

Information Transmission in Coalitional Voting Games*

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Abstract. A core allocation of a complete information economy can be characterized as one that would not be unanimously rejected in favor of another feasible alternative by any coalition. We use this test of coalitional voting in an incomplete information environment to formalize a notion of resilience. Since information sharing is generally implicit in the equilibria of such voting games, this approach makes it possible to derive core concepts in which information sharing is endogenous. Our results lend support to credible core of Dutta and Vohra (2003) and the core proposed by Myerson (2003) as two that can be justified in terms of coalitional voting.

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1 Introduction

In the complete information setting, the core provides a natural way of formalizing coalitional stability. Simple as the core concept is, there is no unique, unambiguous way of extending it to an exchange economy with incomplete information; see Forges, Minelli and Vohra (2002) for a survey. At the interim stage an important issue that remains unresolved is that of making endogenous the amount of information that agents share in the process of cooperation.

At the interim stage, there are two sets of assumptions to be made (amount of information sharing, and whether or not incentive constraints are imposed) before the standard definition of the core can be extended to the incomplete information model. For example, we may assume that there is no sharing of information; agents in a coalition can only rely on common knowledge events to construct potential objections. This approach leads to the notion of the coarse core, first formalized by Wilson (1978) in a framework without incentive constraints and subsequently extended by Vohra (1999) to include them. The main idea underlying the coarse core, namely that a coalition's objection is focused on a common knowledge event, is motivated by standard issues of adverse selection; see the examples in Wilson (1978) and Vohra (1999).

Not surprisingly, the theory is quite different if one allows coalition members to share information. If information sharing is unrestricted among the members of a coalition, one arrives at the notion of the fine core, also proposed by Wilson (1978). Incentive constraints can be added to a fine objection by requiring that it be incentive compatible over the event that is relevant for the objection. While unrestricted sharing of information may seem arbitrary and, in many instances, unreasonable, there do exist cases in which some amount of information sharing seems natural.

Our aim is to make endogenous the amount of information that is shared among agents when a coalition forms in order to block a status-quo allocation.¹ While this concern is by no means new (see, for example, Dutta and Vohra (2003)), our approach here is based on viewing the core in another, more positive (perhaps more primitive) way. We argue that non-cooperative equilibrium theory is ideally suited to deal with the question of how much

¹Our approach is related to the notions of durability in Holmström and Myerson (1983) and credibility in Dutta and Vohra (2001), as we shall discuss below.

private information agents transmit to each other. This is so even though we are interested in modeling cooperative behavior.² This leads to the idea of formalizing coalitional stability by making use of non-cooperative equilibrium behavior in a voting game.

Suppose a status-quo allocation is in the core, and a coalition compares it to some other outcome that is feasible given the coalition's resources. Then it must be the case that if the two alternatives were to be voted upon by the agents in the coalition, we would not expect a unanimous acceptance of the alternative over the status-quo. Indeed, in the complete information setting this is a defining property of the core. We take this simple test of stability and apply it systematically to a model with incomplete information. The amount of information sharing as well as the incentive constraints will then emerge as equilibrium conditions of a voting game. It should be clear that matters will now depend crucially on the precise details of the voting game. Indeed, one may expect these details to lead to a plethora of different notions of core stability. We show that this is not the case; the coalitional voting approach is quite sharp in its conclusions. We show that there are only a few important aspects of the voting game in terms of core stability, leading to a few possible core concepts.

If incentive constraints are not important (because types become verifiable) we show that the coalitional voting approach yields the fine core of Wilson (1978) as the natural concept. In the general case where types are not verifiable, resilience to coalitional voting yields the credible core of Dutta and Vohra (2003). In this sense, the credible core can be seen as the appropriate generalization of the fine when incentive constraints are relevant.

An interesting modification of this simple voting game allows a mediator to construct a more sophisticated challenge to a status-quo by asking for a vote as a function of the agents' reported types. Effectively, the only difference between this game and the simpler one corresponding to the credible core is that in this case a coalition can challenge a status-quo over an informational event that is not necessarily a product event. We call the corresponding set of resilient allocations the mediated core.

Finally, we allow a mediator to randomize over the coalitions that it approaches for a vote. This yields a form of resilience that yields a the

²While the ability to cooperate makes it reasonable to allow for unfettered, frictionless communication, it does not mean that agents will necessarily share their private information with others or believe others' claims that cannot be verified.

randomized mediated core, a notion that corresponds closely with a recent core concept suggested by Myerson (2003).

2 Preliminaries

The basic model of an exchange economy with asymmetric information can be formulated as follows. Let T_i denote the (finite) set of agent i 's types. The interpretation is that $t_i \in T_i$ denotes the *private information* possessed by agent i . With $N = \{1, \dots, n\}$ as the finite set of agents, let $T = \prod_{i \in N} T_i$ denote the set of all information states. We will use the notation t_{-i} to denote $(t_j)_{j \neq i}$. Similarly $T_{-i} = \prod_{j \neq i} T_j$, $T_S = \prod_{j \in S} T_j$ and $T_{-S} = \prod_{j \notin S} T_j$. We assume that agents have a common prior probability distribution defined on T . We assume that no type is redundant, i.e., $q(t_i) > 0$ for all $t_i \in T_i$ for all i . At the interim stage, nature chooses $t \in T$, and each agent i knows her type, t_i . Hence, conditional probabilities will be important: for each $i \in N$ and $t_i \in T_i$, the conditional probability of $t_{-i} \in T_{-i}$, given t_i is denoted $q(t_{-i} | t_i)$.

We assume that there are a finite number of commodities, and that the consumption set of agent i is $X_i = R_+^L$. Agent i 's utility function in state t is denoted $u_i(\cdot, t) : X_i \times T \mapsto R$. We shall assume that $u_i(x, t) \geq u_i(0, t)$ for all i , all $t \in T$ and all $x \in R_+^L$. The endowment of agent i of type t_i is ω_i (assumed to be independent of the state – with this assumption, all private information concerns agents' preferences.)

We can now define an exchange economy as $\mathcal{E} = \langle (u_i, X_i, \omega_i, T_i)_{i \in N}, q \rangle$.

For coalition $S \subseteq N$, a feasible (state contingent) S -allocation, $x : T \mapsto R^{ls}$ (where s denotes the cardinality of S), consists of a commodity bundle for each consumer in S in each state such that $\sum_{i \in S} x_i(t) \leq \sum_{i \in S} \omega_i$ for all $t \in T$, and satisfying that $x(t_S, t'_{-S}) = x(t_S, t''_{-S})$ for all $t_S \in T_S$ and for all $t'_{-S}, t''_{-S} \in T_{-S}$. (The latter assumption is made to exclude these basic externalities across coalitions, i.e., the set of feasible allocations to a coalition is independent of the information held by the complement). We will denote by \mathcal{A}_S the set of feasible state contingent allocations of S . We shall use A_S to denote the set of feasible allocations in a given state: $A_S = \{(x_i) \in R^{ls} | \sum_i x_i \leq \sum_i \omega_i\}$. Similarly, state contingent N -allocations are simply referred to as allocations, and the set of state contingent allocations is denoted by \mathcal{A} .

Given $x \in \mathcal{A}_S$, the interim utility of agent $i \in S$ of type t_i is:

$$U_i(x|t_i) = \sum_{t_{-i} \in T_{-i}} q(t_{-i} | t_i) u_i(x_i(t_{-i}, t_i), (t_{-i}, t_i)).$$

If agent i of type t_i pretends to be of type t'_i (while all other agents are truthful), she gets interim utility:

$$U_i(x, t'_i | t_i) = \sum_{t_{-i} \in T_{-i}} q(t_{-i} | t_i) u_i(x_i(t_{-i}, t'_i), (t_{-i}, t_i)).$$

An S -allocation $x \in \mathcal{A}_S$ is *incentive compatible* if for every $i \in S$ and for every $t_i, t'_i \in T_i$:

$$U_i(x | t_i) \geq U_i(x, t'_i | t_i).$$

We shall denote the set of incentive compatible S -allocations by \mathcal{A}_S^* , and the set of incentive compatible allocations by \mathcal{A}^* .

