Consumer Time Inconsistency: Evidence from an Experiment in the Credit Card Market

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January 24, 2004

Abstract

This paper analyzes a unique dataset, which contains results of a large-scale experiment in the credit card market. Two strange phenomena that suggest time inconsistency in consumer behavior are observed: First, consumers prefer an introductory offer which has a lower interest rate with a shorter duration to that of a higher interest rate with a longer duration, even though they would benefit more should they choose the latter. Second, consumers are very reluctant to switch, and even those consumers, who have switched before, fail to switch again later. A multi-period model with complete information is studied to show that the standard exponential preferences can't explain the observed behavior but the hyperbolic preferences can. Furthermore, we study a dynamic model where realistic random shocks are incorporated. Estimation results show that consumers have severe self-control problem, with a present-bias factor $\beta = 0.8$, and that the average switching cost is \$150. With the estimated parameters, the dynamic model can replicate quantitative features of the data.

1 Introduction

Does consumer behavior exhibit time inconsistency? This is an essential, yet difficult question to answer. Since the pioneering contribution of Samuelson (1937), it has become a standard assumption that consumers have an exponential time discount function, $\{1, \delta, \delta^2, ...\}$, which implies that consumers behave *time consistently.* A significant body of evidence in experimental psychology and economics literatures, however, suggests that consumers discount future hyperbolically, not exponentially. The essential feature of hyperbolic discounting is that consumers are *time inconsistent.* In the last decade, many researchers have applied hyperbolic discounting to explain various economic anomalies, such as procrastination, retirement, addiction and credit card borrowing.[1] In particular, the quasi-hyperbolic discount function, $\{1, \beta\delta, \beta\delta^2, ...\}$, has been widely studied.[2] And we shall simply refer it as hyperbolic discounting in later discussion.

Regardless of its enormous impact, the hyperbolic discounting has been criticized for lack of convincing empirical evidence.[3] An ideal test is to compare consumers' long run plans with their later actions, which will be consistent for exponential consumers but inconsistent for hyperbolic consumers. In the real world, it is difficult to track both under controlled conditions. This paper examines time inconsistency using a large-scale randomized experiment in the credit card market, with which we have a unique opportunity to conduct an almost ideal test of hyperbolic discounting.

In the experiment, 600,000 consumers were randomly mailed one out of six different credit card offers, denoted as Market Cell A to F. The six offers have different introductory interest rates and different durations: Market Cell A (4.9% for 6 months), B (5.9% for 6 months), C (6.9% for 6 months), D (7.9% for 6 months), E (6.9% for 9 months) and F (7.9% for 12 months). All other characteristics of the solicitations are identical across the six market cells. Consumer responses and subsequent usage of respondents for 24 months are observed.

There are two phenomena in this dataset suggestive of time inconsistency. First, significantly more consumers in Market Cell A are found to accept their offers than in Market Cell F. This *ex*

[1]See Akerlof (1991), Diamond and Koszegi (1998), Gruber and Koszegi (2001), Harris and Laibson (2001), Laibson (1997), Laibson *et al.* (1998), O'Donoghue and Rabin (1999,2001).

[2]The quasi-hyperbolic discounting accomodates three different hyperbolic time preferences as special cases: naive, sophisticated and partial naivety. We will dicuss their difference in more detail later. Naive and sophisticated hyperbolic discounting are commonly applied in theoretic studies.

[3]For example, Rubinstein (2000).

ante preference is puzzling after observing that respondents, *ex post*, kept on borrowing on this card well after the short introductory period. We will show in later sections that respondents in Market Cell A would pay less interest if their accounts were repriced as the offer F. We termed this phenomenon "rank reversal". Second, consumer switching behavior is not consistent over time. The majority of respondents (60%) stay with this bank after the introductory period, even though their balances remain at the same level as when they accepted this card. Given the same debt level, it should be worthwhile to switch a second time since it was optimal to accept this offer before.[4]

There are two possible explanations for consumer behavior that on the surface appears to be time inconsistent. First, consumers may behave in a time inconsistent fashion, such as hyperbolic consumers, because they discount at a higher rate in the short run than in the long run. Second, consumers are subject to random shocks, the *ex post* realizations of which can generate divergences between consumers' initial plans and later actions, even if their preferences are time consistent.

In this paper, we first develop a multi-period complete information model. We prove that exponential consumers will never exhibit "rank reversal" in this setting. However, it is possible for hyperbolic consumers.

To fully explore the possibility of explaining behavior with exponential discounting, we develop a dynamic model which incorporates three important random processes. First, consumer income has both persistent and transitory shocks. Second, receiving new introductory offers is probabilistic. Third, accepting a new offer incurs a switching cost which is a random draw from a known distribution.

We find that it is impossible to reconcile low switching rates and a preference for the shorter offer in an exponential model even with realistic random shocks. An exponential individual may, *ex ante,* choose the wrong plan based on the realized *ex post* random shocks. However a sufficiently large group of exponential consumers, as we observe, should prefer the offer that *on average* provides the lowest interest payment.

Hyperbolic time preferences are examined as an alternative to exponential discounting. There

[4]Obviously, there would be no puzzle if the respondents did not receive new low-rate solicitations from other banks after the end of the introductory period. However, the number of solicitations averaged three per qualified household per month during this period. The typical solicitation included a 5.9% introductory interest rate for 6 months for the observed issuer, and the vast majority of respondents remained creditworthy, which will be discussed in more detail in the data section.

are two kinds of hyperbolic preferences which are widely studied in the literature: *sophisticated* and *naive*. Our studies show that both types are able to explain the second puzzle, although, the underlying economic stories are different. A sophisticated hyperbolic consumer who recognizes her time inconsistency problem would like to precommit to avoid overspending in the future. Accepting a shorter introductory offer, rather than a longer one, serves as a commitment device, even though she would pay less interest if she accepted the longer offer. A naive hyperbolic consumer, however, trades the longer offer for a shorter one because she underestimates the amount she will borrow after the current period. This underestimation is due to the fact that she naively believes that her future preference would be as patient as she desires now.

Estimation results show that the second puzzle can only be explained by the stochastic nature of switching costs, which are normally assumed to be constant. The random switching cost is actually a more realistic treatment of individual consumers, because it captures fluctuations due to subjective, psychological factors that strongly affect realized switching costs.[5] Under this interpretation, respondents of this experiment accept the offers due to their low realized switching costs at the time of solicitation. However, their mean switching costs are much higher, which can be partially inferred from the low response rate (1%). This high mean will keep the majority of respondents from switching a second time after the introductory period. There are some empirical studies that estimate the magnitude of consumer switching costs, such as Sorensen (2001) and Kim, Kliger and Vale (2003). However, they cannot estimate a probability distribution for the switching costs because, unlike this study, they don't have data on transactions at the individual-level.

There have been many empirical studies in support of hyperbolic discounting, both from lab experiments and field studies, as will be discussed in detail in the following section. Our study has two advantages compared with previous studies. First, the dataset provides a unique opportunity to identify hyperbolic preferences. Consumer relative preferences between credit card offers with different durations convey information about their borrowing plans for the next 12 months, which are determined by their expected future time preferences at solicitation. We can also infer time discounting functions consumers actually use after acceptance, since we observe the subsequent borrowing behavior of respondents. The second advantage is that we allow for realistic random shocks. A realistic dynamic model is required because some researchers argue that exponential

[5] For example, Loewenstein (1996) studied emotional influences on people's behavior.

discounting can explain anomalies if "even a small degree of" uncertainty is incorporated,[6] which we show is not necessarily the case here.

The paper is organized as follows. In section 2 we give a brief review of previous empirical studies of hyperbolic discounting and how they compare with this paper. In section 3, the dataset is introduced and the two puzzles are elaborated. Section 4 rigorously define what is "rank reversal". And we prove it is impossible in an exponential model with complete information. A simple 3-period model illustrates that "rank reversal" is possible for hyperbolic agents. The dynamic model with incomplete information, which accommodates both exponential and hyperbolic time preferences, is presented in Section 5. The estimation strategy and results are discussed in section 6, 7 respectively. Lastly, we conclude this study in section 8.

2 Related Empirical Studies of Hyperbolic Discounting

The most cited empirical evidence on hyperbolic discounting is from laboratory experiments.[7] One major problem with laboratory evidence is that most experiments only elicit consumer time preferences once. In Ainslie and Haendel (1983), experimental subjects are asked the following two questions:

Question 1: Would you rather receive $50 today or $100 in 6 months?
Question 2: Would you rather receive $50 in one year or $100 in 1 year plus 6 months?

Many subjects chose the smaller-sooner reward in the first question and the larger-later reward in the second. This phenomenon has been termed as "preference reversal." and is cited as empirical evidence against exponential discounting. The argument is that subjects apparently apply a larger discount rate for a six-month delay as the delay becomes closer, while exponential time preferences assume that consumers use the same discount rate for any equal-distance period. However, this "preference reversal" can also be explained by foreseeable preference changes in an exponential model.

Essential difference between an exponential and hyperbolic consumer concerns that whether "current self" and "future self" agree on the desired discount factor in the future, not whether the

[6] For example Fernandez-Villaverde (2002).

[7] For example, Ainslie and Haendel (1983), Loewenstein and Thaler (1989) and Thaler (1981).

discount factor is exactly the same for any equal-distance period. An exponential consumer has the same discount factor (δ) between period t and $t+1$ no matter which period she is at. However, a hyperbolic consumer has a discount factor δ between period t and $t+1$ as period $\tau < t$, and a discount factor $\beta\delta$ at period t. Because of this, a hyperbolic consumer would like to revise her consumption plan for period t when period t arrives. This revision does not exist in the exponential model. Therefore, to identify hyperbolic discounting, it is vital to solicit consumer time preferences in multiple periods.

Several dynamic experiments have been conducted, such as Read and van Leeuwen (1998), in which subjects were asked to choose between healthy and unhealthy food both in advance and immediately before the snacks were given. They found that subjects were more likely to make the unhealthy choice when asked immediately before the snacks were to be given than when asked a week in advance. However, this evidence is also questionable, subjects may not tell the truth when they were first asked, because they knew they could always change their mind later[8].

Since eliciting consumer true time preferences from laboratories is difficult, some researchers have attempted to infer consumer time preferences from their economic behavior in the real world. Researchers have analyzed consumer behavior in different markets, such as the credit card market (Laibson, Repetto and Tobacman (2000)), the health club market (Della Vigna and Malmendier (2001)), and the labor market (Fang and Silverman (2001) , Paserman (2001)).

