Bargaining with History Dependent Preferences

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Abstract

We study perfect information bilateral bargaining game with an infinite alternating-offers procedure, in which we add an assumption of history dependent preference. A player will devalue a share which gives her strictly lower discounted utility than what she was offered in earlier stages of the bargaining, namely, a "worse off" outcome. In a strong version of the assumption, each player prefers impasse to any "worse off" outcome. We characterize the essentially unique subgame perfect equilibrium path under the assumption. The equilibrium entails considerable delay and efficiency loss. As the players become infinitely patient, the efficiency loss goes to one half, and the equilibrium share goes to Nash solution. The assumption can also be weakened. We provide a sufficient condition on the extent of devaluation under which the feature of the equilibrium from strong assumption remains.

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1 Introduction

Rubinstein (1982) analyzed a bargaining model with an infinite alternating-offers procedure, and established uniqueness of subgame perfect equilibrium (SPE) when both parties' time preferences are represented by exponential discounting. He also showed that agreement is immediately reached. This contradicts our daily observation that bargaining almost always takes some time, and rarely appears to be efficient. Delays and even impasses are common in real bargaining. Much theoretic work has been done to account for this, and most of it relies on incomplete information as the driving force behind delays, i.e., the players have incomplete information about the "fundamentals" of the bargaining, such as the other party's patience, outside option, etc.. Under incomplete information settings, there are always multiple equilibria, and refinement criteria for the equilibria becomes a major issue.

In this paper, we take a different approach to study delay in bargaining. We assume that players have history dependent preferences, that is, players' payoffs not only depend on the outcome of the bargaining, but also depend on the specific bargaining process leading to the outcome. What they get from a final agreement, together with what they have been offered and rejected determine their current payoff, which is then discounted due to impatience.

In our model, players still have intrinsic preferences over the outcome. As usual, this preference admits a separable representation, i.e., a concave utility function measuring current value of a share and a time-discounting term. This representation allows us to introduce a strong version of history dependent preference. We shall assume each player prefers impasse to any outcome, which is worse, in terms of discounted utility, than any offer she has rejected; and her preference over improving outcome is always measured by the intrinsic utility function. This preference is assumed to be common knowledge between the players. With such preferences, the players will strategically hold back in making offers, because once an offer is rejected the proposer has to keep improving it in order to reach agreement. Yet players are impatient, so they have a countervailing incentive to reach an early agreement. These two counteracting forces induce some interesting results. We still have essentially unique SPE, but it involves considerable delay. The equilibrium has a flavor of reciprocity, that is, the players will start from two extreme positions, each player will make some small concession at the beginning, after the opponent makes reciprocal responses, the step of the concession will increase, and they finally reach an agreement somewhere in the middle. Equilibrium play thus exhibits realistic features.

A weaker version of history dependent preference will also be studied. Before we proceed, we provide two different interpretations of the history dependent preferences, delegated bargaining with justification and reference dependent preference.

Many real life bargaining situations are delegated bargaining, for instance, political bargaining. In such kind of situations, the agent usually has to justify her performance ex post. For the agent, the most important thing is to convince her principal that her performance was successful, and she has done the best given the situation. Based on this, the agent also cares about the material outcome. Basically, the justification is combined with a contract between the principal and the agent. The contract is monotone, i.e., the better is the discounted utility of the principal from the outcome of the bargaining, the better is the payment to the agent. The monotone contract prevents the agent from simply taking the first offer. About the justification, we think it is natural to consider a criterion involving following rule: a performance is considered to be unsuccessful if the agent has forgone an offer and ends up with a worse offer in terms of discounted utility for the principal. Intuitively, the principal's logic is: the agent did not do a satisfactory job because she had chance(s) to get a better result, but did not take it. The justification requirement gives the agent a commitment advantage, which induces delay.

We borrow the concept of *Justifiability* from Spiegler (2002), but apply the idea in a different way. In his model, the agent, who plays the game, has to be able to justify her strategy choice based on a plausible conjecture of the opponent's strategy in an ex post debate-like justification procedure between the agent and the principal. Being justifiable means that for any potential criticism on the agent's strategy, there is a counter-argument based on the same logic. Spiegler applied a simplicity-based criterion on plausibility, and obtained interesting outcomes for a number of finite horizon games with paradoxical SPEs, such as chain-store game and centipede game.

We can also interpret the assumed preference as preference with loss aversion as defined in Tversky and Kahneman (1991). Each player takes the best ever offer in terms of discounted utility as her reference point. A share providing higher discounted utility will be valued as before, but a share with lower discounted utility, namely a loss, will be devalued. Our strong assumption corresponds to an extreme case of devaluation of the shares in the region of loss, any such share brings a negative payoff to the player. The magnitude of this negative payoff does not matter, the player simply prefer impasse to any loss. In other words, the players here are assumed to be extremely loss averse.

One notable feature here is that the reference point evolves endogenously. An improving offer from one side change the reference point of the other side in the future play, and without the improving offer, the reference point changes over time due to the discounting. There is a similar formation of reference level in Barberis, et al. (2001). In their model, investor gets utility from fluctuations of risky asset, and they take the reference level to be the current value of the asset scaled up by the risk-free interest rate.

Under this interpretation, it is natural to weaken the strong assumption. We can replace the discontinuity at the reference point by a kink. In the case of linear utility, what we do is simply to give the utility function a larger slope rate in the loss region. In section 4, we will show that when the kink is sharp enough, or, the loss aversion effect is large enough, we can get similar unique SPE path with considerable delay. Thus, the discontinuity in the strong assumption is not necessary to obtain delay.

1.1 Related Literature

Experiments in game theory have been providing results inconsistent with game theoretic predictions. It has been argued that human subjects do not behave according to these predictions in many situations, and an alternative view is that the preferences of the human subjects are not always determined by the material outcome of the game. When two different plays bring the same material outcome for a player, she may prefer one play over the other. Weibull (2004) introduced a notion of "game protocol", and analyzed context-dependent preference and interpersonal preference dependence. Our preference assumption is in favor of the latter view mentioned above. It can be viewed as a special form of context-dependent preference.

Abreu and Gul (2000) introduced a small fraction of behavioral type into their bargaining model and studied two-sided reputation formation in bargaining. The behavioral bias they used was that the players might commit to a fixed share and only accept that amount or higher[1]. There is a unique sequential equilibrium in their model, and it entails delay, consequently efficiency loss. The type of each player is not revealed before the bargaining, and the efficiency loss in their

[1] The behavioral type with obstinate demand was first introduced into a bargaining model with one-sided reputation formation in Myerson (1991).

model is effectively information-induced. The behavioral bias can be interpreted as being caused by an aberration of preference.

