Alternative Central Bank Credit Policies for Liquidity Provision in a Model of Payments

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I explore alternative credit policies in a theoretical model where (i) money is necessary as a means of payment, (ii) there is a shortage of liquidity that a central bank addresses through the extension of credit, (iii) money is necessary to repay debts, and (iv) the incentives to default are explicit and contingent on the credit policy designed. Using a mechanism design approach, I compare a credit policy of charging an interest rate on credit with that of requiring the posting of collateral. I find that the pricing policy can implement good allocations while the collateral policy cannot whenever collateral bears an opportunity cost.

Key Words: Payments systems, central banking, liquidity, collateral

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1. INTRODUCTION

A primary role of a central bank is to facilitate a safe and efficient payments system. One source of inefficiency in payments systems is a potential shortage of liquidity. Central banks often respond by providing liquidity through the extension of credit. Because of this, a central bank must manage its exposure to the risk that an agent does not repay. Some central banks, such as the European Central Bank, manage this risk by requiring borrowers to post collateral. Others, such as the Federal Reserve in the U.S., charge an explicit interest rate on credit and limit the amount any particular agent can borrow. In this paper, I explore these alternative credit policies in a theoretical model of payments and offer a rationale for why some central banks may choose one credit policy over another. I do this in a mechanism design framework, paying particular attention to the moral hazard issues associated with the repayment of debt that alternative credit policies aim to mitigate.

The payment systems most relevant to this paper are large-value payment systems which are mainly intraday, interbank payment systems. Many large-value payment systems are operated by central banks and are often real-time gross settlement (RTGS) systems. In an RTGS system, payments are made one at a time, with finality, during the day. Examples of RTGS systems include Fedwire operated by the Federal Reserve in the U.S. and TARGET, operated by the European Central Bank in the EMU.[3] Because payments are made one at a time, liquidity is needed to complete each transaction. If participants do not have enough liquidity to make a payment at a particular point in time, they can typically borrow funds

[3] TARGET is the collection of inter-connected domestic payment systems of the EMU that settle cross-border payments denominated in euros.

from the central bank, which they then pay back at the end of the day. The central bank faces a trade-off between supplying this intraday liquidity at little or no cost to enhance the efficiency of the system and accounting for moral hazard issues associated with the extension of credit. Of fundamental interest in this paper is how a central bank should design a credit policy for the provision of liquidity in an RTGS system to deal with moral hazard associated with debt repayment.

The main contribution of this paper is a framework with which to study the alternative credit policies of central banks. The key features of the framework are (i) default decisions of agents are endogenous, and (ii) mechanism design. The first is important to rigourously introduce a moral hazard problem when debt is extended. The second is a useful approach to evaluate what good outcomes are achievable under alternative credit policies taking into account agents' incentives to default.

This framework is applied to a model of payments that is similar to that of Freeman (1996). Such a model is attractive because it captures some of the key ingredients of actual large-value payments systems. First, fiat money must be necessary as a means of payment. Second, there must be a need to acquire liquidity (in the form of fiat money) during the day to make such payments. Third, money must be necessary to repay debts at the end of the day.

An important abstraction in Freeman's original model is that there is costless enforcement that exogenously guarantees that debts are repaid. Such an abstraction has led to conclusions by Freeman (1996), Green (1997), Zhou (2000), Kahn and Roberds (2001) and Martin (2003) that a credit policy of free liquidity provision is optimal. These conclusions are immediate given that there is no explicit moral hazard problem in most of these models[4]. Moreover, Mills (2004) endogenizes the repayment decision of agents under costless enforcement in Freeman's model and shows that money is not necessary to repay debts if enforcement is too strong and so the need for liquidity in the model is questioned. As in Mills (2004), I shall depart from this abstraction so that the default decision of agents is not trivial.

In the context of the background environment, I look at two alternative credit policies that resemble some of the features of such policies in actual large-value payment systems. The first such policy is that of costly enforcement and pricing. The central bank invests in a costly enforcement technology that allows it to punish defaulters by confiscating some consumption goods. The second policy is that of requiring those who borrow from the central bank to post collateral. The central bank does not charge an explicit interest rate on debt. Collateral, however, may have an opportunity cost in that it cannot earn a return that it otherwise would have.

I use a mechanism design approach to see if the credit policies can achieve good allocations, which I define to be Pareto-optimal allocations. It is possible for both types of credit policies to implement these good allocations. In the case of the pricing policy, I find an example of where the optimal intraday interest rate is positive because of a requirement for the central bank to recover its costs of enforcement. This differs from the aforementioned literature and supports a suggestion made by Rochet and Tirole (1996) that the intraday interest rate be positive because monitoring and enforcement is costly. In the case of collateral, if it does not have an opportunity cost, such a policy can implement a good allocation that is first-best. If, on the other hands, there is a positive opportunity cost

[4] Martin (2003) is an exception. See below.

of collateral, requiring collateral adds binding incentive constraints that distort the allocation away from Pareto-optimality because collateral serves as a credit constraint.

