# A Decision-Theoretic Basis for Choice Shifts in Groups

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## Abstract

The phenomenon of *choice shifts* in group decision-making is fairly ubiquitous in the social psychology literature. Faced with a choice between a "safe" and "risky" decision, group members appear to move to one extreme or the other, relative to the choices each member might have made on her own. Both risky and cautious shifts have been identified in different situations. This paper demonstrates that from an individual decision-making perspective, choice shifts may be viewed as a systematic violation of expected utility theory. We propose a model in which a well-known failure of expected utility — captured by the Allais paradox — is equivalent to a particular configuration of choice shifts. Thus, our results imply a connection between two well-known behavioral regularities, one in individual decision theory and another in the social psychology of groups.

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# **1** Introduction

How do groups confront risky decisions? Prior to 1961, the conventional wisdom on the subject was fairly unambiguous: relative to the members of that group, the group itself would be likely to favor compromises and avoid risk. This view was challenged in a series of experiments by Stoner (1961), which identified what became to be known as the "risky shift": when faced with the same decision problem, individuals within a group make *riskier* decisions, compared with the actions they would take outside the group. Stoner's results were confirmed again and again, not only in the United States with the usual experimental setting of college students, but also in different countries, cultures and social and occupational settings (see the special issue of the *Journal of Personality and Social Psychology*, **20**(3), 1971).

This isn't to say that risky shifts were ubiquitous. Not long after Stoner, a number of studies (most notably, Nordhøy (1962) and Stoner (1968)) provided some evidence for *cautious* shifts: a group tendency to exhibit greater restraint in risk-taking relative to the proclivities of individuals in that group. To accommodate both directions of change, the general phenomenon was ultimately referred to as a *choice shift*. Today, choice shifts in group decision-making are universally viewed as a consistent and robust phenomenon (Davis *et al.* (1992)). Obviously, this phenomenon has important implications for fields such as law, political science, sociology and — of course — economics.

From the perspective of economic theory, choice shifts present an interesting challenge. The reason is that the standard paradigm of group decision-making focuses on *pivotal events*, situations in which a particular individual's "vote" does affect the final outcome. But in such an event, the individual must act as he would in isolation. Therefore, the usual theory does not explain why an individual facing the same decision problem would make one decision on his own and another decision in a group, at least in situations in which all the relevant information is publicly known.<sup>1</sup>

But pivotality must embody an independence axiom for decision-making (or some variant thereof), so it is plain that the standard theory rests to some extent on the axiomatic foundations of expected utility. Indeed, our goal in this paper is to demonstrate that from an individual decision-making perspective, choice shifts may be viewed as a systematic violation of expected utility theory. But more than this, we propose a model in which a well-known failure of expected utility — captured by the Allais paradox is *equivalent* to a particular configuration of choice shifts (which include both risky and cautious shifts, but in a specific pattern). [In the model we describe, the Allais paradox is also equivalent to the notion of disappointment aversion.] Thus, our results imply a

<sup>&</sup>lt;sup>1</sup>To be sure, in situations in which individuals have private information about some common value, their behavior might differ between when they are deciding alone or in a group (see Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998)). Here we abstract from issues of information aggregation as most of the experimental settings did.

connection between two well-known behavioral regularities, one in individual decision theory and another in the social psychology of groups.

In its most general form, a choice shift refers to a situation in which an individual prefers lottery *A* to *B*, yet votes for lottery *B* in a group decision. The simplest parsing of this action suggests that the individual may be reluctant to choose a particular lottery on his own, yet some of his reluctance may be alleviated when the choice of lottery is affected by the decisions of others. Indeed, this sort of reasoning underlies the initial explanation of the risky shift, which emphasized the "diffusion of responsibility" (DOR) created by a group decision (see Section 2.2).

The DOR argument is motivated by the idea that when an individual does not win the highest prize in the risky lottery, he experiences a "discomfort" which is distinct from his direct utility from the prize. The source of discomfort can be interpreted as a "feeling of responsibility for not obtaining the high prize". An alternative interpretation, perhaps more relevant to the present discussion, is that the failure creates a feeling of disappointment (not necessarily a sense of guilt or failure of responsibility). By the same token, winning the high prize in the risky lottery should bring on a sense of "elation" over and above the utility that the prize produces directly. The DOR explanation can be interpreted as saying that individuals dislike disappointment more than they like elation (in other words, they are *disappointment-averse*), but that their feelings of disappointment are diffused when they participate in a group decision; hence, they are willing to take greater risks within the group.

This last assertion is, however, highly contingent on the nature of the group decision process. Specifically, much depends on what the predicted group outcome would be, conditional on the choices made by the agent (in this sense pivotal events still matter). If it is felt that the risky outcome is likely one way or the other, then the arguments in the previous pargraph apply. If, on the other hand, the safe outcome can be easily precipitated by our individual (as would happen, for instance, if unanimous consent is required to move a decision from safe to risky), then the very same disappointment-aversion might generate additional incentives for caution. In short, informal discussion suggests that disappointment-aversion should be related to a *family* of shifts, each contingent on the ambient environment. Our formal analysis supports this guess, and is able to describe the exact class of shifts that is equivalent to disappointment-aversion.

