# A Dynamic Stochastic Analysis of International Patent Application and Renewal Processes

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# Abstract

This paper develops a dynamic stochastic model to examine the joint patent application and renewal behavior under an international patent protection regime. This framework makes it possible to utilize both the cross-sectional (multi-country filing) and the time-series (patent renewal) dimensions of available international patent data to evaluate the private value of patent protection, and allows one to distinguish more aspects of patent value. The private value of European patents in the pharmaceutical and the electronics industries in the 1980's are examined, and the estimation results indicate substantial differences in the patent application and renewal patterns in these two industries. Pharmaceutical patents on average are endowed with higher initial values, and the patent holders seek for protection in more countries than the electronic patent holders. However, pharmaceutical patents depreciate faster than electronics patents, and consequently have lower renewal rates and shorter patent lives.

JEL Classification: C13, C15, C61, D23, K11, L63, L65

# 1 Introduction

The search for accurate methods to evaluate patent rights has a long history in both academia and industry. Such evaluation methods have been demanded for a wide variety of purposes. At the firm level, patents are a proxy for the firm's inventive output. Therefore a better estimation of the private value of patents measures the productivity of the firm's R&D expenditures better, which provides useful information for the firm's strategic R&D management. On the other hand, accurate evaluations of a specific firm's patent portfolios are also commercially valuable in securities analysis as well as merger and acquisition activities. This is especially crucial for start-ups in industries that heavily rely on technological innovations such as pharmaceuticals, biotech and computer. At the industry and country level, accurate evaluation methods for patents are needed in international competitiveness assessment as well as the associated policy analyses. Moreover, the rapidly increasing international trade flows of intellectual property rights and licensing also underscore the need for patent evaluation, which can be directly applied to analyzing the international trade patterns of patent rights and other kinds of intellectual property rights as well as their causes and implications.

However, available patent evaluation methods are not very satisfactory. While simple patent counts are easily implemented and have been used extensively (Griliches 1990), the quality of innovations protected by patents varies widely. This makes patent counts too noisy an indicator of innovative output in many cases. For instance, Evenson (1984) has documented a dramatic decline in patenting relative to measures of research for a broad set of countries. Furthermore, Kortum (1993) finds that all U.S. manufacturing industries have experienced a decline in patenting per unit of real R&D input. This leads Evenson (1984, 1991) to conclude that exhaustion of technological opportunities has reduced the productivity of the research sector. However, Schankerman and Pakes (1986) argue that the quality of patents varies. They construct a patent value index and compare it with the aggregate patent count for each of the U.K., France and Germany for the period of 1955 to 1975, and conclude that "...one cannot draw inferences on changes in the value of cohorts of patents during this period from changes in the quantity of patents, for there have been large (and largely offsetting) changes in the 'quality' (or mean values) of patents..."

Trajtenberg (1990) suggests weighting patent counts by the number of forward citations (citations by later patents) and using it as an indicator of the *social* value of patents. Citations to previous innovations are a part of the patent document that is published when the patent is granted. They specify the previous patents of relevant technology, and are identified by the examiner and patent applicant during the patent examination process. Trajtenberg (1990) argues that since patent citations contain extensive information about the technological antecedents of the invention, they are a good proxy for the complexity of the technological innovations. Therefore, patents of relatively high economic value are cited more frequently than low-value patents. Lanjouw and Schankerman (1999) find that patent citations are strongly related to the market value of patent, as well as the years of patent renewal and the family size of the patent (the number of countries in which the patent is applied for) in a sample of about 8000 U.S. patents applied for during 1960-1991. Harhoff, Narin, Scherer and Vopel (1999) also record a high correlation between the counts of citations and private value estimates of patents in a survey study performed in Germany and U.S. A short list of recent studies using this approach includes Sampat (1998), Shane (1999, 2001) and Hall, Jaffe and Trajtenberg (2001).

However, although in practice patent citation data are widely used in measuring the quality of patents, it should be recognized that more citations do not necessarily indicate higher market or *private* value of the patent. This is because more forward citations may indicate more subsequent competitors in the same or similar technological field, which may decrease the monopoly profits the earlier patent otherwise enjoys. Hall, Jaffe and Trajtenberg (2001) report that, in their estimation on a sample of over 4,800 U.S. manufacturing firms during 1965-1995, self-citations (i.e., citations made by the same patentee on later patents of his own) are worth about twice as much to the patentees as ordinary citations. This informs us of not only the spillover effects of internal knowledge *within* the firms, but also the extent of technological competition *among* different firms. In other words, more forward citations may not always increase the market value of the antecedents, and the technological competition in the industry has to be taken into account. While more self-citations may strengthen a firm's monopolistic position and identify the cited patent as a more valuable one (in private value), such effects of citations by competitors are more obscure.

Patent renewal studies were originated by Pakes and Schankerman (1984) and Pakes (1986). In particular, assuming that the patent renewal process is an optimizing process during which patent holders compare the annual renewal fees to the *expected* future returns from keeping the patent alive, Pakes (1986) develops a stochastic patent renewal model and estimates the distributions of patent value in post-war France, United Kingdom and Germany. Lanjouw (1998) recognizes the existence of possible patent infringements and the subsequent litigation process, develops a stochastic discrete-choice model of patent renewal incorporating the possibility of infringements,[1] and estimates the private value of patent protection in different technology areas in West Germany during 1953 to 1988.

Pakes (1986) and Lanjouw (1998) primarily focus on analyzing the optimal patent renewal processes. On the other hand, the patent application is also an optimizing process. Facing a lump sum application cost, the inventor forms his expectation of future returns, and makes a binary decision of whether to file a patent application in any single country. Therefore, the international patent family size (the number of countries in which the patent is applied for) also contains information about the private value of patent protection. Putnam (1997) is the first in the literature to utilize the multi-country patent application data to study the private value of patents, and the first chapter of this dissertation analyzes the joint filing decision-making problem faced by an EPO (European Patent Office) patent applicant while choosing which contracting states to designate for future patent protection. However, in formulating the patent applicants' expectation of patent value, both studies adopt deterministic patent renewal models. They assume that patents are endowed with a distribution of initial current returns which decay deterministically at a fixed rate thereafter. Therefore, the patent applicants possess perfect information about the future patent returns with certainty, and are able to make their renewal decisions even at the beginning of the patent life, when they decide to whether file the initial patent applications. This is a highly simplified assumption, neglecting the fact that inventors usually apply for patents at an early stage in their innovation process, and therefore will still gradually gather market information and explore alternative opportunities for earning returns after the initial applications. Moreover, both of them utilize only the cross-sectional data on patent applicants' choice of designated countries of protection to evaluate the patent value, but not the time-series observations on the patent renewal decision over the entire

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[1] Lanjouw and Schankerman (2001) examine a sample of 5,452 U.S. patents and find that more valuable patents are considerably more likely to be involved in litigation. This underscores the necessity of taking possible patent infringement and litigation into consideration when estimating the private value of patent protection.

patent life.

This paper extends the previous patent evaluation studies by developing a joint patent application-renewal model as well as incorporating the possibility of infringements and the subsequent litigation costs. The joint examination of the patent application and renewal behaviors may shed light on some puzzling observations of the inventors' filing and renewal patterns. For instance, in studying patentee's behavior under the EPO (European Patent Office) regime, Deng (2001) finds that on average patent applicants in the "pharmaceutical and health" technology group file in more countries for protection than patentees in other technology groups, while the "electronics" patents are filed in the fewest countries. However the "electronics" patent holders tend to pay the renewal fees and keep the patents alive for a longer life than patentees in other technology groups, while the "pharmaceutical and health" patents have the shortest life among all technology groups on average. While this renewal pattern suggests a higher average value of the electronics patents than the pharmaceuticals, the application pattern suggests the opposite. This disparity cannot be identified by either a multi-country patent application model or a renewal model alone, and calls for the examination of a joint application-renewal model. Moreover, a joint application-renewal model enables the utilization of both cross-sectional (multi-country filing) and time-series (patent renewal) dimensions of the available international patent data, and allows one to distinguish more aspects of patent value.

The model is estimated using the EPO patent application and renewal data in the pharmaceutical and electronics industries. Founded in 1977, the European Patent Office (EPO) provides a unified patent application and examination procedure for the member countries. Instead of filing a patent application and going through the tedious examination and granting process in each and every country in which the inventor intends to seek patent protection, an EPO patent applicant only needs to file a single application and, upon paying a per-country designation fee, chooses which countries to designate for future patent protection. Once the application is approved, the patentee can then transfer the EPO patent to the national patent office of the designated countries and enjoy the same patent protection as a national patentee. The patent designation records in the EPO, when combined with the patent renewal records in all of the EPO member countries as well as data on the application and renewal costs and litigation expenses, make it feasible to examine the patentee's joint application-renewal decision-making process.

The outline of this paper is as follows. Section 2 formulates the dynamic stochastic discrete-choice model to analyze the international patent application and renewal behaviors. The EPO patent data set is described in Section 3, along with a summary of some of the characteristics of the patents in the pharmaceutical and electronics industries. In Section 4 a renewal model is estimated, using the EPO patent renewal data in Germany, France and the U.K. Section 5 displays the estimation results of the joint application-renewal model in the pharmaceutical and electronics industries. The Monte Carlo simulation results are also reported in this section. Section 6 concludes.

# 2 The Patent Application-Renewal Model

This section first develops the dynamic stochastic discrete-choice model that is used to analyze a representative inventor's patent application and the subsequent renewal decisions. It then solves for the joint application-renewal decision rules and concludes with a discussion of the properties of the method of simulated moments (SMM) estimator.

The dynamic discrete-choice problem faced by the representative patent applicant is to decide at the beginning of year 1 whether to file a patent application on invention $i$, $i = 1, 2, ..., I$, in the multi-national patent protection regime (here the EPO), and if so, whether to designate the patent protection in each of the $J$ contracting states, given the fact that the future returns to patent protection are uncertain. Once the patent application is granted, at the beginning of each year thereafter the patent holder has to decide whether to pay a renewal fee in each country which will keep the patent in force over the coming period. The inventor is assumed to be aiming at maximizing the expected discounted value of the net returns from his action. He is uncertain about the sequence of returns that will be generated in later periods if the patent is to be kept in force. At the beginning of each year he receives new information about patent returns and makes his renewal decision accordingly.

## 2.1 Model Setup and the Renewal Decision Rule in a Single Country

I start by analyzing the renewal decision faced by the holder of patent $i$ in a single contracting state $j$ in age $t$, conditional on the patentee having filed for patent protection in country $j$ and having paid the renewal fees up to age $t-1$.

If the patent holder decides to pay the renewal fee for age $t$, the net value of patent will be the sum of two parts — the returns $r_{i,j,t}$ to be collected in current age $t$, and the expected value of the option of continuing to renew the patent in the future — minus the renewal cost $c_{j,t}$. If the total benefit from keeping the patent alive is less than the renewal cost, the patent holder will simply choose not to renew the patent and let it permanently lapse, in which case the value of the patent becomes zero forever. Therefore, the value of patent $i$ in country $j$ can be expressed as:

$$V_j(t, r_{i,j,t}) = \max\{0, r_{i,j,t} + \beta E_t V_j(t+1, r_{i,j,t+1}) - c_{j,t}\}, \qquad t = 1, 2, .., T \tag{1}$$

or, by neglecting the subscripts $i$ and $j$,

$$V(t, r_t) = \max\{0, r_t + \beta E_t V(t+1, r_{t+1}) - c_t\}, \qquad t = 1, 2, .., T \tag{1'}$$

where $\beta$ is the discount rate, $T$ is the statutory limit to patent life, and $E_t$ is the expectation operator conditional on the information available up to age $t$. $V(T+1, .) = 0$ because the patent expires after age $T$.

The evolution of the returns of the patent is assumed to follow a stochastic Markov process. Following Pakes (1986) and Lanjouw (1998), I assume that there are three distinct factors governing this process:

First, in each year with probability $(1-\theta)$ the patent is subject to obsolescence.[2] Obsolescence occurs when there is any major technological breakthrough from competitors which makes the current patented technology totally worthless, at least commercially. If this happens the patent holder will naturally choose not to pay the renewal fee from now on and let the patent lapse.

Secondly, even if there is no major technological breakthrough which totally obsoletes the patent, the existence of competing innovations of smaller technological progress will still gradually erode the profitability of the patented

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[2] In Pakes (1986) the probability of obsolescence is assumed to vary with the current return of the patent $r_t$, and therefore is varying over the patent's life. However, Lanjouw (1998) finds from the data that the obsolescence does not have a noticeable trend over age and seems to be constant. Therefore a constant obsolescence probability $(1-\theta)$ is assumed throughout this paper.

technology, and I assume that this will depreciate the return $r_t$ at a constant rate $\delta$ over time.

Finally, to capture the fact that most patent holders constantly collect new information on market and experiment with new commercial strategies to exploit the profits from patent protection over time, I assume that in each year the patent holder draws a random variable $z$ as the patent return generated by the new commercial strategies. Note that new commercial strategies may not necessarily result in a more profitable use of the patent, and if this is the case the current year's patent return will simply be the depreciated return from last year.

In summary, the patent return in age $t$ is given by

$$\begin{aligned} r_t &= \max\{\delta r_{t-1}, z_t\} \qquad \text{with probability } \theta \\ &= 0 \qquad\qquad\qquad\quad\; \text{with probability } 1-\theta \end{aligned} \tag{2}$$

and $z_t$ is assumed to follow a two-parameter exponential distribution:

$$q_t(z_t) = \sigma_t^{-1} \exp\{-(z_t\sigma_t^{-1} + \gamma)\}, \qquad z_t \geq -\gamma\sigma_t \tag{3}$$

where $\gamma \geq 0$ and $\sigma_t = \phi^{t-1}\sigma$ with $0 < \phi \leq 1$.

As noted by Lanjouw (1998), patent holders tend to experiment with the marketing strategies which they believe to be most lucrative first, and accordingly here smaller $\sigma_t$'s are assumed over time to make sure that the probability of uncovering a use which leads to returns greater than a given number declines over the patent life.

A patent grants its holder an exclusive right to utilize the patented technology and gather monopoly profits. However, patents are subject to possible challenges and have to be defended by their owners. As a result the patentee will not be able to receive all of the potential returns with certainty. Lanjouw (1998) recognizes the possibility of patent infringements and analyzes the patentee's willingness to prosecute the infringers and defend his patent.[3] A patentee has strong incentives to defend his patent, because if he chooses not to go to the court or drops the case during the litigation process (which

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[3] In practice there are two kinds of litigation in terms of patent challenges: the infringement suits initiated by the patent holders against the infringers, and revocation suits initiated by patent challengers against patent holders. However they are not distinguished in this paper.

normally takes three years),[4][5] then others may infringe with impunity, and returns to patent protection will become zero. Moreover, if common knowledge is assumed, then the patentee will only renew the patent when he is willing to prosecute the infringers, since if he is not then all the potential competitors will certainly infringe.