Martin (2003) models moral hazard differently. He endogenizes some agents' choice of risk arising from a central bank's free provision of liquidity, but not the choice of default as is done here. Agents can choose a safe production technology or a risky one that exogenously leads to some default as in Freeman (1999). He finds that a collateral policy is preferred to debt limits in mitigating this risk. The collateral in his model is debt issued by private agents who exogenously commit to repayment and does not bear an opportunity cost.

Finally, in this paper the default decision of agents is endogenous, but the liquidity shortage is exogenous. This is complementary to an area in the literature by Bech and Garratt (2003), Angelini (1998) and Kobayakawa (1997). These papers endogenizes the liquidity shortage by focusing on the incentives agents have to coordinate the timing of payments given alternative credit policies, but do not endogenize the need for such credit policies.

The paper is organized as follows. Section 2 presents the environment while Section 3 provides a benchmark of optimal allocations. Sections 4 and 5 contain the main results as pertains to the credit policy with pricing and collateral, respectively. Section 6 concludes.

2. THE ENVIRONMENT

The model is a variation of both Freeman (1996) and Mills (2004). It is a pure exchange endowment model of two-period-lived overlapping generations with two goods at each date, *good 1* and *good 2*. The economy starts at date $t = 1$. There is a $[0,1]$ continuum of each of two types of agents, called *creditors* and *debtors*, born at every date.[5] These two types are distinguished by their endowments and preferences.

Each creditor is endowed with y units of good 1 when young and nothing when old. Each debtor is endowed with x units of good 2 when young and nothing when old.

Let $c^t_{zt'}$ denote consumption of good $z \in \{1,2\}$ at date t' by a creditor of generation t. The utility of a creditor is $u(c^t_{1t}, c^t_{2,t+1})$, where $u : \Re^2_+ \to \Re$. Notice that a creditor wishes to consume good 1 when young and good 2 when old. The function u is strictly increasing and concave in each argument, is C^1, and $u'(0) = \infty$ and $u'(\infty) = 0$.

Let $d^t_{zt'}$ denote consumption of good $z \in \{1,2\}$ at date t' by a debtor of generation t. The utility of a debtor born at date t is $v(d^t_{1t}, d^t_{2t})$ where $v : \Re^2_+ \to \Re$. Hence, a debtor wishes to consume both good 1 and good 2 when young. A debtor does not wish to consume either good when old. The function v is strictly increasing and concave in each argument, is C^1, and $v'(0) = \infty$ and $v'(\infty) = 0$.

At date $t = 1$, there is a [0,1] continuum of initial old creditors. These creditors are each endowed with M divisible units of fiat money.

It is assumed that agents cannot commit to trades and that there is no public memory of trading histories. It is also assumed that agents do not consume any goods until the end of the period.

There is also an institution called a central bank that has three technologies unique to it.[6] The first technology is the ability to print fiat money. The second is

[5] The name given to creditors is a bit misleading because these agents never lend in equilibrium.

[6] What I call a central bank may also be interpreted as a private clearinghouse that is separate from the other agents. As noted in Green (1997), it remains an open question as to whether the liquidity-providing institution in the model should be a public or private one, and is beyond the

a record-keeping technology that enables the central bank to keep track of individual balances of both money and goods that a private agent may have with it. The third technology is an enforcement technology that can be acquired at a real resource cost $\gamma > 0$ per period.[7] The enforcement technology allows the central bank to punish defaulters by confiscating goods. The resource cost γ can be thought of as the cost of monitoring and the use of channels to confiscate goods to satisfy repayment.

There are four stages within a period. At the first stage, young debtors meet the central bank. As we shall see, young debtors may seek liquidity from the central bank at this time. At the second stage, young debtors and young creditors meet. This is the only opportunity for young debtors to acquire good 1. At the third stage, young debtors and old creditors meet. This is the only opportunity for old creditors to acquire good 2. Finally, at the fourth stage, young debtors are reunited with the central bank. At this time, young debtors have an opportunity to repay the central bank for any liquidity provided by it at the first stage.

Debtors are endowed with an investment technology that allows them to invest some of their endowment ($I \leq x$) at the end of the first stage, that yields with certainty, RI units of good 2 at the beginning of the third stage, where $R \geq 1$. The sequence of events for each date is summarized in Figure 1.

Because of the timing of trading opportunities within a period and the fact that there is no commitment and no public memory, money is necessary as a means of payment if trade is to take place.[8] Because young debtors are not endowed with fiat money they must first acquire some via a credit relationship with the central bank. These young debtors must then repay the central bank at the final stage

scope of this paper.

[7] In Freeman (1996), $\gamma = 0$.

[8] See Kocherlakota (1998) for a general discussion and Mills (2003) for one in the context of this type of model.

within a period so that it may retire an equivalent amount of money that it injected into the economy at the beginning of the period. Moreover, as in Townsend (1989) money serves as a communication device that signals to the central bank the past behavior of debtors. Thus, money is essential for the repayment of debt.