Among these studies, the closest relative to our work is Della Vigna and Malmendier (2001), who utilize a clear identification strategy. They identify consumer time inconsistency by comparing initial contractual choices among an annual contract, a monthly contract and a pay-per-visit, with subsequent actual attendance. The disadvantage of that study is that they focus on first-time users. Inexperienced users may choose the wrong contract because they have incorrect expectations about their future attendance. [9] For example, Miravete(2003) found that consumers often chose the wrong calling plan when first faced with new choices, but they switch to the right calling plan after they learn more about their own telephone usage. Therefore, an experienced sample is very important when identifying consumer time-inconsistency from behavior at different dates. In the next section

[8]See Besharov and Coffey (2003) for more details.

[9]Della Vigna and Malmendier (2001) show that exponential but inexperienced users will choose the correct contract on average. However this result is based on a strong assumption that users have correct expectations about the distribution of their attendence.

we will show that consumers in our sample are very familiar with credit card offers.

3 A Unique Dataset

A substantial portion of credit card marketing today is done via direct-mailed preapproved solicitations. The typical solicitation includes a low introductory interest rate for a known duration, followed by a much higher post-introductory interest rate. Sophisticated card issuers decide on the terms of their solicitations by conducting large-scale randomized trials. The dataset used is the result of such a "market experiment" conducted by a major United States issuer of bank credit cards in 1995. The issuer generated a mailing list of 600,000 consumers and randomly assigned the consumers into six equal-sized market cells (A-F). The market cells have different introductory offers as mentioned above but are otherwise identical (including the same post-introductory interest rate of 16%).

In each market cell, between 99860 and 99890 observations are actually obtained, out of the 100,000 consumers. About half of the missing observations are due to one known data problem: approximately 5% of the individuals who respond to the preapproved solicitation but are declined (due to a deterioration of credit condition or failure to report adequate information or income) are deleted from the dataset for unknown reasons. Nevertheless, over 99.8% of the sample is still included. Ausubel (1999) offers statistical evidence that this is still a good random experiment among the remaining observations. Financial statistics of the remaining 599,257 consumers are observed at the time of solicitation and their responses to their offers are recorded.

For consumers who accept their credit offers (respondents), we observe detailed information about their monthly account activities for subsequent 24 months. For a month t, we observe the amount paid on the account at the beginning of the month, the amount of new charges during the month, any finance charge (such as interest, late-payment fee and over-credit-limit fee) and the total balance owed at the end of the month. Based on the information, we distinguish convenience charges, no interest billed, from credit card debt. In later analysis, we will focus on debt.

Besides these quantitative measures, we also observe two interesting qualitative measures. The first measures the delinquency status: whether the account is delinquent this month or not and the duration of the delinquency. The second measures whether the account owner has filed for a personal bankruptcy or not. These two measures offer important information about the respondent's credit status over time.

Important financial statistics for the whole sample and for respondents are reported in Table 1. Most observables of respondents are statistically worse than the whole sample. Nevertheless, both groups are of good credit quality. Majority of consumers have more than a ten-year credit history. Very few have been past due in last two years, which is shown in "Number of Past-due". And none of them has had a sixty-day past due, which is considered to be a severe delinquency. For both groups, every consumer has at least one existing credit card and 75% have more than two credit cards. "Revolving Limit" is the total credit limit a consumer has on her revolving accounts. Revolving accounts are the accounts on which consumers can borrow with no prespecified repayment plan. The majority of revolving accounts are credit cards. "Revolving Balance" is the total balance on these revolving accounts, including both convenience charges and credit card debt. For a better description, a *utilization rate* is introduced, defined as the ratio of revolving balance to its limit. The average utilization rate for the whole sample is only 16% and for respondents only 27%.

There are two puzzling phenomena observed in this dataset. The first puzzle is that significantly more consumers in Market Cell A accept their offers than in Market Cell F. However, respondents would *ex post* pay less interest in Market Cell F than in Market Cell A. This phenomenon is called "rank reversal". Consumer responses are recorded in the third column of Table 2. Only about one percent consumers accept their credit card offers, which is also the average response rate for the whole economy in the sample period.[10] Among offers A, B, C and D, consumers optimally prefer a lower introductory interest rate given the same introductory duration. However, significantly more consumers accept the shorter offer A than the longer offers, E and F. This preference is suboptimal if one compares the effective interest rate under different offers. The effective interest rate is the annual interest rate respondents actually pay in each market cell, which equals the ratio of the total interest payment to the total credit card debt and is shown in the fifth column of Table 2. The effective interest rate is two percentage points lower in Market Cell F than in Market Cell A and one percentage point lower in Market Cell E. Since the average debt among borrowers is $2000, an average respondent in Market Cell A pays $40 more interest than in Market Cell F and $20 than in Market Cell E.

To make sure this "rank reversal" phenomenon is not driven by outliers, we calculate a "what

[10] According to BAI Global Inc., the response rate to solicitations is 1.4% in 1995.

if" interest payment for each respondent. We ask how much more or less a member of Market Cell A would pay if her account were repriced according to the formula of Market Cell F. Consumer behavior is assumed unchanged under the new cell. 42% of them would save more than \$10, 34% would save more than \$20 and 26% would save more than \$40. Only 21% of respondents would get worse in this exercise.

The Second puzzle is that respondents don't switch after the introductory offer expires even though their debts remain at the same level as before. We observe a stable debt distribution over time among respondents who borrow. The median debt among borrowers stabilizes around \$2000 in the twenty-four months, shown in Fig.1. The first quartile remains around \$3500 and the third quartile is around \$500. The proportion of respondents who borrow doesn't decrease much over time. As shown in Fig.2, 60% of respondents borrow during introductory periods and over 35% continues to carry balances after two years. Of course, this is not a puzzle if respondents haven't received new offers after this one expires. Credit card companies will not send a consumer new solicitations if she is either more than 60 days past due or she declares a personal bankruptcy. Among respondents, about 1% declare bankruptcy and 4% experience a severe delinquency after accepting this card. Apparently, this cannot explain why 35% respondents don't switch.

4 A Multi-period Complete Information Model

In this section, we will analyze a multi-period model with complete information to prove that "Rank Reversal" is impossible in an exponential model. Time consistent agents will always choose a credit offer which provides the lowest interest payment. However, this possibility exists for hyperbolic agents, both naive and sophisticated, which is illustrated by a simple three-period model. Regardless of its simplicity, the three-period model illustrates essential differences between exponential and hyperbolic models.

Besharov and Coffey (2003) concluded that hyperbolic time preferences are not identifiable using financial rewards. The reward they considered is a specific type: giving a certain amount of money to agents at different dates, as is commonly observed in laboratory experiments. The below model provides a specific example that hyperbolic discounting is identifiable, if the formula of financial rewards is carefully designed. Our later estimation work, which is based on a realistic dynamic model, shows that this identification still holds when uncertainty and liquidity constraint are incorporated.

4.1 Model Set-up

The representative agent lives for T periods. At the beginning of period τ, she chooses an optimal consumption level by maximizing a weighted sum of her utilities from this period on:

$$\max_{C_\tau} u(C_\tau) + \beta_0 \sum_{t=\tau+1}^{T} \delta^{t-\tau} u(C_t), \tag{1}$$

where the relative weights are determined by her *current* discount function. β_0 represents "a bias for the present", how much the agent favors this period versus later and δ is a long-term discount factor. When $\beta_0 = 1$ the agent has an exponential time preferences and is time consistent. However, when $\beta_0 < 1$ the agent is time inconsistent. Inconsistency is in two aspects. First, she has a larger discount rate in the short run than in the long run. She has a lower discount factor $(\beta_0\delta)$ between period τ and period $\tau+1$ and, however, δ between any two consecutive periods in the future. Second, the desired discount factor between period τ and period $\tau+1$ is changing over time, decreasing from δ to $\beta_0\delta$ as period τ arrives. $u(C_t)$ is a concave instantaneous utility function.

The agent receives an income y_t at period t and she lives in a complete market, where she can borrow or save at the save gross interest rate r_t and there is no credit limit on her credit card.[11]

$$C_t = y_t - A_t + r_{t-1}A_{t-1} \tag{2}$$

The boundary condition is that she pays off all her debt in the Tth period, i.e. $A_T = 0$. And she has an initial debt A_0 at the beginning of period one. The interest rates $\{r_t\}_{t=1}^{T}$ are determined by her credit card choice in the first period.

In the first period, she receives two introductory offers A (r_A, Γ_A) and B (r_B, Γ_B), where $r_A < r_B$ and $\Gamma_A < \Gamma_B$. Offer A provides a lower introductory interest rate, however, for fewer periods. She chooses one credit offer, and then she makes the optimal consumption choice conditional on her card offer. Her later selves have no control over which card to choose. To simplify the model, there is no more new offers in later periods.

[11]The complete market assumption is only for exposition purpose. We have relaxed this assumption, where the agent can't save. The results are similiar. This assumption will be further relaxed in the later dynamic model, where the agent faces credit limit and the borrowing and saving rates are not equal.

Definition 1: *"Rank Reversal" is that offer A is chosen over offer B, however* $PDV_{A,B}\left(\left\{A_t^A\right\}_{t=1}^T\right) < PDV_{B,B}\left(\left\{A_t^A\right\}_{t=1}^T\right)$*, where* $\left\{A_t^k\right\}_{t=1}^T$ *denotes the optimal asset under offer k and* $PDV_{j,i}\left(\left\{A_t^k\right\}_{t=1}^T\right) = \sum_{t=1}^T \frac{A_t^k\left(r_t^j-1\right)}{\prod_{s=1}^{t-1} r_s^i}$.

The "Rank Reversal" means that the agent's preference order in the utility space is different from that in the financial payment space. She prefers the short offer even though she would have paid less interest (received more interest income) for the same asset path if she have chosen the longer offer B.

4.2 "Rank Reversal" Impossible for Exponential Agents

Definition 2: *A static game is that self 1 chooses an optimal consumption plan according to her preference and all later selves are required to follow the plan. Self 1's problem, given an offer i, is the following:*

$$\begin{aligned} &\max_{\{C_\tau\}_{\tau=1}^T} u\left(C_1\right) + \delta \sum_{t=2}^T \delta^{t-2} u\left(C_t\right) \\ s.t. \quad &\sum_{t=1}^T \frac{C_t}{\prod_{s=1}^{t-1} r_s^i} = \sum_{t=1}^T \frac{y_t}{\prod_{s=1}^{t-1} r_s^i} \end{aligned}$$

Lemma 1: $PDV_{A,B}\left(\left\{A_t^A\right\}_{t=1}^T\right) < PDV_{B,B}\left(\left\{A_t^A\right\}_{t=1}^T\right)$ *implies that* $\sum_{t=1}^T \frac{C_t^A}{\prod_{s=1}^{t-1} r_s^B} < \sum_{t=1}^T \frac{C_t^B}{\prod_{s=1}^{t-1} r_s^B} = \sum_{t=1}^T \frac{y_t}{\prod_{s=1}^{t-1} r_s^B}$*, i.e. the consumption path* $\left\{C_t^A\right\}_{t=1}^T$ *is also feasible under offer B in the static game.*

Proof for the lemma is straightforward, plugging Eq.(2) into the PDV condition.