Some work has been done to explain the strategic delay in bargaining with complete information models. Ma and Manove (1993) studied a bargaining game with deadline and imperfect control over the timing of offers. They obtained a symmetric Markov-Perfect Equilibrium involving delay and positive probability of impasse. Perry and Reny (1993) and Sakovics (1993) considered continuous time case, simultaneous offer is the necessary condition to get delay in their models. More closely related, Fershtman and Seidman (1993) combined "endogenous commitment" assumption and "deadline effect" in their model. When players are sufficiently patient, there is unique and inefficient SPE, in which the agreement is delayed until the last period, and no concessions are made before that. A fair lottery determines one player who gets the whole pie in the last period. The endogenous commitment assumption, a player cannot accept any offer lower than what she has rejected, is similar to our assumption of history dependent preference. The key difference is that we scale up the commitment levels by the discount factors. More importantly, the "endogenous commitment" assumption in their model itself cannot induce delay without "deadline effect".

In our model, rejection of an offer from the opponent can also be view as a commitment tactic. There are other theoretic models concerning the strategic commitment, for example, Crawford (1982) and Muthoo (1996). The commitment in our model is different from theirs. They both assumed that once a player makes a demand for a specific share of the pie, she will have to pay a retreat cost if she settles with a share lower than her original demand. Crawford (1982) explained impasses in real bargaining based on this possibility of commitment, and Muthoo (1996) studied how the retreat cost functions affect the equilibrium share. In our model, the commitment comes from the rejection of offers. An offer to the opponent also means a demand from the proposer, thus, the key difference here is where the commitment power comes from. If we consider the behavior of committing as a kind of reputation concern, the question will be which one hurts the reputation more, retreat or regret? From our point of view, there is no yes or no answer for this question, they are just different perspectives to understand real life bargaining. Different commitment tactics work in different situations.

Another closely related paper is Admati and Perry (1991). Our model and some of the main results under the strong assumption share features with their contribution game. Both models have essentially unique SPE path and considerable delay in the equilibrium. The way we characterize the equilibrium is also similar to theirs. One important difference between the two models is about the inefficiency. In their model, the inefficiency is in the sense that the socially desirable project may not be completed. The delay is not the major concern of their model. Actually, given the convex investment cost functions, the delay is to some extent socially desirable. In our model, we discuss the inefficiency only in the sense of strategic delay as most of the bargaining literature does. Moreover, we also consider a more general setting, in which the players may have different concave utility functions and different discount factors.

More recently, Compte and Jehiel (2003) studied bargaining and contribution games under the assumption of history dependent outside options. The ideas of their paper and the current paper are similar, both explain not only the delay, but also the important feature of gradualism in real bargaining.

The rest of the paper is organized as follows. In section 2, we lay out the model. We obtain the main results under the strong version of history dependent preference in section 3. Results under the weaker version of history dependent preference is presented in section 4. Section 5 includes an informal discussion, in which we explore, without the assumption of history dependent preferences, the possibility of providing epistemic foundation for the equilibrium play we have obtained. Section 6 concludes.

2 The Model

We analyze a perfect information bargaining game. Two players bargain over a pie with size 1, the bargaining takes the infinite alternating-offers procedure introduced by Rubinstein (1982). Each player in turn makes an offer to her opponent, who decides to accept or reject. The bargaining ends if one player accepts an offer from the other player. If no one ever accepts an offer, it is an impasse. An offer is defined as the nominal share the proposer agrees to give to her opponent, and the proposer gets whatever is left.

We call any play path a history. Every history leads to either a terminal node or a decision node, and a decision node starts a subgame. A startegy specifies an action at every decision node of a player. We call the part of the strategy in a subgame as subgame strategy.

Initially, players have intrinsic utility functions $u(\cdot)$ and $\upsilon(\cdot)$ defined on $[0,1]$, $u(\cdot)$ and $\upsilon(\cdot)$ are strictly increasing and concave, and we also normalize $u(0) = \upsilon(0) = 0$, $u(1) = \upsilon(1) = 1$. The utility function specifies the current value of a share for the player. It is how the players value the share of the pie before they start to bargain. We can also define the initial set of feasible utility pairs as $U = \left\{(\alpha, \beta) \in [0,1]^2 : u^{-1}(\alpha) + v^{-1}(\beta) \leq 1\right\}$.

An agreement can be reached at $t = 0, 1, 2, \ldots$. The players are impatient, and the nominal utility from the share of the pie will be discounted over periods by $0 < \delta_1, \delta_2 < 1$, i.e., the discounted utility for player 1 (resp. player 2) from a share x (resp. y) attained in t^{th} period is $\delta_1^t u(x)$ (resp. $\delta_2^t v(y)$). As usual, the (discounted) utility from an impasse is simply zero. The key difference we have in the current model is that the payoff to the players will be history dependent. Basically, we do not always take the discounted utility from the share of the pie as the payoff of a player.

We assume that if a player ends up with a share which gives her the highest discounted utility from whatever she has been offered, her payoff is just the discounted utility from that share. But if the player ends up with a share strictly worse in terms of discounted utility than any offer she rejected along the history path, i.e., a "worse off" outcome or an unsuccessful bargaining performance, her payoff will be negative. The payoff from impasse is zero. Therefore, impasse is strictly preferred to any "worse off" outcome. We can also put it this way, the players have lexicographic preferences over the outcomes of the bargaining, in which a 0/1 indicator of being successful or not is the first argument, the discounted utility is the second argument, and the first one has the higher priority. We think it makes sense in many political bargaining situations. Under such setting, it is obvious that in a possible subgame perfect equilibrium, each player will never accept an offer which gives her lower discounted utility than what she was offered before. This is what we call the Strong Assumption of History Dependent Preference. Before we formally state the assumption, we need one more notation, the state variables for the bargaining game.

Definition *State variable x_t (resp. y_t) at t^{th} period of the bargaining game is the smallest share player 1 (resp. player 2) needs to keep her discounted utility not lower than what she could get from any previous offer.*

We denote by c_s^i the offer made by player i in s^{th} period, then
$x_t = \max_{(s<t)}\{u^{-1}[u(c_s^2)/\delta_1^{t-s}]\}$, for any odd number $s < t$;
$y_t = \max_{(s<t)}\{v^{-1}\left[v(c_s^1)/\delta_2^{t-s}\right]\}$, for any even number $s < t$;
and $x_0 = y_0 = 0$.

In our model, all relevant information in a specific history is included in the state variables. We can denote a subgame starting with a decision node for one of the players to make an offer as $(x_t, y_t)_i$, the subscript $i \in \{1, 2\}$ refers to the player who makes the first offer in the subgame.