3. BENCHMARK: OPTIMAL ALLOCATIONS

Before describing the alternative credit policies, I first define some optimal allocations. A first-best allocation is one that maximizes ex-ante expected steady-state utility of debtors and creditors subject to a limited set of feasibility constraints. This limited set abstracts from incentive constraints which will be important for implementation.

Denote the steady state levels of consumption of both good 1 and good 2 by d_z for a debtor and c_z for a creditor for $z \in \{1, 2\}$. The problem is then to maximize

$$u(c_1, c_2) + v(d_1, d_2) \tag{1}$$

with respect to I, d_1, d_2, c_1, c_2 and subject to the following feasibility constraints:

$$x \geq I \tag{2}$$

$$y \geq d_1 + c_1 \tag{3}$$

$$RI + (x - I) \geq d_2 + c_2. \tag{4}$$

Define u_z as the partial derivative of creditor utility with respect to good z and v_z as the partial derivative of debtor utility with respect to good z for $z \in \{1, 2\}$. Optimal allocations require that (2)-(4) are satisfied at equality. The first order conditions, then, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to:

$$\frac{u_1}{u_2} = \frac{v_1}{v_2} \tag{5}$$

Condition (5) states that optimal allocations are those that are Pareto optimal. Thus, in what follows, I shall look for implementable allocations (ones that take into account the incentives of agents) that satisfy (5).

4. LIQUIDITY PROVISION WITH COSTLY ENFORCEMENT AND PRICING

In this section, I provide an example of a payment mechanism where a central bank provides liquidity with a credit policy of paying $\gamma > 0$ for the enforcement technology and charging an intraday interest rate (or price) for liquidity. I characterize a set of implementable allocations via the mechanism as those that satisfy a set of incentive constraints. Allocations are implementable if they are sub-game perfect equilibrium allocations. Finally, I show that the second-best optimal allocation implementable via the pricing mechanism is Pareto-optimal.

Recall that investment in the enforcement technology enables the central bank to confiscate goods from a defaulting debtor. The central bank can effectively choose some combination of goods 1 and 2 to confiscate so that, in equilibrium, debtors will choose not to default. This costly enforcement is meant to model the opportunities a central bank may have when it monitors the behavior of payments system participants. The exogenous parameter, γ, is a proxy for the cost of monitoring agents and the costs associated with the potential liquidation of assets in the event of a default.

The central bank charges an intraday interest rate, $r \geq 0$ units of good 2. The interest rate is quoted in terms of good 2 so as to provide an easy comparison with the collateral policy of the next section. Some central banks, such as the Federal Reserve, have a mandate to fully recover costs of the operation of its payment services. Such an assumption in the context of the model is that the central bank charge an intraday interest rate, r, equal to γ units of good 2. I shall describe the payment game that agents play under the assumption that the central bank must fully recover its costs.

The game is as follows for any date t. At the first stage of a period, generation-t debtors choose whether or not to seek liquidity from the central bank. Those that seek such credit acquire M units. Let $\delta_1^t \in [0,1]$ be the fraction of debtors who seek credit from the central bank. The generation-t debtor then invests the entire amount of good 2 ($I = x$). At the second stage, the mechanism suggests that generation-t creditors who want to participate in exchange each offer $\overline{d_1}$ of good 1 and that generation-t debtors who want to participate offer M units of fiat money. The creditors and debtors simultaneously choose whether to participate in exchange or not. Let $\kappa_2^t \in [0,1]$ be the fraction of generation-t creditors who agree to offer $\overline{d_1}$ of good 1 and $\delta_2^t \in [0, \delta_1^t]$ be the fraction of generation-t debtors who agree to exchange M units of money for some consumption of good 1. Each debtor who agrees to trade M units of money for consumption receives $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1. Each creditor that agrees then receives $\frac{\delta_2^t}{\kappa_2^t}M$ units of money and has $c_1^t = y - \overline{d_1}$ units of good 1 left for consumption. Those that disagree leave with autarky.

At the third stage of date t, the generation-t debtors' investments pay off and each now has Rx units of good 2. The mechanism then suggests that generation-t debtors who want to participate in exchange each offer $\overline{c_2}$ of good 2 and that generation-$t-1$ creditors who have $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and who want to participate in exchange offer up $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. Generation-$t-1$ creditors with no money are not able to participate in exchange. The debtors and those creditors able to participate in exchange simultaneously choose to participate or not. Let $\delta_3^t \in [0, \delta_2^t]$ be the fraction of generation-t debtors who agree to offer $\overline{c_2}$ of good 2 and $\kappa_3^{t-1} \in [0, \kappa_2^{t-1}]$ be the fraction of generation-$t-1$ creditors who agree to exchange money for some consumption of good 2. Each creditor who agrees to trade money for consumption receives $\frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}$ units of good 2. Each debtor that agrees then receives $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and has $Rx-\overline{c_2}$ units of good 2 left over. Those that disagree leave with autarky.