Definition 3: *A dynamic game is that self* τ *chooses her optimal consumption given the initial asset and she has no control over future selves' choices. The problem is:*

$$\begin{aligned} V_\tau\left(A_{\tau-1}\right) &= \max_{C_\tau} u\left(C_\tau\right) + \delta V_{\tau+1}\left(A_\tau\right) \\ s.t. \quad & C_\tau = y_\tau - A_\tau + r_{\tau-1} A_{\tau-1} \end{aligned}$$

Lemma 2: *For the exponential model, the solutions are the same for the static game and the dynamic game.*

The lemma is true due to the Principle of Optimality, Bellman (1957).

Proposition: *Exponential agents will never exhibit " Rank Reversal".*

Proof:

In the reality, consumers' credit card usage is determined by a dynamic game, defined in Definition 3. For exponential agents, the asset path is also the optimal plan for the static game, given

Lemma 2. Therefore *the asset path should provide the highest utility among all financially feasible plans.* If $PDV_{A,B}\left(\left\{A_t^A\right\}_{t=1}^T\right) < PDV_{B,B}\left(\left\{A_t^A\right\}_{t=1}^T\right)$, the optimal consumption path under A is also feasible under B, given Lemma 1. Hence, the offer B should be no worse than offer A, i.e. in period one B must be chosen in stead of A. Thus, $PDV_{A,B}\left(\left\{A_t^A\right\}_{t=1}^T\right) < PDV_{B,B}\left(\left\{A_t^A\right\}_{t=1}^T\right)$ and preference for offer A won't coexist.

4.3 "Rank Reversal" Possible for Hyperbolic Agents

However, the "Rank Reversal" is possible in hyperbolic models. The key reason is that hyperbolic time preference is not stationary so that some consumption plans are not implementable even though they are financially feasible. It is possible that some consumption plan which may incur higher costs but yield a higher utility at the same time.

Hyperbolic agents have time inconsistent preferences. Their current discount function is $\{1, \beta_0\delta, \beta_0\delta^2, ...$. Their preference between any two consecutive periods, period t and period $t+1$, is changing. If they are asked in any period earlier than period t, the desired patience is δ. However, when period t arrives, they would prefer to discount at $\beta\delta$. Therefore, one agent at different time points would prefer different consumption plans. The dynamic consumption problem is really an interpersonal game, in which the agent at different dates are different players. We solve hyperbolic models for a subgame perfect equilibrium. Each self chooses a level of consumption which maximizes her own utility (Eq.(1) given the utility maximizing strategies of all future selves, which is formally defined as *a Strotz-Pollak equilibrium* in Peleg and Yaari (1973).

The *expectation of the strategies of future selves* are determined by the expected discount function, which is assumed to be $\{1, \beta_1\delta, \beta_1\delta^2, ...\}$ for all subsequent periods. Depending on the expected strategies, the hyperbolic model has three interesting special cases, which have been commonly studied[12]. When $\beta_0 = \beta_1 = \beta < 1$, the agent (sophisticate hyperbolic) has a correct expectation about her future. Self τ realizes that the discount factor between period t and period $t+1$ will become $\beta\delta$ when period t arrives. When $\beta_0 < \beta_1 = 1$, the agent is called a naive hyperbolic agent since she has an incorrect expectation about her future. She naively believes that she would behave herself

[12] In particular, naive and sophisticated hyperbolic models have been studied. Strotz (1956) and Phelps and Pollak (1968) carefully distinguished the two assumptions, and O'Donoghue and Rabin (1999) studied different theoretic implications from these two. Laibson (1994, 1996, 1997) assumed consumers are sophisticated. On the other hard, Akerlof (1991) adopted the naive hyperbolic assumption.

from the next period on ($\beta_1 = 1$). In between the sophisticate and naive hyperbolic agent, a partial naive agent can be defined when $0 < \beta_0 < \beta_1 < 1$. Such an agent underestimates the impatience she has in the later periods like a naive agent. However, she anticipates a difference between today's desired patience and tomorrow's actual one. In the following discussion, we will focus on the first two types.

We analytically solve the above model, where $T = 3$ and $u(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$. The optimal asset decision is the following:

$$
\begin{aligned}
A_1 &= \frac{X(y_1+A_0)-Z^{-\frac{1}{\rho}}(y_3+r_2y_2)}{X+r_1r_2Z^{-\frac{1}{\rho}}} \\
A_2^{\exp} &= \frac{y_2+r_1A_1-(\beta_1\delta r_2)^{-\frac{1}{\rho}}y_3}{X}, \\
A_2^{real} &= \frac{y_2+r_1A_1-(\beta_0\delta r_2)^{-\frac{1}{\rho}}y_3}{1+(\beta_0\delta r_2)^{-\frac{1}{\rho}}r_2}, \\
where \quad X &= 1+(\beta_1\delta r_2)^{-\frac{1}{\rho}}r_2, \\
Z &= (\beta_0\delta)(\beta_1\delta)r_1r_2 - \frac{(\beta_0\delta)(\beta_1\delta)r_1r_2}{X} + \frac{\beta_0\delta^2r_1r_2}{X}.
\end{aligned}
$$

where $A_2^{\exp}$ is the expected behavior of self 2 from self 1's point of view and A_2^{real} is the real behavior of self 2.

We will use numerical examples to illustrate some interesting findings, which are not easy to see from the analytic solution.

Assume $\rho = 2$, $y_1 = y_2 = y_3 = 1$ and $A_0 = 0$. Offer A carries an interest rate of 5% for the first period and 20% for the second period. Offer B has a flat interest rate schedule: 10% for both periods.

We plot the rank reversal region (the shaded area) in Fig.(3), based on the above numerical example. Apparently, there is no rank reversal when $\beta = 1$, which is the exponential model. However, there exists a wide rank reversal area for hyperbolic models.

A naive agent exhibits "rank reversal" because she underestimates her future borrowing. For example, suppose $\beta = 0.82$ and $\delta = 1$. In the first period, she prefers offer A because she expects that $A_1 = -0.0207$ and $A_2^{\exp} = 0.0312$. However, when the second period arrives she gives in to her instantaneous desire and borrows again, $A_2^{real} = -0.0135$. Base on her actual behavior, she has made a suboptimal choice in the first period. However, her decision is optimal based on her expectation.

A sophisticated agent behaves suboptimally not because she has incorrect expectation, but because she tries to align her future behavior with her current preference. She may reduce her debt in period one to take into account of overspending in period two. Or, she may constrain herself in period two by choosing the shorter offer A. A much higher post-introductory interest rate will damp her desire to borrow, which is conform to her preference in period one. The two strategies are called *the strategy of consistent planning* and *the strategy of precommitment* respectively in Strotz (1956).

We will illustrate the two strategies by numerical examples. Still suppose $\beta = 0.82$ and $\delta = 1$. If she can commit her future behavior, she will choose offer A and $A_1 = -0.0207$ and $A_2 = 0.0312$. However, she anticipates that this plan will not be followed in the second period. She decides to still accept offer A but borrow less at period one, -0.0199, to accommodate tomorrow's borrowing, $A_2 = -0.0131$. Based on her reduced debt, the interest payment under A is more than B. However it is not optimal to choose B since this consumption plan will not be implementable if B is chosen. Given $A_1 = -0.0199$, she will borrow much more under offer B in the second period, -0.0347.

To illustrate the strategy of precommitment, suppose $\beta = 0.28$ and $\delta = 0.64$. If the sophisticated agent can commit, she would choose offer B since she would like to borrow in both periods. However, she decides to choose A, $A_1 = -0.5688$ and $A_2 = -0.4888$, even though she has to pay more interest. If she chosen B, the best plan is $A_1 = -0.5657$ and $A_2 = -0.539$, which is worse.

Given any δ, a smaller β will be more likely in the rank reversal region. As the difference between the long term desired discount factor, δ, and the short term temptation, $\beta\delta$, is larger, an naive agent's underestimation error is larger and more desperate the sophisticated agent wants to constrain herself. Both will lead to financially suboptimal behavior.

Only when β is really small, less than 0.8 in this numerical example, rank reversal region for sophisticated agents separates from that of naive agents. The same is true for asset choices. Whether the agent recognizes self-control problem or not makes a difference only when the self-control problem is severe. As $\beta \to 1$, both models converge to the exponential model. Laibson (2001) shows that when β is close enough to 1, the sophisticated model behaves qualitatively like the exponential model, even though it is an intrapersonal game.

5 A Multi-period Incomplete Information Model

A dynamic model, which captures consumer decision problem in the market experiment more realistically, is presented in this section. Comparing with the previous model, this dynamic model has two realistic institution features. First, consumers face both transitory and persistent income shocks. Second, receiving new offers is an probabilistic event. One possible explanation of "rank reversal" is that realized random events make consumer *ex ante* choice suboptimal. It is optimal based on their expectation about future, though not according to the true realization. For example "Rank reversal" may due to the fact that some consumers don't receive another offer even though another offer is very likely *ex ante*.

5.1 Model Setup

The model is inspired by standard "buffer-stock" life-cycle models, Carroll (1992, 1997a), and Deaton (1991). This model is set in discrete time. One period in the model represents one quarter in the real world. The consumer lives for T periods. The boundary condition is that the consumer consumes all her cash-on-hand at the final period.[13] The consumer receives stochastic income every period. She can either save in her saving account or borrow on credit cards to smooth her consumption. However, she is liquidity constrained in two respects. First, she is restricted in her ability to borrow. The upper bound is the total credit limit of her credit cards, denoted as $\bar{L}$, which is exogenously given. However nothing prevents her from accumulating liquid assets. Second, she faces different interest rates depending on whether she is savings (r^s) or borrowing (r), where $r > r^s$, and r is the regular interest rate on credit cards.