Taking patent infringements and litigation into consideration will change the patentees' renewal decision, not only because the expected benefits to the patentees of renewing becomes smaller, but also for the fact that pursuing prosecution incurs litigation expenses, although such expenses may be at least partially compensated if the patentee finally wins the case. Recent survey studies (see for example, Hamburg (2001), Meller (2001)) indicate that in European countries like Germany and Austria litigation expenses are calculated based on the "value-of-the-case" (VOC): the patent courts apply rough estimates when trying to find out what the VOC should be, and the litigation expenses increases approximately linearly in VOC[6]:

$$\begin{aligned}\text{Litigation costs } (LC) &= \alpha_0 + \alpha_1 * VOC \\ &= \alpha_0 + \alpha_1[r_t + \beta E_t V(t+1, r_{t+1})] \qquad (4)\end{aligned}$$

Assume that an infringement suit will take three years before a ruling, and with probability $w$ the patentee wins the case. The value of the patent in age $t$ can then be expressed as

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[4] An alternative to prosecuting the infringers is seeking for settlements outside the court. However, as Lanjouw and Lerner (1998) point out, the patent holders have more to gain from winning the suit than the infringers have to lose. The infringers are unable to adequately compensate the patent holders simply because monopoly prices cannot be sustained in the final goods market with two firms. Moreover, winning a case by the patent holders may generate reputational benefits in threatening the possible infringers in the future. Therefore, patent holders often turn to courts to resolve disputes.

[5] As Lanjouw (1998) notes, patent suits in Germany typically are completed within three years. The estimations on the duration of such cases in other European countries, however, are currently not available. In the later sections of this paper, a three-year duration is assumed in all other EPO member countries.

[6] On the other hand, in other countries like France there is not a clear relationship between the litigation costs and the court-estimated value of infringement cases. In the model estimation in Section 4 and 5 it is then assumed that in these countries the patentees always expect to pay a fixed amount of *minimal* litigation costs, i.e., $\alpha_1$ in equation (4) equals zero.

$$
\begin{aligned}
V(t, r_t) \;\; = \;\; & \max.\{0, [w - \alpha_1(1-w)]\theta^2 r_t + \beta\theta^2[w - \alpha_1(1-w)]E_t VL(t+1, r_{t+1}) \\
& -c_t - \beta\theta c_{t+1} - (\beta\theta)^2 c_{t+2} - \alpha_0(1-w)\} \qquad (5)
\end{aligned}
$$

where $E_t VL(t+1, r_{t+1})$ is the expected value of the future returns given that the patentee is in the second year of litigation process[7], and is defined as

$$
\begin{aligned}
E_t VL(t+1, r_{t+1}) \;\; = \;\; & \int r_{t+1} G_{t+1}(dr_{t+1}|t) + \beta \iint [r_{t+2} + \beta EV(t+3, r_{t+3})] \\
& G_{t+2}(dr_{t+2}|t+1) G_{t+1}(dr_{t+1}|t) \qquad (6)
\end{aligned}
$$

where $G_{t+1}(s|t) = prob(r_{t+1} \leq s|t)$ defines the *c.d.f.* of the Markov process $(r_{t+1}|t)$ described in equations (2) and (3).

Pakes (1986) provides the regularity conditions[8] for the existence of a unique solution to the patent renewal problem and discusses the general form of the solution. Specifically, there exists a threshold minimal return $r_t^*$ for each age of the patent depending on the renewal fee schedule $\{c_t\}_{t=1}^T$, and the representative patentee pays the renewal fee $c_t$ if and only if the current return $r_t$ equals or exceeds the threshold minimal return $r_t^*$: $r_t \geq r_t^*$. Moreover, $r_t^*$ is non-decreasing in $t$, and is implicitly defined by:

$$
r_t^* + \beta E_t V(t+1, r_{t+1}) - c_t = 0 \qquad (7)
$$

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[7] Equation (5) assumes that, during the three years of the litigation, once the patent becomes obsolete, the patentee will stop renewing the patent. This may not be necessarily true, however, since non-renewal automatically leads to abandonment of the law suit, and consequently the patentee forgoes the possibility of finally winning the suit and being compensated for the damages ($r_t$ and maybe $r_{t+1}$, if the obsolescence occurs in period $t+2$). Therefore, during the litigation process the patentee may still choose to renew the patent in age $t+1$ or $t+2$ even if the patent becomes obsolete, and such renewal decisions are contigent on the returns in the early stage of the litigation. It is theorectically straightforward to solve the renewal rule in this case, however, it increases the computational burdens enormously. Following Lanjouw (1998) I assume that out of the patent value in the latter two years of the litigation period, the option of keeping the patent alive is sufficiently important that once obsolescence occurs, the patentee chooses to let the patent lapse since the option value becomes zero.

[8] The regularity conditions are: 1). The renewal fee schedule in every year is nondecreasing in age; 2). There exists an $\varepsilon$ such that $E(r_t^{1+\varepsilon}|r_1) < \infty$; 3). The conditional *c.d.f.* $G_{t+1}(s|r_t)$ is nonincreasing in $r_t$ and nondecreasing in $t$; 4). $G_{t+1}(s|r_t)$ is continuous in $r_t$ at every $s$ except when $G_{t+1}(s|r_t)$ is discontinuous in $s$. These regularity conditions are checked before solving the following renewal problems and the application problems.

for each age $t$ from equation (1), or, in the present model, after taking into account of the possible infringement and the subsequent litigation,

$$[w-\alpha_1(1-w)]\theta^2 r_t^* + \beta\theta^2[w-\alpha_1(1-w)]E_t VL(t+1, r_{t+1}) - c_t$$
$$-\beta\theta c_{t+1} - (\beta\theta)^2 c_{t+2} - \alpha_0(1-w) = 0 \qquad (8)$$

The series of the minimal renewal return $\{r_t^*\}_{t=1}^T$ in this renewal problem could be solved through integrating equation (6) backwards with the terminal condition $V(T+1, r_{T+1}) = 0$. Appendix $A$ gives the details of derivation and the formulae of the solution.

If combined with a specification of distribution of the patent values in an early age, for instance right after the patent is transferred to the national patent offices, the above equations will essentially comprise a pure renewal model, like the ones in Pakes (1986) or Lanjouw (1998). In Section 4, such a model will be estimated using the renewal observations after the EPO patents have been transferred to Germany, France and the United Kingdom. However, to jointly examine the application and the renewal behaviors, the decision rules for initial application and the subsequent transfer processes have to be analyzed.

## 2.2 The Application and Designation Decision Rule

Patent application with the EPO is a two-stage process. The patent applicant has to decide at first whether to file an initial application with EPO, and if so, which EPO contracting countries he would like to designate by paying the corresponding designation fees, in order to keep alive the option of transferring the EPO patent into a national patent in such countries on the later stage. The application then goes through an examination process that usually takes three to four years (Deng 2001). Once the patent application is granted by the EPO, the patentee has to decide whether to pay additional lump-sum transfer fees in each of the designated states and enjoy the patent protection in that state. Therefore, the joint application and designation problem faced by the patent applicant is to

$$\max_{R}. \ \{\sum_{j=1}^{J} 1_{j1}(R^*)[h_j\theta^2 r_{j,1} + \beta\theta^2 h_j E_1(r_{j,2} + \beta r_{j,3} + \beta^2 prob_{gr} V(4, r_{j,4})) - C_j$$

$$-\alpha_{j,0}(1-w_j)] - C_{EPO}, 0\} \tag{9}$$

where $C_j$ is the per-country designation cost, $C_{EPO}$ is the initial application fee due at the EPO, and $h_j = w_j - \alpha_{1,j}(1-w_j)$ is determined by the litigation cost parameters $\alpha_{j,0}$, $\alpha_{j,1}$ and the winning probability $w_j$ in country $j$. $R$ represents the patent applicant's decision rule. For instance, $1_{j1}(R) = 1$ means that he chooses to designate country $j$ at the time of the initial filing. Moreover, I assume here that the official examination is an exogenous process and the final granting decision is out of the applicant's control, and that the patent applicants recognize a constant probability of the application being granted $prob_{gr}$.[9]

The above problem is solved backward. At the beginning of the fourth year the patentee has to decide whether to transfer the granted patent to the national patent office in contracting state $j$, conditional on the patent application having been approved and that country $j$ was designated three years ago. Therefore, the patent value in the current age is:

$$\begin{aligned} V(4, r_{j,4}) &= \max\{0, h_j\theta^2 r_{j,4} + \beta\theta^2 h_j E_4 VL(5, r_{j,5}) - C_{j,4} \\ &\quad -\beta\theta c_{j,5} - (\beta\theta)^2 c_{j,6} - \alpha_{j,0}(1-w_j)\} \end{aligned} \tag{10}$$

where $C_{j,4}$ is the sum of the lump sum transfer fee and the renewal fee due at the national patent office. $E_4 VL(5, r_{j,5})$, the expected value of the future returns in the following two years, has a functional form given in Appendix $A$. Therefore the patentee will pay $C_{j,4}$ if and only if $r_{j,4} \geq r^*_{j,4}$, with $r^*_{j,4}$ solving the following function:

$$h_j\theta^2 r_{j,4} + \beta\theta^2 h_j E_4 VL(5, r_{j,5}) - C_{j,4} - \beta\theta c_{j,5} - (\beta\theta)^2 c_{j,6} - \alpha_{j,0}(1-w_j) = 0 \tag{11}$$

and the patent value in country $j$ in age 1 becomes:

$$V(1, r_{j,1}) = \max\{0, h_j\theta^2 r_{j,1} + \beta\theta^2 h_j E_1(r_{j,2} + \beta r_{j,3} + \beta^2 prob_{gr} V(4, r_{j,4})) - C_j - \alpha_{j,0}(1-w_j)\} \tag{12}$$

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[9] An alternative is to view the patent application and examination in whole as a multi-period bargaining process between the applicant and the examiner, and the probability of grant is thus endogenously determined. The width of the patent claim is a control variable that the applicant can choose: wider claim brings higher expected future returns, but the probability of being granted becomes smaller. In each period the applicant chooses to pay a filing or review fee, makes the claim and bargins with the examiner, or simply abandons his application. However this bargaining process is not the primary focus of the current paper, and instead an exogenously determined constant grant probability is assumed here.

Similarly, there exists a unique minimal designation return $r^*_{j,1}$ above which the patent applicant is going to designate in country $j$, conditional on having filed the patent application at EPO. The specific solution of $r^*_{j,1}$ is derived in Appendix $A$.

Finally, with the conditional designation decision rule $1_{j1}(R)$, the inventor will now be able to decide whether he would like to file the initial patent application at EPO. He will choose to file the application if by doing so the sum of the net returns in all designated countries is enough to cover the large application cost $C_{EPO}$. Otherwise he will not resort to the EPO patent protection regime. The details of the solution are provided in Appendix $A$.

## 2.3 Moment Conditions and Estimation Algorithm

The above two subsections have defined the stochastic Markov process generating the distribution of $\{r_t\}_{t=2}^T$ from the distribution of $r_1$ and solved for the conditional decision rules. What is left to be specified in the model is the initial distributions of the patent returns. Here I assume that the initial return of any patent $i$ in country $j$ is lognormally distributed[10], and

$$r_{ij1} = \exp(\alpha_i + bX_i + \upsilon \log GDP_j + \varepsilon_{ij}) \tag{13}$$

That is, the initial return $r_{ij1}$ is determined by a common (across different country $j$'s) factor $\alpha_i$, a list of patent-specific characteristics $X_i$, the real GDP of the country $j$ (to incorporate the potential market size of the patented innovation in the designated country),[11] and an idiosyncratic (to each country) factor $\varepsilon_{ij}$. Moreover, both the common and idiosyncratic factors follow lognormal distributions:[12]

$$\begin{aligned} \alpha_i &\backsim N(\mu_\alpha, \sigma^2_\alpha) \\ \varepsilon_{ij} &\backsim N(0, \sigma^2_\varepsilon), \quad \textit{i.i.d.} \text{ across country } j\text{'s} \end{aligned} \tag{14}$$

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[10] Similarly, in formulating and estimating the patent renewal model in a single country in Section 4, the patent value right after the patent being granted and transferred is also assumed to follow a lognormal distribution: $log(r_4) \backsim N(\mu_4, \sigma^2_4)$.

[11] Notice that $\upsilon$ is an indicator of returns to scale of the economy, and is *ad hoc* assumed to be 1 in Putnam (1996). However the estimate in Deng (2001) of $\upsilon$ is about 0.5, showing a decreasing returns to scale in general.

[12] Schankerman and Pakes (1986) find that the log-normal distribution fits the renewal data better than any other kind of distribution they have tried.

and $\alpha_i$ and $X_i$ are independent of $\varepsilon_{ij}$: $E(\alpha_i \varepsilon_{ij}) = 0$, $E(X_i \varepsilon_{ij}) = 0$. Condition (14) transforms the original formulation into a random-coefficient model.

Finally, to capture the effects of the market size on the magnitude of the learning process, the *p.d.f.* of the independently learned values in any specific country $j$ in age $t$, $z_{ijt}$, is defined as

$$q_{jt}(z_{ijt}) = \sigma_{jt}^{-1} \exp\{-(z_{ijt}\sigma_{jt}^{-1} + \gamma)\}, \qquad z_{ijt} \geq -\gamma\sigma_{jt}$$

where $\sigma_{jt} = \phi^{t-1}\sigma_j$ with $0 < \phi \leq 1$, and $\sigma_j = (GDP_j)^v \sigma$. In other words, the realizations of $z_{ijt}$ in any specific country $j$ are assumed to be proportional to the scale of the economy, as in defining the distributions of the initial returns in equation (13).

Given the conditional distribution of $r_{t+1}$, $G_{t+1}(s|t) = prob(r_{t+1} \leq s|t)$ and the initial distributions of the patent returns, it is straightforward to derive the *c.d.f.* of $r_{ijt}$. (Note that once the patent lapses there are no returns to the patent protection thereafter):

$$1 - F_j(r, t) = \Pr\{r_{ijt} \geq r, r_{ij,t-1} \geq r^*_{j,t-1}, ..., r_{ij,2} \geq r^*_{j,2}, r_{ij,1} \geq r^*_{j,1}\} \qquad (15)$$

Therefore, in any cohort of patents, the proportion of patent holders who pay the renewal fees at age $t$ is simply the proportion with current return $r_{ijt}$ exceeding the minimal renewal return $r^*_{jt}$, or $1 - F_j(r^*_{jt}, t)$. The proportion of patents lapsing (the hazard rate) at age $t$ in country $j$ is simply the proportion not paying the renewal fee at age $t$ out of those having paid the renewal fee at age $t-1$:

$$\pi_j(t) = [F_j(r^*_{jt}, t) - F_j(r^*_{j,t-1}, t-1)]/[1 - F_j(r^*_{j,t-1}, t-1)], \qquad t = 5, ..., T \quad (16)$$

Similarly, the hazard rate between age one (the initial filing) and age four (when the patents are to be transferred to national office $j$), conditional on the country having been designated when the initial EPO application was filed, is

$$\pi_j(4) = [F_j(r^*_{j4}, 4) - F_j(r^*_{j1}, 1)]/[1 - F_j(r^*_{j1}, 1)] \qquad (17)$$

And finally, the proportion of patents not designated in country $j$ at the time of initial filing is

$$\pi_j(1) = F_j(r^*_{j1}, 1) = \Pr\{r_{ij,1} < r^*_{j,1}\} \qquad (18)$$

Equations (15) to (18) provide the moment conditions required for the estimation. Specifically, I have

$$E[\pi_N(\omega)] = \pi(\omega) \tag{19}$$

where $\pi(\omega)$ is a vector stacking up $\pi_j(t)$, $j = 1, ..., J$, and $t = 1, 4, .., T$ (or $T_L$ since for some cohorts the renewal data is truncated so that final ages are not observed in the data). $\pi_N(\omega)$ is the vector of hazard rates from the sample, where the subscript N denotes the sample size. $\omega$ is a vector consisting of all the parameters.