For an event $E \subseteq T$ and $t_i \in T_i$, let

$$E_{-i}(t_i) = \{t_{-i} \in T_{-i} \mid (t_i, t_{-i}) \in E\}$$

and

$$E_i = \{t_i \in T_i \mid E_{-i}(t_i) \neq \emptyset\}.$$

Consider an allocation rule $x \in \mathcal{A}$, agent i of type t_i and an event E . Suppose $q(E_{-i}(t_i)) > 0$. Then the interim utility conditional on E can be expressed as:

$$U_i(x|t_i, E) = \sum_{t_{-i} \in E_{-i}(t_i)} q(t_{-i} | t_i) u_i(x_i(t_{-i}, t_i), (t_{-i}, t_i)).$$

(Strictly speaking, the expression on the right hand side should be divided by $q(E_{-i}(t_i))$, but we will find it convenient not to do so.)

The corresponding interim utility (conditional on E) if type t_i pretends to be of type t'_i is:

$$U_i(x, t'_i | t_i, E) = \sum_{t_{-i} \in E_{-i}(t_i)} q(t_{-i} | t_i) u_i(x_i(t_{-i}, t'_i), (t_{-i}, t_i)).$$

Given $E \subseteq T$, an S -allocation $x \in \mathcal{A}_S$ is *incentive compatible over E* if for every $i \in S$ and for every $t_i, t'_i \in E_i$:

$$U_i(x | t_i, E) \geq U_i(x, t'_i | t_i, E).$$

The set of such allocations is denoted $\mathcal{A}_S^*(E)$.

We will often consider an event that refers to independent subsets of T_i , and is therefore a product event of the form $E = \prod_i E_i$, where $E_i \subseteq T_i$ for all i .

3 Coalitional Voting

One way to assess the coalitional stability of a status-quo is to ask whether or not some coalition would vote unanimously in favor of another feasible ‘outcome’. This is a somewhat pedantic exercise in the complete information framework because there it is obvious that the core is precisely the set of allocations which no coalition would vote (unanimously) to give up in favor of some other feasible allocation. Moreover, this conclusion does not depend on precisely how the coalitional voting game is constructed. Our aim is to take this simple characterization of core stability in the complete information case and extend it to a model with incomplete information.³ Since several new issues arise in this transition, it is useful to begin by illustrating the coalitional voting approach in the simple setting where types are verifiable. (The main focus of the paper, however, is the more general case in which types are not verifiable).

3.1 Verifiable Types

We take verifiability of types to mean that eventually all private information becomes public, and contracts contingent on types can be enforced (by using prohibitive penalties if necessary). In this setting agents cannot misrepresent their types, and incentive constraints become unnecessary. Despite this simplicity, it is by no means obvious how the core should be defined in such a model. Indeed, Wilson [] suggested two distinct notions of the core: the fine core (allowing arbitrary information sharing) and the coarse core (allowing agents to coordinate only on common knowledge events); see Forges et al [] for precise definitions based on the types formulation. Wilson’s notion of the core, and others that have since been suggested, are all based on the traditional approach based on dominance and feasibility of a potential objection. The main conceptual issue concerns whether *some* of the types or *all* the types should be better-off in an ‘objection’. The coalitional voting approach is different in that it seeks to justify the notion of an objection from more primitive principles: an equilibrium in a voting game is used as a means for

³The test being performed here is weaker than the implementation exercise. In the classical framework it is possible to construct a game form that implements the core of an economy; see, for example, Perry and Reny (1994), and Serrano and Vohra (1997). The present paper may be seen as a preliminary step in developing game forms to implement core in incomplete information economies.

defining what an ‘objection’ is. As we shall see, in the verifiable types case, this approach leads to just one corresponding notion of the core - the fine core. We see this as an illustration of the power of the coalitional voting approach in selecting an appropriate notion of core stability.

Suppose $x \in \mathcal{A}$ is the status-quo and coalition S can consider making use of a voting game to discard x in favor of $y \in \mathcal{A}_S$. More precisely, members of S vote to either “accept” or “reject” the new proposal y . It is important that y is not seen as a proposal by a particular agent, in order to avoid complications of signaling that such an act would imply. The coalition is simply asked to make a choice between x and y . In case of a unanimous “accept” vote, the outcome assigned will be $y(t)$ to coalition S , where t is the profile of actual types. Note that type announcements are not necessary because we are assuming that the types eventually become publicly known.⁴ In case of a “reject” vote from any member of S the mediator assigns $x(t)$ to the grand coalition. This defines a game $\Gamma^0(x, (S, y))$, in which the only active players are all the players in S , who simultaneously must choose from $\{a, r\}$, (“accept” or “reject” the alternative y). Then, y is implemented if and only if it is unanimously accepted; otherwise, x is the outcome. A strategy for player i is a function $v_i : T_i \mapsto \{a, r\}$. Given a profile of strategies $(v_i(\cdot))$, the outcome, $\phi(v(t))$, for members of S , is defined as follows:

$$\phi_i(v(t)) = \begin{cases} y_i(t) & \text{if } v_i(t_i) = a \text{ for all } i \in S \\ x_i(t) & \text{otherwise} \end{cases}$$

A strategy profile \bar{v} of the voting game $\Gamma^0(x, (S, y))$ is a *Bayesian Nash equilibrium* if, for all $i \in S$ and $t_i \in T_i$,

$$U_i(\phi_i(\bar{v}(t))|t_i) \geq U_i(\phi_i(\bar{v}_{-i}, v_i)|t_i), \text{ for all } v_i : T_i \mapsto \{a, r\}.$$

Given a Bayesian Nash equilibrium \bar{v} , define for each i the set of types who vote to accept the alternative, i.e.,

$$E_i(\bar{v}) = \{t_i \in T_i | v_i(t_i) = a\}.$$

For $i \in S$, let $E_{-i} = \prod_{j \in S, j \neq i} E_j \times T_{-S}$.

⁴A model in which agents have to announce their types in order to implement the allocation, but types become known at a later stage (and prohibitive penalties are available), is operationally identical to this because it can be assumed that agents announce their types truthfully.

In a voting game $\Gamma^0\{x, (S, y)\}$, a Bayesian Nash equilibrium \bar{v} is said to be an *equilibrium rejection* of x if there is positive probability that in equilibrium all agents vote to accept the alternative mechanism, i.e., $q(\prod_{i \in S} E_i(\bar{v})) > 0$, and $\phi(\bar{v})$ is not interim equivalent to x for all $i \in S$ (in the sense that there exists $i \in S$ and $t_i \in T_i$ such that $U_i(\phi_i(\bar{v}) | t_i) > U_i(x | t_i)$)).

A status-quo x is *resilient to coalitional voting with verifiable types* if there does not exist a voting game $\Gamma^0(x, (S, y))$ with an equilibrium rejection of x .

Proposition 1 *Given $x \in \mathcal{A}$, a coalition S and $y \in \mathcal{A}_S$, the following statements are equivalent:*

- (a) *There is an equilibrium rejection of x in the voting game $\Gamma^0(x, (S, y))$.*
- (b) *There exists $y' \in \mathcal{A}_S$ and $E_i \subseteq T_i$ for all $i \in S$, where $q(E) > 0$ for the product event $E = \prod_{i \in S} E_i \times T_{-S}$, such that:*

$$U_i(y' | t_i, E) \geq U_i(x | t_i, E) \text{ for all } i \in S \text{ and all } t_i \in E_i,$$

with strict inequality for some i and t_i .