The consumer can reduce the interest payment on her debt if she accepts an introductory offer. At the beginning of period 1, the credit card company that conducted this market experiment, denoted as Red, offers the consumer an introductory interest rate $r^r < r$ with a duration τ^r periods, and a credit limit l. The consumer may also receive credit card solicitations from other credit card companies which are not observed in this dataset. These unobservable companies are simplified as one company, Blue. Blue provides an introductory interest rate $r^b < r$ with an introductory

[13]The model is chosen to have finite horizon because the standard contraction map theory fails for sophisticated hyperbolic models. See Laibson (1997,1998) and for more details. We choose T large enough, so that results will not vary with it.

duration of τ^b and a credit limit also l. The consumer's total credit limit $\bar{L}$ is held constant even after accepting a new offer to simplify computation. This is an innocuous assumption since respondents have so much idle credit limit: the average debt is only \$2,000 while the average credit limit is \$6,000 on this observed card and \$15,000 on other cards. In every period, the consumer receives a Blue offer with a probability q, which is positive and finite if the consumer has no existing introductory offer from Blue, otherwise zero.[14]

Simultaneously with the acceptance of credit card offer(s), the consumer decides how much to consume at the beginning of period t. There is a switching cost, k_t, associated with accepting every introductory offer. The switching cost is indexed by t because it is assumed that the consumer has a time-varying switching cost. This assumption is required because respondents with similar credit card debt fail to switch after this red offer expires, the second puzzle. Simulation results show that respondents, no matter hyperbolic or exponential agents, will definitely switch again after the offer expires if their credit card debt remains the same, they have fixed switching costs and there are new offers available. As mentioned before, respondents must have some other offers available given their credit status and the economy environment. The only possible explanation is that consumers's switching costs are changing over time. Respondents of this experiment accepted the offers due to their low realized switching costs at the time of solicitation. However, their mean switching costs are much higher, which can be partially inferred from the low response rate (1%). This high mean will keep the majority of respondents from switching a second time after the introductory periods. The switching cost captures the (expected) time and effort required in filling out an application for a new card. It is assumed that there is no extra cost for transferring balance after the consumer accepts a new offer. Once she accepts introductory offer(s), she has immediate access to the credit.

The consumer in period t maximizes a weighted sum of utilities from current period on which is summarized in the following Eq.(3).

$$\begin{aligned} V_{t,t}\left(\Lambda_t\right) &= \max_{C_t, d_t^b, d_t^r} \frac{C_t^{1-\rho}}{1-\rho} - d_t^b k_t - d_t^r k_t + \beta_0 \delta E\left\{V_{t,t+1}\left(\Lambda_{t+1}\right)\right\}, \quad \text{for } t = 1, \\ V_{t,t}\left(\Lambda_t\right) &= \max_{C_t, d_t^b} \frac{C_t^{1-\rho}}{1-\rho} - d_t^b k_t + \beta_0 \delta E\left\{V_{t,t+1}\left(\Lambda_{t+1}\right)\right\}, \quad \text{for } t \geq 2. \end{aligned} \tag{3}$$

The instantaneous utility is the sum of the consumption utility and the disutility (the switching

[14] This assumption effectively excludes that consumers have more than one introductory offer from Blue. We believe relaxing it will only complicate the problem with little benefit.

cost) from accepting an introductory offer. C_t and d_t^b are the consumption choice and the decision to accept an introductory offer from Blue at period t respectively. d_1^r is the decision to accept the Red offer at period 1. k_t is the current switching cost. The consumption function is assumed to be CRRA and ρ is the coefficient of relative risk aversion. Eq.(3) is similar to Eq.(1) in the previous section. Λ_{t+1} denotes the vector of state variables: $\left\{X_{t+1}, \varphi_{t+1}, k_{t+1}, \tau_{t+1}^b, \Gamma_{t+1}^r, s_{t+1}\right\}$. X_{t+1} is cash-on-hand at the beginning of period t+1, which is a sum of income, y_{t+1}, and wealth, A_{t+1}. φ_{t+1} is the realized persistent income shock at period t+1, which will be discussed in more detail later. k_{t+1} is the realized switching cost in period t+1. τ_{t+1}^b and Γ_{t+1}^r denote the number of introductory periods left on the Blue and Red card at period $t+1$ respectively. s_{t+1} denotes whether a new introductory offer is received at period $t+1$. The expectation is taken with respect to the distributions of y_{t+1}, φ_{t+1}, k_{t+1} and s_{t+1}.

$V_{t,t+1}$ is a weighted sum of self t's excepted future utilities and the weights are determined by self t's long-run preference δ. It is recursively defined as:

$$V_{t,t+1}\left(\Lambda_{t+1}\right) = \frac{\widetilde{C}_{t+1}^{1-\rho}}{1-\rho} - \widetilde{d}_{t+1}^b k_{t+1} + \delta E\left\{V_{t+1,t+2}\right\} \tag{4}$$

$V_{t,t+1}$is the optimal utility of "expected self $t+1$" from self t's point of view. $\widetilde{C}_{t+1}$ and $\widetilde{d}_{t+1}^b$ are the behavior of a hypothetical self $t+1$. Exponential and sophisticated hyperbolic agents have correct expectation. Hence the hypothetical behavior is decided by solving Eq.(3). For hyperbolic agent, the hypothetical behavior is determined by a problem similar to Eq.(3) with $\beta_0 = 1$. Note $V_{t+1,t+2}$ is used instead of $V_{t,t+2}$ because they are the same. Self t and self $t+1$ have the same expectation about periods later than $t+1$. This special feature of quasi-hyperbolic models makes it easier to compute. This consumer problem is solved numerically by backward induction (iterating Eq.3 and Eq.4).

5.2 Model Prediction

Can random shocks explain "Rank Reversal"? Simulation results reveals that the conflict between preference for the short offer A and later low switching is still unexplainable in the exponential model. In the top panel of Table (3), exponential agents' response to offer A and F are reported for different δ, given other parameters. The more patient the agents are, the more response to the short offer A compared with that to offer F. On the contrary, agents are more likely to stay with the card only if they become more impatient. The corresponding average debt over time are shown in Fig.(4). The time consistent agents always prefer an offer incurring the least cost. The short offer costs less only if the debt declines rapidly over time. Under that scenario, earlier interest saving can compensate for the later higher interest rate.

However, both sophisticated and naive models exists some β, where agents prefer the short offer and they keep on borrowing on the card for a long period. Corresponding results for sophisticates are reported in Table (3) and Fig.(5). When β is very low, the longer offer is preferred because it really saves much more interest than the short one, which is outweighing the benefit of constraining future selves. However, when β is moderate, like 0.8, agents would like the short offer which will save them money in the near future and constrain them to borrow less in the future.

In the next section, we will apply the dynamic model to the empirical data and estimate related parameters.

6 Estimation Strategy

To estimate the above dynamic model by matching every consumer's behavior, we need much more individual-level dataset than we actually have, such as $y_{i,t}$, $s_{i,t}$. To circumvent the data problem, the parameters of the model are estimated by matching empirical moments with simulated moments from the dynamic model. The estimation method used is *Simulated Minimum Distance Estimator* (SMD), proposed in Hall and Rust (2002). [15] There is one important feature of this method, which deserves mentioning. It recognizes the fact that there is severe endogenous sampling at the first period, i.e. we only observe respondents' subsequent borrowing behavior.

Total 216 moments are used, 36 for each market cell. The 36 moments are the response rate plus five debt distribution statistics for seven quarters: proportion of consumers who borrow, mean, median, forty and sixty percentiles among borrowers. The debt statistics for the first quarter is omitted because they are exceptionally low due to the fact that it takes about 2-3 months to accumulate debt on this account. This time lag is not modeled in the dynamic model.

To make estimation feasible, we calibrate a subset of parameters, using related literature and our dataset, and make assumptions about exogenous variables' distributions.

[15] This method is similiar to Simulated Moments Estimator (SME) of McFadden (1989) and Pakes and Pollard (1989).

First is the income process, which is modeled as a time series with changes in two possible states: a good state and a bad state.[16] In a given state, income is a random draw from a lognormal distribution, $LN\left(\eta^{j}, \varepsilon^{j}\right)$, where $j \in \{g, b\}$. The distribution parameters depend on whether it is in the good state or the bad state. The evolution of the two states is governed by a discrete random variable, $\varphi_t \in \{1, 0\}$, where 1 and 0 represent the good and bad state respectively. φ_t is a two-state Markov chain with transition probabilities: $\{p_{i,j}\}$, where $i, j \in \{1, 0\}$, $p_{i,j} = prob(\varphi_t = i/\varphi_{t-1} = j)$. To get reasonable estimates for the income distribution, we use estimates from Laibson et al. (2000) as a starting point. we describe the calibration in more details in Appendix.

We assume consumer switching cost, k_t, is an identical and independent random draw from a uniform distribution with a range $[0, k]$. We assume, at the time of solicitation, consumer liquid asset/credit card debt follows a normal distribution with a mean of the mean μ and the variance ϵ^2.

We calibrate the total credit limit, $\overline{L}$, and the credit limit for each credit card (recall that they are assumed the same), l, using the information in the dataset. The calibrated $\overline{L}$ are \$15,000 and l is \$6,000. In addition, the regular interest rate for credit cards r is assumed to be 1.16%, and the saving interest rate $r_s = 1.01\%$. The relative risk aversion coefficient, ρ, is assumed to be 2.

The introductory interest rates and durations of the Red offers, r_r and τ_r are given in the experiment dataset. However, we don't observe introductory offers consumers received in subsequent periods. We assume the duration for the Blue offer is 6 months, which is the typical duration in the company we observed. The interest rate on the Blue offer is assumed to be 8%.

We assume consumers have a probability of 90% to receive Blue offers. As argued before, we believe respondents should receive new offers every quarter with a probability of almost one in the sample period. It is assumed 1% consumers have an ongoing Blue offer at the time of the Red solicitation which was the average response rate to credit card solicitations at the sample period.

Given the calibrated parameters and the distribution assumptions, we estimate remaining parameters by minimizing a weighted[17] distance between simulated moments of consumers' behaviors

[16] The standard way of modeling the income randomness is as a sum of a persistent and transitory shock. In this study, the persistent income shock is modeled as a two-state Markov process, rather than as an AR(1). The method was introduced in Laibson *et al.* (2000) to save computation time.

[17] The weighting matrix is chosen by the authors, which gives much weight to the response rate. Shui (2003) provides more detail on this.

and their empirical counterparts. We estimate parameters for three models: exponential, naive and sophisticated hyperbolic. The estimated parameters are the time discount factors, β and δ, the switch cost distribution parameter k, and the parameters of the liquid asset distribution at the beginning of period 1, the mean μ and the variance ϵ^2. For the exponential model, $\beta = 1$. For the naive model, $\beta_1 = 1$ and $\beta_0 = \beta$ is estimated. For the sophisticated model, $\beta_1 = \beta_0 = \beta$.

