in that there are potential gains going both ways. If macroeconomic theory has some empirical success, it should help price securities more efficiently.

This paper aims at presenting a unified empirical framework where a small structural model of the macro economy is combined with an arbitrage-free model of bond yields. In doing so, we build on the work of Piazzesi (2001) and Ang and Piazzesi (2003), who introduce macroeconomic variables into the standard affine term structure framework based on latent factors – e.g. Duffie and Kan (1996) and Dai and Singleton (2000). The main innovative feature of our paper is that we use a structural macroeconomic framework rather than starting from a reduced-form VAR representation of the data. One of the advantages of this approach it to allow us to relax Ang and Piazzesi's restriction that inflation and output be independent of the policy interest rate, thus facilitating an economic interpretation of the results. Our framework is similar in spirit to that in Wu (2002), who prices bonds within a calibrated rational expectations macro-model. The difference is that we estimate our model and allow a more empirically oriented specification of both the macro economy and the parametrization of the market price of risk. A framework similar to ours is employed in a recent paper by Rudebusch and Wu (2003), who interpret latent term structure factors in terms of macroeconomic variables, while Bekaert, Cho and Moreno (2003) mix a structural macro framework with unobservable term structure factors.

Our estimation results, based on German data, show that macroeconomic factors affect the term-structure of interest rates in different ways. Monetary policy shocks have a marked impact on yields at short maturities, and a small effect at longer maturities. Inflation and output shocks mostly affect the curvature of the yield curve at medium-term maturities. Changes in the perceived inflation target have more lasting effects and tend to have a stronger impact on longer term yields.

Our results also suggest that including macroeconomic variables in the information set helps to forecast yields. The out-of-sample forecasting performance of our model is in superior to that of the best available affine term structure models for most maturities/horizons.

Finally, we show that the risk premia generated by our model are sensible. First, the model can account for the features of the data which represent a puzzle for the expectations hypothesis, namely the finding of a negative and large – rather than positive and unit – coefficients obtained, for example by Campbell and Shiller (1991), in regressions of the yield change on the slope of the curve. Second, regressions based on risk-adjusted yields do, by and large, recover slope coefficients close to unity, i.e. the value consistent with the rational expectations hypothesis.

The rest of the paper is organized as follows. Section 2 describes the main features of our general theoretical approach and then provides a brief overview of our estimation method. It also discusses the specific macroeconomic model which we employ in our empirical application. The estimation results, based on our application to German data is described in Section 3. Section 4 then discusses the forecasting performance of our model, compared to leading available alternatives. The ability of the model to solve the expectations puzzle is tested in Section 5. Section 6 concludes.

2 The approach

In recent years, the finance literature on the term structure of interest rates has made tremendous progress in a number of directions (see e.g. Dai and Singleton, 2003). Following the seminal paper by Duffie and Kan (1996), one of the most successful avenues of research has focused on models where the yields are affine functions of a vector of state variables. This literature, however, has typically not investigated the connections between term structure and macroeconomic dynamics. In the rare cases in which macroeconomic variables–notably, the inflation rate–have been included in estimated term-structure models, those variables have been modelled exogenously (e.g. Evans, 2003, Zaffaroni, 2001; Ang and Bekaert, 2004). The interactions between macroeconomic and term structure dynamics have also been left unexplored in the macroeconomic literature, in spite of the fact that simple "policy rules" have often scored well in describing the dynamics of the short-term interest rate (e.g., Clarida, Galí and Gertler, 2000).

An attempt to bridge this gap within an estimated, arbitrage-free framework has recently been made by Ang and Piazzesi (2003). Those authors estimate a term structure model based on the assumption that the short term rate is affected partly by macroeconomic variables, as in the literature on simple monetary policy rules, and partly by unobservable factors, as in the affine term-structure literature.[1] Ang and Piazzesi's results suggest that macroeconomic variables have an important explanatory role for yields and that the inclusion of such variables in a term structure model can improve its one-step ahead forecasting performance. Nevertheless, unobservable factors without a clear economic interpretation still play an important role in their model. Moreover, Ang and Piazzesi's two-stage estimation method relies on the assumption that the short term interest rate does not affect macroeconomic variables.

In order to redress these shortcomings, we construct a dynamic term structure model entirely based on macroeconomic factors, which allows for an explicit feedback from the short term (policy) rate to macroeconomic outcomes. The joint modelling of three key macroeconomic variables–namely, inflation, the output gap and the short term "policy" interest rate–should allow us to obtain a more accurate (endogenous) description of the dynamics of the short term rate. At the same time, our explicit modelling or risk premia should also help us in capturing the dynamics of the entire term-structure.

In this section, we present our approach to model jointly the macroeconomy and the term structure. The main assumption we impose is that aggregate macroeconomic relationships can be described using a linear framework. To motivate our approach, we start with an outline of the macroeconomic model that we use in our empirical analysis. We then cast this macro-model in a more general framework and show how to price bonds within such a framework based on the assumption of absence of arbitrage opportunities.

2.1 A simple backward/forward looking macroeconomic model

We rely on a structural macroeconomic model, whose choice is motivated by the fact that it could be derived from first principles. The model is certainly too stylised – for example in its ignoring foreign variables or the exchange rate – to provide a fully-satisfactory account of German macroeconomic dynamics. Nevertheless, it does include the minimal structure of a macroeconomic model proper. Our results in sections in Sections 4 and 5 suggest that such minimal structure does capture the

[1] In related papers, Dewachter and Lyrio (2002) and Dewachter, Lyrio and Maes (2002) also estimate jointly a term structure model built on a continuous time VAR.

central features of the dynamics of yields.

The model of the economy includes just two equations which describe the evolution of inflation, π_t, and the output gap, x_t:

$$\begin{aligned}\pi_t &= \mu_\pi E_t[\pi_{t+1}] + (1-\mu_\pi)\pi_{t-1} + \delta_x x_t + \varepsilon_t^\pi, \\ x_t &= \mu_x E_t x_{t+1} + (1-\mu_x) x_{t-1} - \zeta_r (r_t - E_t[\pi_{t+1}]) + \varepsilon_t^x.\end{aligned}$$

The inflation equation implies that prices will be set as a markup on marginal cost, captured by the output gap term in the equation. The assumption of price stickiness generates the expected inflation term, while the lags capture inflation inertia. The output gap equation provides a description of the dynamics of aggregate demand, which is assumed to be affected by movements in the short term real interest rate. The forward looking term captures the intertemporal smoothing motives characterising consumption, the main component of aggregate demand.[2]

The two equation above are often interpreted as appropriate to describe yearly data. Since we will employ monthly data in estimation, we recast the model at the monthly frequency along the lines of Rudebusch (2002). The equations that we will actually estimate are therefore

$$\pi_t = \frac{\mu_\pi}{12}\sum_{i=1}^{12} E_t[\pi_{t+i}] + (1-\mu_\pi)\sum_{i=1}^{3}\delta_{\pi i}\pi_{t-i} + \delta_x x_t + \varepsilon_t^\pi \tag{1}$$

$$x_t = \frac{\mu_x}{12}\sum_{i=1}^{12} E_t[x_{t+i}] + (1-\mu_x)\sum_{i=1}^{3}\zeta_{xi}x_{t-i} - \zeta_r (r_t - E_t[\pi_{t+11}]) + \varepsilon_t^x \tag{2}$$

Note that all variables are now expressed at the monthly frequency (notably,

[2] Both equations can be derived from first principles. More precisely, the inflation equation can be derived as the first order condition of the price-setting decision of firms acting in an environment with monopolistic competition. Monopolistic competition implies that prices will be set as a markup on marginal cost, which explains the presence of the output gap term in the equation. The assumption of sticky prices generates the expected inflation term, as firms do not know when their prices will adjust next and therefore need do maximize the sum of current and expected future profits. The additional lagged inflation rate has been motivated through the assumption of partial price indexation (Christiano, Eichenbaum and Evans, 2001) or the presence of a set of firms that use a backward-looking rule of thumb to set prices (Galí and Gertler, 1999). The output gap equation can be derived from an intertemporal consumption Euler equation. The first term on the right-hand side is essentially Hall's (1978) random walk hypothesis which states that consumption is equal to expected consumption tomorrow (in simple, closed-economy models, consumption equals output in equilibrium). This hypothesis is supplemented with two additional terms. First, a real interest rate (which Hall assumed to be constant) shifts the consumption profile such that a real rate increase tends to discourage current consumption. The second term is lagged consumption, whose presence can be motivated by habit persistence and/or the presence of rule of thumb consumers (Campbell and Mankiw, 1989; Fuhrer, 2000; McCallum and Nelson, 1999).

inflation is defined as the 12-month change of the log-price level). In particular, the two equations include a forward-looking term capturing expectations over the next twelve months of inflation and output, respectively. The backward-looking components of the two equations are restricted to include only 3 lags of the dependent variable. This choice results in a more parsimonious empirical model. In the estimation, we impose $\mu_\pi + (1 - \mu_\pi) \sum_i \delta_{\pi i} = 1$, a version of the natural rate hypothesis.

Finally, we need an assumption on how monetary policy is conducted in order to solve for the rational expectations equilibrium. Since our estimates will include also bond prices, we focus on private agents' perceptions of the monetary policy rule followed by the central banks, rather than solving the models under full commitment or discretion. Accordingly, the "simple rule" supposedly followed by the central bank is to set the nominal short rate according to

$$r_t = (1 - \rho) \left(\beta \left(E_t \left[\pi_{t+11} \right] - \pi_t^* \right) + \gamma x_t \right) + \rho r_{t-1} + \eta_t \tag{3}$$

where π_t^* is the perceived inflation target and η_t is a "monetary policy shock".

This is consistent with the formulation in Clarida, Galí and Gertler (1998, henceforth CGG), which is a natural benchmark for comparison because the rule has been estimated for Germany, the country which we focus on in the empirical implementation. The first two terms represent a typical Taylor-type rule (in this case forward looking), where the rate responds to deviations of expected inflation from the inflation target. The second part of the rule is motivated by interest rate smoothing concerns, which seem to be an important empirical feature of the data.

The main difference with respect to the rule estimated by CGG is that we also allow for a time-varying, rather than constant, inflation target π_t^*. We adopt this formulation because the Bundesbank modified its "medium term price norm" over the sample period used in our analysis and the modifications were public knowledge. At the same time, we do not want to impose that the announced price norm was "credible," and reflected in bond prices, by assumption. For this reason, we treat the time-varying inflation target π_t^* as an unobservable variable, which should capture markets' perceptions reflected in equilibrium bond yields. This formulation allows us to exploit the full available sample period, without having to assume a break in the policy rule at some point in the late seventies, as done by CGG.

Finally, we need to specify the processes followed by the stochastic variables of the model, i.e. the perceived inflation target and the three structural shocks. We assume that our 3 macro shocks are serially uncorrelated and normally distributed with constant variance. The only factor that we allow to be serially correlated is the unobservable inflation target, which will follow an AR(1) process

$$\pi_t^* = \phi_\pi \pi_{t-1}^* + u_{\pi,t} \tag{4}$$

where $u_{\pi,t}$ is a normal disturbance with constant variance uncorrelated with the other structural shocks.

2.2 A general macroeconomic set-up

In order to solve the model we write it in the general form

$$\begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t \mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{K} r_t + \begin{bmatrix} \boldsymbol{\xi}_{1,t+1} \\ \mathbf{0} \end{bmatrix}, \tag{5}$$

where $\mathbf{X}_1$ is a vector of predetermined variables, $\mathbf{X}_2$ includes the variables which are not predetermined, r_t is the policy instrument and $\boldsymbol{\xi}_1$ is a vector of independent, normally distributed shocks (see the appendix for the exact definitions of all these variables in our example). The short-term rate can be written in the feedback form

$$r_t = -\mathbf{F} \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix}. \tag{6}$$

This linear structure is nevertheless general enough to accommodate a large number of standard macroeconomic models, potentially much more detailed than the one we adopt here. The main restriction we impose, for simplicity, is that only the short-term interest rate, which is controlled by the central bank, affects the macro economy, whereas longer rates do not.

The solution of the (5)-(6) model can be obtained numerically following standard methods. We choose the methodology described in Söderlind (1999), which is based on the Schur decomposition. The result are two matrices $\mathbf{M}$ and $\mathbf{C}$ such that $\mathbf{X}_{1,t} = \mathbf{M}\mathbf{X}_{1,t-1} + \boldsymbol{\xi}_{1,t}$ and $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$.[3] Consequently, the equilibrium short term interest rate will be equal to $r_t = \boldsymbol{\Delta}'\mathbf{X}_{1,t}$, where $\boldsymbol{\Delta}' \equiv -(\mathbf{F}_1 + \mathbf{F}_2\mathbf{C})$ and $\mathbf{F}_1$ and $\mathbf{F}_2$ are partitions of $\mathbf{F}$ conformable with $\mathbf{X}_{1,t}$ and $\mathbf{X}_{2,t}$. Focusing on the short-term

[3] The presence of non-predetermined variables in the model implies that there may be multiple solutions for some parameter values. We constrain the system to be determinate in the iterative process of maximizing the likelihood function.