Corollary 1 *x is resilient to coalitional voting with verifiable types if and only if it belongs to the fine core.*

The corollary follows from the fact that condition (b) is essentially the same as the definition of a fine objection; see, for example, Forges et al (2001). The only difference is that an ‘objection’ is usually defined with a strict inequality for all i . It should be clear that the characterization of equilibria of voting games will be in the form of weak inequalities. For this reason, throughout this paper, when we refer to a certain core concept it should be understood to be the ‘strong’ version of that core concept.⁵

Proof: An equilibrium $\bar{\sigma}$ of the voting game $\Gamma^0(x, (S, y))$ can be characterized equivalently in terms of y and x as follows. For all $i \in S$ and $t_i \in E_i$:

$$\sum_{t_{-i} \in E_{-i}} q(t_{-i}|t_i)u_i(y|t_i) + \sum_{t_{-i} \notin E_{-i}} q(t_{-i}|t_i)u_i(x|t_i) \geq \sum_{t_{-i} \in T_{-i}} q(t_{-i}|t_i)u_i(x|t_i),$$

and for all $i \in S$, $t_i \notin E_i$:

$$\sum_{t_{-i} \in E_{-i}} q(t_{-i}|t_i)u_i(y|t_i) + \sum_{t_{-i} \notin E_{-i}} q(t_{-i}|t_i)u_i(x|t_i) \leq \sum_{t_{-i} \in T_{-i}} q(t_{-i}|t_i)u_i(x|t_i).$$

⁵Monotonicity of preferences will suffice to eliminate this difference as well, in the present result. However, when we turn to the case of non-verifiable types, this difference may be important.

These two conditions are equivalent to the following two conditions:

$$U_i(y \mid t_i, E) \geq U_i(x \mid t_i, E) \text{ for all } i \in S \text{ and } t_i \in E_i \quad (1)$$

and

$$U_i(y \mid t_i, E) \leq U_i(x \mid t_i, E) \text{ for all } i \in S \text{ and } t_i \notin E_i. \quad (2)$$

Clearly, this means that (a) implies (b).

To see that (b) implies (a) consider y' and E satisfying condition (b), and define:

$$y(t) = \begin{cases} y'(t) & \text{if } t \in E \\ 0 & \text{otherwise.} \end{cases}$$

Since $U_i(y' \mid t_i, E) = U_i(y \mid t_i, E)$ for all $i \in S$ and $t_i \in E_i$, it follows that y satisfies both (1) and (2), and this completes the proof. ■

3.2 Non-Verifiable Types

When types are not verifiable, we need to be more careful about precisely a status-quo means. Henceforth, we shall take a *status-quo*, x , to refer to an incentive compatible, state contingent allocation (thus $x \in \mathcal{A}^*$) with the interpretation that in every state t , the outcome is $x(t)$, unless there is an agreement to change it. This means, in particular, that even if there is an attempt to change the status-quo but the attempt fails, the outcome in state t is $x(t)$, i.e., any *discussion* about a possible change does not by itself allow any agent to strategically manipulate the status-quo x . Since x was assumed to be incentive compatible, this can be justified by a scenario in which agents have already sent their truthful reports to implement the status-quo via a direct mechanism (Maskin-Tirole, Myerson??). And this cannot be changed if the status-quo prevails. This assumption greatly simplifies the analysis, and will be used throughout the rest of this paper.

As before, coalitional voting may involve a comparison between a status-quo (a feasible allocation rule for the grand coalition) and an allocation rule that is feasible (both on informationally and physically) for the given coalition. However, when types are not verifiable, an allocation rule is only the simplest example of what a coalition may consider as an alternative. More generally, we may think of a coalition constructing a communication mechanism of its own to map out its feasible set of alternatives. Indeed, this will capture the idea that a coalition is permitted as much latitude in constructing

a competing proposal as is within the bounds of feasibility. A voting game can now be used to describe the situation in which members of a coalition vote to reject a status-quo in favor of a competing mechanism. The new mechanism is applied if all agents vote for it; otherwise the status-quo remains as the outcome. We will say that a status-quo is resilient to coalitional voting if there does not exist a coalition and a Bayesian Nash equilibrium of a voting game in which an alternative mechanism is unanimously accepted. Note that our approach yields a very strong notion of stability; a resilient allocation rule cannot be rejected in *any* Bayesian Nash equilibrium of *any* competing mechanism.

We now turn to formal definitions.

A mechanism for coalition S consists of message sets M_i for each $i \in S$ and an outcome function $g : M \mapsto A_S$.

A status-quo, $x \in \mathcal{A}^*$, and a mechanism for S , $((M_i)_{i \in S}, g)$, defines a voting game in which each player i chooses an action in $\{a, r\} \times M_i$. The interpretation being that a refers to an ‘accept’ vote in favor of the new mechanism while r refers to a rejection of the new proposal. A strategy for player i is therefore $\sigma_i : T_i \mapsto \{a, r\} \times M_i$. We will find it convenient to denote $\sigma_i(t_i) = (v_i(t_i), m_i(t_i))$, where the first element denotes the vote of agent i of type t_i . Given a strategy profile σ , let $\phi(\sigma)$ denote the corresponding outcome for coalition S . In state t , the commodity bundle assigned to each $i \in S$ is denoted $\phi_i(\sigma(t))$, and defined as follows:⁶

$$\phi_i(\sigma(t)) = \begin{cases} g_i(m(t)) & \text{for all } t \text{ such that } v_i(t_i) = a \text{ for all } i \in S \\ x_i(t) & \text{otherwise} \end{cases}$$

A coalitional voting game for coalition S corresponding to status-quo, x , and a mechanism $((M_i), g)$ can now be denoted $\Gamma(x, S, M, g)$. This is the formulation first used by Holmstrom and Myerson [], for the grand coalition, in defining their notion of durability. And as they pointed out, this way of formulating a voting game is more general than it may first seem. It includes any complex voting procedure for various alternatives, as long as each agent has the option, at the outset, to anonymously veto the new proposal and revert to the status-quo. The mechanism can then be interpreted as the normal form of the game following the accept/reject vote.

⁶Note that $\phi(\sigma(t))$ does not necessarily belong to A_S since $(x_i(t))_{i \in S}$ need not belong to A_S .

A *Bayesian Nash equilibrium* of the voting game Γ is a strategy profile, σ , for members of S such that, for all $i \in S$ and $t_i \in T_i$,

$$U_i(\phi_i(\sigma)|t_i) \geq U_i(\phi_i(\sigma_{-i}, \sigma'_i)|t_i), \text{ for all } \sigma'_i : T_i \mapsto M_i.$$

In a direct mechanism $((T_i), y)$, a *truthful equilibrium* is an equilibrium where $m_i(t_i) = t_i$ for all $i \in S$ and all $t_i \in T_i$.

Given a Bayesian Nash equilibrium $\sigma = ((v_i(t_i), m_i(t_i)))$, define for each i the set of types who vote a , i.e.,

$$E_i(\sigma) = \{t_i \in T_i | v_i(t_i) = a\}.$$

Let $E(\sigma) = \prod_{i \in S} E_i(\sigma) \times T_{-S}$ denote the states in which the alternative is adopted.

In a voting game $\Gamma(x, S, M, g)$, a Bayesian Nash equilibrium σ is said to be an *equilibrium rejection* of x if there is positive probability that in equilibrium all agents vote to accept the alternative mechanism, i.e., $q(\prod_i E_i(\sigma)) > 0$, and $\phi(\sigma)$ is not interim equivalent to x for all $i \in S$.