To establish this link we seek a model of individual decision-making that allows for (but does not insist on) disappointment aversion. The literature offers two candidates: the theory of disappointment aversion developed in Gul (1991) and the anticipated utility theory of relative disappointment aversion in Grant and Kajii (1998).<sup>2</sup> Both models characterize preferences that could be specified by a class of functionals one parameter

<sup>&</sup>lt;sup>2</sup>For a detailed discussion of different notions of disappointment aversion see Grant, Kajii and Polak (2001).

richer than expected utility (hence, both satisfy the desideratum of straying minimally from the standard theory). Gul's model studies a special case of preferences satisfying the so-called *betweenness* property, while Grant and Kajii's model is a special case of *rank-dependent preferences*.<sup>3</sup>

It turns out, however, that Gul's theory cannot account for choice shifts in our setup.<sup>4</sup> Indeed, this is true of any decision theory based on the betweenness axiom (we record this formally as Observation 2 in Section 4.3). Oddly enough — and quite parenthetically — there is a large body of evidence (a review of which can be found in Camerer and Teck (1994)) suggesting that choices in general are inconsistent with the implications of the betweenness axiom.<sup>5</sup> Moreover, there is mounting evidence in favor of non-expected utility models that are based on decision weights (see Starmer (2000)). The only model belonging to this class, which does not violate monotonicity and has a firm axiomatic foundation, is the model of rank-dependent preferences. In the light of this discussion, we adopt the Grant-Kajii framework. The framework is somewhat more specific than general rank-dependence because it defines disappointing outcomes in a particular way: intuitively, an outcome of a lottery is *relatively disappointing* if it is not the best outcome offered by that lottery. Thus in the Grant-Kajii approach, the mixing of lotteries with the best outcome in those lotteries has no effect on rankings, but the other mixes can have varied effects, depending on whether the individual in question is disappointmentaverse or not. It is also the case that in the Grant-Kajii framework, disappointmentaversion is equivalent to the display of the Allais paradox (see our formal statement of the paradox and the assertion of equivalence in Observation 1).

So much for individual decision theory. The other half of the connection involves group decisions. To this end, consider two lotteries, one of which is "risky" (r) and the other is "safe" (s); formal definitions will follow soon. When deciding on her own, an individual uniquely determines the choice between r and s. In contrast, when deciding in a group, that individual will generally cast her vote or express an opinion on the choice to be made between this pair, while remaining uncertain of the final outcome. We summarize the resulting group decision problem by a pair of numbers (a, b), where  $a \in (0,1)$  is the probability that our individual is *pivotal* (i.e., decides the outcome) and  $b \in [0,1]$  is the probability with which the group decides on s, conditional on our individual not being pivotal. Observe that this description admits a large class of aggregation rules within the group. We say that an individual exhibits a risky shift (cautious shift) over r and s within some group decision if she is indifferent between r and s, yet strictly prefers to "vote" for r ("vote" for s).

Our main result, Theorem 1, establishes an equivalence between disappointment-

<sup>&</sup>lt;sup>3</sup>See Starmer (2000) for a discussion and references on these two classes of preferences.

<sup>&</sup>lt;sup>4</sup>We consider environments in which a risky lottery over two outcomes is compared with a safe outcome. <sup>5</sup>In his survey of non-expected utility models, Chris Starmer writes "A second general lesson in the data seems to be *don't impose betweeness*" (Starmer (2000)).

aversion (equivalently, the Allais paradox) and a *particular* pattern of choice shifts. To describe this, consider a pair of lotteries — one risky and one safe — over which the individual is indifferent. Now embed the choice over these two lotteries into a group decision problem (a, b). Suppose that for every  $a \in (0, 1)$ , there exists a threshold  $\bar{b}$  (which may be sensitive to a) such that our individual strictly prefers to support the risky outcome when  $b < \bar{b}$ , and strictly prefers to support the safe outcome when  $b > \bar{b}$ . This assumed behavior generates a family of choice shifts. It is a complicated family, but it has structure: broadly speaking, the more an individual expects his compatriots to vote in a particular direction, the more he prefers that direction himself.

Theorem 1 asserts that disappointment-aversion is precisely equivalent to the above pattern of choice shifts, within the Grant-Kajii framework.

Our result therefore provides an intriguing connection between two disparate yet well-known "paradoxes" in two different literatures: one, the Allais paradox, and the other the phenomenon of choice shifts. In doing so, however, we do more than simply assert the existence of choice shifts; *we predict a particular structure for them* which may be of use in further experimental research.

We should observe that we do not necessarily believe that all choice shifts stem from non-expected utility. In particular, even when choices are over something as objective as monetary lotteries, there may be peculiar group psychologies at work of which our understanding is as yet imperfect. More abstractly, the notion of pivotality may be problematic not just on the grounds of the independence axiom, but for other reasons that may be worth future exploration. The current paper simply makes a particular connection.

Finally, while we do not enter into an "equilibrium analysis" of group effects, the above result has important implications for the equilibrium behavior of individuals engaged in a group decision. If in order to replace a safe (resp. risky) default, all group members need to unanimously agree on the risky (resp. safe) option, then voting for the default is weakly dominating. This suggests that in the special case of unanimity, the direction of the choice shift is uniquely determined by the nature of the default. At the same time, a choice shift towards the default is distinct from a status-quo bias. Two important factors determine whether or not a choice shift occurs. The first is the attitude of individuals towards risk. In particular, if preferences satisfy the betweeness axiom, no shift would occur. The second factor is the relation between the default and the alternative option. No shift would occur if the default were to be first-order stochastically dominated by the alternative option. As we explain in Section 4.6, this conclusion is consistent with much of the experimental evidence, most of which has focused on unanimous decision rules.