The model is estimated using a simulated method of moment (SMM) estimator, $\widehat{\omega}_N$, of the true parameter vector $\omega_0$. Specifically, $\widehat{\omega}_N$ is chosen so as to minimize

$$||G_N(\omega)|| = ||\pi_N(\omega) - \widetilde{\pi}_N(\omega)||_{W_N(\omega)} \tag{20}$$

where $\widetilde{\pi}_N(\omega)$ is a vector of simulation estimates of the aggregate hazard rates implied by the parameter $\omega$. $W_N(\omega)$ is a weighting matrix. Normally one would use the sample estimates of the inverse of the asymptotic variances of the moment conditions, or the simulated estimates of it, as in other SMM estimations. However, calculating such a weighting matrix is computationally unfeasible due to the large dimension of the moment conditions, since there are 18 (number of ages) times 10 (number of EPO contracting states) times 6 (number of cohorts) of them. The weighting matrix used in the estimation is thus

$$W_N(\omega) = diag(\sqrt{n/N}), \tag{21}$$

where $n$ is the number of patents still alive in the specific country-cohort-age cell. In other words, the simulated moment conditions are weighted by the sample size in calculating the objective function $||G_N(\omega)||$.

# 3 Data Description

The patent application and renewal data set compiled from information provided by the European Patent Office (EPO) will be used in the following study. I will focus on analyzing the patent cohorts of 1980 through 1985. Years 1978 and 1979 are considered a transition period during which potential patent applicants may not have been aware of the establishment of the new patent-protection regime of the EPO. This may bias the model estimation and thus the applications filed in these two years are excluded from the sample of interest. The patents applications filed after 1986 are also excluded because the maximal age of these patents that is observable in the data set is 12 years. Since the lapse of patents is a gradual process after the patents are granted, the variations in the renewal patterns during early ages may not be significant across different groups of patents. Another advantage of limiting the sample to patents applied during 1980 to 1985 is that the member countries of EPO were unchanged during this period. Having constant member countries during the chosen period for study avoids generating an unbalanced sample, and thus simplifies the analysis.

Section 3.1 summarizes the characteristics of the data used in estimating the renewal model in Germany, France and the U.K., and Section 3.2 illustrates the different filing and renewal patterns in the pharmaceutical and electronics patent groups.

## 3.1 Patent Renewal Pattern in Germany, France and the U.K.

Table 1 summarizes the characteristics of the data used in estimating the renewal model in Germany, France and the U.K. Row 1 and row 2 report the time of filing and transfer of patents. The renewal model uses data on all of the patents in cohorts 1980 to 1985 that are transferred to any of the three countries of interest across all technology groups, a total of 120,768 patents. The statutory limit of patent life is 20 years in all the three countries during

**Table 1: Characteristics of the Data in the Renewal Model Estimations**

| | Germany | France | U.K. |
|---|---|---|---|
| Application dates at the EPO | 1980-85 | 1980-85 | 1980-85 |
| Years of renewal | 1984-1996 | 1984-1996 | 1984-1996 |
| Number of cohort-age cells | 63 | 42 | 63 |
| Number of granted patents (out of 120, 768 patents) | 113,053 | 108,587 | 110,651 |

Note: Table 1 describes the characteristics of the data used in estimating the patent renewal model in Germany, France and the U.K. The renewal records before 1990 are incomplete for France, and therefore are excluded from the estimation sample.

the sample period. Hence the sample contains patent renewal information in most years of their lives (for cohort 1980, 18 years, and for cohort 1985, 13 years). Consequently, there are 63 cohort-age cells in each of the German and U.K. sample (the sum of number of observed renewal ages since age 4 in all cohorts), as shown in row 3. However, the patent renewal records in France before 1990 are incomplete and are excluded from the estimation sample. As a result, there are only 42 cohort-age cells in the French sample. Row 4 of the table reports the number of patents transferred to each of the three countries. There is a high proportion of overlap among patents transferred to different countries, *i.e.*, most of the EPO patents studied in this section were designated and transferred to combinations of these three countries.

**Figure 1: Average Hazard Rates in Germany, France and the United Kingdom**

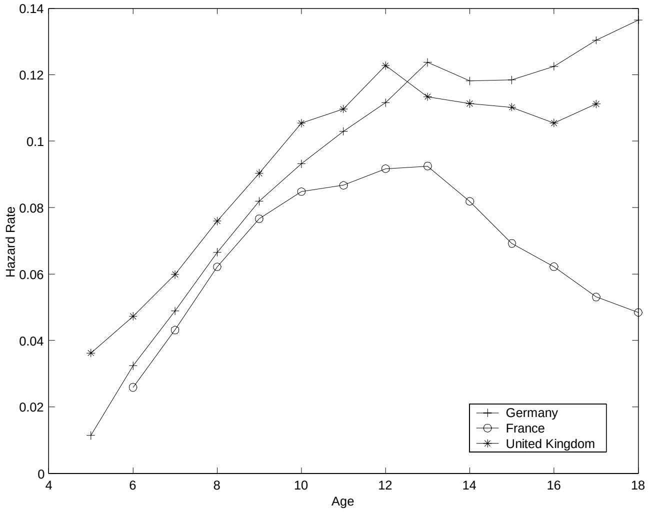
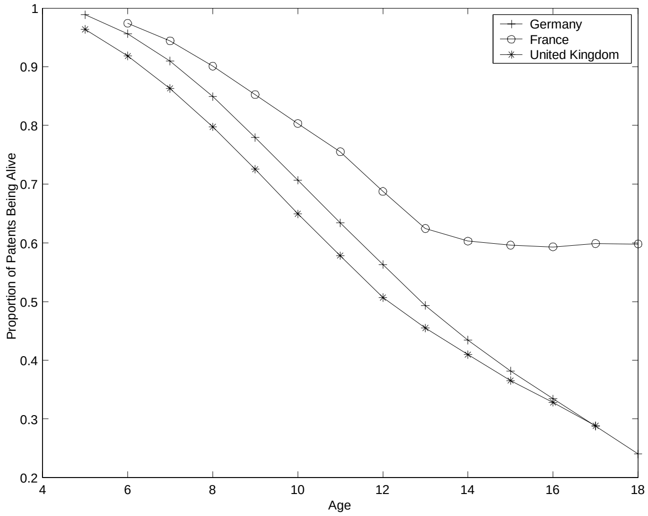


Figure 1 provides the average hazard rate at each age in each of Germany, France and U.K., weighted over cohorts by the number of patents in each cohort. The hazard rate in age $t$ is defined as the proportion of patents dropped at age $t$ out of the patents alive up to age $t-1$. The estimation procedure compares these hazard rates to those implied by different values of the model's parameter vector, and finds the set of parameters that minimizes their difference. Figure 2 displays the associated patent renewal rate at each age, averaged over different cohorts in each country. Figure 3 plots the average renewal fee schedule in each of the three countries. The renewal fee schedules were obtained originally in nominal domestic currency, converted to real domestic currency using the country's own GDP price deflator, and then converted to 1997 U.S. dollar values using the official exchange rate in 1997. All monetary values are therefore in 1997 U.S. dollars. From Figure 1 a distinct difference in the renewal patterns can be seen across countries. Compared with the U.K., the hazard rate in Germany is substantially lower at earlier ages and higher at later ages. The hazard rate in France is consistently lower in all ages than that in Germany and the U.K. Correspondingly, the patent renewal rate in Germany is larger than that in the U.K. at earlier ages, and the renewal rate in France is consistently the highest at all ages.

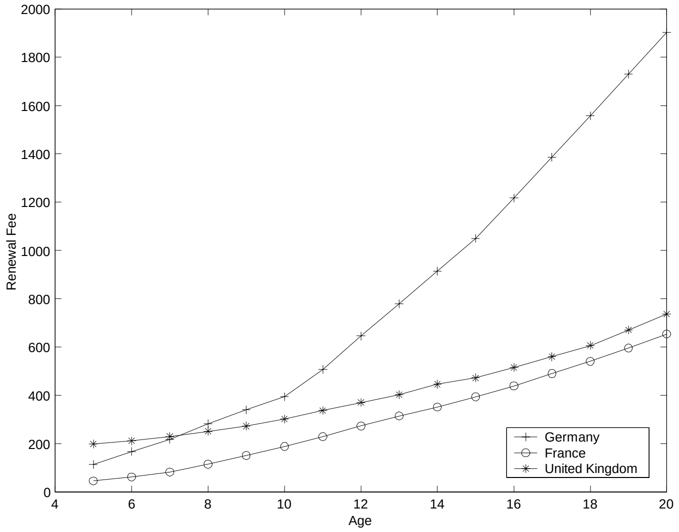
**Figure 2: Average Renewal Rates in Germany, France and the United Kingdom**



As noted earlier, there is high proportion of overlap among the patents in different countries, therefore the quality of patents should be similar across countries. Consequently, the heterogeneity of the intrinsic quality of these patents can only have limited effects on the different renewal patterns observed. There should be other explanations for the variations in the renewal pattern across different countries.

One possible explanation could be the different renewal costs across countries. As Figure 3 reveals, the renewal costs in France are the lowest at all ages among these three countries, which helps explain the high proportions of patent renewal in France. Compared with the U.K., the renewal fees in Germany are lower at earlier ages, but increase at a much faster pace at subsequent ages. This should, *ceteris paribus*, generate lower hazard rates at earlier ages and higher ones at the later ages in Germany than in the U.K., and we see this in Figure 1.

**Figure 3: Average Renewal Fee Schedules in Germany, France and the United Kingdom**



Difference in the strength of patent protection across countries may also contribute to the difference in the renewal patterns. For one thing, a patentee may form different expectations of the probability of winning a infringement suit in different countries, and this is closely related to institutional details such as different judicial systems in each country. Litigation cost is another factor that should be taken into account. A survey performed by Bouju (1987) demonstrates that the litigation costs of patent infringement cases vary substantially in European countries. In France, the litigation costs associated with the patentees in infringement cases[13] before the court of first

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[13] The figures on litigation costs obtained from Bouju (1987) are only the minimal costs in patent infringement cases concerning the plaintiff, i.e., the patentee or his successor,

instance[14] would be about \$33,515 in 1997 U.S. dollars. However, in U.K., such costs would be \$164,768, nearly four times larger than in France. The litigation costs in Germany are the lowest among the three countries. The average costs for a case valued \$68,400 would be \$6,525, for a case valued \$222,300 would be \$14,095, and even for a case of value of \$684,020 would be \$30,074, significantly lower than similar costs in France.

## 3.2 Designation and Renewal Pattern of Pharmaceutical and Electronics Patents Under the EPO Regime

Table 2 reports the number of patent applications and grant rates of the pharmaceutical and electronics patents[15] used in the joint filing-renewal model estimation. In the original data set some patent applications are assigned with more than one IPC codes and consequently are categorized to both technology groups. In order to avoid repetitive counting, all of the patent applications carrying both pharmaceutical and electronics IPC codes are deleted from the sample used in the joint filing-renewal model estimation. This reduces the total sample size from 69,407 to 69,077, or by 0.48%.

The records on initial designation and subsequent transfer of patent ap-

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assignee or any other party entitled to sue for infringement, hereinafter called "the patentee". The nominal amounts include 1) "the fees to be paid to attorneys and other agents authorized to deal with the court"; and 2) "the legal costs, if any, to be paid to courts". However, They do not take into account: 3) "the fees paid to attorneys and consultants for studying the case and checking the validity of the patent and the reality of the infringement before the suit is initiated", or 4) "the costs to be paid by the patentee if he loses the suit, especially the damages awarded to the defendant"; or 5) "the costs incurred by the defendant during the suit to build up the defence, not the damages to be paid by the defendant if he is declared infringer and loses the suit".

[14] The court of first instance refers to the Tribunal of First Instance (TGI) in France, High Court in United Kingdom, or District Court in Germany. Consequently, the litigation costs discussed here only include the costs incurred during proceedings in the court of original jurisdiction. They do not include the costs of going through the court of appeal or the supreme court.

[15] Following Deng (2001), the EPO patent applications are categorized into different technology groups according to their International Patent Classification (IPC) codes. In particular, in this study the "pharmaceutical" patent group includes the patent applications in the following sub-IPC groups: medical or veterinary science; hygiene (IPC code A61) and preservation of bodies of humans or animals or plants or parts thereof; biocides; pest repellants or attractants; plant growth regulators (IPC code A01N). The "electronics" patent group includes both the electronics instruments and electricity.

plications are complete in all EPO member countries. However, the records on patent renewal are incomplete in some countries. In Italy and Luxembourg, such records are not available throughout the whole sample period. In France, such records are incomplete before 1990. In these cases, the patent renewal rates in the associated cohort-age-country cells cannot be calculated and are not included in the model estimation. On the other hand, due to possible data missing in the last year of the data compiling, which is 1997, only the renewal records up to 1996 are used in the estimation.

It can be seen from Table 2 that the electronics patent group is significantly larger than the pharmaceutical group, with a total number of applications about 3 to 4 times larger in all cohorts. During 1980 to 1985, 12,334 pharmaceutical patent applications were filed with the EPO, while 56,743 application were filed by electronics inventors. The patent grant rates, on the other hand, are not very far apart in these two technology groups. For instance, 73% of the pharmaceutical patent applications in cohort 1981 are finally approved, and it is 71% for cohort 1983 and 64% for cohort 1985. In the electronics group, 71% of the patent applications in cohort 1981 are finally granted, and it is 72% for cohort 1983 and 69% for cohorts 1985. When averaged over all cohorts, the grant rate for pharmaceutical patents is 69%, and is 71% for electronics patents. In model estimation the patent applicant is assumed to form an expectation of the grant probability which is set to be

**Table 2: Number of Patent Applications and Grant Rates of Pharmaceutical and Electronics Patents**

| | Pharmaceutical | | Electronics | |
|---|---|---|---|---|
| | Number | Grant Rate | Number | Grant Rate |
| 1980 | 1,400 | 74.14% | 6,057 | 71.97% |
| 1981 | 1,710 | 73.45% | 7,472 | 71.35% |
| 1982 | 1,860 | 71.08% | 8,879 | 71.54% |
| 1983 | 2,158 | 70.71% | 9,925 | 71.88% |
| 1984 | 2,444 | 64.81% | 11,698 | 70.50% |
| 1985 | 2,762 | 63.90% | 12,712 | 68.56% |
| 1980-85 | 12,334 | 68.84% | 56,743 | 70.74% |
| Number of cohort-age-country cells | | 583 | | 583 |

Note: Table 2 reports the number of patent applications and grant rates of the pharmaceutical and electronics patents, as used in the joint filing-renewal model estimation.

the average approval rate of the technology group his application belongs to, as shown on row 9 of Table 2.