(policy) interest rate, the solution can be written as

$$\begin{aligned} r_t &= \boldsymbol{\Delta}'\mathbf{X}_{1,t} \\ \mathbf{X}_{1,t} &= \mathbf{M}\mathbf{X}_{1,t-1} + \boldsymbol{\xi}_{1,t}. \end{aligned} \tag{7}$$

2.3 Adding the term structure to the model

The system (7) expresses the short term interest rate as a linear function of the vector $\mathbf{X}_1$, which in turn follows a first order Gaussian VAR. This structure is formally equivalent to that on which affine models are normally built. To derive the term structure, we only need to impose the assumption of absence of arbitrage opportunities, which guarantees the existence of a risk neutral measure, and to specify a process for the stochastic discount factor.

Behind this formal equivalence, however, our model has the distinguishing feature that both the short rate equation and the law of motion of vector $\mathbf{X}_1$ have been obtained endogenously, as functions of the parameters of the macroeconomic model. This contrasts with the standard affine set-up based on unobservable variables, where both the short rate equation and the law of motion of the state variables are postulated exogenously.

This feature also differentiates our approach from Ang and Piazzesi's (2003). More specifically, Ang and Piazzesi (2003) still rely on an exogenously postulated model of the short-term rate, which they interpret as the monetary policy rule. In any macroeconomic model, however, the dynamics of the short term rate will be obtained endogenously. We show that this property of macro-models does not prevent the specification of a dynamic arbitrage-free term structure model. Provided that one's favourite macroeconomic model can be cast in the linear (5)-(6) form, arbitrage-free pricing is possible.

In fact, rather than building the term structure directly on equations (7), we allow for the possibility to write bond yields as functions of a different vector, $\mathbf{Z}_t$, which can include any variable in $\mathbf{X}_t$ or the short term rate. The new vector $\mathbf{Z}_t$ is defined as $\mathbf{Z}_t = \mathbf{D}\mathbf{X}_t$, where $\mathbf{D}$ is a selection matrix. Obviously, $\mathbf{Z}_t$ can also be rewritten as a function of the predetermined vector $\mathbf{X}_{1t}$ using the result $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$. This yields $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$, where $\hat{\mathbf{D}}$ is a matrix described in the appendix. Specifically, in the empirical application, we choose $\hat{\mathbf{D}}$ so that bond yields are expressed as functions of the levels of the macro variables, rather than of their shocks.

Given the solution equation for the short term interest rate written as a function of the $\mathbf{Z}_t$ vector, $r_t = \overline{\boldsymbol{\Delta}}'\mathbf{Z}_t$, we follow the standard dynamic arbitrage-free term structure literature and define the (nominal) pricing kernel m_{t+1}, which prices all nominal bonds in the economy, as $m_{t+1} = \exp(-r_t)\,\psi_{t+1}/\psi_t$, where ψ_{t+1} is the Radon-Nikodym derivative, which is assumed to follow the log-normal process $\psi_{t+1} = \psi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\boldsymbol{\xi}_{1,t+1}\right)$.

We then make an assumption on the dynamics of λ_t, the vector of market prices of risk associated with the underlying sources of uncertainty in the economy. These have commonly been assumed to be constant (in the case of Gaussian models) or proportional to the factor volatilities (e.g. Dai and Singleton, 2000), but recent research has highlighted the clear benefits in allowing for a more flexible specification of the risk prices (e.g. Duffee, 2002; Dai and Singleton, 2002). We therefore assume that the market prices of risk are affine in the state vector $\mathbf{Z}_t$

$$\lambda_t = \lambda_0 + \lambda_1 \mathbf{Z}_t, \tag{8}$$

so that the market's required compensation for bearing risk can vary with the state of the economy.

It should be pointed out here that, in a micro-founded framework, the pricing kernel (or stochastic discount factor) would be linked to consumer preferences, rather than being postulated exogenously as we do here. The pricing kernel would be obtained from the intertemporal consumption Euler equation, essentially consisting of the discounted ratios of marginal utility between two consecutive periods, scaled by expected inflation in the case of the nominal kernel. In standard consumption-based formulations of asset pricing models, the prices of risk would be related to the agents' risk aversion and to the curvature of the indirect utility function with respect to the state variables of the problem. We would obtain a micro-founded pricing kernel if we specified a utility function, set $\lambda_1 = 0$ and restricted λ_0 to be consistent with the selected utility function.

We prefer our exogenous specification (8) for two main reasons. The first is that we want to employ an empirically plausible formulation and the state-dependent specification in equation (8) is not straightforward to obtain from first principles.[4] The second reason is that, even if we found a sufficiently flexible formulation of the utility function, the yield premia would always be zero in a log-linearised solution

[4] Dai (2003) argues that preferences embodying a particular specification of habit formation would be consistent with pricing kernel that, to a first order approximation, would be of the form (8) with a non-zero λ_1.

of the model, such as the one we implicitly adopt here (see also Kim et al., 2003). Higher order approximations could obviously be employed to deal with this problem, but they would imply leaving the convenient affine world, in which both the bond prices and the likelihood can be specified in closed-form.

In the appendix we show that the reduced form (7) of our macroeconomic model, coupled with the aforementioned assumptions on the pricing kernel, implies that the continuously compounded yield y_t^n on an n-period zero coupon bond is given by

$$y_t^n = A_n + B_n' \mathbf{Z}_t, \tag{9}$$

where the A_n and B_n' matrices can be derived using recursive relations. Stacking all yields in a vector $\mathbf{Y}_t$, we write the above equations jointly as $\mathbf{Y}_t = \mathbf{A} + \mathbf{B}'\mathbf{Z}_t$ or, equivalently, $\mathbf{Y}_t = \mathbf{A}_n + \tilde{\mathbf{B}}_n' \mathbf{X}_{1,t}$, where $\tilde{\mathbf{B}}_n' \equiv \mathbf{B}_n' \hat{\mathbf{D}}$.

2.4 Maximum likelihood estimation

In order to estimate the model, we need to distinguish first between observable and unobservable variables in the $\mathbf{X}_t$ vector. We adopt the approach which is common in the finance literature and which involves inverting the relationship between yields and unobservable factors (Chen and Scott, 1993). In our case, the method needs to be extended to take into account that the observable variables include not just the yields, $\mathbf{Y}_t$, but also some of the non-predetermined variables. We also use the common approach of assuming that some of the yields are imperfectly measured to prevent stochastic singularity.

Using the assumption of orthogonality of measurement error shocks and shocks to the unobservable states, we show in the appendix that the log-likelihood function to maximize takes the form

$$\begin{aligned} \pounds(\boldsymbol{\theta}) \quad = \quad & -(T-1)\left(\ln|J| + \frac{n_p}{2}\ln(2\pi) + \frac{1}{2}\ln\left|\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right| + \frac{n_m}{2}\ln(2\pi) + \frac{1}{2}\sum_{i=1}^{n_m}\ln\sigma_i^2\right) \\ & -\frac{1}{2}\sum_{t=2}^{T}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right)'\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right)^{-1}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right) - \frac{1}{2}\sum_{t=2}^{T}\sum_{i=1}^{n_m}\frac{\left(u_{t,i}^m\right)^2}{\sigma_i^2}. \end{aligned}$$

where $\mathbf{X}_{1,t}^u$ are the unobservable variables included in the $\mathbf{X}_{1,t}$ vector, u_t^m are the measurement error shocks, $\mathbf{J}$ is a Jacobian matrix defined in the appendix, $\boldsymbol{\Sigma}\boldsymbol{\Sigma}'$ is the variance-covariance matrix of the four macroeconomic shocks, σ_i are the standard deviations of measurement error shocks, T is the sample size, n_m is the number of measurement errors and n_p is the number of variables measured without error.[5]

When, as in the model used by Ang and Piazzesi (2003), there is no feedback from interest rates to the macro variables, estimation can be performed with a two-step procedure. In the more general case analysed here this is not possible and we must estimate the whole system jointly.

In theory, this is of course preferable. The problem is that the parameter space is quite large and therefore the optimization problem of maximizing the likelihood function is non-trivial and time consuming. We employ the method of simulated annealing, introduced to the econometric literature by Goffe, Ferrier and Rogers (1994). The method is developed with an aim towards applications where there may be a large number of local optima.[6]

One disadvantage of the simulated annealing method is that it does not provide us with an estimate of the derivatives, evaluated at the maximum, of the likelihood function with respect to the parameter vector, i.e. $\partial \ln (\pounds(\boldsymbol{\theta})) / \partial \boldsymbol{\theta}'$. These derivatives are necessary to compute asymptotic estimates of the variance-covariance matrix of the parameters. The derivatives could be evaluated numerically, but the computation would be based on arbitrarily selected step-lenghts $\partial \boldsymbol{\theta}$, with ensuing risks of spurious results because of the highly nonlinear fashion in which the parameters enter the likelihood function.

To deal with this problem, we rely on analytical results to calculate the Jacobian $\partial \ln (\pounds(\boldsymbol{\theta})) / \partial \boldsymbol{\theta}'$. The evaluation of the analytical derivatives is quite involved. The key steps are described in the appendix.

3 An application to German data

3.1 Data

Our data set runs from January 1975 to December 1998. The term structure data consists of monthly German zero-coupon yields for the maturities 1, 3 and 6 months,

[5] So far, we have not imposed any restrictions on the $\mathbf{X}_{1t}$ vector. In the estimation, however, care must be taken to avoid that the unobservable variables included in $\mathbf{X}_{1t}$ be linearly dependent. If this were the case, the Jacobian matrix would not be invertible.

[6] The key parameters of the simulated annealing method were set as follows: $T_0 = 15$; $r_T = 0.9$; $N_T = 20$. The convergence criterion ε was set at $\varepsilon = 1.0E - 8$. In a preliminary estimation, the starting values were taken from CGG's results (for the policy rule) and from the parameters of an unrestricted VAR in output, inflation, and the short term nominal rate. The estimates reported in the text correspond to a maximum value of the likelihood function found in a process of 100 estimations using simulated annealing, starting from randomised initial values.

as well as 1, 3, and 7 years.[7] We assume that the 1-month rate and the 3-year yield are perfectly observable, while the other rates are subject to measurement error. Yields have been bootstrapped from on an original Bundesbank dataset of end-of-month raw prices, coupons and maturities.[8]

Concerning the macro data, we construct the year-on-year inflation series using the CPI (all items). For the output gap, we simply follow CGG and detrend the log of total industrial production (excluding construction) using a quadratic trend. We only deviate from CGG in constructing the series recursively, so that each datapoint is obtained by fitting a quadratic trend to the original series up to that point. We adopt this approach to ensure that our forecast at time t does not rely on information unavailable at that point in time. Both series refer to unified Germany from 1991 onwards and to West Germany prior to this date. The macroeconomic and term-structure series are shown in Figure 1.

3.2 Estimation results

To reduce the parameter space in our empirical application, we impose a number of restrictions on the coefficients of the market prices of risk. In the general set-up, we showed that the risk prices can be specified as $\lambda_t = \lambda_0 + \lambda_1 \mathbf{Z}_t$. In our application, $\mathbf{Z}_t$ includes the perceived inflation target and contemporaneous and lagged values of inflation, output and the short term rate. Given $\mathbf{Z}_t$, λ_t can obviously have nonzero elements only corresponding to time t variables, as lagged variables are no longer subject to surprise changes. This leaves only four potentially non-zero rows in the λ_0 and λ_1 matrices, corresponding to the perceived inflation target, the policy interest rate, inflation and the output gap. Next, we restrict λ_0 and λ_1 in the sense of allowing interactions only between prices of risk of contemporaneous variables, which leaves us with a 4×4 non-zero submatrix in λ_1. Finally, we follow Duffee (2002) and set to zero all entries whose elements have a t-statistic lower than 1 in preliminary estimations.

As a result, we are left with the following non-zero elements in the matrices of

[7] We do not use 10-year bonds because these are only available without breaks as of April 1986.

[8] The methodology is equivalent to that employed by Fama and Bliss (1987). We wish to thank Thomas Werner for providing us with the raw data and Vincent Brousseau for bootstrapping the term structures of zero-coupon rates.

prices of risk

$$\lambda_t = \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ \lambda_{03} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & 0 \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & 0 \\ 0 & \lambda_{42} & 0 & \lambda_{44} \end{pmatrix} \begin{pmatrix} \pi_t^* \\ r_t \\ \pi_t \\ x_t \end{pmatrix}.$$

3.2.1 Parameter estimates

Table 1 presents the parameter estimates with associated asymptotic standard errors (based on the analytical outer-product estimate of the information matrix).

The results are broadly consistent with the evidence of Clarida, Galì and Gertler (1998) regarding the Taylor rule in Germany and, as far as the other macro-parameters are concerned, with existing evidence based on structural models or identified VARs.

For example, our point estimate of the degree of forward-lookingness of inflation (μ_π) is within the range of values found by Jondeau and Le Bihan (2001), who estimate on German data a Phillips curve based on quarterly data using a variety of specifications and two different estimation methods. Kremer, Lombardo and Werner (2003), who estimate a structural macroeconomic model with explicit microfoundations, estimate a much higher value of μ_π. Their estimate, however, is not directly comparable to ours due to the fact that they capture the persistence of inflation through highly persistent exogenous shocks (whereas our shocks are white noise).

