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# DIVISIBLE MONEY IN AN ECONOMY WITH VILLAGES

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## Abstract

This paper provides a tractable search model with divisible money that encompasses the two frameworks currently used in the literature. In the model, individuals belong to many villages. Inside a village, individuals are not altruistic as in a representative household, but they share information so financial contracts are feasible. Money is essential in the model to facilitate trade with individuals outside the village. The framework proposed by Lagos and Wright (2002) arises as a special case if some goods trade in competitive markets while others trade in search markets, and preferences are quasi-linear. The framework proposed by Shi (1997) arises as a special case if individuals can insure trading risks inside the village. In general, if preferences are not quasi-linear and trading risks cannot be insured, the distribution of money holdings is non-degenerate and monetary transfers have distributional effects. However, neither quasi-linear preferences nor insurance of trading risks are necessary for tractability. Indeed, this paper advances a tractable benchmark with an endogenous frequency of shopping in which all buyers choose to carry the same amount of money even if preferences are not quasi-linear and trading risks cannot be insured.

# 1 Introduction

Monetary search models have provided rich insights on the foundations of money, and they have become the dominant paradigm in this field of economics. To facilitate tractability, early monetary search models made strong assumptions on the properties of money (indivisibility and limited storage capacity). These strong assumptions prevented the study of many interesting issues such as the effects of inflation. Thanks to the work of Shi (1997) and Lagos and Wright (2002), we have now two distinct frameworks that yield tractable monetary search models with divisible money. Both frameworks use a trick to obtain a tractable distribution of money balances. In the case of Shi, the trick is the assumption that individuals belong to large households. In the case of Lagos and Wright, the trick is the assumption that utility is linear on a good traded in a competitive market. Despite the many accomplishments on the rich literate that has followed the seminal contributions of Shi and Lagos and Wright, there is a major unresolved problem. Quite often a model in one of these two frameworks obtains a conclusion that contradicts the findings of a model in the other framework. For example, Shi (1997) argues that a positive rate of inflation is likely to be optimal, while Lagos and Wright (2002) argues that the costs of inflation are much larger than previously thought. Since both frameworks are built around different tricks, it is difficult to trace the ultimate reasons for these disparate findings. The present paper introduces a comprehensive framework that encompasses those advanced by Shi (1997), and Lagos and Wright (2002). This framework is useful not only to compare these earlier models, but also to enrich the set of issues that can be dealt with monetary search models.

In the model of this paper, individuals belong to villages.[1] Each village contains a large number of individuals, but it is only a small part of the global economy. In a village, individuals are not altruistic as in a household, but they know their neighbors affairs. Therefore, financial contracts such as insurance and credit are feasible among individuals of the same village. Despite the existence of a rich set of financial contracts inside the village, money is still essential to facilitate trade with anonymous individuals from other villages.

Lagos and Wright (2002) is a special case of this paper's model if individuals trade competitively in the village during the day, trade in search markets outside the village during the night, and the utility for one of the commodities traded during the day is linear (quasi-linear preferences). Relaxing the quasi-linearity of preferences is important for several reasons. Quasi-linear preferences imply risk neutrality, so these preferences rule out most of the issues dealt in financial economics. Also, quasi-linear preferences rule out wealth effects on all goods except for the one that yields linear utility. This hinders the study of many interesting issues such as balanced growth and the endogenous frequency of shopping studied in Section 3.

Shi (1997) is a special case of this paper's model if there is complete information inside a village, so individuals can perfectly insure risky trading opportunities. The idea that the large household construct is a way to implicitly insure trading risks goes as far back as Lucas (1990). The present contribution takes seriously Lucas's idea and fleshes out

[1] Jin and Temzelides (2004) advance also a random search model with villages to generate equilibria where money is used in some exchanges while credit is used in some others. However, they do not attempt to use villages to make tractable the divisibility of money. In their model, both money and goods are indivisible, and credit is a gift giving equilibrium with trigger strategies.

the mechanisms that arise in the village to insure trading risks. There are several advantages on explicitly designing these mechanisms. Typically, the mechanisms for insuring trading risks are financial contracts which are interesting on themselves. Moreover, once we are explicit about the extend of risk sharing in the village, there is no ambiguity on the objectives of buyers and sellers when they interact in the money market. Finally, and most importantly, one can vary the information that is shared inside the villages to find out if the abstraction of trading risks is relevant or not to a particular issue.

If preferences are not quasi-linear and trading risks cannot be insured, the distribution of money holdings in the search market is non-degenerate. In general, this reduces the tractability of the model dramatically. However, neither quasi-linear preferences nor insurance of trading risks are necessary for tractability. Indeed, this paper advances a simple benchmark with an endogenous frequency of shopping in which all buyers choose to carry the same amount of money even if preferences are not quasi-linear and trading risks cannot be insured. In this simple benchmark, the distributional effects of monetary policy are irrelevant for the performance of monetary markets.

In summary, the present model provides a unifying framework for tractable search monetary models with divisible money. This framework fleshes out the financial contracts required to obtain a degenerate distribution of money holdings. This framework also leads to new results such as the simple benchmark with an endogenous frequency of shopping presented in this paper. The present framework should be particularly useful to combine money and finance and to embed a tractable monetary search market into macroeconomic models where typically risk aversion and wealth effects are respectively important.

The rest of the paper is organized as follows. The basic model of this paper is analyzed in Section 2. This model generalizes the attractive version of Lagos and Wright (2002) found in Rocheteau and Wright (2003). Section 3 studies a variation of the model in Section 2 in which all goods are traded in monetary search markets and individuals decide how often they go shopping. The solution to this model is equivalent to the solution by Rauch (2000) of the Shi (1997) model. Section 4 shows that an endogenous frequency of shopping makes the insurance of trading risks redundant under quite general conditions. Section 5 contains concluding remarks.

# 2 The Model

The economy is composed of a large number of symmetric villages. Each village contains many individuals (a continuum) who can be of two types: buyers and sellers. All members of a village have complete information about their mutual actions and their histories, but individuals are anonymous outside their village. Time is discrete and the horizon is infinite. Each period consists of two subperiods: day and night. During the day, all the individuals can produce and consume a general nondurable good, which is traded in a frictionless competitive market. Also, during the day individuals trade competitively with their fellow villagers a set of financial contracts to be specified below. During the night, individuals trade nondurable goods specific to each village. These goods are traded in search markets. Sellers are able to produce the good specific to their own village. Buyers get utility from the specific goods produced in other villages. Neither buyers nor sellers get utility from the specific good from their own village. The measures of buyers and sellers are exogenous and normalized to 1. The probabilities that a buyer and a seller meet a suitable trading partner are respectively $\alpha^b$ and $\alpha^s$.

In this environment, there is a role for money to facilitate trade because at night there is a lack of double coincidence of wants in all possible bilateral trading meetings and buyers are anonymous outside their village. Money is an intrinsically useless, perfectly divisible, and storable asset. The money supply grows at a constant factor $\gamma$ such that $M_{+1} = \gamma M$, where $M$ is the quantity of money per buyer. The subscript $t$ is omitted in most expressions of the paper, so, for example, $M$ stands for $M_t$ and $M_{+1}$ stands for $M_{t+1}$. New money is injected via lump-sum transfers to buyers at the beginning of each day. Money is the numeraire.