# 2 Choice Shifts in Groups

#### 2.1 Experimental Evidence

The discovery of choice shifts in groups is commonly attributed to James A. F. Stoner who in his doctoral dissertation (Stoner (1961)) introduced the *risky-shift phenomenon:* group members tend to take on higher levels of risk compared with the levels they accept as individuals outside the group. Stoner's study was based on a questionnaire with 12 hypothetical "life situations", which were originally designed by Wallach and Kogan (1959,1961) to investigate individual risk-taking propensities. In each "life situation" an individual faces the problem of choosing one of two courses of action, one of which is more risky than the other but also more rewarding if successful. Using a simple "test-retest" design in which individuals recorded their decisions in private and then reached unanimous decisions in six-member groups, Stoner found that group decisions were significantly more risky than the mean of the individual group members' prior decisions.

Evidence that group members might also make more cautious decisions was first demonstrated by Nordøy (1962). Nordøy noted that group effects were not consistent for all the 12 items used by Stoner. In particular, on one item — a question dealing with marriage — Stoner's subjects were consistently more cautious in their group decisions. Subsequently, other researchers (Rabow, Fowler, Bradford, Hofeller and Shibuya (1966), Stoner (1968), Vidmar and Burdeny (1969), Fraser, Gouge and Billig (1970)) have been able to construct situations in which groups shift toward caution. Therefore, it seems more appropriate to speak of a "choice shift" on the risk dimension, without committing to a particular direction of that shift.

The choice shift phenomenon was found to be remarkably robust, even to the point that the same decision problems produced shifts of comparable direction and size (Pruitt and Teger (1967)). The shifts are even consistent across countries, including Canada (Vidmar (1970)), England (Bateson (1966), Fraser *et al.* (1970)), France (Kogan and Doise (1969)), Germany (Lamm and Kogan (1970)), Israel (Rim (1963)) and New Zealand (Bell and Jamieson (1970)). Comparable shifts have been found for both men and women (Wallach, Kogan and Bem (1962)), for workers (Jamieson (1968)) and for people in professional and managerial capacities (Marquis (1962), Rim (1963), Siegel and Zajonc (1967)), as well as for college students.

Decision problems other than life situations have also been used. Early concern about generality led to the use of procedures that provided subjects with material incentives. Wallach *et al.* (1964) demonstrated a risky shift using choices among ability-test items with different probabilities of success, in which successful answers were rewarded with monetary payments. Shifts have also been found in studies using choices between lotteries with monetary prizes, though the direction of the shift has varied with the

study. Kogan and Zaleska (1969) and Pruitt and Teger (1969) demonstrated risky shifts, Zajonc, Wolosin, Wolosin and Sherman (1968) demonstrated shifts towards caution, while a series of studies by James H. Davis and his collaborators demonstrated shifts in both directions (see Davis, Hoppe and Hornseth (1968), Davis et al. (1974), Davis and Hintz (1982), and Johnson and Davis (1972)).

## 2.2 Explanations

The social psychology literature offers several competing theories for choice shifts in groups. The proponents of each theory present experimental results, which they interpret as evidence in favor of their proposed theory. However, many of these studies do not control for various factors that are absent from the proposed theory, but which could potentially explain the experimental findings. Moreover, in most cases, experimental evidence that supports one study seems to refute another. Consequently, it is difficult to judge which theory provides the correct explanation for the source of choice shifts. Nevertheless, it is clear that depending on the situation, various factors from all the different theories play a role in the emergence of choice shifts. In this section we discuss some of the leading theories that have been proposed in the literature. We explain why none of these theories provide a complete picture of the underlying cause of choice shifts in groups.

#### 2.2.1 Familiarization

This theory originated with Bateson (1966), who argued that group discussion led to increased familiarity with the available options. According to Bateson, this should lead to a higher willingness to take risk because of a general reduction in uncertainty. To validate his claim, Bateson demonstrated that subjects in isolation took greater risks when asked to write down the pros and cons associated with each alternative in hypothetical choice dilemmas. However, there are two main problems with this theory. First, not only does it fail to account for cautious shifts, it also fails to account for choice shifts when the available alternatives are objective monetary lotteries.<sup>6</sup> Second, many subsequent studies have failed to replicate the familiarization effect (Bell and Jamieson (1970), Fraser (1970) and St. Jean (1970)).

#### 2.2.2 Leadership

The basic tenet of this approach is that individuals whose attitude towards risk is extreme (in either direction) are more persuasive in group discussions, and this creates a choice

<sup>&</sup>lt;sup>6</sup>Of course, it is possible that individuals "learn" to "evaluate" such objective lotteries in group discussion, but this requires a deeper theory to accompany the familiarization hypothesis.

shift (Collins and Guetzkow (1964), Marquis (1962) and Rabow *et al.* (1966)). The main evidence for this theory is that in group discussion that produced a risky shift (or cautious shift) subjects perceived the members who advocated a high level of risk (or caution) to be more influential. The problem with interpreting this finding as evidence in favor of a leadership theory is that subjects may have viewed the highest risk-taker (or lowest risk-taker, for cautious shifts) as particularly influential because they shifted towards her to start with, rather than because she was particularly influential. Furthermore, Teger and Pruitt (1967) have demonstrated risky-shifts in group decisions without discussion. This seems to suggest that persuasive argumentation may not be necessary for producing choice shifts.