A result which casts doubts on the ability of our macro-model to provide an accurate characterisation of the dynamics of output and inflation in Germany is that the elasticity of inflation to the output gap is very small ($\delta_x = 0.0004$ and insignificantly different from zero). This is not entirely surprising. Jondeau and Le Bihan (2001) also find values of δ_x close to zero for some specification/estimation method (Kremer, Lombardo and Werner, 2003, calibrate, rather than estimate, this parameter). Identified VARs estimated at the monthly frequency (e.g. Sims, 1992) also tends to find a very small and insignificant responses of inflation to, e.g., monetary policy shocks, which is consistent with our results of a vey small δ_x and also a small ζ_r.

To assess whether our macro-parameter estimates are affected by our inclusion of term structure information in the model, we re-estimated the macroeconomic model separately. In order to work with a more conventional set-up, we also eliminated the stochastic inflation target from the policy rule and replaced it with the

Bundesbank's announced price norm. Apart from a small increase in ζ_r from 0.03 to 0.06, the other parameter estimates (including δ_x) were virtually unchanged.

The macro-model performance may be affected by the fact that volatile, monthly data are noisy and make it harder to identify the link between inflation, output and interest rates. Another possibility is that our output gap definition, which plays a crucial role in the analysis, is an imperfect proxy for the theoretical notion of real marginal costs. Or else, as already emphasised, our 2-variable macro-model may be too parsimonious to describe German macroeconomic dynamics, which are possibly affected also by variables such as the exchange rate or, as in Kremer, Lombardo and Werner (2003), a monetary aggregate. Since our main interest is not that of finding the macroeconomic model most suited for German policy analysis, we do not perform further specification search. We only test for a potential missing variable bias by examining the residuals' autocorrelation. We find little evidence of serial correlation in our preferred specification.[9]

As to the other parameters, the autocorrelation coefficient of the inflation target process is very close to 1.[10] Concerning the term structure, our estimates of the standard deviations of the measurement errors are between 23 basis points for the 3-month rate and 28 basis points for the other yields. These values are broadly in line with the results of models based solely on unobservable factors and also those of an unrestricted VAR including inflation, the output gap and the bond yields.[11] The standard errors of the 1-month and 3-month rate equations are equal to 43 and 32 basis points in the VAR, respectively, compared to 48 and 23 in our model; for 1-year and 7-year yields, the VAR equations have a standard error of 29 and 24 basis points, respectively, compared to 28 and 28 in our model. Obviously, our model has the advantage of describing, at the same time, the yields on all other possible maturities (and it also does better than the VAR at fitting output and inflation).

Finally, one of the benefits of our model is that of providing us with a measure of the central bank's inflation target as reflected in the prices of long term bonds. One of the tests of the model is therefore to check whether the filtered series "looks" reasonable. For this purpose, Figure 2 compares it to the Bundesbank medium term price norm.[12] The two series are quite close to each other in the volatile seventies

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[9] More precisely, looking at the correlograms of the estimated residuals we find no evidence of statisticallly significant first or higher order correlation in the output and inflation equations.

[10] This parameter is constrained to be strictly smaller than 1 in the estimation.

[11] The VAR is estimated over the same sample period and includes 3 lags of the variables.

[12] Until 1981 and from 1997 to 1998, the Bundesbank actually announced a range, rather a point

and in the sharp decline of the beginning of the eighties. A large discrepancy can be observed mostly at the beginning of the nineties, when the estimated target increases sharply while the price norm remains unchanged. The increase in the estimated target is, however, not unreasonable, as it coincides with an increase in actual inflation following the expansionary policies that accompanied German unification.[13] The perceived inflation target is also less variable than actual inflation, both in terms of its sample standard deviation and of its minimum and maximum sample values.

3.2.2 Impulse response functions

Our structural model allows us to compute impulse response functions of macro variables and yields to the underlying macro shocks.

Figures 3 to 6 show the impulse responses of selected variables to the structural shocks. The responses of the macroeconomic variables and of the short term interest rate are broadly in line with existing VAR evidence based on German (monthly) data and we will not delve on them here. We concentrate instead on the responses of yields.

We start from Figure 3, which displays the impulse responses to a shock to the perceived inflation target, which increases on impact by approximately 0.2 percentage points. The shock is obviously expansionary and very persistent, due to the high serial correlation of the inflation target process. The response of the yield curve is an almost parallel and very persistent upward shift at all maturities, except the very short ones (which move slowly because of the high interest rate smoothing coefficient in the policy rule). The size of the shift corresponds roughly to that of the initial inflation target shock and it is significantly different from zero for maturities around 1-year.

Figure 4 shows the effect of a 45 basis points increase in the 1-month interest rate because of a monetary policy shock (the disturbance η_t). The response of the yield curve is decreasing in the maturity of yields, which factor in the slow return to baseline of the policy rate. Hence, a monetary policy shock tends to cause a

value, for the price norm. In these years, the mid-point of the range is displayed in Figure 2. No values were announced pre-1976 and in 1979.

[13] In spite of the unchanged price norm, this may have sparked fears of a waning in the Bundesbank anti-inflationary determination because of domestic – due to unification – and European-wide – due to the impact of any monetary policy tightening on ERM partner countries – political pressures (see Issing, 2003, for a concise account of the Bundesbank's policy at the time of German unification).

statistically significant change in the slope of the yield curve. The shape of this response is quite similar to that obtained by Evans and Marshall (1996) for the US.

An inflation shock, shown in Figures 5, tends to increase the curvature of the yield curve. Yields move little and slowly at the short end, more around the 1-year maturity, then little again at the long end. While statistically significant for maturities below 7 years, the responses appear to be very small from a quantitative viewpoint.

Finally, Figures 6 shows the impulse responses to an output shock. Due to the small policy response, the yield curve increases little, but significantly, over maturities up to 1 year. Yields on 3 and 7-year bonds, however, fall as a result of the shock and in spite of the fact that the response of the short-term rate always remains above the baseline. This surprising pattern is to a large extent shaped by the dynamics of risk premia.

3.3 Macro shocks and risk premia

Another advantage of our joint treatment of macroeconomics and term-structure dynamics is that we are able to derive the impulse response of theoretical risk premia to macro shocks, including the monetary policy shock. These are shown in Figure 7.

The inflation target shock is immediately followed by an increase of the yield premium for maturities up to 4 years, with a peak effect of 10 basis points at the 1-year maturity. The premium then turns negative for longer maturities. Such increase in the yield premium is highly significant from an economic viewpoint, as it plays a large quantitative role in shaping the total yield response displayed in Figure 3.

The monetary policy shock gives rise to a large fall, on impact, at the short end of the term structure of yield premia, thus reducing significantly the size of the impact response of the yields. The impact response of the 1-year yield to the monetary policy shock, for example, would increase by a half if yield premia were set equal to a constant.

Similar considerations hold for the impact response of yield premia to inflation and output shocks. The latter is notable, since the premia embody most of the action in the response. The impact response of the 7-year rate, for example, would change sign and essentially maintain the same absolute value, if risk premia were constant.

We conclude that, in general, the dynamics of yield premia have a nonnegligible effect on the impulse responses of yields to all macroeconomic shocks. An interpretation of the yield responses based on the expectations hypothesis may therefore be significantly biased.

The general features of the yield premia are that their level and volatility are increasing in maturity. The premia also tend to be decreasing over the sample in parallel to the fall in inflation, but then shoot up again, temporarily, in 1992-93. To investigate their determinants more closely (using equation (15) in the appendix), we can decompose the premia in the components due to risk of changes in the inflation target, in the short-term rate, in inflation and in the output gap.[14] Figure 8 shows the most important components for 1 and 7-year maturities.

The most striking outcome of this decomposition is that premia linked to inflation risk are almost perfectly constant over time and negligible in size across maturities. Even at their peaks, they never reach the level of 10 basis points. This number should be compared, for example, to the maximum level of 1 percentage point reached by the premium due to output gap uncertainty for 7y bonds.

Variations in yield premia arise by and large from fluctuations in the other three variables, with an importance that changes across maturities. Figure 8 shows that at the 1-year horizon, the largest fraction of the time-varying yield premium is due to interest rate risk, i.e. the possibility of monetary policy surprises. Interest rate risk, in turn, is decreasing in the level of the interest rate: when the latter is very high, yield premia are lower than average and 1-year bonds appear to be a very appealing form of investment; when interest rates are low, on the contrary, the risk of unexpected changes in the short-term rate appears high and 1-year bond command a higher than average premium. The second most important component of the time varying yield premium at 1-year maturities is inflation target risk. The target premium is increasing in the level of the inflation target. A high inflation target makes 1-year bonds riskier and increases the premium investors require to hold them.

At the long 7-year horizon, the time varying component of the yield premium is almost entirely due to inflation target risk until the end of 1988. At this maturity, the inflation target premium is negatively correlated with the level of the inflation target.

[14] This decomposition is not exact, because the term premium is also affected by the lags of inflation, output and the interest rate. We disregard these additional effects for two reasons. First, given our assumption on the prices of risk λ_t, they are due to convexity effects, rather than a pure risk premium. Second, they are quantitatively minor.

When the target is high, the yield premium is lower than average and investors are relatively more willing to hold 7-year bonds. This may be taken as a signal of investors' confidence in the ultimate return to a low inflation target environment and of the low probability of further increases in the target. As of 1989, with inflation and the policy interest rate increasing after the German unification and the recession of 1992-93 ensuing, the variable yield premium becomes significantly affected also by output gap risk. In other words, booms tend to make investors more willing to hold long term bonds, while they require a larger bond premium during recessions.

4 Forecasting

The forecasting performance is a particularly interesting test of our macroeconomic-based term-structure model. Due to the relatively large number of parameters that needs to be estimated, the model could be expected to perform poorly with respect to more parsimonious representations of the data. In fact, the random walk model has been shown to provide yield forecasts that are particularly difficult to beat (Duffee, 2002). An important test of our model is therefore to check whether the information contained in macro variables can improve the performance of a standard essentially-affine model including only term-structure information. For completeness, we also check whether the inclusion of yields in the information set can improve the performance of the macro-only model in terms of forecasting the macro variables.

The forecasting tests for macroeconomic variables and yields are presented in turn in the next two sections. Our results suggest that term structure information helps little in forecasting macroeconomic variables. Our structural framework including macroeconomic variables does, however, help to forecast yields. The out-of-sample forecasting performance of our model up to 12-month ahead is almost always superior to all the alternatives we consider, and the difference is often statistically significant.

4.1 Do yields help to forecast macroeconomic variables?

Given the imperfect ability of our macroeconomic model to describe the joint dynamics of German macroeconomic variables, we do not expect it to be very successful in forecasting inflation and the output gap. This is consistent with existing evidence. In a thorough study of inflation forecasting in the G7 countries, for example, Canova (2002) concludes that theory-based models are not always better than atheoretical univariate models.

Our test on macroeconomic variables is therefore very focused to assess whether including yields in the analysis can help in forecasting. The results are presented in Table 2, whcih shows that the full model including term structure information and the stochastic inflation target does marginally better than the macro-only model at forecasting inflation. The latter model, however, prevails as far as output forecasts are concerned. Both models are beaten by the random walk or a 3-variable VAR.

We conclude that yields are unlikely to provide useful information for macroeconomic forecasting within our framework. This result may be due to our assumption that long term yields do not affect the dynamics of inflation and the output gap.

4.2 Do macroeconomic variables help to forecast yields?

To assess the yields forecasting performance of our model, we compare it to a number of benchmarks.

The first is the random walk. In addition, we also consider forecasts based on three other models. One is a canonical $A_0(3)$ essentially affine model based on unobservable factors.[15] Provided that risk premia are specified to be linear functions of the states, Duffee (2002) finds this model most successful in the class of admissible affine three factor models in terms of forecasting US yields. Apart from providing a benchmark for comparison, our results on the $A_0(3)$ model are of independent interest, since they highlight the performance of this model on a different data-set. The second model we take into account is the Ang and Piazzesi (2003) model, which we reestimate on our data-set. Based on Ang and Piazzesi's results, we use their favorite "Macro model" in this exercise, i.e. a model in which the interest rate responds to current inflation and output gap, as well as to 3 unobservable factors. A potentially important difference in our application of their model, however, is that we use inflation and the output gap directly in the estimation, rather than the principal components of real and nominal variables employed by Ang and Piazzesi (2003), thereby facilitating comparison to our results. Finally, we use an unrestricted VAR including all the variables in our structural model, in order to gauge the importance of structural and no-arbitrage restrictions to improve the performance of our model.

For all models, out-of-sample forecasting performances are reported based on estimates over the period February 1975 - December 1994, and a series of 1 to 12

[15] For a definition of the $A_0(3)$ class of affine models, see Dai and Singleton (2000).

step ahead forecasts for all yields used in the estimation over the period January 1995 to December 1998. Each month, we update the information set, but we do not reestimate the model. Instead, we rely on the estimates up until end-1994. We choose this approach to limit the computational burden of the exercise. All results are therefore based on the same estimated parameters.