7 Estimation Results

Estimation Results for the dynamic model are reported in Table 4. "Goodness-of-Fit" is the weighted distance between empirical moments and simulated moments. Allowing for hyperbolic time preferences significantly improves it, reducing the distance by more than half. As explained above, the failure of exponential discounting is because it is time consistent but the behavior observed is inconsistent. Even after random shocks are incorporated into the model, time consistent consumers on average exhibit consistent behavior. Only allowing consumers have time inconsistent preferences, the model prediction can match the empirical data.

An inspection of Table 4 shows that all parameters are estimated precisely. The parameters for both hyperbolic models are very close, nevertheless those of the exponential model are quite different. The similarity of naive and sophisticated parameters is because the optimal β is close to 1. The dynamic model also predicts that only when β is small, less than 0.7, the sophisticated model will behave qualitatively different from its naive counterpart. There is a small quantitative difference that: given β, δ, naive consumers borrow more and are more eager to accept new offers because they don't realize their self-control problem. Therefore, the naive model needs a larger β to match the consumer debt level and a larger switching cost to keep consumers from switching out.

Exponential consumers have a much larger switching cost (\$861 vs. \$150). Such a large k is required to better match the debt path over time. In Fig.6, the predicted debt paths of Market Cell A, E and F by three models are compared with empirical data. Comparing to the exponential model, the two hyperbolic model match the debt path much better, which is the reason why their "Goodness-of-Fit" are much lower. Despite a very large switching cost, the predicted debt path by the exponential model declines much faster than the data. Exponential consumers borrow too much at the beginning, an average of \$3500 compared with \$2700 empirically, and too little at the end, an average of $900 instead of $2600 empirically. Such a debt path is because that exponential consumers are so patient ($\delta = 0.9999$) that they will pay off their debt even without switching. However such a large δ is required to match consumer preference for the short offer.

Consumer responses to six different introductory offers are shown in Table 5. All three models match the response rates because we put a large weight on this moment. Hyperbolic models are better than the exponential model because they also match the relative preferences among the six offers.

The magnitude of k deserves some discussion. Is the average switching cost $150 outrageously high? The magnitude of k here is consistent with anecdotal evidence in the credit card market. First, credit card issuers spend lots of money to acquire one customer. Credit card companies send out billions of solicitations every year and 99% of them end up in trash cans. Many solicitations offer a very low introductory rate, as low as 0%. The behavior of issuers will only be rational if majority consumers don't switch. Second, it is a widespread view that switching between different financial products is more hassle than it's worth. A 2002 survey by WHICH? revealed that 66% respondents kept the same credit card and majority of them said that they thought it was too difficult to change. the k captures not only the time it takes to fill out one application form, but also any psychological disutility that consumers associate with the whole process.

A close inspection of Fig.6 reveals that there are two features of the data, which are not explained by the hyperbolic models. First, there are only 65% respondents borrowed during introductory duration. However both hyperbolic models predict that almost all respondents borrow at the beginning. Second, the predicted debt distribution is more concentrated than the empirical data, because there are significantly more respondents borrow more than the sixty percentile, about 75%. We suspect this is due to the restrictive assumption of homogenous consumers. We observe that there are 20% respondents are convenience users, who never borrowed even in the introductory period. Their time preferences must be different from those revolvers, who borrow even under 16% interest rate. In the future research, we plan to explore the effect of consumer heterogeneity.

8 Conclusion

This study uses a rich individual-level dataset in the credit card market to answer the question: whether consumers are time consistent not. From this unique dataset, two puzzles are identified.

First, at the time of solicitation, consumers prefer an offer with a lower introductory interest rate (4.9%) and a shorter duration (6 months), to an offer with a higher introductory interest rate (7.9%) but a longer duration (12 months). The relative preference is puzzling since consumers would benefit more, *ex post*, from the longer introductory offer. We call it "rank reversal". Second, the majority of respondents do not switch out after the expiration of their introductory offers, even though their debt remains at the same level as when they accept the offer. This is puzzling because there are so many other offers available and the benefit of switching is as large as before.

We first use a multi-period complete information model to illustrate that standard exponential consumers will not exhibit "rank reversal". However, if consumers are assumed to have time inconsistent preferences, such as the newly developed hyperbolic discounting, "rank reversal" is not a puzzle any more. It is actually rational behavior by time inconsistent consumers.

Can a exponential model with realistic random shocks explain the above two puzzles? To address this question, a dynamic model is developed, in which consumers have both time consistent (exponential) and time inconsistent (hyperbolic) preferences and they are subject to realistic random shocks. According to the estimation results based on this dynamic model, only the hyperbolic model can explain the first puzzle. The exponential model fails because that time consistent consumers would always prefer an offer which provides the lowest interest payment. We have explored two extreme types of hyperbolic discounting: naive and sophisticated. Both of them can explain the data, however the underlying stories are different. Naive consumers mistakenly prefer the shorter offer because they underestimate their future borrowing. Sophisticated consumers prefer the shorter offer because it offers a self-commitment device. Unfortunately, the two hyperbolic models are indistinguishable in this experiment. A time-varying switching cost is required to explain the second puzzle. Accepting one offer only implies that respondents have low switching costs at the time of acceptance. Most of the time consumers face much higher switching costs. Therefore majority of them fail to switch a second time even though their debt remains large.

Consumer time consistency is an important question since different models have vastly different normative implications. For example a consumer piles up debt on her credit cards. She may do so because the pleasure of consumption today outweighs the interest payment tomorrow. Or she may do so because she has an impulse to overspend which is not valued from the long-run perspective, like the sophisticated agent. The two stories have different public policy implications. The first consumer just borrows the right amount. However, the second consumer would like somebody to bind her hands. It is crucial to distinguish between the two hypotheses.

Consumer behavior identified here also facilitates the understanding of two competition anomalies in the credit card market. The first is that consumer credit card loans earn a higher capital return than other bank assets, and credit card interest rates are excessively downward sticky compared with fund costs, as well established in Ausubel (1991). The other is, instead of lowering interest rates, credit card issuers fiercely compete with each other by sending out "junk mail". Consumer irresponsiveness to interest rates has been offered as a reason for this in Ausubel (1991). This study not only provides individual-level evidence of this inertia, but also identify two separate forces behind it: self-control problems and high switching costs.[18]

[18] Calem and Mester (1995) also find that consumers have high switching cost in the credit card market.

Appendix

Laibson *et al.* (2000) models the idiosyncratic income shock, ξ_t, as a sum of a persistent shock, μ_t, and a transitory shock, ν_t. The persistent shock follows an AR(1) process with a coefficient α.

$$\xi_t = \mu_t + \nu_t,$$
$$\mu_t = \alpha\mu_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ and $v_t \sim N\left(0, \sigma_v^2\right)$. He estimated α, σ_ε^2, σ_v^2 for three different education levels. The parameters for "completed college" are used in the estimation.

Define a quarterly income shock, η_q, such that $\xi_t = \sum_{q=4(t-1)+1}^{4t} \eta_q$.

$$\eta_q = s_q + \epsilon_q,$$
$$s_q = f s_{q-1} + \gamma_q,$$

where s_q is a quarterly persistent shock with a coefficient of f. $\gamma_q \sim N\left(0, \sigma_r^2\right)$ and $\epsilon_q \sim N\left(0, \sigma_\epsilon^2\right)$.

It can be shown that:

$$4\sigma_\epsilon^2 = \sigma_v^2$$
$$\frac{1}{1-\alpha^2}\sigma_\varepsilon^2 = (4 + 6f + 4f^2 + 2f^3)\frac{\sigma_r^2}{1-f^2}$$
$$\frac{\alpha}{1-\alpha^2}\sigma_\varepsilon^2 = (f + 2f^2 + 2f^3 + 4f^4 + 3f^5 + 2f^6 + f^7)\frac{\sigma_r^2}{1-f^2}$$

After obtaining parameters for the quarterly shock, I use a two-state Markov process to replace the s_q which follows an AR(1), following Laibson *et al.* (2000). The Markov process is symmetric taking two values $\{\theta, -\theta\}$, where $\theta = \sqrt{\frac{\sigma_r^2}{1-f^2}}$ and the transition probability $p = \frac{1+f}{2}$. In this way the Markov process matches the variance covariance of s_q.

Recall the income process in the dynamic model, $y_t = \varphi_t y_t^g + (1-\varphi_t)\, y_t^b$. y_t^j is lognormal random variable, where $j \in \{g, b\}$ and φ_t is a signal whether the income state is good or bad.

$$\log\left(y_t^g\right) = c + \theta + \epsilon_t$$
$$\log\left(y_t^b\right) = c - \theta + \epsilon_t$$

where c is a constant to capture the permanent income. To determine c, I assume the mean income is \$10,000 per quarter.

In summary, the income process in the good state has a mean of 10,000 and a variance of 3.5×10^5. The income process in the bad state has a mean of 7645 with a variance of 2.05×10^5. The transition probability matrix is:

$$p = \begin{pmatrix} 0.9939 & 0.0061 \\ 0.0061 & 0.9939 \end{pmatrix}.$$

References

[1] **Ainslie, George and Varda Haendel**, "The Motives of the Will," In E. Gotteheil *et al.* (eds.), *Etiologic Aspects of Alcohol and Drug Abuse*, Springfield, Il, Thomas, 1983.

[2] **Akerlof, George A,** "Procrastination and Obedience," *American Economic Review, Papers and Proceedings,* May, 1991 1-19.

[3] **Ausubel, Larry M.,** "The Failure of Competition in the Credit Card Market," *American Economic Review,* March, 1991, 50-81.

[4] **Ausubel, Larry M.,** 1999. "Adverse Selection in the Credit Card Market," working paper, University of Maryland.

[5] **Besharov, Gregory and Bentley Coffey, "**Reconsidering the Experimental Evidence for Quasi-Hyperbolic Discounting," working paper, Duke University, 2003.

[6] **Carroll, Christopher D.,** "The Buffer Stock Theory of Saving: Some Macroeconomic Evidence," BPEA, 1992, *2*, 61-156.

[7] **Carroll, Christopher D.,** "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, 1997, *112*, 1-57.

[8] **Deaton, Augus**, "Saving and Liquidity Constraints," *Econometrica,* 1991, 59, 1221-48.