Now we can give the Strong Assumption of History Dependent Preference formally as following.

Strong Assumption of History Dependent Preference

At t^{th} period of the bargaining game, the (current) utility functions from the share of the pie are:

Player 1: $\widetilde{u}^t(x) = \begin{cases} u(x) & if\ x \geq x_t, \\ -\varepsilon < 0 & if\ x < x_t; \end{cases}$

Player 2: $\widetilde{v}^t(y) = \begin{cases} v(y) & if\ y \geq y_t, \\ -\varepsilon < 0 & if\ y < y_t. \end{cases}$

What we do here is to truncate the utility functions, $u(\cdot)$ and $v(\cdot)$, according to the state variables every period. This is equivalent to putting a dynamic restriction on the feasible set of actions. It can be either one of the following two alternatives.

1. A player is not allowed to accept any offer which is worse (for herself) than what she has rejected along the history path in terms of discounted utility.

2. A player is not allowed to make any offer which is worse (for her opponent) than what she has offered and been rejected along the history path in terms of discounted utility.

It is obvious that this different way to make the assumption has no effect on the SPE path. We would like to make the assumption on the preference instead of on the feasible action set because it is more natural to weaken the assumption as we will do in section 4.

3 Equilibrium under the Strong Assumption

3.1 A Simple Case

In this section, we discuss the benchmark case, i.e., two players have linear initial utility function, $u(x) = x$ and $v(y) = y$, and they also have common discount factor, $\delta \in (0, 1)$. In this case, we simply have $x_t = \max\left[c_s^2/\delta^{t-s}\right]$, for any odd number $s < t$; $y_t = \max\left[c_s^1/\delta^{t-s}\right]$, for any even number $s < t$; and $x_0 = y_0 = 0$. c_s^i is the offer made by player i in s^{th} period.

Our purpose now is to characterize the SPE(s). The following straightforward lemmas lead to the main result.

Lemma 1 *Impasse is not a SPE outcome.*

Both players get zero payoff from impasse, while player 1 can simply offer anything larger than δ at the beginning, it will not be rejected and bring both players positive payoffs.

Lemma 2 *In the subgame $(x_t, y_t)_i$ with $x_t + y_t \leq 1$, the highest offer i will make in a SPE is* $\max[\delta - x_t, y_t]$ *for* $i = 1$, $\max[\delta - y_t, x_t]$ *for* $i = 2$.

Proof. *We consider $i = 1$, it is the same for $i = 2$.*

It is obvious that in any subgame $(x_t, y_t)_i$ with $x_t + y_t > 1$, impasse is the only SPE outcome. Thus, when $x_t + y_t > \delta$, it is infeasible to get an agreement in next period, and player 2 will accept any offer higher or equal to y_t. Thus,it is optimal for player 1 to make offer y_t.

When $x_t + y_t \leq \delta$, any feasible agreement in next period has to give player 1 a share no lower than x_t/δ, by which player 2 gets the highest possible share $1 - x_t/\delta$. In terms of discounted utility, such a share is equivalent to a current share $\delta - x_t$. In other words, player 2 will not reject any offer higher than $\delta - x_t$ in a SPE, thus, it is also the upper bound of the offer player 1 will make.

Finally, $x_t + y_t \leq \delta$ is just $y_t \leq \delta - x_t$. ■

Lemma 3 *Any strategy involving acceptance of a non-highest offer is not a SPE strategy.*

Proof. This is also straightforward. We only need to consider a subgame $(x_t, y_t)_1$ with $x_t + y_t < \delta$, if player 1 offers c_t^1, with $y_t \leq c_t^1 < \delta - x_t$, player 2 will reject because she can at least counteroffer with $\max\left[\delta - c_t^1/\delta, x_t/\delta\right]$, the highest offer at $t + 1^{th}$ period, and get a payoff higher than c_t^1.

To see this, if $\delta - c_t^1/\delta \geq x_t/\delta$, player 2's payoff in terms of utility in t^{th} period is

$$\delta(1 - (\delta - c_t^1/\delta)) = \delta - \delta^2 + c_t^1 > c_t^1;$$

if $\delta - c_t^1/\delta < x_t/\delta$, it is $\delta(1 - x_t^1/\delta) = \delta - x_t > c_t^1$. Thus, player 2 will reject any offer lower than $\delta - x_t$ in a SPE. ■

We notice that the lower bound of the offer not being rejected coincides with the upper bound of the offer being rejected. We name this offer as 'clinching offer'[2] with respect to the corresponding subgame. Obviously, a SPE of the bargaining game will end with a clinching offer being made and accepted. We are now ready for the main result of this section.

Proposition 1

(i) There exists an essentially unique subgame perfect equilibrium (SPE) path. When $\delta > \frac{\sqrt{5}-1}{2}$ *(the golden number), there is delay in SPE.*

(ii) The SPE delay, measured as the number n *of time periods until agreement is reached, is a non-decreasing function of the common discount factor* δ*. As* δ *goes to* 1*,* n *converges to infinity.*

(iii) The SPE share to player 1 (who makes the first offer) is $x = 1 - \delta^{n+1}$*, while player 2 receives the share* $y = \delta^{n+1}$*. The associated payoffs are thus* $\delta^n - \delta^{2n+1}$ *and* δ^{2n+1}*, respectively.*

(iv) As δ *goes to* 1*,* δ^n *(where* n *is a function of* δ*) converges to* $1/2$ *from above, i.e., the efficiency loss converges to* $1/2$*.*

Proof. (i) From the lemmas, we know any SPE ends the bargaining with acceptance of the clinching offer with respect to the state variables, thus, we need to see when it is optimal for a player to make the clinching offer.

When $x_t + y_t > \delta$, it is obvious that players would make the clinching offer defined as above.

When $x_t + y_t \leq \delta$, by making the clinching offer, the proposer, say, player 1 gets a current share $1 - (\delta - x_t)$; if player 1 does not make the clinching offer, the best outcome for her is to get a share $\delta - y_t/\delta$, the clinching offer, in next period. Then it is obvious that when $x_t + y_t \geq \delta^2 + \delta - 1$, for the current proposer[3], it is better, in terms of discounted utility, to make the clinching offer (being accepted immediately) than to wait for the clinching offer in next period. Therefore when the state variables get into this grid (See Figure 1), say, grid 1, the corresponding subgame will end with an immediate agreement in any SPE. We define grid 1 as

$$G_1 = \left\{ (\alpha, \beta) \in [0,1]^2 : \delta^2 + \delta - 1 \leq \alpha + \beta \leq 1 \right\}.$$

Formally speaking, any subgame $(x_t, y_t)_i$ with $(x_t, y_t) \in G_1$ has unique SPE, in which the clinching offer is made by i, and accepted by $-i$. Moreover, the

[2] Credit goes to Jeff Ely for the use of this intuitive term.