Figure 4 shows the designation rate of the EPO patent applicants across different technology groups in each of the 10 EPO member countries. On average the pharmaceutical patent applicants designate more countries for patent protection than electronics patent applicants. For instance, 90% of the pharmaceutical patent applicants choose to designate Switzerland, while only 42% of the electronics patent applicants do. Belgium is designated by 84% of the pharmaceutical patent applicants and only 36% of the electronics patent applicants. However, the designation rates in countries with larger market size, for example Germany, France, and the U.K., are similar.

Figure 5 summarizes the above designation pattern in two different ways. The first two bars display the simple counts of EPO member countries that a typical EPO patent applicant chooses to designate, averaged over cohorts 1980 to 1985. On average pharmaceutical patent applicants choose to designate 8.4 out of the 10 EPO member countries, while electronics patent applicants designate an average of 5.6 countries.

The number of designated countries can be a measure of how valuable a patent is to its inventor. The patent applicant compares the net value of his invention with the designation costs, and chooses to designate countries wherever the net value exceeds the designation costs. However, as pointed out in the first chapter, gauging the private value of patents by simply counting the number of designated countries can be misleading. The revenue a patentee expects to gather from a large economy is presumably higher than that from a small economy. To account for heterogeneity in the size of economy among the EPO member countries, I weight the simple counts of designated countries by the average real GDP ratio for these countries during 1980 to 2000. Again pharmaceutical patent applications have higher weighted number of designated countries than the electronics patent applications, by a margin of about 20%. Therefore, a patent evaluation model utilizing cross-sectional data on the size of patent family, such as Putnam (1997) or the structural model formulated in the first chapter of this dissertation, would conclude that the expected value of an average pharmaceutical patent is higher than an average electronics patent, because the former is designated in more countries (both in simple and weighted counts) than the latter.

However, examining the time series data on patent renewal provides further insight into the value of patent protection. Figure 6 displays the patent renewal rate of the two technology groups in Germany, averaged over cohorts

**Figure 4: Designation Rates of Pharmaceutical and Electronics Patents in the EPO Member Countries**

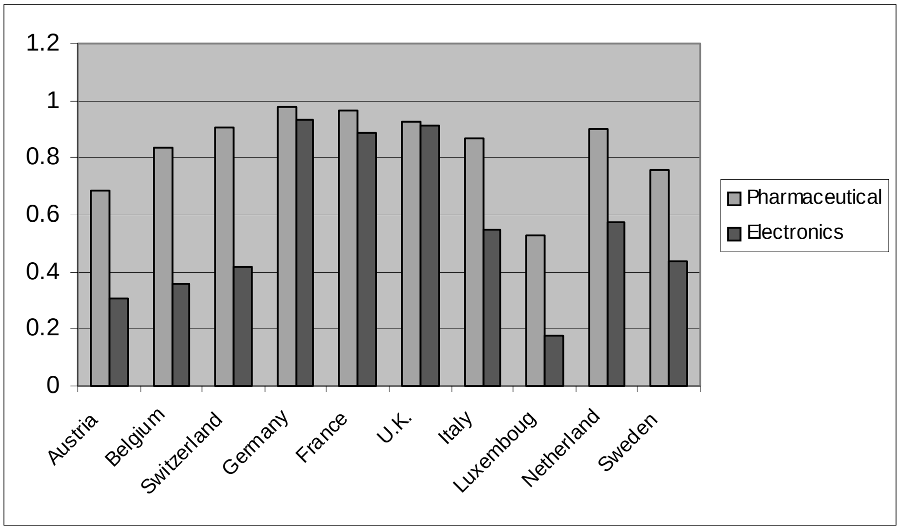


**Figure 5: Average Number of Designated Countries by Pharmaceutical and Electronics Patent Applications**

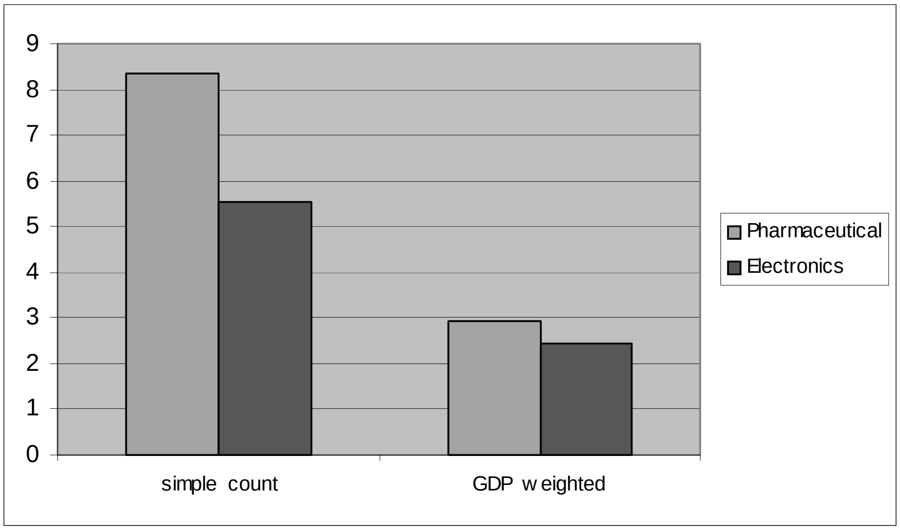


Figure 6: Average Renewal Rate in Germany: Pharmaceutical and Electronics Industries

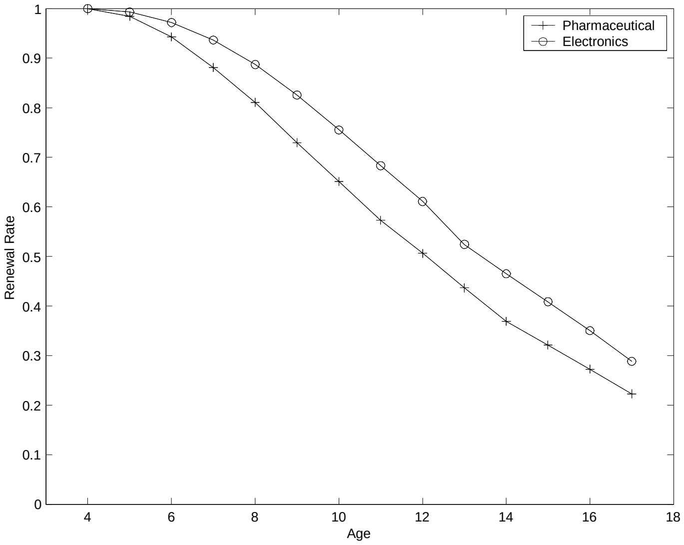
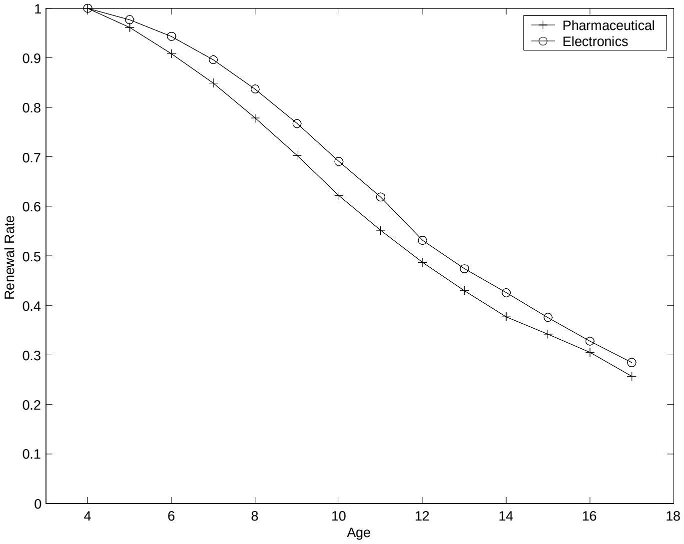
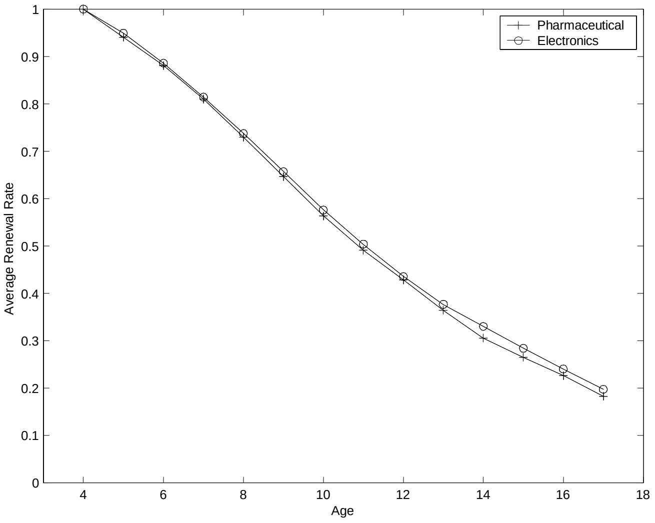


Figure 7: Average Renewal Rate in the U.K: Pharmaceutical and Electronics Industries



1980 through 1985, and Figure 7 shows the average renewal rate in the U.K. In both figures the line describing the renewal rate of pharmaceutical patents lies below the line of electronics patents at all ages. For instance, of all the pharmaceutical patents transferred to Germany, only 73% are still alive by the end of age 9, while it is 83% for the electronics patents. 22% of the pharmaceutical patents live up to age 17, the latest age observed in the sample, whereas 29% of the electronics patents live up to this age. In the U.K., 76% of the electronics patents live up to age 9 and 29% live up to age 17, while it is 70% and 25% for the pharmaceutical patents, respectively. Given these facts, a patent renewal model estimation would suggest a higher average value for the electronics patents than pharmarceutical patents in Germany and the U.K., because the owners of the electronics patents are willing to keep them alive for a longer period of time and pay higher renewal costs.

**Figure 8: Average Renewal Rate in the EPO Member Countries: Pharmaceutical and Electronics Patents in the Sample**



The average renewal rate of the two technology groups at each age, weighted across all cohorts and countries (by the number of patents alive through that age), is displayed in Figure 8. As it shows, the average renewal rates are close at earlier ages for the two technology groups. But at later ages, the average renewal rate of electronics patents is higher than pharmaceutical patents. By the end of age 17, only 18% of the pharmaceutical patents are still alive, while more than 20% of the electronics patents live up to this age.

# 4 Patent Renewal Model Estimation

Table 3 reports the parameter estimates of the renewal model for each of Germany, France and the U.K. To alleviate the dimensionality problem in the numerical optimizations, all the estimations are performed conditional on setting the real discount factor $\beta$ equal to 0.95. Lanjouw (1998) made the same assumption, and the structural model estimation in the first chapter of this dissertation reports a $\beta$ estimate of similar magnitude.

The parameter estimates in three countries are all positive and highly significant. The Mean Square Error (MSE), constructed as the sum of squared residuals divided by the number of cohort-age cells, is reported on row C1 of the table. By comparing the MSE to the variances in the actual hazard rates across different cohort-age cells as reported in row C2, it can be seen that the model-implied hazard rates seem to fit the German data quite well.[16] However, in the French sample the renewal behavior in earlier ages was not observed. As a result there are not enough variations in the renewal pattern, which may explain why the model's performance is less satisfactory.

The estimated probabilities of a patentee winning an infringement suit in all three countries are fairly high, ranging from 90% in Germany to 98% in France and the U.K. However, it should be noted that these estimates cannot be directly interpreted as the winning probabilities *once* an infringement suit actually occurs in these countries. As noted in Section 2, these probability estimates are based on the assumption of common knowledge. If this assumption is relaxed, then the patentee will recognize that infringements may not necessarily occur even if he chooses not to defend his patent. In such cases, the estimated $w$ could be essentially interpreted as a composite

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[16] The reported variance in the actual hazard rates can be viewed as the MSE of a "naive" model which predicts that in all cohort-age cells the hazard rates would be constant and is identical to the average hazard rate. Therefore, the differences between the variance of the actual hazard rates and the MSE implied by the current model estimation can serve as a measure of the improvement of the model performance over such a "naive" model.

probability, consisting of the winning probability once an infringement occurs plus the probability of the patent not being infringed. Therefore the winning probability once an infringement suit actually occurs would be lower than the estimated $w$ in Table 3.

**Table 3: Patent Renewal Model Estimation Results**

| | Country | | | | | |
|---|---|---|---|---|---|---|
| | Germany | | France | | U.K. | |
| A. Parameter[a] | | | | | | |
| $\theta$ | 0.9490 | (0.1395) | 0.9741 | (0.0212) | 0.9462 | (0.0413) |
| $\delta$ | 0.9233 | (0.0854) | 0.9245 | (0.2014) | 0.8967 | (0.0534) |
| $\sigma$ | 9,980.0 | (374.0) | 4,929.8 | (754.3) | 6,999.1 | (397.5) |
| $\phi$ | 0.5994 | (0.0524) | 0.6200 | (0.1352) | 0.5969 | (0.0504) |
| $\gamma$ | 0.1496 | (0.0519) | 0.4135 | (0.0902) | 0.1992 | (0.0715) |
| $w$ | 0.8991 | (0.1318) | 0.9704 | (0.2116) | 0.9777 | (0.0578) |
| $\mu_4$ | 8.8368 | (0.4695) | 7.9434 | (0.7324) | 8.0908 | (0.8103) |
| $\sigma_4$ | 1.4550 | (0.4173) | 1.9589 | (0.4265) | 1.8984 | (0.6273) |
| B. Size of | | | | | | |
| B1. Sample | 113,053 | | 108,587 | | 110,651 | |
| B2. Simulation | 226,106 | | 217,174 | | 221,302 | |
| B3. Cohort-Age Cells | 63 | | 42 | | 63 | |
| C. Summary Statistics[b] | | | | | | |
| C1.MSE($\widetilde{\pi}$) | $4.56\times10^{-4}$ | | $3.88\times10^{-4}$ | | $5.31\times10^{-4}$ | |
| C2. V($\pi$) | $1.5\times10^{-3}$ | | $4.40\times10^{-4}$ | | $9.97\times10^{-4}$ | |
| C3. MSE($\widetilde{\pi}$)/V($\pi$) | 0.3040 | | 0.8818 | | 0.5326 | |

a. Estimated standard errors are reported in parentheses.

b. MSE is calculated as the sum of squared residuals weighted by the number of patents in each cohort-age-country cell. V($\pi$) is the sample variance from the data.