A status-quo x is *resilient to coalitional voting* if there does not exist a voting game, $\Gamma(x, S, (M_i), g)$, with an equilibrium rejection of x .

Proposition 2 *Given $x \in \mathcal{A}^*$, and a coalition, S , the following statements are equivalent:*

- (a) *There is an equilibrium rejection of x in voting game $\Gamma(x, S, (M_i), g)$.*
- (b) *There is a truthful equilibrium rejection of x in a (direct) voting game $\Gamma(x, S, (T_i), y)$.*
- (c) *There exists $y' \in \mathcal{A}_S$ and $E_i \subseteq T_i$ for all $i \in S$, where $q(E) > 0$ for the product event $E = \prod_{i \in S} E_i \times T_{-S}$, such that:*
 - (i) $U_i(y' | t_i, E) \geq U_i(x | t_i, E)$ for all $i \in S$ and all $t_i \in E_i$, with strict inequality for some i and t_i .
 - (ii) $U_i(y' | t_i, E) \leq U_i(x | t_i, E)$ for all $i \in S$ and all $t_i \notin E_i$.
 - (iii) $y' \in \mathcal{A}_S^*(E)$.

It is instructive to compare this proposition to the revelation principle. The equivalence between (a) and (b) follows from the usual argument, and allows us to restrict attention to direct mechanisms, without loss of generality.

It shows that a wide variety of coalitional voting games are strategically equivalent to one in which agents vote and report their types. In terms of Bayesian Nash equilibria, it is unimportant whether the vote and type reports are simultaneous or sequential (votes followed by type reports or vice versa). In equilibrium, rejection of a status-quo is equivalent to truthful rejection in a direct voting game.

The equivalence of these with (c) has a more novel interpretation: it can be viewed as a form of a revelation principle concerning mechanisms in the context of a status-quo. The existence of an equilibrium rejection (through *some* mechanism) of a status-quo is equivalent to the existence of a feasible allocation rule satisfying the three inequalities in condition (c): (i) there is an event over which all members of the coalition gain, (ii) it is reasonable for them to believe this event since those types who do not belong to this event would not vote to accept the alternative mechanism, and (iii) the new proposed mechanism is incentive compatible over the event E . Interestingly, these inequalities are precisely the ones used in Dutta and Vohra (2003) to define a *credible objection*. This observation yields the following corollary.

Corollary 2 x is resilient to coalitional voting if and only if it belongs to the credible core.

Proof of Proposition 2. Suppose $\sigma = (v_i(t_i), m_i(t_i))$ is an equilibrium of $\Gamma(x, S, M, g)$. Let $E_i = \{t_i \in T_i \mid v_i(t_i) = a\}$, and let $y(t) = g(m(t))$ for all t . The equilibrium interim utility of agent i of type t_i is:

$$U_i(\sigma \mid t_i) = \begin{cases} U_i(y \mid t_i, E) + U_i(x \mid t_i, T \setminus E) & \text{if } t_i \in E_i \\ U_i(x \mid t_i) & \text{otherwise.} \end{cases}$$

The fact that σ is an equilibrium means:

(1) An agent $i \in S$ of type $t_i \in E_i$ cannot gain by rejecting the alternative,

$$U_i(y \mid t_i, E) \geq U_i(x \mid t_i, E) \text{ for all } i \in S \text{ and all } t_i \in E_i, \quad (1)$$

or by continuing to accept but changing the choice of $m_i(t_i)$ to $m_i(t'_i)$,

$$U_i(y, t_i \mid t_i, E) \geq U_i(y \mid t'_i, E) \text{ for all } i \in S, t_i \in E_i, t'_i \in T_i. \quad (2)$$

(2) An agent i of type $t_i \notin E_i$ cannot gain by accepting the alternative, and choosing $m_i(t'_i)$,

$$U_i(x \mid t_i, E) \geq U_i(y, t'_i \mid t_i, E) \text{ for all } i \in S, t_i \notin E_i, t'_i \in T_i. \quad (3)$$

Since i of type $t_i \notin E_i$ cannot change the outcome by continuing to reject the alternative, this exhausts all possible unilateral deviations. Thus conditions (1), (2) and (3) characterize an equilibrium of the voting mechanism. If the equilibrium involves a rejection of the status-quo, then in addition to these three conditions we must have $q(E) > 0$ and at least one inequality in (1) being strict. Clearly, then (a) implies (b) as well as (c). Of course, (b) implies (a), and so it remains only to be shown that (c) implies (b).

To see that (c) implies (b), consider y' and E satisfying condition (c). Let

$$y(t) = \begin{cases} y'(t) & \text{if } t \in E \\ 0 & \text{otherwise.} \end{cases}$$

Since $U_i(y' | t_i, E) = U_i(y | t_i, E)$ for all $i \in S$ and $t_i \in E_i$, it follows from (i) of condition (c) that y satisfies (1), with at least one strict inequality. From (ii) of condition (c) we know that y' (or y) satisfies (3) for all $t'_i \in E_i$. The fact that y also satisfies it for all $t'_i \in E_i$ now follows from construction. Similarly, (2) follows from (iii) of condition (c). ■

A final observation is in order. With a very similar proof, one can show the following: A strictly incentive compatible allocation x in the relative interior of the credible core is resilient to coalitional voting in Γ^* (or Γ^{*1} or Γ^{*2}), and every allocation x resilient to coalitional voting in Γ^* (or Γ^{*1} or Γ^{*2}) belongs to the credible core. Thus, we learn that the strong credible core is “sandwiched” between the credible core and the strictly incentive compatible allocations of its relative interior. This is meaningful when incentive constraints do not restrict first-best efficiency, because then no type will have a binding incentive constraint at an interim efficient allocation. In such circumstances, the characterizations in terms of resilience in these classes of voting games are very close to yield the credible core.

We proceed to give an example that illustrates the difference between the credible core and the incentive compatible fine core. It is a simple version of a “lemon” asymmetric information economy, and it is taken from Vohra (1999).

Example 1: Let $N = \{1, 2\}$. Agent 1 is fully informed and has two possible types: $T_1 = \{t_H, t_L\}$. Agent 2’s set of types is $T_2 = \{t_2\}$, i.e., he is uninformed about the true type of agent 1 and assigns equal probability to both. Let $H = (t_H, t_2)$ denote the high state and $L = (t_L, t_2)$ the low state. Agent 1 is the seller of an indivisible good to be traded for money, and agent 2 is the buyer. In state H , agents’ valuations for the indivisible good

are v_1 and v_2 , respectively, while they are 0 in state L . We shall assume that $v_2/2 < v_1 < v_2$. Under these assumptions, the credible core consists exclusively of no-trade contracts in state H and no transfers of money in state L . To see this, suppose there were trade in state H . Individual rationality for the seller implies that the money transfer should be at least v_1 . Using incentive compatibility, one can establish that the same transfer must happen in state L , which makes it impossible for the buyer to meet his individual rationality constraint. Note how any fine objection (constructed in state H) would not be credible because type t_L would like to join. The strong credible core continues to specify these same contracts. However, as just observed, any such contract would not be in the incentive compatible fine core.

4 Mediated Voting/Blocking

There is one respect in which the voting mechanism of the previous section is *not* general enough. While agents are allowed a lot of flexibility in choosing a communication mechanism, the fact that the application of the alternative mechanism is based on independent accept/reject decisions may turn out to be an important restriction. One can argue that if a mediator can be used to translate messages into actions, presumably the mediator could also be delegated the accept/reject decision (perhaps as a function of additional inputs into the mechanism).