### 2.2.3 Values

Various theories, which we shall refer to collectively as "value theories", have postulated that groups shift in a direction toward which most members of the group are already attracted as individuals. The name "value theories" reflects the fact that these theories identify the cause of attraction, and hence the underlying cause of choice shifts, in widely held values. The evidence that support these theories comes from studies that employ hypothetical "life situation" choice dilemmas, which we mentioned in the previous section. These studies demonstrate that group members shift toward the option associated with the highest social value, where the social value of each option was obtained from each subject in a separate treatment that preceded the group decision (e.g., Brown (1965), Stoner (1968), Fraser (1970) and Vidmar (1970)). The difficulty in interpreting these findings is that it is unclear whether a particular option was risky or cautious. On the one hand, marriage may seem a risky venture because one may be uncertain about sharing his life with his future spouse. On the other hand, marriage may seem like a safe option when one considers the prospects of finding a suitable spouse in the future. Value theories are also limited in scope since they do not apply to choice shifts in situations that do not involve social and cultural values (as in a choice between monetary lotteries).

#### 2.2.4 Diffusion of Responsibility

This theory, advanced by Wallach, Kogan and their collaborators (Bem *et al.* (1965) and Wallach *et al.* (1962, 1964)) attributes the risky shift to reduced concern about the possible negative consequences of making a risky decision, which happens because the responsibility for negative consequences can be psychologically shifted from one's own shoulders to those of the other group members. The main source of support for this theory is the finding that group discussion produces a shift toward attempting more difficult test items (and thus toward greater risk) when success or failure on these items affects the welfare of the entire group than when it affects only the welfare of a single

group member. The main argument raised against this theory was that it could not explain cautious shifts (see, e.g., Pruitt (1971a,b)). However, as we have argued in the introduction, by appropriately extending the DOR theory to account for strategic risk, one may be able to accommodate both risky and cautious shifts.

# **3 Related Literature**

The approach we take in this paper is related to Battaglini, Benabou and Tirole (2001) (BBT). Both our paper and theirs apply a model of individual decision-making to explain differences between an individual's behavior in isolation and within a group. While BBT focus on the effect of observing one's peers on an individual's private actions, we focus on group decision-making. In addition, BBT's explanation is based on a model of dynamic inconsistency in individual decision-making. In contrast, our explanation is based on a model of an individual's attitude towards risk.

A key insight of our paper is the importance of the independence axiom in determining the behavior of individuals in groups. We show that when this axiom is violated, an individual does not behave as if he were pivotal, a standard assumption made in the literature. A similar insight can be found in Nakajima (2003). He shows that individuals who fall for the Allais paradox bid differently in a first price auction and in a Dutch auction, two auction designs that are considered strategically equivalent in standard auction theory.

# 4 Model

### 4.1 Preferences

Throughout, we shall be concerned with lotteries over (at most) three numerical outcomes: the given rewards x, y, and z, with x > y > z. The notation  $\mathbf{p} \equiv (p_x, p_y, p_z)$ will denote a generic lottery over these three outcomes, assigning probability  $p_i$  to the outcome i. Let  $\mathbf{P}$  be the space of all such lotteries. The individual in question will have a preference ordering  $\succeq$  defined on  $\mathbf{P}$ .

For reasons discussed in the introduction, we shall assume that  $\succeq$  lies in the space of preferences described in Grant and Kajii (1998). Each preference ordering in this class is known to have the following representation: there exists  $\alpha > 0$  and a "utility indicator" u such that  $\mathbf{p} \succeq \mathbf{q}$  if and only if

$$\sum_{k \in \mathbf{supp}(\mathbf{p})} \left( \left[ p_k + w_k(\mathbf{p}) \right]^{\alpha} - w_k(\mathbf{p})^{\alpha} \right) u(k) \ge \sum_{k \in \mathbf{supp}(\mathbf{q})} \left( \left[ q_k + w_k(\mathbf{q}) \right]^{\alpha} - w_k(\mathbf{q})^{\alpha} \right) u(k)$$
(1)

where the notation  $w_k(\mathbf{p})$  stands for the probability of a worse outcome than outcome k, under the lottery p. With this functional form pinned down, the entire space of Grant-Kajii preferences is parameterized by the utility indicator u and the index  $\alpha$ . Note that expected utility corresponds to the case of  $\alpha = 1$ . An individual is said to be *disappointment-averse* whenever  $\alpha < 1$  (see Grant and Kajii (1998)).

## 4.2 The Allais Paradox

Consider a *risky lottery*  $\mathbf{r} = (p, 0, 1 - p)$  that places weight only on x and z, and a *safe lottery*  $\mathbf{s} = (0, 1, 0)$  that places weight only on the intermediate value y. Assume that our individual is indifferent between these two options. Now mix each option using the worst outcome z with some weight  $q \in (0, 1)$ . The Allais Paradox refers to a situation in which the individual prefers the risky lottery (mixed with z) to the safe lottery (mixed with z) for some weight q.<sup>7</sup>

Formally say that an individual *exhibits the Allais paradox* if whenever she is indifferent between **r** and **s** she strictly prefers the lottery ((1-q)p, 0, q+(1-q)(1-p)) to (0, 1-q, q) for some  $q \in (0, 1)$ . [Note, obviously, that an expected utility maximizer would remain indifferent over the latter pair of choices if she is indifferent over the former pair.]