The estimates of the decay parameters $\delta$ and $\theta$ do not vary much among the three countries. Annual depreciation in returns is fairly low in France (7.5%) and Germany (7.7%), and the highest in the U.K. (10.3%). On the other hand, about 2.6% to 5.4% of the patents become obsolete each year in all three countries. This alone means that over 73% of patents die simply due to obsolescence by the end of age 13 in France, and 53% in Germany. In an estimation of a similar model using industry-level data in Germany from

1953 to 1988, Lanjouw (1998) reports even higher annual obsolescence probabilities, ranging from 7% to 12%. Pakes (1986) has a different specification of obsolescence process, therefore his estimates are not directly comparable.

Parameter $\sigma$, $\phi$ and $\gamma$ together define the exponentially distributed stochastic learning process $z_t$. In particular, other things being equal, a high $\sigma$ implies that the probability of the patent becomes more valuable is high. A low $\phi$ means that the potential learning opportunities recede quickly over time, and a high $\gamma$ decreases the probability of the newly learned returns being greater than a given value.

The dynamics of learning processes implied by these estimates are displayed in Table 4, which reports the results of a simulation run of 100,000 draws based on the average fee schedule across different cohorts in each country, as well as the parameter estimates reported in Table 3. Column 2 displays the learning probabilities in Germany. At the beginning of age 5, about 10% of the patent holders discover a use which generates higher subsequent profits than known before. At the beginning of age 6, however, much less learning occurred, only about 4.5% discover more profitable ways to utilize the patented idea. It should be noted that the reduction in learning comes not only from a smaller $\sigma_t$ of the learning process, but also from the fact that the increase in patent value at age 5 due to learning makes it more difficult to draw a new $z_6$ exceeding the existing value. The learning probability continues to decline over the ages, and in age 10 the probability of learning has dropped to a mere 0.05%, indicating that the learning process is almost

**Table 4: Percentage of Patents Learning a Higher Value**

| | Country | | |
|---|---|---|---|
| | Germany | France | U.K. |
| Pr. ($z_5 > \delta r_4$) | 10.24 | 14.01 | 17.19 |
| Pr. ($z_6 > \delta r_5$) | 4.51 | 8.56 | 13.58 |
| Pr. ($z_7 > \delta r_6$) | 1.86 | 4.63 | 6.64 |
| Pr. ($z_8 > \delta r_7$) | 0.71 | 2.35 | 3.15 |
| Pr. ($z_9 > \delta r_8$) | 0.20 | 0.99 | 1.17 |
| Pr. ($z_{10} > \delta r_9$) | 0.05 | 0.38 | 0.40 |
| Pr. ($z_{11} > \delta r_{10}$) | 0.01 | 0.15 | 0.01 |

Note: Table 4 reports the learning probabilities from a simulation run of 100,000 draws of patents in Germany, France and U.K., based on the average fee schedule across different cohorts in each country, as well as the parameter estimates reported in Table 3.

over by then, and the obsolescence process starts to dominate the renewal decisions. The situation in France and the U.K. is similar to that in Germany. In France, starting with a learning probability of 14% in age 5, this probability steadily decreases with time and by age 11 only 0.15% of the simulated patents successfully increase their value through learning. In the U.K., 17% of the patentees find more profitable ways to exploit their patented idea in age 5, 14% in age 6, 7% in age 7, and only 0.01% by age 11. Therefore, in all three countries, most of the learning activities occur in the early ages, and by age 10 or 11 the learning probability already becomes negligible.

With a sample of German and French patents in the 1950s to 1970s, Pakes (1986) estimates that the learning process is essentially over by the age of 5. Lanjouw (1998) shows that the learning stops by age 6 or 7 in all technology groups in her sample of German patents in 1953 to 1988. By contrast, my estimates imply a longer learning process during the life of EPO patents. As indicated by Table 4, even at age 7, about 2% EPO patentees in Germany, 5% in France and 7% in the U.K. still discover new ways to increase the profits from utilizing the patents, and the learning probability does not become zero until age 10 or 11 in these countries. This suggests that the sample of EPO patents analyzed in this study have very different characteristics from patents studied in previous literature. The quality and the private value of the EPO patents are significantly higher than those of the national patents studied in Pakes (1986) and Lanjouw (1998), and the higher expected value makes it worthwhile for the patentee to invest more resources on finding new commercialization strategies in order to exploit the patented idea.

The above finding may be explained by a close look at the EPO application fee schedule. Compared with the application fees at the national patent offices, the relatively higher EPO application cost prohibits the patents with lower private values from initiating EPO applications from the start. As a result, the EPO patents are on average more valuable and would be justifiably considered "elite patents". Pakes (1986) provides an estimation of net values of the simulated patents in Germany, France and the U.K. (see columns 3, 5 and 7 in Table 6 below). In his simulated French sample, 25% of the patents have a net value of \$119 (in 1997 U.S. dollars, same below) or less, 50% have a net value of \$844 or less, and 75% have a net value of \$5,896 or less. In other words, even at the top 25% percentile of this French sample, the net value of the patent is still not enough to cover the initial EPO application and examination cost of 8,660 DM in 1985 (or \$6,794 in 1997 U.S. dollars).

His simulated U.K. and German patents have higher values than the French ones on average, with a 50% percentile of \$2,397 and \$9,880, respectively. Nevertheless, one may expect that the initial EPO application and examination cost excludes a considerable proportion of Pakes' simulated sample from entering the EPO sample.

Table 5 reports the percentiles and Lorenz curve coefficients from the distribution of realized patent values from the simulation. The realized patent value is defined as the discounted sum of patent returns after the patent is granted, net of all kinds of administrative expenses including the transfer expenses at age 4 and the annual renewal fees in the subsequent years, but excluding any litigation costs. Column 2 of the table shows that the distribution of the realized patent value in Germany is quite skewed. For instance, 25% of German patents have a realized value of \$9,592 or less, while they contribute about 1.25% of the total value of all simulated German patents. The lower 50% of the distribution contributes about 6% of the total value of the simulated German patents, and the lower 90% of simulated patents only accounts for 41% of the total value. On the other hand, the top 1% most

**Table 5: Distribution of Realized Patent Values in Germany, France and the United Kingdom**

| | Country | | | | | |
|---|---|---|---|---|---|---|
| | Germany | | France | | U.K. | |
| Percentile | Value | LC | Value | LC | Value | LC |
| 25% | 9,592 | 1.25 | 3,202 | 0.24 | 3,331 | 0.36 |
| 50% | 27,657 | 6.04 | 12,376 | 2.04 | 11,682 | 2.45 |
| 75% | 80,883 | 19.53 | 51,710 | 8.97 | 46,188 | 9.95 |
| 85% | 141,123 | 31.38 | 107,612 | 16.73 | 94,628 | 18.12 |
| 90% | 205,265 | 40.78 | 176,898 | 23.84 | 153,401 | 25.51 |
| 95% | 355,512 | 55.54 | 362,609 | 36.80 | 310,911 | 38.79 |
| 98% | 651,442 | 71.14 | 816,066 | 53.21 | 677,268 | 55.36 |
| 99% | 967,509 | 79.86 | 1,370,912 | 63.97 | 1,129,647 | 66.07 |
| maximum | 2,283,163 | —— | 87,450,357 | —— | 70,028,060 | —— |
| mean | 90,221 | —— | 96,768 | —— | 81,351 | —— |

Note: Columns 2, 4 and 6 report the percentiles of the distribution of realized patent values from a simulation of 100,000 draws in each country. columns 3, 5 and 7 report the Lorenz curve coefficients of the simulated distribution. Monetary values are in units of 1997 U.S. dollars, and Lorenz curve coefficients (LC) are in percentage points.

valuable patents, with a minimal value of \$967,509, accounts for about 20% of the total value. The value distribution in France, as reported in column 4, is even more skewed. The lower 90% of simulated patent only accounts for 24% of the total value of all simulated French patents, and the top 1% accounts for about 36% of the total value, with a minimal value of \$1,370,912. The distribution in the U.K., as shown in columns 6 and 7, is less skewed than in France, but significantly more skewed than in Germany.

Table 6 compares the simulated values obtained by this study with those obtained by Pakes (1986). Columns 2, 4 and 6 are taken from Table 5 above, and columns 3, 5 and 7 are taken from Table V of Pakes (1986). It should be noted that the monetary values in Pakes (1986) are expressed in units of 1980 U.S. dollars. For convenience of comparison I convert them into 1997 U.S. dollar values by multiplying them with the ratio of GDP price deflators. As expected, the patents simulated in this study are more valuable than those in Pakes (1986). In particular, the median value of the simulated EPO patents in Germany, \$27,657, is about three times as large as that of German patents obtained by Pakes (\$9,880). A comparison of the values of columns 2 and 3

**Table 6: Comparison of Simulated Patent Values**

| | Country | | | | | |
|---|---|---|---|---|---|---|
| | Germany | | France | | U.K. | |
| Percentile | EPO | Pakes | EPO | Pakes | EPO | Pakes |
| 25% | 9,592 | 3,160 | 3,202 | 119 | 3,331 | 562 |
| 50% | 27,657 | 9,880 | 12,376 | 844 | 11,682 | 2,397 |
| 75% | 80,883 | 30,932 | 51,710 | 5,896 | 46,188 | 12,558 |
| 85% | 141,123 | 51,238 | 107,612 | 16,262 | 94,628 | 24,265 |
| 90% | 205,265 | 69,905 | 176,898 | 27,529 | 153,401 | 35,087 |
| 95% | 355,512 | 103,890 | 362,609 | 49,945 | 310,911 | 54,891 |
| 98% | 651,442 | 149,860 | 816,066 | 80,924 | 677,268 | 81,021 |
| 99% | 967,509 | 187,010 | 1,370,912 | 105,100 | 1,129,647 | 102,820 |
| maximum | 2,283,163 | 662,390 | 87,450,357 | 410,540 | 70,028,060 | 590,965 |
| mean | 90,221 | 25,549 | 96,768 | 8,897 | 81,351 | 11,625 |

Note: Table 6 compares the simulated values obtained by this study with those obtained by Pakes (1986). Columns 2, 4 and 6 are taken from Table 5 above, and columns 3, 5 and 7 are taken from Table V of Pakes (1986). All monetary values are in units of 1997 U.S. dollars.

on all percentile levels shows similar results. This indicates a significant difference in the average quality of the two patent groups.

This difference comes from several sources. First, as discussed above, the EPO route of applying for patent protection is more cost-effective to patents intending to file in multiple countries and of higher quality. Secondly, Pakes (1986) studies patents of cohorts 1950 to 1972, whereas this study focuses on cohorts 1980 to 1985. Therefore, the EPO patent sample is on average 15 to 20 years younger. The difference in patent value may reflect a general trend of increasing patent value over time. Growth in the scale of the economy, as well as improvement in the business environment over this period, for instance, may enable the patentees to better exploit the patented idea and obtain higher profits even with unchanged patent quality. Moreover, advance in science and technology in general increases the average quality of inventions over time.

Columns 4 through 7 reveal that the differences between the average value of the EPO patents and that of the national patents are even larger in France and the U.K. While this could be interpreted as a reflection of higher average quality and value in the EPO patent group, it should be kept in mind that the French and U.K. patent sample in Pakes (1986) includes both the applications finally granted and those declined, while his German patent sample as well as the EPO patents analyzed in this section are only the granted ones. Therefore, the difference between the average value of the EPO patents and those of the French and the U.K. patents in Pakes (1986) also reflects the difference between the quality of the granted patent group and that of the declined group

# 5 Joint Filing-Renewal Model Estimation

Table 7 reports the parameter estimates of the multi-country patent filing-renewal model for the pharmaceutical and electronics patent groups. As in the renewal model estimation in Section 4, the real discount rate $\beta$ is assumed to be 0.95. Meanwhile, the probability of the patentee winning an infringement suit is assumed to be fixed at 0.95 in all countries. While it sounds appealing to estimate the winning probability in each of the 10 EPO member countries and thus reveal more details of patent litigation system in these countries, such practice would add 10 more parameters to the model estimation which greatly increase the computational burden.

The model estimation fits the data reasonably well. The weighted Mean Square Error (MSE), constructed as the sum of squared residuals weighted by the number of patents in each cohort-age-country cell and divided by the total number of cohort-age-country cells, is reported in row C1. As noted in Section 4, the difference between the variances of the actual hazard rates and the MSE implied by the model estimation can be interpreted as a measure of improvement of the model performance over a "naive" model which predicts a constant hazard rate among all cohort-age-country cells. As shown in rows C3, the joint filing-renewal model improves the data fitting by about 46% in fitting the designation and renewal pattern of the pharmaceutical patents and about 39% in fitting that of the electronics patents. I then decompose the total weighted MSE into two parts, one in matching the designation rates and one in matching the renewal rates, as reported in rows C4 and C5 of Table 7, in order to separately examine the model's performance in fitting designation and renewal patterns. By comparing them with the variances of the corresponding actual designation and renewal rates in the sample, I conclude that the estimated model performs well in both dimensions. It improves over the "naive" model in fitting the designation rates by 53% and 50% in the pharmaceutical and electronics patent groups respectively, and in fitting the renewal rates by 26% and 15% in these two groups respectively.

The model parameter estimates are all positive and highly significant. The estimates of the annual obsolescence rate $\theta$ are close in the two patent groups: each year about 5% of patents become obsolete in both industries. This means that over 40% of patents die simply due to obsolescence by the end of age 10, and over 55% die by the end of age 15. These estimates of the obsolescence rate are also close to the estimates of 4.9% to 6.4% obtained in estimating the renewal model using patent renewal data in Germany, France and the U.K., as reported in Table 3.

The estimates of the deterministic depreciation rate $\delta$, however, are much different for the two patent groups. If there is neither obsolescence observed nor new values learned, the expected value of pharmaceutical patents would depreciate at an annual rate of 13%, much faster than the electronics patents (5%). That is to say, other things being equal, pharmaceutical patents tend to have a shorter life than electronics patents, since at later ages the pharmaceutical patentees are more likely to find that the depreciated patent value is not enough to cover the increasing annual renewal fee, and choose to let the patent lapse.

On the other hand, the estimate of the parameter $\sigma$ for the pharmaceutical patents, which characterizes the stochastic learning processes, is 10,814, significantly higher than that of the electronics patents (4,519). For patent of a given value, a large $\sigma$ implies that the probability of the patent becoming more valuable through learning is high. Therefore, while deterministically the pharmaceutical patents depreciate faster, stochastically they benefit from a more fruitful learning process, which may boost their expected values as time goes by.