Recall that in the voting game $\Gamma(x, S, M, g)$ of the previous section, the outcome function of the mechanism (M, g) maps from M to A_S . However, an outcome of the game includes that the possibility that the alternative is rejected, and the status-quo survives. A more general communication mechanism would allow the outcome function to map from M to $A_S \cup \{x\}$.⁷ This is the interpretation of a mechanism we shall adopt in this section. As before, an agent votes for or against a new mechanism and also chooses a message to communicate in the mechanism. Given a status-quo x , coalition S and a mechanism (M, g) , where $g : \mapsto A_S \cup \{x\}$, we denote the coalitional voting game $\Gamma'(x, S, M, g)$.

Given a strategy profile σ , let $\phi(\sigma)$ denote the corresponding outcome for coalition S . Let $E(\sigma)$ denote the states in which the mechanism effectively

⁷There is some abuse of notation here because rejecting the new mechanism leads to $x(t)$ in state t .

adopts a new alternative, i.e.,

$$E(\sigma) = \{t \in T \mid \phi(\sigma(t)) \neq x(t)\}.$$

Note that $t \in E(\sigma)$ implies that $v_i(t_i) = \{a\}$ for all $i \in S$, but the converse may not hold, i.e., acceptance of an ‘alternative’ by all agents in S does not necessarily mean that the *mechanism* accepts an alternative. Let

$$E_i(\sigma) = \{t_i \in T_i \mid (t_i, t_{-i}) \in E \text{ for some } t_{-i} \in T_{-i}\}.$$

In a voting game $\Gamma'(x, S, M, g)$, a Bayesian Nash equilibrium σ is said to be an *equilibrium rejection* of x if $q(E(\sigma)) > 0$ and $\phi(\sigma)$ is not interim equivalent to x for all $i \in S$.

A status-quo x is *resilient to mediated coalitional voting* if there does not exist a voting game, $\Gamma'(x, S, M_i, g)$, with an equilibrium rejection of x .

Proposition 3 *Given $x \in \mathcal{A}^*$, and a coalition, S , the following statements are equivalent:*

- (a) *There is an equilibrium rejection of x in a mediated voting game $\Gamma'(x, S, M, g)$.*
- (b) *There is a truthful equilibrium rejection of x in a (direct) mediated voting game $\Gamma'(x, S, T, y)$.*
- (c) *There exists $y' \in \mathcal{A}_S$ and $E \subseteq T$ (with $q(E) > 0$) such that:*
 - (i) $U_i(y' \mid t_i, E) \geq U_i(x \mid t_i, E)$ for all $i \in S$ and all $t_i \in E_i$, with strict inequality for some i and t_i .
 - (ii) $\sum_{t_{-i} \in E_{-i}(t_i)} q_i(t) [u_i(y_i(t), t) - u_i(x_i(t), t)] \geq \sum_{t_{-i} \in E_{-i}(t'_i)} q_i(t) [u_i(y_i(t_{-i}, t'_i), t) - u_i(x_i(t), t)]$ for all $i \in S$, $t_i, t'_i \in T_i$.

Again, we can use the equivalence between (a) and (b) to claim that several versions of the coalitional voting game are strategically equivalent. One version, that will we will find convenient in terms of fixing ideas is the following. All agents in S report their types, and then the mediator asks for a vote if and only if the reported types belong to $E_S \subseteq T_S$.

Proof of Proposition 3. Suppose $\sigma = (v_i(t_i), m_i(t_i))$ is an equilibrium of $\Gamma'(x, S, M, g)$. Let $y(t) = \phi(m(t))$ for all t . Of course, $v_i(t_i) = \{a\}$ for all $t_i \in E_i(\sigma)$. Moreover, if we modify σ to σ' only by having all types vote

a , σ' remains an equilibrium. It now follows that in the direct mediated mechanism $\Gamma'(x, S, (T_i), y)$, it is an equilibrium for all types of agents in S to accept, and report truthfully. To see this, note that voting to reject the alternative mechanism yields the status-quo - an option that was feasible in the original mechanism. A unilateral deception also yields an outcome that is feasible in the original mechanism. Clearly then, (a) and (b) are equivalent.

Suppose σ is a truthful equilibrium rejection of x in the game $\Gamma'(x, S, (T_i), y)$. This means that:

- No $i \in S$ of type t_i can gain by rejecting the alternative mechanism. For any $t_i \notin E_i(\sigma)$ the accept/reject decision is outcome equivalent. For $t_i \in E_i(\sigma)$, we must have $U_i(y | t_i, E) \geq U_i(x | t_i, E)$. Thus,

$$U_i(y | t_i, E) \geq U_i(x | t_i, E) \text{ for all } i \in S \text{ and all } t_i \in E_i,$$

with strict inequality for some i of some type.

- No $i \in S$ of type t_i can gain by pretending to be of type t'_i (incentive compatibility). The interim utility from truthful reporting is:

$$U_i(\sigma | t_i) = \sum_{t_{-i} \in E_{-i}(t_i)} q(t) u_i(y_i(t), t) + \sum_{t_{-i} \notin E_{-i}(t_i)} q(t) u_i(x_i(t), t).$$

If i of type t_i reports t'_i , the resulting interim utility is:⁸

$$U_i(\sigma, t'_i | t_i) = \sum_{t_{-i} \in E_{-i}(t'_i)} q(t) u_i(y_i(t_{-i}, t'_i), t) + \sum_{t_{-i} \notin E_{-i}(t'_i)} q(t) u_i(x_i(t), t).$$

And we must have,

$$U_i(\sigma | t_i) \geq U_i(\sigma, t'_i | t_i) \text{ for all } i \in S, \text{ for all } t_i, t'_i \in T_i.$$

This general incentive compatibility constraint can also be expressed in a form that compares utilities only over the relevant subset of $E(\sigma)$. To do this, subtract $U_i(x | t_i)$ from the right hand side of both $U_i(\sigma | t_i)$ and $U_i(\sigma, t'_i | t_i)$ to re-state the above inequality as:

$$\begin{aligned} \sum_{t_{-i} \in E_{-i}(t_i)} q_i(t) [u_i(y_i(t), t) - u_i(x_i(t), t)] \geq \\ \sum_{t_{-i} \in E_{-i}(t'_i)} q_i(t) [u_i(y_i(t_{-i}, t'_i), t) - u_i(x_i(t), t)] \text{ for all } i \in S, t_i, t'_i \in T_i. \end{aligned} \tag{IC}$$

⁸Note how the different type report affects the implementation of the blocking plan, but not that of the status-quo.

This proves that (b) implies (c). The converse follows easily from the same arguments as above by considering the direct mediated mechanism y' where:

$$y'(t) = \begin{cases} y(t) & \text{if } t \in E \\ x(t) & \text{otherwise} \end{cases}$$

■

To compare Proposition 3 to Proposition 2, consider $x \in \mathcal{A}^*$ and y, E satisfying condition (c) of Proposition 3. For $t'_i \notin E_i$, $E_{-i}(t'_i) = \emptyset$ (or more precisely, consists of types that have 0 probability given t_i). The RHS of (c.ii) is then 0, and the condition becomes the same as (c.i). Thus, we need only consider (c.ii) for those cases in which $t'_i \in E_i$.

It may now be useful to write (c.ii) explicitly for the remaining two cases:

(i) $t_i \notin E_i$ and $t'_i \in E_i$: in this case, the LHS is 0, and (c.ii) becomes the same as (c.iii) of Proposition 2; the self selection condition used in defining a credible objection.

(ii) $t_i \in E_i$ and $t'_i \in E_i$: in this case, if E has a product structure, $E_{-i}(t_i) = E_{-i}(t'_i)$ it is easy to see that (c.ii) of Proposition 3 is the same as (c.ii) of Proposition 2.