The notion of disappointment aversion was introduced as an intuitive explanantion for the Allais paradox (see Gul (1991)). In the Grant-Kajii framework, disappointement aversion is tightly related to the Allais paradox, as the following observation shows.

Observation **1** Assuming Grant-Kajii preferences, an individual exhibits the Allais Paradox if and only if he is disappointment-averse.

The above observation follows from Lemma 3 in the Appendix.

## 4.3 Choice Shifts

We develop the notion of choice shifts by considering the starkest possible descriptions of risk and safety. As in the case of the Allais paradox, we are interested in choices over a risky lottery **r** that places weight p on x and 1 - p on z, and a safe lottery **s**, given by the realization of y for sure. However, owing to the nature of group interaction, the individual must confront more complex (compound) lotteries.

Specifically, a key distinction between individual decision-making and group decisionmaking is that the latter introduces strategic uncertainty. The individual decision problem is just a choice between **r** and **s**. In contrast, in a group situation an individual will

<sup>&</sup>lt;sup>7</sup>This is not Allais' (1979) most famous example. This particular variant is sometimes referred to as the common ratio effect. For more on this systematic violation of the independence axiom, see Starmer (2000).

generally cast her vote or express an opinion on the choice to be made between this pair,<sup>8</sup> while usually remaining uncertain of the final outcome. In its most abstract and general form, a group decision problem (from the point of view of a given individual) may be represented by a pair  $\mathbf{g} \equiv (a, b)$ , where  $a \in (0, 1)$  is the probability that our individual is *pivotal* (i.e., decides the outcome) and  $b \in [0, 1]$  is the probability with which the group decides on  $\mathbf{s}$ , conditional on our individual not being pivotal. The great advantage of this description is, of course, that it admits a large class of aggregation rules within the group. [A possible disadvantage is that *a* and *b* don't simply depend on the nature of the group problem but on the behavior of other group members. This is not an "equilibrium analysis".] Note that the restriction a > 0 means that our individual must have *some* say within the group, and the restriction a < 1 means that she cannot be a dictator.

We will say that an individual exhibits a *risky shift* over **r** and **s** within the group problem **g** if she is indifferent between **r** and **s**, yet *strictly* prefers to "vote" for **r** in the context of that group problem. Likewise, she exhibits a *cautious shift* over **r** and **s** (within the group problem **g**) if she is indifferent between **r** and **s**, yet *strictly* prefers to "vote" for **s** in the context of that group problem. A shift — risky or cautious — is generally referred to as a *choice shift*.

Recall from the introduction that an alternative notion of disappointment aversion is offered by Gul (1991). Gul's notion is based on the *betweeness axiom*, which states that a probability mixture of two lotteries is intermediate (in preference space) to the original lotteries. However, in our formulation of a group decision, an individual with preferences satisfying the betweeness axiom cannot exhibit a choice shift:

Observation **2** If  $\succeq$  satisfies the betweeness axiom, then in any group decision g, an individual weakly prefers to vote for **r** (**s**) if and only if she weakly prefers **r** to **s** (**s** to **r**).

Intuitively, the betweeness axiom reduces to the independence axiom when there are only two available lotteries (i.e., **r** and **s**).

## 4.4 The Main Result

**Theorem 1** Assuming Grant-Kajii preferences, the following statements are equivalent:

1. An individual is disappointment averse, or equivalently exhibits the Allais Paradox.

2. Over any pair **r** and **s** between which the individual is indifferent, and any  $a \in (0, 1)$ , there exists  $b^* \in (0, 1)$  such that for any group decision problem g = (a, b) with  $b < b^*$ , she exhibits a risky shift, while if  $b > b^*$ , she exhibits a cautious shift.

<sup>&</sup>lt;sup>8</sup>Note that individuals don't necessarily vote within the group problem. Depending on the context, one may be modelling votes, advice, command or suggestion.

The above result establishes that in the Grant-Kajii framework, disappointment aversion is equivalent to a preference reversal that generates both risky and cautious shifts. Recall from the introduction that disappointment aversion is intuitively related to the DOR explanation of risky shifts. Therefore our theorem can be viewed as a formalization of DOR, at least in the context of an explanation for risky shifts.

But what we have is more than a formalization. The DOR explanation was dismissed when several studies reported evidence of cautious shifts. The common view was that DOR can only explain risky shifts but not their cautious counterparts.<sup>9</sup> We argue that by properly formalizing the notion of DOR through disappointment aversion, we are able to accommodate both types of choice shifts. At the same time, this eclecticism is sharper than no prediction at all, because a *particular* pattern of choice shifts is described in (2) of the proposition.

To illustrate how a cautious shift may arise due to disappointment aversion consider the following example. Suppose that a unanimity vote is required to replace a sure status-quo, *s*, with a risky alternative, *r*. Then by voting "risky" an individual effectively generates a mix between the safe and risky alternative, a lottery in which the probability of disappointment is higher than in the original risky alternative. Hence, if the statusquo and the risky alternative are nearly indifferent when the individual decides on his own, voting for the risky alternative becomes *less* preferred in the unanimity vote.