[9] **Della Vigna, Stefano, and M. Daniele Paserman** 2000. "Job Search and Hyperbolic Discounting," Harvard University mimeo.

[10] **Della Vigna, Stefano and Ulrike Malmendier** 2002. "Self-Control in the Market: Evidence from the Health Club Industry," Harvard University mimeo.

[11] **Diamond, Peter and Botond Koszegi** 1998. "Hyperbolic Discounting and Retirement," MIT mimeo.

[12] **Fernandez-Villaverde, Jesus and Arijit Mukherji** 2002. "Can We Really Observe Hyperbolic Discounting?" University of Pennsylvania mimeo.

[13] **Fang, Hanming and Dan Silverman** 2001. "Time-inconsistency and Welfare Program Participation: Evidence from the NLSY," Yale University mimeo.

[14] **Gruber, Jonathan and Botond Koszegi** 2001. "Is Addiction 'Rational'? Theory and Evidence," *Quarterly Journal of Economics,* November, 116(4), 1261-1303.

[15] **Hall, George and John Rust** 2002. "Econometric Methods for Endogenously Sampled Time Series: The Case of Commodity Price Speculation in the Steel Market," Yale University mimeo.

[16] **Hansen, L. P.** 1982. "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica,* 50, 1929-1954.

[17] **Harris, Christopher and David Laibson** 2001. "Dynamic Choices of Hyperbolic Consumers," forthcoming in *Econometrica.*

[18] **Imrohoroglu, Ayse, Selahattin Imrohoroglu and Douglas H. Joines** 2003. "Time-inconsistent Preferences and Social Security," *Quarterly Journal of Economics,* 118(2), 745-84.

[19] **Laibson, David I.** 1997. "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics,* 62(2), 443-478.

[20] **—–, Andrea Repetto, and Jeremy Tobacman** 1998. "Self-Control and Saving for Retirement," *Brookings Papers on Economic Activity,* 1, 91-196.

[21] **—–, —–, and —–** 2000. "A Debt Puzzle," in eds. Philippe Aghion, Roman Frydma, Joseph Stiglitz, Michael Woodford, *Knowlege, Information and Expectation in Modern Economics: In Honor of Edmund S. Phelps,* forthcoming.

[22] **Loewenstein, Geoge and Richard H. Thaler 1989.**

[23] **Loewenstein, G.** "Out of control: visceral influences on behavior," *Organizational Behavior and Human Decision Processes*, 1996, 65, 272-92.

[24] **Miravete, Eugenio J.** 2003. "Choosing the Wrong Calling Plan? Ignorance and Learning," *American Economic Review,* 93(1), 297-310.

[25] **Mulligan, Casey.** 1997. "A Logical Economist's Argument Against Hyperbolic Discounting," University of Chicago mimeo.

[26] **O'Donoghue, Ted and Matthew Rabin** 1999. "Doing it Now or Later," *American Economic Reviewb,* March, 89(1), 103-124.

[27] —– 2001 "Choice and Procrastination." *Quarterly Journal of Economics,* 116(1), 121-160.

[28] **Paserman, Marco Daniele** 2003. "Job Search and Hyperbolic Discounting: Structural Estimation and Policy Implications." Manuscript, Hebrew University of Jerusalem.

[29] **Read, Daniel and Barbara van Leeuwen**, "Predicting Hunger: The Effects of Appetite and Delay on Choice." *Organizational Behavior and Human Decision Processes*, 1998, 76(2), 189-205.

[30] **Rubinstein, Ariel.** 2000. "Is It 'Economics and Psychology'?: The Case of Hyperbolic Discounting," Princeton University mimeo.

[31] **Thaler, Richard.** 1981 "Some Empirical Evidence on Dynamic Inconsistency," *Economics Letters,* 8, 201-207.

Table 1: Sample Statistics[a]

	N	Months On File	Number of Past-due	Revolving Balance	Revolving Limit	Credit Score	Number of Credit Cards	Income
All[b]	599,257	174	0.0197	$2,509	$17,481	643	3.77	NA
		(71)	(0.139)	(4058)	(11388)	(89)	(1.88)	
A	1073	126	0.0308	$3,927	$15,473	584	3.94	$44,180
		(76)	(0.1727)	(4979)	(10573)	(96)	(2.057)	(24051)
B	903	128	0.0266	$3,474	$15,137	592	3.81	$43,170
		(79)	(0.1609)	(4725)	(11112)	(96)	(2.101)	(25175)
C	687	114	0.0247	$3,543	$14,230	579	3.598	$42,253
		(77)	(0.1555)	(4901)	(11268)	(95)	(2.068)	(24437)
D	645	112	0.0248	$3,584	$14,075	582	3.557	$41,215
		(76)	(0.1557)	(4988)	(11703)	(104)	(2.07)	(25274)
E	992	125	0.0363	$3,694	$15,176	590	3.729	$43,830
		(76)	(0.1871)	(5066)	(11313)	(100)	(2.076)	(28733)
F	944	123	0.0222	$4,042	$15,107	581	3.807	$43,697
		(77)	(0.1476)	(5469)	(10688)	(100)	(1.98)	(26725)

[a]Standard deviations are in parentheses.

[b]Sample statistics are reported for all six market cells to save space. Due to randomization, the statistics are similiar across different market cells.

Table 2: Rank Reversal[a]

Market Cell	Number of Observations	Effective Response Rate	Rank by Response Rate	Effective Interest Rate	Rank by Interest Rate
A: 4.9% 6 months	99,886	1.073% (0.00033)	1	10.23%	3
B: 5.9% 6 months	99,872	0.903% (0.00030)	4	11.35%	4
C: 6.9% 6 months	99,869	0.687% (0.00026)	5	11.86%	5
D: 7.9% 6 months	99,880	0.645% (0.00025)	6	12.35%	6
E: 6.9% 9 months	99,890	0.992% (0.00031)	2	9.23%	2
F: 7.9% [b] 12 months	99,860	0.944% (0.00031)	3	8.32%	1
T-TEST		P-VALUES			
A vs. E		7.23%			
A vs. F		0.29%			

[a]Standard errors in parentheses.

[b]It should be briefly be explained why the calculated effective interest rate for market cell F (8.32%) slightly exceeded the stated APR of 7.9%. First, the author's calculations incorporated the first 13 months of the potential life of the account, in order to deal with some timing problems in the data. Second, the APR is twelve times the monthly interest rate and, so, omits monthly compounding. Third, the introductory interest rate is conditional on the card holder remaining current on his account; each market cell includes customers who went delinquent and lost the introductory rate.

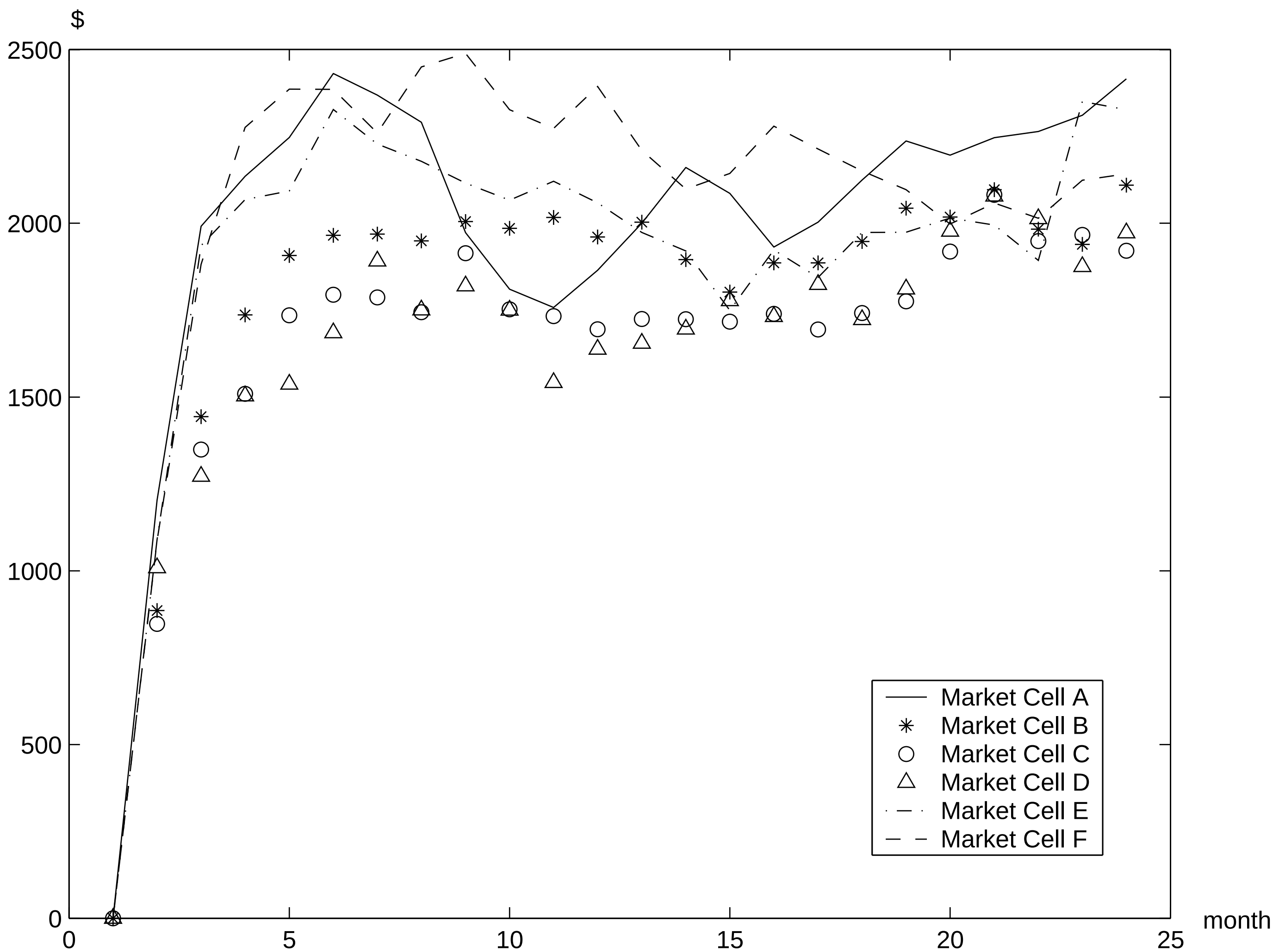

Figure 1: Medians of borrowers' debt distributions over time.