The instantaneous utility of a buyer is:

$$U^b(x, y, q) = v(x) - w(y) + u(q); \tag{1}$$

where $x$ and $y$ are respectively quantities consumed and produced of the general good, $q$ is the quantity consumed of specific goods from other villages. Likewise, the instantaneous utility of a seller is:

$$U^s(x, y, q) = v(x) - w(y) - c(q); \tag{2}$$

Buyers and sellers maximize their expected utilities: $E \sum_{t=0}^{\infty} \beta^t U^i(x_t, y_t, q_t)$, for $i = b$ and $s$, and $\beta \in (0, 1)$ is the discount factor. The functions $v$, $w$, $u$, and $c$ are all bounded, continuously differentiable, and increasing. The function $v$ is concave, $u$ is log-concave, and $w$ and $c$ are convex. Moreover, $v(0) = w(0) = u(0) = c(0) = 0$, $w'(0) = c'(0) = 0$, and $v'(0) = u'(0) = \infty$. The discount and money growth factors obey $\gamma > \beta$.

Individuals are not risk neutral as in Rocheteau and Wright (2003), so there are gains from insuring with fellow villagers the risks faced in the search markets. To this end, buyers can purchase a contract for the delivery of $\mu^b$ dollar next morning contingent upon meeting a seller during the night. Likewise, sellers can purchase a contract for the delivery of $\mu^s$ dollars next morning contingent upon failing to meet a buyer during the night. The fair premiums to acquire these contracts are respectively $\mu^b\alpha^b$ and $\mu^s(1 - \alpha^s)$. These premiums are also payable next morning.

In addition to insurance contracts, credit is viable inside the village. For concreteness, I assume that individuals can issue one period risk free nominal discount bonds that promise one unit of money next morning. These bonds trade in competitive markets inside the village at a price $(1 + r)^{-1}$. Outside the village, these personal bonds are not valued because potential buyers do not know the issuer of these bonds. Initial bond holdings are assumed to be zero.

The concept of equilibrium combines competitive markets during the day with Nash bargaining at night. In equilibrium, buyers and sellers make optimal choices given the environment where they live. This environment includes the sequence of general good prices: $\{p_t\}_{t=0}^{\infty}$ and the rules describing how the bargaining outcomes depend on the assets held by buyers and sellers. These prices and these rules must be consistent with market clearing and Nash bargaining. For simplicity, I focus on symmetric and stationary equilibria where all individuals of the same type behave identically and real allocations are constant over time.

To find an equilibrium, I adopt the following widely used strategy. First, I make a conjecture on the rate of inflation, other individuals behavior, and the bargaining rules. Given this conjecture, I then solve for the optimal behavior of a representative buyer and a representative seller. Finally, I check that when individuals behave optimally and symmetrically the initial conjecture is consistent with market clearing, Nash bargaining, and stationarity.

The conjecture about inflation is that the price of the general good increases at the same rate as the money supply: $p_{+1} = \gamma p$. The conjecture about other individuals is that, at the night market, sellers carry no money balances, while buyers carry an amount of money $M^b$ that grows at the rate $\gamma$ over time. Finally, the conjecture about bargaining is that the output and the money exchanged in a trading meeting $(q, d)$ depend only on the quantity of money that the buyer carries and the price $p_{+1}$. The quantity $q$ is a function of $m^b/p_{+1}$, and the payment $d$ is a linerarly homogeneous function of $m^b$ and $p_{+1}$:

$$q = \tilde{q}\left(\frac{m^b}{p_{+1}}\right), \text{ and } d = \tilde{d}(m^b, p_{+1}), \tag{3}$$

In addition, these functions satisfy the following properties. If $m^b/p_{+1}$ is smaller or equal than a fixed threshold $z^*$, then the buyer spends all the money that carries ($d = m^b$), and $q$ is an increasing, concave, and differentiable function of $m^b/p_{+1}$. If $m^b/p_{+1}$ is greater or equal than $z^*$, then the buyer spends $d = p_{+1}z^* \leq m^b$ to obtain a fixed amount of output $q = q^*$ and keeps the remaining money balances. Finally, at $m^b = 0$, $\tilde{q}(0) = \tilde{d}(0, p_{+1}) = 0$.

## 2.1 The Behavior of Buyers

A representative buyer that starts the morning with wealth $\omega_0^b$ faces the following budget constraint:

$$x^b + \frac{m^b + a^b\,(1+r)^{-1}}{p} = y^b + \omega_0^b, \quad m^b \geq 0, \tag{4}$$

where $x^b$ and $y^b$ are respectively the consumption and the production of general goods during the day, and $m^b$ and $a^b$ are respectively the dollars and bonds acquired during the day. The buyer is aware that the outcome of bargaining in a trade meeting at night obeys the rules in (3), and can purchase fair insurance on the event of meeting a seller. The optimal choice of $\left\{x^b, y^b, m^b, \mu^b\right\}$ solves the following maximization program:

$$V^b(\omega_0^b) = \max_{\{x^b, y^b, m^b, \mu^b\}} v(x^b) - w(y^b) + \alpha^b\left[u(q) + \beta V^b\left(\omega^{b1}\right)\right] + \left(1 - \alpha^b\right)\beta V^b\left(\omega^{b0}\right) \tag{5}$$

subject to (3) and (4). The terms $\omega^{b0}$ and $\omega^{b1}$ denote the wealth next morning. The superscripts $b1$ and $b0$ denote respectively the existence or not of a trading opportunity. Therefore,

$$\omega^{b0} = \frac{a^b + m^b - \mu^b\alpha^b + \tau}{p_{+1}}, \text{ and} \tag{6}$$

$$\omega^{b1} = \frac{a^b + m^b - d + \mu^b\left(1 - \alpha^b\right) + \tau}{p_{+1}}. \tag{7}$$

where $\tau$ denotes the monetary transfers from the government.

Standard recursive dynamic arguments show that $V^b$ is a well defined value function that depends on $\omega_0^b$. Furthermore, $V^b$ is concave and continuously differentiable.

To solve (5), the buyer must behave as follows. Because of the concavity of $V^b$ the buyer must fully insure the risk on trading opportunities:

$$\mu^b = d. \tag{8}$$

Therefore, the quantity of money held in the next morning is $m^b - d\alpha^b + \tau$ regardless of meeting a seller or not, so $\omega^{b0} = \omega^{b1} \equiv \omega^b$. Also, the buyer must equate the marginal utility of consumption to the marginal desutility of production:

$$v'(x^b) = w'(y^b) \tag{9}$$

Finally, using the Envelope Theorem $V^{b\prime}\left(a^b\right) = w'(y^b_{+1})$, the buyer must demand bonds and money to satisfy the following conditions:

$$w'(y^b) = \beta\,(1+r)\,\frac{p}{p_{+1}} w'(y^b_{+1}), \text{ and} \tag{10}$$

$$\frac{w'(y^b)}{p} = \alpha^b u'\,(q)\,\tilde{q}'\left(\frac{m^b}{p_{+1}}\right)\frac{1}{p_{+1}} - \beta w'(y^b_{+1})\left[\frac{\tilde{d}_{m^b}\left(m^b, p_{+1}\right)\alpha^b - 1}{p_{+1}}\right]. \tag{11}$$

Equation (10) is a standard Euler condition equating the marginal utility of consuming the general good with the marginal utility of acquiring bonds to consume tomorrow. Condition (11) equates the marginal cost and the marginal benefit of acquiring money. The cost is the desutility of producing the general good. The benefit is the expected increase in the consumer's surplus at night.