At the final stage, if a generation-t debtor who has borrowed money at the first stage now has $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the debtor may choose to repay the central bank $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money plus the interest rate of $r=\gamma$ units of good 2. The central bank then removes the $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money from circulation and the debtor has $d_2^t = Rx-\gamma-\overline{c_2}$ units of good 2 for consumption. If the young debtor does not have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and does not offer γ units of good 2 to the central bank, then that debtor is punished by surrendering the amount of good 1 he acquired at stage 2.

Notice that equations (2)-(3) are satisfied at equality by the mechanism but that (4) is not because $r=\gamma>0$ represents a real resource cost borne by the private agents. I first characterize the set of allocations that are implementable via the payment mechanism with pricing.

Proposition 1. *A steady-state allocation is implementable if it satisfies the following participation constraints:*

$$v[y-c_1, Rx-\gamma-c_2] \geq v[0, Rx] \tag{6}$$

for debtors and

$$u[c_1, c_2] \geq u[y, 0] \tag{7}$$

for creditors.

Proof. The proof solves for subgame perfect equilibria of the game via backwards induction. The equilibria are those where every agent agrees at every stage.

Begin with stage 4 within a period at date t. Generation-t debtors who have agreed up to this stage have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. They will choose to return the money and pay the interest rate $r = \gamma$ if

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - \gamma - c_2] \geq v[0, Rx - c_2]. \tag{8}$$

Note that the right-hand side of (8) represents utility after the central bank confiscates the debtor's amount of good 1 he previously acquired.

Now turn to stage 3. A creditor from generation $t-1$ enters this stage with either $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ or 0 units of money which is private information. Suppose that all other agents agree in the third stage. If the creditor does not have any money then she cannot trade. If she has $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money then it is trivial that she will want to agree to trade as well because

$$u[y - \overline{d_1}, \frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y - \overline{d_1}, 0]. \tag{9}$$

Thus, $\kappa_3^{t-1} = \kappa_2^{t-1}$.

A generation-t debtor enters the third stage with either $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ or 0 units of good 1, which is private information. Suppose that all other agents that can participate in trade will agree in the third stage. If the debtor has $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1, he will also agree if:

$$\max\{v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - \gamma - \overline{c_2}], v[0, Rx - c_2]\} \geq v[0, Rx]. \tag{10}$$

The left hand side of the expression represents a debtor's stage 4 decision. The right hand side takes into account the fact that if a debtor disagrees, he will not receive money and will then be punished by losing $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ at stage 4. Because $v[0, Rx - c_2]\} < v[0, Rx]$ (10) reduces to

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - \gamma - \overline{c_2}] \geq v[0, Rx] \tag{11}$$

and (8) is satisfied if (11) is satisfied. If the debtor has none of good 1, it is trivial that he chooses not to agree to trade.

Now consider an arbitrary generation-t debtor at stage 2. If the debtor disagrees at this stage, he enters the third stage with 0 units of good 1 and will disagree in the third stage as well. Thus, he will receive only autarkic utility, $v[0, Rx]$. If the debtor agrees when everyone else does, then his second-stage participation constraint is identical to his third-stage participation constraint, (11). This implies that all of the generation-t debtors who agree at stage 2 will also agree at stage 3, that is, $\delta_3^t = \delta_2^t$.

Now, consider an arbitrary generation-$t-1$ creditor at the second stage of date $t-1$. If the generation $t-1$ creditor disagrees at the second stage when young, she enters the third stage when old with no money and, therefore, receives autarkic utility, $u[y, 0]$. She also knows that $\delta_3^t = \delta_2^t$. If she agrees at the second stage of date $t-1$, she will enter the third stage of date t with money and agree so that she receives $u[y-\overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}]$. She will agree if

$$u[y-\overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y,0] \tag{12}$$

where δ_2^t is substituted for δ_3^t.

Finally, consider generation-t debtors at stage 1 of date t. Here, if all other debtors agree to borrowing from the central bank, then $\delta_1^t = \delta_2^t = \delta_3^t = 1$ and an arbitrary debtor also agrees if

$$v[\kappa_2^t\overline{d_1}, Rx-\gamma-\overline{c_2}] \geq v[0, Rx] \tag{13}$$

which turns out to be the debtor participation constraint for stage 2 and stage 3. As a result, $\kappa_2^t = \kappa_3^{t-1} = 1$ because (12) is now satisfied because (7) is satisfied by hypothesis. Thus, generation-t debtor constraints at stages 2 and 3 reduce to (6), the stage 1 generation-t constraint is then trivially satisfied so that $\delta_4^t = 1$, and all nontrivial creditor constraints reduce to (7), both of which are satisfied. ∎

Proposition 1 gives two simple conditions that an allocation must meet for it to be implementable. They essentially ensure that both debtors and creditors wish to participate in exchange. The next proposition characterizes the second-best optimal allocation via the payment mechanism with pricing and shows that it is Pareto-optimal. The optimal allocation is always second-best because the enforcement technology combined with the cost-recovery constraint reduces the amount of good 2 available to the agents.