The root mean squared errors (RMSEs) of the forecast evaluation exercise are summarized in Tables 3. Lower values of the RMSE denote better forecasts, and the best forecast at each maturity/horizon is highlighted in bold. The exercise shows that our model performs better than the alternatives for all maturities, at least beyond the very shortest forecast horizon. In particular, our model beats the predictions of the random walk benchmark in almost all cases. Table 4, which displays the trace MSE statistic − a multivariate summary measure of the forecasting performance across yields for each horizon − confirms this picture.[16]

To understand the reasons for this success, compare first the performance of the $A_0(3)$ model in Table 3 to that of the VAR. The former model includes no-arbitrage restrictions and, as a result, it appears to be more efficient at forecasting long yields, especially at longer forecasting horizon. The $A_0(3)$ model, however, is not always superior to the VAR, which is a first suggestion that macroeconomic information could be important in forecasting yields. The AP may be expected to improve the performance of the $A_0(3)$ model, because it includes macroeconomic information on top of the no-arbitrage restrictions. The AP model includes, however, a very large number of parameters to estimate, since it is based on a reduced-form representation of the macroeconomic variables. This may be the cause for its less satisfactory performance over forecasting horizons beyond 1 month. Its good performance in 1-step ahead forecasts is, incidentally, consistent with the results reported by Ang and Piazzesi (2003). Our model appears to strike a good balance in incorporating macroeconomic information without becoming overparameterised.

Concerning, more specifically, the market prices of risk, a crucial role in affecting the forecasting performance of our model is played by risk premia associated to inflation target risk. If we re-estimate our model restricting to zero the elements in equation (8) associated to the inflation target, i.e. λ_{21} and λ_{31}, the forecasting performance of the model worsens dramatically, especially for long maturities. This appears to be consistent with the evidence on the main components of the risk

[16]The trace MSE statistic is due to Christoffersen and Diebold (1998). For each forecast horizon, it is simply computed as the trace of the covariance matrix of the forecast errors of all yields considered. Hence, a lower trace MSE statistic signals more accurate forecasts across yields.

premia presented in section 3.3.

In order to formally test the out-of-sample yield forecasting performance of our model, we apply White's (2000) "reality check" test. This test, which builds on the work of Diebold and Mariano (1995) and West (1996), involves examining whether the expected value of the difference between the forecast loss (e.g. the squared forecast error) of one or several models is significantly greater than the forecast loss of a benchmark model. We choose this test mainly for two reasons. First, in contrast to many other forecast performance tests, White's method tests for superior predictive ability rather than equal predictive ability. Second, White's test allows us to examine whether a specific model is significantly outperformed by any model among a number of alternatives, whereas other tests typically do not permit this.

We implemented the test first using our model as the benchmark and, over all five maturities considered and 12 forecast horizons, we found that in only 3 out of the 60 cases could we reject the null hypothesis that none of the four models is better than our model. While encouraging, this result does not necessarily imply that our model is superior to the alternatives. To test this, we turned around the null hypothesis and proceeded to test for superior predictive ability of our model vis-à-vis each of the four alternative models separately. The results are displayed in Table 5, where bold figures indicate rejection of the null that our model does not have superior predictive ability compared to the benchmark used, at the 5% level. In over 60% of the cases we reject the null, meaning that for most of the combinations of maturities and forecast horizons considered here, the forecasting performance of our model is significantly better than the alternatives. Looking at the results in more detail, we see that, somewhat surprisingly, the VAR model seems to be harder to beat than the other alternatives, although for longer horizons and maturities the HTV model consistently outperforms the VAR. With respect to the performance of our model at different forecast horizons, we seem to do roughly equally well across all horizons, except for the one-month ahead case, where the null is rejected less often.

We therefore conclude that the inclusion of macroeconomic variables within a structural framework contributes to sharpening our ability of forecasting yields accurately out of sample. The improvement is due both to the inclusion of additional information in the model, and to the structural restrictions imposed on its macroeconomic and term structure sections.

5 Expectations hypothesis tests

According to the expectations hypothesis, the yield on an n-period zero-coupon bond should increase when the spread between the same yield and the short term rate (the "slope of the yield curve") widens. In fact, the projection of the yield change $y_{t+1}^{n-1} - y_t^n$ on the yield spread $(y_t^n - r_t) / (n-1)$ should yield a coefficient of 1. A number of empirical tests of this implication of the theory have, however, found a negative relationship. This pattern represents a puzzle for the expectations hypothesis, and it appears to be particularly clear for United States data. The relevant regression coefficient can be as big as -5 for 10-year bonds, according to e.g. Campbell and Shiller (1991), while the expectations hypothesis predicts a value of one for all maturities.

One interpretation of these results is that the large deviation from 1 in the estimated coefficient on the yield spread is due to large and time varying risk premia (not permitted by the expectations hypothesis). Using a highly stylised model, McCallum (1994) conjectures that an exogenous, stochastic term premium is, in principle, capable of causing deviations from 1 in the slope coefficient of the aforementioned regression. The actual size of the deviation will depend on both the stochastic properties of the term premium (see also Roberds and Whiteman, 1999) and the monetary policy rule followed by the central bank. These papers, however, work by "reverse engineering." Given the results of projections of the yield change on the yield spread, they derive the properties that risk premia *should* have to explain those results. This is different from deriving the risk premia from a certain model and checking *ex-post* if they are capable of solving the expectations puzzle.

In this section, we follow instead the latter strategy. We do not test if the yield premia consistent with our model are capable of solving the expectations puzzle *for some parameter values*. This is likely to be the case, given that our model includes a relatively flexible specification of the market prices of risk. We ask instead a more stringent question, namely whether the premia generated by our model can solve the expectations puzzle *for the specific set of parameter values which maximises the likelihood.*

In so doing, we follow closely Dai and Singleton (2002) who ask the same question within a number of dynamic affine term structure models based on unobservable factors. More specifically, we ask whether the model-implied, population coefficients

ϕ_n in the regression

$$y_{t+1}^{n-1} - y_t^n = const. + \phi_n \left(y_t^n - r_t \right) / \left(n - 1 \right) + residual \tag{10}$$

match the values obtained from an OLS regression on actual yield data. The population coefficient are obtained assuming that the model parameters are true and then deriving the ϕ_n coefficients analytically based on the stochastic properties of the model.[17] Following Dai and Singleton (2002), we also examine the small-sample counterparts of these coefficients. Some correction for small sample bias is desirable because of the persistent nature of yields. For this purpose, we generate 1000 samples of the same length of our data (287) and calculate the mean estimate of the ϕ_n coefficients.

Dai and Singleton (2002) denote the above test as LPY(i). LPY(i) is a test of the capacity of the model to replicate the historical dynamics of yields as generated by a combination of the dynamics of risk premia and expectations of future short rates. As already emphasised, a successful model should be capable of generating the negative intercept coefficient of Campbell and Shiller-type regressions.

In addition to LPY(i), Dai and Singleton (2002) also suggest running a second sort of test, defined as LPY(ii), which focuses on the realism of the dynamic properties of risk premia. If the model captures these dynamics well, a Campbell and Shiller-type regression based on risk-premium-adjusted yield changes should recover the coefficient of unity consistent with the expectations hypothesis. LPY(ii) therefore tests that the sample coefficient ϕ_n^* in the regression

$$y_{t+1}^{n-1} - y_t^n + e_{n,t} / \left(n - 1 \right) = const. + \phi_n^* \left(y_t^n - r_t \right) / \left(n - 1 \right) + residual \tag{11}$$

is equal to its population value of 1 (in the above regression, $e_{n,t}$ is the excess holding period return $e_{n,t} \equiv E_t \left[\ln \left(p_{t+1}^{n-1} / p_t^n \right) - r_t \right]$ – see appendix).

Dai and Singleton (2002) show that an affine 3-factor model with Gaussian innovations and including a risk-premium specification of the type suggested by Duffee (2002) scores extremely well in terms of both LPY(i) and LPY(ii). Our model also includes a flexible specification of the risk-premium as in Duffee (2002). Unlike in pure finance models, however, our risk-premia are partly functions of observable variables, namely lags of output and inflation. This feature represents an

[17] This amounts to evaluating analytically $\phi_n \equiv \frac{cov\left(y_{t+1}^{n-1} - y_t^n, \left(y_t^n - r_t\right)/(n-1)\right)}{var\left(\left(y_t^n - r_t\right)/(n-1)\right)}$.

additional constraint, which makes the LPY tests more stringent than in the pure finance literature.

5.1 $\boldsymbol{LPY}$(i)

Since the evidence on Campbell and Shiller-type regressions based on European data is less compelling than for the US (e.g. Hardouvelis, 1994, Gerlach and Smets, 1997, Bekaert and Hodrick, 2001), we start by replicating Campbell and Shiller's analysis on our data. The results of the sample estimates of equation (10) are shown in Figures 11 and 12 as dots under the label "Sample". Consistently with the puzzle, the estimated intercept coefficient is always negative and decreasing in maturity. We confirm, however, that the puzzle appears less severe for German yield data: the estimated coefficient hovers around -0.7 for 7-year yields, compared to a value of less than -3 reported by Dai and Singleton (2002) for US 7-year yields.

In Figure 11 we show the results of the LPY(i) test. The population coefficients follow quite closely the pattern of the sample coefficients, although less so for short maturities. The population coefficients also have the downward sloping feature emphasised also by Dai and Singleton. The small-sample values of the ϕ_n coefficients (labelled "Model-implied MC" in Figure 11 and drawn together with 95% confidence bands of their small-sample distribution) confirm and strenghten this result. Our model appears to match strikingly well the pattern of the sample coefficients for essentially all maturities included in the regression.

The success of the model in matching LPY(i) depends crucially on our assumptions related to the market prices of risk. Our parameterisation of the λ_1 matrix permits variations of the prices attached to the various sources of risk depending on the level of the state variables of the model. For example, the risk premium required for the possible occurrence of inflation target shocks varies with the levels of inflation and the output gap (see the first row of the λ_1 matrix). In fact, it turns out that the statistically significant dimension of the inflation target premium is not related to the occurrence of "own shocks" (the first element in the matrix is zero). What matters is whether inflation and the output gap are high once the target is also high because of *past* inflation target shocks.

In specifications not allowing for such interactions – for example if the λ_1 matrix were diagonal – we experienced a worsening of the the performance of our model in terms of the LPY tests. The importance of the interactions generated by the off-diagonal terms in the λ_1 matrix is related to the fact that these increase the persistence of the yield premia. Once the premium related to inflation target shocks has gone up, it will possibly remain high not only because of the persistence of the inflation target, but also because of increases in the output gap or inflation driven by any other shock in the system. The persistence of the yield premia, in turn, is crucial to generate significant deviations in the yields levels from the values consistent with the expectations hypothesis.

5.2 LPY(ii)

Figure 12 shows the results of the LPY(ii) tests. Once again, the model does remarkably well in fitting the data. The risk-premium correction always goes in the right direction and the model can generate a coefficient very close to unity for maturities of 4 years or longer.

For shorter maturities the model does less well, but we still recover coefficients that are positive and larger than 0.5, which is a dramatic improvement with respect to the implications of the expectations hypothesis. The reduced degree of success of the model at the short end of the yield curve is also consistent with standard results that 3-factor models are unable to capture short-lived money-market dynamics and that a fourth factor may be necessary for this purpose. Alternatively, such dynamics may be captured allowing for jumps in the short-term rate, as in Piazzesi (2001).

To summarize, our model appears to do as well as the essentially affine $A_0(3)$ class in tests of the expectations hypothesis, in spite of the further constraints imposed by the dependence of risk premia on observed variables. The results of LPY(i) are very positive, in that the model can replicate the estimated coefficient of Campbell and Shiller-type regressions at all maturities. The test of LPY(ii) are also positive, and especially so for long maturities.

6 Conclusions

This paper presents a general set-up allowing to jointly model and estimate a macroeconomic-plus-term-structure model. The model extends the term-structure literature, since it shows how to derive bond prices using no-arbitrage conditions based on an explicit structural macroeconomic model, including both forward-looking and backward-looking elements. At the same time, we extends the macroeconomic literature by studying the term structure implications of a standard macro model within a dynamic no-arbitrage framework.

In an empirical application, we show that there are synergies to be exploited from current advances in macroeconomic and term-structure modelling. The two approaches can be seen as complementary and, when used jointly, give rise to sensible results. Notably, we show that our estimates of macroeconomic parameters, that are partly determined by the term-structure data, are consistent with those that would be estimated using only macroeconomic information. At the same time, our model's explanatory power for the term-structure is comparable to that of term-structure models based only on unobservable variables.

We assess the performance of our model mainly along two dimensions: forecasting and ability to solve the expectations hypothesis puzzle.

While yields do not seem to provide useful additional information in forecasting macroeconomic variables, our model performs very well in forecasting yields. We argue that this is due to both the inclusion of macroeconomic variables in the information set and to the imposition of a large number of no-arbitrage and structural restrictions on the reduced form representation of the model.

Our macro-based term-structure model can also match features of yield curve data which represent a puzzle for the expectations hypothesis. These results confirm that the dynamics of stochastic risk premia are important determinants of yield dynamics, and that all such dynamics can be ultimately reconducted to underlying macroeconomic dynamics within a consistent framework.