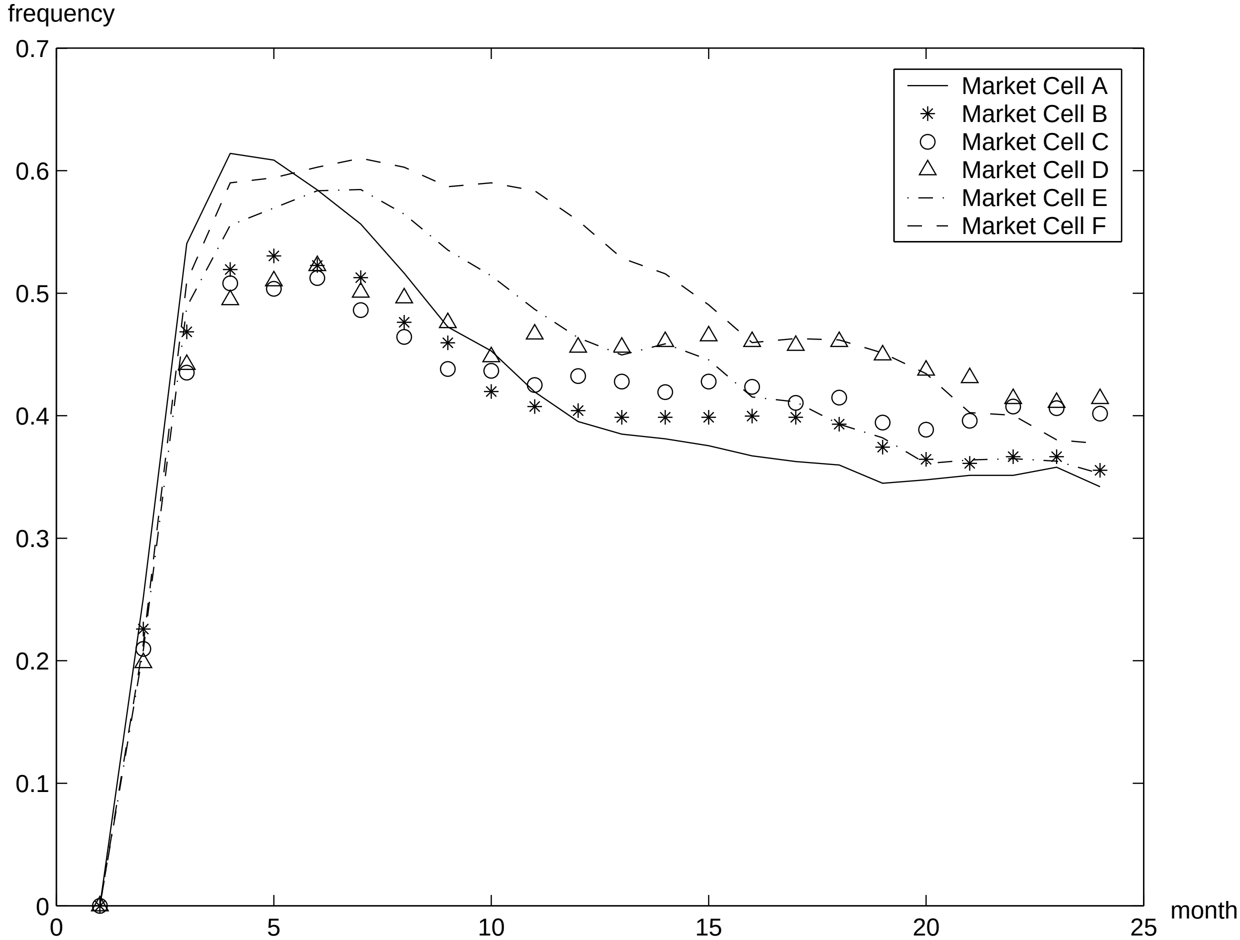

Figure 2: Borrowing Frequencies of respondents over time.

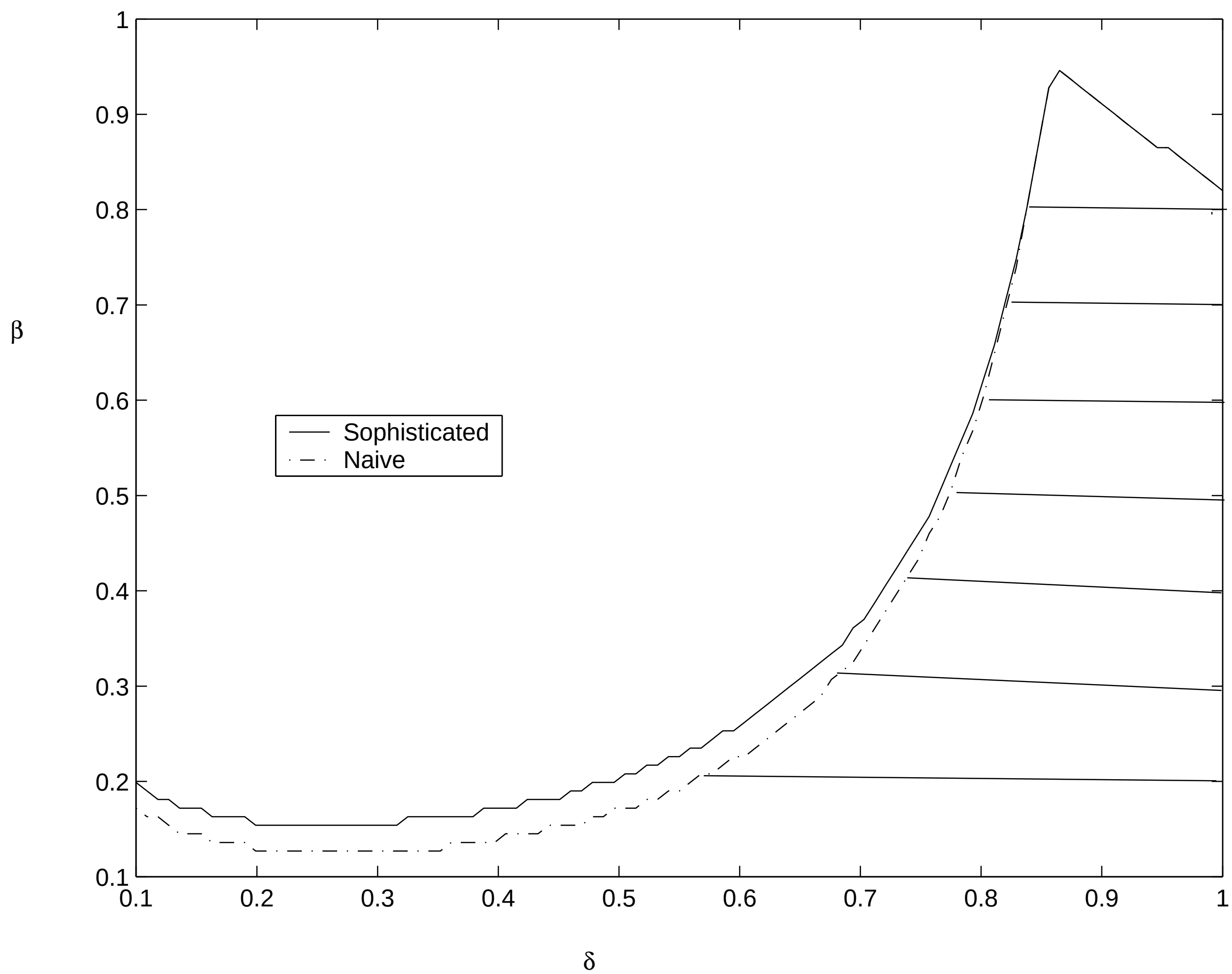

Figure 3: Rank Reverse Area of Sophisticated and Naive Hyperbolic Agents

Table 3: Response [a]

Exponential				
δ	0.96	0.98	0.99	0.9999
A	152	72	53	41
F	476	100	71	36
Sophisticated				
β	0.5	0.8	0.9	0.95
A	987	80	54	47
F	2134	72	41	49

[a] Response is out of 10,000 simulations. β,δ are discount factors. k, the switching cost parameter, is 0.03. Mean of A_1 is 5000 and variance is 8e6, which is liquid assets at the time of solicitation.

Table 4: Estimated Parameters [a]

	(1) Sophisticated	(2) Naive	(3) Exponential
β	0.7863 (0.039)	0.8172 (0.02343)	
δ	0.9999 (0.003575)	0.9999 (0.002534)	0.9999 (0.000827)
k	0.02927 ($146) (0.006196)	0.0326 ($163) (0.00899)	0.1722 ($861) (0.004)
μ	1.0088 ($5044) (0.04485)	0.9584 ($4792) (0.02348)	1.5836 ($7918) (0.0098)
ϵ^2	0.831 ($2883) (0.0438)	0.8167 ($2858) (0.01913)	4.278 ($6541) (0.009812)
Goodness-of-Fit	$2.5202e-4$	$2.8183e-4$	$6.0534e-4$

[a] β,δ are discount factors. k is the switching cost parameter. A_1 is liquid assets at the time of solicitation. Standard errors are in parenthese.