[3] An important point here is that the condition is same for both players, which differs with the general case we will discuss later.

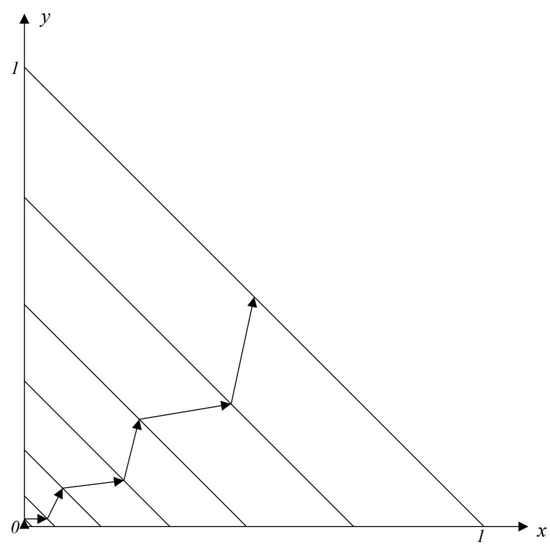

Figure 1: Simple Case

equilibrium share will be $(1-\delta+x_t, \delta-x_t)$ when $x_t+y_t<\delta$ and it is player 1's turn $(i=1)$ to make an offer, and vice versa.

When (x_t, y_t) is outside G_1, the current proposer, say, player 1, will compare the following two choices: one is to make the smallest offer such that (x_{t+1}, y_{t+1}) will be in G_1; the other is to make the basic offer, y_t, or a lousy offer, lower than y_t, and in the next period player 2 makes an offer such that (x_{t+2}, y_{t+2}) will be in G_1. We know once a proposer realizes she is in G_1, she is willing to make the clinching offer in any SPE. It is easy to see when $\delta(\delta^2+\delta-1) \leq x_t+y_t < \delta^2+\delta-1$, there is no difference between the two alternatives; and when $\delta^4+\delta^3-\delta \leq x_t+y_t < \delta(\delta^2+\delta-1)$, the former choice gives a higher discounted utility. Thus we can again define grid 2 as

$$G_2 = \left\{(\alpha, \beta) \in [0,1]^2 : \delta^4+\delta^3-\delta \leq \alpha+\beta < \delta^2+\delta-1\right\}$$

For $(x_t, y_t)_1 \in G_2$, the SPE path will start with player 1's making offer $\max\left[\delta(\delta^2+\delta-1-x_t/\delta), y_t\right]$ and for player 2, it is still to reject any non-clinching offer. After the rejection from player 2, we will have $(x_{t+1}, y_{t+1})_2 \in G_1$. If $x_t+y_t < \delta(\delta^2+\delta-1)$, the equilibrium share will be $\left(1-\delta^2+x_t/\delta, \delta^2-x_t/\delta\right)$. The analysis will be similar for $(x_t, y_t)_2 \in G_2$.

By doing this recursively, we can find a series of such grids with critical values of form $A_n = \delta^{2n}+\delta^{2n-1}-\delta^{n-1}$, i.e.,

$$G_n = \left\{(\alpha, \beta) \in [0,1]^2 : A_n \le \alpha + \beta < A_{n-1}\right\}.$$

When $(x_t, y_t)_1 \in G_n$, the SPE path will start with player 1's making offer

$$\max\left[\delta(\delta^{2(n-1)} + \delta^{2(n-1)-1} - \delta^{n-2} - x_t/\delta), y_t\right],$$

and rejection of any non-clinching offer by player 2, then we have $(x_{t+1}, y_{t+1})_2 \in G_{n-1}$, and so on. When

$$x_t + y_t < \delta(\delta^{2(n-1)} + \delta^{2(n-1)-1} - \delta^{n-2}),$$

the equilibrium share will be $\left(1 - \delta^n + x_t/\delta^{n-1}, \delta^n - x_t/\delta^{n-1}\right)$. The analysis will be similar for $(x_t, y_t)_2 \in G_n$.

Now we can characterize the equilibrium path as following:

(a) If for some n, we have

$$\delta^{2n} + \delta^{2n-1} - \delta^{n-1} > 0 > \delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n,$$

player 1 makes her first offer $c_0^1 = \delta(\delta^{2n} + \delta^{2n-1} - \delta^{n-1})$, which makes $x_1 = 0$ and $y_1 = \delta^{2n} + \delta^{2n-1} - \delta^{n-1}$, then it follows the SPE path we have defined. In this case, it is the unique SPE path[4];

(b) If we have $\delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n = 0$, in addition to the SPE path we specified in (a), we have another SPE path, in which player 1's first offer is $c_0^1 = 0$, followed by the previous SPE path starting with player 2. Indifference of player 1 between the two outcomes may induce one more period of delay.

When $\delta^2 + \delta - 1 > 0$, i.e., $\delta > \frac{\sqrt{5}-1}{2}$, there will be at least one period of delay. In the case with delay, the equilibrium path is basically that the players take turn to jump from the lower boundary of one grid to another.

(ii) When

$$\delta^{2n} + \delta^{2n-1} - \delta^{n-1} > 0 \ge \delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n,$$

n is the number of periods delayed in the essentially unique SPE path (the more efficient one in case of two). In other words, in the SPE, it takes $n+1$ periods of offering and counteroffering to reach an agreement.

[4] We call it the unique SPE path instead of SPE because off equilibrium path, when a player only need to make basic offer (specified by state variables) to get into next grid in next period, she can also make any lousy offer, say, the offers lower than the state variable. It has no effect on the outcome.

Claim n is a nondecreasing function of δ.

Suppose not, $\exists\ 1 > \delta_1 > \delta_2 > 0$, and $n_1 < n_2$, that is,

$$\delta_1^{2n_2} + \delta_1^{2n_2-1} - \delta_1^{n_2-1} \leq 0 \ \ and \ \ \delta_2^{2n_2} + \delta_2^{2n_2-1} - \delta_2^{n_2-1} > 0,$$

then

$$\delta_1^{n_2} \leq \frac{1}{1+\delta_1} \ \ and \ \ \delta_2^{n_2} > \frac{1}{1+\delta_2}.$$

Since $\delta_1 > \delta_2$, we have a contradiction.

It is straightforward to show that number of periods delayed goes to infinity as δ goes to 1.