### 4.5 Discussion

Theorem 1 states that (within the Grant-Kajii framework) disappointment aversion — or the Allais paradox — is equivalent to a fairly complex set of choice shifts. In particular, the more likely is the group to settle on the risky outcome conditional on the nonpivotality of our individual, the more likely is our individual to react with a risky shift. Otherwise, if the group is likely to choose the safe outcome in the non-pivotal event, a cautious shift ensues.

As an example, consider the case of unanimity, and suppose that in the absence of unanimous agreement the fallback or default option is the risky outcome. Then the pivotal event is one in which every compatriot of our individual votes for the safe option; she then decides the outcome with her vote. In the nonpivotal event, therefore, there has been at least *one* vote for the risky outcome, so that the safe outcome can never be chosen in this event. In other words, b = 0 in this example. Theorem 1, part 2, states

<sup>&</sup>lt;sup>9</sup>In his introduction to the special issue of the *Journal of Personality and Social Psychology* on Choice Shifts (Vol. 20, No. 3, 1971), Dean G. Pruitt lists five arguments against the DOR explanation. The first argument is raised against the common practice of using hypothetical dilemma problems. The next three arguments criticize the role of group discussion on the occurrence of shifts. The final argument is concerned with the inability of DOR to explain the cautious shift. Since our setup abstains from group discussions and uses monetary lotteries, the first four arguments cannot be raised against our approach.

that a risky shift will occur.

If the default option in the unanimity example is the safe outcome, then by the same logic, the nonpivotal event will generate the safe option for sure, so that b = 1. A cautious shift is then the predicted outcome.

However, it is important to note that a choice shift is distinct from a simple statusquo bias. To see this, note that we have assumed that individuals are initially indifferent between s and r. If, for example, s paid x dollars for sure, then voting for r when it is the default is weakly *dominated*. In addition, when preferences satisfy the independence axiom, no choice shifts can occur. Finally, note that the theorem captures situations which are more general than unanimity voting.

One interpretation of the default is that it is determined by social convention. For example, a football team, huddled in the final seconds of an important game, would choose a risky play that may lead to victory over a safe play that would guarantee a tie, unless all the teammates favor the safe play.<sup>10</sup> Another example concerns a team of physicians that recommends the use of a new experimental drug (which was successful in some cases but led to complications in others) only if there is unanimous agreement that this drug is preferred to the conventional option (which, say, alleviates the pain but does not cure the illness). In these examples and in many similar situations, one course of action is usually considered the social norm: in one situation the norm may call for risk-taking, while in another the norm may involve caution. Hence, in a context where *s* (resp. *r*) is considered to be the social norm, a group decision can be interpreted as a situation in which a group of individuals must reach a consenus in order to replace *s* with *r* (resp. *r* with *s*).

Another interpretation of the default is not that it represents a social convention, but simply a fallback option (given by majority voting, for instance), when consensus is impossible. This is typically the case when no overt social or cultural values are at stake (e.g. a decision between investment opportunities, job candidates). In these problems, it is common for groups who cannot reach a consensus to regard the alternative favored by the majority as the socially desirable default (or compromise). Under this interpretation, a decision rule can be viewed as a situation in which a group needs to reach a consensus, otherwise, the majority would implement its favorite outcome.

Finally, Theorem 1 also implies that individuals exhibit a choice shift if and only if they exhibit the Allais paradox. Thus, our results imply a connection between two well known behavioral regularities, one in individual decision theory and another in the social psychology of groups.

<sup>&</sup>lt;sup>10</sup>Wallach, Kogan and Bem (1962) use this decision problem as an example of a group decision in which the risky option is considered to be the social norm.

#### 4.6 Relation to Experimental Evidence

Given that choice shifts have been extensively studied in the experimental literature, it is important to relate that literature to the hypotheses raised in the previous sections. The first observation we make is that virtually all studies on unanimous decision rules have reported a unique choice shift. Moreover, studies that employed the same decision problem reported the same type of choice shift. In addition, our hypotheses regarding the nature of the shift has also been confirmed in many of the studies.

Our first hypothesis applies to decision problems in which social and cultural values are embedded (as in the first interpretation of the preceding section). Some of the pioneering studies on decision problems of this type include Stoner (1968), Fraser *et al.* (1970) and Vidmar (1971). The basic finding in these studies was that for items on which the widely held values favored the risky alternative, unanimous group decisions were more risky than the average of the initial individual decisions. Group decisions tended to be more cautious on items for which widely held values favored the cautious alternative. These results were replicated in later studies, many of which are discussed in Pruitt (1971a,b).

Our second hypothesis applies to decision problems devoid of overt social content. Given our interpretation of default options in decision problems of this type, we hypothesize that shifts would occur in the direction of the outcome, which is believed to be favored by the majority of group members. Numerous studies have demonstrated choice shifts using lotteries with monetary prizes where subjects' choices determined their earnings (e.g. Davis, Hoppe and Hornseth (1968), Davis *et al.* (1974), Davis and Hintz (1982), Johnson and Davis (1972), Zajonc *et al.* (1968)). Of particular interest to us are the studies by Davis *et al.* (1974) and Davis and Hintz (1982). These studies report evidence suggesting that in binary decision problems, the direction of choice shifts in groups is largely predicted by the preferences of the majority of individuals.