Proposition 2. *The optimal allocation implementable via the payment mechanism with pricing satisfies (5).*

Proof. The optimization problem can be written as maximizing (1) with respect to I, d_1, d_2, c_1, c_2 and subject to (2)-(3), (6)-(7), and

$$\begin{aligned} RI + (x - I) - r &\geq d_2 + c_2 & (14) \\ r &= \gamma & (15) \end{aligned}$$

where (14) replaces (4) and (15) is the cost recovery constraint of the central bank. Given that (2)-(3) and (14) hold at equality, and substituting these relationships into the optimization problem, the first order conditions, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to

$$\begin{aligned} \frac{(1+\lambda_d)v_1}{(1+\lambda_d)v_2} &= \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} & (16) \\ \lambda_c\{u[c_1, c_2] - u[y, 0]\} &= 0 & (17) \\ \lambda_d\{v[y - c_1, Rx - \gamma - c_2] - v[0, Rx]\} &= 0 & (18) \\ \lambda_c, \lambda_d &\geq 0 & (19) \end{aligned}$$

where λ_c and λ_d are the multipliers for the creditor and debtor participation constraints, respectively. Inspection of (16) reveals that (5) is satisfied regardless of whether the participation constraints (6) and (7) bind or not. ∎

While the intraday interest rate may influence whether or not debtors participate in trade (for trade to take place at all under this credit policy, it is important that $r = \gamma$ is not too high that constraint 6 is violated), it does not create a wedge between the ratios of marginal rates of substitution and so is Pareto-optimal. This is because debtors do not have to pay the interest rate until stage 4 so that the cost in terms of good 2 can be shared among debtors and creditors.

One interpretation of $r = \gamma$ is that it is the optimal risk-free intraday interest rate. This is because (i) a positive interest rate is necessary for central bank liquidity provision (because of the cost recovery constraint) and (ii) investment in the enforcement technology eliminates the risk that a debtor defaults. This is a departure from the case where $r = 0$ (free intraday liquidity), which has been found to be optimal in papers such as Freeman (1996), Green (1997), Zhou (2000) Kahn and Roberds (2001) and Martin (2003). In each of those cases, it is implicitly assumed that $\gamma = 0$ so that there was no social cost attached to providing intraday liquidity. The positive optimal risk-free interest rate found here seems to support a recommendation suggested by Rochet-Tirole (1996); costly monitoring of agents is necessary, and liquidity providers should be compensated.

5. LIQUIDITY PROVISION WITH COLLATERAL

In this section, I provide an example of a payment mechanism where a central bank provides liquidity with a credit policy of requiring collateral. As in the previous section, I characterize a set of implementable allocations via the mechanism as those that satisfy a set of constraints. Finally, I show that the second-best optimal allocation implementable via the collateral policy is not Parteo-optimal if collateral bears an opportunity cost. The first-best optimal allocation is achieved, however, if there is no opportunity cost to posting collateral.

The young debtors, when they seek liquidity from the central bank, pledge some of their endowment of good 2 as collateral which they will then buy back from the central bank at the end of the period (during the fourth stage). Recall that young debtors can invest their endowment of good 2 and receive a certain return of $R \geq 1$. Because the amount of good 2 they pledge is transferred to the central bank, there is an opportunity cost in that the collateral is no longer available to invest whenever $R > 1$.

In terms of actual large-value payment systems, one can think of the opportunity cost of collateral in the following way.[9] Suppose that participants of the system can post only a limited set of assets as collateral. These assets are generally viewed as safe from the point of view of the liquidity-provider. Typically, such safe assets have lower (expected) returns. To the extent that participants seeking intraday liquidity hold more of these safe assets than they otherwise would without the need to post them as collateral, one could argue that there is an opportunity cost to pledging collateral.

The game is as follows for any date t. At the first stage of a period, generation-t debtors choose whether or not to seek liquidity from the central bank. Those that seek such credit acquire M units and deposit $\sigma \leq x$ units of good 2 at the central bank as collateral. Generation-t debtors then invest their remaining supply of good 2 ($I = x - \sigma$). Let $\delta_1^t \in [0, 1]$ be the fraction of debtors who seek credit from the central bank. At the second stage, the mechanism suggests that generation-t creditors who want to participate in exchange each offer $\overline{d_1}$ of good 1 and that generation-t debtors who want to participate offer M units of fiat money. The creditors and debtors simultaneously choose whether to participate in exchange or not. Let $\kappa_2^t \in [0, 1]$ be the fraction of generation-t creditors who agree to offer $\overline{d_1}$ of good 1 and $\delta_2^t \in [0, \delta_1^t]$ be the fraction of generation-t debtors who agree to exchange M units of money for some consumption of good 1. Each debtor who agrees to trade M units of money for consumption receives $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1. Each creditor that agrees then receives $\frac{\delta_2^t}{\kappa_2^t}M$ units of money and has $c_1^t = y - \overline{d_1}$

[9] Zhou (2000) also makes this argument.

units of good 1 left for consumption. Those that disagree leave with autarky.