In a stationary equilibrium ($y^b = y^b_{+1}$), condition (10) implies that the demand for bonds is perfectly elastic (across steady states) at $\beta^{-1} = (1+r)p/p_{+1}$. Therefore,

$$r = \frac{\gamma - \beta}{\beta} \tag{12}$$

If the buyer chose to carry money in excess of the amount ever spent in a trade meeting ($\tilde{q}'\left(m^b/p_{+1}\right) = \tilde{d}_{m^b}\left(m^b, p_{+1}\right) = 0$), then stationarity and condition (11) would also imply $\beta = \gamma$. The assumption $\gamma > \beta$ rules out such a possibility. Hence, in a stationary equilibrium $\tilde{d}_{m^b}\left(m^b, p_{+1}\right) = 1$ and condition (11) simplifies to:

$$\frac{u'(q)}{\beta w'(y^b)}\tilde{q}'\left(\frac{m^b}{p_{+1}}\right) = 1 + \frac{r}{\alpha^b}. \tag{13}$$

## 2.2 The Behavior of Sellers

A representative seller that starts the morning with wealth $\omega^s_0$ faces the following budget constraint:

$$x^s + \frac{m^s + a^s(1+r)^{-1}}{p} = y^s + \omega^s_0, \quad m^s \geq 0, \tag{14}$$

where $x^s$ and $y^s$ are respectively the consumption and the production of general goods during the day, and $m^s$ and $a^s$ are respectively the dollars and bonds acquired during the day. The seller is aware that the outcome of bargaining in a trade meeting at night obeys (3), and she can purchase fair insurance on the event of meeting a seller. The optimal choice of $\{x^s, y^s, m^s, \mu^s\}$ solves the following maximization program:

$$V^s(\omega^s_0) = \max_{\{x^s, y^s, m^s, \mu^s\}} v(x^s) - w(y^s) + (1-\alpha^s)\,\beta V^s\left(\omega^{s0}\right) + \alpha^s\left[-c(q) + \beta V^s\left(\omega^{s1}\right)\right] \tag{15}$$

subject to (15) and (3). The terms $\omega^{s0}$ and $\omega^{b1}$ denote the wealth next morning. The superscripts $s1$ and $s0$ denote respectively the existence or not of a trading opportunity. Therefore,

$$\omega^{s0} = \frac{a^s + m^s + \mu^s \alpha^s}{p_{+1}}, \text{ and} \tag{16}$$

$$\omega^{s1} = \frac{a^s + m^s + d - \mu^s (1 - \alpha^s)}{p_{+1}}. \tag{17}$$

Recursive dynamic arguments show that $V^s$ is a well defined value function that depends on $\omega_0^s$. Furthermore, $V^s$ is concave and continuously differentiable.

To solve (14), the seller must fully insure the risk of failing to meet a buyer because $V^s$ is concave:

$$\mu^s = d. \tag{18}$$

Therefore, the quantity of money held next morning is $m^s + d\alpha^s$ regardless of meeting a buyer or not, so $\omega^{s0} = \omega^{s1} \equiv \omega^s$. Also, the seller must equate the marginal utility of consumption to the marginal desutility of production:

$$v'(x^s) = w'(y^s) \tag{19}$$

Finally, using the Envelope Theorem $V^{s\prime}(\omega^s) = w'(y^s_{+1})$, the seller must demand bonds and money to satisfy the following conditions:

$$w'(y^s) = \beta (1 + r) \frac{p}{p_{+1}} w'(y^s_{+1}). \tag{20}$$

$$\left[ \frac{\beta w'\left(y^s_{+1}\right) p}{p_{+1}} - w'\left(y^s\right) \right] m^s = 0. \tag{21}$$

In a stationary equilibrium, $y^s_{+1} = y^s$ and $\beta p < p_{+1}$, so the demand for bonds is perfectly elastic at $\beta^{-1} = (1 + r) p / p_{+1}$, and the demand for money is zero.

## 2.3 Generalized Nash Bargaining

This subsection characterizes the outcome of Nash bargaining when a buyer and a seller meet at night. Let $S^b$ and $S^s$ be respectively the trading surpluses of the buyer and the seller. The generalized Nash bargaining outcome $(q, d)$ solves the following program:

$$\max_{(q,d)} \left(S^b\right)^{\theta} \left(S^s\right)^{1-\theta} \tag{22}$$

subject to the rationality constraints ($S^b \geq 0$ and $S^s \geq 0$) and the constraint that the buyer can only pay the money he carries $\left(d \leq m^b\right)$. From (4) and (14), the trading surpluses are

$$S^b = u(q) + \beta \left[ V^b \left(\omega^b\right) - V^b \left(\omega^b + d\right) \right] \tag{23}$$

and

$$S^s = \beta \left[ V^s \left(\omega^s\right) - V^s \left(\omega^s - d\right) \right] - c(q) \tag{24}$$

To solve the model analytically, the following Taylor approximations are very convenient:

$$V^b\left(\omega^b + d\right) - V^b\left(\omega^b\right) \approx V^{b\prime}(\omega^b)\frac{d}{p_{+1}} \tag{25}$$

$$V^s\left(\omega^s\right) - V^s\left(\omega^s - d\right) \approx V^{s\prime}(\omega^s)\frac{d}{p_{+1}} \tag{26}$$

Without using these approximations, the model is still well defined, but the solution must be numerical. These approximations turn into equalities, and so their use is fully justified, in the following three cases. The first justification is that the value functions are afin. Lagos and Wright (2002) and Rocheteau and Wright (2003) provide an example of this justification with quasi-linear preferences. Next section provides another example with endogenous shopping, but the value functions $V^s$ and $V^b$ in this section are in general not afin. The second justification is that individuals have insurance not only against the risk of meeting or not a trading partner, but also against the risk of failing to reach a bargaining agreement with the trading partner. Of course, such an insurance generates all kinds of moral hazard problems. To avoid these problems, insurance contracts must then specify in detail how individuals must act in all eventualities. In the framework of a representative household, such a modeling strategy has lead to the critique that individuals turn into machines. The third justification, which is the one to be adopted here, is that payment $d$ is an infinitessimal fraction of the comprehensive wealth of individuals. This comprehensive wealth, which includes the present discounted value of all future production, is arbitrarily large as $\beta \longrightarrow 1$. Consequently, (25) and (26) are justified when $\beta$ is in a neighborhood of one, or equivalently, when the trading period is short.