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A Appendix

A.1 State-space form

We write the model (1)-(3) in the state-space form (5)-(6), we define the vectors $\mathbf{X}_{1t}$, $\mathbf{X}_{2t}$ and $\mathbf{F}$ and the matrix $\mathbf{H}$ as follows:

$$\begin{aligned}
\mathbf{X}_{1t} &= [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, \eta_t, \varepsilon_t^{\pi}, \varepsilon_t^{x}, r_{t-1}]', \\
\mathbf{X}_{2t} &= [E_t x_{t+11}, ..., E_t x_{t+1}, x_t, E_t \pi_{t+11}, ..., E_t \pi_{t+1}, \pi_t]', \\
\mathbf{X}_t &= \left[\begin{array}{cc} \mathbf{X}'_{1,t} & \mathbf{X}'_{2,t} \end{array}\right]', \\
\mathbf{F} &= \left[\underset{1\times 6}{\mathbf{0}}, \beta\left(1-\rho\right), -1, 0, 0, -\rho, \underset{1\times 11}{\mathbf{0}}, -\gamma\left(1-\rho\right), -\beta\left(1-\rho\right), \underset{1\times 11}{\mathbf{0}}\right],
\end{aligned}$$

$$\mathbf{H} = \left[\begin{array}{cc} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{array}\right],$$

$$\mathbf{H}_{11} = \left[\begin{array}{ccc} & \underset{1\times 11}{\mathbf{0}} & \\ \underset{2\times 2}{\mathbf{I}} & \underset{2\times 9}{\mathbf{0}} & \\ & \underset{1\times 11}{\mathbf{0}} & \\ \underset{2x3}{\mathbf{0}} & \underset{2x2}{\mathbf{I}} & \underset{2x6}{\mathbf{0}} \\ \underset{1x6}{\mathbf{0}} & \phi_{\pi^*} & \underset{1x4}{\mathbf{0}} \\ & \underset{4x11}{\mathbf{0}} & \end{array}\right],$$

$$\mathbf{H}_{12} = \left[\begin{array}{ccc} \underset{1\times 11}{\mathbf{0}} & 1 & \underset{1\times 12}{\mathbf{0}} \\ & \underset{2\times 24}{\mathbf{0}} & \\ \underset{1\times 23}{\mathbf{0}} & 1 & \\ & \underset{7\times 24}{\mathbf{0}} & \end{array}\right],$$

$$\mathbf{H}_{21} = \left[\begin{array}{cccccc} & -\frac{12(1-\mu_x)}{\mu_x}\boldsymbol{\zeta}'_x & \underset{1\times 6}{\mathbf{0}} & -\frac{12}{\mu_x} & 0 & \\ & & \underset{11\times 11}{\mathbf{0}} & & & \\ \underset{1\times 3}{\mathbf{0}} & -\frac{12(1-\mu_\pi)}{\mu_\pi}\boldsymbol{\delta}'_\pi & \underset{1\times 2}{\mathbf{0}} & -\frac{12}{\mu_\pi} & \underset{1\times 2}{\mathbf{0}} & \\ & & \underset{11\times 11}{\mathbf{0}} & & & \end{array}\right],$$

$$\mathbf{H}_{22} = \left[\begin{array}{ccccc} -\underset{1\times 11}{\mathbf{1}} & \frac{12}{\mu_x} & -\frac{12\zeta_r}{\mu_x} & \underset{1\times 11}{\mathbf{0}} & \\ & \underset{11\times 11}{\mathbf{I}} & \underset{11\times 13}{\mathbf{0}} & & \\ \underset{1\times 11}{\mathbf{0}} & -\frac{12\delta_x}{\mu_\pi} & -\underset{1\times 11}{\mathbf{1}} & \frac{12}{\mu_\pi} & \\ \underset{11\times 12}{\mathbf{0}} & \underset{11\times 11}{\mathbf{I}} & \underset{11\times 1}{\mathbf{0}} & & \end{array}\right],$$

where $\boldsymbol{\zeta}_x = [\zeta_{x1}, \zeta_{x2}, \zeta_{x3}]'$ and $\boldsymbol{\delta}_\pi = [\delta_{\pi 1}, \delta_{\pi 2}, \delta_{\pi 3}]'$,

Finally, we define $\mathbf{K} = \left[\begin{array}{cccc} \underset{1\times 10}{\mathbf{0}} & 1 & \frac{12\zeta_r}{\mu_x} & \underset{1\times 23}{\mathbf{0}} \end{array}\right]'$ and $\boldsymbol{\xi}_{t+1} = \left[\boldsymbol{\xi}'_{1,t+1}, \underset{1x24}{\mathbf{0}}\right]'$, where $\boldsymbol{\xi}_{1,t+1} = \left[\underset{1x6}{\mathbf{0}}, u_{\pi^*,t+1}, u_{\eta,t+1}, u^{\pi}_{\varepsilon,t+1}, u^{x}_{\varepsilon,t+1}, 0\right]'$. We can therefore write the system as

$$\mathbf{X}_{t+1} = \mathbf{Q}\mathbf{X}_t + \boldsymbol{\xi}_{t+1} \tag{12}$$

where $\mathbf{Q} \equiv \mathbf{H} - \mathbf{KF}$.

A.2 Bond prices

For the pricing of bonds, we work with the transformed vector $\mathbf{Z}_t$ defined as $\mathbf{Z}_t = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi^*_t, r_t, \pi_t, x_t, r_{t-1}]'$. Using the solution $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$, $\mathbf{Z}_t$ can be written as $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$, where $\hat{\mathbf{D}}$ is

$$\hat{\mathbf{D}} \equiv \begin{bmatrix} \left(\underset{7\times 7}{\mathbf{I}}, \underset{7\times 4}{\mathbf{0}}\right) \\ -\mathbf{FG} \\ \mathbf{C}_{\{24,.\}} \\ \mathbf{C}_{\{12,.\}} \\ \left(\underset{1\times 10}{\mathbf{0}}, 1\right) \end{bmatrix},$$

$\mathbf{G} \equiv \begin{bmatrix} \mathbf{I}_{11} & \mathbf{C}' \end{bmatrix}'$ and $\mathbf{C}_{\{j,.\}}$ denotes row j of the matrix $\mathbf{C}$.

Given the definition of r_t and ξ_{t+1}, the pricing kernel $m_{t+1} = \exp(-r_t)\frac{\xi_{t+1}}{\xi_t}$ can be written as

$$m_{t+1} = \exp\left(-\overline{\boldsymbol{\Delta}}'\mathbf{Z}_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{1,t+1}\right), \tag{13}$$

where we used $r_t = \overline{\boldsymbol{\Delta}}'\mathbf{Z}_t$ with $\overline{\boldsymbol{\Delta}}' = \left[\underset{1\times 7}{\mathbf{0}}, 1, \underset{1\times 3}{\mathbf{0}}\right]$.

We know that this set-up will deliver bond prices that are exponential affine functions of $\mathbf{X}_1$. Since $\mathbf{Z}_t$ is an affine transformation of $\mathbf{X}_1$, we can write the bond prices as

$$p^n_t = \exp\left(\bar{A}_n + \bar{B}'_n\mathbf{Z}_t\right) \tag{14}$$

where the coefficients $\bar{A}_n$ and $\bar{B}_n$ have to be determined.

Note first that the price of a one-period bond at time t is $p^1_t = E_t[m_{t+1}] = \exp\left(-\overline{\boldsymbol{\Delta}}'\mathbf{Z}_t\right)$, so that $\bar{A}_1 = 0$ and $\bar{B}_1 = -\overline{\boldsymbol{\Delta}}$. We can now use the pricing kernel (13) and the postulated form of bond prices (14) to rewrite the equation for the price of an $(n+1)$-period bond $p^{n+1}_t = E_t\left[m_{t+1}p^n_{t+1}\right]$ as

$$p^{n+1}_t = \exp\left(\bar{A}_n - \bar{B}'_n\hat{\mathbf{D}}\Sigma\lambda_0 + \frac{1}{2}\bar{B}'_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n + \left(\bar{B}'_n\hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1} - \overline{\boldsymbol{\Delta}}' - \bar{B}'_n\hat{\mathbf{D}}\Sigma\lambda_1\right)\mathbf{Z}_t\right)$$

where we also used the properties of a lognormal variable ε_{1t} such that $E[\exp(a + b\varepsilon_{1t+1})] = \exp\left(a + \frac{1}{2}b^2 var[\varepsilon_{1t+1}]\right)$. The bond-pricing coefficients for any maturity n can therefore be found using the recursion

$$\begin{aligned}
\bar{A}_{n+1} &= \bar{A}_n - \bar{B}_n'\hat{\mathbf{D}}\Sigma\lambda_0 + \frac{1}{2}\bar{B}_n'\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n, \\
\bar{B}_{n+1}' &= \bar{B}_n'\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\boldsymbol{\Delta}}',
\end{aligned}$$

initialised at $\bar{A}_1 = 0$ and $\bar{B}_1 = -\overline{\boldsymbol{\Delta}}$.

A.3 Likelihood function

To implement ML estimation of the model, we first partition the state vector $\mathbf{X}_{1,t}$ into a vector $\mathbf{X}_{1,t}^u$ that includes only unobservable variables and a vector $\mathbf{X}_{1,t}^o$ of observable variables. Similarly, we define a vector $\mathbf{X}_{2,t}^o$ of observables from $\mathbf{X}_{2,t}$. Moreover, to prevent stochastic singularity, we assume that some of the yields are subject to measurement errors, that are assumed to be serially uncorrelated and mean-zero. We denote these by $\mathbf{Y}_t^m$, while $\mathbf{Y}_t^p$ will denote the (perfectly observed) remaining yields. If we denote by n_p the number of unobservable variables (i.e. the dimension of vector $\mathbf{X}_{1,t}^u$), then the sum of the dimensions of $\mathbf{X}_{2,t}^o$ and $\mathbf{Y}_t^p$ must also equal n_p. Correspondingly, we will denote by n_m the number of variables subject to measurement error (i.e. the dimension of $\mathbf{Y}_t^m$).

In our application, we have

$$\begin{aligned}
\mathbf{X}_{1,t}^o &= \left[x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, r_{t-1}\right]', \\
\mathbf{X}_{1,t}^u &= \left[\pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x\right]', \\
\mathbf{X}_{2,t}^o &= \left[x_t, \pi_t\right]', \\
\mathbf{Y}_t^p &= \left[y_t^1, y_t^{36}\right]', \\
\mathbf{Y}_t^m &= \left[y_t^3, y_t^6, y_t^{12}, y_t^{84}\right]',
\end{aligned}$$

where y_t^n denotes the yield on a zero-coupon bond with n-month maturity.

Next, we follow Chen and Scott (1993), Duffee (2002), and Ang and Piazzesi (2003), among others, and use the perfectly observed yields and macro variables to back out the vector of unobservable state variables, $\mathbf{X}_{1,t}^u$. To do this, we use the fact that $\mathbf{Y}_t^p$ can be expressed as $\mathbf{Y}_t^p = \mathbf{A}^p + \tilde{\mathbf{B}}^{op}\mathbf{X}_{1,t}^o + \tilde{\mathbf{B}}^{up}\mathbf{X}_{1,t}^u$, where the superscript p denotes the selection of factor loadings corresponding to $\mathbf{Y}^p$. Similarly, given the relationship $\mathbf{X}_{2t}^o = \tilde{\mathbf{C}}\mathbf{X}_{1,t}$, where $\tilde{\mathbf{C}} \equiv \left[\begin{array}{cc} \mathbf{C}_{\{12,.\}}' & \mathbf{C}_{\{24,.\}}' \end{array}\right]'$, we can write $\mathbf{X}_{2t}^o = \tilde{\mathbf{C}}^o\mathbf{X}_{1,t}^o + \tilde{\mathbf{C}}^u\mathbf{X}_{1,t}^u$.

Given the vector of parameters $\boldsymbol{\theta}$, these equations can be inverted to form an implied vector $\hat{\mathbf{X}}_{1,t}^u$. More specifically, let $\mathbf{W}_t$ denote the vector stacking $\mathbf{Y}^p$ and $\mathbf{X}_{2,t}^o$, i.e. $\mathbf{W}_t \equiv \left[\begin{array}{cc} \mathbf{Y}^{p\prime} & \mathbf{X}_{2,t}^{o\prime} \end{array}\right]'$. We obtain

$$\hat{\mathbf{X}}_{1t}^u = \mathbf{J}^{-1}\left(\mathbf{W}_t - \left[\begin{array}{c} \mathbf{A}^p \\ \mathbf{0} \end{array}\right] - \left[\begin{array}{c} \tilde{\mathbf{B}}^{op} \\ \tilde{\mathbf{C}}^o \end{array}\right]\mathbf{X}_{1,t}^o\right),$$

where the Jacobian term is given by

$$\mathbf{J} \equiv \left[\begin{array}{c} \tilde{\mathbf{B}}^{up} \\ \tilde{\mathbf{C}}^{u} \end{array} \right].$$

Finally, given the vector $\hat{\mathbf{X}}_{1t}^{u}$, implied yields for the remaining n_m bonds can be computed using $\hat{\mathbf{Y}}_t^m = \mathbf{A}^m + \tilde{\mathbf{B}}^{om}\mathbf{X}_{1,t}^o + \tilde{\mathbf{B}}^{um}\hat{\mathbf{X}}_{1,t}^u$, where the superscript m denotes the selection of factor loadings corresponding to $\mathbf{Y}^m$. In general, these implied yields will not exactly correspond to the observed yields. The difference produces the vector of measurement errors, $\mathbf{u}_t^m = \hat{\mathbf{Y}}_t^m - \mathbf{Y}_t^m$, which is assumed to have a constant diagonal variance covariance matrix with element i given by $\sigma_{m,i}^2$.