Thus, we see that the (c) of Proposition 2 implies (c) of Proposition 3, and the converse holds if E is a product event. This leads us to define the *mediated core* as the set of all allocations in \mathcal{A}^* for which there does not exist a coalition S , and event E and an allocation $y \in \mathcal{A}_S$ satisfying condition (c) of Proposition 3. Clearly then, the mediated core is a subset of the credible core.

The difference between these two cores is analogous to that between mixed strategy Nash equilibria and correlated equilibria. The next example illustrates this difference and shows that the mediated core can be strict subset of the credible core.

Example 2: Consider an exchange economy with three agents and three commodities. Agent 3 is uninformed and agents 1 and 2 have two types each: $T_1 = \{t_1, t'_1\}$, $T_2 = \{t_2, t'_2\}$ and each state is equally likely. Let $t = (t_1, t_2)$ and $t' = (t'_1, t'_2)$. The state independent endowments are:

$$\omega_1 = (1, 0, 0), \quad \omega_2 = (0, 1, 0), \quad \omega_3 = (0, 0, 2).$$

The utility functions are:

$$\begin{aligned} u_1((x, y, z), t) &= 0.9x + y + 1.05z \\ u_1((x, y, z), t') &= 5z \\ u_1((x, y, z), (t'_1, t_2)) &= u_1((x, y, z), (t_1, t'_2)) = 0.9x + y \end{aligned}$$

$$\begin{aligned}
u_2((x, y, z), t) &= x + 0.9y + 1.05z \\
u_2((x, y, z), t') &= 5z \\
u_2((x, y, z), (t'_1, t_2)) &= u_2((x, y, z), (t_1, t'_2)) = x + 0.9y \\
u_3((x, y, z), t) &= x + y \text{ for all } t.
\end{aligned}$$

The status-quo allocation x , along with ex-post utilities, is the following:

	t_2	t'_2
t_1	$x_1 = (0, 0, 1); u_1 = 1.05$ $x_2 = (0, 0, 1); u_2 = 1.05$ $x_3 = (1, 1, 0); u_3 = 2$	$x_1 = (0, 0.8, 0); u_1 = 0.8$ $x_2 = (1, 0.1, 0); u_2 = 1.09$ $x_3 = (0, 0.1, 2); u_3 = 0.1$
t'_1	$x_1 = (0.1, 1, 0); u_1 = 1.09$ $x_2 = (0.8, 0, 0); u_2 = 0.8$ $x_3 = (0.1, 0, 2); u_3 = 0.1$	$x_1 = (0, 0, 1); u_1 = 5$ $x_2 = (0, 0, 1); u_2 = 5$ $x_3 = (1, 1, 0); u_3 = 2$

This allocation rule is incentive compatible, individually rational and interim efficient. Furthermore, coalition $\{1, 2\}$ does not have a coarse objection because type t'_1 and t'_2 cannot be better-off without commodity z . Also, it is easy to see that coalitions $\{1, 3\}$ or $\{2, 3\}$ do not have a coarse objection: there are not enough units of goods 1 or 2 to improve upon player 3's expected utility. This shows that x is in the incentive compatible coarse core.

Moreover, there is no credible objection to x either. To see this suppose there is a credible objection by agents 1 and 2 over some event E . If the event is t or t' they cannot do better. If the event is (t'_1, t_2) they can do better, but then type t_1 will want to lie; the preferences of agent 1 are the same in (t_1, t_2) and (t'_1, t_2) . The event E cannot be the first column either, because agent 1 will have to be given utility at least 1.09 in each state (to make sure that she is better-off in state (t'_1, t_2) , and to maintain incentive compatibility). But then t_2 would be worse off, getting no more than a utility of 0.9 in each state. For similar reasons, the argument holds for state (t_1, t'_2) or the first row. This exhausts all possibilities for a credible objection. Thus x belongs to the credible core.

Now consider a mediated objection in which the mediator will use the mechanism δ for coalition $\{1, 2\}$ in states other than t' :

	t_2	t'_2
t_1	$y_1 = (0, 1, 0); u_1 = 1$ $y_2 = (1, 0, 0); u_2 = 1$	$y_1 = (0, 0.895, 0); u_1 = 0.895$ $y_2 = (1, 0.105, 0); u_2 = 1.0945$
t'_1	$y_1 = (0.105, 1, 0); u_1 = 1.0945$ $y_2 = (0.895, 0, 0); u_2 = 0.895$	

Type t_1 's expected utility from the blocking plan is higher than at the status-quo: he gets a utility equal to 1 in state (t_1, t_2) and equal to 0.895 in state (t_1, t'_2) (instead of status-quo utilities 1.05 and 0.8, respectively): since he does not learn anything from the call of the mediator, he maintains the same beliefs to sustain a higher expected utility in the blocking plan. On the other hand, type t'_1 learns from the call of the mediator that the state is (t'_1, t_2) , and in that state the blocking plan utility is 1.0945, higher than the status-quo utility of 1.09. The arguments for types t_2 and t'_2 are identical to those for t_1 and t'_1 , respectively.

It only remains to check for incentive compatibility of the blocking plan. Let us begin with type t_1 . Again, by being truthful, his expected utility will be $(1/2)(1 + 0.895)$. If he misreports his type by announcing t'_1 to the mediator, he will be invited to the blocking plan only with probability $1/2$ when the type of player 2 is t_2 and he will then be offered to consume $(0.105, 1, 0)$, whereas with the rest of probability he will not get the phone call and the status-quo will result. However, in this case, his false report will not interfere with the status-quo since, to implement it, the very first (truthful) reports are used. In conclusion, in this case he is allocated $(0, 0.8, 0)$. The corresponding expected utility is $(1/2)(1.0945 + 0.8)$, smaller than what he gets by being truthful. In other words, type t_1 does not gain by lying, because the gain in the first column is 0.0945, but the loss in the second column is 0.095. Finally, let us check the incentives of type t'_1 . If he reports truthfully, with probability $1/2$ he will be invited and be allocated $(0.105, 1, 0)$ [in state (t'_1, t_2)], while with probability $1/2$ he will be allocated $(0, 0, 1)$ [in state (t'_1, t'_2) from the status-quo, where he really values good z]. This is better than being allocated $(0, 1, 0)$ and $(0, 0.895, 0)$, respectively. Of course, the incentive compatibility arguments for types t_2 and t'_2 are identical to the corresponding types of player 1, and we omit them.

Since the blocking inequalities we have written are all strict, we therefore conclude that x is not in the mediated core.

5 Randomized Mediation

In this section we consider coalitional voting in which the voting mechanism may involve an even more sophisticated role for the mediator. A mediator may construct a blocking mechanism in which the coalition that is asked to vote on a new alternative mechanism is chosen at random. This corresponds

closely to a core concept recently suggested by Myerson (2003).⁹

Given Propositions 2 and 3, it should be clear that without loss of generality we may concentrate on direct voting games in which agents vote and report their types. A particular version of such a voting that we will find convenient to refer to is the one in which agents first report their types, and are then asked for a vote (as a function of the type reports).

Now, proposal by a mediator, μ , consists of a probability distribution of feasible allocation mechanisms for various coalitions. In particular, $\mu(S, y, t)$, where $y \in A_S$, denotes the probability with which coalition S is invited by the mediator to vote for y against the status-quo, when the (reported) state is t . For ease of comparisons with the previous sections we will consider randomization only over coalitions, i.e., $\mu(S, y, t) > 0$ implies $\mu(S, y', t) = 0$ for all $y' \neq y$. Thus, we associate with each coalition S one proposed allocation y^S . The mediator may choose with positive probability not to invite any coalition, thereby imposing the status-quo. Thus, for each $t \in T$, $0 \leq \mu(S, y^S, t) \leq 1$ for all coalitions S , and $\sum_S \mu(S, y^S, t) \leq 1$. This describes a ‘blocking plan’ used in the definition (and characterization) of the *inner core*; see Myerson (1991), Qin (1989) and de Clippel and Minelli (2002).