(iii)&(iv)

It is easy to see the equilibrium shares will be $1-\delta^{n+1}$ and δ^{n+1} for player 1 and 2 respectively, and the payoffs are $\delta^n - \delta^{2n+1}$ and δ^{2n+1}. Moreover, $\delta^n > \frac{1}{1+\delta} > \frac{1}{2}$, as δ goes to one, δ^n goes to $\frac{1}{2}$ from above. The delay, or, the efficiency loss in the current model is substantial. More specifically, when the players become infinitely patient, half of the pie will be wasted due to delay[5]. ■

In Rubinstein's model, the equilibrium share for this simple case is $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$; and in our model, the equilibrium share lies between $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ and $\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right)$. $\frac{\delta}{1+\delta}$ is the lower bound of player 1's equilibrium share, and $\frac{1}{1+\delta}$ is the upper bound. Thus, there is no first mover advantage in our model. There is not necessarily first mover disadvantage either. When the $(0,0)$ is close to or on the lower boundary of G_n, there is first mover disadvantage; when it is close to the upper boundary, the first mover disadvantage gets smaller and even becomes advantage. As δ goes to 1, both $\frac{\delta}{1+\delta}$ and $\frac{1}{1+\delta}$ go to $\frac{1}{2}$, which indicates that in the limit the equilibrium share goes to Nash solution.

The equilibrium play we characterized is indeed a concession process. We can treat the bargaining game as a continuous concession game. The two players start from two extreme positions, and take turn to make concession. If the total amounts of concessions reach 1 in finite time, there is an agreement with corresponding delay; otherwise it is an impasse. It is not difficult to see that the step of the concession is increasing. This is intuitive, the higher is the current offer, the higher is the waiting cost from the proposer's point of view since she has to keep her opponent at least the same discounted utility to get an agreement. Thus, players have incentive to improve their offers by increasing steps. We think it is also a realistic prediction. In political bargaining, we always observe

[5] Although the efficiency loss converges to $\frac{1}{2}$ from below as δ goes to 1, it is not an increasing function of δ due to the reason that the number of delay, n, is not strictly increasing with δ.

two parties start from two extreme positions, and at the beginning they are very insistent on their positions, as times goes by, they start to make small concessions, the step of concession tends to increase over time until they meet somewhere in the middle.

3.2 General Case

We have studied the case with homogeneous and risk neutral players. We wonder if the main result is robust to the introduction of asymmetry and risk aversion. Thus, we consider a general case now, in which two players have different concave utility functions and different discount factors. We will follow the same logic as in the benchmark model. Given any subgame $(x_t, y_t)_i$, we first specify the clinching offer in the following lemma. Same as in the simple case, clinching offer is the lower bound of the offers, which the opponent cannot reject in a SPE; and it is also the upper bound of the offers, which the opponent always rejects in a SPE. Any SPE path ends the bargaining with a clinching offer being made and accepted.

Lemma 4 *For any subgame $(x_t, y_t)_i$, with $u^{-1}(u(x_t)/\delta_1) + \upsilon^{-1}(\upsilon(y_t)/\delta_2) \leq 1$[6], the clinching offer is:*

$$\upsilon^{-1}(\delta_2 \upsilon(1 - u^{-1}(u(x_t)/\delta_1))), \ if \ i = 1;$$
$$u^{-1}(\delta_1 u(1 - \upsilon^{-1}(\upsilon(y_t)/\delta_2))), \ if \ i = 2.$$

For tractability, we focus on the family of concave power functions, i.e.,

$$u(x) = x^{\lambda_1} \ and \ \upsilon(y) = y^{\lambda_2}, \ \lambda_1, \ \lambda_2 \in (0, 1].$$

The set of feasible utility pairs now is

$$U = \left\{ (\alpha, \beta) \in [0, 1]^2 : \alpha^{\frac{1}{\lambda_1}} + \beta^{\frac{1}{\lambda_2}} \leq 1 \right\}.$$

The discount factors remain as $\delta_1, \ \delta_2 \in (0, 1)$. Now we pin down the conditions on the state variables under which the players will make the clinching offer. For player 1, making the clinching offer means payoff is $u(1 - \upsilon^{-1}(\delta_2 \upsilon(1 - u^{-1}(u(x_t)/\delta_1))))$, waiting for the clinching offer in next period means payoff is $\delta_1^2 u(1 - \upsilon^{-1}(\upsilon(y_t)/\delta_2^2))$. Comparing the payoffs, we can define the grid 1 for player 1, i.e., G_1^1.

[6] This condition means that without making any further concession in the current period, an agreement is still feasible in next period.

$$G_1^1 \equiv \left\{ (\alpha, \beta) \in [0,1]^2 : A_1^1 \leq \delta_2^{\frac{3}{\lambda_2}} \alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3}{\lambda_1}} \beta^{\frac{1}{\lambda_2}} \leq 1 \right\},$$

where

$$A_1^1 = \delta_1^{\frac{1}{\lambda_1}} \delta_2^{\frac{3}{\lambda_2}} + \delta_2^{\frac{2}{\lambda_2}} \delta_1^{\frac{3}{\lambda_1}} - \delta_1^{\frac{1}{\lambda_1}} \delta_2^{\frac{2}{\lambda_2}}.$$

For player 2, making the clinching offer means payoff is $\upsilon(1 - u^{-1}(\delta_1 u(1 - \upsilon^{-1}(\upsilon(y_t)/\delta_2))))$, waiting for the clinching offer in next period means payoff is $\delta_2^2 \upsilon(1 - u^{-1}(u(x_t)/\delta_1^2))$. Comparing the payoffs, we can define the grid 1 for player 2, i.e., G_1^2.

$$G_1^2 \equiv \left\{ (\alpha, \beta) \in [0,1]^2 : A_1^2 \leq \delta_2^{\frac{3}{\lambda_2}} \alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3}{\lambda_1}} \beta^{\frac{1}{\lambda_2}} \leq 1 \right\},$$

where

$$A_1^2 = \delta_2^{\frac{1}{\lambda_1}} \delta_1^{\frac{3}{\lambda_2}} + \delta_1^{\frac{2}{\lambda_2}} \delta_2^{\frac{3}{\lambda_1}} - \delta_2^{\frac{1}{\lambda_1}} \delta_1^{\frac{2}{\lambda_2}}.$$