The comparison of renewal dynamics between these two industries are further complicated when the decay rate of $\sigma_t$, $\phi$, is taken into account. Recall that the parameter $\sigma_t$ of the exponential distribution that characterizes the

**Table 7: Joint Patent Filing-Renewal Model Estimation Resluts**

| | Pharmaceutical | | Electronics | |
|---|---|---|---|---|
| A. Parameter[a] | | | | |
| $\theta$ | 0.9498 | (0.0224) | 0.9523 | (0.0361) |
| $\delta$ | 0.8651 | (0.0304) | 0.9457 | (0.0212) |
| $\sigma$ | 10,814 | (408.77) | 4,519 | (219.28) |
| $\phi$ | 0.5584 | (0.0212) | 0.6977 | (0.0220) |
| $\gamma$ | 0.4749 | (0.0231) | 0.4421 | (0.0198) |
| $\upsilon$ | 0.9759 | (0.0965) | 1.3880 | (0.1084) |
| $\mu_\alpha$ | 10.9755 | (0.9227) | 9.7903 | (0.3558) |
| $\sigma_\alpha$ | 0.7539 | (0.0295) | 1.3549 | (0.1815) |
| $\sigma_\varepsilon$ | 2.4916 | (0.1462) | 2.0654 | (0.4344) |
| B. Size of | | | | |
| B1. Sample | 12,334 | | 56,743 | |
| B2. Simulation | 37,002 | | 170,229 | |
| B3. Cohort-Age-Country Cells | 583 | | 583 | |
| C. Summary Statistics[b] | | | | |
| C1. MSE($\widetilde{\pi}$) | $3.1837\times10^{-4}$ | | $4.3214\times10^{-4}$ | |
| C2. V($\pi$) | $5.8621\times10^{-4}$ | | $7.1099\times10^{-4}$ | |
| C3. MSE($\widetilde{\pi}$)/V($\pi$) | 0.5431 | | 0.6078 | |
| C4.MSE($\widetilde{\pi}_{desig}$)/V($\pi_{desig}$) | 0.4732 | | 0.5028 | |
| C5. MSE($\widetilde{\pi}_{renewal}$)/V($\pi_{renewal}$) | 0.7419 | | 0.8485 | |

a. Estimated standard errors are reported in parentheses.

b. MSE is calculated as the sum of squared residuals weighted by the number of patents in each cohort-age-country cell. V($\pi$) is the sample variance from the data.

learning process is defined as $\sigma_t = \sigma\phi^{t-1}$ in equation (3). The estimate of the decay rate $\phi$ is 0.56 for pharmaceutical patents and 0.70 for electronics patents. In other words, although a pharmaceutical patent has a higher initial learning probability (a higher $\sigma$), such probability decreases more quickly. And starting from age 6, the learning probability for pharmaceutical patents become smaller than that for electronics patents. The estimates of the other parameter of the exponential distribution, $\gamma$, are similar for these two technology groups.

The implications of the parameter estimates of the learning process are illustrated in Table 8, which reports the results of a simulation run of 50,000 draws of pharmaceutical patents and 100,000 draws of electronics patents, based on the average fee schedule across different cohorts as well as the parameter estimates reported in Table 7. Columns 2 to 4 display the percentage of pharmaceutical patents which learn a higher value at each age in Germany, France and the U.K., out of all patents that live up to this age. For instance, at the beginning of age 2, 10% of the pharmaceutical patent applicants discover a use which generates higher subsequent profits than known before in Germany and in France, and 8% in the U.K. At the beginning of age 3, such percentage drops to 6% in Germany, 5% in France and in the U.K. The proportions of patents learning a higher value continue to decline over the ages. By age 5, only 1.6% of the pharmaceutical patent holders find more profitable ways to exploit their patented idea in Germany, and even fewer in France (0.7%) and the U.K. (0.5%). By age 7, none of the pharmaceutical patent holders from the simulation find an increased patent value in the U.K. And the learning process is essentially over by age 8 in France and by age 10 in Germany. The deterministic depreciation and obsolescence processes start to dominate the renewal decisions after that.

Columns 5 to 7 of Table 8 report the learning dynamics of the electronics patents in Germany, France and the U.K. Similar to the case of pharmaceutical patents, the learning probability of the electronics patent group gradually declines over the ages: in Germany from 13% at age 2 to 3% at age 5, and the learning is over by age 11. In France, learning probability drops from 13% at age 2 to 1% at age 5, and to essentially zero at age 9. Such probability is 7% in the U.K. at age 2, 0.13% at age 5, and the learning is over by age 7. The fact that the dynamics of learning probability is similar in pharmaceutical and electronics patent groups reflects the offsetting effects of different parameters of the learning processes in these two groups. As noted above, the parameter $\sigma_t$ in the learning process of pharmaceutical patents are initially higher than that of electronics patents, which generates higher probabilities of discovering a higher value for any given level of patent value. However, because the initial value of pharmaceutical patents is on average higher than that of electronics patents (as shown below, in Table 9), the actual probability of finding a value exceeding the present level by pharmaceutical patent holders may not be necessarily higher than that of the electronics patents. The first few rows of Table 8 show that the learning probability of pharmaceutical patents at early ages is higher than that of electronics patents in the U.K., but slightly lower in Germany and France. Moreover, over ages the parameter $\sigma_t$ of pharmaceutical patents declines faster, and by age 5 it becomes lower than that of electronics patents. From then on, the learning probability of pharmaceutical patents is consistently lower than the corresponding probability of electronics patents.

**Table 8: Percentage of Pharmaceutical and Electronics Patents Learning a Higher Value**

| | Pharmaceutical (%) | | | Electronics (%) | | |
|---|---|---|---|---|---|---|
| Age | Germany | France | U.K. | Germany | France | U.K. |
| 2 | 10.27 | 10.43 | 8.43 | 12.81 | 12.73 | 6.72 |
| 3 | 5.69 | 5.42 | 5.37 | 7.68 | 7.39 | 3.35 |
| 4 | 4.62 | 4.58 | 3.25 | 7.03 | 7.77 | 1.57 |
| 5 | 1.61 | 0.65 | 0.53 | 2.94 | 0.87 | 0.13 |
| 6 | 0.71 | 0.17 | 0.02 | 1.62 | 0.33 | 0.02 |
| 7 | 0.24 | 0.03 | 0.00 | 0.83 | 0.10 | 0.00 |
| 8 | 0.04 | 0.00 | 0.00 | 0.38 | 0.02 | 0.00 |
| 9 | 0.01 | 0.00 | 0.00 | 0.10 | 0.00 | 0.00 |
| 10 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 |
| 11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Note: Table 8 reports the learning probability from a simulation run of 50,000 draws of pharmaceutical patents and 100,000 draws of electronics patents, based on the average fee schedule across different cohorts as well as the parameter estimates reported in Table 7.

Model estimation also reveals that the expected value of patent protection in any country increases as the size of the economy increases, i.e., larger market brings more returns to the patentees. But the estimated degree of returns to scale in the two patent groups differ. In particular, the expected value of pharmaceutical patents has an approximately constant returns to scale, while electronics patents show increasing returns to scale. For instance, while the market size of Austria is 9.5% of that of Germany, as measured by the ratio of average real GDP in these two countries, the model estimates imply that the expected value of an average pharmaceutical patent in Austria is 10% of that in Germany, yet the expected value of an average electronics patent in Austria would be only 4% of that in Germany. Previous literature has provided little evidence regarding the degree of returns to scale of patent value in different countries. Putnam (1997) assumes constant returns to scale in his study. The structural model estimation in the first chapter of this dissertation obtains decreasing returns to scale of patent value in estimating a patent filing model.

Table 9 reports the distribution of the initial patent values in each of the 10 EPO member countries in the two simulated patent groups, before the designation and granting decisions are made. It reveals that, within the same technology group, the initial patent value varies a lot across countries. For instance, the median of the initial value of simulated pharmaceutical patents is \$59,200 in Germany, \$38,285 in France, and only \$440 in Luxembourg, the smallest economy. For the electronics patents, the median initial value is \$18,086 in Germany, \$9,794 in France, and only \$17 in Luxembourg. On the other hand, the initial value of the pharmaceutical patents are on average much higher than that of the electronics patents. For example, the median initial value of pharmaceutical patents is 2.3 times larger than that of electronics patents in Germany, 3 times larger in France, and almost 8 times larger in Austria.

The distribution of the initial values determines the patent applicants' designation decision across countries. As Figure 9 shows, almost all simulated pharmaceutical patents choose to designate Germany, France and Italy at the time of initial filing, but only 88% choose to designate Sweden and 85% choose to designate Austria. The designation rate for Luxembourg is 49%, the lowest among all EPO member countries. Corresponding to lower initial values, the designation rate of electronics patents is also lower than that of the pharmaceutical patents in almost all countries: almost 100% in Germany and France, but only 83% in Italy, 56% in Sweden, and 53% in Austria. The average number of designated countries is 8.7 for the simulated pharmaceutical patents and 6.3 for the electronics patents, very close to the average number in the actual sample (8.4 for pharmaceutical and 5.6 for electronics patents as shown in Figure 5).

## Table 9: Distribution of the Initial Value of Simulated Patents

| | Real GDP Ratio | Pharmaceutical | | | | | |
|---|---|---|---|---|---|---|---|
| | | 50% | | 75% | | 90% | |
| | | Value | Cum. % | Value | Cum. % | Value | Cum. % |
| Austria | 0.0947 | 6,025 | 0.47% | 34,305 | 2.74% | 166,450 | 9.44% |
| Belgium | 0.1133 | 6,851 | 0.49% | 40,428 | 2.87% | 195,900 | 10.01% |
| Switzerland | 0.1310 | 8,174 | 0.48% | 46,564 | 2.74% | 221,420 | 9.37% |
| Germany | 1.0000 | 59,200 | 0.46% | 346,360 | 2.72% | 1,608,900 | 9.30% |
| France | 0.6419 | 38,285 | 0.47% | 224,700 | 2.77% | 1,098,100 | 9.65% |
| U.K. | 0.4579 | 27,739 | 0.50% | 158,200 | 2.88% | 765,730 | 9.97% |
| Italy | 0.4508 | 27,567 | 0.47% | 158,460 | 2.73% | 748,960 | 9.39% |
| Luxembourg | 0.0066 | 440 | 0.51% | 2,524 | 2.98% | 12,657 | 10.37% |
| Netherlands | 0.1682 | 10,664 | 0.43% | 60,470 | 2.46% | 293,930 | 8.48% |
| Sweden | 0.1013 | 6,141.9 | 0.47% | 35,745 | 2.71% | 176,790 | 9.51% |
| Standard Deviation | —— | 18,708 | —— | 109,440 | —— | 513,960 | —— |

| | Electronics | | | | | |
|---|---|---|---|---|---|---|
| | 50% | | 75% | | 90% | |
| | Value | Cum. % | Value | Cum. % | Value | Cum. % |
| Austria | 687 | 0.68% | 3,624.0 | 3.67% | 16,284 | 11.89% |
| Belgium | 859 | 0.73% | 4,657.6 | 3.93% | 20,872 | 12.74% |
| Switzerland | 1,070 | 0.69% | 5,631.5 | 3.63% | 25,050 | 11.71% |
| Germany | 18,086 | 0.67% | 95,252 | 3.65% | 413,060 | 11.67% |
| France | 9,794 | 0.72% | 52,364 | 3.89% | 236,880 | 12.68% |
| U.K. | 6,048 | 0.70% | 31,858 | 3.74% | 146,800 | 12.25% |
| Italy | 6,002 | 0.69% | 31,794 | 3.74% | 142,180 | 12.08% |
| Luxembourg | 17 | 0.68% | 90 | 3.69% | 411 | 11.97% |
| Netherlands | 1,526 | 0.69% | 8,053 | 3.69% | 35,718 | 11.88% |
| Sweden | 738 | 0.69% | 3,954 | 3.74% | 17,964 | 12.22% |
| Standard Deviation | 5,773 | —— | 30,473 | —— | 133,360 | —— |

Note: Table 9 reports the distribution of the initial patent value (prior to the designation decision being made) in each of the 10 EPO member countries in the simulated pharmaceutical patent group and the electronics patent group. The results are from a simulation run of 50,000 draws of pharmaceutical patents and 100,000 draws of electronics patents. Columns 3, 5, 7, 9, 11 and 13 display the initial value of the patents, and columns 4, 6, 8, 10, 12 and 14 display the cumulative proportions of the patent value in the total value of the simulated patent group in each country. All monetary values are in units of

1997 U.S. dollars.

Figure 9: Designation Rates of the Simulated Patents in the EPO Member Countries

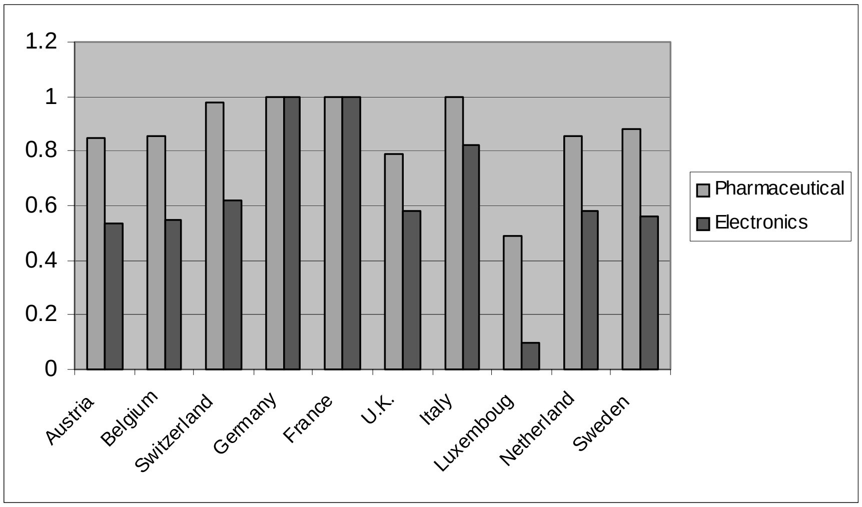

Figure 10: Average Renewal Rate in the EPO member Countries: Pharmaceutical and Electronics Patents from the Simulation

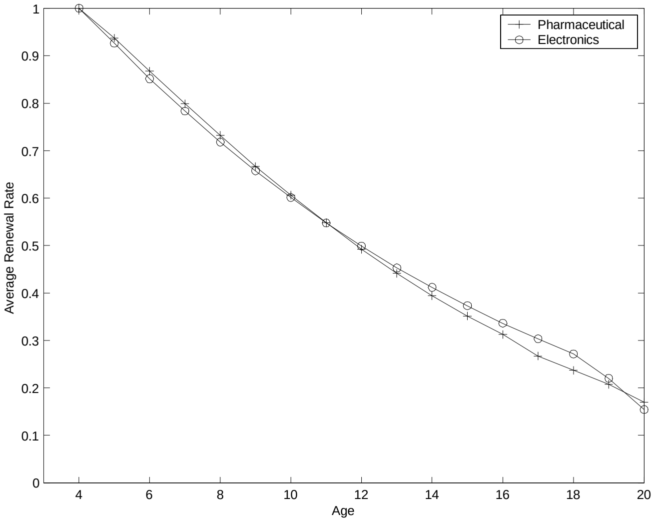

Table 9 also reveals that the distribution of the initial patent values is highly skewed. For instance, in Germany, the sum of initial values of the bottom 50% of pharmaceutical patents applications contributes less than 0.5% of the total initial value of the whole pharmaceutical group, and over 90% of the total initial value is attributed to the top 10% most valuable pharmaceutical patents. The bottom 50% of electronics patent applications contributes only 0.7% of the total initial value of the whole group in Germany, while the top 10% contributes 88% of the total initial value. The distribution of the initial value in other countries has a similar pattern.