At the third stage of date t, the generation-t debtors' investments pay off and each now has $R(x-\sigma)$ units of good 2 available at this stage. The mechanism then suggests that generation-t debtors who want to participate in exchange each offer $\overline{c_2} \leq R(x-\sigma)$ of good 2 and that generation-$t-1$ creditors who have $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and who want to participate in exchange offer up $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. Generation-$t-1$ creditors with no money are not able to participate in exchange. The debtors and those creditors able to participate in exchange simultaneously choose to participate or not. Let $\delta_3^t \in [0, \delta_2^t]$ be the fraction of generation-t debtors who agree to offer $\overline{c_2}$ of good 2 and $\kappa_3^{t-1} \in [0, \kappa_2^{t-1}]$ be the fraction of generation-$t-1$ creditors who agree to exchange money for some consumption of good 2. Each creditor who agrees to trade money for consumption receives $\frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}$ units of good 2. Each debtor that agrees then receives $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and has $R(x-\sigma)-\overline{c_2}$ units of good 2 left. Those that disagree leave with autarky.

At the final stage, if a generation-t debtor who has borrowed M units at the first stage, now has $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the debtor may choose to repay the central bank $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money in exchange for the return of the σ units of good 2 that served as collateral. The central bank then removes the $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money from circulation and the debtor has $d_2^t = R(x-\sigma)-\overline{c_2}+\sigma$ units of good 2 for consumption. If the young debtor does not have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the central bank does not return the collateral.

Notice that (3) and (4) are satisfied at equality by the mechanism but that (2) is not when there is an opportunity cost of collateral ($R > 1$). Rather $I = x-\sigma$. The opportunity cost of collateral is then $(R-1)\sigma$ which is the difference between Rx and $R(x-\sigma)+\sigma$. There is also an additional feasibility constraint that requires

$c_2 \leq R(x-\sigma)$. This constraint reflects the fact that the amound of good 2 that generation-$t-1$ creditors can consume must be less than the total amount available at the third stage.

I now characterize the set of allocations that are implementable via the payment mechanism with pricing.

PROPOSITION 3. *A steady-state allocation is implementable if it satisfies the following incentive constraints:*

$$v[y-c_1, R(x-\sigma)-c_2+\sigma)] \geq v[0, Rx] \tag{20}$$

$$v[y-c_1, R(x-\sigma)-c_2+\sigma)] \geq v[y-c_1, R(x-\sigma)] \tag{21}$$

for debtors and

$$u[c_1, c_2] \geq u[y, 0] \tag{22}$$

for creditors.

Proof. The proof solves for subgame perfect equilibria of the game via backwards induction. The equilibria are those where every agent agrees at every stage.

Begin with stage 4 within a period at date t. Generation-t debtors who have agreed up to this stage have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. They will choose to return the money in exchange for collateral if

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)-c_2+\sigma] \geq v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)-c_2] \tag{23}$$

which trivially holds.

Now turn to stage 3. A creditor from generation $t-1$ enters this stage with either $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ or 0 units of money which is private information. Suppose that all other agents agree in the third stage. If the creditor does not have any money then she cannot trade. If she has $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money then it is trivial that she will want to agree to trade as well because

$$u[y - \overline{d_1}, \frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y - \overline{d_1}, 0]. \tag{24}$$

Thus, $\kappa_3^{t-1} = \kappa_2^{t-1}$.

A generation-t debtor enters the third stage with either $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ or 0 units of good 1, which is private information. Suppose that all other agents that can participate in trade will agree in the third stage. If the debtor has $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1, he will also agree if:

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - \overline{c_2} + \sigma] \geq v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)]. \tag{25}$$

The right hand side of the expression takes into account the fact that if a debtor disagrees, he will not receive money and will then not be able to reclaim his collateral at stage 4. If the debtor has none of good 1, it is trivial that he chooses not to agree to trade.

Now consider an arbitrary generation-t debtor at stage 2 who has borrowed from the central bank. If the debtor disagrees at this stage, he enters the third stage with 0 units of good 1 and will disagree in the third stage as well. Thus, he will receive only autarkic utility, $v[0, R(x-\sigma)]$. If the debtor agrees when everyone else does, then his second-stage participation constraint is

$$\max\{v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - \overline{c_2} + \sigma], v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)]\} \geq v[0, R(x-\sigma)] \tag{26}$$

which is trivially satisfied. Thus, $\delta_2^t = \delta_1^t$. Those that have not borrowed from the central bank will not be able to agree to trade.

Now, consider an arbitrary generation-$t-1$ creditor at the second stage of date $t-1$. If the generation $t-1$ creditor disagrees at the second stage when young, she enters the third stage when old with no money and, therefore, receives autarkic utility, $u[y, 0]$. If she agrees at the second stage of date $t-1$, she will enter the third stage of date t with money and agree so that she receives $u[y - \overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}]$. She will agree if

$$u[y - \overline{d_1}, \frac{\delta_3^t}{\kappa_2^{t-1}}\overline{c_2}] \geq u[y, 0] \tag{27}$$

where κ_2^{t-1} is substituted for κ_3^{t-1}.