Using (25) and (26) with equality, together with the Envelope Theorem, the trading surpluses of buyers and sellers are the following:

$$S^b = u(q) - \beta w'(y^b)\frac{d}{p_{+1}} \tag{27}$$

$$S^s = \beta w'(y^s)\frac{d}{p_{+1}} - c\left(q\right) \tag{28}$$

Substituting (27) and (28) into (22), the first order interior conditions that characterize generalized Nash bargaining are:

$$\frac{\theta}{1-\theta}\frac{w'(y^b)}{w'(y^s)} = \frac{u(q) - \beta w'(y^b)z}{\beta w'(y^s)z - c(q)} \tag{29}$$

$$\frac{\theta u'(q)}{\left(1-\theta\right)c'(q)} = \frac{u(q) - \beta w'(y^b)z}{\beta w'(y^s)z - c(q)}. \tag{30}$$

where $z \equiv d/p_{+1}$. Let $(q^*, z^*)$ be the solution to the system (29) and (30). Using the Kuhn-Tucker theorem, if $m^b \geq z^* p_{+1}$, then the Nash bargaining solution is unconstrained by $d \leq m^b$ and the outcome is $(q^*, z^*)$. In contrast, if $m^b \leq z^* p_{+1}$, then the Nash bargaining solution is constrained, so $d = m^b$ and $q$ obeys (30) with $z = m^b/p_{+1}$.

As we saw in the previous section, the constrained solution is the relevant one in equilibrium. In this case, it is convenient to rewrite (30) in the following form:

$$z = \frac{\theta u'(q)c(q) + \left(1-\theta\right)c'(q)u(q)}{\theta u'(q)\beta w'(y^s) + \left(1-\theta\right)c'(q)\beta w'(y^b)} \equiv g(q). \tag{31}$$

In the three justifications provided for turning (25) and (26) into equalities, the values $w'(y^s)$ and $w'(y^b)$ are treated as constant in a particular trading match. Also, in a constrained solution $z = m^b/p_{+1.}$ Consequently, the function $g(q)$ in (31) is the inverse function of the terms of trade rule $\tilde{q}\left(m^b/p_{+1}\right)$. Using the inverse function theorem, we obtain

$$\tilde{q}'\left(\frac{m^b}{p_{+1}}\right) = \left(\frac{\partial z}{\partial q}\right)^{-1} = \frac{1}{g'(q)} \tag{32}$$

The concavity of $\tilde{q}$ is equivalent to the convexity of $g$. Using the log-concavity of $u$, Lagos and Wright (2002) show that $g$ is convex for the special case $w'(y^b) = w'(y^s) = 1$. This result easily generalizes to $w'(y^b)$ and $w'(y^s)$ being positive numbers, because if $u$ is log-concave, then $u$ times the value of the ratio $w'(y^s)/w'(y^b)$ is also log-concave.

## 2.4 Equilibrium

The previous subsection shows that the conjectured properties on the terms of trade rules that we employed to solve the optimization problems of buyers and sellers are consistent with generalized Nash bargaining if $\beta$ is in a neighborhood of one. The conjecture that prices increase at the rate $\gamma$ is a direct implication of stationarity. Finally, optimal behavior is consistent with the conjecture that sellers carry no money and buyers carry an amount that grows at the rate $\gamma$. Therefore, we are ready to collect the equations that characterize the equilibrium allocation $\left(x^b, x^s, y^b, y^s, q\right)$.

Combining the first order conditions (9), (19), and (13) with the constrained solution of Nash bargaining (32), the market clearing condition for the general good, and the budget constraint of sellers with $m^s = 0$, $a^s = 0$, and $\omega_0 = \alpha^s g(q)$, we obtain

$$v'(x^b) = w'(y^b) \tag{33}$$

$$v'(x^s) = w'(y^s) \tag{34}$$

$$x^b + x^s = y^b + y^s \tag{35}$$

$$y^s - x^s = \alpha^s g(q) \tag{36}$$

$$\frac{u'(q)}{\beta w'(y^b)g'(q)} = 1 + \frac{\gamma - \beta}{\beta \alpha^b} \tag{37}$$

$$g(q) = \frac{\theta u'(q)c(q) + (1-\theta)\, c'(q)u(q)}{\theta u'(q)\beta w'(y^s) + (1-\theta)\, c'(q)\beta w'(y^b)} \tag{38}$$

A simpler system characterizes an equilibrium in which bond positions of buyers and sellers are such that all individuals share a common marginal value of general goods. In this simpler case, the equilibrium is described by

$$x^b = x^s = y^b = y^s, \tag{39}$$

together with (33), (37), and (38)

# 3 Endogenous Shopping

This section presents a modified version of the previous model where individuals choose which role they play at the search market. This analysis complements the previous section in two ways. In the previous section, credit contracts were not used along the equilibrium path. In this section, credit contracts are the key financial arrangement between the members of a village. In the previous section, the model extends the Rocheteau and Wright (2003) version of the Lagos and Wright (2002) model. In this section, an equilibrium is equivalent to the Rauch (2000) solution to the Shi (1997) model.

In the version of the model presented in this section, there are no general goods, and all the individuals have symmetric preferences and abilities. All individuals are able to produce a good specific to their village. Also, they all get utility from the goods produced in other villages. However, nobody desires to consume the good produced in their own village, so trade across villages is necessary to consume. This trade makes money essential because outside their own village individuals are anonymous. Production must take place around the village of origin, so at the beginning of each period each individual has to choose to either stay around the village as a seller or visit other villages as a buyer. In each period, some individuals are buyers while the others are sellers. Over time, an individual alternates between these two roles because he cannot afford being always a buyer, and he never consumes being always a seller. The emphasis of the present model is to endogenize how often an individual plays each one of these roles.

The measure of individuals is one. The endogenous fraction of sellers is denoted as $n$, so the fraction of buyers is $1-n$. Each period, buyers and sellers are randomly matched. The measure of trading matches, in which a buyer meets a seller, is a function of $n : \alpha(n)$. The function $\alpha$ is assumed to be non-negative, continuously differentiable, concave, and satisfies the terminal conditions $\alpha(0) = \alpha\,(1) = 0$. The probability that a buyer finds a seller is:

$$\alpha^b(n) = \frac{\alpha(n)}{1-n}. \tag{40}$$

The probability that a seller finds a buyer is

$$\alpha^s(n) = \frac{\alpha(n)}{n}. \tag{41}$$

Money is injected through lump-sum transfers received by all individuals at the beginning of a period. The growth factor of the money supply is $\gamma : M_{+1} = \gamma M$. Because of the absence of the general good, $p$ has no meaning as a price. However, to facilitate the comparison with the the previous model, I continue using $p$ to deflate the value of money. It turns out that a convient and simple deflator is the aggretate quantity of money, so I define $p \equiv M$. With this deflator, the ratio $m/p$ denotes the fraction of the money supply $M$ that an individual holds. As it would be seen, this fraction measures the purchasing power of the individual's money holdings.