## 5 **Proofs**

### 5.1 **Proof of Theorem 1**

We begin with a risky lottery, described by  $\mathbf{r} = (p, 0, 1 - p)$  and a safe option  $\mathbf{s} = (0, 1, 0)$ , between which our individual is indifferent. The decision problem she faces in a group decision  $\mathbf{g} = (a, b)$  is a choice between the two compound lotteries

$$\mathbf{r}^* \equiv a [r] + (1-a) [b [s] + (1-b) [r]]$$

and

$$\mathbf{s}^* \equiv a[s] + (1-a)[b[s] + (1-b)[r]]$$

This means that  $\mathbf{r}^*$  may be viewed as the lottery  $(r_x, r_y, r_z)$ , where

$$r_x = [a + (1 - a)(1 - b)]p,$$
  

$$r_y = (1 - a)b,$$
  

$$r_z = [a + (1 - a)(1 - b)](1 - p).$$
(2)

Similarly,  $\mathbf{s}^*$  may be viewed as the lottery  $(s_x, s_y, s_z)$ , where

$$s_x = (1-a)(1-b)p,$$
  

$$s_y = a + (1-a)b,$$
  

$$s_z = (1-a)(1-b)(1-p).$$
(3)

Consider the (Grant-Kajii) utility value of the lottery  $r^*$ ; using (2), it is given by

$$V(\mathbf{r}^*) \equiv u(x) \left[1 - (1 - \{a + (1 - a)(1 - b)\}p)^{\alpha}\right] + u(y) \left[(1 - \{a + (1 - a)(1 - b)\}p)^{\alpha} - \{[a + (1 - a)(1 - b)](1 - p)\}^{\alpha}\right] + u(z) \{[a + (1 - a)(1 - b)](1 - p)\}^{\alpha}$$

We may rewrite this as follows:

$$V(\mathbf{r}^*) = u(x) - \Delta_1 \left[ J(a,b) - ap \right]^{\alpha} - \Delta_2 \left[ K(a,b) + a(1-p) \right]^{\alpha},$$
(4)

where  $\Delta_1 \equiv u(x) - u(y) > 0$ ,  $\Delta_2 \equiv u(y) - u(z) > 0$ ,  $J(a, b) \equiv 1 - (1 - a)(1 - b)p$ , and  $K(a, b) \equiv (1 - a)(1 - b)(1 - p)$ .

In a similar vein, consider the utility value of  $s^*$ ; using (3), it is

$$V(\mathbf{s}^*) \equiv u(x) \left[ 1 - (1 - (1 - a)(1 - b)p)^{\alpha} \right] + u(y) \left[ (1 - (1 - a)(1 - b)p)^{\alpha} - \{(1 - a)(1 - b)(1 - p)\}^{\alpha} \right] + u(z) \{(1 - a)(1 - b)](1 - p)\}^{\alpha}$$

Rewriting,

$$V(\mathbf{s}^*) = u(x) - \Delta_1 J(a, b)^{\alpha} - \Delta_2 K(a, b)^{\alpha}.$$
(5)

Combining (4) and (5), we may conclude that

$$V(\mathbf{r}^*) - V(\mathbf{s}^*) = \Delta_1 \left[ J(a,b)^{\alpha} - \{ J(a,b) - ap \}^{\alpha} \right] - \Delta_2 \left[ \{ K(a,b) + a(1-p) \}^{\alpha} - K(a,b)^{\alpha} \right].$$
(6)

**Lemma 2**  $V(\mathbf{r}^*) - V(\mathbf{s}^*)$  is continuous in *b*. It is strictly decreasing in *b* if  $0 < \alpha < 1$ , and is nondecreasing in *b* if  $\alpha \ge 1$ .

**Proof.** Continuity is trivial. For any given  $a \in (0,1)$  notice that J(a,b) is strictly increasing in *b* while K(a,b) is strictly decreasing in *b*. Therefore, if  $0 < \alpha < 1$ ,  $J(a,b)^{\alpha} - \{J(a,b) - ap\}^{\alpha}$  is strictly decreasing in *b* and  $\{K(a,b) + a(1-p)\}^{\alpha} - K(a,b)^{\alpha}$  is strictly increasing in *b* by a standard property of strictly concave functions. It follows that  $V(\mathbf{r}^*) - V(\mathbf{s}^*)$  is strictly decreasing in *b* if  $0 < \alpha < 1$ . The remaining assertion follows in similar fashion.

Lemma 3 The following statements are equivalent:

[1] An individual exhibits the Allais paradox;

[2]  $\alpha < 1$ , and

[3]  $\Delta_1 + q^{\alpha}\Delta_2 - [1 - p(1 - q)]^{\alpha}(\Delta_1 + \Delta_2) > 0$  for every p and q strictly between 0 and 1 such that (p, 0, 1 - p) is indifferent to (0, 1, 0).