Finally, consider generation-t debtors at stage 1 of date t. Here, if all other debtors agree to borrowing from the central bank, then $\delta_1^t = \delta_2^t = 1$ and an arbitrary debtor also agrees if

$$\max\{v[\kappa_2^t\overline{d_1}, R(x-\sigma) - \overline{c_2} + \sigma], v[\kappa_2^t\overline{d_1}, R(x-\sigma)]\} \geq v[0, Rx]. \tag{28}$$

Now it remains to be shown that $\kappa_2^{t-1} = \kappa_2^t = \delta_3^t = 1$ is supported in an equilibrium. If $\kappa_2^{t-1} = \kappa_2^t = 1$, then (28) trivially holds given that (20) holds by hypothesis. Thus, $\delta_1^t = \delta_2^t = 1$ and (25) reduces to (21) which is satisfied and $\delta_3^t = 1$. If $\delta_1^t = \delta_2^t = \delta_3^t = 1$, then it is obvious that $\kappa_2^{t-1} = \kappa_2^t = 1$ because (27) is satisfied given that (22) is. ∎

Compared with Proposition 1, Proposition 3 has an additional incentive constraint beyond participation. This constraint, (21), essentially requires that the amount of collateral that a debtor buys back from the central bank must be at least as much as the amount of good 2 a creditor is expected to receive ($\sigma \geq \overline{c_2}$).

Otherwise, a debtor, after acquiring some of good 1, would prefer not to exchange with old creditors to acquire money and so default on his debt to the central bank.

The following proposition states that the payment mechanism under a credit policy with collateral cannot achieve Pareto-optimal allocations when there is an opportunity cost of collateral.

PROPOSITION 4. *When $R > 1$, the optimal allocation implementable via the payment mechanism with collateral does not satisfy (5).*

Proof. The optimization problem can be written as maximizing (1) with respect to $I, d_1, d_2, c_1, c_2, \sigma$ and subject to (3)-(4), (20)-(22), and

$$x - \sigma \geq I \tag{29}$$

$$RI + (x - \sigma - I) \geq c_2. \tag{30}$$

where (29) replaces (2) from the benchmark problem and (30) is an additional feasibility constraint for stage 3. Given that (3),(4), and (29) will hold at equality, and substituting these relationships into the optimization problem, the first order conditions, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to

$$\frac{(1+\lambda_d)v_1}{(1+\lambda_2+\lambda_d)v_2+\lambda_1} = \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} \tag{31}$$

$$(1+\lambda_d)v_2(R-1)+\lambda_1 R = \lambda_2 v_2 \tag{32}$$

$$\lambda_1\{R(x-\sigma)-c_2\} = 0 \tag{33}$$

$$\lambda_2\{v[d_1, R(x-\sigma)-c_2+\sigma)] - v[d_1, R(x-\sigma)]\} = 0 \tag{34}$$

$$\lambda_c\{u[c_1, c_2] - u[y, 0]\} = 0 \tag{35}$$

$$\lambda_d\{v[y-c_1, R(x-\sigma)-c_2+\sigma] - v[0, Rx]\} = 0 \tag{36}$$

$$\lambda_1, \lambda_2, \lambda_c, \lambda_d \geq 0 \tag{37}$$

where λ_1 is the multiplier for (30), λ_2 is the multiplier for (21), and λ_c and λ_d are the multipliers for the creditor and debtor participation constraints, (20) and (22), respectively. Condition (32) is the first-order condition with respect to σ.

For (31) to equal (5), it must be the case that $\lambda_1 = \lambda_2 = 0$ which is the case if (21) and (29) do not bind. If $\lambda_1 = \lambda_2 = 0$, then (32) reduces to $(1+\lambda_d)v_2(R-1) = 0$ implying that $v_2 = 0$, which violates the assumptions about debtor preferences.[10] Therefore, a solution to this optimization problem cannot have both λ_1 and λ_2 be equal to 0. ∎

The intuition for Proposition 4 is as follows. Because there is an opportunity cost to pledging collateral, a solution to the optimization problem should minimize the amount of collateral required. For such an allocation to be incentive feasible for debtors, $\sigma \geq \overline{c_2}$. Thus, an optimal allocation should have $\sigma = \overline{c_2}$ so that (21) binds. But if (21) binds, then it turns out that debtors are credit constrained.

[10] $v_2 = 0$ if and only if $d_2 = \infty$ which is not feasible.

That is to say they cannot borrow "enough" from the central bank to acquire the desired amount of good 1 from young creditors. Thus, when collateral bears an opportunity cost, it serves as an endogenous credit constraint. This result is consistent with other papers on the use of collateral, such as Lacker (2001).