To compute the log-likelihood value, we start from the knowledge that the one-period ahead conditional distribution of the unobservable state variables has a multivariate normal distribution $f_{\mathbf{X}_1^u}\left(\mathbf{X}_{1,t}^u \mid \mathbf{X}_{1,t-1}^u, \mathbf{X}_{1,t-1}^o\right)$. This distribution is known, since the conditional mean of $\mathbf{X}_t^u$ is given by the theoretical model and its variance-covariance matrix $\mathbf{\Sigma\Sigma}'$ is assumed to be constant and diagonal. The distribution of $\mathbf{W}_t$ conditional on $\mathbf{W}_{t-1}$ and $\mathbf{X}_{t-1}^o$ is then

$$f_{\mathbf{W}}\left(\mathbf{W}_t \mid \mathbf{W}_{t-1}, \mathbf{X}_{t-1}^o\right) = \frac{1}{|\mathbf{J}|} f_{\mathbf{X}_1^u}\left(\hat{\mathbf{X}}_{1,t}^u \mid \hat{\mathbf{X}}_{1,t-1}^u, \mathbf{X}_{1,t-1}^o\right).$$

Assuming that the yield measurement errors are jointly normal with distribution $f_{\mathbf{u}^m}\left(\mathbf{u}_t^m\right)$, the log-likelihood of observation t will be given by the sum $\ln f_{\mathbf{W}}\left(\mathbf{W}_t \mid \mathbf{W}_{t-1}, \mathbf{X}_{t-1}^o\right) + \ln f_{\mathbf{u}^m}\left(\mathbf{u}_t^m\right)$, which can be written as

$$\begin{aligned}
\sum_{t=2}^{T} \pounds_t(\boldsymbol{\theta}) = & -(T-1)\ln|J| - \frac{(T-1)\,n_p}{2}\ln(2\pi) - \frac{T-1}{2}\ln\left|\mathbf{\Sigma\Sigma}'\right| \\
& -\frac{1}{2}\sum_{t=2}^{T}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right)'\left(\mathbf{\Sigma\Sigma}'\right)^{-1}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right) \\
& -\frac{(T-1)\,n_m}{2}\ln(2\pi) - \frac{T-1}{2}\sum_{i=1}^{n_m}\ln\sigma_i^2 - \frac{1}{2}\sum_{t=2}^{T}\sum_{i=1}^{n_m}\frac{\left(u_{t,i}^m\right)^2}{\sigma_i^2}.
\end{aligned}$$

Our maximum likelihood estimate is the vector $\boldsymbol{\theta}^*$ which maximises the above expression.

A.4 Analytical derivatives

The calculation of the analytical derivatives of the log-likelihood function with respect to the parameter vector involves two key steps. First, the derivatives of the $\mathbf{A}$ and $\tilde{\mathbf{B}}$ matrices with respect to the $\mathbf{M}$ and $\mathbf{C}$ matrices; second, the derivatives of the $\mathbf{M}$ and $\mathbf{C}$ matrices with respect to the $\mathbf{Q}$ matrix.

For the first step, it can be shown that

$$\mathrm{d}\mathbf{A} = -\frac{1}{n}\mathrm{d}\bar{A}_n,$$

where

$$
\begin{aligned}
\mathrm{d}\bar{A}_n &= \mathrm{d}\bar{A}_{n-1} - \left(\mathrm{d}\bar{B}'_{n-1}\hat{\mathbf{D}} + \bar{B}'_{n-1}\mathrm{d}\hat{\mathbf{D}}\right)\Sigma\lambda_0 - \bar{B}'_{n-1}\hat{\mathbf{D}}\left(\mathrm{d}\Sigma\lambda_0 + \Sigma\mathrm{d}\lambda_0\right) \\
&\quad + \frac{1}{2}\left(\left(\mathrm{d}\bar{B}'_{n-1}\hat{\mathbf{D}} + \bar{B}'_{n-1}\mathrm{d}\hat{\mathbf{D}}\right)\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1} + \bar{B}'_{n-1}\hat{\mathbf{D}}2\Sigma\mathrm{d}\Sigma\hat{\mathbf{D}}'\bar{B}_{n-1}\right. \\
&\quad \left. + \bar{B}'_{n-1}\hat{\mathbf{D}}\Sigma\Sigma'\left(\mathrm{d}\hat{\mathbf{D}}'\bar{B}_{n-1} + \hat{\mathbf{D}}'\mathrm{d}\bar{B}_{n-1}\right)\right),
\end{aligned}
$$

and

$$\mathrm{d}\tilde{\mathbf{B}}_n = -\frac{1}{n}\left(\mathrm{d}\bar{B}'_n\hat{\mathbf{D}} + \bar{B}'_n\mathrm{d}\hat{\mathbf{D}}\right),$$

where

$$\mathrm{d}\bar{B}'_{n+1} = \mathrm{d}\bar{B}'_n\mathbf{S} + \bar{B}'_n\mathrm{d}\mathbf{S}.$$

$$\underset{n_1 \times n_1}{\mathbf{S}} \equiv \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right),$$

$$\mathrm{d}\mathbf{S} = \mathrm{d}\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) + \hat{\mathbf{D}}\left[\mathrm{d}\mathbf{M}\hat{\mathbf{D}}^{-1} - \mathbf{M}\hat{\mathbf{D}}^{-1}\left(\mathrm{d}\hat{\mathbf{D}}\right)\hat{\mathbf{D}}^{-1} - \mathrm{d}\Sigma\lambda_1 - \Sigma\mathrm{d}\lambda_1\right].$$

To compute $\mathrm{d}\mathbf{C}$ and $\mathrm{d}\mathbf{M}$ in the second step, we adapt the methodology described in Anderson, McGrattan, Hansen and Sargent (1996) and obtain

$$
\begin{aligned}
\mathrm{vec}\left(\mathrm{d}\mathbf{C}\right) &= \left[\left(\mathbf{Q}'_{11} \quad \mathbf{I}\right) + \left(\mathbf{I} \quad \mathbf{C}\mathbf{Q}_{12}\right) + \left(\mathbf{C}'\mathbf{Q}'_{12} \quad \mathbf{I}\right) - \left(\mathbf{I} \quad \mathbf{Q}_{22}\right)\right]^{-1} \cdot \\
&\quad \left[\begin{array}{c} -\left(\mathbf{I} \quad \mathbf{C}\right)\mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{11}\right) - \left(\mathbf{C}' \quad \mathbf{C}\right)\mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{12}\right) \\ + \mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{21}\right) + \left(\mathbf{C}' \quad \mathbf{I}\right)\mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{22}\right) \end{array}\right]
\end{aligned}
$$

and

$$\mathrm{vec}\left(\mathrm{d}\mathbf{M}\right) = \mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{11}\right) + \left(\mathbf{C}' \quad \mathbf{I}\right)\mathrm{vec}\left(\mathrm{d}\mathbf{Q}_{12}\right) + \left(\mathbf{I} \quad \mathrm{d}\mathbf{Q}_{12}\right)\mathrm{vec}\left(\mathrm{d}\mathbf{C}\right).$$

A.5 Risk premia

A.5.1 Holding premia

We define the one-period holding premium $e_{n,t}$ on an n-period bond purchased at t as the expected holding return of that bond over one period, less the risk-free rate:

$$e_{n,t} = E_t\left[\ln\left(p_{t+1}^{n-1}\right) - \ln\left(p_t^n\right)\right] - r_t.$$

Using the bond pricing equation, this can be written as

$$e_{n,t} = \left(\bar{B}'_{n-1}\hat{\mathbf{D}}\Sigma\lambda_0 - \frac{1}{2}\bar{B}'_{n-1}\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1}\right) + \left(\bar{B}'_{n-1}\hat{\mathbf{D}}\Sigma\lambda_1\right)\mathbf{Z}_t$$

A.5.2 Forward premia

The one-period forward premium $\psi_{n,t}$ at t for maturity n is defined as the difference between the implied one-period forward rate n periods ahead, $f_{n,t}$, less the corresponding expected one-period interest rate:

$$\psi_{n,t} = f_{n,t} - E_t\left[r_{t+n}\right].$$

The implied forward rate is given by

$$
\begin{aligned}
f_{n,t} &= \ln\left(p_t^n\right) - \ln\left(p_t^{n+1}\right) \\
&= \left(\bar{B}_n' \hat{\mathbf{D}} \Sigma \lambda_0 - \frac{1}{2} \bar{B}_n' \hat{\mathbf{D}} \Sigma \Sigma' \hat{\mathbf{D}}' \bar{B}_n\right) + \left[\bar{B}_n' - \bar{B}_n' \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) + \overline{\boldsymbol{\Delta}}'\right] \mathbf{Z}_t
\end{aligned}
$$

while the expected short rate is

$$
E_t\left[r_{t+n}\right] = \overline{\boldsymbol{\Delta}}' \hat{\mathbf{D}} \mathbf{M}^n \hat{\mathbf{D}}^{-1} \mathbf{Z}_t.
$$

The one-month forward premium is therefore

$$
\begin{aligned}
\psi_{n,t} &= f_{n,t} - E_t\left[r_{t+n}\right] \\
&= \left(\bar{B}_n' \hat{\mathbf{D}} \Sigma \lambda_0 - \frac{1}{2} \bar{B}_n' \hat{\mathbf{D}} \Sigma \Sigma' \hat{\mathbf{D}}' \bar{B}_n\right) + \\
&\quad \left[\bar{B}_n' - \bar{B}_n' \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) + \overline{\boldsymbol{\Delta}}'\left(I - \hat{\mathbf{D}} \mathbf{M}^n \hat{\mathbf{D}}^{-1}\right)\right] \mathbf{Z}_t
\end{aligned}
$$

A.5.3 Yield risk premia

The n-maturity yield premium at t, $\omega_{n,t}$, can be defined as the average of the forward premia up until $t+n-1$, i.e. $\omega_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} \psi_{n,t}$. This is given by

$$
\begin{aligned}
\omega_{n,t} &= \frac{1}{n} \sum_{i=0}^{n-1} \left[\bar{B}_i' \hat{\mathbf{D}} \Sigma \lambda_0 - \frac{1}{2} \bar{B}_i' \hat{\mathbf{D}} \Sigma \Sigma' \hat{\mathbf{D}}' \bar{B}_i + \right. \\
&\quad \left. \left(\bar{B}_i' - \bar{B}_i' \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) + \overline{\boldsymbol{\Delta}}'\left(I - \hat{\mathbf{D}} \mathbf{M}^i \hat{\mathbf{D}}^{-1}\right)\right) \mathbf{Z}_t\right]. \qquad (15)
\end{aligned}
$$

Table 1: Parameter estimates
(Sample period: Feb 1975-Dec 1998)

Parameter	Point estimate	Standard error
ρ	0.976	0.015
β	2.087	0.855
γ	1.243	0.925
μ_π	0.132	0.011
$\delta_x \times 10^2$	0.038	0.054
μ_x	0.303	0.029
ζ_r	0.027	0.023
ϕ_{π^*}	0.999	–
$\sigma_{\pi^*} \times 10^2$	0.014	0.001
$\sigma_\eta \times 10^2$	0.040	0.001
$\sigma_x \times 10^2$	0.022	0.001
$\sigma_\pi \times 10^2$	0.097	0.004
$\sigma_1^m \times 10^2$	0.019	0.011
$\sigma_2^m \times 10^2$	0.025	0.014
$\sigma_3^m \times 10^2$	0.023	0.001
$\sigma_4^m \times 10^2$	0.023	0.001
$\lambda_{0,1}$	−0.421	0.306
$\lambda_{0,2}$	−0.587	0.345
$\lambda_{0,3}$	4.431	2.565
$\lambda_{0,4}$	−1.693	1.438

$\lambda_1 \times 10^{-2}$

	π^*	r	π	x
π^*	0 (–)	0 (–)	0.976 (0.264)	0.912 (0.175)
r	−35.354 (12.920)	18.955 (7.184)	−8.535 (3.396)	0 (–)
π	152.232 (56.674)	−88.432 (29.876)	41.470 (12.488)	0 (–)
x	0 (–)	2.094 (1.070)	0 (–)	2.559 (0.908)

Standard errors in parentheses

Asymptotic standard errors are based on the outer-product estimate of the information matrix. The estimates of the lag coefficients for inflation and output are not reported.

Table 2: Out-of-sample output and inflation forecasting performance: RMSEs

	Forecast horizon							
	3-month		6-month		9-month		12-month	
variable	HTV-M	HTV	HTV-M	HTV	HTV-M	HTV	HTV-M	HTV
x	**1.242**	1.508	**1.776**	2.416	**2.064**	2.889	**2.154**	2.980
π	0.393	**0.379**	0.519	**0.490**	0.679	**0.602**	0.900	**0.751**

RMSEs for out-of-sample forecasts between 1995:01 and 1998:12, based on parameter estimates for 1975:02 - 1994:12. VAR is a 3-variable unrestricted VAR(3) including inflation, the output gap and the 1-month rate, HTV-M denotes the macroeconomic model represented by equations (1)-(3) in the text (this model is estimated using the the Bundesbank's price norm as the inflation target in the policy rule), and HTV denotes our structural macro model.