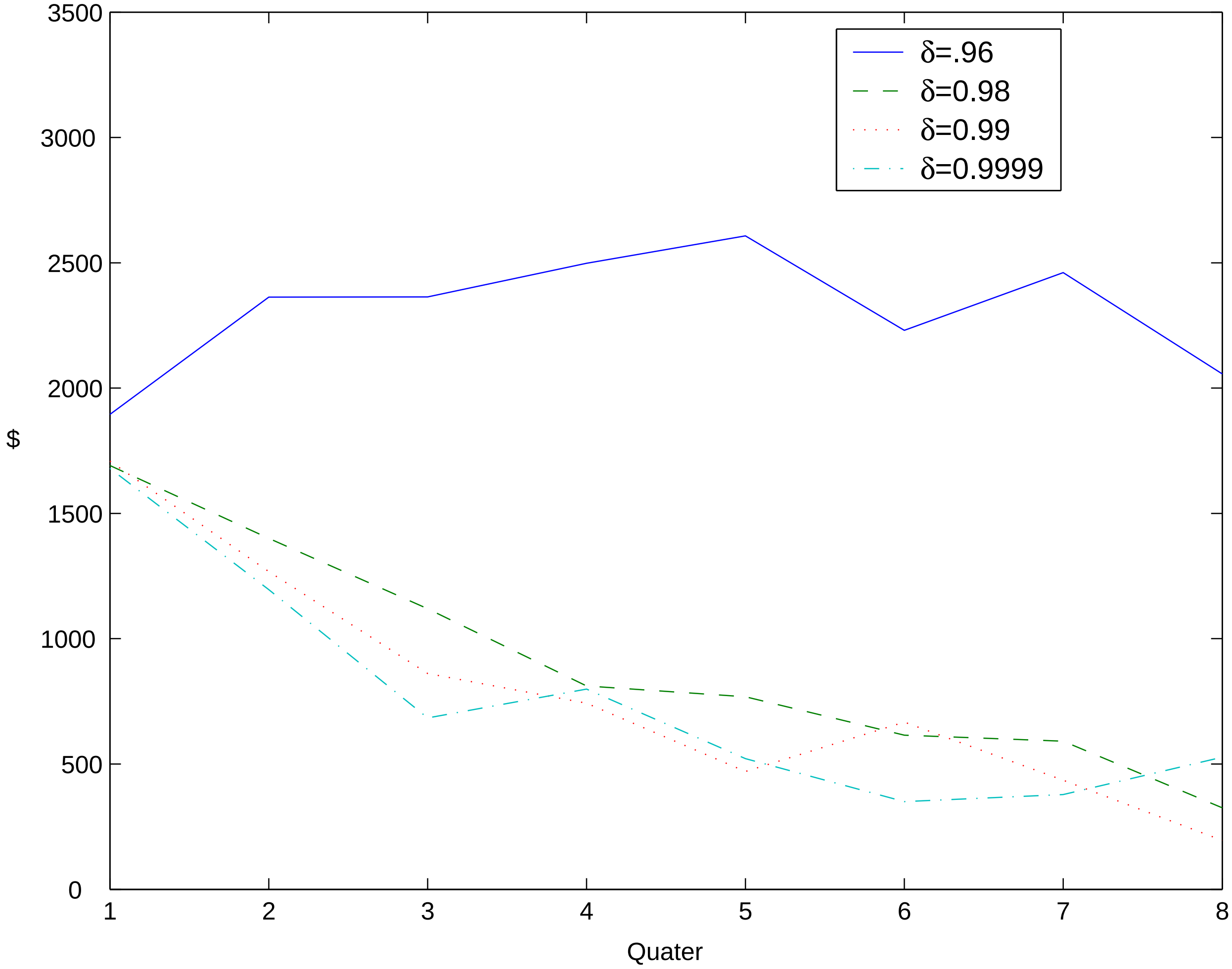

Figure 4: Simulated Dynamic Debt Path for Exponential

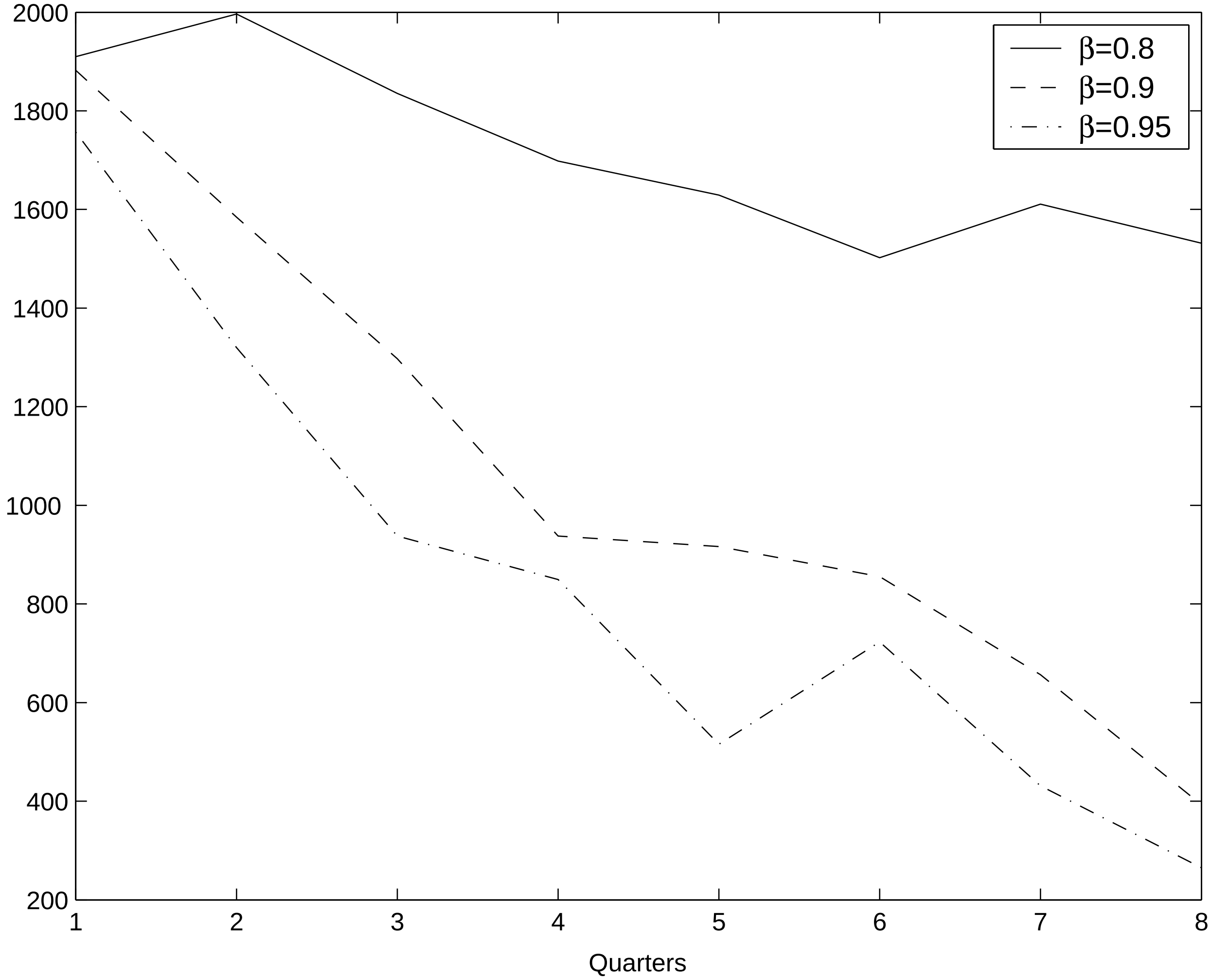

Figure 5: Simulated Dynamic Debt Path for Sophisticated

Table 5: Simulated Response

Market Cell	Total	Empirical	Naive Hyperbolic	Sophisticated Hyperbolic	Exponential
A: 4.9% 6 months	99,886	1073	1013	1001	951
B: 5.9% 6 months	99,872	903	911	888	845
C: 6.9% 6 months	99,869	687	810	793	764
D: 7.9% 6 months	99,880	645	701	652	672
E: 6.9% 9 months	99,890	992	997	980	1005
F: 7.9% 12 months	99,860	944	978	947	1047

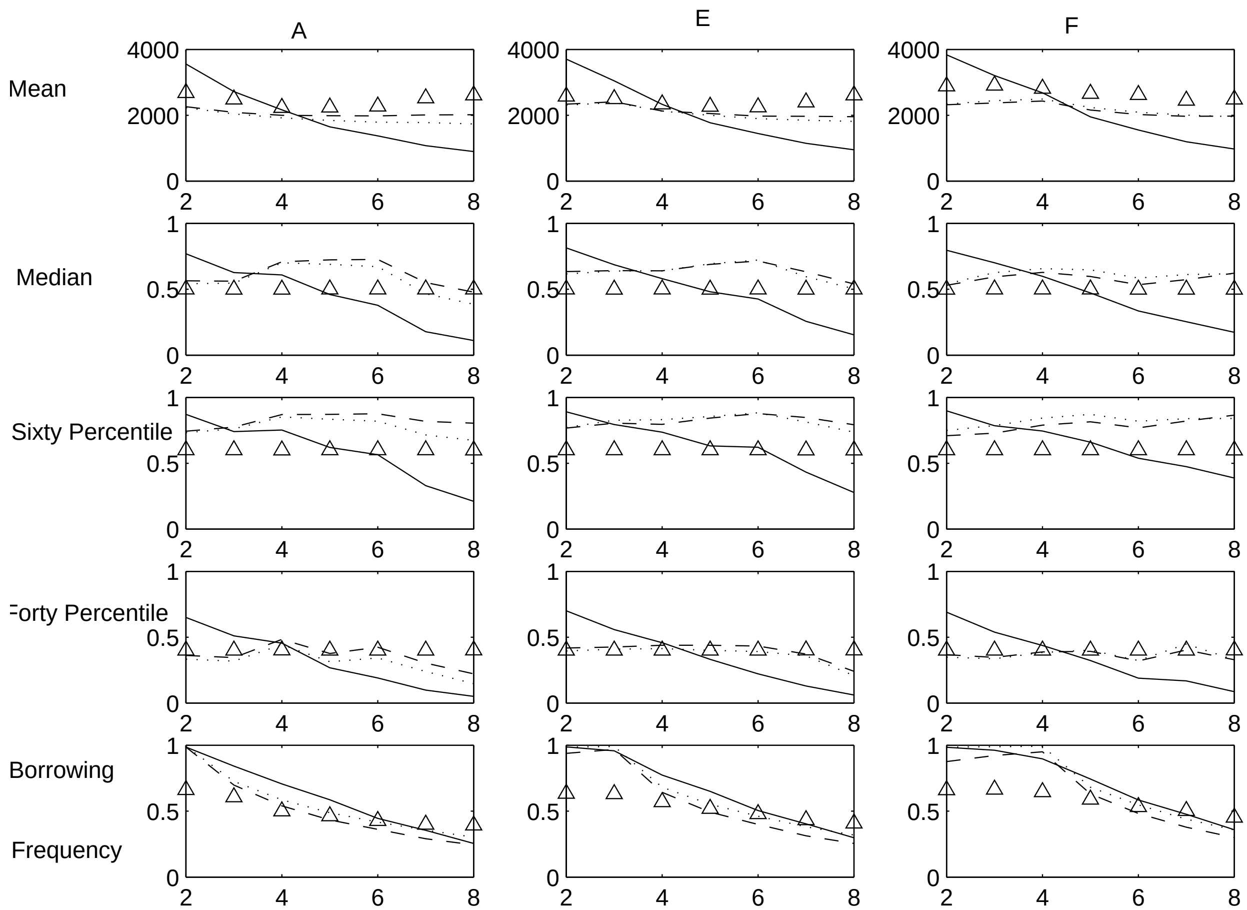

Figure 6: Simulated Debt Moments. The triangle line is the empirical data. The solid line, the dash line and the dotted line are predicted by the exponential, sophisticated and naive hyperbolic models respectively.