To apply this idea to coalitional voting with incomplete information, we consider the following extension of the game Γ' of the previous section. Given a status-quo x and a blocking plan μ , all agents report their types to the mediator. Given these reports, t , the mediator chooses coalition S with probability $\mu(S, y^S, t)$ and asks all agents in S to vote for the proposed y^S . If all voters accept, the new proposal is implemented, otherwise the status-quo survives.

Based on this interpretation of a mediator, we are led to modifying the notion of mediated core of the previous section as follows.

An allocation $x \in \mathcal{A}_S$ is said to belong to the *randomized mediated core* if there does not exist a blocking plan μ such that

$$\sum_{t_{-i}} q_i(t) \sum_{S \supseteq \{i\}} \mu(S, y^S, t) [u_i(y_i^S(t), t) - u_i(x_i(t), t)] \geq 0 \text{ for all } i \in N \text{ and } t_i \in T_i$$

and

$$\begin{aligned} & \sum_{t_{-i}} q_i(t) \sum_{S \supseteq \{i\}} \mu(S, y^S, t) [u_i(y_i^S(t), t) - u_i(x_i(t), t)] \geq \\ & \sum_{t_{-i}} q_i(t) \sum_{S \supseteq \{i\}} \mu(S, y^S, t_{-i}, t'_i) [u_i(y_i^S(t_{-i}, t'_i), t) - u_i(x_i(t), t)] \text{ for all } i \in N \text{ and } t_i, t'_i \in T_i. \end{aligned}$$

⁹To maintain comparability with the previous sections we neglect the possibility that alternative mechanism may be a random state contingent allocation. Myerson (2003) allows for this form of randomization as well.

Suppose μ is a blocking plan that involves one coalition S and one alternative, y , i.e., $\mu(S', y') = 0$ if $S' \neq S$ or $y' \neq y$. Let $E(\mu) = \{t \in T \mid y(t) \neq x(t)\}$. Then it follows that μ satisfies the above inequalities if and only if (S, y, E) satisfy condition (c) of Proposition 3. Thus equilibria of deterministic blocking plans are precisely the equilibria of voting games Γ' defined in the previous section. Thus, we have:

Observation: The randomized mediated core is a subset of the mediated core, itself a subset of the credible core.

As in the previous sections, we can consider resilience to randomized mediated voting to get an alternative characterization of the randomized mediated core.

The concept is very similar to the core concept defined by Myerson (2003). The only differences are: (1) he assumes that transfers are possible and (2) feasibility is weakened to require expected feasibility of the transferable commodity.

We close the section by going over another example, to illustrate the difference between the mediated core and other cores previously mentioned.

Example 3: This example is taken from Myerson (2003, Example 1). Let $N = \{1, 2\}$, $T_1 = \{t_H, t_L\}$, $T_2 = \{t_2\}$, the common prior is $q(t_H) = q(t_L) = 1/2$. Agent 1 is a potential seller and 2 is a potential buyer of a commodity. There is a second commodity (money) and the utility functions are quasi linear. Ex post utilities are 5 and 6 in state t_H , and 1 and 2 in state t_L , respectively. Agent 1 has an endowment 1 unit of the commodity.

Let $\theta = x_1(t_H)$ be the dollars paid to agent 1 in state t_H , and π be the amount of commodity 1 sold in state t_H .

The following allocation belongs to the credible core. Sell 1 unit at price = 1.5 in state t_L and sell 1/9 at price 5.5 in state t_H . Also belongs to the coarse core and the mediated core. But it doesn't belong to the randomized mediated core of Myerson. The blocking plan

Let us begin by seeing that interim incentive efficiency implies that the probability of selling in state t_L is $\pi_L = 1$ and the expected money transfer to the seller in state t_L is $\theta_L = \theta + 1 - \pi$. By incentive compatibility applied to type t_L of the seller:

$$\theta_L + (1 - \pi_L) \geq \theta + (1 - \pi).$$

First, if $\pi_L < 1$, one can raise it by ϵ , accompany it by an increase in θ_L also of ϵ . This leaves this incentive compatibility constraint unaffected. Furthermore, after the ϵ change, the incentive constraint for type t_H is satisfied easier. And the buyer's expected utility increases as a result by $\epsilon = 2\epsilon - \epsilon$ in state t_L . This shows that $\pi_L = 1$.

Now suppose that the incentive constraint written above does not bind:

$$\theta_L > \theta + (1 - \pi).$$

If this were true, one could not have $\pi = 1$ because of incentive compatibility for type t_H . Then, we can improve all types as follows: for $\epsilon > 0$ small enough, raise θ by ϵ and raise π by $\epsilon' = \epsilon/5$. With this, the incentive constraints are met, and both types of the seller are indifferent. The buyer, though, is increasing his payoff in state t_H : he pays ϵ more, but the increase of trade probability raises his payoff by $(6/5)\epsilon$, so there is room for interim improvement. Therefore, the incentive constraint for type t_L binds.

We can therefore write the following payoffs in each state: In state t_L : the seller gets

$$\theta + (1 - \pi);$$

and the buyer

$$2 - [\theta + (1 - \pi)].$$

In state t_H , the seller gets

$$\theta + 5(1 - \pi),$$

and the buyer

$$6\pi - \theta.$$

The interim individual rationality constraints are as follows: for type t_L seller: $\theta + 1 - \pi \geq 1$, or $\theta \geq \pi$. For type t_H seller: $\theta + 5(1 - \pi) \geq 5$, or $\theta \geq 5\pi$, which implies the earlier one because $\pi \in [0, 1]$. For the buyer: $2 - (\theta + 1 - \pi) + 6\pi - \theta \geq 0$, or $1 - 2\theta + 7\pi \geq 0$.

Thus, the two inequalities that characterize interim individual rationality and interim incentive efficiency (the incentive compatible coarse core) in this case are:

$$0.5 + 3.5\pi \geq \theta \geq 5\pi.$$

If one draws θ on the horizontal axis, and π on the vertical, this set is a triangle whose three vertices are the points $(0, 0)$, $(0.5, 0)$ and $(5/3, 1/3)$.

Moreover, in economies such as this one, because of the presence of one fully informed agent, one can easily argue that the credible core and the non-random mediated core coincide with the incentive compatible coarse core. On the other hand, the fine core is empty: for any profile (θ, π) corresponding to a coarse core allocation, $(\theta + 5.5\epsilon, \pi + \epsilon)$ is a fine objection for the event consisting of state t_H . However, none of these are credible because type t_L would like to join.

This is to be compared with the strong mediated core, which as done in Myerson (2003), is given by the constraints

$$0.5 + 3.5\pi \geq \theta \geq 6\pi.$$

This is also a triangle whose vertices are the points $(0, 0)$, $(0.5, 0)$ and $(6/5, 1/5)$.

As an illustration of the mediator's blocking plans, consider any allocation (θ, π) that is in the coarse core, but not in the strong mediated core. The blocking plan against those allocations has no block in the low state; while in the high state, with probability π the coalition $\{1, 2\}$ is invited to trade, the seller receives a transfer θ/π and the buyer pays $\theta + 6(1 - \pi)$, and with probability $1 - \pi$ only agent 1 is invited, and keeps his endowment. Note that the buyer and type t_H are indifferent between the status-quo and the blocking plan, whereas the mediator obtains a positive expected payoff.