Figure 10 compares the renewal rate averaged across countries of the simulated pharmaceutical and electronics patents at each age, weighted by the number of patents transferred to each country. Endowed with higher initial value and more potential opportunities to learn and exploit higher returns, the pharmaceutical patents designate more countries for patent protection on average, as displayed in Figure 9, and have a higher renewal rate in the early ages. However, a high depreciation rate (13%) and a more rapidly decaying learning process diminish the expected value of pharmaceutical patents more quickly, and as a result the renewal rate of pharmaceutical patents becomes lower in the later ages. As Figure 10 shows, the average renewal rate of pharmaceutical patents is about 1 to 2 percentage points higher than that of electronics patents at each age until age 10, and after age 11 the electronics patent group has a higher renewal rate. 28% of the simulated electronics patents live up to age 18, while only 23% of the simulated pharmaceutical patents are still alive by then. Interestingly, by age 20, the average renewal rate of pharmaceutical patents again exceeds that of electronics patents. This may reflect a larger variation of values in the pharmaceutical patent group, which means that although pharmaceutical patents have a shorter average life, proportionately there are more high-valued “elite patents” in this group, whose owners choose to renew for a full 20 years.

Table 10 reports the percentiles and Lorenz curve coefficients from the distribution of realized patent values from the simulation, conditional on the patent applications will be granted. The realized patent value is defined as the discounted sum of patent returns at all ages in all designated countries, net of all kinds of administrative expenses including designation cost and transfer expenses as well as the annual renewal fees, but excluding any litigation costs. Columns 2 and 3 of the table show that the distribution of the realized pharmaceutical patent values is highly skewed. For instance, 25% of the pharmaceutical patents have a realized value of $27,400 or less, while they contribute about 0.02% of the total value of all simulated pharmaceutical patents. The total value of the bottom 50% of pharmaceutical patents accounts for only 0.40% of the total value of the whole pharmaceutical group, and the lower 90% contributes about 16% of the total value. On the other hand, the top 1% most valuable patents, with a minimal value of $142 million, accounts for 46% of the total value of the pharmaceutical group. Similarly, the distribution of the realized value of the electronics patent groups, as reported in columns 4 and 5, is also highly skewed. For instance, the lower 90% of the electronics patents contributes less than 15% of the total value of the electronics group, whereas the top 1% contributes 51% of the total value. On the other hand, electronics patents have significantly lower value than pharmaceutical patents, especially at the high end of the value distribution. For instance, the 85% percentile of the value of pharmaceutical patents is $8.4 million, nearly 4 times of that of electronics patents, which is $2.2 million.

**Table 10: Distribution of the Total Realized Values of Simulated Pharmaceutical and Electronics Patents**

|  | Pharmaceutical |  | Electronics |  |
|---|---|---|---|---|
| Percentile | Value ($million) | LC | Value ($million) | LC |
| 25% | 0.0274 | 0.02 | 0.0111 | 0.04 |
| 50% | 0.4078 | 0.40 | 0.1155 | 0.46 |
| 75% | 3.5906 | 4.42 | 0.8792 | 3.95 |
| 85% | 8.4145 | 10.28 | 2.1831 | 9.17 |
| 90% | 14.6810 | 16.14 | 3.9041 | 14.57 |
| 95% | 31.6340 | 27.45 | 8.8216 | 25.39 |
| 98% | 77.1490 | 42.73 | 21.6860 | 40.49 |
| 99% | 142.3800 | 53.71 | 39.6020 | 51.15 |

Note: Columns 2 and 4 report the percentiles of the distribution of the total realized patent values in all 10 EPO member countries from the simulation. Columns 3 and 5 report the Lorenz curve coefficients of the simulated distribution. Monetary values are in units of 1997 U.S. dollars, and Lorenz curve coefficients (LC) are in percentage points.

# 6 Concluding Remarks

This chapter of the dissertation develops a dynamic stochastic model to examine joint patent application and renewal behavior under an international patent-protection regime. The model takes a first step in utilizing both the cross-sectional (multi-country filing) and the time-series (patent renewal) dimensions of international patent data to evaluate the private value of patents, allowing one to distinguish more aspects of patent value.

The model is estimated using the filing and renewal data of the pharmaceutical and electronics patents filed with the EPO during 1980 to 1985. Estimation result shows that pharmaceutical patents on average are endowed with higher initial values, and the patent holders seek for protection in more countries than the electronics patent holders. However, pharmaceutical patents depreciate faster than electronics patents, and consequently they have lower renewal rates and shorter patent life.

A direct application of the model results would be the construction of a simple “weighting index” that measures the relative value of different patents using the size of the international patent family and the number of years of renewals, which is more accurate than simple patent counts as a measure of innovative output. On the other hand, although combining the patent filing and renewal data reveals more aspects of patent value, the patent renewal data are not available until later stage of a patent’s life. For evaluation of patents at earlier ages, it is useful to exploit other characteristics of patents available at or near the patent’s “birth”, such as the number of patent claims or patent citations. A study of the linkage between these characteristics and the estimated patent values from this study would also provide further insights into the value of patents. These will be topics for future research.

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# Appendix A. Model Solution

Part I of Appendix A, based on the solution algorithm developed in Pakes (1986) and Lanjouw (1998), solves the renewal problem faced by the representative patentee in any contracting state $j$, $j = 1, ..., J$. Part II characterizes the representative patentee's application and designation decision rules and gives the estimator's moment conditions.

**Part I. The Renewal Decision Rule** The value of the patent at age $t$, as rewritten in equation (A1)

$$\begin{aligned} V(t, r_t) &= \max\{0, h\theta^2 r_t + \beta\theta^2 h E_t VL(t+1, r_{t+1}) - c_t - \beta\theta c_{t+1} \\ &\qquad - (\beta\theta)^2 c_{t+2} - \alpha_0(1-w)\} \end{aligned} \tag{A1}$$

where $h = w - \alpha_1(1-w)$, is dependent on the expected gross returns of the patent in the coming periods, $VL(t+1, r_{t+1})$, which is further contingent on the realization of $z_{t+1.}$

Starting from age $T$, the value function is given by

$$V(T, r_T) = \{0, hr_T - c_T - \alpha_0(1-w)\} \tag{A2}$$

This is because $T$ is the maximal age that the patent can possibly be kept in force and $E_T VL(T+1, r_{T+1})$ is simply zero. The minimal return $r_T^*$ to justify the renewal at age $T$ can be obtained by setting equation (A2) to zero: $V(T, r_T) = 0$, or $r_T^* = h^{-1}[c_T + \alpha_0(1-w)]$.

To solve the renewal decision rule at age $T-1$, the functional form of $E_{T-1}VL(T, r_T)$ is needed:

$$\begin{aligned} E_{T-1}VL(T, r_T) &= \int r_T G_T(dr_T | T-1) \\ &= \int_{-\gamma\sigma_T}^{\delta r_{T-1}} \delta r_{T-1} dQ_T(z_T) + \int_{\delta r_{T-1}}^{+\infty} z_T dQ_T(z_T) \\ &= \delta r_{T-1} + \sigma_T[1 - Q_T(\delta r_{T-1})] \end{aligned} \tag{A3}$$

The value function of the patent at age $T-1$ is then solved as

$$V(T-1, r_{T-1}) = \max\{0, h\theta r_{T-1} + \beta\theta h E_{T-1} VL(T, r_T) - c_{T-1} - \beta\theta c_T - \alpha_0(1-w)\} \tag{A4}$$

and by setting $V(T-1, r_{T-1})$ to zero, the minimal return $r^*_{T-1}$ can be obtained by numerically solving

$$h\theta r^*_{T-1} + \beta\theta h E_{T-1} VL(T, r_T) - c_{T-1} - \beta\theta c_T - \alpha_0(1-w) = 0 \qquad \text{(A5)}$$

Similarly, for age $t = T-2$, since $V(t+3, r_{t+3}) = V(T+1, r_{T+1}) = 0$, $E_t VL(t+1, r_{t+1})$ can be obtained as:

$$\begin{aligned}
& E_{T-2} VL(T-1, r_{T-1}) \\
= & \int r_{T-1} G_{T-1}(dr_{T-1}|T-2) + \beta \iint r_T G_T(dr_T|T-1) G_{T-1}(dr_{T-1}|T-2) \\
= & \int_{-\gamma\sigma_{T-1}}^{\delta r_{T-2}} \delta r_{T-2} dQ_{T-1}(z_{T-1}) + \int_{\delta r_{T-2}}^{+\infty} z_{T-1} dQ_{T-1}(z_{T-1}) \\
& \quad + \beta \int [\delta r_{T-1} + \sigma_T(1 - Q_T(\delta r_{T-1}))] G_{T-1}(dr_{T-1}|T-2) \\
= & \delta r_{T-2} + \sigma_{T-1}[1 - Q_{T-1}(\delta r_{T-2})] + \beta\{\delta^2 r_{T-2} \\
& \quad + [1 - Q_{T-1}(\delta r_{T-2})][\delta\sigma_{T-1} + \xi^1_{T-1}(1 - Q_T(\delta^2 r_{T-2}))] \\
& \quad + \sigma_T Q_{T-1}(\delta r_{T-2})[1 - Q_T(\delta^2 r_{T-2})]\} \qquad \text{(A6)}
\end{aligned}$$

where $\xi^1_{T-1} = \sigma^2_T/(\sigma_T + \delta\sigma_{T-1})$, and the value function $V(T-2, r_{T-2})$ is obtained by substituting $E_{T-2}VL(T-1, r_{T-1})$ into equation (A1) when $t = T-2:$

$$\begin{aligned}
V(T-2, r_{T-2}) \;=\; & \max\{0, h\theta^2 r_{T-2} + \beta\theta^2 h E_{T-2} VL(T-1, r_{T-1}) - c_{T-2} \\
& -\beta\theta c_{T-1} - (\beta\theta)^2 c_T - \alpha_0(1-w)\} \qquad \text{(A7)}
\end{aligned}$$

and the minimal return required for renewal is the $r^*_{T-2}$ that solves

$$h\theta^2 r_{T-2} + \beta\theta^2 h E_{T-2} VL(T-1, r_{T-1}) - c_{T-2} - \beta\theta c_{T-1} - (\beta\theta)^2 c_T - \alpha_0(1-w) = 0 \qquad \text{(A8)}$$

The renewal problem for age $t \geq T-3$ is more complicated. Recall that by definition

$$E_{T-3} VL(T-2, r_{T-2})$$

$$\begin{aligned} &= \int r_{T-2}G_{T-2}(dr_{T-2}|T-3) + \beta \iint [r_{T-1} + \beta E_{T-1}V(T, r_T)] \\ &\quad G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3) \qquad \text{(A9)} \end{aligned}$$

However, the exact functional form of $E_{T-1}V(T, r_T)$ depends on whether $\delta r_{T-1}$ is larger than $r_T^*$ or not:

If $\delta r_{T-1} \leq r_T^*$, the renewal decision at age $T$ has to depend on the realization of $z_T$, and the expected value of $V(T, r_T)$ becomes

$$\begin{aligned} E_{T-1}V(T, r_T) &= \theta \int_{r_T^*}^{+\infty} (hz_T - c_T - \alpha_0(1-w))dQ_T(z_T) \\ &= \theta[hr_T^* + h\sigma_T - c_T - \alpha_0(1-w)][1 - Q_T(r_T^*)] \\ &= \theta k_{T-1}^0 \qquad \text{(A10)} \end{aligned}$$

However, if $\delta r_{T-1} > r_T^*$, the patentee knows that he will definitely pay the renewal fee at age $T$ as long as obsolescence does not occur ($r_T = \max(\delta r_{T-1}, z_T) > r_T^*$), and the expected return will be

$E_{T-1}V(T, r_T)$

$$\begin{aligned} &= \theta \int_{-\gamma\sigma_T}^{\delta r_{T-1}} (h\delta r_{T-1} - c_T - \alpha_0(1-w))dQ_T(z_T) \\ &\qquad + \theta \int_{\delta r_{T-1}}^{+\infty} (hz_T - c_T - \alpha_0(1-w))dQ_T(z_T) \\ &= \theta\{k_{T-1}^0 + \int_{-\gamma\sigma_T}^{r_T^*} (h\delta r_{T-1} - c_T - \alpha_0(1-w))dQ_T(z_T) \\ &\qquad + \int_{r_T^*}^{\delta r_{T-1}} (h\delta r_{T-1} - hz_T)dQ_T(z_T)\} \\ &= \theta\{k_{T-1}^0 + k_{T-1}^1(r_{T-1})\} \qquad \text{(A11)} \end{aligned}$$

Therefore, in computing $\iint E_{T-1}V(T, r_T)G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3)$, it differs depending on whether $r_{T-3}$ is greater than $r_T^*/\delta^3$ or not:

If $r_{T-3} \leq r_T^*/\delta^3$, then

$$\begin{aligned}
& \iint E_{T-1}V(T, r_T)G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3) \\
= \;& \theta\int_{-\gamma\sigma_{T-2}}^{r_T^*/\delta^2}\{\int_{-\gamma\sigma_{T-1}}^{r_T^*/\delta} k_{T-1}^0 dQ_{T-1}(z_{T-1}) \\
& \quad + \int_{r_T^*/\delta}^{+\infty}[k_{T-1}^0 + k_{T-1}^1(z_{T-1})]dQ_{T-1}(z_{T-1})\}dQ_{T-2}(z_{T-2}) \\
& \quad +\theta\int_{r_T^*/\delta^2}^{+\infty}\{\int_{-\gamma\sigma_{T-1}}^{\delta z_{T-2}}[k_{T-1}^0 + k_{T-1}^1(\delta z_{T-2})]dQ_{T-1}(z_{T-1}) \\
& \quad + \int_{\delta z_{T-2}}^{+\infty}(k_{T-1}^0 + k_{T-1}^1(z_{T-1})dQ_{T-1}(z_{T-1})\}dQ_{T-2}(z_{T-2}) \qquad \text{(A12)}
\end{aligned}$$