## 3.1 Optimal Behavior

As in the previous section, I describe the optimal behavior of individuals in a conjectured environment. In this conjecture, the terms of trade rules are summarized with the functions $\tilde{q}$ and $\tilde{d}$ in (3) that have the same properties as those conjectured in the previous section. These properties imply that $p$ correctly deflates the value of money in terms of specific goods. The interest rate on bonds satisfies:

$$\frac{(1+r)\,p}{p_{+1}} = \beta^{-1}, \tag{42}$$

so $\beta^{-1}-1$ is the real interest rate. Finally, other individual carry no money balances to the market if they decide to be sellers, and carry a certain amount $M^b$ that grows over time at the rate $\gamma$ ($M^b/p$ is constant). This conjectured environment is later shown to be consistent with generalized Nash bargaining and stationarity, when individuals behave optimally.

The optimal program of an individual is described as follows.

$$V(\omega_0) = \max\left\{V^b(\omega_0), V^s(\omega_0)\right\}; \tag{43}$$

where

$$V^b(\omega_0) = \max_{\{m^b,a^b,\mu^b\}} \alpha^b\left[u\left(q^b\right) + \beta V\left(\omega^{b1}\right)\right] + \left(1-\alpha^b\right)\beta V\left(\omega^{b0}\right), \text{ and} \tag{44}$$

$$V^s(\omega_0) = \max_{\{m^s,a^s,\mu^s\}} \alpha^s\left[-c(q^s) + \beta V\left(\omega^{s1}\right)\right] + (1-\alpha^s)\, V\left(\omega^{s0}\right). \tag{45}$$

The wealth $\omega$ at the beginning of next period depends on if the individual chooses to be a buyer (superscript $b$) or a seller (superscript $s$). It also depends on if the individual found a trading partner (superscript 1) or not (superscript 0):

$$\omega^{b1} = \frac{a^b + m^b - d^b + \mu^b\left(1-\alpha^b\right) + \tau}{p_{+1}}, \tag{46}$$

$$\omega^{b0} = \frac{a^b + m^b - \mu^b\alpha^b + \tau}{p_{+1}}, \tag{47}$$

$$\omega^{s1} = \frac{a^s + m^s + d^s - \mu^s(1-\alpha^s) + \tau}{p_{+1}}, \text{ and} \tag{48}$$

$$\omega^{s0} = \frac{a^s + m^s + \mu^s\alpha^s + \tau}{p_{+1}}. \tag{49}$$

The program (43) faces the following constraints: $q^b = \tilde{q}(m^b/p_{+1})$, $d^b = \tilde{d}(m^b, p_{+1})$, $q^s = \tilde{q}(M^b/p_{+1})$, $d^s = \tilde{d}(M^b, p_{+1})$, $m^b \geq 0$, $m^s \geq 0$, and the budgets:

$$m^b + a^b(1+r)^{-1} = \omega_0 p, \text{ and } m^s + a^s(1+r)^{-1} = \omega_0 p. \tag{50}$$

In addition, next period wealth is bounded below by the condition that individuals must be able to repay their debts with probability one without reliance to unbounded borrowing (No-Ponzi game condition):

$$\omega^i \geq \underline{\omega}, \text{ for } i \in \{b1, b0, s1, s0\}, \tag{51}$$

where $\underline{\omega}$ is an endogeneous lower bound on wealth. The value of $\underline{\omega}$ depends crucially on the government transfers and the existence of insurance. Without transfers ($\tau = 0$) and insurance, the lower bound $\underline{\omega}$ is zero. With these assumptions, an individual cannot guarantee having positive income in the future because sellers may have a long stream of bad outcomes in the market. Therefore, an individual cannot guarantee to be able to repay a positive debt with probability one without runing a Ponzi game. Positive transfers and insurance on trading risks allow individuals to issue risk free bonds up to the point that the interest on these bonds is equal to the guaranteed income of a seller purchasing full insurance ($\mu^s = d^s$). Therefore, using (50) and (49) with $\omega_0 = \omega^{s0} = \underline{\omega}$, $\mu^s = d^s$, $m^s = 0$, $\tau = (\gamma - 1) M$, and $p_{+1} = \gamma M$, we obtain

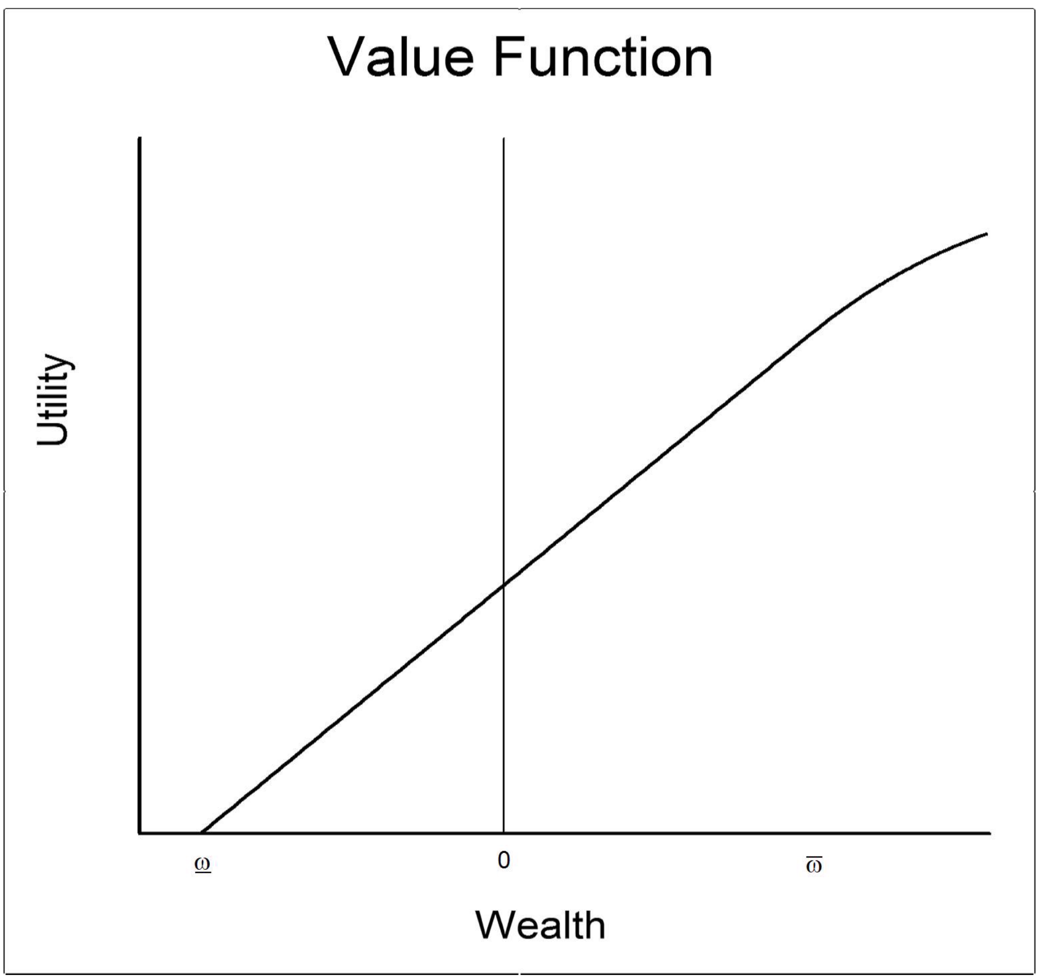
$$\underline{\omega} = -\frac{\frac{\gamma-1}{\gamma} + \frac{\alpha^s d^s}{\gamma M}}{1-\beta}. \tag{52}$$