This is the main difference between the current case and the benchmark case, G_1^1 does not coincide with G_1^2. Nevertheless, there is still an inclusion relation between G_1^1 and G_1^2, which depends on $A_1^1 \gtrless A_1^2$. Without loss of generality, we assume $A_1^1 > A_1^2$, then $G_1^1 \subset G_1^2$. For any subgame $(x_t, y_t)_i$, with $(u(x_t), \upsilon(y_t)) \in G_1^1$, the SPE path will start with that player i makes the clinching offer, which will be accepted by player $-i$. For $(x_t, y_t)_i$, with $(u(x_t), \upsilon(y_t)) \in G_1^2 \backslash G_1^1$, if $i = 2$, it is still that player 2 makes the clinching offer, and player 1 accepts; if $i = 1$, player 1 will choose to wait, i.e., making an offer equal to y_t or less, and player 2 will reject and make the clinching offer in the following period. Similar to the simple case, we can define two families of grids, G_n^1 and G_n^2, recursively as following:

$$G_n^1 \equiv \left\{ (\alpha, \beta) \in [0,1]^2 : A_n^1 \leq \delta_2^{\frac{3n}{\lambda_2}} \alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3n}{\lambda_1}} \beta^{\frac{1}{\lambda_2}} < \min[A_{n-1}^1, A_{n-1}^2] \right\},$$

and

$$G_n^2 \equiv \left\{ (\alpha, \beta) \in [0,1]^2 : A_n^2 \leq \delta_2^{\frac{3n}{\lambda_2}} \alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3n}{\lambda_1}} \beta^{\frac{1}{\lambda_2}} < \min[A_{n-1}^1, A_{n-1}^2] \right\},$$

where

$$\begin{aligned} A_n^1 &= \delta_1^{\frac{2n-1}{\lambda_1}} \delta_2^{\frac{3n}{\lambda_2}} + \delta_2^{\frac{2n}{\lambda_2}} \delta_1^{\frac{3n}{\lambda_1}} - \delta_1^{\frac{2n-1}{\lambda_1}} \delta_2^{\frac{2n}{\lambda_2}}, \\ A_n^2 &= \delta_2^{\frac{2n-1}{\lambda_1}} \delta_1^{\frac{3n}{\lambda_2}} + \delta_1^{\frac{2n}{\lambda_2}} \delta_2^{\frac{3n}{\lambda_1}} - \delta_2^{\frac{2n-1}{\lambda_1}} \delta_1^{\frac{2n}{\lambda_2}}. \end{aligned}$$

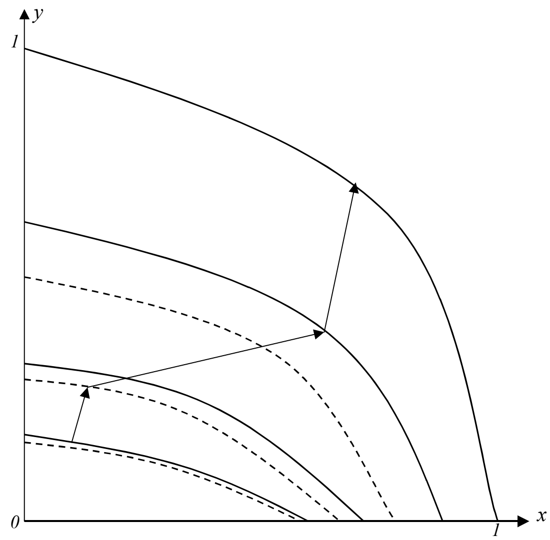

Figure 2: General Case

The inequality between A_n^1 and A_n^2, thus, the inclusion relation between G_n^1 and G_n^2, will not change with n. This is given in the following Lemma.

Lemma 5 *$A_n^1 \geqslant A_n^2$ for any n if and only if $\delta_1^{\frac{1}{\lambda_1}} \geqslant \delta_2^{\frac{1}{\lambda_2}}$.*

Proof. See appendix.

Given the inclusion relation of G_n^1 and G_n^2, we can specify the SPE path for any subgame $(x_t, y_t)_i$. Here we only consider the case with $G_n^1 \subset G_n^2$. If $(u(x_t), \upsilon(y_t)) \in G_n^1$, player i will make the offer such that $(u(x_{t+1}), \upsilon(y_{t+1}))$ is on the lower boundary of G_{n-1}^2,player $-i$ will reject the offer unless $n = 1$. If $(u(x_t), \upsilon(y_t)) \in G_n^2 \backslash G_n^1$, and $i = 2$, it is same to the above; if $i = 1$, player 1 will now choose to wait, i.e., making an offer no larger than y_t, and it will become the same as above from next period. Finally we can find some n such that $(0, 0) \in G_{n+1}^i$, and define the essentially unique SPE path for the whole game as before. When $(0, 0)$ is on the lower boundary of G_n^1 and $G_n^1 \subset G_n^2$, we have two SPE paths with player 1's choice of waiting or not at the beginning of the game.

Another important feature (See Figure 2) is that for each n, the lower boundaries of G_n^1 and G_n^2 are parallel, but they are not parallel to the boundary of the set of feasible utility pairs. The leaning of the lower boundaries will keep to the same direction as n increases, and the direction of leaning depends on $\delta_1^{\frac{1}{\lambda_1}} \lesseqgtr \delta_2^{\frac{1}{\lambda_2}}$.

From Figure 2, we can see that the direction of the leaning is in favor of the player with the larger $\delta_i^{\frac{1}{\lambda_i}}$ in terms of the final share in the SPE. The inclusion relation between G_n^1 and G_n^2 also has this effect. It is therefore natural to take $\delta_i^{\frac{1}{\lambda_i}}$ as the measure of 'bargaining power'. It is increasing with respect to both δ_i and λ_i.

The results for the general case are summarized in the following Proposition.

Proposition 2

(i) There is an essentially unique SPE path, the SPE strategies are specified as above given the location of (x_0, y_0) *or* $(0,0)$*;*

(ii) Either δ_1 *or* δ_2 *goes to* 1*, the number of periods delayed goes to infinity;*

(iii) If $G_{n+1}^1 \subseteq G_{n+1}^2$*, when* $(0,0) \in G_{n+1}^1$*, the equilibrium share is* $(1 - \delta_2^{\frac{n+1}{\lambda_2}}, \delta_2^{\frac{n+1}{\lambda_2}})$ with n periods of delay; *when* $(0,0) \in G_{n+1}^2 \backslash G_{n+1}^1$*, the equilibrium is* $(\delta_1^{\frac{n+1}{\lambda_1}}, 1 - \delta_1^{\frac{n+1}{\lambda_1}})$ with $n+1$ periods of delay. *If* $G_{n+1}^2 \subseteq G_{n+1}^1$ *and* $(0,0) \in G_{n+1}^1$*, the equilibrium share is* $(1 - \delta_2^{\frac{n+1}{\lambda_2}}, \delta_2^{\frac{n+1}{\lambda_2}})$ with n periods of delay.