Table 3: Out-of-sample yield forecast performance: RMSEs

	1-month forecast horizon				
maturity	RW	VAR	$A_0(3)$	AP	HTV
1 month	0.148	0.182	0.151	0.146	**0.129**
3 months	**0.173**	0.177	0.181	0.178	0.220
1 year	**0.194**	0.211	0.319	0.271	0.270
3 years	0.252	0.267	0.254	0.256	**0.236**
7 years	**0.220**	0.237	0.331	0.320	0.384

	3-month forecast horizon				
maturity	RW	VAR	$A_0(3)$	AP	HTV
1 month	0.299	0.337	0.303	0.263	**0.210**
3 months	0.345	0.358	0.426	0.446	**0.292**
1 year	0.395	0.433	0.582	0.569	**0.383**
3 years	0.448	0.544	0.462	0.475	**0.397**
7 years	**0.379**	0.454	0.428	0.465	0.447

	6-month forecast horizon				
maturity	RW	VAR	$A_0(3)$	AP	HTV
1 month	0.458	0.425	0.452	0.410	**0.296**
3 months	0.512	0.477	0.652	0.743	**0.404**
1 year	0.574	0.604	0.829	0.873	**0.529**
3 years	0.624	0.765	0.669	0.885	**0.534**
7 years	0.521	0.684	0.521	0.706	**0.479**

	9-month forecast horizon				
maturity	RW	VAR	$A_0(3)$	AP	HTV
1 month	0.610	0.548	0.588	0.523	**0.432**
3 months	0.666	0.656	0.828	0.975	**0.559**
1 year	0.733	0.786	1.012	1.138	**0.689**
3 years	0.782	0.986	0.873	0.950	**0.678**
7 years	0.678	0.916	0.719	0.848	**0.580**

	12-month forecast horizon				
maturity	RW	VAR	$A_0(3)$	AP	HTV
1 month	0.747	0.678	0.706	0.608	**0.604**
3 months	0.793	0.857	0.956	1.154	**0.730**
1 year	0.854	1.002	1.140	1.333	**0.829**
3 years	0.842	1.198	1.002	1.139	**0.744**
7 years	0.806	1.184	0.920	1.091	**0.680**

RMSEs for out-of-sample forecasts between 1995:01 and 1998:12, based on parameter estimates for 1975:02 - 1994:12. "RW" are random walk forecasts, "VAR" is an unrestricted VAR(3) including the same variables as our model, "$A_0(3)$" is a canonical essentially affine Gaussian three-factor model, "AP" denotes the Ang-Piazzesi (2003) Macro Model (estimated using our macro data, but with inflation expressed in y-o-y terms), and "HTV" denotes our structural macro model.

Table 4: Out-of-sample yield forecast performance: Trace MSEs

forecast horizon (months)	RW	VAR	$A_0\,(3)$	AP	HTV
1	7.2	8.5	11.9	10.6	12.3
2	16.4	19.2	23.8	23.1	17.7
3	25.5	33.5	36.3	37.2	22.8
4	33.6	44.4	47.4	51.1	26.3
5	41.0	55.7	58.1	65.7	29.8
6	52.7	65.7	73.2	85.6	37.6
7	64.0	81.3	88.3	105.7	45.5
8	74.4	94.2	102.8	125.9	52.9
9	87.3	113.8	120.0	149.1	63.7
10	99.9	135.8	136.7	171.9	75.0
11	109.6	159.4	150.9	193.0	84.4
12	117.9	181.2	164.2	214.9	93.6

The trace MSE statistics of Christoffersen and Diebold (1998) are for out-of-sample forecasts between 1995:01 and 1998:12, based on parameter estimates for 1975:02 - 1994:12. "RW" are random walk forecasts, "VAR" is an unrestricted VAR(3) including the same variables as our model, "$A_0\,(3)$" is a canonical essentially affine Gaussian three-factor model, "AP" denotes the Ang-Piazzesi (2003) Macro Model (estimated using our macro data, but with inflation expressed in y-o-y terms), and "HTV" denotes our structural macro model.

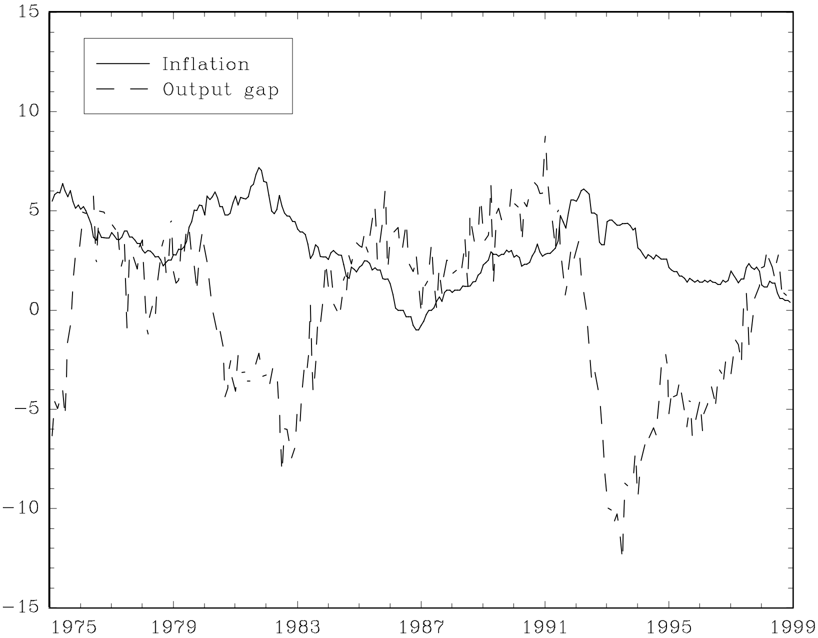
Table 5: Tests for superior out-of-sample predictive ability of yield forecasts from the HTV model compared to four different benchmarks

maturity	RW	VAR	$A_0(3)$	AP
1 month forecast horizon				
1 month	**0.031**	**0.097**	**0.036**	0.028
3 months	−0.112	−0.104	−0.095	−0.100
1 year	−0.212	−0.169	**0.172**	0.002
3 years	**0.046**	0.094	**0.052**	**0.058**
7 years	−0.597	−0.550	−0.231	−0.271
3 month forecast horizon				
1 month	**0.271**	**0.414**	**0.285**	0.150
3 months	**0.202**	0.256	**0.575**	**0.684**
1 year	0.053	0.246	**1.154**	**1.064**
3 years	**0.255**	**0.829**	**0.334**	**0.404**
7 years	−0.341	0.036	−0.100	0.095
6 month forecast horizon				
1 month	**0.733**	0.555	**0.701**	0.481
3 months	**0.595**	0.384	**1.569**	**2.328**
1 year	**0.296**	0.512	**2.439**	**3.023**
3 years	**0.629**	**1.802**	**0.974**	**1.286**
7 years	0.251	**1.431**	0.251	0.880
9 month forecast horizon				
1 month	**1.110**	0.683	**0.950**	0.523
3 months	**0.789**	0.711	**2.238**	**3.834**
1 year	**0.376**	0.853	**3.289**	**4.920**
3 years	**0.910**	**3.068**	**1.815**	**2.653**
7 years	**0.742**	**3.022**	**1.082**	**2.297**
12 month forecast horizon				
1 month	**1.159**	0.566	0.795	0.029
3 months	0.582	1.209	**2.290**	**4.804**
1 year	0.248	1.900	**3.671**	**6.528**
3 years	**0.930**	**5.291**	**2.699**	**4.465**
7 years	**1.123**	**5.634**	**2.302**	**4.375**

The table shows test statistics for superior forecast ability of the HTV model, compared to each of the four different benchmarks listed in the tables, calculated according to White's (2000) "reality check." We use a squared forecast error loss function when implementing the test. The null hypothesis is that the expected differential between the forecast loss of the benchmark and that of the HTV model is smaller than or equal to zero. Bold figures denote rejection of the null at the 5 percent level, based on a stationary bootstrap approach, with 50,000 resamples of the loss differential series (using a smoothing parameter of 1/12).

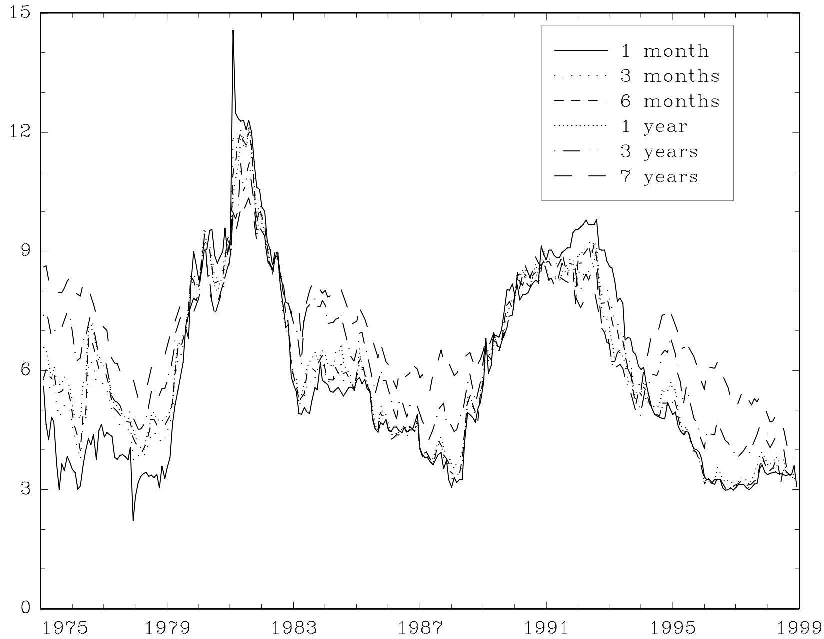
Figure 1: Data used in the estimations

(a) Macro data



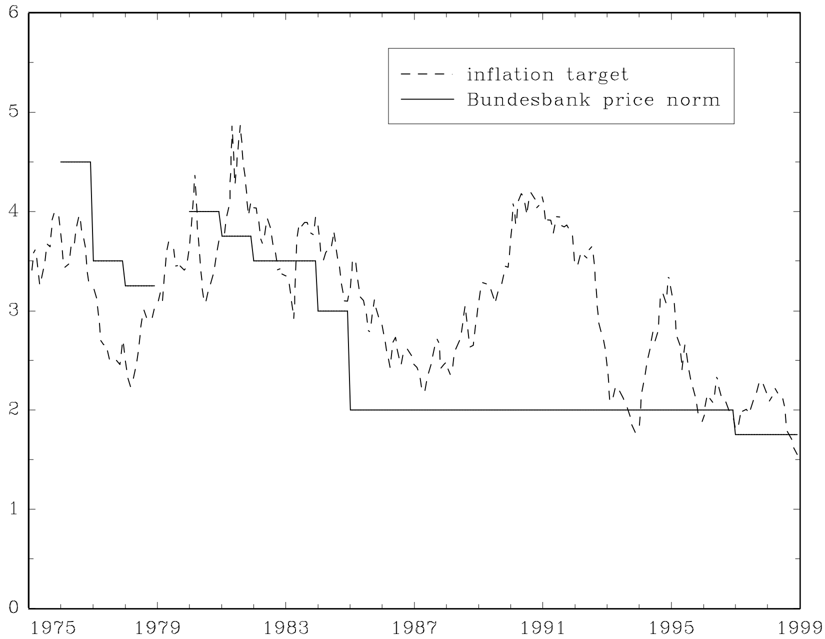
The inflation and output gap series have been multiplied by 100. The sample period is January 1975 to December 1998.

(b) Yield data



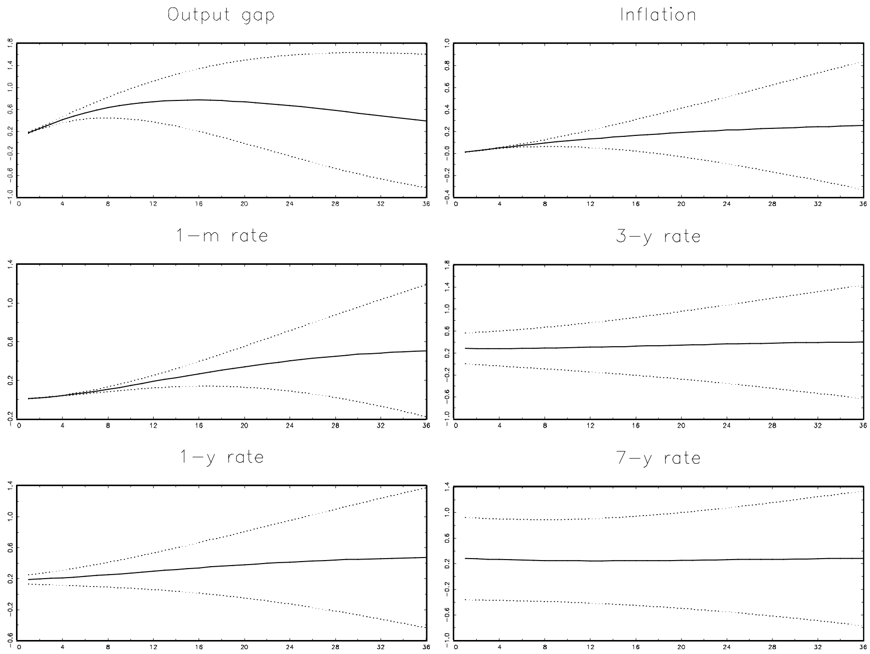
German term structure data over the sample period January 1975 to December 1998 (percent per year).