**Proof.** Construct the Grant-Kajii utility value of the lottery  $\mathbf{r}_q \equiv ((1-q)p, 0, q + (1-q)(1-p))$ , which is

$$V(\mathbf{r}_q) = u(x)\{1 - [1 - p(1 - q)]^{\alpha}\} + u(z)[q + (1 - q)(1 - p)]^{\alpha}$$
  
=  $u(x) - [1 - p(1 - q)]^{\alpha}(\Delta_1 + \Delta_2).$ 

Similarly, the value of the lottery  $\mathbf{s}_q \equiv (0, 1 - q, q)$  is

$$V(\mathbf{s}_q) = u(y)\{1 - q^{\alpha}\} + u(z)q^{\alpha}$$
  
=  $u(y) - q^{\alpha}\Delta_2,$ 

so that

$$V(\mathbf{r}_q) - V(\mathbf{s}_q) = \Delta_1 + q^{\alpha} \Delta_2 - [1 - p(1 - q)]^{\alpha} (\Delta_1 + \Delta_2).$$

At q = 0 the value above is zero by the initial indifference condition (in the definition of the Allais Paradox), and for q = 1 the value above is zero anyway. Routine computation shows that the value is strictly positive for all intermediate values of q if  $\alpha < 1$ , and that it is nonpositive for all intermediate values of q if  $\alpha \ge 1$ . Consequently, the Allais Paradox holds if and only if  $\alpha < 1$ , and indeed, then, the stricter form [3] in the statement of the lemma also applies.

We now prove that part 2 of the theorem implies part 1. By part 2,  $V(\mathbf{r}^*) - V(\mathbf{s}^*) > 0$  for some value of *b* and  $V(\mathbf{r}^*) - V(\mathbf{s}^*) < 0$  for some larger value of *b*. By Lemma 2, this can only happen if  $0 < \alpha < 1$ . By Lemma 3, our individual exhibits the Allais Paradox.

To establish the converse, assume that the Allais paradox holds. Then [2] of lemma 3 is satisfied, so that  $0 < \alpha < 1$ . By Lemma 2,  $V(\mathbf{r}^*) - V(\mathbf{s}^*)$  as given in (6) must be

continuous and strictly decreasing in *b*. It remains to examine the endpoints. First, set b = 0; then J(a, 0) = 1 - (1 - a)p and K(a, 0) = (1 - a)(1 - p). Consequently,

$$V(\mathbf{r}^*) - V(\mathbf{s}^*) = \Delta_1 \left[ \{1 - (1 - a)p\}^\alpha - (1 - p)^\alpha \right] - \Delta_2 \left[ (1 - p)^\alpha - \{(1 - a)(1 - p)\}^\alpha \right] \\ = \{1 - (1 - a)p\}^\alpha \Delta_1 + \{(1 - a)(1 - p)\}^\alpha \Delta_2 - (1 - p)^\alpha (\Delta_1 + \Delta_2),$$

so that

$$\frac{V(\mathbf{r}^*) - V(\mathbf{s}^*)}{\{1 - (1 - a)p\}^{\alpha}} = \Delta_1 + \left[\frac{(1 - a)(1 - p)}{1 - (1 - a)p}\right]^{\alpha} \Delta_2 - \left[\frac{1 - p}{1 - (1 - a)p}\right]^{\alpha} (\Delta_1 + \Delta_2)$$
(7)

Define  $q \equiv \frac{(1-a)(1-p)}{1-(1-a)p}$ ; then  $q \in (0,1)$ . Routine computation allows us to rewrite (7) as

$$\frac{V(\mathbf{r}^*) - V(\mathbf{s}^*)}{\{1 - (1 - a)p\}^{\alpha}} = \Delta_1 + q^{\alpha} \Delta_2 - [1 - p(1 - q)]^{\alpha} (\Delta_1 + \Delta_2).$$

But [3] of Lemma 3 is in force, so the right hand side of this expression is strictly positive. So we've shown that at b = 0,  $V(\mathbf{r}^*) > V(\mathbf{s}^*)$ .

Finally, set b = 1; then J(a, 1) = 1 and K(a, 1) = 0. Consequently,

$$V(\mathbf{r}^*) - V(\mathbf{s}^*) = \Delta_1 \left[ 1 - (1 - ap)^{\alpha} \right] - \Delta_2 \left[ a(1 - p) \right]^{\alpha}$$
(8)

Write down the indifference condition between  $\mathbf{r}$  and  $\mathbf{s}$ ; this is easily seen to be the restriction that

$$\Delta_1 \left[ 1 - (1-p)^{\alpha} \right] - \Delta_2 (1-p)^{\alpha} = 0.$$
(9)

But a < 1, so given (9), the expression in (8) must be strictly negative.

Combining these observations, we have shown that there exists  $b^* \in (0,1)$  such that  $V(\mathbf{r}^*) - V(\mathbf{s}^*) > 0$  if  $b < b^*$  and  $V(\mathbf{r}^*) - V(\mathbf{s}^*) < 0$  if  $b > b^*$ , which establishes part 2 of the theorem.

### 5.2 Proof of Observation 1

Assume that  $\mathbf{r} \succeq \mathbf{s}$ . The betweeness axiom implies that for any  $b \in [0, 1]$ ,

$$\mathbf{r} \succeq b[s] + (1-b)[r] \succeq \mathbf{s}$$

Employing betweeness once again, we obtain that for any  $a \in [0, 1]$ ,

$$a[r] + (1-a)[b[s] + (1-b)[r]] \succeq b[s] + (1-b)[r]$$

and

$$b[s] + (1-b)[r] \succeq a[s] + (1-a)[b[s] + (1-b)[r]]$$

Hence, using the notations of Section 5.1,

 $r^* \succeq s^*$ 

The symmetric argument applies to the case of  $\mathbf{s} \succeq \mathbf{r}$ .

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