Standard recursive dynamic arguments show that $V$ is a well defined concave value function that depends on $\omega_0$. Using the method of undetermined coefficients, I will show that $V$ has the following afin functional form for the interval of wealth individuals hold in a stationary equilibrium:

$$V(\omega_0) = v_0 + \omega_0, \text{ for } \omega_0 \in [\underline{\omega}, \overline{\omega}]; \tag{53}$$

where $\underline{\omega}$ is defined in (52) and $\overline{\omega}$ is determined below. Figure 1 displays this value function.

Figure 1



Intutitively, the function $V$ depicted in Figure 1 is consistent with the following behavior. At $\underline{\omega}$, an individual can only afford being a seller forever, so the utility is zero. In the upper interval $[\overline{\omega}, \infty)$ , an individual is sufficiently rich that will be a buyer in all future periods. In this interval, the money holdings $m^b$ and the output purchased $q^b$ are increasing with wealth. The strict concavity of $u$ implies the strict concavity of $V$ if wealth is above $\overline{\omega}$. In the interval $(\underline{\omega}, \overline{\omega})$ , the individual alternates between being a buyer and a seller. Each period, the individual is indifferent between these two roles as long as next period wealth remains in $[\underline{\omega}, \overline{\omega}]$ with probability one. However, it is not optimal or feasible behavior to escape this interval. At the lower bound, the individual is forced to either purchase insurance, or go to the market as a seller, or both in order to satisfy (51). Crossing the upper bound $\overline{\omega}$ violates the Euler equation, which in this model is equivalent to the buyer-seller choice. To avoid crossing $\overline{\omega}$, it is optimal for an individual to either purchase insurance, or go to the market as a buyer, or both. As long as wealth is in the interval $(\underline{\omega}, \overline{\omega})$ , whenever an individual goes to the market as a buyer carries an amount of money $m^b$ independent from wealth. Consequently, richer individuals can expect to consume more often in the rest of their lives, but they consume the same amount each time they do so.

As long as next period wealth remains in $[\underline{\omega}, \overline{\omega}]$ with probability one, the functional form (53) implies that the optimization problems for buyers (44) and sellers (45) simplify into:

$$V^b(\omega_0) = \frac{\beta(1+r)p}{p_{+1}}\omega_0 + \beta\left(v_0 + \frac{\tau}{p_{+1}}\right) + \max_{m^b}\left\{\alpha^b\left[u\left(q\right) - \beta\frac{d}{p_{+1}}\right] - \beta\frac{m^b}{p_{+1}}r\right\}, \text{ and} \tag{54}$$

$$V^s(\omega_0) = \frac{\beta(1+r)p}{p_{+1}}\omega_0 + \beta\left(v_0 + \frac{\tau}{p_{+1}}\right) + \max_{m^s}\left\{\alpha^s\left[\beta\frac{d}{p_{+1}} - c(q)\right] - \beta\frac{m^s}{p_{+1}}r\right\}. \tag{55}$$

The values of $\mu^b$ and $\mu^s$ drop from (54) and (55). Therefore, individuals are indifferent between insuring their trading risks as long as next period wealth remains in $[\underline{\omega}, \overline{\omega}]$ with probability one. The optimal choice of $m^b$ and $m^s$ is characterized by

$$m^s = 0 \text{ since } r = \frac{\gamma - \beta}{\beta} > 0, \text{ and} \tag{56}$$

$$\frac{u'(q)\tilde{q}'\left(z\right)}{\beta} = 1 + \frac{r}{\alpha^b}. \tag{57}$$

where $z \equiv m^b/p_{+1}$. The optimal values of $m^s$ and $m^b$ implied by (56) and (57) are independent of $\omega_0$.Therefore, in an environment where (42) holds, we have

$$V^b\left(\omega_0\right) = v_0^b + \omega_0, \text{ and } V^s\left(\omega_0\right) = v_0^s + \omega_0; \tag{58}$$

where the values $v_0^b$ and $v_0^s$ are constants because $\tau/p_{+1} = (\gamma - 1)/\gamma$, and the properties of $\tilde{q}$ and $\tilde{d}$ imply that $m^b/p_{+1}$, $q^b$, $q^s$, $d^b/p_{+1}$, $d^s/p_{+1}$ are constant. Therefore, the functions in (**??**) together with (53) implies that $V\left(\omega_0\right)$ has the functional form in (53) with $\overline{\omega}$ being the wealth that permits an individual to be a buyer in perpetuity when (57) holds and the individually fully insures.

To confirm our conjecture about the validity of (53), it remains to show that if $\omega_0 \in [\underline{\omega}, \overline{\omega}]$ , then it is not feasible or not optimal to make a choice that would lead to $\omega \notin [\underline{\omega}, \overline{\omega}]$. Choices that lead to $\omega < \underline{\omega}$ with positive probability violate the No-Ponzi game condition (51). Furthermore, if $\omega_0 > \underline{\omega}$, the individual can always avoid violating (51) by choosing to be a seller and insuring trading risks. Choices that lead to $\omega > \overline{\omega}$ with positive probability cannot be optimal for the following reason. If there is a positive probability of entering the strictly concave region of $V$, optimal behavior implies full insurance of trading risks. With full insurance, the Euler equation of program (43) is the following:

$$V'(\omega_0) = \frac{\beta(1+r)p}{p_{+1}} V'(\omega). \tag{59}$$

In an environment where (42) holds, this equation is violated if $V'(\omega_0) > V'(\omega)$, so $\omega \leq \overline{\omega}$.

## 3.2 Equilibrium

In a monetary equilibrium, there must be both buyers and sellers. For the coexistence of both roles, it is necessary that $v_0^b = v_0^s$. This implies that the following condition must hold:

$$\alpha^b(n)\left[u(q) - \beta z\right] - zr = \alpha^s(n)\left[\beta z - c(q)\right]. \tag{60}$$

That is, $n$ must be such that an individual must be indifferent between being a buyer or a seller.

Nash bargaining proceeds along the same lines as the previous section with the advantage that $w'(y^b) = w'(y^s)$ and the approximations (25) and (26) can be replaced with equalities even if $\beta$ is low. From this analysis we obtain:

$$\beta z = \frac{\theta u'(q)c(q) + (1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)} \equiv g(q) \tag{61}$$

This validates the conjecture on terms of trade rules.