Otherwise, if $r_{T-3} > r_T^*/\delta^3$, then

$$\begin{aligned}
& \iint E_{T-1}V(L, r_T)G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3) \\
= \;& \theta\int_{-\gamma\sigma_{T-2}}^{\delta r_{T-3}}\{\int_{-\gamma\sigma_{T-1}}^{\delta^2 r_{T-3}}[k_{T-1}^0 + k_{T-1}^1(\delta^2 r_{T-3})]dQ_{T-1}(z_{T-1}) \\
& \quad + \int_{\delta^2 r_{T-3}}^{+\infty}[k_{T-1}^0 + k_{T-1}^1(z_{T-1})]dQ_{T-1}(z_{T-1})\}dQ_{T-2}(z_{T-2}) \\
& \quad +\theta\int_{\delta r_{T-3}}^{+\infty}\{\int_{-\gamma\sigma_{T-1}}^{\delta z_{T-2}}[k_{T-1}^0 + k_{T-1}^1(\delta z_{T-2})]dQ_{T-1}(z_{T-1}) \\
& \quad + \int_{\delta z_{T-2}}^{+\infty}(k_{T-1}^0 + k_{T-1}^1(z_{T-1})dQ_{T-1}(z_{T-1})\}dQ_{T-2}(z_{T-2}) \qquad \text{(A13)}
\end{aligned}$$

Consequently, the functional forms of $E_{T-3}VL(T-2, r_{T-2})$ and the value function $V(T-3, r_{T-3})$ differ accordingly:

$$E_{T-3}VL(T-2, r_{T-2})$$

$$
\begin{aligned}
= \quad & \int r_{T-2}G_{T-2}(dr_{T-2}|T-3) \\
& +\beta \iint [r_{T-1}+\beta E_{T-1}V(T,r_T)]G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3) \\
= \quad & \delta r_{T-3}+\sigma_{T-2}[1-Q_{T-2}(\delta r_{T-3})]+\beta\{\delta^2 r_{T-3} \\
& +[1-Q_{T-2}(\delta r_{T-3})][\delta\sigma_{T-2}+\xi^1_{T-2}(1-Q_{T-1}(\delta^2 r_{T-3}))] \\
& +\sigma_{T-1}Q_{T-2}(\delta r_{T-3})[1-Q_{T-1}(\delta^2 r_{T-3})]\} \\
& +\beta^2 \iint E_{T-1}V(T,r_T)G_{T-1}(dr_{T-1}|T-2)G_{T-2}(dr_{T-2}|T-3) \qquad \text{(A14)}
\end{aligned}
$$

$$
\begin{aligned}
V(T-3,r_{T-3}) \quad = \quad & \max\{0, h\theta^2 r_{T-3}+\beta\theta^2 hE_{T-3}VL(T-2,r_{T-2})-c_{T-3} \\
& -\beta\theta c_{T-2}-(\beta\theta)^2 c_{T-1}-\alpha_0(1-w)\} \qquad \text{(A15)}
\end{aligned}
$$

and $r^*_{T-3}$ is implicitly defined by the following equation:

$$
h\theta^2 r_{T-3}+\beta\theta^2 h[k_{T-3}(r_{L-3})]-c_{T-3}-\beta\theta c_{T-2}-(\beta\theta)^2 c_{T-1}-\alpha_0(1-w)=0 \quad \text{(A16)}
$$

where function $[k_{T-3}(r_{T-3})]$ is defined as

$$
\begin{aligned}
& [k_{T-3}(r_{T-3})] \\
= \quad & \delta r_{T-3}+\sigma_{T-2}[1-Q_{T-2}(\delta r_{T-3})]+\beta\{\delta^2 r_{T-3}+[1-Q_{T-2}(\delta r_{T-3})][\delta\sigma_{T-2} \\
& \quad +\xi^1_{T-2}(1-Q_{T-1}(\delta^2 r_{T-3}))]+\sigma_{T-1}Q_{T-2}(\delta r_{T-3})[1-Q_{T-1}(\delta^2 r_{T-3})]\} \\
& \quad +\beta^2\theta k^0_{T-1}+\beta^2\theta\int_{r^*_T/\delta}^{+\infty} k^1_{T-1}(z_{T-1})dQ_{T-1}(z_{T-1}) \\
& \quad +\beta\int_{r^*_T/\delta^2}^{+\infty} k^2_{T-2}(z_{T-1})dQ_{T-2}(z_{T-2})
\end{aligned}
$$

with $\xi^1_{T-2}=\sigma^2_{T-1}/(\sigma_{T-1}+\delta\sigma_{T-2})$ and

$$
k^2_{T-1}=\beta\theta\{k^1_{T-1}(\delta r_{T-2})Q_{T-1}(r^*_T/\delta)+\int_{r^*_T/\delta}^{+\infty}[k^1_{T-1}(\delta r_{T-2})-k^1_{T-1}(z_{T-1})]dQ_{T-2}(z_{T-2})\}.
$$

In general, define

$$\begin{aligned}
k_t^0 &= [h\delta r^*_{t+1} + h\sigma_{t+1} - \widetilde{c}_{t+1}][1 - Q_{t+1}(r^*_{t+1})] + \beta\theta \sum_{v=1}^{T-(t+1)} \int_{r^*_{t+v+1}/\delta^v}^{+\infty} k^v_{t+1} dQ_{t+1}(z_{t+1}) \\
k_t^1(r_t) &= [h\delta r_t - \widetilde{c}_{t+1} + \beta\theta k^0_{t+1}]Q_{t+1}(r^*_{t+1}) + \int_{r^*_{t+1}}^{\delta r_t} [h\delta r_t - h z_{t+1}] dQ_{t+1}(z_{t+1}) \\
k_t^v(r_t) &= \beta\theta\{k^{v-1}_{t+1}(\delta r_t) Q_{t+1}(r^*_{t+v}/\delta^{v-1}) + \int_{r^*_{t+v}/\delta^{v-1}}^{\delta r_t} [k^{v-1}_{t+1}(\delta r_t) - k^{v-1}_{t+1}(z_{t+1})] dQ_{t+1}(z_{t+1})\}, \\
& \qquad\qquad\qquad\qquad \text{for } 2 \leq \text{v} \leq \text{20-t}
\end{aligned}$$

where $\widetilde{c}_t = c_t + \beta\theta c_{t+1} + (\beta\theta)^2 c_{t+2} + \alpha_0(1-w)$, then the minimal return $r^*_t$ for any age $t < T-3$ can be obtained by recursively solving the following equation:

$$h\theta^2 r_t + \beta\theta^2 h k_t(r_t) - \widetilde{c}_t = 0 \tag{A17}$$

where

$$\begin{aligned}
& k_t(r_t) \\
= \; & \delta r_t + \sigma_{t+1}[1 - Q_{t+1}(\delta r_t)] + \beta\{\delta^2 r_t + [1 - Q_{t+1}(\delta r_t)][\delta\sigma_{t+1} \\
& \quad + \xi^1_{t+1}(1 - Q_{t+2}(\delta^2 r_t))] + \sigma_{t+2} Q_{t+1}(\delta r_t)[1 - Q_{t+2}(\delta^2 r_t)]\} \\
& \quad + \beta^2\theta k^0_{t+2} + \beta^2\theta \sum_{v=1}^{T-(t+2)} H^v_{t+2} + \beta \sum_{v=2}^{T-(t+1)} H^v_{t+1},
\end{aligned}$$

$$H^v_t = \int_{r^*_{t+v}/\delta^v}^{+\infty} k^v_t dQ_t(z_t),$$

and $\xi^v_t = \sigma^2_{t+v}/(\sigma_{t+v} + \delta^v \sigma_t)$.

**Part II. The Application and Designation Decision Rule** The patent value in country $j$ at the beginning of the fourth year, conditional on that the patent application has been approved and that country $j$ was designated three years ago, is

$$V(4, r_{j,4}) = \max\{0, h_j\theta^2 r_{j,4} + \beta\theta^2 h_j E_4 VL(5, r_{j,5}) - C_{j,4} - \beta\theta c_{j,5} - (\beta\theta)^2 c_{j,6} - \alpha_{j,0}(1 - w_j)\} \tag{A18}$$

where $C_{j,4}$ is the lump sum transfer cost needed, and $h_j = w_j - \alpha_{1,j}(1 - w_j)$ determined by the litigation costs and the winning probability in country $j$.

Note that one of the regularity conditions listed in Pakes (1986) is not satisfied any more, namely, the requirement of a non-decreasing renewal fee schedule over the ages, because the lump sum transfer fee $C_{j,4}$ is larger than the renewal fee at the subsequent age $c_{j,5}$. However, this does not affect the existence of a unique minimal return $r^*_{j,4}$ beyond which the patentee is willing to transfer his patent to country $j$. What this changes is only the fact that now $r^*_{j,4}$ may not necessarily be smaller than $r^*_{j,5}$ and so on. Therefore, given that country $j$ has been designated and that the patent application has been granted, the patent holder will choose to pay the transfer cost $C_{j,4}$ if and only if $r_{j,4} \geq r^*_{j,4}$, where $r^*_{j,4}$ is implicitly defined by

$$h_j\theta^2 r^*_{j,4} + \beta\theta^2 h k_t(r^*_{j,4}) - C_{j,4} - \beta\theta c_{j,5} - (\beta\theta)^2 c_{j,6} - \alpha_{0,j}(1 - w_j) = 0 \quad \text{(A19)}$$

where $k_t(r_{j,4}) = E_4 VL(5, r_{j,5})$, and the functional form of $k_t(r_{j,4})$ is recursively given in equation (A17). The value function at age 4 is

$$V(4, r_{j,4}) = \max\{0, h_j\theta^2 r_{j,4} + \beta\theta^2 h_j k_t(r_{j,4}) - C_{j,4} - \beta\theta c_{j,5} - (\beta\theta)^2 c_{j,6} - \alpha_{j,0}(1 - w_j)\} \quad \text{(A20)}$$

Once the functional form of patent value after the examination process $V(4, r_{j,4})$ is specified, patent value in country $j$ in period 1 becomes:

$$V(1, r_{j,1}) = \max\{0, h_j\theta^2 r_{j,1} + \beta\theta^2 h_j E_1(r_{j,2} + \beta r_{j,3} + \beta^2 prob_{gr} V(4, r_{j,4})) - C_j - \alpha_{j,0}(1 - w_j)\} \quad \text{(A21)}$$

and the minimal filing return $r^*_{j,1}$ can be obtained by solving the following equation:

$$h_j\theta^2 r_{j,1} + \beta\theta^2 h_j E_1(r_{j,2} + \beta r_{j,3} + \beta^2 prob_{gr} V(4, r_{j,4})) - C_j - \alpha_{j,0}(1 - w_j) = 0$$

or,

$$h_j\theta^2 r^*_{j,1} + \beta\theta^2 h_j k_1(r^*_{j,1}) - C_j - \alpha_{j,0}(1 - w_j) = 0 \quad \text{(A22)}$$

Finally, in choosing whether to file an initial patent application with the EPO, the inventor compares the lump sum filing cost $C_{EPO}$ with the total net return, which is the sum of net return in all countries he chooses to designate, and files the EPO patent application if

$$\sum_{j=1}^{J} 1_{j1}(R^*)\{h_j\theta^2 r_{j,1} + \beta\theta^2 h_j E_1(r_{j,2} + \beta r_{j,3} + \beta^2 prob_{gr} V(4, r_{j,4})) - C_j - \alpha_{j,0}(1 - w_j)\} \geq C_{EPO} \quad \text{(A23)}$$

where $1_{j1}(R^*) = 1$ is an indicator function of the applicant's designation decision in country $j$. If the sum of net return in all to-be-designated countries is still not enough to cover the large filing cost $C_{EPO}$, the inventor will not resort to EPO for patent protection.

## Appendix B. Numerical Estimation Procedures

The parameter estimate $\widehat{\omega}_N$ is obtained by minimizing $||G_N(\omega)|| = ||\pi_N(\omega) - \widetilde{\pi}_N(\omega)||_{W_N(\omega)}$ from equation (20). However, the usual numerical optimization algorithms such as variants of Newton's method may not be directly applicable here. This is because the models to be estimated are highly nonlinear, and consequently the surface of the objective function value $||G_N(\omega)||$ in the parameter space is highly "rugged". Thus the convergence of the numerical search would be very sensitive to the starting point, and as a result, unless we are fairly certain about where in the parameter space the "true" parameters lie, we cannot comfortably take any local minimum and claim it is the global minimum, simply because the search can be easily "trapped" anywhere in the parameter space.

To estimate the models in this study, a two-step numerical optimization strategy is devised. In the first step, several rounds of grid search are performed. For instance, in estimating the renewal models using the data in Germany, France and the U.K., the first-round grid search was performed over the following grid points with a total of 4,860 combinations for each country:

$\theta = 0.90 \quad 0.95$
$\delta = 0.90 \quad 0.95$
$\sigma = 2500 \quad 4000 \quad 5500 \quad 7000 \quad 8500$
$\phi = 0.2 \quad 0.4 \quad 0.6$
$\gamma = 0.2 \quad 0.4 \quad 0.6$
$w = 0.9 \quad 0.95 \quad 0.98$
$\mu_4 = 7.0 \quad 8.5 \quad 10$
$\sigma_4 = 1.0 \quad 1.5 \quad 2.0$

The choice of these grid points is based on examination of the data and results from previous studies such as Pakes(1986), Lanjouw(1998) and the structural model estimation in the first chapter of this dissertation. When the first-round grid search is over, the second-round grid search is then performed in the neighborhood of the optimal point(s)[17] from the first round, with smaller grid scales. And upon the completion of the second round, if necessary, a third round of grid search is also conducted.

After a few rounds of grid search, a number of optimal points can be identified, each of which is located in a tight neighborhood defined in the last round of grid search. A quasi Newton's method is then used to search over these tight neighborhoods, using the optimal points from the grid search as starting points of the algorithm. Finally, in the case where several convergence points are found, the one with the lowest functional value $||G_N(\omega)||$ is chosen to be the final estimate. The estimation results obtained using such a combination of grid search and numerical optimization algorithm are expected to be more robust than the results from using either of them individually.

This strategy is also adopted in estimating the joint filing-renewal model. Based on the estimation results of the renewal models in Germany, France and the U.K., the following first-round grid points are chosen (with a total of 105,840 combinations):

$$
\begin{array}{lllllll}
\theta = 0.90 & 0.95 & & & & & \\
\delta = 0.90 & 0.95 & & & & & \\
\sigma = 3000 & 4500 & 6000 & 7500 & 9000 & 12000 & 15000 \\
\phi = 0.5 & 0.6 & 0.7 & & & & \\
\gamma = 0.15 & 0.30 & 0.45 & & & & \\
\upsilon = 0.5 & 0.75 & 1.0 & 1.25 & 1.5 & 1.75 & 2.0 \\
\mu_\alpha = 8.0 & 10 & 12 & & & & \\
\sigma_\alpha = 0.75 & 1.5 & 2.25 & 3.0 & & & \\
\sigma_\varepsilon = 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & &
\end{array}
$$

Four more rounds of grid search are performed before the last step in which a quasi Newton's method is used to find the convergence. The final estimation results are reported in Section 5.

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[17] To make the estimation results more robust, when each round of grid search is over, not only the grid point with the lowest functional value is chosen, other grid points with similar and slightly higher functional value are also selectd and passed on to the next round grid search or the numerical optimization run.