4 Results under Weaker Assumption

In this section, we explore the possibility of obtaining similar results under weaker assumption. Under the interpretation of loss aversion, we are interested in the effects on the SPE path of weakening the assumption from the extremely loss aversion to some moderate level of loss aversion. For the sake of simplicity, we work with linear utility function and common discount factor.

Given the state variables (x_t, y_t), the (current) utility functions from the share of the pie are:

$$\textit{Player 1: } \widetilde{u}^t(x) = \begin{cases} x & if \;\; x \geq x_t, \\ x_t + \beta(x - x_t) & if \;\; x < x_t; \end{cases}$$

$$\textit{Player 2: } \widetilde{v}^t(y) = \begin{cases} y & if \;\; y \geq y_t, \\ y_t + \beta(y - y_t) & if \;\; y < y_t. \end{cases}$$

When $\beta = 1$, it is the standard setting in Rubinstein(1982); when β goes to infinity, it goes to the case with strong assumption as we discussed in section 3. We are now interested in the case with $1 < \beta < \infty$. Basically, what we are doing here is to replace the discontinuity of the utility function under the strong assumption by a kink. Reducing the sharpness of the kink is equivalent to weakening the assumption, or reducing the extent of the loss aversion. Our next Proposition says when β is large enough, we can still have essentially unique

SPE path with considerable delay, which is similar to the case under the strong assumption.

Proposition 3

When $\beta \geq \frac{1}{1-\delta}$,there is an essentially unique SPE path with considerable delay. The equilibrium share and the number of periods delayed is same to the case under the strong assumption for the same δ.

Proof. For $\beta \geq \frac{1}{1-\delta}$, we have $\frac{\beta-1}{\beta\delta} \geq 1$.

Given the current utility functions with loss aversion, impasse is the only SPE in a subgame $(x_t, y_t)_i$ with $\frac{\beta-1}{\beta}(x_t + y_t) > 1$. For subgame $(x_t, y_t)_i$ with $\frac{\beta-1}{\beta}(x_t + y_t) = 1$, we have either impasse or the immediate agreement $(\frac{\beta-1}{\beta}x_t, \frac{\beta-1}{\beta}y_t)$, both have payoff $(0, 0)$.

Consider a possible clinching offer $(A(x_t) - x_t)$ for player 1 in $(x_t, y_t)_1$, where $\delta \leq A(x_t) < 1$.

If $\frac{\beta\delta}{\beta-1} < A(x_t) < 1$:

$A(x_t) - x_t$ will be accepted since $\frac{\beta-1}{\beta}(x_{t+1} + y_{t+1}) = \frac{\beta-1}{\beta}\frac{A(x_t)}{\delta} > 1$. (Impasse if rejected)

If $\delta \leq A(x_t) \leq \frac{\beta\delta}{\beta-1}$:

if rejected, player 2 has to offer $\frac{\beta-1}{\beta}\frac{x_t}{\delta}$(zero payoff) in next period, which will not be rejected by player 1 since there will be impasse in following period. Player 2 then has $y^* = 1 - \frac{\beta-1}{\beta}\frac{x_t}{\delta}$. Choose minimum $A(x_t)$ such that $y^* \leq \frac{[A(x_t)-x_t]}{\delta}$, we get $A(x_t) \geq \delta + x_t/\beta$.

Now we can specify the clinching offer as following: $A(x_t) - x_t$,

where

$$A(x_t) = \begin{cases} \delta + x_t/\beta & if\ x_t \leq \frac{\beta\delta}{\beta-1} \\ x_t & if\ x_t \geq \frac{\beta\delta}{\beta-1}. \end{cases}$$

In any $(x_t, y_t)_1$, once the offer $A(x_t) - x_t$ is made by player 1, player 2 will accept immediately. On the other hand, any offer lower will be rejected because a similar counteroffer by player 2 in next period will not be rejected by player 1, and the payoff will be better for player 2. Similar clinching offer can be defined for player 2.

When will a player make the clinching offer?

Given $(x_t, y_t)_i$:

$$1 - [A(x_t) - x_t] \geq \delta[A(y_t/\delta) - y_t/\delta], for\ i = 1,$$
$$1 - [A(y_t) - y_t] \geq \delta[A(x_t/\delta) - x_t/\delta], for\ i = 2.$$

We get the following condition:

$$x_t + y_t \geq \frac{\beta}{\beta - 1}(\delta^2 + \delta - 1).$$

After this, it will be same to the case under strong assumption. A family of grids can be defined recursively as following:

$$G_n = \left\{(\alpha, \beta) \in [0,1]^2 : A_n \leq \alpha + \beta < A_{n-1}\right\},$$

where

$$A_n = \frac{\beta}{\beta - 1}(\delta^{2n} + \delta^{2n-1} - \delta^{n-1}).$$

Therefore, we get essentially unique SPE path again.

It is easy to check the equilibrium share and the number of periods delayed are the same as before. ■

It is natural to ask the following questions now: (1) Will there be delay on SPE path when β is small? (2) Will the SPE path converge to Rubinstein's result when β goes to 1? We do not have clean results on these questions. Our conjecture is that at least for not very large β, and for some δ, there is still delay in the bargaining game parameterized by (β, δ).

5 Discussion

One interesting observation is that the equilibrium play we obtain has a flavor of forward induction. When a player strongly believes that her opponent is a Bayesian maximizer, she has a reason to believe that her opponent is looking forward to a better payoff after observing a rejection, therefore, it is also likely that she believes that she has to improve the offer in order to reach an agreement. However, this is not exactly the forward induction as stated in Van Damme (1989). In his informal definition, if a player chooses between an outside option and a subgame with a unique and viable equilibrium, which is strictly better than the outside option, the equilibrium with players' playing the subgame is the only self-enforcing one. The SPE in Rubinstein's model satisfies this forward induction requirement, and an important reason is that the bargaining game in his model is isomorphic at every decision node where an offer is made. The reason that uniqueness is needed is that there maybe a coordination problem in the subgame. Our bargaining model is a perfect information game, this should not be a problem. The viability concerns the situation, in which there maybe further moves by both players in the subgame after the deviation from the outside option. In the bargaining game with infinite alternating-offers procedure, if we want to justify the deviation, i.e., the rejection of the SPE offer in Rubinstein's model, as a signal about future play, the viability requirement cannot be satisfied. Another problem is that there exists an outcome in the subgame, which is indifferent with the current offer for the player who has to decide to accept or not. To incorporate the forward induction analysis into bargaining model is not new. Dekel (1990) discussed the power of forward induction and stability in a two-period simultaneous bargaining game. We want to understand how the idea of forward induction could work in an infinite horizon game such as the bargaining game with alternating-offers procedure.