Finally, it is worth exploring the case when there is no opportunity cost of collateral. This may be the case in actual large-value payment systems when the central bank accepts a wide range of assets as collateral, mitigating the need to have an asset portfolio with a heavier than optimal weight on safe assets.

PROPOSITION 5. *When $R = 1$, the optimal allocation implementable via the payment mechanism with collateral satisfies (5) if the allocation has $\overline{c_2} < \frac{x}{2}$.*

Proof. When $R = 1$, the first order conditions from Proposition 4 simplify to

$$\frac{(1+\lambda_d)v_1}{(1+\lambda_2+\lambda_d)v_2+\lambda_1} = \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} \tag{38}$$

$$\lambda_1 = \lambda_2 v_2 \tag{39}$$

$$\lambda_1\{x-\sigma-c_2\} = 0 \tag{40}$$

$$\lambda_2\{v[d_1, x-c_2] - v[d_1, x-\sigma]\} = 0 \tag{41}$$

$$\lambda_c\{u[c_1, c_2] - u[y, 0]\} = 0 \tag{42}$$

$$\lambda_d\{v[y-c_1, x-c_2] - v[0, x]\} = 0 \tag{43}$$

$$\lambda_1, \lambda_2, \lambda_c, \lambda_d \geq 0 \tag{44}$$

As before, I need $\lambda_1 = \lambda_2 = 0$ which is the case if (21) and (29) do not bind. Such a condition does not violate (39) and is met when $\overline{c_2} < \sigma < x - \overline{c_2}$ or $\overline{c_2} < \frac{x}{2}$. ∎

This gives sufficient conditions for which the debtor incentive constraint (21) does not bind. In this case, because there is no opportunity cost of collateral, the optimum does not require σ to be small. Thus it is possible to choose from a range of σ that does not lead to any credit constraints.

Notice that when there is no opportunity cost of collateral, the Pareto-optimal allocations are first-best. This is because the use of collateral in this case does not add any additional social cost. Only a subset of such allocations, however, are achievable because of the need to satisfy the incentive constraint of debtors.

6. CONCLUSION

The above analysis sheds some light on why different central banks may have different credit policies for RTGS systems. Collateral is preferred if there is no opportunity cost of collateral, such as may be the case when a wide range of assets are accepted as collateral. This is because it can achieve first-best allocations. If collateral does have an opportunity cost, comparison of the relative cost (in terms of good 2 in the model) is important. For example, the European Central Bank does not have monitoring authority over participating banks (although individual national central banks do). Thus, it may be difficult to coordinate monitoring and enforcement authorities across borders. In the context of the model, this is a high enough γ so that collateral may be the preferred option. On the other hand, the Federal Reserve already has supervisory authority over depository institutions it serves over Fedwire, so that economies of scope are likely to yield a low γ so that pricing may be the preferred option. In the case where the cost of both policies would be the same ($\gamma = (R-1)\sigma$), the pricing policy would clearly be preferred due to the result that collateral adds a binding endogenous borrowing constraint that does not permit a Pareto-optimal allocation.

This paper takes a first step in understanding optimal credit policies for liquidity provision. There are several possible extensions for further research. For example, only one form of credit risk has been explored here, namely that arising from moral hazard in response to the design of credit policies. One would also like to add aggregate default risk such as in Freeman (1999) so that not all uncertainty concerning default can be eliminated by the credit policy. Given that the central bank could not fully insure itself against such risk, it may be the case that the intraday interest rate under a pricing policy is $r > \gamma$ so that the difference between the two represents a risk premium.

One additional credit policy tool that has not been modeled here is that of setting limits or caps to the amount a debtor can borrow. The Federal Reserve, for example, sets net debit caps that limit the amount that Fedwire participants can borrow to limit the Fed's exposure to credit risk. In the context of the model, such binding constraints would reduce welfare under the pricing regime in the same manner that collateral does when it has an opportunity cost. This is because the central bank can effectively eliminate its credit risk exposure by investing in the enforcement technology so that such caps are unnecessary. In the presence of aggregate default risk, it is not immediately obvious that caps are unwarranted.

The results of the paper suggest that the existence of an opportunity cost of collateral is key to that type of credit policy leading to inefficient allocations. Thus, it is important to empirically understand whether or not there is an effective opportunity cost of collateral intraday.

Collateral in this model is used exclusively as an incentive device to encourage debtors not to default. An additional use of collateral not modeled here is to compensate the central bank in the event of a default (the central bank does not have any preferences regarding good 2). This complicates matters in that the central bank may have to decide what types of risky assets are acceptable as collateral. The conjecture here is that as a central bank accepts a wider range of assets, the opportunity cost to the participant of posting collateral is less, but collateral provides less protection to the central bank in the event of defaults unless the value of the collateral is discounted appropriately.

Finally, this paper restricts itself only to two credit policies designed to replicate actual central bank policies. A more generalized study may reveal that a third policy may be more appropriate especially when some of the complications listed above are present in the model.

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FIG. 1 Sequence of Events in a Period