Moreover, there does not exist any strict blocking: because of incentive compatibility, if one improves type t_H strictly, type t_L will also want to join. This means that he needs to be part of the blocking plan and also strictly improve, which renders improving the buyer impossible. Thus, in this example, the mediated core coincides with the incentive compatible coarse core.

6 An Open Question

In terms of investigating other variants of voting games, one could consider a game based on Serrano and Vohra (1997). The dynamic mechanism proposed in that paper implements the core of complete information economies in subgame perfect equilibrium. In adapting its rules to a context of incomplete information, we shall find it convenient, as in the previous sections, to define a class of games where the status quo x is exogenously given. The dynamic procedure will test x against any y that could be endogenously proposed by

any player. This is in the spirit of Lagunoff's (1994) test for core outcomes under complete information.

Consider the voting procedure $\Gamma_x^D(\pi)$. An allocation $x \in A$ is given exogenously as the status-quo. Before the game begins, every agent reports his type privately to the mechanism. Let the report profile be \hat{t} , unobservable to agents before the end of the game. The game consists of sequential moves. Each player moves only once, following the protocol π . The first player announces (S, y) where $i \in S$ and $y \in A_S$. All other players in S vote sequentially, following the order induced by π , to accept or reject this proposal. If all accept, the outcome is $y(\hat{t})$, while $N \setminus S$ receive their initial endowments. Otherwise, the outcome is $x(\hat{t})$.¹⁰ Let $\Gamma_x^D = \cup_{\pi} \Gamma_x^D(\pi)$ and $\Gamma^D = \cup_{x \in A} \Gamma_x^D$.

We shall say that an allocation x is resilient to voting in the class Γ^D if it is incentive compatible and if there is no perfect Bayesian equilibrium of $\Gamma_x^D(\pi)$ for any π where an alternative y is accepted with positive probability.

We first conjecture that, under some reasonable assumptions, resilience in the class Γ^D yields a refinement of the credible core. One such assumption would be the existence of a "bad state" with zero aggregate endowment. In this case, if one has an equilibrium of the game $\Gamma_x^{*D}(y, S, \pi)$ in which y is accepted with positive probability, one can turn it into a perfect Bayesian equilibrium of $\Gamma_x^D(\pi)$ by stipulating that, following a deviation from (S, y) made by the proposer, responders' beliefs concentrate on the bad state and therefore are justified in rejecting any deviation.

On the other hand, our second conjecture about this game in which the proposal is endogenously made by one of the players in N is that resilience to coalitional voting in this class will yield a set that will contain the mediated core. The reason is that one can think of a blocking plan where to complete the event over which it takes place into a product set one needs to add states at least along two dimensions. For example, suppose there are three players and each of them has two possible independent types. Suppose further that the only blocking plan against a status-quo x takes place over the event

$$E = \{(t_1, t_2, t_3), (t_1, t'_2, t_3), (t'_1, t_2, t_3), (t_1, t'_2, t'_3), (t'_1, t_2, t'_3), (t'_1, t'_2, t'_3)\}.$$

Then, it would be impossible to construct an equilibrium of $\Gamma_x^D(\pi)$ where the blocking plan is accepted, because the set of types of the responders

¹⁰To simplify the analysis of the dynamic game, we do not consider a fresh type report if y is accepted. Given the robustness of our previous results in this respect, we do not anticipate that this would make a substantial difference.

accepting the proposal will constitute an event with a product structure. However, note that if one fixes one player, the set of states that have that type fixed does not have a product structure over the other two players' sets of types.

6.1 Relationship to Durability

In their analysis of interim efficiency, Holmström and Myerson (1983) point out that agents may sometimes discard an interim (incentive) efficient allocation in favor of another allocation.¹¹ Their notion of *durability* is meant to formalize the following idea:

“The essential idea is that an incentive compatible decision rule δ should be considered *durable* iff the individuals in the economy would never unanimously approve a change from δ to any other decision rule.” (Holmström and Myerson (1983, p. 1811))

Their actual definition of durability proceeds as follows. Given an allocation x and an alternative feasible allocation y , they consider a voting game $\Gamma_x(y, N)$, in which type reports are made after it is determined whether or not y is unanimously accepted. An allocation x is said to *endure* y if there exists an equilibrium in which y is rejected in every state. Since there always exists a Bayesian equilibrium in which everyone rejects the alternative because they do not expect to be pivotal, it becomes necessary to refine the equilibrium notion. They do this by using some of the conditions used in defining a trembling hand perfect equilibrium. An allocation $x \in A^*$ is said to be durable if it endures every alternative mechanism in A . Thus a durable allocation has the property that for every alternative, there exists an equilibrium rejection of the alternative.

To compare our approach with durability, we apply the stability notion implicit in Γ_x^* (or equivalently Γ_x^{*1} or Γ_x^{*2}) to the grand coalition. An allocation $x \in A^*$ is said to be *resilient to grand coalition voting*, or simply *resilient* in a given class of voting games (e.g., Γ_x^*), if there does not exist $y \in A$ such that in the game $\Gamma_x^*(y, N)$, there exists a Bayesian equilibrium in which y is unanimously accepted with positive probability.¹²

¹¹This phenomenon should be seen as an important consequence of incomplete information since it is, of course, impossible in the complete information setting.

¹²The same set of allocations would result if one used the class Γ_x^{*1} or Γ_x^{*2} .

Proposition 4 *An allocation is resilient in Γ^* if and only if it is strongly interim incentive efficient.*

The proof of this result is similar to that of Proposition 2, and therefore, we omit it. Also, one can prove the following result, providing bounds to the strong interim incentive frontier.

Proposition 5 *A uniformly incentive compatible interim incentive efficient allocation is resilient, and a resilient allocation is interim incentive efficient.*

The proof of the first statement is similar to Dutta and Vohra (2001)???. The second part is obvious.

The reader will notice that our notion of resilience seems to be more faithful to the idea described by the above Holmström-Myerson quote. A resilient allocation has the property that there is no equilibrium in which the alternative is ever unanimously accepted in place of the given status-quo. To meet the durability test it is enough that there exist a (trembling-hand perfect) equilibrium such that in every positive probability state the alternative is rejected. It does not rule out the possibility that there is also an equilibrium in which the alternative is accepted. Clearly, if there exists no equilibrium (perfect or not) in which the alternative is accepted (with positive probability), then provided an equilibrium exists, it must involve a rejection of the alternative in each (positive probability) state. In this sense, a resilient allocation is durable. But the converse need not be true. This can be illustrated by the example in section 9 of Holmström and Myerson (1983) in which an inefficient allocation is durable because there is an equilibrium in which the status-quo endures the alternative. But there also exists, for an alternative that is an interim improvement, an equilibrium in which the alternative is unanimously accepted. Of course, this implies that the original allocation was not resilient. We should be careful in pointing out that this discussion ignores the other differences in the two approaches. In particular, the voting game considered by Holmström and Myerson (1983) is such that types are announced only in the last stage and are applied to the implementation of x , if x endures an alternative. We have not analyzed the voting game in Holmström-Myerson played by coalitions, although we expect one could also find there support for the strong credible core. However, the basic idea in our approach, given our interest in a cooperative solution such as the core, is that a coalition may choose any feasible allocation as long as it can be supported by an equilibrium. In particular, a coalition should be able to resolve

any coordination problem (as in the example of section 9, Holmström and Myerson (1983)). Compared to durability, this makes it easier to challenge a status-quo, and leads therefore to a smaller set of stable allocations. Not surprisingly, this may lead to problems of non-existence. But in studying the core, unlike efficient allocations, this is a problem that we cannot avoid in general, anyway (see Vohra(1999)).

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