For existence of an equilibrium both sides of the equation in (60) must be non-negative otherwise an individual would be better off dropping out of the market, this places a limit on $r$ and so $\gamma$. Also, for an equilibrium to exists, individuals must be willing to alternate between being buyers and sellers over time. Since individuals save when then they are sellers and disave then they are buyers, the afinity of the value function $V$ implies that individuals must be indifferent to save or disave in a given period. Hence, the following Euler condition for an interior saving decision must hold:

$$1 = \frac{\beta(1+r)p}{p_{+1}}. \tag{62}$$

Furthermore, (62) is consistent with optimal individual behavior, generalized Nash bargaining, and stationarity. This validates the conjecture (42).

Using (12), (40), and (41) to simplify (60), we obtain the following two equations that determine $n$ and $q$ together with (61) in an equilibrium:

$$\frac{n}{1-n} = \frac{g(q) - c(q)}{u(q) - g(q)\left(1 + \frac{\gamma-\beta}{\alpha^b(n)\beta}\right)} \tag{63}$$

$$\frac{u'(q)}{g'(q)} = 1 + \frac{\gamma - \beta}{\alpha^b(n)\beta} \tag{64}$$

These equations are equivalent to the solution presented by Rauch (2000) of Shi (1997).[2] The distribution of assets across individuals in a stationary equilibrium is undetermined except for the fact that the wealth of individuals is in the interval $[\underline{\omega}, \overline{\omega}]$ . However, this distribution is irrelevant to find the values of $n$ and $q$. This implies that as long as the wealth of all inidividuals remains in $[\underline{\omega}, \overline{\omega}]$, random redistributions of money at the beginning of a period are irrelevant to characterize aggregate variables.

# 4 Financial Contracts versus Lotteries

One of the nice features of achieving tractability with the existence of financial contracts is that the effects of this mechanism can be evaluated by simply assuming imperfect information inside the village. In general, the absence of financial contracts leads to models where the amounts of money carried by buyers are heterogeneous. Once buyers hold divers money balances, models are typically much more difficult to solve and quite often their analyses require numerical methods. Interestingly, this is not the case in the simple model analyzed in the previous section. Even if individuals are anonymous both inside and outside the village, fair lotteries are able to replace the role of financial contracts. These lotteries demand no memory about personal histories, yet they lead to the same system of equations (61) to (64) that characterizes an equilibrium.

The model in this section makes two modifications to the model presented in the previous section. First, the government can give lump-sum transfers but cannot impose lump-sum taxes, so the money supply can only grow: $\gamma \geq 1$. The importance of this modification is discussed below. Second, individuals have no access to credit and insurance contracts, but individuals can play a fair lottery. A lottery ticket delivers an amount of money $m$ with probability $\pi$ at a cost $\pi m$.

The absence of financial contracts implies that money is the only asset in the economy. Consequently, the state variable measuring an individuals wealth is the real quantity of money at the beginning of a period. As it will be confirmed with the method of undetermined coefficients, the value function has still the afin form: $V(\omega) = v_0 + \omega$ in the relevant interval of wealth. This inveral is $\left[0, m^b/p\right]$ , where $m^b/p$ is calculated below. Using this functional form for next period value function, the utilities attained by being a buyer or a seller are:

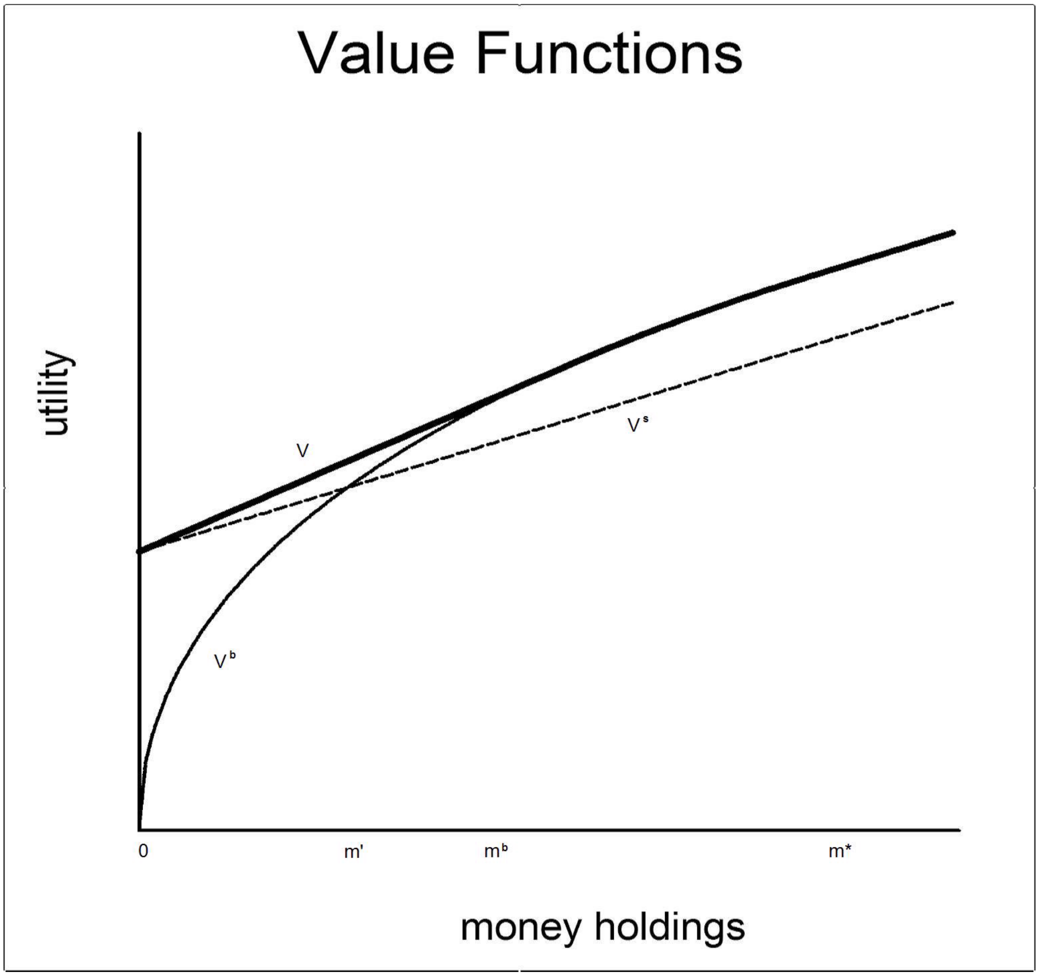
$$V^b(\frac{m}{p}) = \beta\left(v_0 + \frac{\tau}{p_{+1}}\right) + \alpha^b\left[u\left(q\right) - \beta\frac{d}{p_{+1}}\right] + \beta\frac{m}{p_{+1}} \tag{65}$$

$$V^s(\frac{m}{p}) = \beta\left(v_0 + \frac{\tau}{p_{+1}}\right) + \alpha^s\left[\beta\frac{d}{p_{+1}} - c(q)\right] + \beta\frac{m}{p_{+1}} \tag{66}$$

subject to the terms of trade rules (3). Figure 2 represents the shape of these functions if the individual optimizes in an equilibrium environment with both buyers and sellers active in the market.