Figure 2: Estimated inflation target and announced Bundesbank price norm



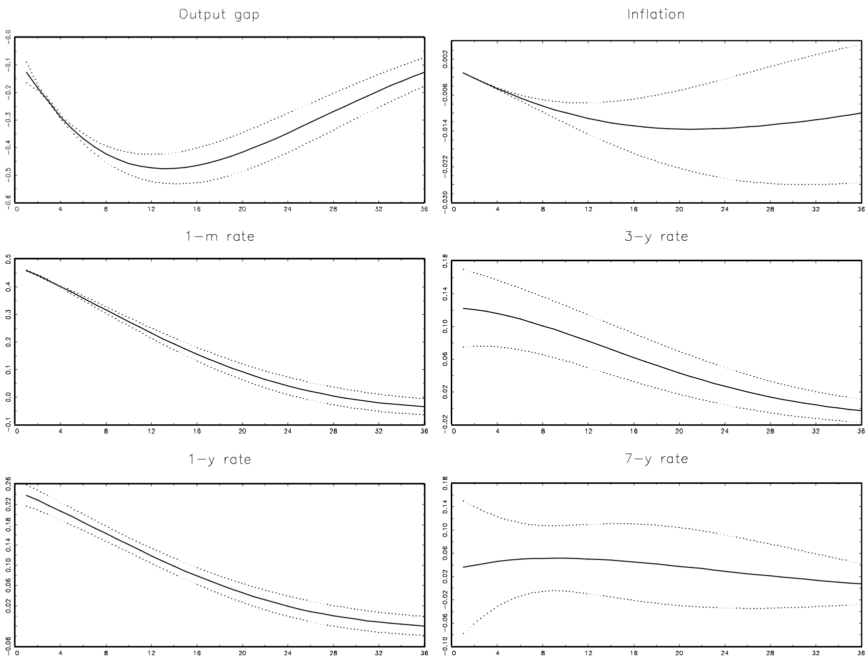
Percent per year. For those periods when the Bundesbank announced upper and lower bounds for the price norm, an average of these is shown in the figure.

Figure 3: Impulse responses from inflation target shock



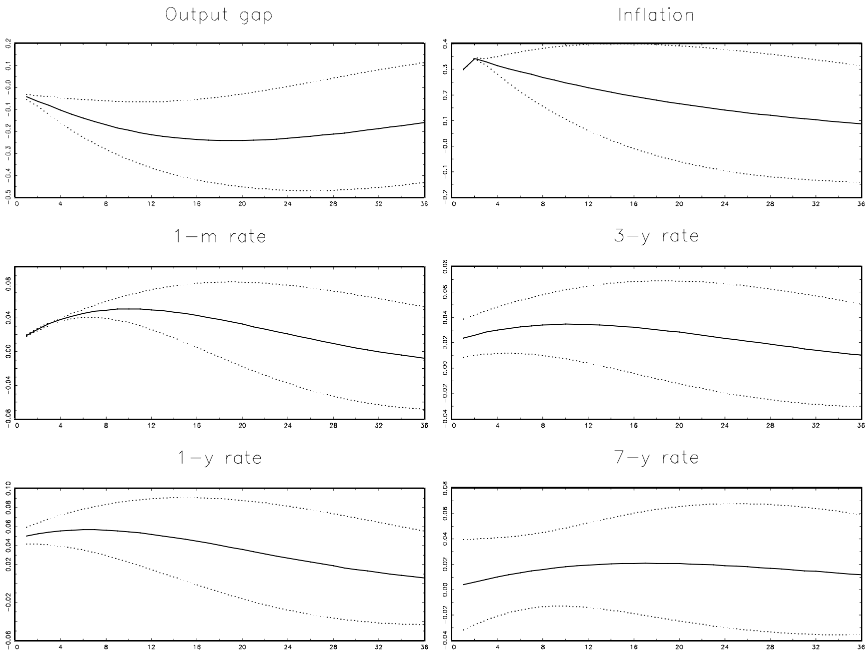
All responses are expressed in percentage terms. The inflation and short rate responses are expressed in annual terms. The inflation target was shocked by one standard deviation (around 0.2% p.a.).

Figure 4: Impulse responses from monetary policy shock



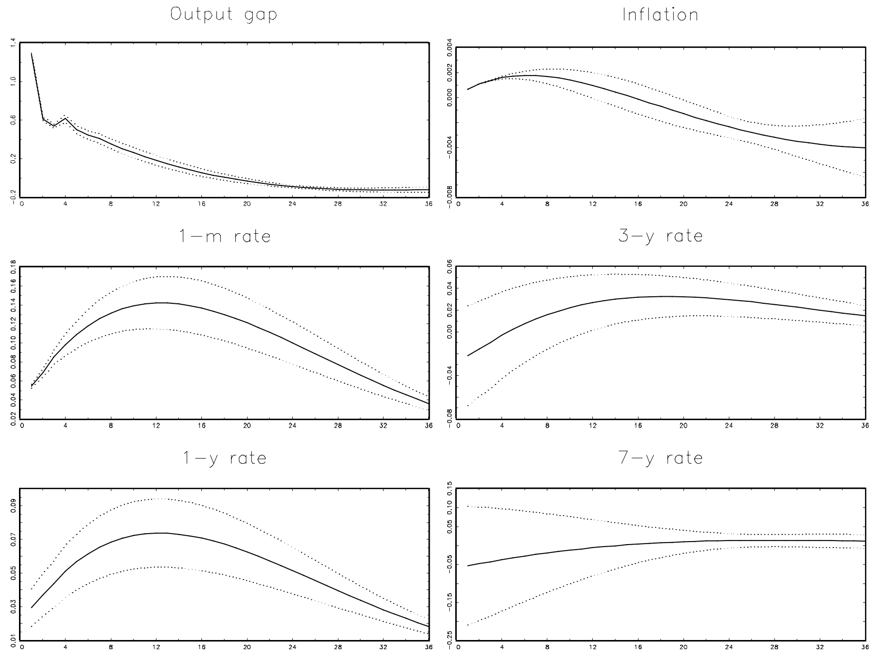
All responses are expressed in percentage terms. The inflation and short rate responses are expressed in annual terms. The inflation target was shocked by one standard deviation (around 0.2% p.a.).

Figure 5: Impulse responses from inflation shock



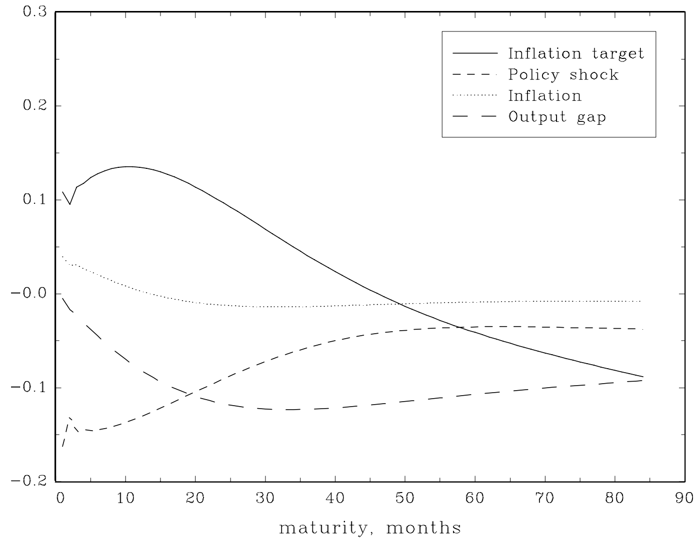
All responses are expressed in percentage terms. The inflation and short rate responses are expressed in annual terms. Inflation was shocked by one standard deviation (around 0.26% p.a.).

Figure 6: Impulse responses from output shock



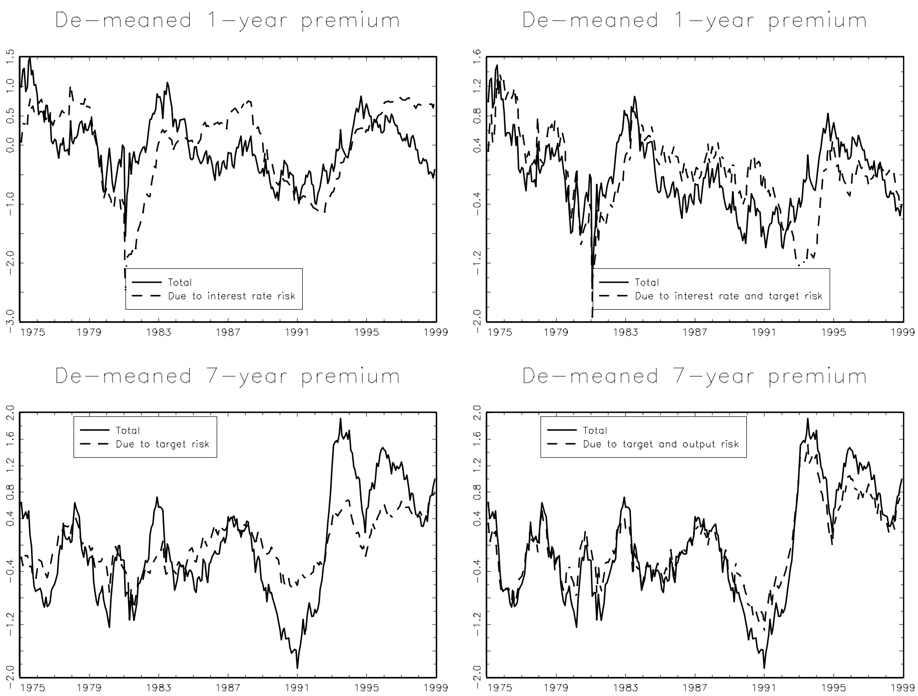
All responses are expressed in percentage terms. The inflation and short rate responses are expressed in annual terms. The output gap was shocked by one standard deviation (around 1.2%).

Figure 7: Initial response of yield premia to macro shocks



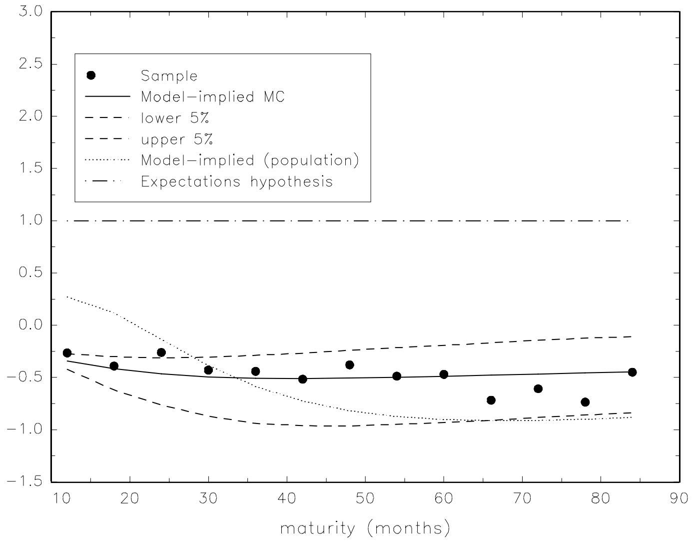
The figure shows the one-month ahead response of the yield premia ω_n, at maturities n up to 84 months, to one standard deviation shocks to the four macro factors. The premia are expressed in annual percentage terms.

Figure 8: Estimated yield premia and components of premia



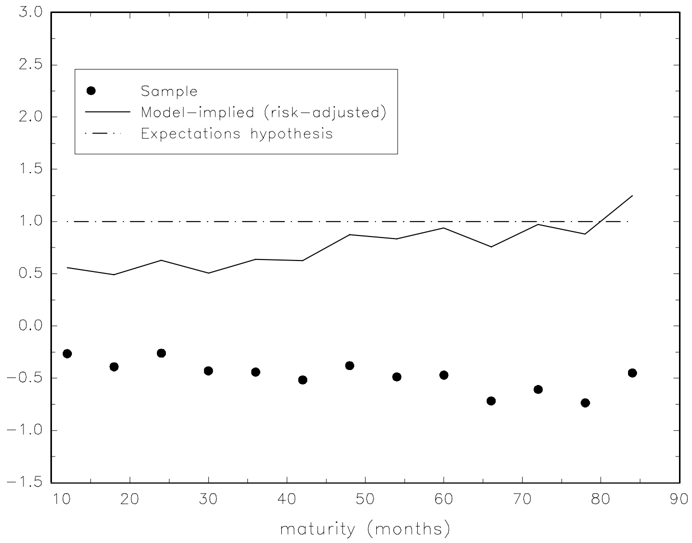
The solid lines are the estimated (de-meaned) yield premiums ω_n during the sample period, for maturities $n = 12$ and 84 months, expressed in annual percentage terms. The dashed lines show the portions of the premia that are due to selected macro factors or combinations of such factors.

Figure 9: Model-implied CS projection coefficients: "LPY(i)"



Empirical estimates of the CS long-rate coefficients ϕ_n in $y_{t+1}^{n-1} - y_t^n = \phi_n \left(y_t^n - r_t\right) / \left(n-1\right)$, plus corresponding model-implied cofficient values. The "population" coefficients are the theoretical values based on our estimates; the MC coefficients are the mean estimates from 1000 series of the same size as the sample, simulated from our model. The bands around the MC mean estimates are 5% confidence bands.

Figure 10: Model-implied risk-premium adjusted CS coefficients: "LPY(ii)"



The figure shows the estimates of the Campbell and Shiller (1991) long-rate coefficients ϕ_n in the regression $y_{t+1}^{n-1} - y_t^n = \phi_n \left(y_t^n - r_t\right) / \left(n-1\right)$ for our sample, along with the corresponding risk-premium adjusted model-implied cofficient values based on our parameter estimates.