One closely related issue is about the common belief of rationality in extensive games. It is well known that we have counterintuitive SPEs in many cases, for example, the chain-store paradox in Selten (1978) and the centipede game in Rosenthal (1981). Kreps *et al.* (1982) obtain considerable cooperations in a finitely repeated prisoner's dilemma game by adding a small dose of "Tit-for-Tat" players. An exogenous lack of common belief of rationality results in a rational play which is inconsistent with the SPE. Reny (1992) argued that even there is common belief of rationality at the beginning of the game, it might still be possible to have a rational play inconsistent with the SPE. The lack of common belief of rationality may arise endogenously. In a perfect information game, the concept of SPE depends on the assumption of common knowledge of rationality, or, common belief of rationality at every information set. Reny (1992) showed that in the games with paradoxical SPEs, such as the "Take it or leave it" game, once a player deviates from the SPE path, it is impossible to retain common belief of rationality. Reny (1993) showed that in most two-player games with perfect information, it is impossible to have common belief of rationality everywhere. This challenged the salience of the concept of SPE and the theory of rationalizability in extensive form games developed by Bernheim (1984) and Pearce (1984) as well.

We think it is also reasonable to include the bargaining game into this family of extensive form games with counterintuitive SPEs. The unique SPE in Rubinstein's bargaining game also depends on the common belief of rationality at every decision node. This point can be made clear by looking at the concept of iterated conditional dominance and its application on Rubinstein's bargaining game introduced in Fudenberg and Tirole (1991). By iterative elimination of conditional dominated actions, the unique SPE can be obtained in Rubinstein's bargaining game. If we state the belief about rationality explicitly, the logic will be like this: player 1 believes player 2 is rational, thus she will not make an offer higher than δ; player 1 believes that player 2 believes player 1 is rational, thus she will not make an offer lower than $\delta(1-\delta)$; etc. This logic goes as an infinite sequence, the unique SPE will be the convergent point, which corresponds to the argument, *player 1 believes player 2 believes player 1...is rational*, with infinite length. Since the common belief of rationality gives the unique prediction of the play, the SPE path, the deviation from it will be the violation of the common belief of rationality, it will not hold any more. Once there is no common belief of rationality, it will be an important issue that how the players update their belief, belief about belief, etc., then given a specific belief updating rule, it may be rational to choose a play inconsistent with the SPE at the first place[7]. It will be of our interest for further study to formally model the idea included in this discussion. Our inclination now is that the relation between the Abreu and Gul (2000) and this possible line of research on bargaining will be parallel to that between Kreps *et al.* (1982) and the sequel papers by Reny (1992a, 1992b and 1993).

6 Conclusion

In this paper, we provide an explanation for the delay in real life bargaining as an alternative of the usual incomplete information approach. Our point of view is that in many real life bargaining situation, information asymmetry may not be the main underlying force for the delay, endogenous preferences or strategic commitment effects should be taken into consideration. Our preference assumption will be taken as incredible threat in classic analysis, and we try to provide a rationale for such kind of preference to make the incredible threat credible. We think both delegated bargaining with justification and preference with loss aversion have some explaining power.

The equilibrium play we obtain has some appealing features, especially, it has a flavor of forward induction. We also want to explore the possibility of providing epistemic foundation for such a play without relying on the preference assumption. This will involve the issues about rationality, belief system, and belief updating. The pioneering work on these topics focused on finite extensive form games, while as the bargaining game has two dimensional infinity, infinite

[7] In Rubinstein's model, there is no such updating, the common belief of rationality retains after any history, we take it as an extreme case of the belief updating.

horizon and infinite actions at each decision node for an offer to be made. This imposes the great difficulty on analysis. However, it will be important to extend the analysis to infinite horizon games, for example, the forward induction reasoning in Rubinstein's bargaining game or infinitely repeated games. We leave it for further investigation.

APPENDIX

Proof of Lemma 4

Let $\theta_1 = \delta_1^{\frac{1}{\lambda_1}}$, $\theta_2 = \delta_2^{\frac{1}{\lambda_2}}$.

$A_n^1 - A_n^2$

$= (\theta_1^{3n}\theta_2^{2n} - \theta_2^{3n}\theta_1^{2n}) + (\theta_1^{2n-1}\delta_2^{3n} - \theta_2^{2n-1}\theta_1^{3n}) - (\theta_1^{2n-1}\theta_2^{2n} - \theta_2^{2n-1}\theta_1^{2n})$

Divide both sides by $\theta_1^{2n-1}\theta_2^{2n-1} > 0$

$(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1}$

$= (\theta_1^{n+1}\theta_2 - \theta_2^{n+1}\theta_1) + (\theta_1 - \theta_2) - (\theta_1^{n+1} - \theta_2^{n+1})$

$= \theta_1\theta_2(\theta_1 - \theta_2)\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-1-i} + (\theta_1 - \theta_2) - (\theta_1 - \theta_2)\sum_{i=0}^{n}\theta_1^i\theta_2^{n-i}$

$= (\theta_1 - \theta_2)(\theta_1\theta_2\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-1-i} + 1 - \sum_{i=0}^{n}\theta_1^i\theta_2^{n-i})$

$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_1^{i+1}\theta_2^{n-i} - \sum_{i=0}^{n}\theta_1^i\theta_2^{n-i})$

$= (1) = (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}(\theta_1^{i+1}\theta_2^{n-i} - \theta_1^i\theta_2^{n-i}) + 1 - \theta_1^n)$

$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-i}(\theta_1 - 1) + (1 - \theta_1)\sum_{i=0}^{n-1}\theta_1^{n-1-i})$

$= (\theta_1 - \theta_2)(1 - \theta_1)\sum_{i=0}^{n-1}(\theta_1^{n-1-i} - \theta_1^i\theta_2^{n-i})$

or $= (2) = (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}(\theta_2^{i+1}\theta_1^{n-i} - \theta_2^i\theta_1^{n-i}) + 1 - \theta_2^n)$

$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_2^i\theta_1^{n-i}(\theta_2 - 1) + (1 - \theta_2)\sum_{i=0}^{n-1}\theta_2^{n-1-i})$

$= (\theta_1 - \theta_2)(1 - \theta_2)\sum_{i=0}^{n-1}(\theta_2^{n-1-i} - \theta_2^i\theta_1^{n-i})$

If $\theta_1 \geq \theta_2$, $(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1} \geq 0$ by (1);

if $\theta_1 < \theta_2$, $(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1} < 0$ by (2).

Thus, $A_n^1 \geqslant A_n^2$ for any n if and only if $\delta_1^{\frac{1}{\lambda_1}} \geqslant \delta_2^{\frac{1}{\lambda_2}}$.

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