Figure 2

[2] Rauch adds barter and assumes linearity of $u$ and symmetric bargaining.




Both $V^s$ and $V^b$ are increasing functions of $m$. At the vertical axis, $V^s(0) - V^b(0)$ is equal to the expected seller's surplus, which must be positive in equilibrium, so $V^s(0) > V^b(0)$. The function $V^b$ is at least as steep as the function $V^s$, because money holdings increase the buyer's surplus but have no effect on the seller's surplus. Beyond the quantity of money $m^*$, where Nash bargaining ceases to the constrained by the buyer's money, the two functions have the same slope $\beta/p_{+1}$. Finally, $V^s$ and $V^b$ must cross otherwise there would be no buyers in the market.

In the absence of lotteries, an individual would choose to be a buyer if $m < m'$ and to be a seller if $m > m'$.At $m'$ the individual is indifferent. However, it is clear from Figure 2, that the individual can achieve a better outcome if $m \in \left(0, m^b\right)$ by randomizing between $V^s(0)$ and $V^b\left(m^b/p_{+1}\right)$. In this case, the individual uses all his money holdings $\omega_0 p$ at the beginning of each period to buy a lottery ticket with a payoff $m^b$, and probability of winning $\omega_0 p/m^b$. If the individual wins the lottery becomes a buyer and if the individual loses the lottery become a seller. Consequently, if $\omega_0 \in \left[0, m^b/p\right]$, the individual solves the following program:

$$V(\omega_0) = \max_{\{m^b,\pi\}} \pi V^b\left(\frac{m^b}{p}\right) + (1-\pi) V^s(0), \tag{67}$$

subject to (3), the non-negativity of money balances, and the budget constraint.

$$\pi m^b = \omega_0 p. \tag{68}$$

The first order conditions with respect to $\pi$ and $m^b$ yield (57) and (60). These conditions imply that $m^b$ is independent from the initial wealth of an individual $\omega_0$ if $\omega_0 \in \left[0, m^b/p\right]$ . This confirms that $V$ is afin in this interval. As long as the initial wealth is in this interval, $m^b$ is not affected by $\omega_0$ while $\pi$ is a linear function of $\omega_0$.

After an analogous characterization of Nash bargaining, we obtain that $q$ and $n$ are still determined by the system (61) to (64). Since sellers carry no money in equilibrium and all buyers carry the same amount, the equilibrium value of $m^b/p$, which is the share of money held by a buyer, must be $(1-n)^{-1}$ .

In equilibrium, the wealth distribution at the beginning of next period is $\tau/p_{+1}$ for matched buyers and unmatched sellers and $m^b/p_{+1}$ for the rest of individuals. Moreover, in a stationary equilibrium $\tau = (\gamma - 1) M$ and $p_{+1} = \gamma M$, so the wealth distribution at the beginning of each period is:

$$\frac{\tau}{p_{+1}} = \frac{\gamma - 1}{\gamma} \quad \text{with frequency } (1-n)\,\alpha(n) + n\left[1 - \frac{\alpha(n)}{n}\right], \text{ and} \tag{69}$$

$$\frac{m_b}{p_{+1}} = \frac{m_b}{p\gamma} = \frac{1}{(1-n)\,\gamma} \text{ with frequency } (1-\ n)\left[1 - \alpha(n)\right] + n\frac{\alpha(n)}{n}. \tag{70}$$

As long as $\gamma \geq 1$, both mass points in this distribution are in the interval $\left[0, m^b/p\right]$ . Consequently, this confirms that this is the relevant interval where value functions must be evaluated in equilibrium.

In conclusion, this is a very tractable model of divisible money without neither quasi-linear preferences nor implicit or explicit insurance of trading risks. The feature of the model that delivers a constant marginal utility of wealth is the endogeneous shopping frequency.

## 4.1 A Difficulty with the Optimum Quantity of Money

An interesting feature of the model is that monetary authorities cannot implement reductions on the quantity of money with uniform lump-sum taxes $(\gamma < 1)$ . A uniform lump-sum tax means that each individual must pay every period a pre-ordanined amount of money. In the absence of financial contracts, an individual even if he goes to the market as a seller every period, he can fail to earn income for an indefinite number of periods. When this happens the law of motion of real wealth is:

$$\omega_{t+1} = \frac{\tau}{p_{+1}} + \frac{\omega_0 p}{p_{+1}} = \frac{\gamma - 1 + \omega_t}{\gamma}, \text{ for } t = 0, 1, 2... \tag{71}$$

If $\gamma < 1$ and $\omega_0 < 1$, $\omega_t$ drops down to zero asymptotically as $t \to \infty$. In finite time and hence positive probability, the value $\omega_t$ falls below the value of the lump-sum tax. Therefore, for an individual to avoid defaulting on the tax obligations must maintain $\omega_0 \geq 1$ at all times. Since $\omega_0$ is the ratio of money held by an individual over the average quantity of money, for all individuals to be solvent with probability one, we need that they all hold the average quantity of money at all times. This is impossible in this model with trading risks. Hence, there is no equilibrium with $\gamma < 1$ if money must be reduced with a uniform lump-sum tax. This is reminiscent of the difficulty in implementing the optimum quantity of money emphasized by Bewley (1983).

# 5 Conclusion

This paper provides a tractable search model with divisible money that encompasses Lagos and Wright (2002) and Shi (1997). The key feature of the model is that individuals belong to many villages. Inside a village, individuals are not altruistic as in a representative household, but they share information so financial contracts are feasible. Money is essential in the model to facilitate trade with individuals outside the village. Rocheteau and Wright (2003) version of Lagos and Wright (2002) arises as a special case if competitive markets coexist with search markets, and preferences are quasi-linear. Rauch (2000) solution to Shi (1997) arises as a special case if individuals can insure trading risks inside the village. In general, if preferences are not quasi-linear and trading risks cannot be insured, the distribution of money holdings is non-degenerate and the model turns difficult to solve. However, neither quasi-linear preferences nor insurance of trading risks are necessary for tractability. A simple version of the model in which all buyers choose to carry the same amount of money is tractable under quite general conditions even if preferences are not quasi-linear and trading risks cannot be insured.

The type of financial arrangements that arise in the village are not always centered around the insurance of trading risks as conjectured by Lucas (1990). In the version of the model in section 2, insurance is the key financial institution that arises in equilibrium, but this is not the case in the versions in sections 3 and 4. In section 3, credit plays the key role and insurance is only necessary to ensure the solvency of debtors. In section 4, insurance contracts are completely unnecessary.

The coexistence of money and financial contracts allows an integration of the microfoundations of money and finance. In a related paper, Faig (2004), I study a version of the model where the credit market in the village is centered around banks because they are the only agents that are able to implement credit contracts. This provides an integration of money and banking allong quite different lines from previous attempts in monetary search models.[3] The role of banks in this previous attempts is to provide a medium of exchange whereas the primary role of banks in Faig (2004) is to intermediate between borrowers and lenders.

3 See for example Cavalcanti, Erosa, and Temzelides (1999) and He, Huang, and Wright (2003).

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